The 1/Nc Expansion for Baryons

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OUTLINE

- Introduction
- Ground state Baryons in $1/N_c$: mass formulas and relations
- Excited Baryons in $1/N_c$: spin-flavor symmetry
- Baryon masses
- Strong transitions
- Photoproduction amplitudes
- Comments and outlook

Expansions in QCD

pQCD
pert: α_s, $1/Q^2$, $\mathbf{1}/N_c$
n-pert: PDFs, GPDs

Low energy QCD: χPT
pert: p/Λ_χ, m_q, $\mathbf{1}/N_c$
n-pert: LECs

$q\bar{Q}$: HQET
pert: $1/m_Q$, m_q, $\mathbf{1}/N_c$
n-pert: IW FFs, ECs

$Q\bar{Q}$: NRQCD
pert: α_s, v, $\frac{1}{m_Q}$, $\mathbf{1}/N_c$
n-pert: effective pots

QCD

Hadron Resonances
pert: m_q, $1/m_Q$, $\mathbf{1}/N_c$
n-pert: ECs, FFs

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Introduction to 1/Nc

- Introduced in Stat. Mech. in 1968 (Stanley): expansion in 1/N, N: number of internal degrees of freedom.
- Applied to many field theory models to study non-perturbative dynamics.
- Introduced in gauge theories in 1974 ('tHooft): $SU(N_c)$ gauge theory admits expansion in $1/N_c$.
- One "solved" case: QCD_2. Full analysis of meson sector to leading order in $1/N_c$. Sub-leading order still open problem. Too simple to shed light on real world QCD.
- QCD: $1/N_c$ expansion still an open theoretical problem.

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$1/N_c$ Expansion in QCD

Two general cases: 'tHooft expansion: N_f fixed, Veneziano expansion: N_f/N_c fixed.

- 'tHooft expansion:

$$\mu\frac{dg}{\mu} = \beta(g) = -\frac{11N_c - 2N_f}{48\pi^2}g^3 = \mathcal{O}(1/\sqrt{N_c})$$
$$\alpha_s = \mathcal{O}(1/N_c) \quad \Lambda_{QCD} \text{ fixed}$$

Keeps main non-perturbative features:

$$\sigma = \mathcal{O}(N_c^0) \quad \langle\bar{q}q\rangle = \mathcal{O}(N_c) \quad \langle G_{\mu\nu}G^{\mu\nu}\rangle = \mathcal{O}(N_c^2)$$

- Quantitative $1/N_c$ expansion still unsolved problem for QCD after 35 years!.
- Key observation: $1/N_c$ expansion can be identified at hadronic level and built into effective theories.
- Two essentially different sectors: glueballs+mesons, and baryons

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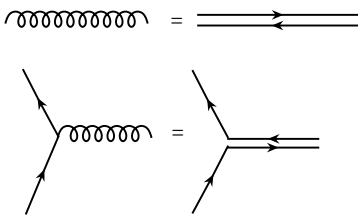
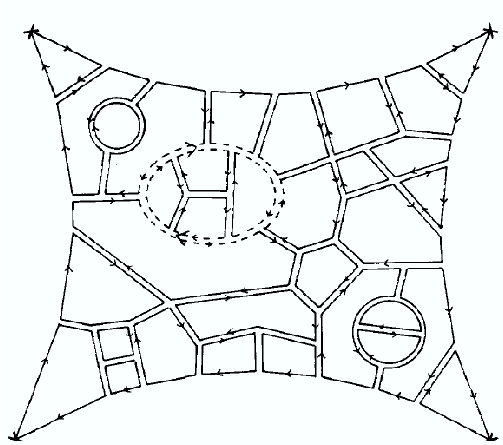
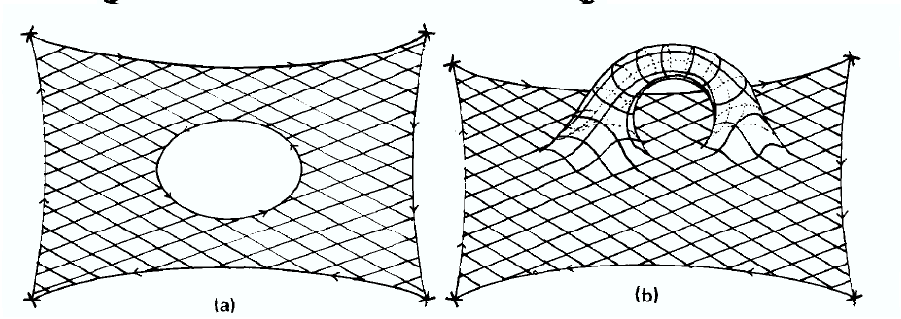
Mesons and Glueballs in $1/N_c$





