On the Electromagnetic Pion Form Factor in an NJL Model

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QCD BOUND STATES: METHODS & PROPERTIES

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Introduction

- QCD is rich in its non perturbative domain.
- Dynamical mass generation for massless quarks; (dynamical chiral symmetry breaking).
- Color degrees of freedom (quarks and gluons) are not observable (confinement).
- Strong interactions generate a multitude of bound states whose complete quantitative understanding alludes us.
- Studying QCD: lattice, Schwinger-Dyson and Bethe-Salpeter equations. Then there are effective models such as the NJL model.

Introduction

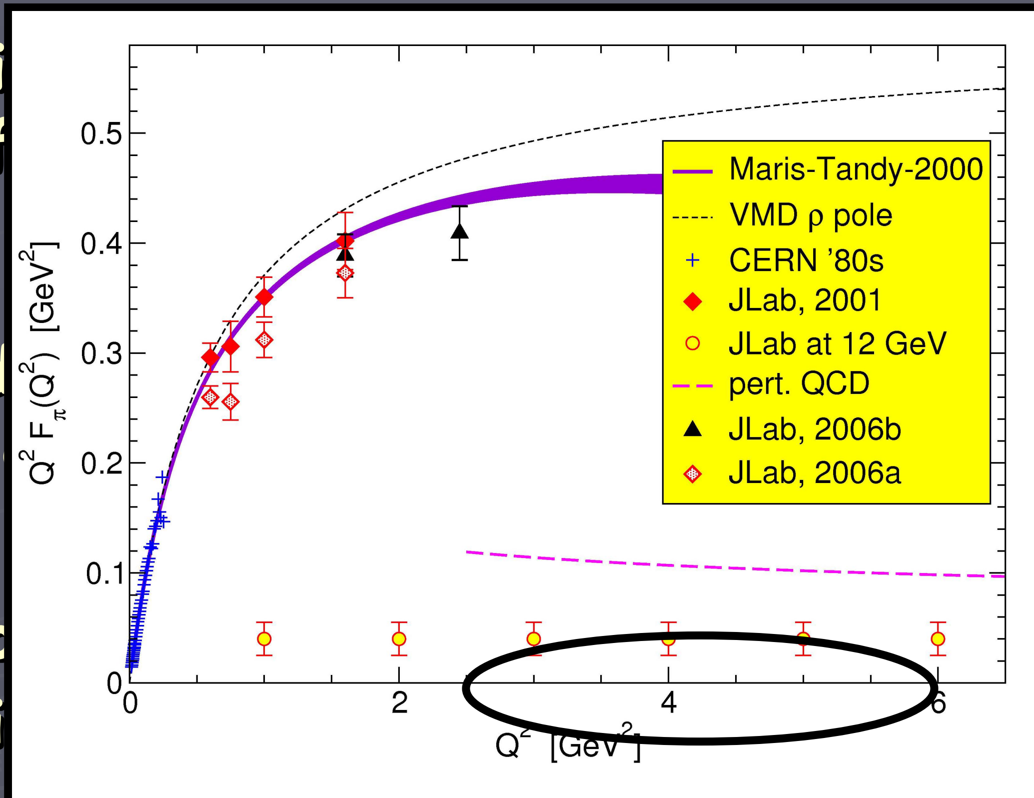
- U. Vogl and W. Weise, Prog. Part. Nucl. Phys. 27 195 (1991).
- T. Hatsuda and T. Kunihiro, Phys. Rept. 247 221 (1994).
- I.C. Cloet, W. Bentz and A. W. Thomas, Phys. Lett. B642 210 (2006); Phys. Rev. Lett. 95 052302 (2005).
- H. Mineo, W. Bentz, N. Ishi, A.W. Thomas and K. Yazaki, Nucl. Phys. A735 482 (2004).
- I.C. Cloet, W. Bentz and A. W. Thomas, Phys. Lett. B621 246 (2005).

Introduction

- Pion occupi… rons in part because of…

- Moreover, … tate of quarks and… with the dynamical …

- Its electro… much studied and experi…

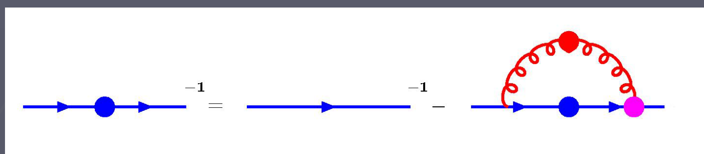


- With the 12 GeV upgrade of the JLab, an exciting new era will unfold where a broad region of transition from nonperturbative to perturbative physics will be explored.

