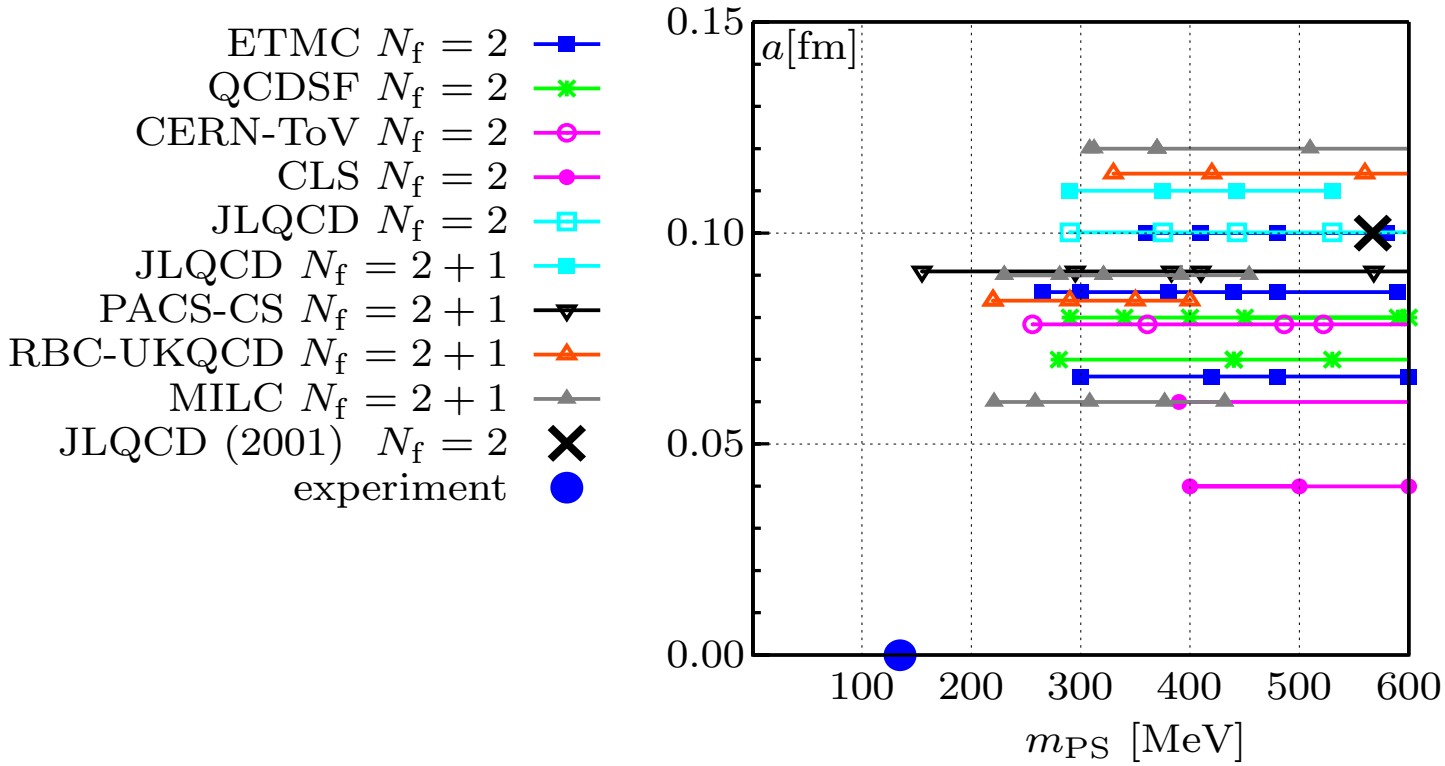
An Overview of Physics Results from Maximally Twisted Mass Fermions

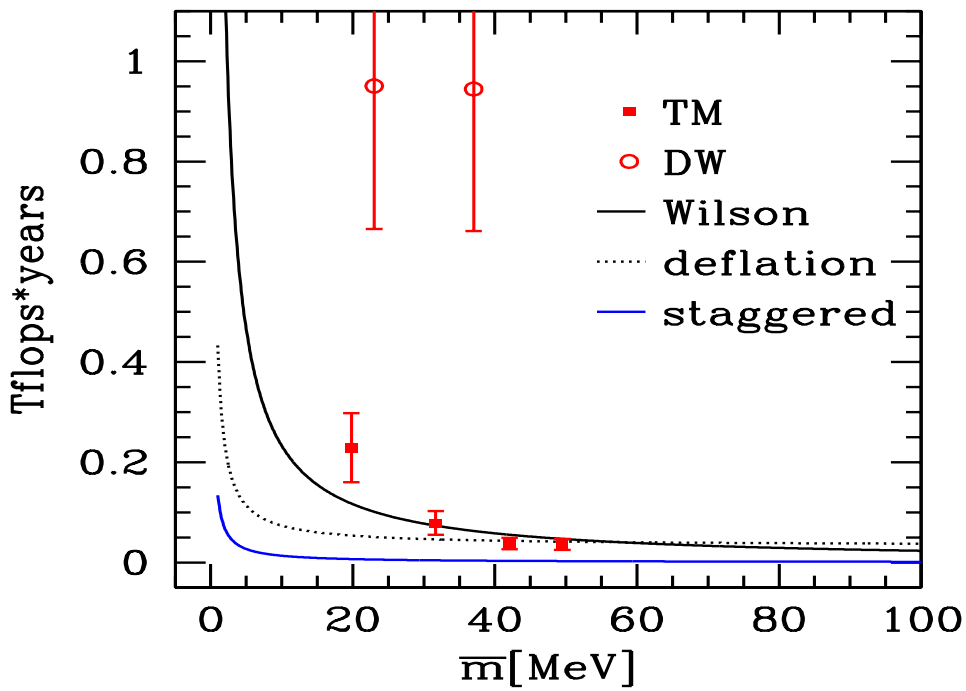
Karl Jansen

- $O(a)$ **improvement with twisted mass quarks**
- **Dynamical Quarks**
- **Precision results from** $N_f = 2$ **dynamical twisted mass fermions**
- **Summary**

Simulation landscape



Algorithm performance



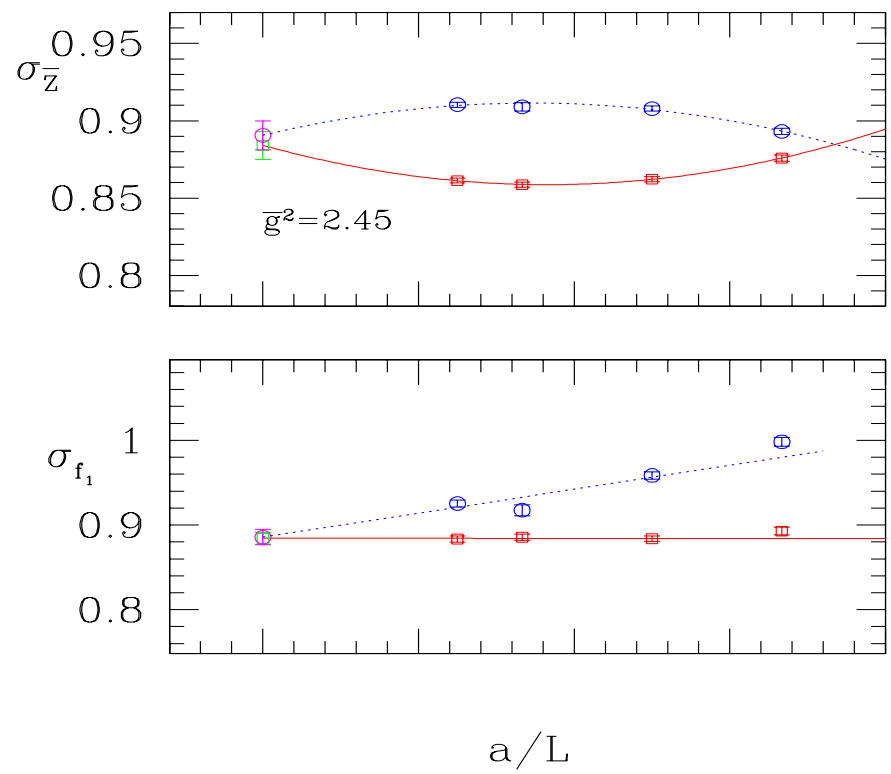
Wilson's Lattice Quantum Chromodynamics

$$S = \underbrace{S_{\mathrm{G}}}_{\mathrm{O}(a^2)} + \underbrace{S_{\mathrm{naive}}}_{\mathrm{O}(a^2)} + \underbrace{S_{\mathrm{wilson}}}_{\mathrm{O}(a)}$$

lattice artefacts appear linear in a

$\longrightarrow$ possibly large lattice artefacts
$\Rightarrow$ need of fine lattice spacings
$\Rightarrow$ large lattices
(want $L = N \cdot a = 1\mathrm{fm}$ fixed)

$\longrightarrow$ simulation costs $\propto 1/a^{6-7}$



present (Wilson-type) solutions:

- clover-improved Wilson fermions
- maximally twisted Wilson fermions
- overlap/domainwall fermions $\longleftarrow$ <u>exact</u> (lattice) chiral symmetry

Realizing $\mathrm{O}(a)$-improvement

Continuum QCD action $S = \bar{\Psi}\left[m + \gamma_\mu D_\mu\right]\Psi$

Continuum *twisted mass* QCD action

$$S = \bar{\Psi}\left[m' + i\mu\gamma_5\tau_3 + \gamma_\mu D_\mu\right]\Psi \equiv \bar{\Psi}\left[m e^{i\omega\gamma_5\tau_3} + \gamma_\mu D_\mu\right]\Psi \equiv$$

$$m = \sqrt{m'^2 + \mu^2}\,, \quad \tan\omega = \mu/m'$$

an *axial transformation*: $\Psi \to e^{i\alpha\gamma_5\tau_3/2}\Psi\,, \quad \bar{\Psi} \to \bar{\Psi}e^{i\alpha\gamma_5\tau_3/2}$

$\to$ leaves measure invariant $\Rightarrow$ same physics

- $\alpha = -\omega$: back to standard QCD action
- $\omega = \pi/2$: $S = \bar{\Psi}\left[i\mu\gamma_5\tau_3 + \gamma_\mu D_\mu\right]\Psi$: (maximally) twisted mass action
  note: $m' = 0$

Wilson (Frezzotti, Rossi) twisted mass QCD (Frezzotti, Grassi, Sint, Weisz)

$$D_{\rm tm} = m_q + i\mu\tau_3\gamma_5 + \tfrac{1}{2}\gamma_\mu\left[\nabla_\mu + \nabla^*_\mu\right] - a\tfrac{1}{2}\nabla^*_\mu\nabla_\mu$$

quark mass parameter m_q , twisted mass parameter μ

difference to continuum situation:
Wilson term not invariant under axial transformations

$\Psi \to e^{i\omega\gamma_5\tau_3/2}\Psi\ , \quad \bar\Psi \to \bar\Psi e^{i\omega\gamma_5\tau_3/2}$

2-point function: $\left[m_q + i\gamma_\mu \sin p_\mu a + \frac{r}{a}\sum_\mu(1-\cos p_\mu a) + i\mu\tau_3\gamma_5\right]^{-1}$

$\propto (\sin p_\mu a)^2 + \left[m_q + \frac{r}{a}\sum_\mu(1-\cos p_\mu a)\right]^2 + \mu^2$

$$\lim_{a\to 0}: \quad p_\mu^2 + m_q^2 + \mu^2 + \underbrace{a m_q \sum_\mu p_\mu}_{\mathrm{O}(a)}$$

- setting $m_q = 0 \ \ (\omega = \pi/2)$: no $\mathrm{O}(a)$ lattice artefacts
- quark mass is realized by twisted mass term alone

Wilson twisted mass QCD

equivalent Wilson-Dirac operator

$$D_{\mathrm{W}} = m_q e^{i\omega\gamma_5\tau^3} + \tfrac{1}{2}\gamma_\mu\left[\nabla_\mu + \nabla_\mu^*\right] - ar\tfrac{1}{2}\nabla_\mu^*\nabla_\mu \equiv m_q e^{i\omega\gamma_5\tau^3} + D_{\mathrm{crit}} + D_{\mathrm{r}}$$

under parity, $R_5 = e^{i\omega\gamma_5\tau^3}$

- D_{crit} invariant
- $m_q e^{i\omega\gamma_5\tau^3} \longrightarrow -m_q e^{i\omega\gamma_5\tau^3}$
- $D_{\mathrm{r}} \longrightarrow -D_{\mathrm{r}}$

$\Rightarrow D_{\mathrm{W}}$ invariant under $R_5 \times (r \longrightarrow -r) \times (m_q \longrightarrow -m_q)$

Wilson twisted mass QCD

it can be shown than

$$\langle \mathcal{O} \rangle|_{(m_q,r)} = (-1)^P \, \langle \mathcal{O} \rangle|_{(-m_q,-r)}$$

- follows from invariance of action under lattice parity tranformation

O(a) improvement

Symanzik expansion

$$\langle \mathcal{O} \rangle|_{(m_q,r)} = [\xi(r) + a m_q \eta(r)] \, \langle \mathcal{O} \rangle|^{\rm cont}_{m_q} + a\chi(r) \, \langle \mathcal{O}_1 \rangle|^{\rm cont}_{m_q}$$

$$\langle \mathcal{O} \rangle|_{(-m_q,-r)} = [\xi(-r) - a m_q \eta(-r)] \, \langle \mathcal{O} \rangle|^{\rm cont}_{-m_q} + a\chi(-r) \, \langle \mathcal{O}_1 \rangle|^{\rm cont}_{-m_q}$$

we use

$$\langle \mathcal{O} \rangle|^{\rm cont}_{m_q} = (-1)^P \, \langle \mathcal{O} \rangle|^{\rm cont}_{-m_q}$$

$$\langle \mathcal{O}_1 \rangle|^{\rm cont}_{m_q} = -(-1)^P \, \langle \mathcal{O}_1 \rangle|^{\rm cont}_{-m_q} \Leftarrow \dim[\mathcal{O}_1] = \dim[\mathcal{O}] + 1$$

for parity even observables $\langle \mathcal{O} \rangle|_{(m_q,r)} = \langle \mathcal{O} \rangle|_{(-m_q,-r)}$

$$\Rightarrow \quad \langle \mathcal{O} \rangle|_{(-m_q,-r)} = [\xi(-r) - a m_q \eta(-r)] \, \langle \mathcal{O} \rangle|^{\rm cont}_{m_q} - a\chi(-r) \, \langle \mathcal{O}_1 \rangle|^{\rm cont}_{m_q}$$

$$\Rightarrow \quad \xi(r) = +\xi(-r) \; , \; a m_q \eta(r) = -a m_q \eta(-r) \; , \; a\chi(r) = -a\chi(-r)$$

Realizing $O(a)$ improvement

- *mass average*

$$\frac{1}{2}\left[\langle\mathcal{O}\rangle|_{m_q,r} + \langle\mathcal{O}\rangle|_{-m_q,r}\right] = \langle\mathcal{O}\rangle|_{m_q}^{\rm cont} + O(a^2)$$

- *Wilson average:*

$$\frac{1}{2}\left[\langle\mathcal{O}\rangle|_{m_q,r} + \langle\mathcal{O}\rangle|_{m_q,-r}\right] = \langle\mathcal{O}\rangle|_{m_q}^{\rm cont} + O(a^2)$$

- *automatic $O(a)$ improvement*
  $\longrightarrow$ special case of mass average: $m_q = 0 \leftrightarrow$ twist angle $\omega = \pi/2$ <u>maximal</u>

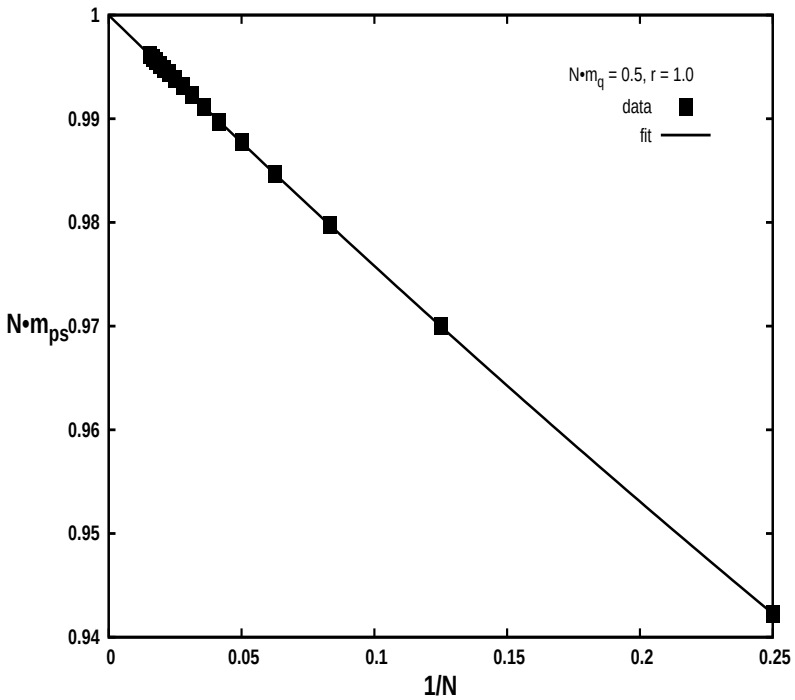
$$\Rightarrow \langle\mathcal{O}\rangle|_{m_q=0,r} = \langle\mathcal{O}\rangle|_{m_q}^{\rm cont} + O(a^2\ )$$