$1/N_c$ as topological expansion



order of diagram: $1/N_c^{\chi}$ $\qquad$ $1/N_c$ $\qquad$ $1/N_c^2$

$$\chi = -2 + \#\text{ holes} + 2\,\#\text{ handles} + \#\text{ edges}$$

Main facts about $1/N_c$ in mesons and glueballs

Mesons	Glueballs
$M = \mathcal{O}(N_c^0)$	$M = \mathcal{O}(N_c^0)$
$\Gamma = \mathcal{O}(1/N_c)$	$\Gamma = \mathcal{O}(1/N_c^2)$
$F_M = \mathcal{O}(\sqrt{N_c})$	$F_M = \mathcal{O}(N_c)$
OZI rule: $\Gamma_{OZI} = \mathcal{O}(1/N_c^3)$	OZI rule: $\Gamma_{OZI} = \mathcal{O}(1/N_c^2)$
$\sigma = \mathcal{O}(1/N_c^2)$	G-meson mixing$= \mathcal{O}(1/\sqrt{N_c})$
nonet symmetry	
$M_{\eta'}^2 = \mathcal{O}(N_f/N_c)$ in χ limit	

- Phenomenological evidence: multiple excited mesons with widths $\mathcal{O}(100\ \mathrm{MeV})$
- OZI: in ϕ meson and quarkonia for instance
- Test of $1/N_c^2$ corrections to $m_G/\sqrt{\sigma}$ in Lattice gluodynamics
- $1/N_c$ suppression of certain LECs in ChPT

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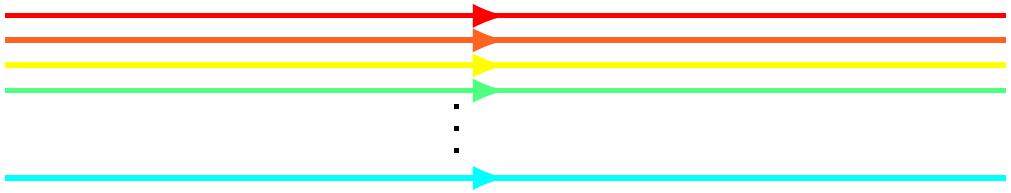
Baryons in $1/N_c$



- Need for N_c valence quarks to form a color singlet
- $M_B = \mathcal{O}(N_c)$, $r_B = \mathcal{O}(N_c^0)$
- Hartree picture of baryons sufficient to figure out $1/N_c$ countings
- π-baryon couplings: $\frac{g_A}{F_\pi}\partial_i \pi_a G^{ia}$
- Axial currents G^{ia} have $\mathcal{O}(N_c)$ MEs $\Longrightarrow$ $g_{\pi BB} = \mathcal{O}(\sqrt{N_c})$
- $\Gamma_B = \mathcal{O}(N_c^0)$
- Model realizations of all of the above: QM, Skyrme model

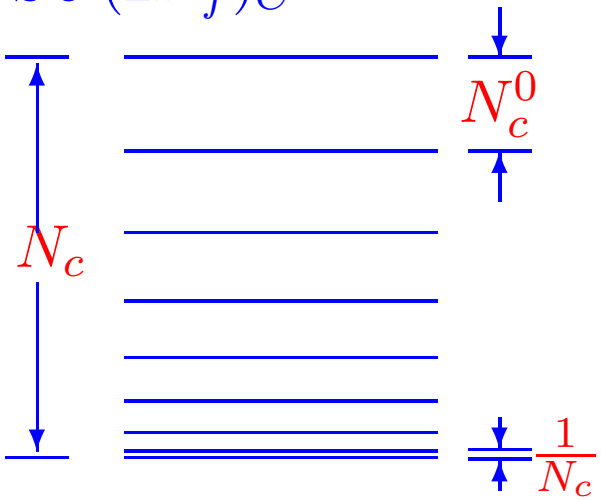
Contracted spin-flavor dynamical symmetry

$$\propto \langle B' \mid [G^{jb}, G^{ia}] \mid B \rangle = \mathcal{O}(N_c^0)$$

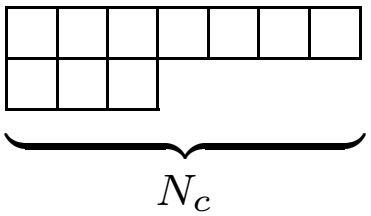
Contracted spin-flavor symmetry: [Gervais & Sakita; Dashen & Manohar]

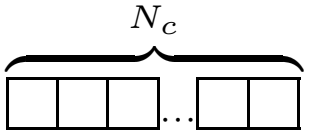
S^i, T^a, $X^{ia} \equiv G^{ia}/N_c$: generators of *contracted* $SU(2N_f)_C$

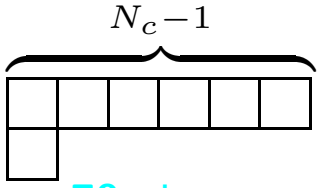
- $SU(2N_f)$ group with generators S^i, T^a, G^{ia} can be used to build effective theory
- Justifies $SU(6)$ symmetry introduced in 1960's as dynamical symmetry in large N_c



Spin-flavor multiplets



S: 

MS: 

56-plet **70**-plet $N_c = N_f = 3$

Known states seem to fit into **56** and **70** plets of $SU(6)$
No evidence for **20**-plet

Approximate $O(3)$ symmetry

Baryons seem to show approximate $O(3)$ symmetry (small spin-orbit effects)

$O(3) \times SU(2N_f)$ a more convenient classification of states than $SU(2N_f)_C$

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- **Effective operators in the $1/N_c$ expansion**
  - Effective operators at baryon level

$$\hat{O}_{\rm QCD} \quad \Rightarrow \quad \hat{O}_{\rm eff} = \sum \left(\frac{1}{N_c}\right)^{\nu(j)} c_j \, \hat{O}_j$$

$$\nu(j) = n_j - 1 \quad \text{for} \quad n_j - \text{body} \quad \text{operator}$$

c_j effective constants or form factors: determined by the QCD dynamics
Operator basis can be ordered in powers of $1/N_c$.