- J. Volmer et. al., Phys. Rev. Lett. 86 1713 (2001) (original red points).
- T. Horn, Phys. Rev. Lett. 97 192001 (2006) (2006b black points).
- V. Tadevosyan, Phys. Rev. C75 055205 (2007) (2006a pink points).
- P. Maris and P. Tandy, Phys. Rev. C62 055204 (2000).

The Gap Equation in an NJL Model

The SDE for the quark propagator in Euclidean space



$$S^{-1}(p) \; = \; Z_2(i\gamma \cdot p + m) + \Sigma(p)$$

where

$$\Sigma(p) \; = \; Z_1 \int d^4q g^2 D_{\mu\nu}(p-q) \, \frac{\lambda^a}{2} \, \gamma_\mu \, S(q) \, \Gamma^a_\nu(q,p)$$

A simple *ansatz* :

$$g^2 D_{\mu\nu}(p-q) \; = \; \delta_{\mu\nu} \, \frac{1}{m_G^2}$$

$$\Gamma^a_\nu(p,q) \; = \; \frac{\lambda^a}{2} \, \gamma_\nu$$

with

$$\int_q^\Lambda \; = \; \frac{1}{4\pi^2} \int d^4q$$

yields NJL model gap equation:

$$S^{-1}(p) \; = \; (i\gamma \cdot p + m) + \frac{1}{3\pi^2 m_G^2} \int_q^\Lambda \gamma_\mu \, S(q) \, \gamma_\mu$$

The Gap Equation in an NJL Model

Define

$$S(p) = -i\gamma\cdot p\,\sigma_V(p^2) + \sigma_S(p^2) = \frac{Z(p^2)}{i\gamma\cdot p + M(p^2)}$$

The gap equation gives the solution: $Z(p) = 1$

And a constant mass:

$$M = m + \frac{M}{3\pi^2 m_G^2}\int ds\,\frac{s}{s+M^2}$$

We use proper time regularization which guarantees confinement and is backed by phenomenology.

$$\frac{1}{s+M^2} \rightarrow \int_{r_{UV}^2}^{r_{IR}^2=1/\Lambda_{IR}^2} d\tau\, e^{-\tau(s+M^2)} = \frac{Z(s)}{s+M^2}$$

$$\frac{1}{X^n} = \frac{(-1)^{n-1}}{(n-1)!}\left(\frac{d}{dX}\right)^{n-1}\left(\frac{1}{X}\right)$$

with

$$Z(s) = \mathrm{e}^{-(s+M^2)r_{UV}^2} - \mathrm{e}^{-(s+M^2)r_{IR}^2}$$

The Gap Equation in an NJL Model

With proper time regularization,

$$M = m + M \frac{1}{3\pi^2} \frac{1}{m_g^2} \mathcal{C}(M^2; r_{IR}^2, r_{UV}^2)$$

$$\mathcal{C}(M^2; r_{IR}^2, r_{UV}^2) = M^2 \Gamma\left(-1, M^2 r_{UV}^2\right) - M^2 \Gamma\left(-1, M^2 r_{IR}^2\right)$$

where

$$\Gamma(\alpha, y) \quad = \quad \int_y^\infty dt\, t^{\alpha-1} \mathrm{e}^{-t}$$

With the phenomenology based values:

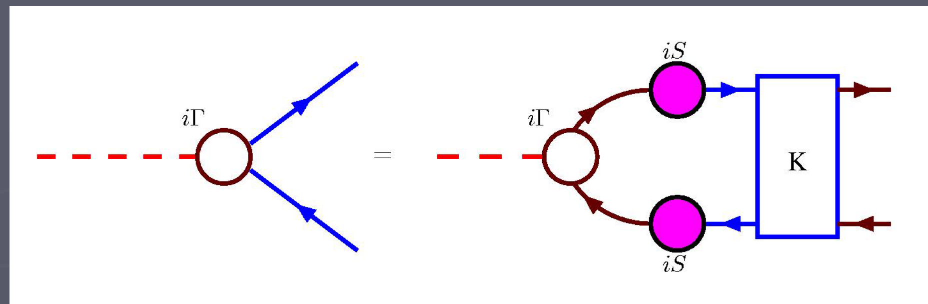
$$m = 0,\ \Lambda_{IR} = 0.24\mathrm{GeV},\ \Lambda_{UV} = 0.64\mathrm{GeV},\ m_g = 0.0738\mathrm{GeV}$$