A first test: experiments in the free theory

(K. Cichy, J. Gonzales Lopez, A. Kujawa, A. Shindler, K.J.)

free fields: imagine study system for $L[\mathrm{fm}] < \infty$

$\Rightarrow \quad L = N \cdot a \quad \rightarrow a \rightarrow 0 \leftrightarrow N \rightarrow \infty$

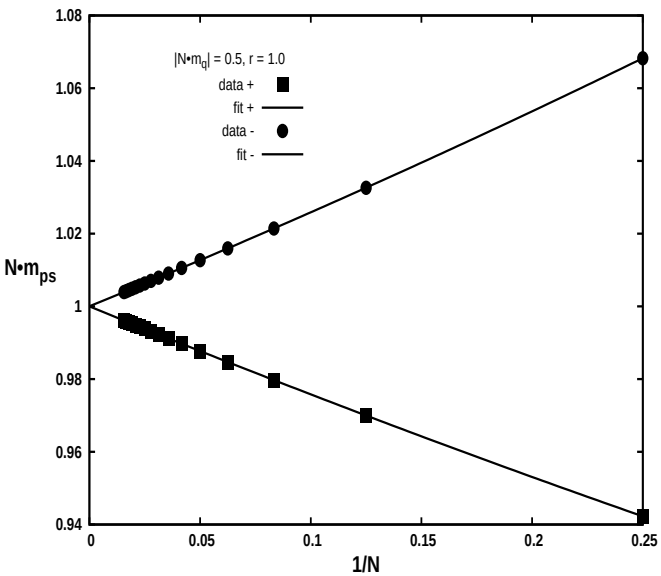


Scaling for pure Wilson fermions

at $Nm_q = 0.5$

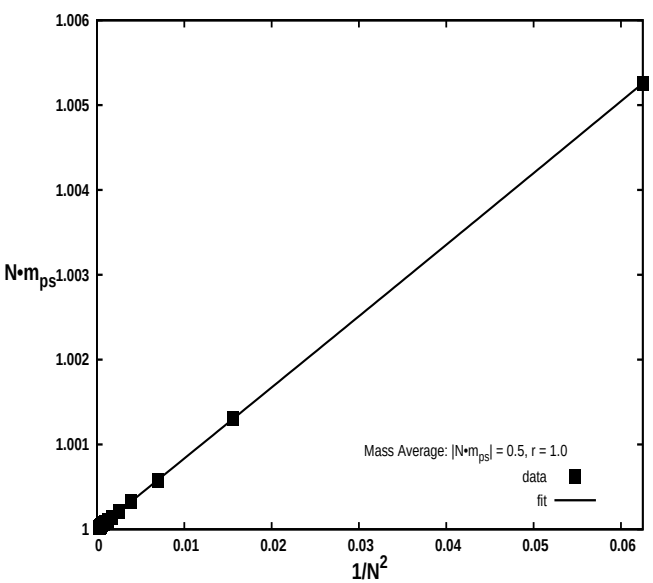
$N \cdot m_\pi$ versus $\mathbf{1/N} = a$

Averaging over the mass



Wilson fermions at $Nm_q = \pm 0.5$

$N \cdot m_\pi$ versus $\mathbf{1/N} = a$

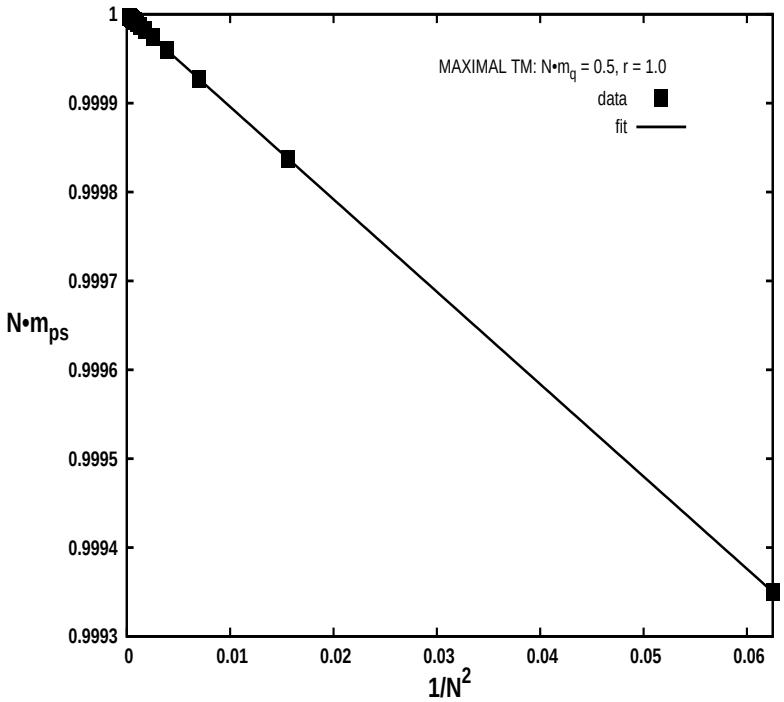


Averaging Wilson fermions at $Nm_q = \pm 0.5$

$N \cdot m_\pi$ versus $\mathbf{1/N^2} = a^2$

Twisted mass at maximal twist

choosing $m_q = 0 \quad \Rightarrow \quad \omega = \pi/2$



maximally twisted mass fermions

at $N\mu = 0.5$

$N \cdot m_\pi$ versus $\mathbf{1/N^2} = a^2$

Summarizing twisted mass fermions

$$D_{\rm tm} = m_q + i\mu\tau_3\gamma_5 + \frac{1}{2}\gamma_\mu\left[\nabla_\mu + \nabla^*_\mu\right] - a\frac{1}{2}\nabla^*_\mu\nabla_\mu$$

quark mass parameter m_q , twisted mass parameter μ

- $m_q = m_{\rm crit} \rightarrow$ O(a) improvement for *hadron masses, matrix elements, form factors, decay constants*

  $\star$ **based on symmetry arguments**

- no need for further operator specific improvement coefficients

- $\det[D_{\rm tm}] = \det[D^2_{\rm Wilson} + \mu^2]$
  $\Rightarrow$ protection against small eigenvalues

- computational cost comparable to Wilson

- expected to simplify mixing problems for renormalization

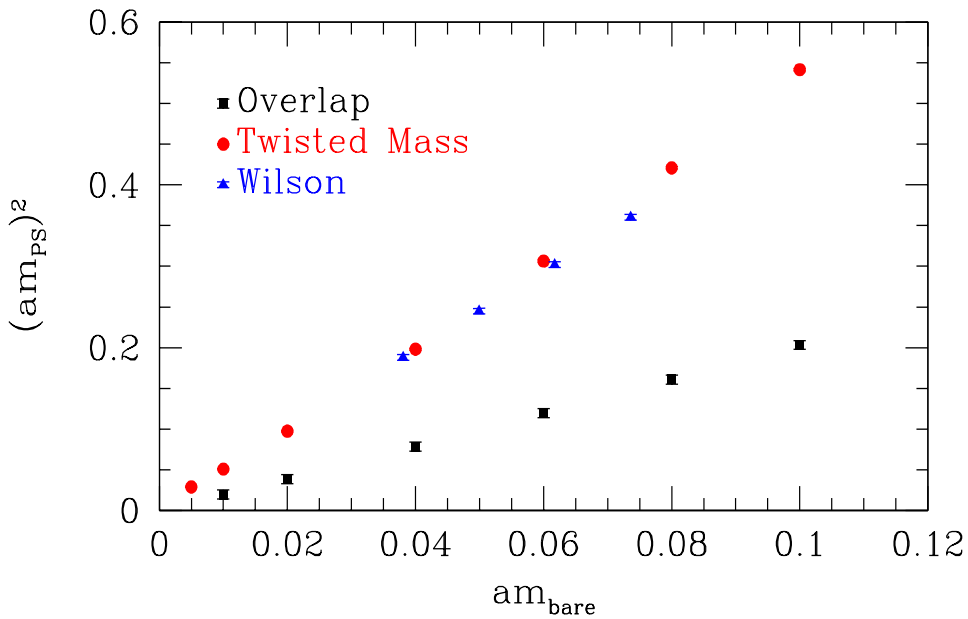
- drawback: explicit violation of isospin
  $\rightarrow$ seems to affect only neutral pion sector

A test in the quenched approximation

twisted mass against overlap fermions: how chiral can we go?

Bietenholz, Capitani, Chiarappa, Christian, Hasenbusch, K.J., Nagai, Papinutto, Scorzato, Shcheredin, Shindler, Urbach, Wenger, Wetzorke

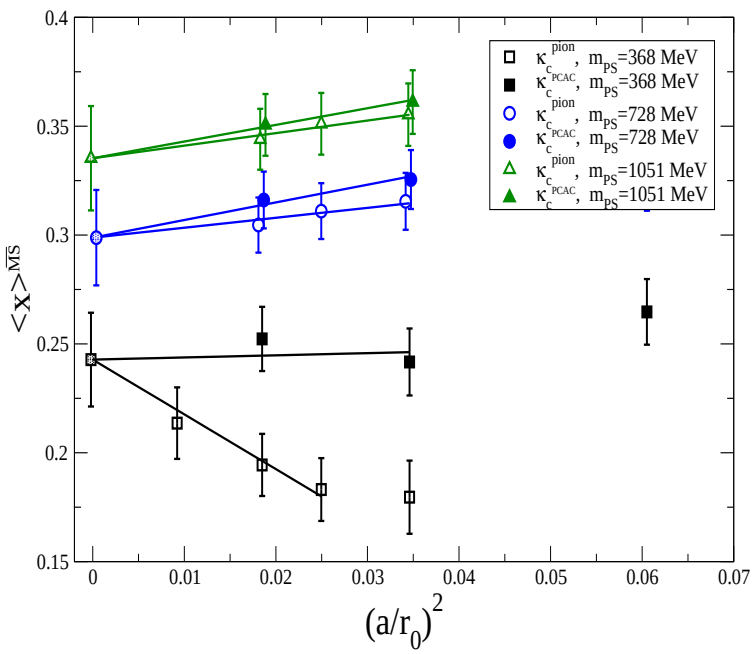
fixed lattice spacing of $a = 0.125$fm, quenched

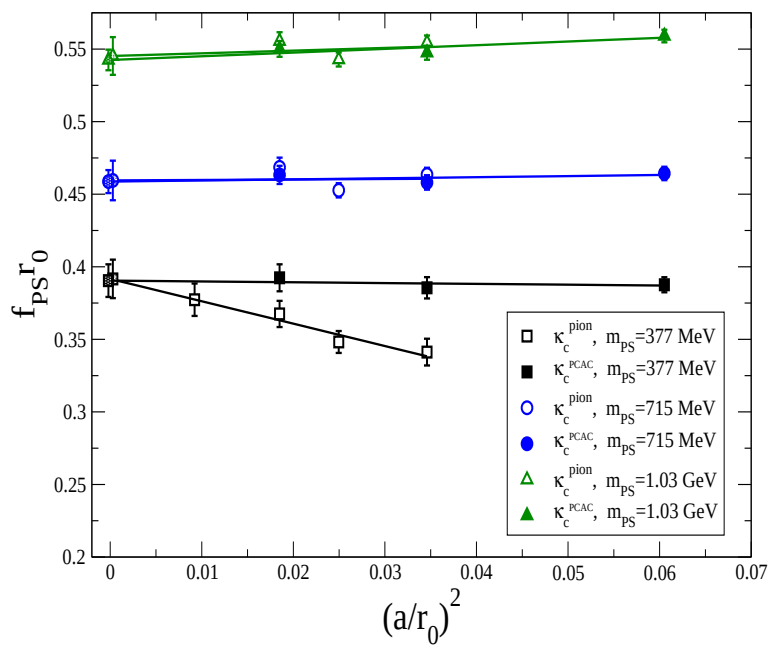


$\Rightarrow$ twisted mass simulations can reach quarks masses as small as overlap substantially smaller than O(a)-improved Wilson fermions

(Quenched) Scaling of $\langle x \rangle$ and $F_{\rm PS}$ with twisted mass fermions

K.J., M. Papinutto, A. Shindler, C. Urbach, I. Wetzorke





→ O(a^2) scaling for two realizations of O(a)-improvement

→ consistent with theoretical considerations

(Frezzotti, Martinelli, Papinutto, Rossi; Sharpe, Wu; Aoki, Bär)

The "No free lunch theorem"

A cost comparison

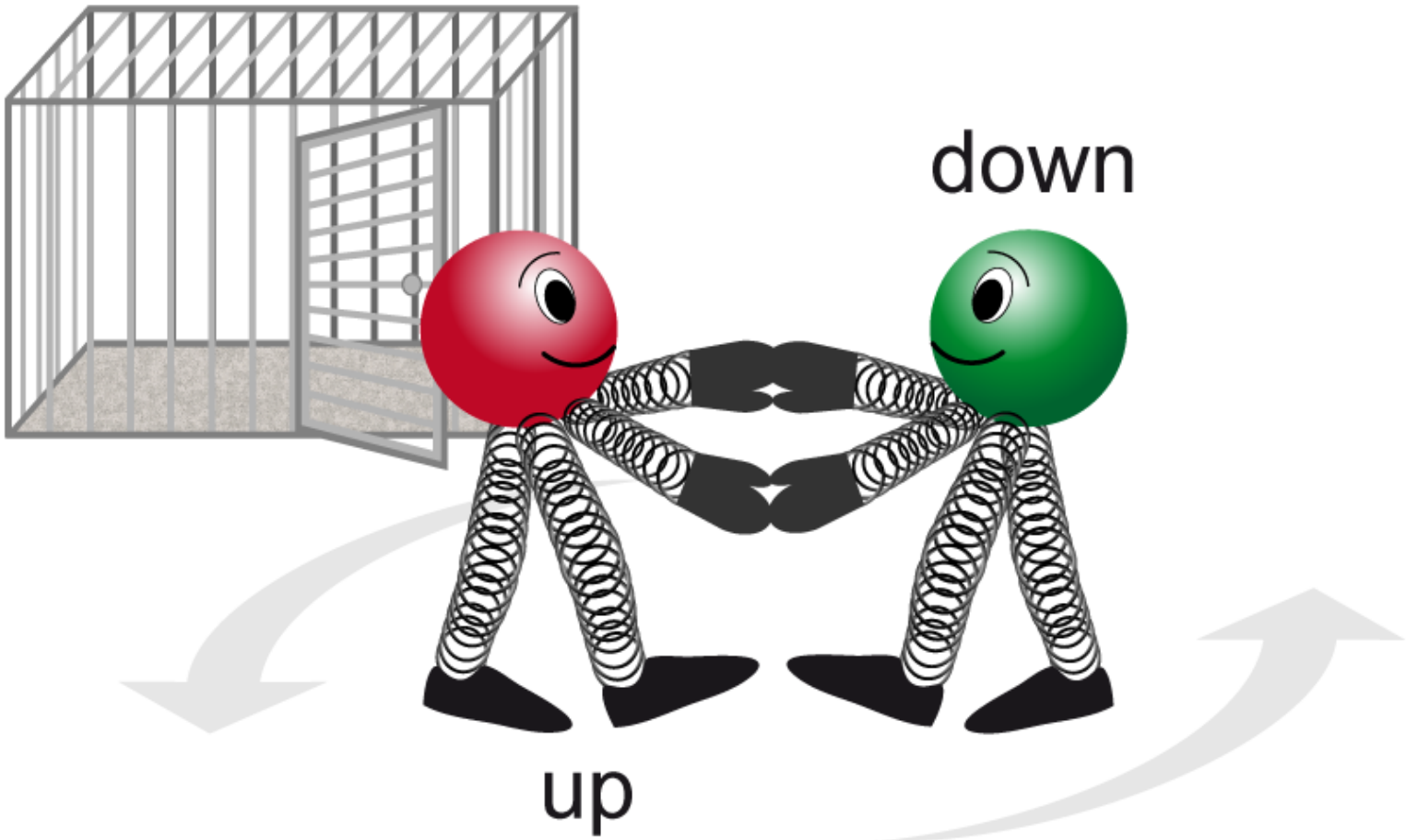
T. Chiarappa, K.J., K. Nagai, M. Papinutto, L. Scorzato,
A. Shindler, C. Urbach, U. Wenger, I. Wetzorke