  - Matrix elements

$$\langle B'_{GS} \mid \hat{O}_j \mid B_{GS} \rangle = \mathcal{O}(N_c^{\kappa(j)}) \; ; \quad \langle B_{GS} \mid \hat{O}_j \mid B^* \rangle = \mathcal{O}(N_c^{-\frac{1}{2}+\kappa(j)})$$

$\kappa(j)$ coherence factor of operator

- General form of effective operators

$$\hat{O}_j = R_j \otimes \mathcal{G}_j$$

R_j: $O(3)$ tensor operator

$\mathcal{G}_j$: spin-flavor tensor operator made out of product of n_j generators of $SU(2N_f)$

$\kappa(\hat{O}_j) = \#G_{ia}$ or T^8 generators in $\mathcal{G}_j$

- Systematic way of building basis of $\mathcal{G}_j$ operators; numerous "reduction rules"

BARYON MASSES

- **GS baryon mass formula**

Write most general scalar, parity even operator with generators of $SU(6)$ and apply various reduction rules valid for MEs in the S representation

Up to $\mathcal{O}(1/N_c)$ and $\mathcal{O}(m_s)$: Gürsey-Radicati mass formula (1964)

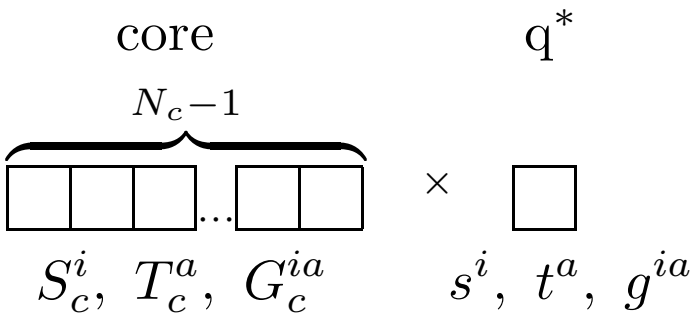
$$M_{GS} = c_1\, N_c + \frac{c_{HF}}{N_c}\,(S^2 - \frac{3}{4}N_c) - c_{\mathcal{S}}\frac{m_s - m_{u,d}}{\Lambda}\,\mathcal{S}$$

Mass relations: deviations $\mathcal{O}(1/N_c^2; m_s^2/N_c)$

GMO	$\Xi_8 - \Sigma_8 = \frac{1}{2}(3\Lambda - \Sigma_8) - N$	128 vs 141	MeV
ES	$\Sigma_{10} - \Delta = \Xi_{10} - \Sigma_{10}$	153 vs 145	
"	$\Omega^- - \Xi_{10} = \Xi_{10} - \Sigma_{10}$	142 vs 145	
8-10	$\Sigma_{10} - \Sigma_8 = \Xi_{10} - \Xi_8$	212 vs 195	
"	$3\Lambda + \Sigma_8 - 2(N + \Xi_8) = \Xi_{10} + \Sigma_{10} - \Omega^- - \Delta$	26 vs 11	

• Excited baryon mass formulas: $[\mathbf{70}, 1^-]$

[JLG; Carlson et al.; JLG, Schat & Scoccola]



$$O_{eff} = \sum \Lambda_c \times \lambda_{q^*} \times \lambda_\ell \qquad M_{\mathbf{70}} = \underbrace{\sum_n c_n O_n}_{\text{SU(3) singlet}} + \underbrace{\sum_m d_m B_m}_{\text{SU(3) octet}}$$

- States in **70**-plet:
  $2\ \mathbf{8}_{J=1/2},\ 2\ \mathbf{8}_{3/2},\ 1\ \mathbf{8}_{5/2},\ 1\ \mathbf{10}_{1/2},\ 1\ \mathbf{10}_{3/2},\ 1\ \mathbf{1}_{1/2},\ 1\ \mathbf{1}_{3/2}$
- Operators:
  SU(3) singlets: 1 $\mathcal{O}(N_c)$ (SU(6) singlet), 4 $\mathcal{O}(N_c^0)$ and 7 $\mathcal{O}(1/N_c)$
  SU(3) octets: 3 $\mathcal{O}(m_s N_c^0)$