Constituent Mass

Chiral Condensate

$$M = 0.4\ \mathrm{GeV} \qquad \text{and} \qquad < \bar{q}q >= -(0.169\ \mathrm{GeV})^3.$$

The Bethe-Salpeter Equation



$$i\Gamma_\pi(P) \quad = \quad \int d^4q\, iS(q+P)\, i\Gamma_\pi(P)\, iS(q)\, \mathcal{K}$$

$$\Gamma_\pi(k,P) = \gamma_5 \left[iE_\pi(k;P) + \gamma\cdot P F_\pi(k;P) + \gamma\cdot k k\cdot P G_\pi(k;P) + \sigma_{\mu\nu} k_\mu P_\nu H_\pi(k;P) \right]$$

k is the relative quark momentum and P the total bound state momentum.

In the NJL Model:

$$\Gamma_\pi(P) \quad = \quad \gamma_5 \left[iE_\pi(P) + \frac{\gamma\cdot P}{M} F_\pi(P) \right]$$

The Bethe-Salpeter Equation

NJL Model:

$$\Gamma_\pi(P) \quad = \quad -\frac{1}{3\pi^2}\,\frac{1}{m_g^2}\,\int_q^\Lambda \gamma_\mu\, S(q+P)\,\Gamma_\pi(P)\,S(q)\,\gamma_\mu$$

BS amplitude:

$$\begin{pmatrix} E_\pi(P) \\ F_\pi(P) \end{pmatrix} \quad = \quad \frac{1}{3\pi^2 m_g^2}\begin{pmatrix} K_\pi^{EE} & K_\pi^{EF} \\ K_\pi^{FE} & K_\pi^{FF} \end{pmatrix}\begin{pmatrix} E_\pi(P) \\ F_\pi(P) \end{pmatrix}$$

$$\begin{aligned}
K_\pi^{EE} &= \mathcal{C}(M; r_{IR}^2, r_{UV}^2) \\
K_\pi^{EF} &= 0 \\
K_\pi^{FE} &= \frac{1}{2}\,\mathcal{C}_1(M; r_{IR}^2, r_{UV}^2) \\
K_\pi^{FF} &= \frac{1}{4}\,\mathcal{C}(M; r_{IR}^2, r_{UV}^2) - \frac{3}{4}\,\mathcal{C}_1(M; r_{IR}^2, r_{UV}^2)
\end{aligned}$$

$$\mathcal{C}_1(M; r_{IR}^2, r_{UV}^2) \quad \equiv \quad -M^2\,\frac{d}{dM^2}\mathcal{C}(M; r_{IR}^2, r_{UV}^2)$$

$E_\pi(P)$	$F_\pi(P)$
0.964575	0.26381

BS amplitude fully defined

The Bethe-Salpeter Equation

Canonical Normalization

$$\mathcal{N}^2 P_\mu = \frac{3}{4\pi^2}\mathrm{Tr}\int_q^\Lambda \Gamma_\pi(-P)\left[\frac{\partial S(q_+)}{\partial P_\mu}\Gamma_\pi(P)S(q_-) + S(q_+)\Gamma_\pi(P)\frac{\partial}{\partial P_\mu}S(q_-)\right]$$

$$\Gamma_\pi^C = \frac{1}{\mathcal{N}}\,\Gamma_\pi(P)$$

$$\begin{aligned} \Gamma_\pi^{\mathrm{C}} &= i\gamma_5\, E_\pi^C + \gamma_5\gamma\cdot P\frac{1}{M}\, F_\pi^C \\ &\equiv i\gamma_5\,\frac{E_\pi}{\mathcal{N}} + \gamma_5\gamma\cdot P\frac{1}{M}\,\frac{F_\pi}{N} \end{aligned}$$

$$E_\pi^C = 6.40288 \;\text{ and }\; F_\pi^C = 1.73387$$

We now have the canonically normalized BS-amplitude for our NJL-model.