V, m_π	Overlap	Wilson TM	rel. factor
$12^4, 720$Mev	48.8(6)	2.6(1)	18.8
$12^4, 390$Mev	142(2)	4.0(1)	35.4
$16^4, 720$Mev	225(2)	9.0(2)	25.0
$16^4, 390$Mev	653(6)	17.5(6)	37.3
$16^4, 230$Mev	1949(22)	22.1(8)	88.6

timings in seconds on Jump

- nevertheless chiral symmetric lattice fermions can be advantageous
  - e.g., Kaon Physics, B_K, $K \to \pi\pi$
  - ϵ-regime of chiral perturbation theory
  - topology
  - use in valence sector

$N_f = 2$ dynamical flavours



- **Cyprus (Nicosia)**
  C. Alexandrou, M. Constantinou, T. Korzec, G. Koutsou
- **France (Orsay, Grenoble)**
  R. Baron, B. Bloissier, Ph. Boucaud, M. Brinet, J. Carbonell, V. Drach, P. Guichon, P.A. Harraud, Z. Liu, O. Pène
- **Italy (Rome I,II,III, Trento)**
  P. Dimopoulos, R. Frezzotti, V. Lubicz, G. Martinelli, G.C. Rossi, L. Scorzato, S. Simula, C. Tarantino
- **Netherlands (Groningen)**
  A. Deuzeman, E. Pallante, S. Reker
- **Poland (Poznan)**
  K. Cichy, A. Kujawa
- **Spain (Valencia)**
  V. Gimenez, D. Palao
- **Switzerland (Bern)**
  U. Wenger
- **United Kingdom (Glasgow, Liverpool)**
  G. McNeile, C. Michael, A. Shindler
- **Germany (Berlin/Zeuthen, Hamburg, Münster)**
  F. Farchioni, X. Feng, J. González López, G. Herdoiza, K. Jansen, I. Montvay, G. Münster, M. Petschlies, D. Renner, T. Sudmann, C. Urbach, M. Wagner

European Twisted Mass Collaboration

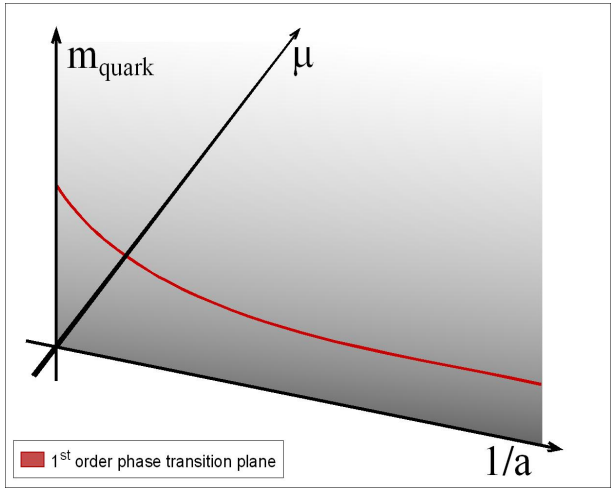
Dynamical Quarks:
The phase structure of lattice QCD

Farchioni, Frezzotti, Hofmann, K.J., Montvay, Münster,
Rossi, Scorzato,Scholz, Shindler, Ukita, Urbach, Wenger, Wetzorke

Let me describe a typical computer simulation:[...]
the first thing to do is to look for phase transitions (G. Parisi)

lattice simulations are done *under the assumption* that the transition is continuum like

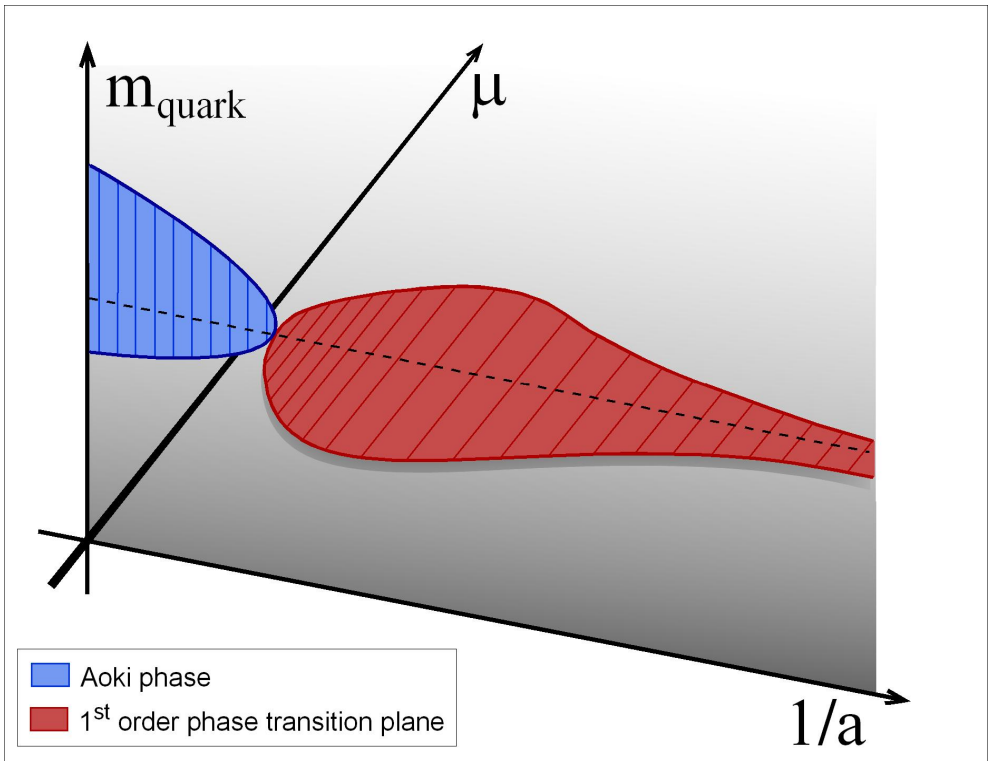
- first order, jump in $< \bar{\Psi}\Psi >$ when quark mass m changes sign
- pion mass vanishes at phase transition point



$\Rightarrow$ *single phase transition line*

$\rightarrow$ twisted mass fermions offer a tool to check this

Revealing the generic phase structure of lattice QCD



Aoki phase: Ilgenfritz, Müller-Preussker, Sternbeck, Stüben

→ Knowledge of phase structure for a particular formulation of lattice QCD: pre-requisite for numerical simulation

Chiral perturbation theory for the phase transition

Sharpe, Wu; Hofmann, Münster; Scorzato; Aoki, Bär

In the regime $m/\Lambda_{QCD} \gtrsim a\Lambda_{QCD}$

$$M = 2B_0/Z_P\sqrt{m^2_{PCAC,\chi} + \mu^2} \qquad \Lambda_R = 4\pi F_0 \qquad \cos\omega = \frac{m_{PCAC,\chi}}{\sqrt{m^2_{PCAC,\chi}+\mu^2}}$$

$$\begin{aligned}
m_\pi^2 &= M + \frac{8}{{F_0}^2}\left\{M^2(2L_{86} - L_{54}) + 4aM\cos\omega(w - \tilde{w})\right\} + \frac{M^2}{32F_0^2\pi^2}\log\left(\frac{M}{\Lambda_R^2}\right) \\
f_\pi &= F_0 + \frac{4}{F_0}\left\{ML_{54} + 4a\cos\omega\tilde{w}\right\} - \frac{M}{16F_0\pi^2}\log\left(\frac{M}{\Lambda_R^2}\right) \\
g_\pi &= B_0/Z_P\left[F_0 + \frac{4}{F_0}\left\{M(4L_{86} - L_{54}) + 4a\cos\omega(2w_s - \tilde{w})\right\} - \frac{M}{32F_0\pi^2}\log\left(\frac{M}{\Lambda_R^2}\right)\right]
\end{aligned}$$

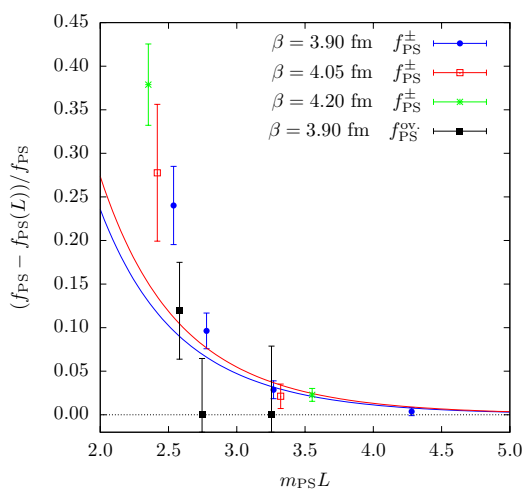
parameters to fit: B_0/Z_P, F_0, L_{86}, L_{54}, w, $\tilde{w}$

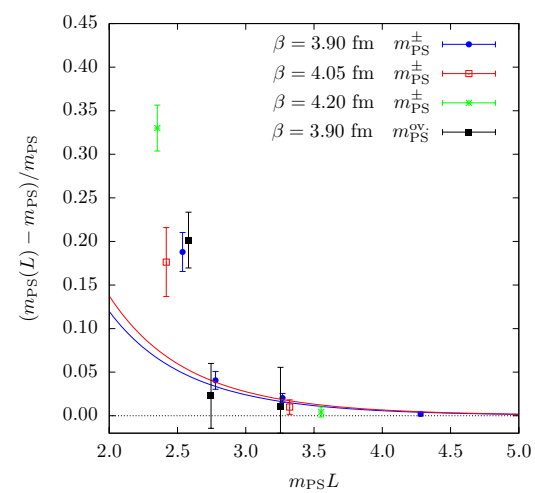
available configurations (free on ILDG)

β	a [fm]	$L^3 \cdot T$	L [fm]	$a\mu$	$N_{\rm traj}$ ($\tau = 0.5$)	$m_{\rm PS}$ [MeV]
4.20	~ 0.050	$48^3 \cdot 96$	2.4	0.0020	5200	~ 300
		$32^3 \cdot 64$	2.1	0.0060	5600	~ 420
4.05	~ 0.066	$32^3 \cdot 64$	2.2	0.0030	5200	~ 300
				0.0060	5600	~ 420
				0.0080	5300	~ 480
				0.0120	5000	~ 600
3.9	~ 0.086	$32^3 \cdot 64$	2.8	0.0030	4500	~ 270
				0.0040	5000	~ 300
		$24^3 \cdot 48$	2.1	0.0064	5600	~ 380
				0.0085	5000	~ 440
				0.0100	5000	~ 480
				0.0150	5400	~ 590
3.8	~ 0.100	$24^3 \cdot 48$	2.4	0.0060	4700×2	~ 360
				0.0080	3000×2	~ 410
				0.0110	2800×2	~ 480
				0.0165	2600×2	~ 580

configurations for checking finite size effects

β	a [fm]	$L^3 \cdot T$	L [fm]	$a\mu$	$N_{\rm traj}$ ($\tau = 0.5$)	$m_{\rm PS}$ [MeV]
4.05	~ 0.066	$32^3 \cdot 64$	2.2	0.0060	5600	~ 420
		$24^3 \cdot 48$	1.6	0.0060	3000×2	~ 420
		$20^3 \cdot 48$	1.3	0.0060	5300×2	~ 420
3.9	~ 0.086	$32^3 \cdot 64$	2.8	0.0040	5000	~ 300
		$24^3 \cdot 48$	2.1	0.0040	10500	~ 300
		$20^3 \cdot 40$	1.9	0.0040	5000	~ 300
		$16^3 \cdot 32$	1.4	0.0040	5000	~ 300

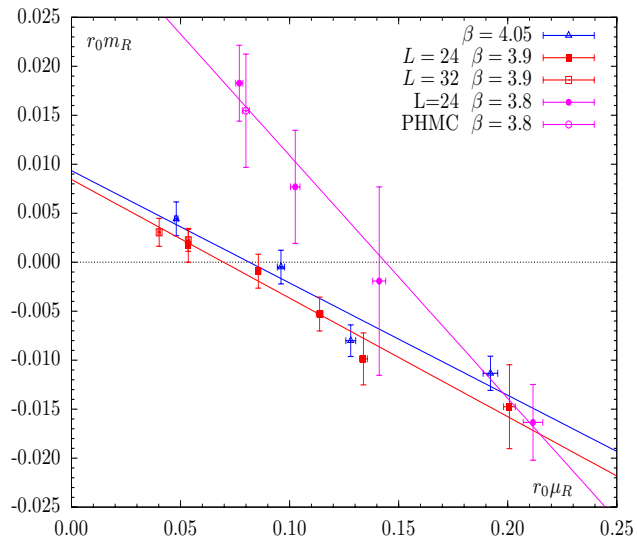




Tuning to maximal twist

Maximal twist: tune m_q such that

$$m_{\mathrm{PCAC}} = \frac{\sum_{\mathbf{x}} \langle \partial_0 A_0^a(x) P^a(0) \rangle}{2 \sum_{\mathbf{x}} \langle P^a(x) P^a(0) \rangle} = 0$$



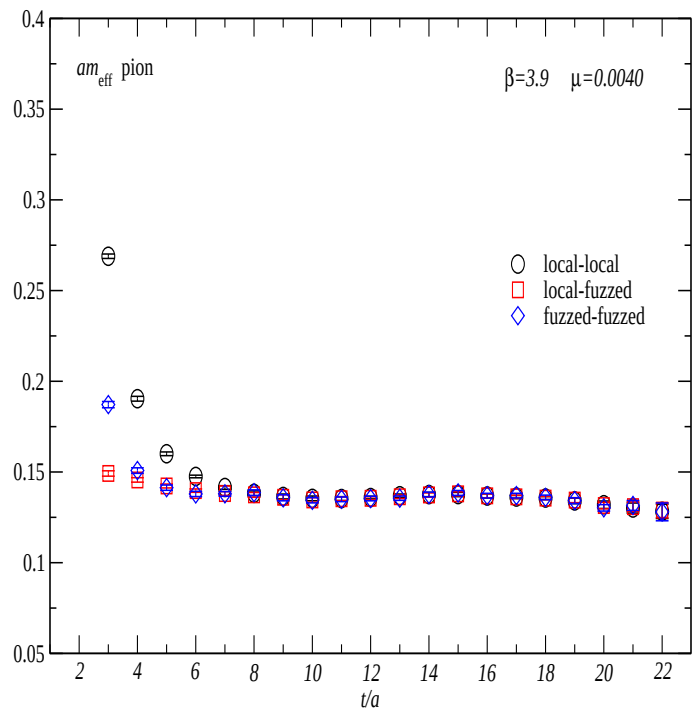
- tuning of m_q at lightest $\mu_{q,\mathrm{min}}$ used
- demand $m_{\mathrm{PCAC}} \lesssim 0.1 \mu_{q,\mathrm{min}}$
- demand $\Delta(m_{\mathrm{PCAC}}) \lesssim 0.1 \mu_{q,\mathrm{min}}$

$\rightarrow$ fullfilled for $\beta = 3.9 (a \approx 0.09\mathrm{fm}), \quad 4.05 (a \approx 0.07\mathrm{fm})$
$4.2 (a \approx 0.05\mathrm{fm}) \leftarrow m_{\mathrm{PCAC}} = 0.00006(6)$

$\rightarrow$ systematic effects for being out of maximal twist can be estimated

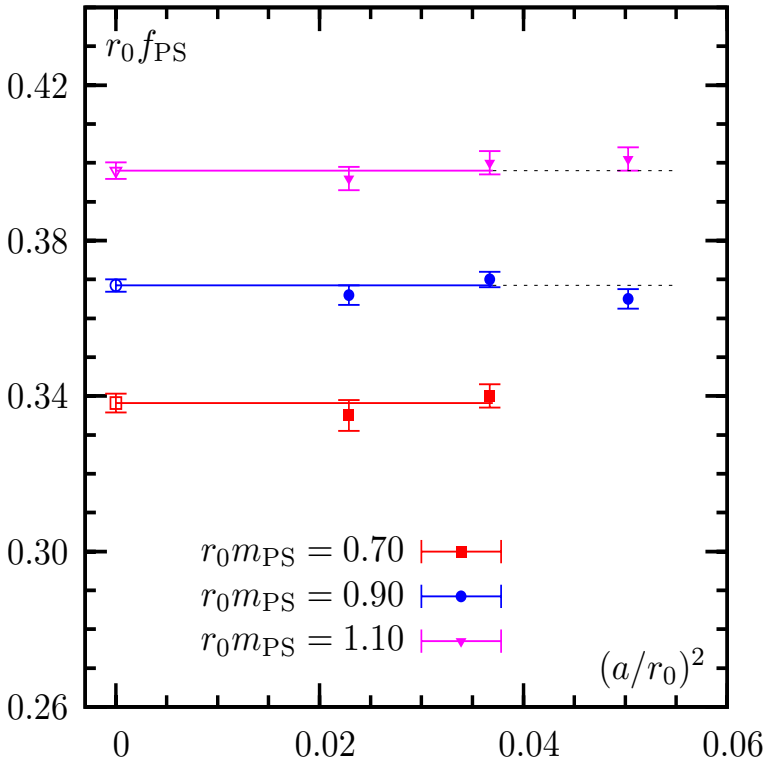
Extraction of Masses

- Quark propagator: stochastic sources to include all spatial sources
- Change the location of the time-slice source: reduce autocorrelations
- Fuzzing