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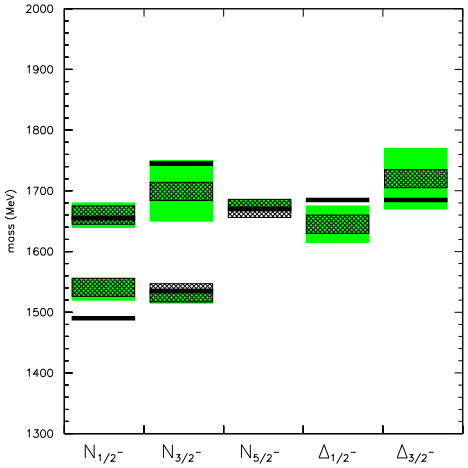
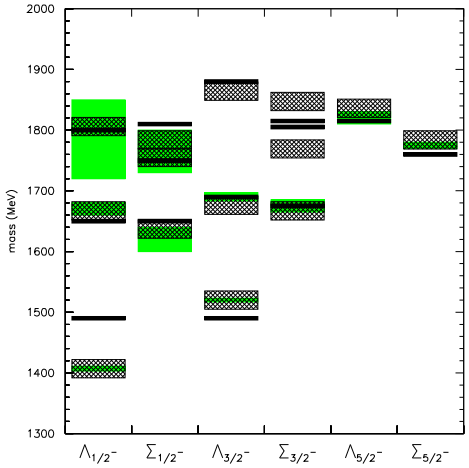
Mass operators for **70**-plets

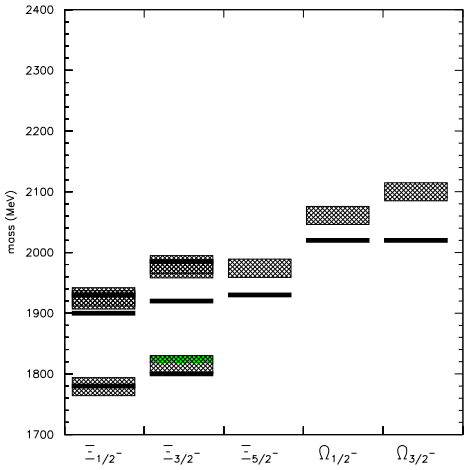
$O_1 = N_c\ 1$	$\mathcal{O}(N_c)$
$O_2 = \ell \cdot s$	
$O_3 = \frac{3}{N_c}\ \ell^{(2)}_{ij}\ g^{ia}\ G_c^{ja}$	$\mathcal{O}(N_c^0)$
$O_4 = \frac{4}{N_c+1}\ \ell^i\ t^a\ G_c^{ia}$	
$O_5 = \frac{1}{N_c} t^a T_c^a - \frac{1}{2\sqrt{3}N_c} O_1$	
$O_6 = \frac{1}{N_c} \ell \cdot\ S_c$	
$O_7 = \frac{1}{N_c} S_c^2$	
$O_8 = \frac{1}{N_c} s \cdot S_c$	
$O_9 = \frac{2}{N_c}\ \ell^{(2)\,ij} s^i\ S_c^j$	$\mathcal{O}(1/N_c)$
$O_{10} = \frac{3}{N_c^2}\ \ell^i\ g^{ja} \{S_c^j, G_c^{ia}\}$	
$O_{11} = \frac{2}{N_c^2} t^a \{S_c^i, G_c^{ia}\}$	
$O_{12} = \frac{3}{N_c^2}\ \ell^i\ g^{ia} \{S_c^j, G_c^{ja}\}$	
$\bar{B}_1 = T_c^8 - \frac{N_c-1}{2\sqrt{3}N_c} O_1$	
$\bar{B}_2 = \frac{10}{N_c}\ d_{8ab}\ g^{ia}\ G_c^{ib} + \frac{5(N_c^2-9)}{8\sqrt{3}N_c^2(N_c-1)} O_1 + \frac{5}{2\sqrt{3}(N_c-1)} O_6 + \frac{5}{6\sqrt{3}} O_7$	$\mathcal{O}(m_s)$
$\bar{B}_3 = 3\ \ell^i\ g^{i8} - \frac{\sqrt{3}}{2} O_2$	

Fit to 70-plet masses to NLO

GMO: 3346 ± 10 vs 3373 ± 10







State	Masses [MeV] Expt.	$1/N_c$	QM [Isgur-Karl]
$N_{1/2}$	1538 ± 18	1541	1490
$\Lambda_{1/2}$	1670 ± 10	1667	1650
$\Sigma_{1/2}$	(1620)	1637	1650
$\Xi_{1/2}$		1779	1780
$N_{3/2}$	1523 ± 8	1532	1535
$\Lambda_{3/2}$	1690 ± 5	1676	1690
$\Sigma_{3/2}$	1675 ± 10	1667	1675
$\Xi_{3/2}$	1823 ± 5	1815	1800
$N'_{1/2}$	1660 ± 20	1660	1655
$\Lambda'_{1/2}$	1785 ± 65	1806	1800
$\Sigma'_{1/2}$	1765 ± 35	1755	1750
$\Xi'_{1/2}$		1927	1900
$N'_{3/2}$	1700 ± 50	1699	1745
$\Lambda'_{3/2}$		1864	1880
$\Sigma'_{3/2}$		1769	1815
$\Xi'_{3/2}$		1980	1985
$N_{5/2}$	1678 ± 8	1671	1670
$\Lambda_{5/2}$	1820 ± 10	1836	1815
$\Sigma_{5/2}$	1775 ± 5	1784	1760
$\Xi_{5/2}$		1974	1930
$\Delta_{1/2}$	1645 ± 30	1645	1685
$\Sigma''_{1/2}$		1784	1810
$\Xi''_{1/2}$		1922	1930
$\Omega_{1/2}$		2061	2020
$\Delta_{3/2}$	1720 ± 50	1720	1685
$\Sigma''_{3/2}$		1847	1805
$\Xi''_{3/2}$		1973	1920
$\Omega_{3/2}$		2100	2020
$\Lambda''_{1/2}$	1407 ± 4	1407	1490
$\Lambda''_{3/2}$	1520 ± 1	1520	1490