Axial Vector Ward Takahashi Identity

The axial vector WTI is given by

$$P_\mu \Gamma_{5\mu}(q_+, q) = S^{-1}(q_+) i\gamma_5 + i\gamma_5 S^{-1}(q)$$

Dressed quark propagator:

$$S(p) = \frac{1}{i\gamma \cdot p A(p^2) + B(p^2)}$$

- P. Maris, C.D. Roberts and P.C. Tandy, Phys. Lett. B420 267 (1998).

$$\begin{aligned}
f_\pi\, E_\pi(k; P=0) &= B(k^2) \\
F_R(k;0) + 2 f_\pi\, F_\pi(k;0) &= A(k^2) \\
G_R(k;0) + 2 f_\pi\, G_\pi(k;0) &= 2A'(k^2) \\
H_R(k;0) + 2 f_\pi\, H_\pi(k;0) &= 0\,.
\end{aligned}$$

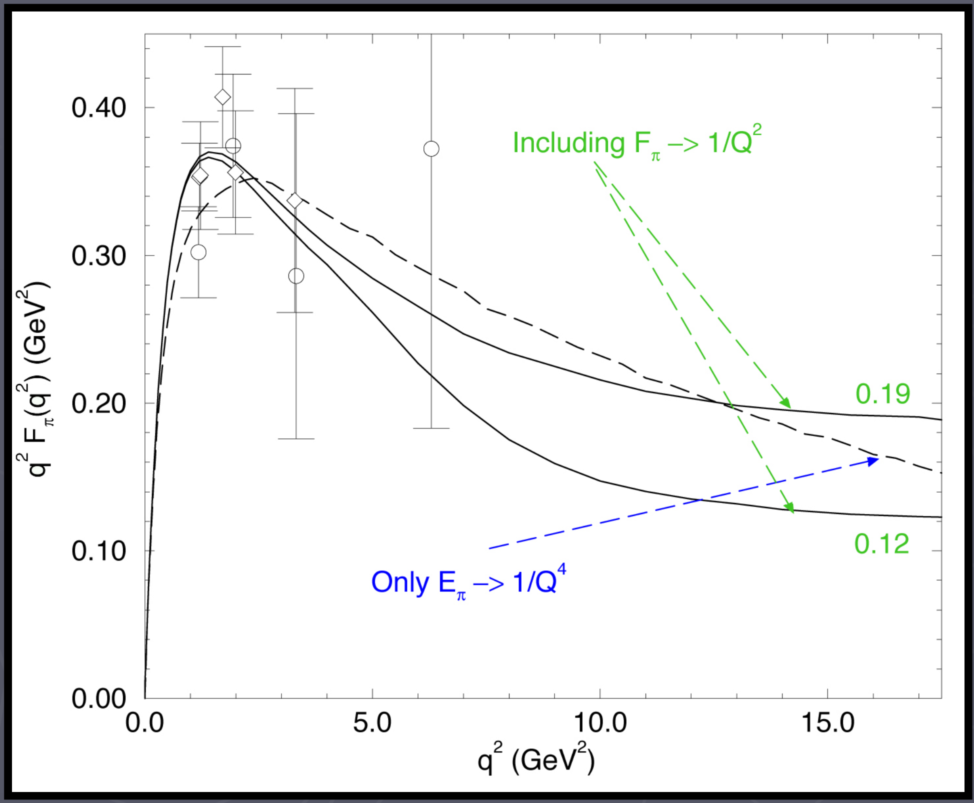
$$f_\pi\, E_\pi(k; P=0) = B(k^2)$$

$$f_\pi\, E_\pi = M \qquad 2\,\frac{F_\pi}{E_\pi} + F_R = 1$$

$$H_R(k;0) + 2 f_\pi\, H_\pi(k;0) = 0\,.$$

Axial Vector Ward Takahashi Identity

Consequences:



P. Maris and C.D. Roberts, Phys. Rev. C58 3659 (1998).

Pseudovector components dominate ultraviolet behaviour of the pion electromagnetic form factor.