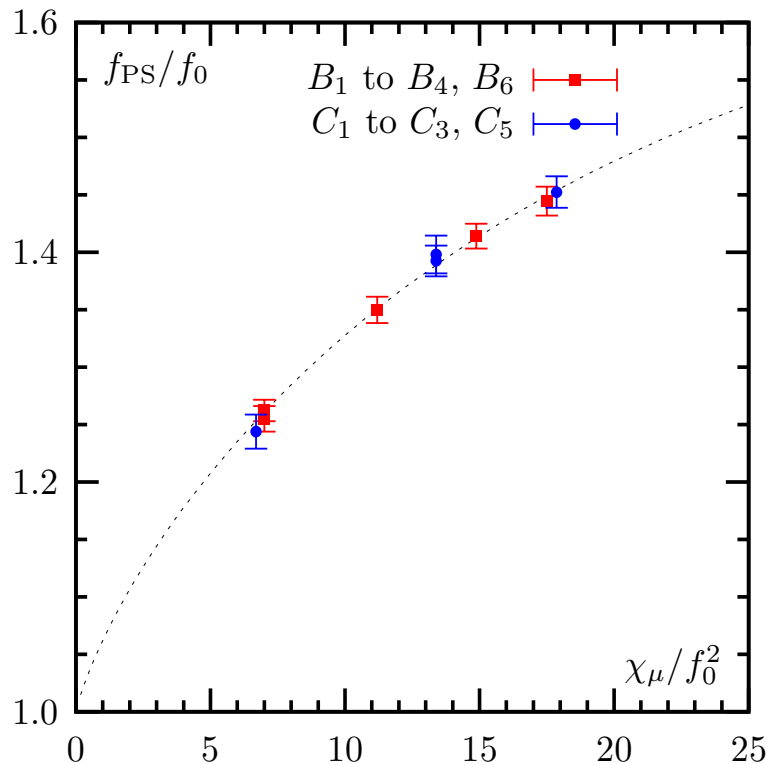


- effective mass of $\pi^{\pm}$
- isolate ground state : small statistical errors

Pion decay constant

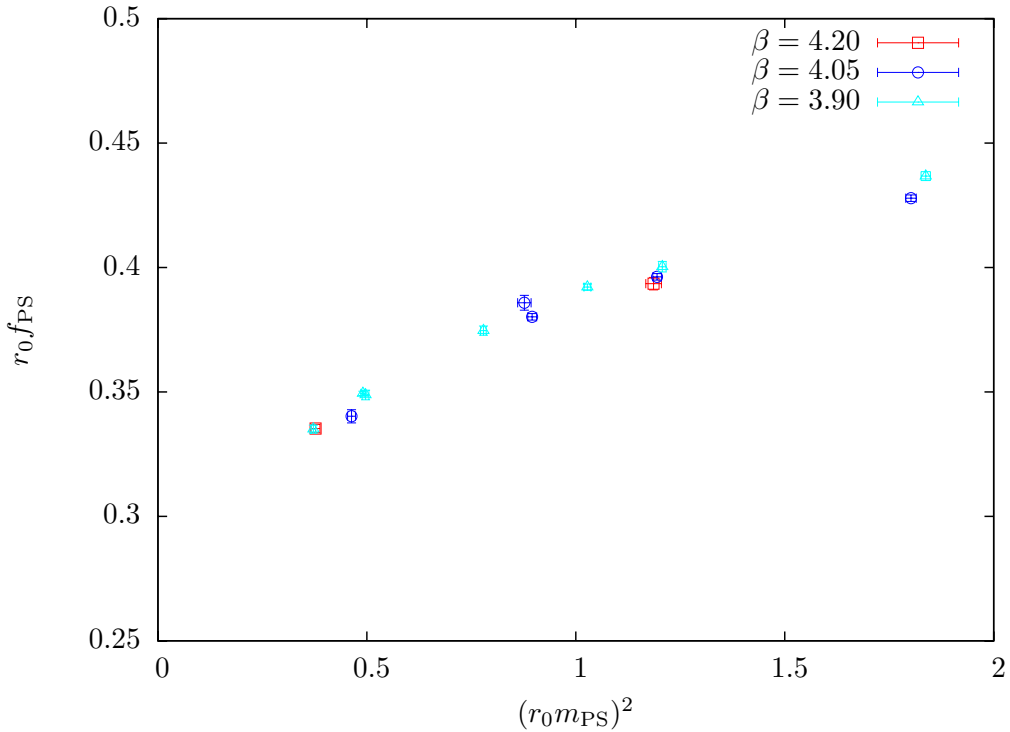


$O(a^2)$ scaling



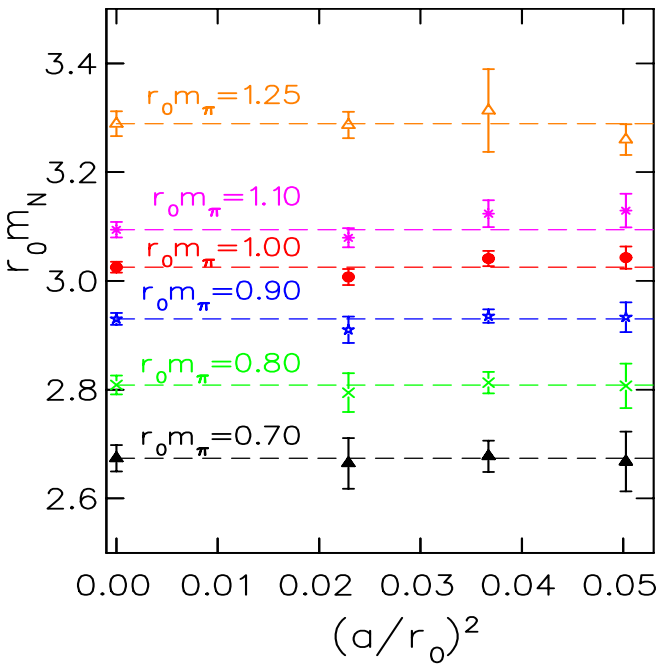
precise determination of $f_{\rm PS}$

Adding $\beta = 4.2$ ($a \approx 0.05$fm) (preliminary)

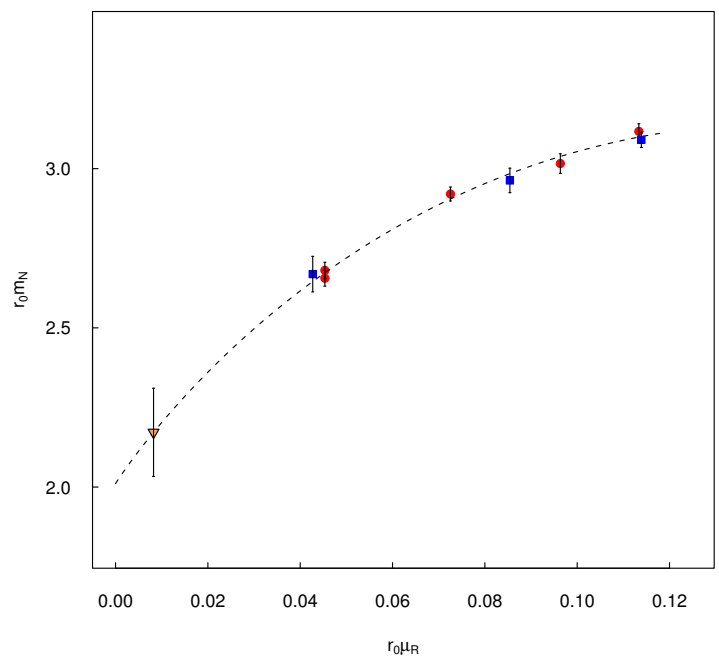


confirmation of very small $O(a^2)$ effects

Proton Mass



$O(a^2)$ scaling



determination of proton mass

Chiral perturbation theory

→ mass, finite volume and lattice spacing dependence:

$$r_0 f_{\rm PS} = r_0 f_0 \left[1 - 2\xi \log\left(\frac{\chi_\mu}{\Lambda_4^2}\right) + D_{f_{\rm PS}} a^2/r_0^2 + T_f^{\rm NNLO} \right] K_f^{\rm CDH}(L)$$

$$(r_0 m_{\rm PS})^2 = \chi_\mu r_0^2 \left[1 + \xi \log\left(\frac{\chi_\mu}{\Lambda_3^2}\right) + D_{m_{\rm PS}} a^2/r_0^2 + T_m^{\rm NNLO} \right] K_m^{\rm CDH}(L)^2$$

$$r_0/a(a\mu_q) = r_0/a + D_{r_0}(a\mu_q)^2$$

where

$$\xi \equiv 2B_0\mu_q/(4\pi f_0)^2, \quad \chi_\mu \equiv 2B_0\mu_R, \quad \mu_R \equiv \mu_q/Z_{\rm P}$$

- $D_{f,m}$ parametrize lattice artefacts
- $K_{f,m}^{\rm CDH}(L)$ Finite size corrections Colangelo *et al.*, 2005
- $T_{f,m}^{\rm NNLO}$ NNLO correction

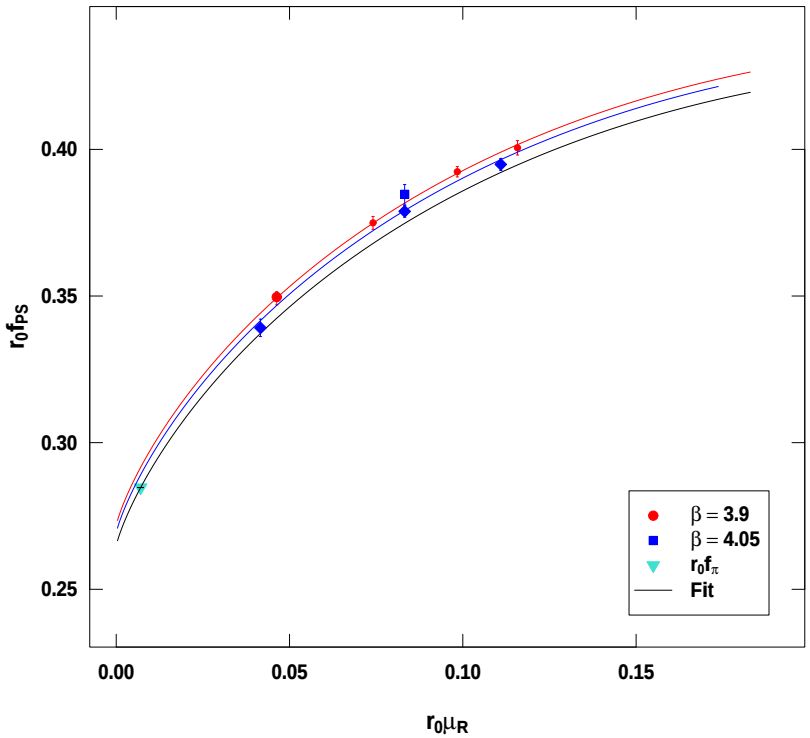
Chiral perturbation theory

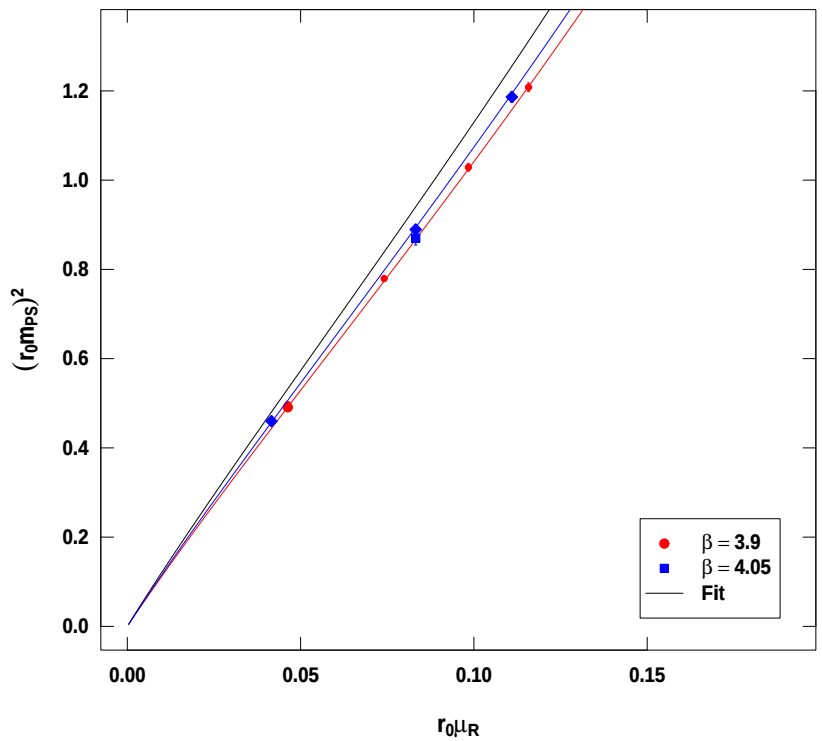
$\longrightarrow$ fit variety:

- Fit A: NLO continuum χPT, $T^{\mathrm{NNLO}}_{m,f} \equiv 0$, $D_{m_{\mathrm{PS}},f_{\mathrm{PS}}} \equiv 0$
- Fit B: NLO continuum χPT, $T^{\mathrm{NNLO}}_{m,f} \equiv 0$, $D_{m_{\mathrm{PS}},f_{\mathrm{PS}}}$ fitted
- Fit C: NNLO continuum χPT, $D_{m_{\mathrm{PS}},f_{\mathrm{PS}}} \equiv 0$
- Fit D: NNLO continuum χPT, $D_{m_{\mathrm{PS}},f_{\mathrm{PS}}}$ fitted

Chiral perturbation theory (preliminary)

- Fit B: NLO continuum χPT, $T_{m,f}^{\mathrm{NNLO}} \equiv 0$, $D_{m_{\mathrm{PS}},f_{\mathrm{PS}}}$ fitted

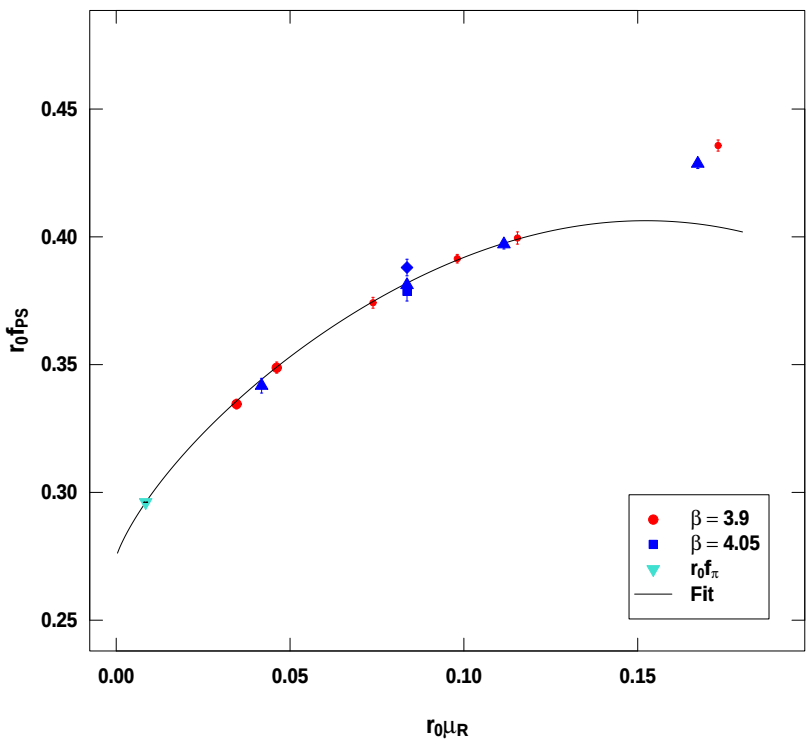


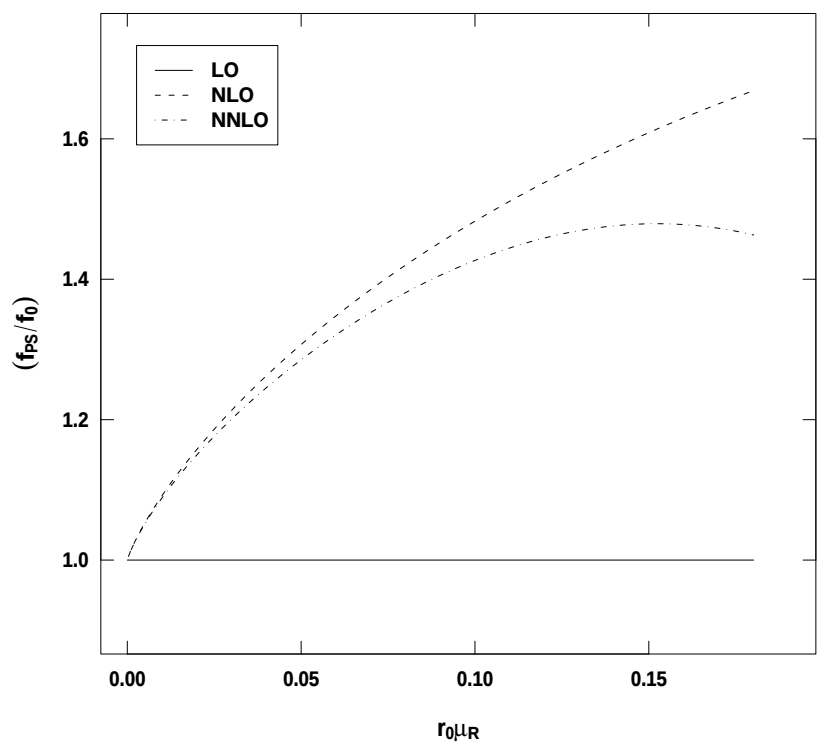


two values of the lattice spacing $\rightarrow$ $D_m = -1.07(97)$, $D_f = 0.71(57)$