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Main results

- Parameter free mass relations: true to $\mathcal{O}(1/N_c^2; m_s^2/N_c)$
  5 GMO, 2 ES, 4 new relations involving **8**'s, **10**'s and **1**'s
  Predict masses of several as yet unknown states in **70**-plet
- Mixing angles

	$N_c \to \infty$	Masses	Strong decays	Helicity Amplitudes
θ_1	54.4^o	30 ± 15°	22 ± 7°	-
θ_3	65.9^o	175 ± 15°	161 or 136 ± 7°	161 ± 7°

- Hyperfine terms S_c^2 dominate spin-flavor breaking
- All $\mathcal{O}(N_c^0)$ operators in **70**-plets have small coefficients: three spin-orbit type and one of pure flavor type: approximate $O(3)$ symmetry
- All $\mathcal{O}(1/N_c)$ operators other than S_c^2 give small corrections
- $SU(3)$ breaking dominated by strangeness operator

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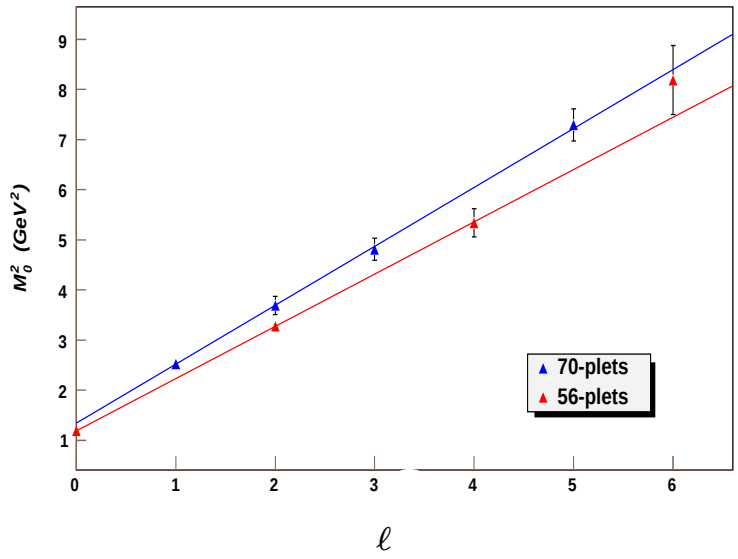
A new take on baryon Regge trajectories $\mathrm{M}_0(\ell)^2$ vs ℓ

[JLG & Matagne]

Spin-flavor singlet component of baryon masses from analyses of all well and not so well known **56**-plets and **70**-plets:

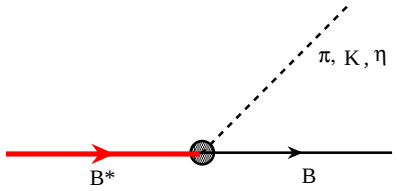
$$[\mathbf{56}, 0^+]_{GS},\ [\mathbf{56}, (2^+, 4^+, 6^+)],\ [\mathbf{70}, (1^-, 2^+, 3^-, 5^-)]$$



- $M_0^2[\mathbf{56}, \ell] = [(1.18 \pm 0.003) + (1.05 \pm 0.01)\,\ell]\ \mathrm{GeV}^2$
- $M_0^2[\mathbf{70}, \ell] = [(1.13 \pm 0.02) + (1.18 \pm 0.02)\,\ell]\ \mathrm{GeV}^2$
- $(M_0[\mathbf{70}, \ell] - M_0[\mathbf{56}, \ell])^2 \simeq (5.7 + 4.2\,\ell) \times 10^{-4}\ \mathrm{GeV}^2$
- Splitting between trajectories $\mathcal{O}(N_c^0)$: due to exchange interaction. In magnitude smaller than expected.
- Regge trajectories with physical masses include contributions which do not have linear behavior.
- Strong indication of small **56**-**70** configuration mixings and good approximate $O(3)$ symmetry

STRONG TRANSITIONS

- **Strong partial wave amplitudes**



$$\mathcal{M}(\ell_\pi, I_\pi) = (-1)^{\ell_\pi} \sqrt{2M_{B^*}} \frac{\sqrt{N_c}}{F_\pi} \langle B_{GS} \mid \mathbf{B}^{\ell_\pi I_\pi}_{m_\pi I_{\pi_3}} \mid B^* \rangle$$

- **$1/N_c$ expansion**

Expand $\mathbf{B}$ in effective operators

$$\begin{aligned} \mathbf{B}^{\ell_\pi I_\pi}_{m_\pi I_{\pi_3}} &= \left(\frac{k_\pi}{\Lambda}\right)^{\ell_\pi} \sum_n c_n \, O_{n\,m_\pi I_{\pi_3}}^{\ell_\pi I_\pi} \\ O_n{}^{\ell_\pi I_\pi} &= [\xi^\ell \, \mathcal{G}_n^{j_n I_\pi}]^{[\ell_\pi, I_\pi]} \end{aligned}$$