Axial Vector Ward Takahashi Identity

$$P_\mu \Gamma_{5\mu}(q_+, q) \quad = \quad S^{-1}(q_+) i\gamma_5 + i\gamma_5 S^{-1}(q)$$

To check it, we start from inhomogeneous BS-equation:

$$\Gamma_{5\,\mu}(P) \quad = \quad \gamma_5\gamma_\mu - \frac{1}{3\pi^2}\,\frac{1}{m_g^2}\,\int_q^\Lambda \gamma_\alpha\; S(q+P)\;\Gamma_{5\,\mu}(P)\;S(q)\;\gamma_\alpha$$

Multiply by P_μ and make use of WTI:

$$\begin{aligned}
P_\mu \Gamma_{5\mu}(k_+, k) \quad &= \quad \gamma_5 \gamma \cdot P - \frac{1}{3\pi^2 m_g^2} \int_q^\Lambda \gamma_\alpha S(q_+) P_\mu \Gamma_{5\mu}(q_+, q) S(q) \gamma_\alpha \\
&= \quad \gamma_5 \gamma \cdot P - \frac{1}{3\pi^2 m_g^2} \int_q^\Lambda \gamma_\alpha S(q_+) \left[S^{-1}(q_+) i\gamma_5 + i\gamma_5 S^{-1}(q) \right] S(q) \gamma_\alpha \\
&= \quad \gamma_5 \gamma \cdot P + \frac{i\gamma_5}{3\pi^2 m_g^2} \int_q^\Lambda \gamma_\alpha S(q) \gamma_\alpha + \frac{1}{3\pi^2 m_g^2} \int_q^\Lambda \gamma_\alpha S(q_+) \gamma_\alpha \, i\gamma_5
\end{aligned}$$

Axial Vector Ward Takahashi Identity

Translational invariance of the regularization scheme and the gap equation reveals:

$$
\begin{aligned}
P_\mu \Gamma_{5\mu}(k_+, k) &= \gamma_5 \gamma \cdot P + i\gamma_5 \left[S^{-1}(k) - i\gamma \cdot k\right] + \left[S^{-1}(k) - i\gamma \cdot k\right] i\gamma_5 \\
P_\mu \Gamma_{5\mu}(k_+, k) &= \gamma_5 \gamma \cdot P + S^{-1}(k) i\gamma_5 + i\gamma_5 S^{-1}(k)
\end{aligned}
$$

or:

$$
P_\mu \Gamma_{5\mu}(q_+, q) = \gamma_5 \gamma \cdot P + S^{-1}(q) i\gamma_5 + i\gamma_5 S^{-1}(q)
$$

1. Thus the axial vector WTI is not satisfied for arbitrary values of P.

2. Goldberger-Treiman relations are not satisfied.

Axial Vector Ward Takahashi Identity

Consequences for the pion leptonic decay constant:

Canonically quantized:

$$f_\pi^C P_\mu = \frac{3}{4\pi^2} \mathrm{Tr} \int_q^\Lambda \gamma_5 \gamma_\mu S(q_+) \Gamma_\pi^C(P) S(q)$$

Ward identity preserving:

$$f_\pi^{\mathrm{WI}} E_\pi^C = M$$

GT-relations:

$$\left(f_\pi^{\mathrm{GT}}\right)^2 P_\mu = \frac{3}{4\pi^2} \mathrm{Tr} \int_q^\Lambda \gamma_5 \gamma_\mu S(q_+) \Gamma_\pi^{GT}(P) S(q)$$

$$\Gamma_\pi^{\mathrm{GT}}(P^2 = 0) = i\gamma_5 E_\pi^{\mathrm{GT}} + \gamma_5 \gamma \cdot P F_\pi^{\mathrm{GT}}$$

$$\begin{aligned} \Gamma_\pi^{\mathrm{GT}} &= f_\pi^{\mathrm{GT}} \Gamma_\pi \\ E_\pi^{\mathrm{GT}} &= M \\ F_\pi^{\mathrm{GT}} &= \frac{F_\pi}{E_\pi} = 0.270795 \end{aligned}$$

Consequence:

	f_π^C	f_π^{GT}	f_π^{WI}
$F_\pi \neq 0$	0.107518	0.0820151	0.062512
$F_\pi = 0$	0.0923286	0.0923286	0.0923286

Axial Vector Ward Takahashi Identity

- We want to regularize the NJL model in such a fashion that WTI is satisfied for all P.