Chiral perturbation theory

- Fit C: NNLO continuum χPT, $D_{m_{\rm PS},f_{\rm PS}} \equiv 0$





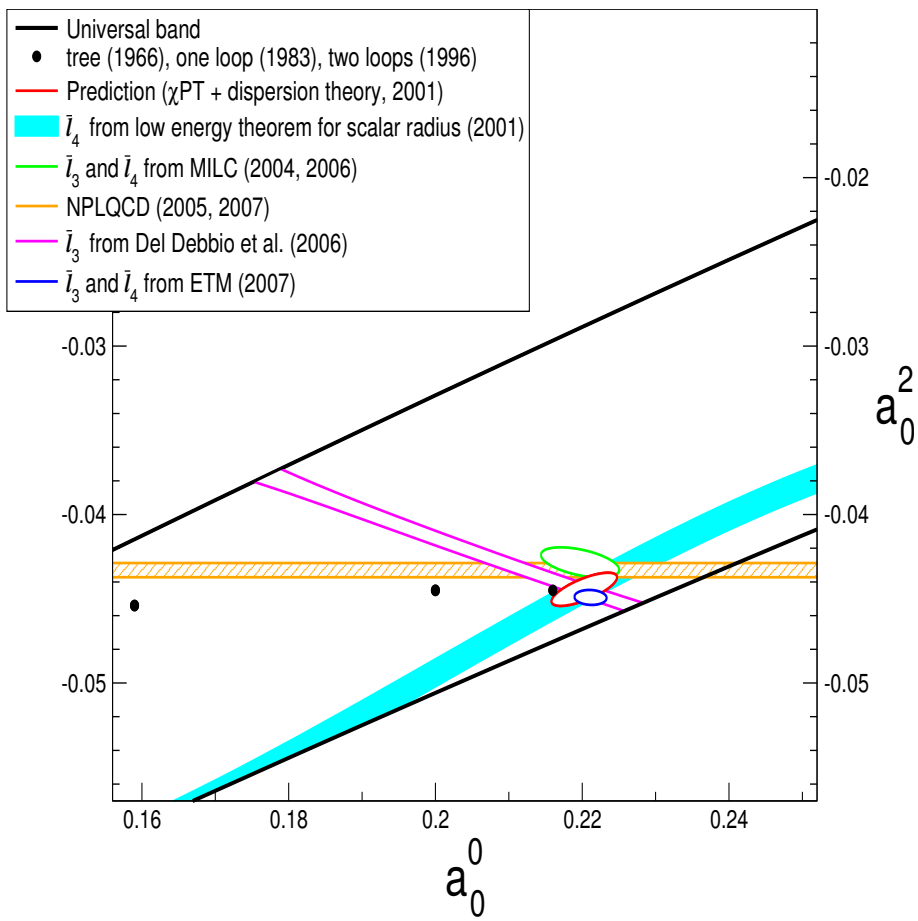
Chiral perturbation theory: results (preliminary)

quantity	value
$m_{u,d}$ [MeV]	3.37(23)
$\bar{\ell}_3$	3.49(19)
$\bar{\ell}_4$	4.57(15)
f_0 [MeV]	121.75(46)
B_0 [GeV]	2774(190)
r_0 [fm]	0.433(14)
$\langle r^2 \rangle_s$	0.729(35)
$\Sigma^{1/3}$ [MeV]	273.9(6.0)
f_π / f_0	1.0734(40)

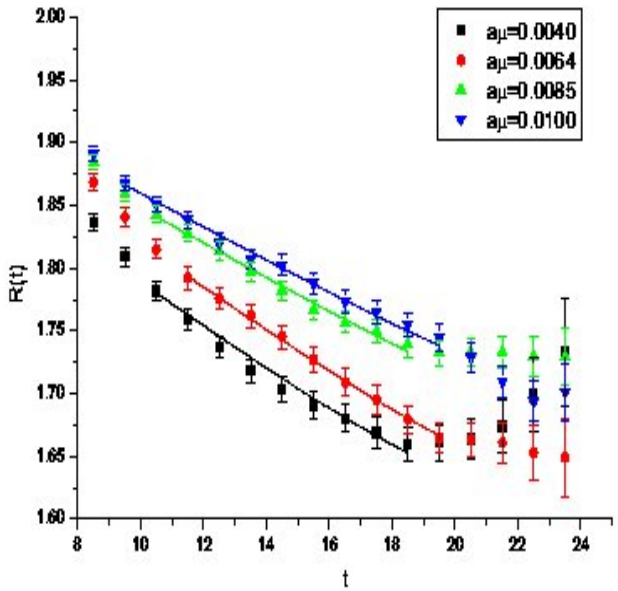
- averaged fit results, weighted with confidence levels
- B_0, Σ, $m_{u,d}$ renormalised in $\overline{\mathrm{MS}}$ scheme at scale $\mu = 2$ GeV

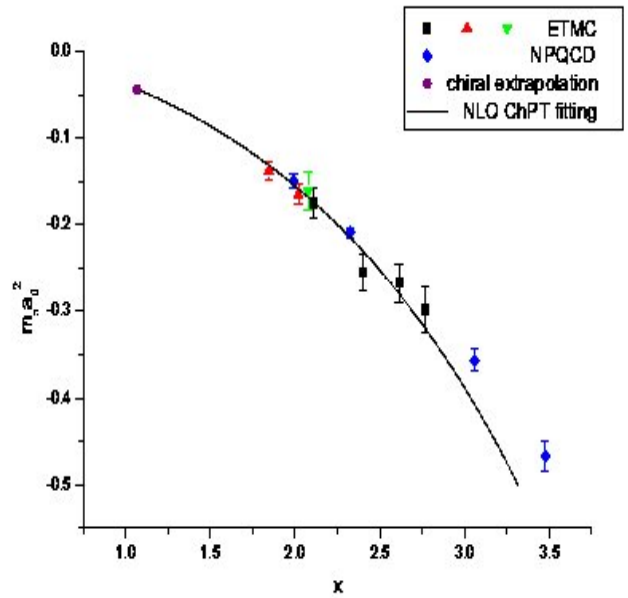
Chiral perturbation theory

S-wave scattering lengths a_0^0 and a_0^2 Leutwyler, 2007



I=2 Pion scattering length





$$C_4(t) = \langle (\pi^+\pi^+)^\dagger(t+t_s)(\pi^+\pi^+)(t_s)\rangle$$

$$C_2(t) = \langle (\pi^+)^\dagger(t+t_s)\pi^+(t_s)\rangle$$

$$R(t) \equiv \frac{C_4(t)}{C_2^2(t)}$$

$$m_\pi a_0^2 = -\frac{m_\pi^2}{8\pi f_\pi^2}\left\{1 + \frac{m_\pi^2}{16\pi^2 f_\pi^2}\left[3\ln\left(\frac{m_\pi^2}{\mu^2}\right) - 1 - l_{\pi\pi}^{I=2}(\mu)\right]\right\}$$

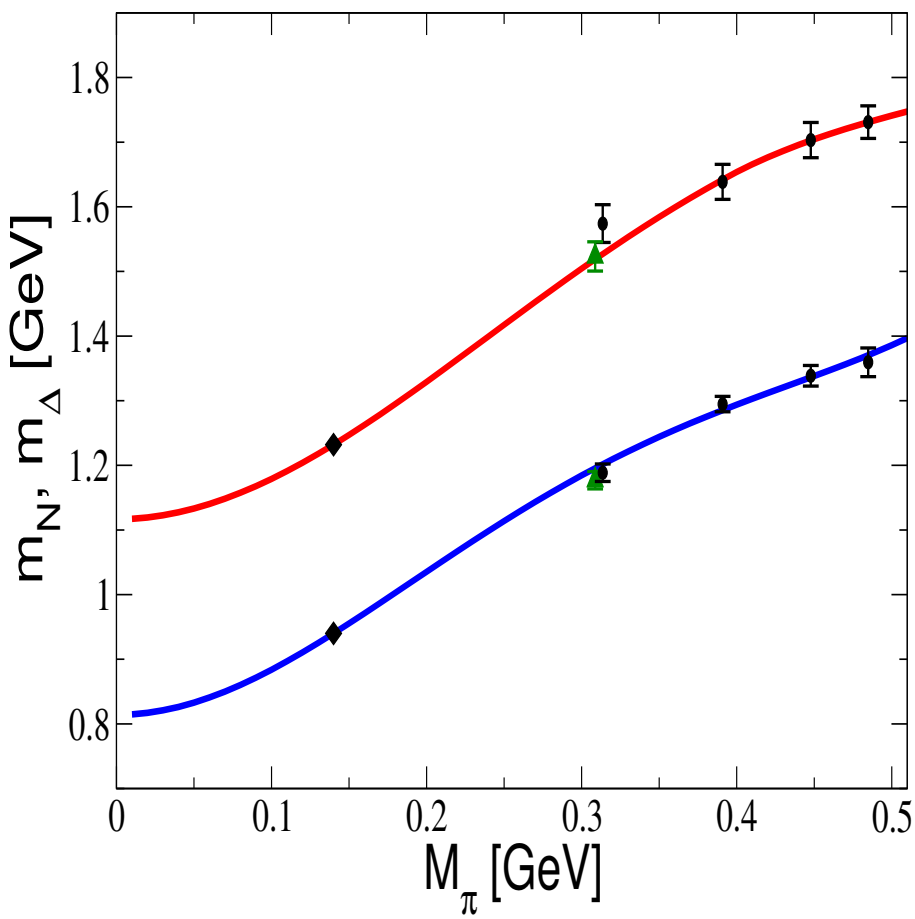
$$m_\pi a_0^2 = -0.04384(28)(23)$$

$$l_{\pi\pi}^{I=2}(\mu = f_{phy}) = 4.14(84)(74)$$

Chiral perturbation theory for Δ resonance

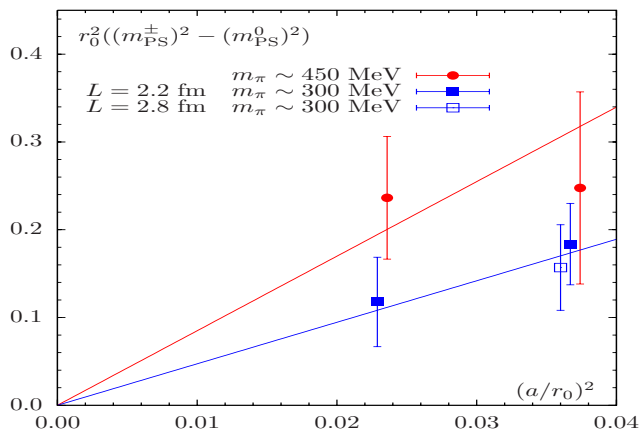
V. Bernard, D. Hoja, U.-G. Meißner, A. Rusetsky

Fit to ETMC data



$O(a^2)$ cutoff effects in neutral pion mass

observation



- $M_{\rm PS}^{\rm neutral}$ shows $O(a^2)$ scaling
- but exhibits large scaling violations

$O(a^2)$ cutoff effects in neutral pion mass

interpretation

analysis a la Symanzik shows that

$$(m_{\rm PS}^0)^2 = m_\pi^2 + a^2\zeta_\pi^2 + \mathcal{O}(a^2 m_\pi^2, a^4)\,, \qquad \zeta_\pi \equiv \langle\pi^0|\mathcal{L}_6|\pi^0\rangle|_{\rm cont}$$

$$(m_{\rm PS}^\pm)^2 = m_\pi^2 \qquad\qquad + \mathcal{O}(a^2 m_\pi^2, a^4)$$

ζ_π has a large contribution:

$$a^2\zeta_\pi^2 \sim a^2|\hat{G}_\pi|^2\,, \qquad \hat{G}_\pi \equiv \langle 0|\hat{P}^3|\pi^0\rangle = \frac{f_\pi m_\pi^2}{2m_q} \sim (550\ {\rm MeV})^2$$

$$|\hat{G}_\pi|^2/\Lambda_{\rm QCD}^4 \sim 25$$

$\rightarrow$ large a^2 effects compared to their expected size $a^2\Lambda_{\rm QCD}^4$

ζ_π enters *only* in quantities related to π^0-mass

$O(a^2)$ cutoff effects in neutral pion mass

other observables

$$R_O = \frac{O^{\pm} - O^0}{O^{\pm}}$$

	β	$a\mu_q$	R_O
$af_{\rm PS}$	3.90	0.004	0.04(06)
	4.05	0.003	−0.03(06)
$am_{\rm V}$	3.90	0.004	0.02(07)
	4.05	0.003	−0.10(11)
$af_{\rm V}$	3.90	0.004	−0.07(18)
	4.05	0.003	−0.31(29)
am_{Δ}	3.90	0.004	0.022(29)
	4.05	0.003	−0.004(45)

- Isospin splittings compatible with zero

Chiral perturbation theory

$\longrightarrow$ mass and finite volume dependence:

$$m_{\rm PS}^2(L) = \chi_\mu \left[1 + \tfrac{1}{2}\xi\tilde{g}_1(\lambda)\right]^2 \left[1 + \xi \ln(\chi_\mu/\Lambda_3^2) + a^2 D_m\right]$$

$$f_{\rm PS}(L) = f_0 \left[1 - 2\xi\tilde{g}_1(\lambda)\right] \left[1 - 2\xi \ln(\chi_\mu/\Lambda_4^2) + a^2 D_f\right]$$

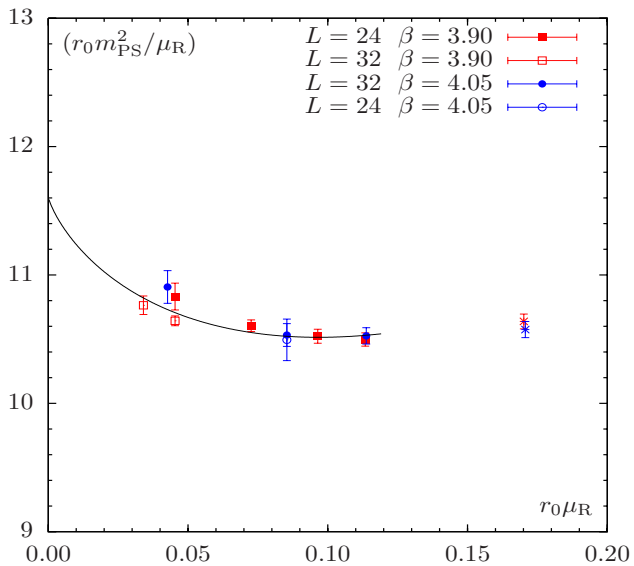
$$m_{\rm N}(L) = M_N - 4c_1\chi_\mu - \frac{6g_A^2}{32\pi f_0^2}\chi_\mu^{3/2} + a^2 D_N$$

where

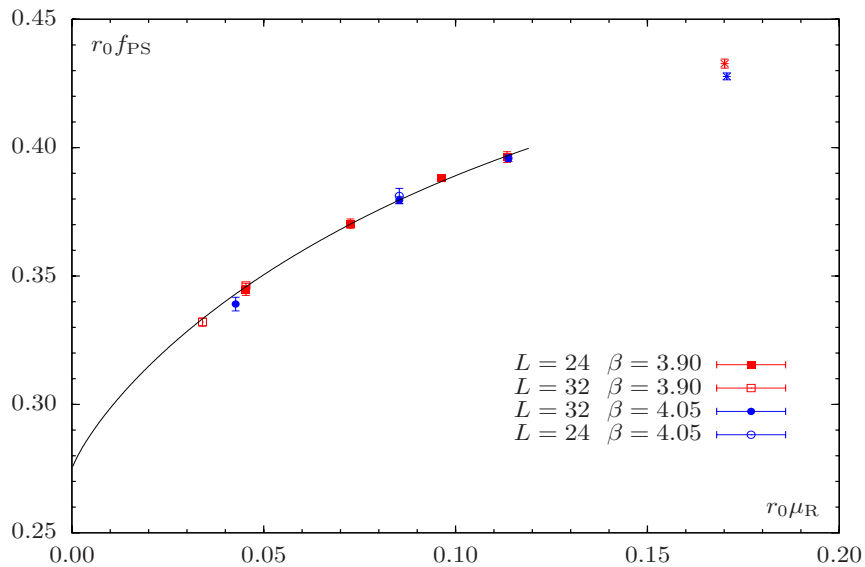
$$\chi_\mu = 2B_R\mu_R\,, \qquad \xi = \chi_\mu/(4\pi f_0)^2\,, \qquad \lambda = \chi_\mu L\,, \qquad f_0 = \sqrt{2}F_0$$