$\mathcal{G}_n^{j_n I_\pi}$ are spin-flavor tensor operators

ℓ the $O(3)$ quantum number of excited baryon

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	Operator	ν
$O_1^{[0,1]}$	$(\xi\ g)^{[0,1]}$	0
$O_2^{[0,1]}$	$\frac{1}{N_c}\left(\xi\,(s\ T_c)^{[1,1]}\right)^{[0,1]}$	1
$O_3^{[0,1]}$	$\frac{1}{N_c}\left(\xi\,(t\ S_c)^{[1,1]}\right)^{[0,1]}$	1
$O_4^{[0,1]}$	$\frac{1}{N_c}\left(\xi\,(g\ S_c)^{[1,1]}\right)^{[0,1]}$	1
$O_1^{[2,1]}$	$(\xi\ g)^{[2,1]}$	0
$O_2^{[2,1]}$	$\frac{1}{N_c}\left(\xi\,(s\ T_c)^{[1,1]}\right)^{[2,1]}$	1
$O_3^{[2,1]}$	$\frac{1}{N_c}\left(\xi\,(t\ S_c)^{[1,1]}\right)^{[2,1]}$	1
$O_4^{[2,1]}$	$\frac{1}{N_c}\left(\xi\,(g\ S_c)^{[1,1]}\right)^{[2,1]}$	1
$O_5^{[2,1]}$	$\frac{1}{N_c}\left(\xi\,(g\ S_c)^{[2,1]}\right)^{[2,1]}$	1
$O_6^{[2,1]}$	$\frac{1}{N_c}\left(\xi\,(s\ G_c)^{[2,1]}\right)^{[2,1]}$	0
$O_7^{[2,1]}$	$\frac{1}{N_c^2}\left(\xi\left(s\,(\{S_c,G_c\})^{[2,1]}\right)^{[2,1]}\right)^{[2,1]}$	1
$O_8^{[2,1]}$	$\frac{1}{N_c^2}\left(\xi\left(s\,(\{S_c,G_c\})^{[2,1]}\right)^{[3,1]}\right)^{[2,1]}$	1
$O_1^{[0,0]}$	$(\xi\ s)^{[0,0]}$	0
$O_2^{[0,0]}$	$\frac{1}{N_c}\left(\xi\,(s\ S_c)^{[1,0]}\right)^{[0,0]}$	1
$O_1^{[2,0]}$	$(\xi\ s)^{[2,0]}$	0
$O_2^{[2,0]}$	$\frac{1}{N_c}\left(\xi\,(s\ S_c)^{[1,0]}\right)^{[2,0]}$	1
$O_3^{[2,0]}$	$\frac{1}{N_c}\left(\xi\,(s\ S_c)^{[2,0]}\right)^{[2,0]}$	1

• $[70, 1^-]$ decays

[JLG, Schat & Scoccola]

Coeff	LO	NLO
$c_1^{[0,1]}$	31 ± 3	23 ± 3
$c_2^{[0,1]}$	-	$(7.4, 32.5) \pm (27, 41)$
$c_3^{[0,1]}$	-	$(20.7, 26.8) \pm (12, 14)$
$c_4^{[0,1]}$	-	$(-26.3, -66.8) \pm (39, 65)$
$c_1^{[2,1]}$	4.6 ± 0.5	3.4 ± 0.3
$c_2^{[2,1]}$	-	-4.5 ± 2.4
$c_3^{[2,1]}$	-	$(-0.01, 0.08) \pm 2$
$c_4^{[2,1]}$	-	5.7 ± 4.0
$c_5^{[2,1]}$	-	3.0 ± 2.2
$c_6^{[2,1]}$	$(-1.86, -2.25) \pm 0.4$	-1.73 ± 0.26
$c_7^{[2,1]}$	-	-
$c_8^{[2,1]}$	-	-
$c_1^{[0,0]}$	11 ± 4	17 ± 4
$c_2^{[0,0]}$	-	-
θ_1	1.56 ± 0.15 0.35 ± 0.14	0.39 ± 0.11
θ_3	$(3.00, 2.44) \pm 0.07$	$(2.82, 2.38) \pm 0.11$
$\chi^2_{\rm dof}$	1.5	0.9
dof	10	3

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Results from fit at NLO

	Transition	PDG [MeV]	LO [MeV]	NLO [MeV]
π S wave	$N(1535) \to \pi N$	68 ± 27	62	(58,68)
	$N(1520) \to \pi\Delta$	10 ± 4	9.7	9.5
	$N(1650) \to \pi N$	121 ± 40	144	122
	$N(1700) \to \pi\Delta$	unknown	175	(259,156)
	$\Delta(1620) \to \pi N$	38 ± 13	35	38
	$\Delta(1700) \to \pi\Delta$	112 ± 53	81	(135,112)
π D wave	$N(1535) \to \pi\Delta$	1 ± 1	.01	0.5
	$N(1520) \to \pi N$	67 ± 9	70	65
	$N(1520) \to \pi\Delta$	15 ± 3	2.8	13
	$N(1650) \to \pi\Delta$	7 ± 5	0.12	8
	$N(1700) \to \pi N$	10 ± 7	10	(11,9)
	$N(1700) \to \pi\Delta$	unknown	4	(4,9)
	$N(1675) \to \pi N$	72 ± 12	85	76
	$N(1675) \to \pi\Delta$	unknown	45	79
	$\Delta(1620) \to \pi\Delta$	68 ± 26	30	87
	$\Delta(1700) \to \pi N$	45 ± 21	49	32
	$\Delta(1700) \to \pi\Delta$	12 ± 10	15	18
η S wave	$N(1535) \to \eta N$	64 ± 28	17	(57,61)
	$N(1650) \to \eta N$	11 ± 6	14	12