- We observe that WTI is satisfied if NJL model is regularized so as to ensure that there are no quadratic or logarithmic divergences.

- Recall

$$\Gamma_{5\,\mu}(P) = \gamma_5\gamma_\mu - \frac{1}{3\pi^2}\,\frac{1}{m_g^2}\int_q^\Lambda \gamma_\alpha\, S(q+P)\,\Gamma_{5\,\mu}(P)\,S(q)\,\gamma_\alpha$$

$$P_\mu\Gamma_{5\mu}(q_+,q) = \cancel{\gamma_5\gamma\cdot P} + S^{-1}(q)i\gamma_5 + i\gamma_5 S^{-1}(q)$$

- then

$$\int_q^\Lambda \frac{\frac{1}{2}\,q^2+M^2}{(q^2+M^2)^2} = 0$$

Axial Vector Ward Takahashi Identity

BS amplitude:

$$\begin{pmatrix} E_\pi(P) \\ F_\pi(P) \end{pmatrix} = \frac{1}{3\pi^2 m_g^2} \begin{pmatrix} K_\pi^{EE} & K_\pi^{EF} \\ K_\pi^{FE} & K_\pi^{FF} \end{pmatrix} \begin{pmatrix} E_\pi(P) \\ F_\pi(P) \end{pmatrix}$$

Symmetry conserving regularization changes the Kernel:

$$\begin{aligned} K_\pi^{EE} &= \mathcal{C}(M; r_{IR}^2, r_{UV}^2) \\ K_\pi^{EF} &= 0 \\ K_\pi^{FE} &= \frac{1}{2}\,\mathcal{C}_1(M; r_{IR}^2, r_{UV}^2) \\ K_\pi^{FF} &= \frac{1}{4}\,\mathcal{C}(M; r_{IR}^2, r_{UV}^2) - \frac{3}{4}\,\mathcal{C}_1(M; r_{IR}^2, r_{UV}^2) \end{aligned}$$

$\Rightarrow$

$$\begin{aligned} K_\pi^{EE} &= \mathcal{C}(M; r_{IR}^2, r_{UV}^2) \\ K_\pi^{EF} &= 0 \\ K_\pi^{FE} &= \frac{1}{2}\,\mathcal{C}_1(M; r_{IR}^2, r_{UV}^2) \\ K_\pi^{FF} &= -\mathcal{C}_1(M; r_{IR}^2, r_{UV}^2) \end{aligned}$$

so that:

	f_π^C	f_π^{GT}	f_π^{WI}
$F_\pi \neq 0$	0.0709	0.0709	0.0709
$F_\pi = 0$	0.0923286	0.0923286	0.0923286

Axial Vector Ward Takahashi Identity

Checking the Goldberger-Treiman relation:

Inhomogeneous BS Eq.

$$\Gamma_{5\,\mu}(P) \;=\; \gamma_5\gamma_\mu - \frac{1}{3\pi^2}\,\frac{1}{m_g^2}\int_q^\Lambda \gamma_\alpha\, S(q+P)\, \Gamma_{5\,\mu}(P)\, S(q)\,\gamma_\alpha$$

$$\begin{pmatrix} E_5(P) \\ F_5(P) \end{pmatrix} = \begin{pmatrix} 0 \\ 1 \end{pmatrix} - \frac{1}{3\pi^2 m_g^2}\begin{pmatrix} K_\pi^{EE} & K_\pi^{EF} \\ K_\pi^{FE} & K_\pi^{FF} \end{pmatrix}\begin{pmatrix} E_5(P) \\ F_5(P) \end{pmatrix}$$

$$\begin{pmatrix} E_5(P) \\ F_5(P) \end{pmatrix} = \begin{pmatrix} 0 \\ 0.590 \end{pmatrix}$$

$$2\,\frac{F_\pi}{E_\pi} + F_5 = 2\,\frac{0.200822}{0.979628} + 0.590 = 0.410 + 0.590 = 1$$

Pion Electromagnetic Form Factor $F_\pi^{em}(Q^2)$

Consider an incoming pion with momentum p_1 that absorbs a photon with spacelike momentum q so that it departs the interaction region with momentum $p_2 = p_1 + q$. We work in the Breit frame, which means we choose $p_1 = K - q/2$.