- low-energy constants $\bar{l}_{3,4} \equiv \log(\Lambda_{3,4}^2/m_{\pi^\pm}^2)$
- Finite size corrections Gasser, Leutwyler,1987; Colangelo *et al.*, 2005
- NNLO: extra parameters : $\Lambda_{1,2}$, k_M, k_F

Chiral perturbation theory



pseudoscalar mass



pseudoscalar deacy constant

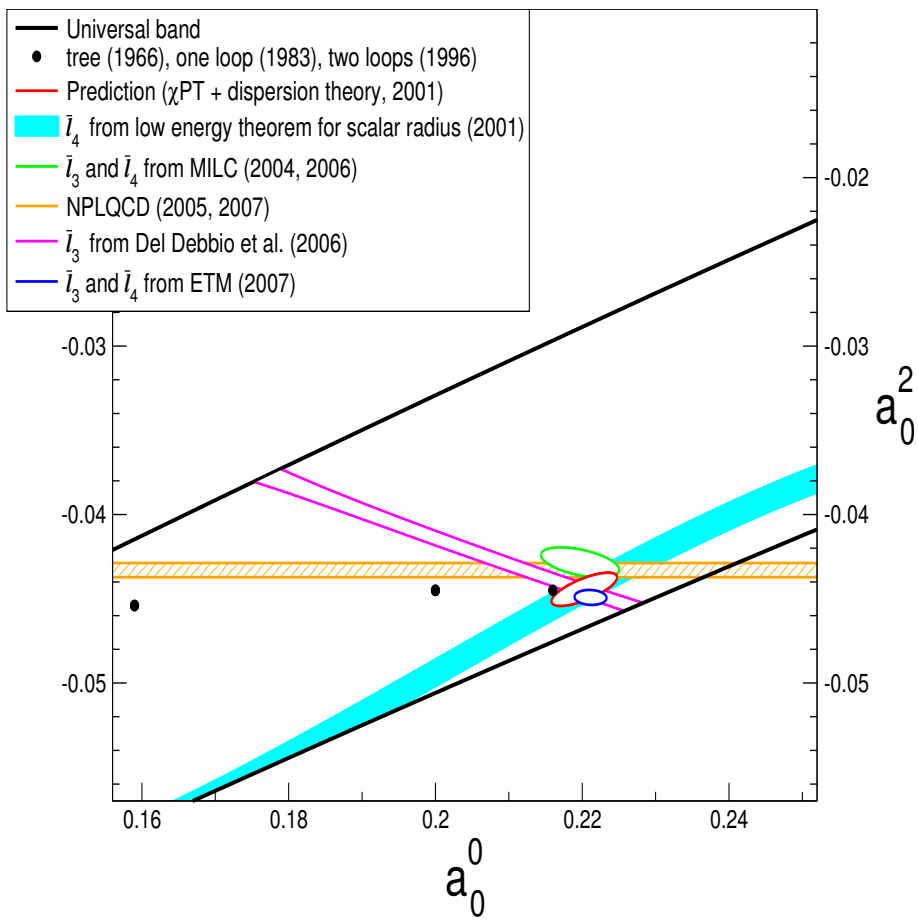
Chiral perturbation theory

- fit of $f_{\rm PS}$, $m_{\rm PS}$, $m_{\rm N}$ combining $\beta = 4.05$ and 3.9
- mass dependence continuum NLO higher masses ($m_{\rm PS} \sim 600$ MeV) not included
- precise results for the χPT low-energy constants : $\bar{l}_{3,4}$

$\bar{l}_3$	$3.41(7)$
$\bar{l}_4$	$4.61(3)$
$m_{\rm N}$ (MeV)	$963(47)$
$a\|_{\beta=3.9}$ (fm)	$0.0855(5)$
$a\|_{\beta=4.05}$ (fm)	$0.0670(5)$
r_0 (fm)	$0.445(3)$
χ^2/dof	$21.4/21$

Chiral perturbation theory

S-wave scattering lengths a_0^0 and a_0^2 Leutwyler, 2007



Finite size effects

Comparison of data at several volumes to :

- NLO χPT : GL [Gasser, Leutwyler, 1987, 1988]
- resummed Lüscher formula : CDH [Colangelo, Dürr, Haefeli, 2005]
- relative deviation : $R_O = (O_L - O_\infty)/O_\infty$

$obs.\ O$	β	$m_{\rm PS}L$	meas. [%]	GL [%]	CDH [%]
$m_{\rm PS}$	3.90	3.3	+1.8	+0.6	+1.1
$f_{\rm PS}$	3.90	3.3	−2.5	−2.5	−2.4
$m_{\rm PS}$	4.05	3.0	+6.2	+2.2	+6.1
$f_{\rm PS}$	4.05	3.0	−10.7	−8.8	−10.3
$m_{\rm PS}$	4.05	3.5	+1.1	+0.8	+1.5
$f_{\rm PS}$	4.05	3.5	−1.8	−3.4	−2.9

- for $R_{\rm CDH}$: parameters estimates from [CDH, 2005] were used as input
- CDH describes data in general better than GL but needs more parameters

Strange quark mass

- renormalization constant : non-perturbative Z_P using RI-MOM scheme

- u-d :

$$m_{u,d}[\overline{\mathrm{MS}}, 2\ \mathrm{GeV}] = 3.65(6)(23)\ \mathrm{MeV}$$

- strange quark mass : $\beta = 3.9$, partially quenched

$$m_s[\overline{\mathrm{MS}}, 2\ \mathrm{GeV}] = 105(3)(9)\ \mathrm{MeV}$$

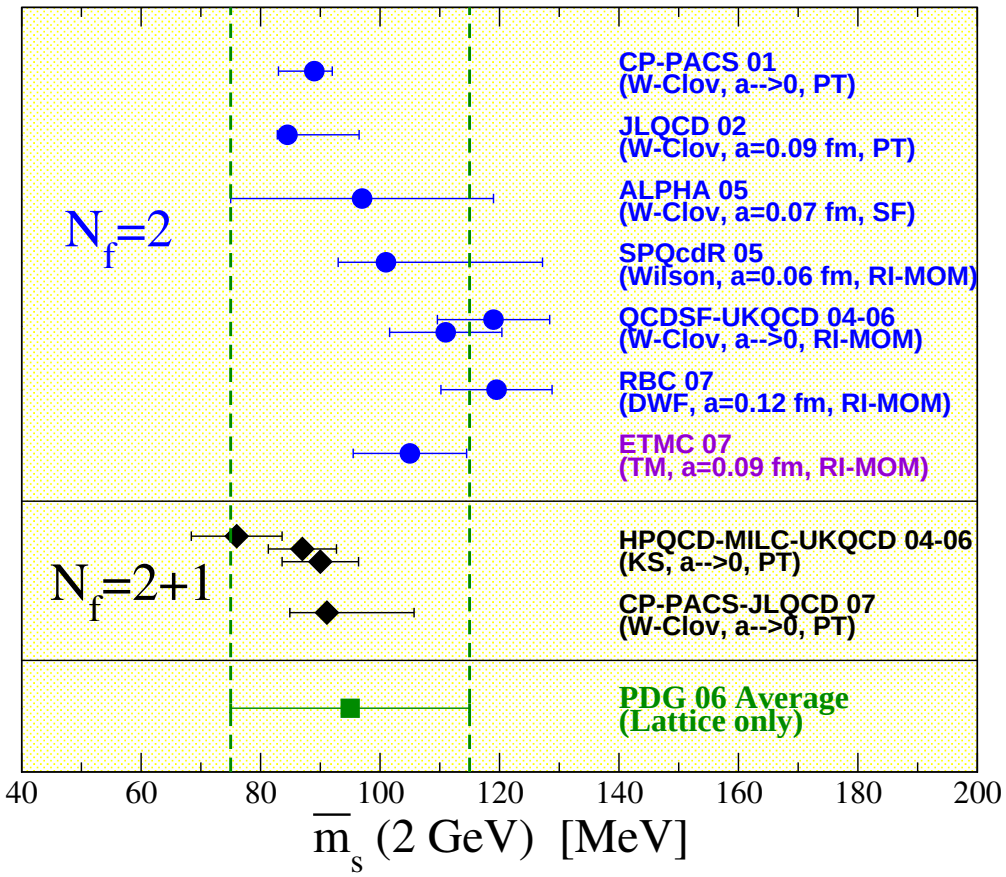
- scalar condensate :

$$\langle \bar{q}q \rangle = -F_0^2\, \widehat{B}_0$$
$$-\langle \bar{q}q \rangle^{1/3} = 267(1)(7)\,\mathrm{MeV}$$

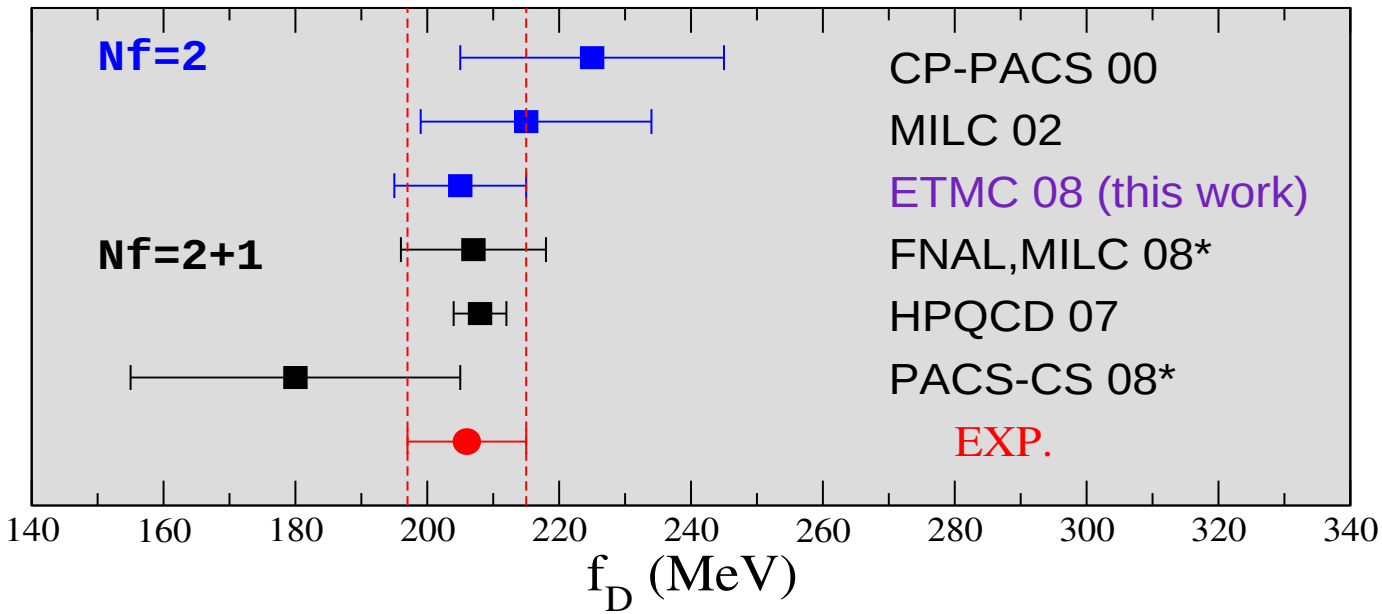
- comparison: ϵ regime

$$-\langle \bar{q}q \rangle^{1/3} = 264(12)(20)\,\mathrm{MeV}$$

Strange quark mass

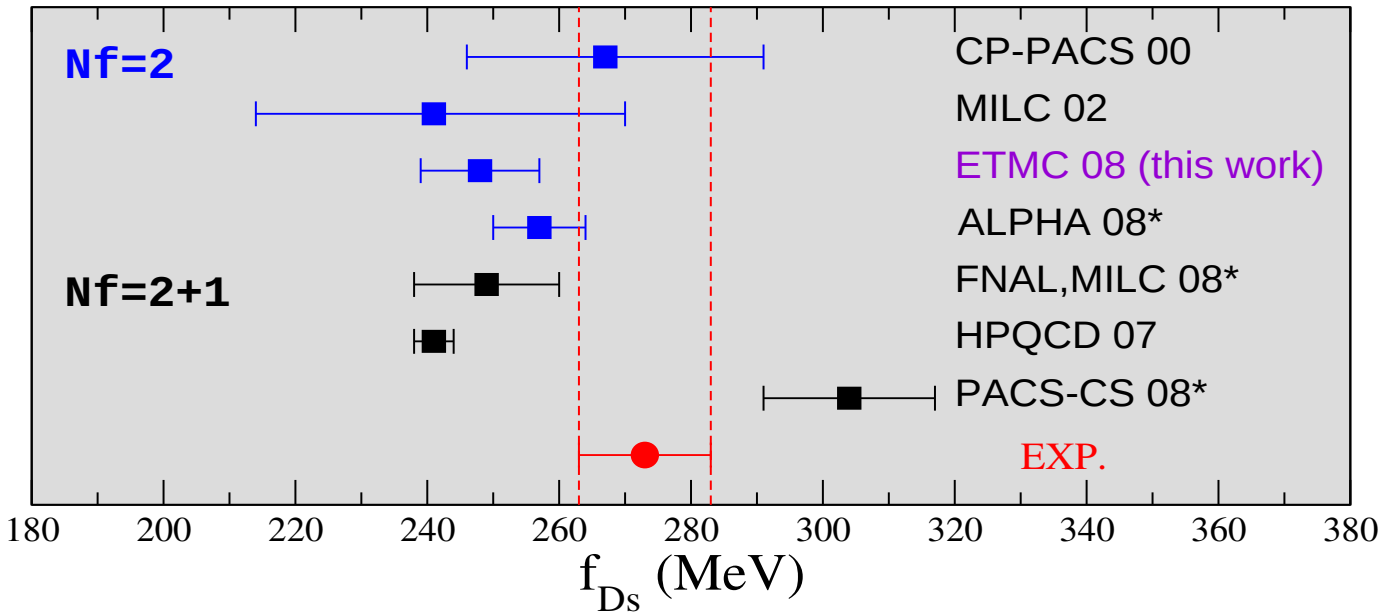


f_D a compilation (ETMC)



- perfect match with experiment

f_{D_s} a compilation (ETMC)



- tension with experiment weakened but remains

Charm physics

$a = 0.09$fm: $m_c^{\overline{\rm MS}}(m_c) = 1.481 \pm 0.022 \pm 0.092$ GeV

(1.286 ± 0.013) GeV (sum rules)

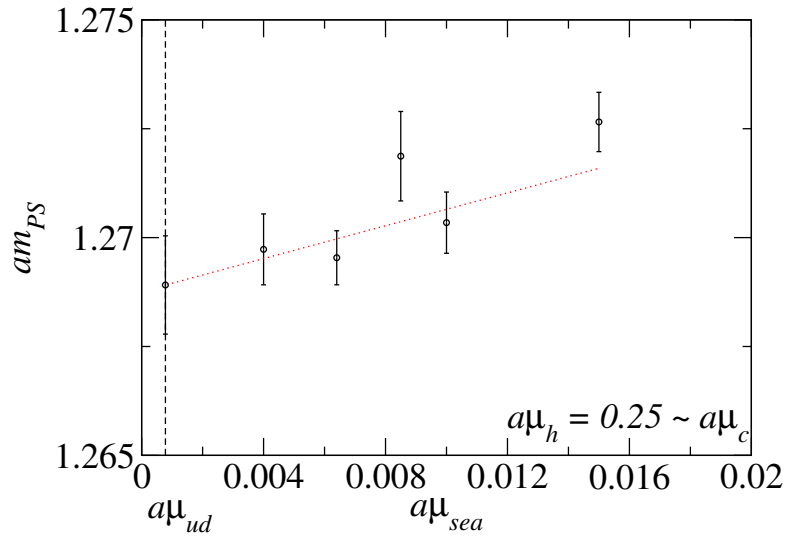
$a = 0.07$fm: $m_c^{\overline{\rm MS}}(m_c) = 1.474 \pm 0.041 \pm 0.132$ GeV