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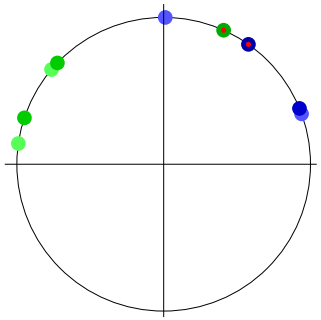
- **Summary of $[70, 1^-]$ decays**
  - Accurate LO fit to π S-wave: only one parameter
  - π D-wave requires NLO operators
  - η modes controlled by θ_1: enhances $N^*(1535) \to \eta N$, suppresses $N^*(1650) \to \eta N$.
    Maximum effect for $\theta_1 = \theta_1(N_c \to \infty)$.
  - Mixing angles θ_1 and θ_3 very different from large N_c limit.
    Reason: LO spin-flavor breaking smaller than NLO!

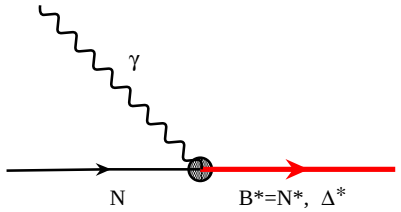


  - NLO coefficients of natural size but have large errors due to uncertainties in input widths

PHOTOPRODUCTION HELICITY AMPLITUDES

[JLG, Schat & Scoccola; Scoccola, JLG & Matagne]

- **Helicity amplitudes (PDG)**



$$\mathcal{A} = -\sqrt{\frac{2\pi\alpha}{\omega}}\,\eta(B^*)\,\langle B^*, \lambda \mid \epsilon_+ \cdot \mathbf{J}_{EM} \mid N, \lambda - 1\rangle$$

$\eta(B^*)$ sign of strong transition reduced amplitude: determined by $1/N_c$ analyses of strong decays

- $1/N_c$ **expansion**

Represent EM current operator in terms of effective operators

$$\begin{aligned} \mathbf{J}_{EM} &= \sqrt{N_c}\sum_n c_n \left(k_\gamma^{[L']} O_n^{[L,I]}\right)^{[1,I]} \\ O_n^{[L,I]} &= \left(\xi^\ell \mathcal{G}_n^{(\ell_n, I)}\right)^{(L,I)} \end{aligned}$$

• $[70, 1^-]$ Helicity Amplitudes

	Operator	Order
$E1_1^{(0)} =$	$(\xi s)^{[1,0]}$	1
$E1_2^{(0)} =$	$\frac{1}{N_c}\left(\xi\,(s\;S_c)^{[0,0]}\right)^{[1,0]}$	$\frac{1}{N_c}$
$E1_3^{(0)} =$	$\frac{1}{N_c}\left(\xi\,(s\;S_c)^{[1,0]}\right)^{[1,0]}$	$\frac{1}{N_c}$
$E1_4^{(0)} =$	$\frac{1}{N_c}\left(\xi\,(s\;S_c)^{[2,0]}\right)^{[1,0]}$	$\frac{1}{N_c}$
$E1_1^{(1)} =$	$(\xi t)^{[1,1]}$	1
$E1_2^{(1)} =$	$(\xi g)^{[1,1]}$	1
$E1_3^{(1)} =$	$\frac{1}{N_c}\left(\xi\,(s\;G_c)^{[2,1]}\right)^{[1,1]}$	1
$E1_4^{(1)} =$	$\frac{1}{N_c}\left(\xi\,(s\;T_c)^{[1,1]}\right)^{[1,1]}$	$\frac{1}{N_c}$
$E1_5^{(1)} =$	$\frac{1}{N_c}\left(\xi\,(s\;G_c)^{[0,1]}\right)^{[1,1]} + \frac{1}{4\sqrt{3}}\;E1_1^{(1)}$	$\frac{1}{N_c}$
$E1_6^{(1)} =$	$\frac{1}{N_c}\left(\xi\,(s\;G_c)^{[1,1]}\right)^{[1,1]} + \frac{1}{2\sqrt{2}}\;E1_2^{(1)}$	$\frac{1}{N_c}$
$M2_1^{(0)} =$	$(\xi s)^{[2,0]}$	1
$M2_2^{(0)} =$	$\frac{1}{N_c}\left(\xi\,(s\;S_c)^{[1,0]}\right)^{[2,0]}$	$\frac{1}{N_c}$
$M2_3^{(0)} =$	$\frac{1}{N_c}\left(\xi\,(s\;S_c)^{[2,0]}\right)^{[2,0]}$	$\frac{1}{N_c}$
$M2_1^{(1)} =$	$(\xi g)^{[2,1]}$	1
$M2_2^{(1)} =$	$\frac{1}{N_c}\left(\xi\,(s\;G_c)^{[2,1]}\right)^{[2,1]}$	1
$M2_3^{(1)} =$	$\frac{1}{N_c}\left(\xi\,(s\;T_c)^{[1,1]}\right)^{[2,1]}$	$\frac{1}{N_c}$
$M2_4^{(1)} =$	$\frac{1}{N_c}\left(\xi\,(s\;G_c)^{[1,1]}\right)^{[2,1]} + \frac{1}{2\sqrt{2}}\;M2_1^{(1)}$	$\frac{1}{N_c}$
$E3_1^{(0)} =$	$\frac{1}{N_c}\left(\xi\,(s\;S_c)^{[2,0]}\right)^{[3,0]}$	$\frac{1}{N_c}$
$E3_1^{(1)} =$	$\frac{1}{N_c}\left(\xi\,(s\;G_c)^{[2,1]}\right)^{[3,1]}$	1