$a = 0.09$fm: $f_D = 205 \pm 13 \pm 17$ MeV

$a = 0.07$fm: $f_D = 230 \pm 31 \pm 8$ MeV

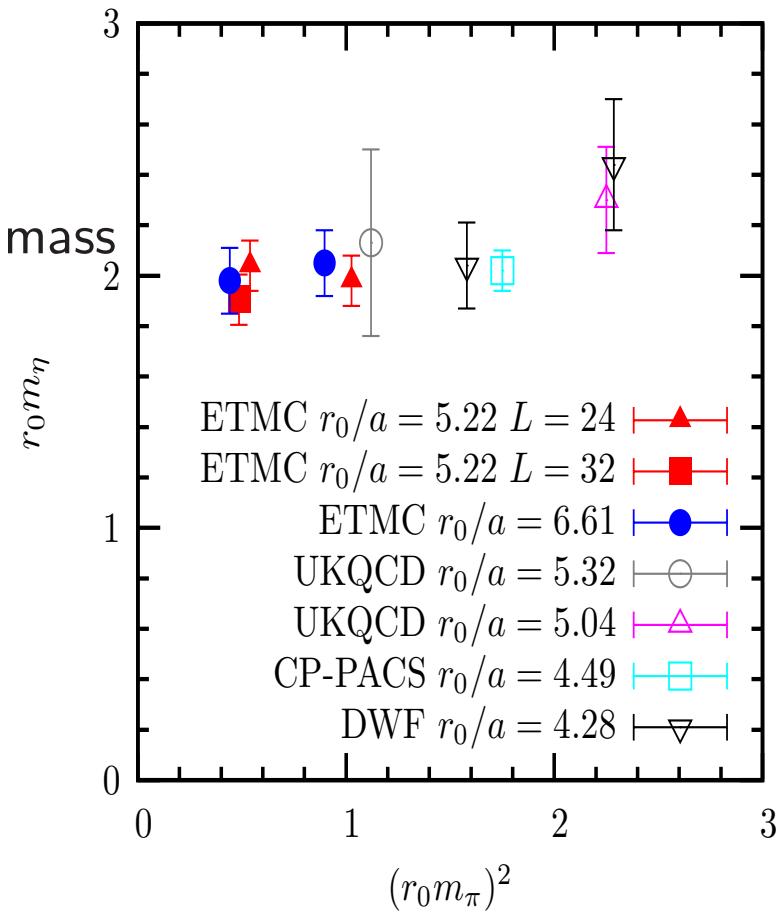
$a = 0.09$fm: $f_{D_s} = 271 \pm 6 \pm 6$ MeV

$a = 0.07$fm: $f_{D_s} = 264 \pm 5 \pm 8$ MeV



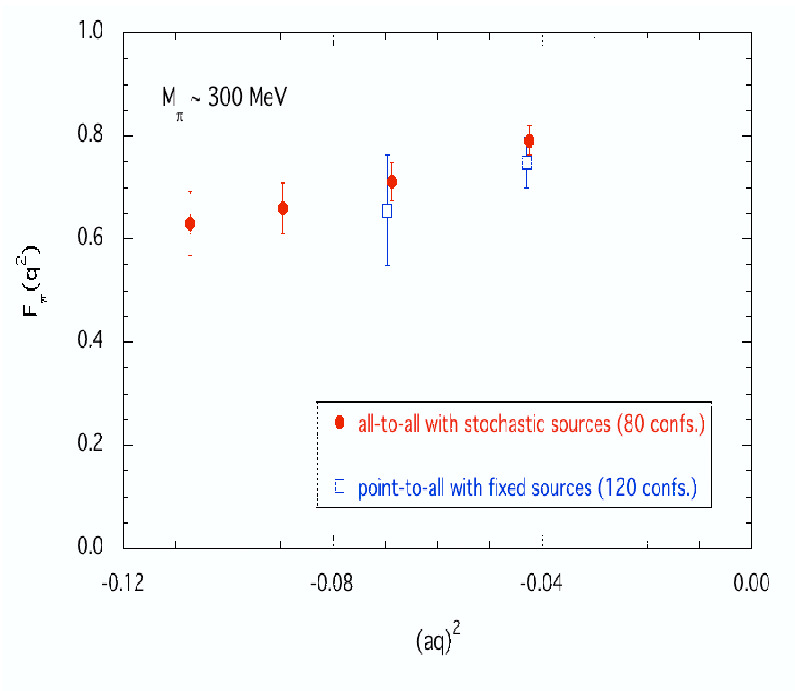
- second lattice spacing consistent

The η' Mass



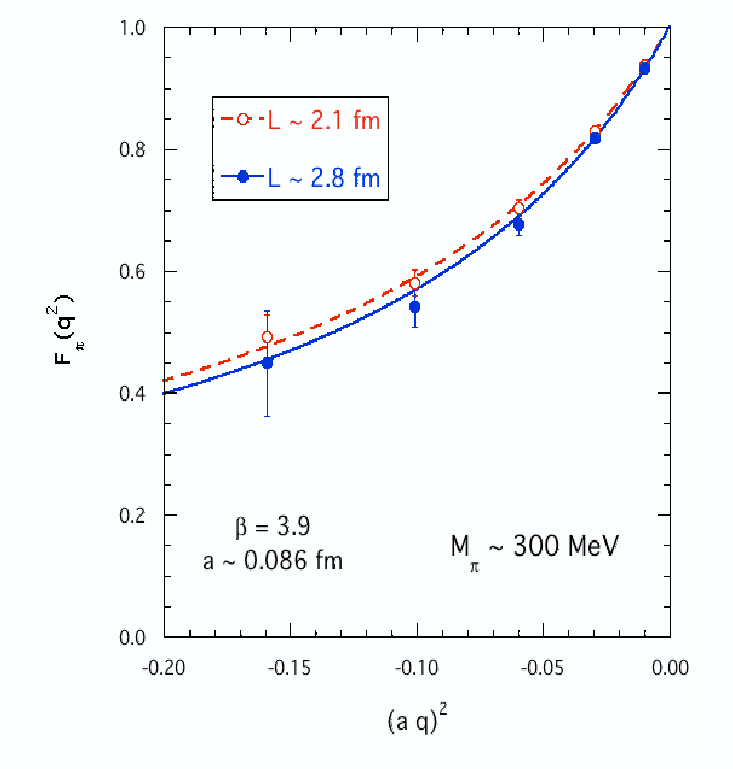
- mysterious particle
- quark content: mass similar to pion
- $m_\eta \approx 150\text{MeV}$
- find: $m_\eta \approx 700\text{MeV}$
- explanation: additional mass of purely topological origin

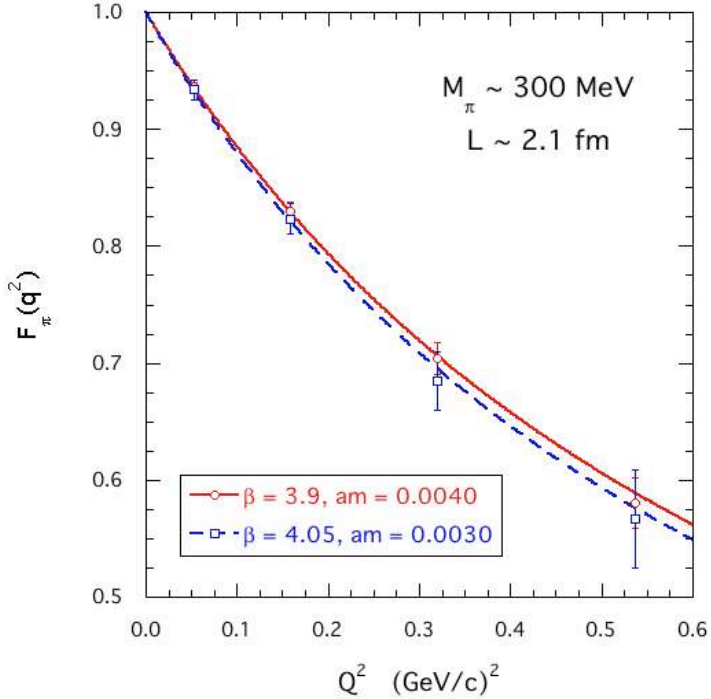
Meson 3-point functions



- error reduction due to stochastic source and randomizing time slice sources

Meson 3-point functions

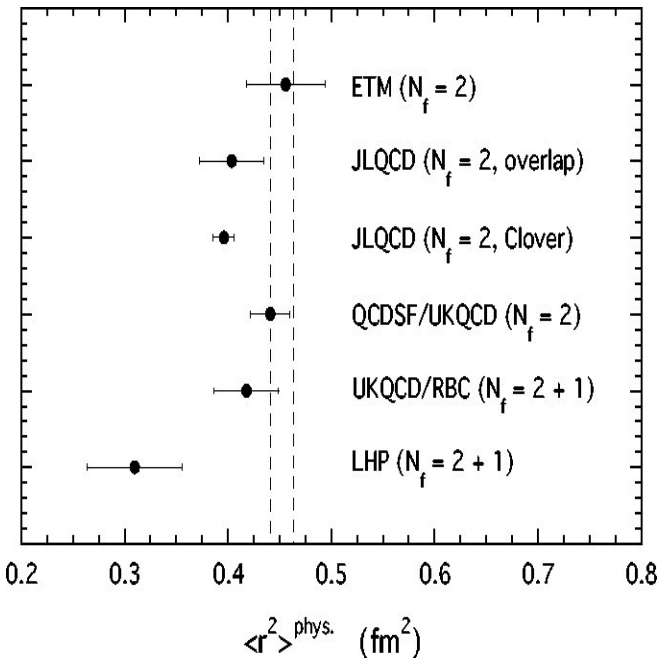




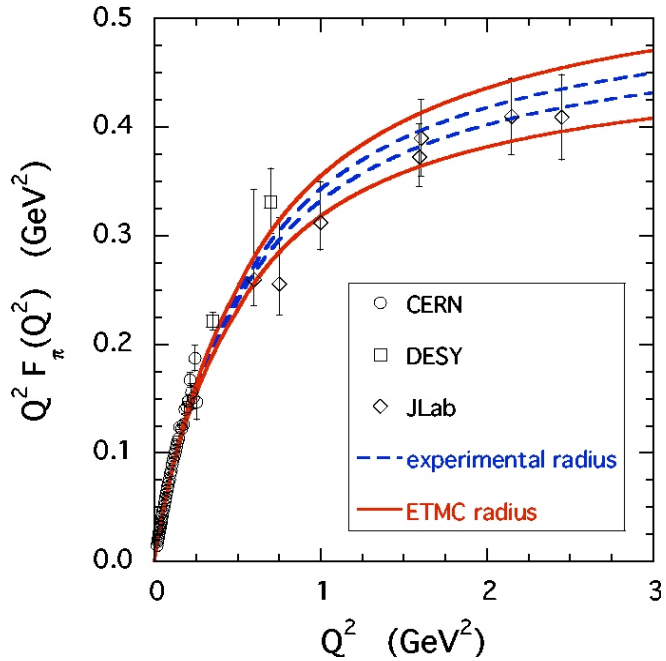
finite size effects

finite a effects

Meson 3-point functions



Results for the pion charge radius



Pion form factor $Q^2 F_\pi(Q^2)$

dashed line: experimental value
$\langle r^2 \rangle^{exp.} = 0.452 \pm 0.011$ fm^2

Importance of non-perturbative renormalization

- strange quark mass **(tm-example, arXiv:0709.4574)**

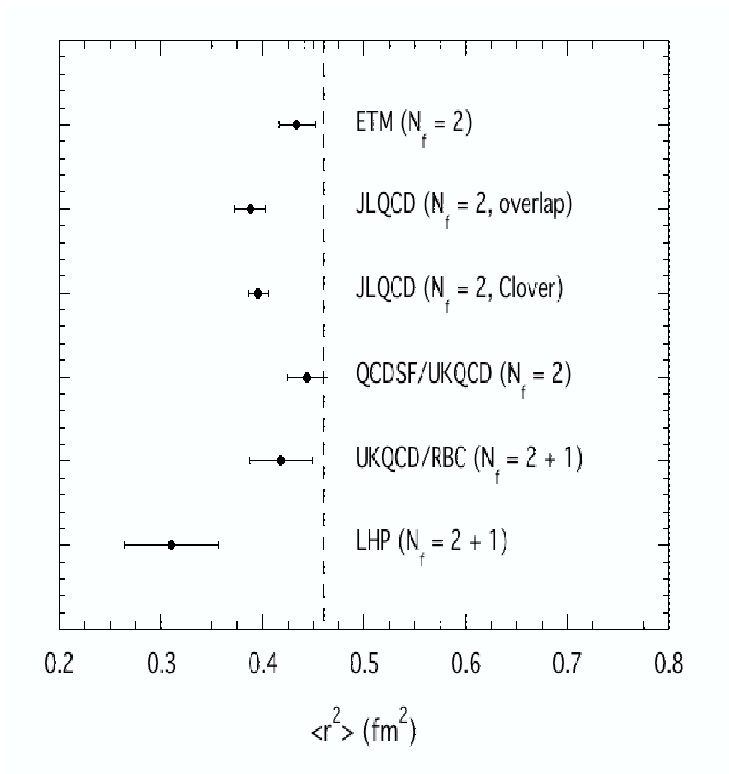
$Z_P^{\mathrm{RI-MOM}}(1/a) = 0.39(1)(2)$ ← non-perturbative RI-MOM method

$Z_P^{\mathrm{BPT}}(1/a) \simeq 0.57(5)$ ← one-loop boosted perturbation theory

$m_q^{\overline{\mathrm{MS}}}(2\mathrm{GeV})\mathrm{MeV}$	perturbative	non-perturbative
m_s	$72 \pm 2 \pm 9$	$105 \pm 3 \pm 9$
m_s (PACS-CS)	72.7 ± 0.8	

- RI-MOM and Schrödinger functional schemes
  UK expertise Dublin (Sint), Liverpool (Shindler)