Amplitude	PDG [MeV]	LO [MeV]	NLO* [MeV]
$A^p_{1/2}[N(1535)]$	90 ± 30	$76(0.2)$	$111(0.5)$
$A^n_{1/2}[N(1535)]$	-46 ± 27	$-54(0.1)$	$-78(1.4)$
$A^p_{1/2}[N(1520)]$	-24 ± 9	$-25(0.0)$	$-20(0.2)$
$A^n_{1/2}[N(1520)]$	-59 ± 9	$-6(8.8)$	$-46(1.9)$
$A^p_{3/2}[N(1520)]$	166 ± 5	$66(4.0)$	$163(0.4)$
$A^n_{3/2}[N(1520)]$	-139 ± 11	$-55(4.0)$	$-143(0.1)$
$A^p_{1/2}[N(1650)]$	53 ± 16	$45(0.3)$	$52(0.0)$
$A^n_{1/2}[N(1650)]$	-15 ± 21	$-12(0.0)$	$-20(0.1)$
$A^p_{1/2}[N(1700)]$	-18 ± 13	$-18(0.0)$	$-20(0.0)$
$A^n_{1/2}[N(1700)]$	0 ± 50	$41(0.7)$	$47(0.9)$
$A^p_{3/2}[N(1700)]$	-2 ± 24	$1(0.0)$	$-10(0.1)$
$A^n_{3/2}[N(1700)]$	-3 ± 44	$47(1.3)$	$47(1.3)$
$A^p_{1/2}[N(1675)]$	19 ± 8	$15\ (0.3)$	$8(2.0)$
$A^n_{1/2}[N(1675)]$	-43 ± 12	$-45(0.0)$	$-50(0.4)$
$A^p_{3/2}[N(1675)]$	15 ± 9	$10(0.3)$	$11(0.2)$
$A^n_{3/2}[N(1675)]$	-58 ± 13	$-53(0.1)$	$-71(1.0)$
$A^N_{1/2}[\Delta(1620)]$	27 ± 11	$53(5.7)$	$32(0.2)$
$A^N_{1/2}[\Delta(1700)]$	104 ± 15	$80(0.6)$	$108(0.1)$
$A^N_{3/2}[\Delta(1700)]$	85 ± 22	$70(0.3)$	$112(1.5)$

[Scoccola, JLG & Matagne]

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Summary of $[70, 1^-]$ Helicity Amplitudes

- LO OK, but misses $n_{1/2,3/2}(1520)$, $p_{3/2}(1520)$, $\Delta_{1/2}(1620)$
- LO fit lifts ambiguity in θ_3: 161 ± 7°, and fixes relative signs between S and D wave strong amplitudes
- LO results similar to "single quark transition model"
- NLO: LO coefficients remain stable. One NLO 2-body $E1$ operator sufficient to have consistency
- Both $E1$ and $M2$ multipoles crucial and of similar magnitude
- Example where QM fails: $\Delta_{1/2}(1620)$ too big: NLO operator takes care of problem
- For a more accurate picture of NLO effects empirical helicity amplitudes need improvement to 10% level

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COMMENTS and OUTLOOK

- Phenomenology of excited baryons indicates consistency of the $1/N_c$ expansion. One exception: $[\mathbf{56}, 0^+]$ Roper multiplet where large $1/N_c$ corrections are necessary in strong decays and in helicity amplitudes.
- QCD dynamics simplifies the picture: LO spin-flavor symmetry breaking and numerous $1/N_c$ corrections are dynamically suppressed.
- Many mass predictions of yet undiscovered excited baryons: e.g. Ξ's.
- Further applications possible: electro-production; $1/N_c$ and baryon ChPT; $1/N_c$ analysis of lattice baryon results.
- Important open theoretical issues: to consistently treat the finite width of excited baryons; understand the physics behind the effective coefficients of the expansion.
- Important empirical issues: more accurate masses, helicity amplitudes and partial widths.
- $1/N_c$ expansion is clearly useful as organizing principle for the physics of excited baryons.

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