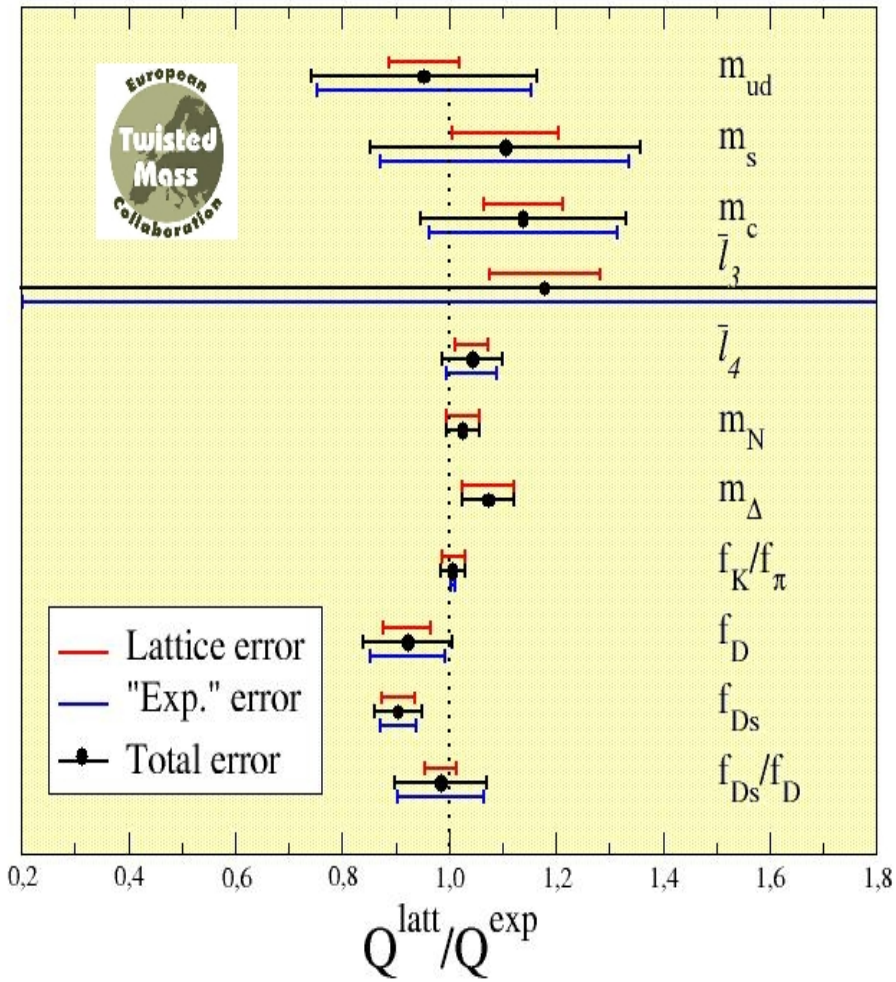
Meson 3-point functions



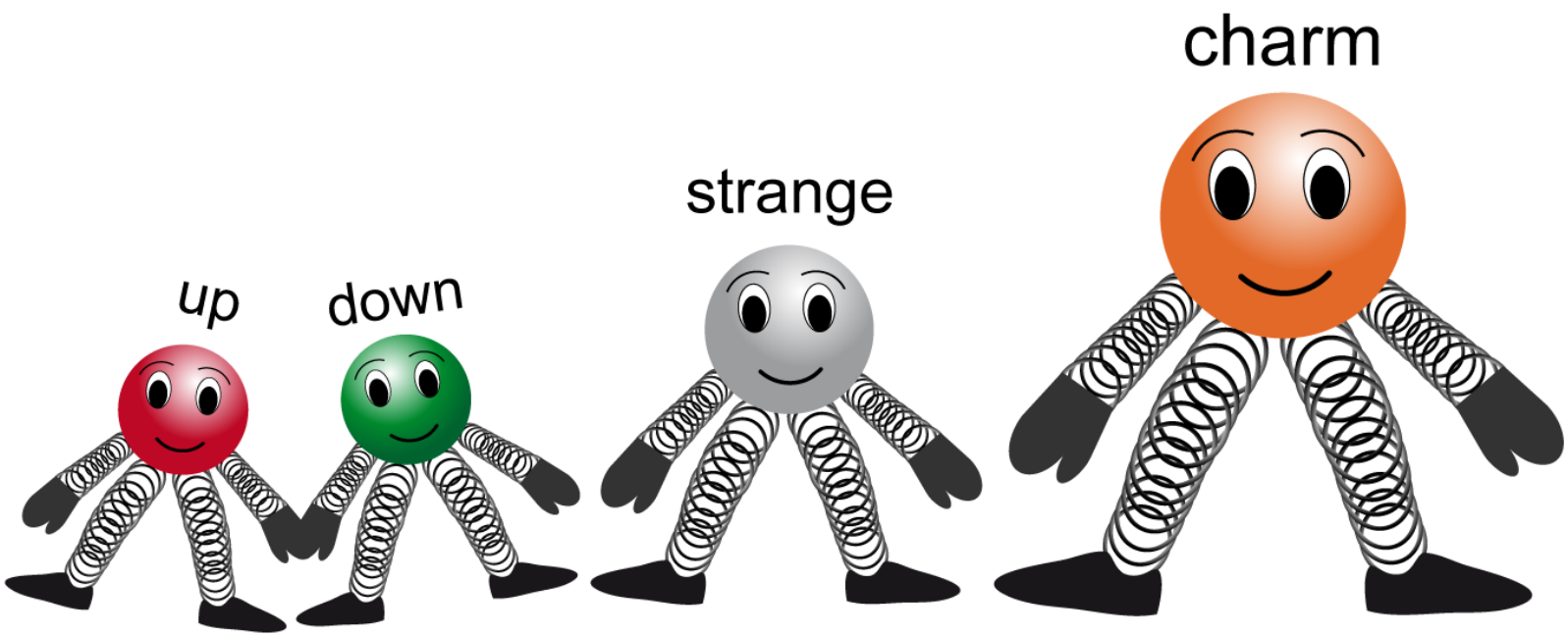
Additional ETMC activities

- nonperturbative renormlaization (RI-MON and SF)
- quark masses (strange, charm)
- static light mesons
- meson (pion and kaon) form factors
- nucleon form factors
- moments of parton distribution functions
- vacuum polarization tensor, e.g. anomalous magnetic moment of muon
- mass splitting of ω and ρ
- width of ρ resonance
- strange baryons
- investigations of ϵ-regime

A comparison to experiment



$N_f = 2 + 1 + 1$ flavours



N_f=2+1+1

Adding dynamical strange and charm quarks

- Mass term

$$S_h = \sum_x \bar{\chi}_x \left[i\gamma_5\tau_1\mu_\sigma + \tau_3\mu_\delta \right] \chi_x \ .$$

- gauge action: RG-improved (Iwasaki)
  - tuning to maximal twist in the light sector
    strategy: tune at each μ_q value separately
  - determinant is still positive

- quark masses

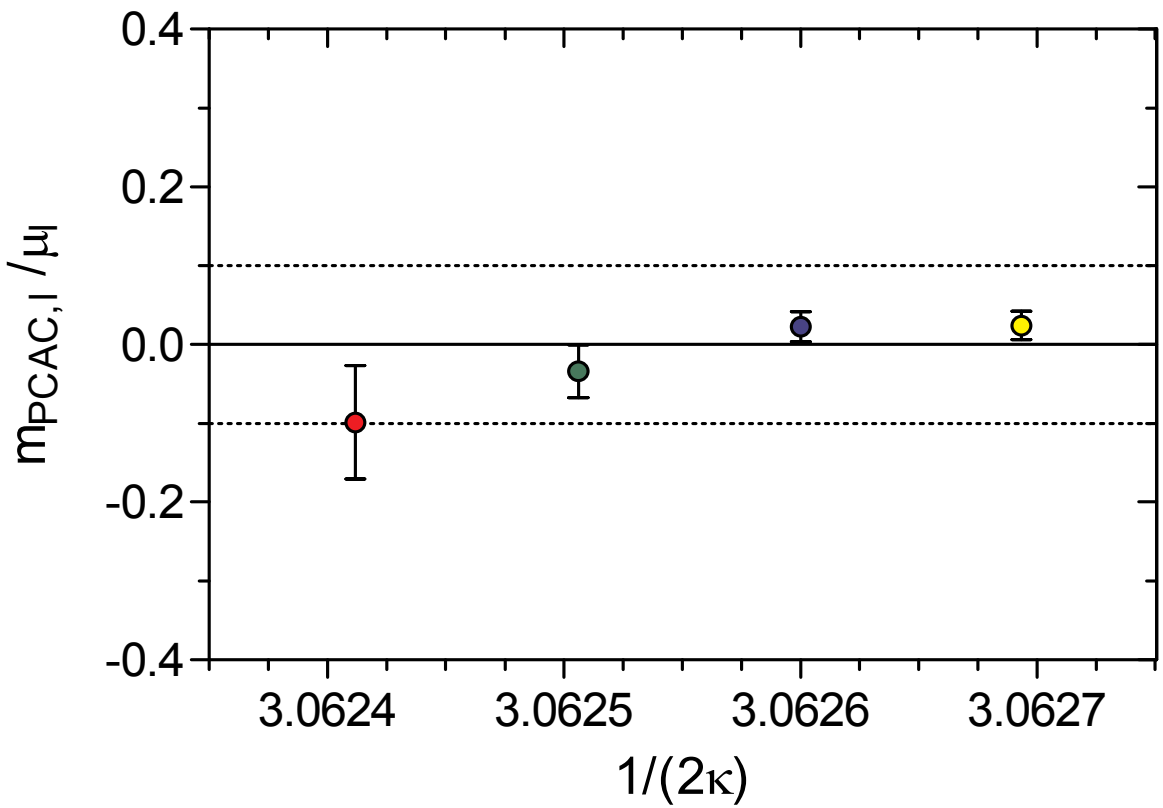
$$m_{s,c} \sim \mu_\sigma \pm Z_P/Z_S \mu_\delta$$

simulation: $Z_P/Z_S \approx 0.65$

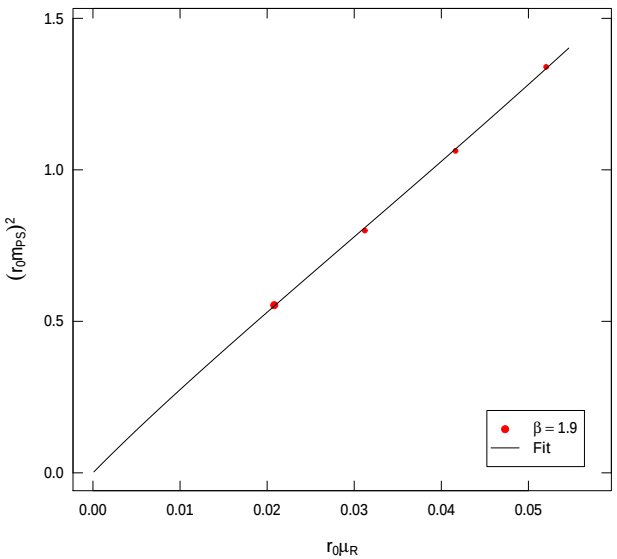
- Algorithm for heavy sector: **PHMC**

Tuning in practise

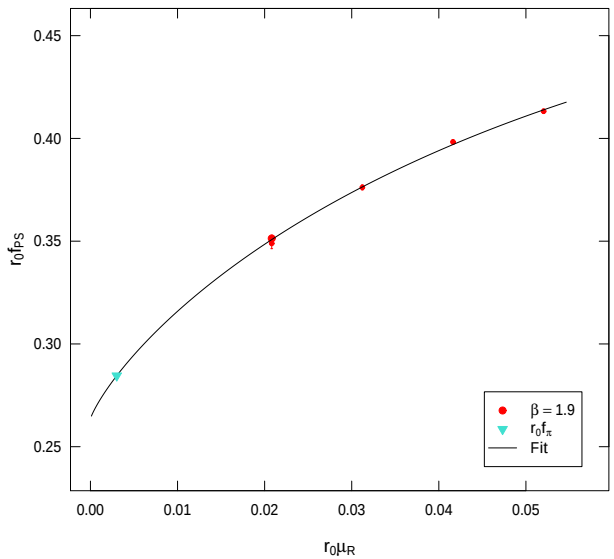
- one value of lattice spacing, $a \approx 0.09\text{fm}$ $(\beta = 1.9)$
- tuning only in light sector



Pseudo scalar mass and decay constant

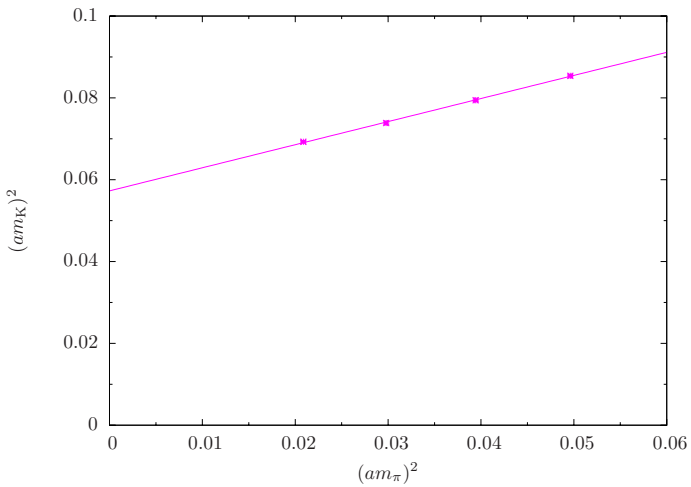


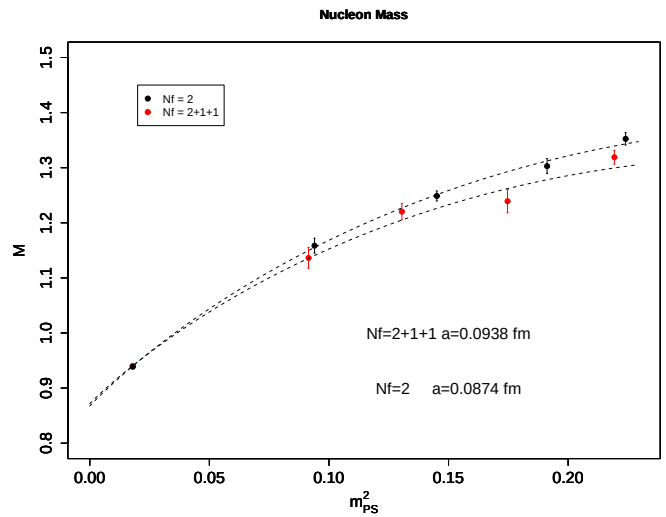
M_{PS}



F_{PS}

Kaon and Nucleon mass





M_{K}

M_{nucleon}

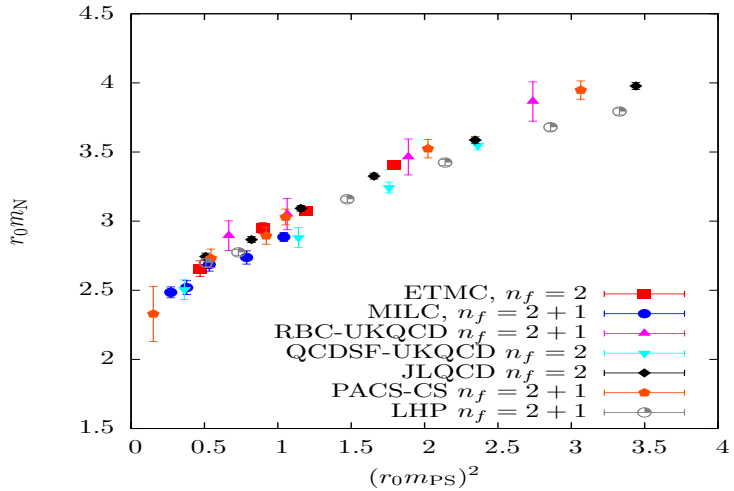
$\rightarrow$ no effect of strange quark

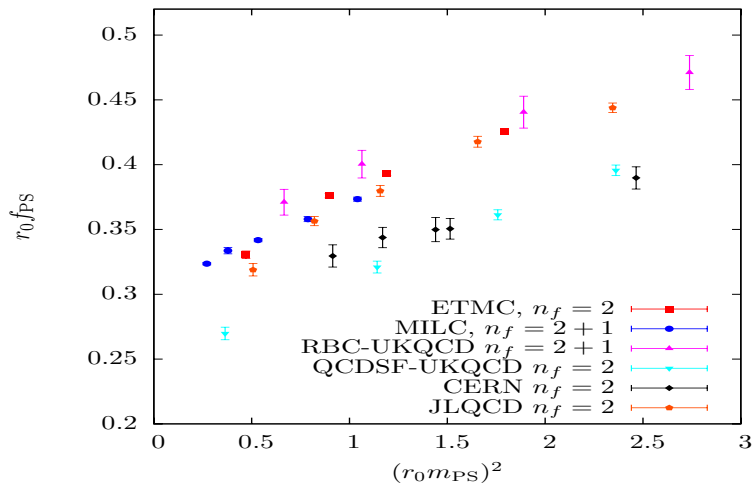
Universality

no ideal fermion action:

- $\mathrm{O}(a)$-*improved Wilson fermions*: breaking of chiral symmetry, non-perturbative operator improvement;
- *rooted staggered fermions*: taste breaking, non-local lattice action;
- *twisted mass fermions*: breaking of chiral symmetry, isospin breaking;
- *overlap fermion*: expense of simulation;
- *domain wall fermions*: expense of simulation and breaking of chirality;
- *fixed topology*: topological finite size effects

Universality: a challenge for lattice QCD





nucleon mass

nice scaling

pseudo scalar decay constant

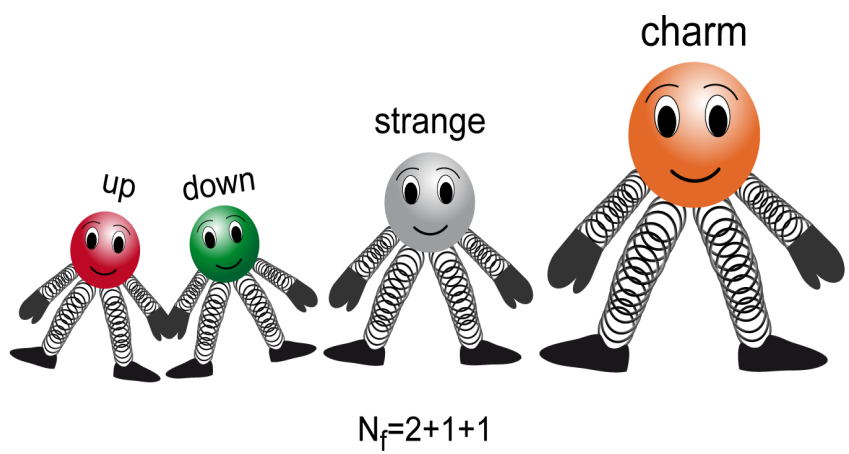
no common scaling visible

Summary

- Maximally twisted mass fermions for $N_f = 2$ flavours
  - very good scaling behaviour
  - smooth and stable simulations
  - obtain very precise results at small quark mass
  - isospin breaking effects only visible in neutral pion mass

- include strange and charm as dynamical degrees of freedom

$\longrightarrow$ Talk by Carsten Urbach



- have established an alternative formulation of lattice QCD