In the chiral limit,

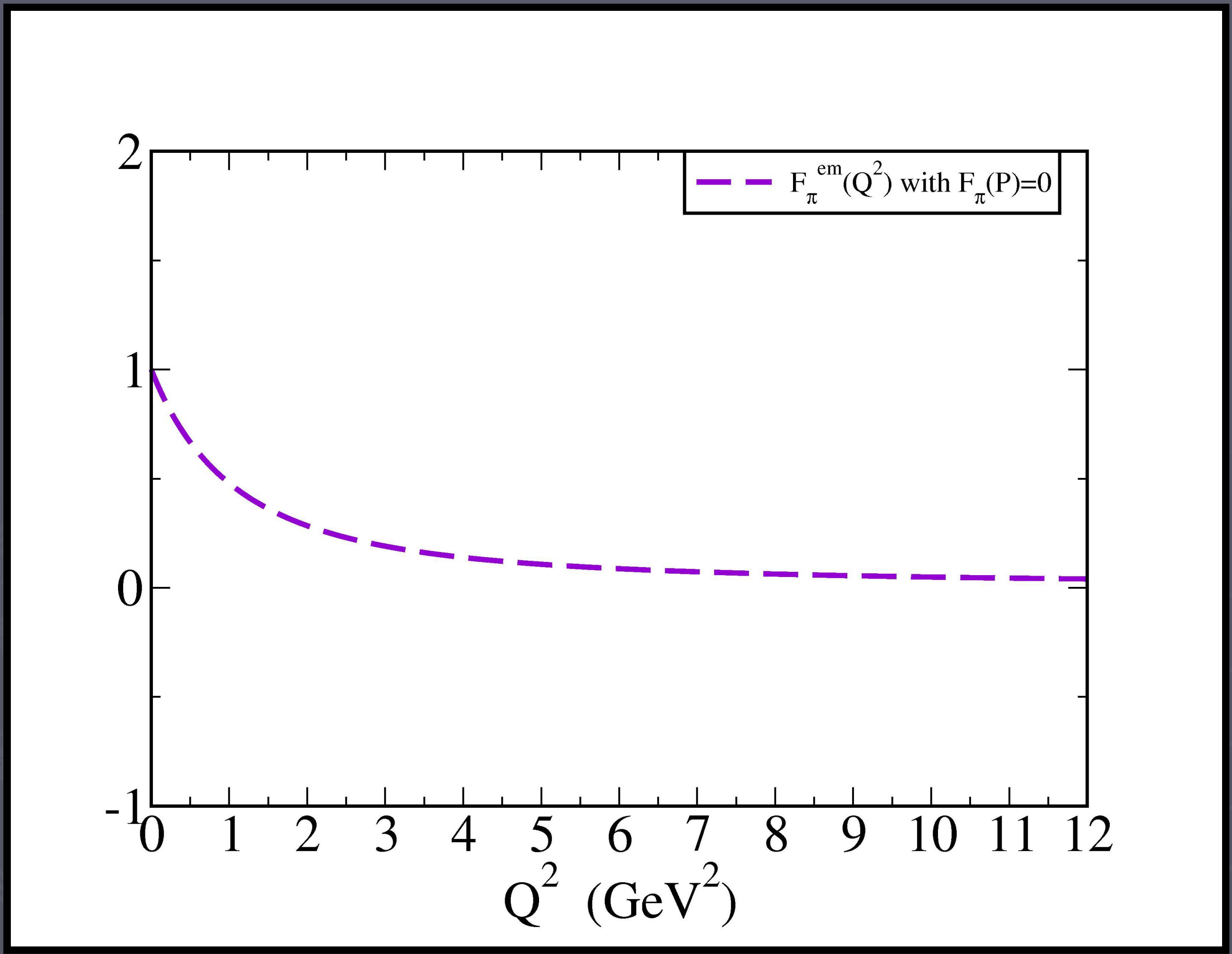
$$p_1^2 = (K - q/2)^2 = 0 = p_2^2 = (K + q/2)^2 \rightarrow K \cdot q = 0,\ K^2 = -q^2/4$$

so that we can choose $q = (0, 0, Q, 0),\ K = (0, 0, 0, iQ/2)$

In the impulse approximation, we have

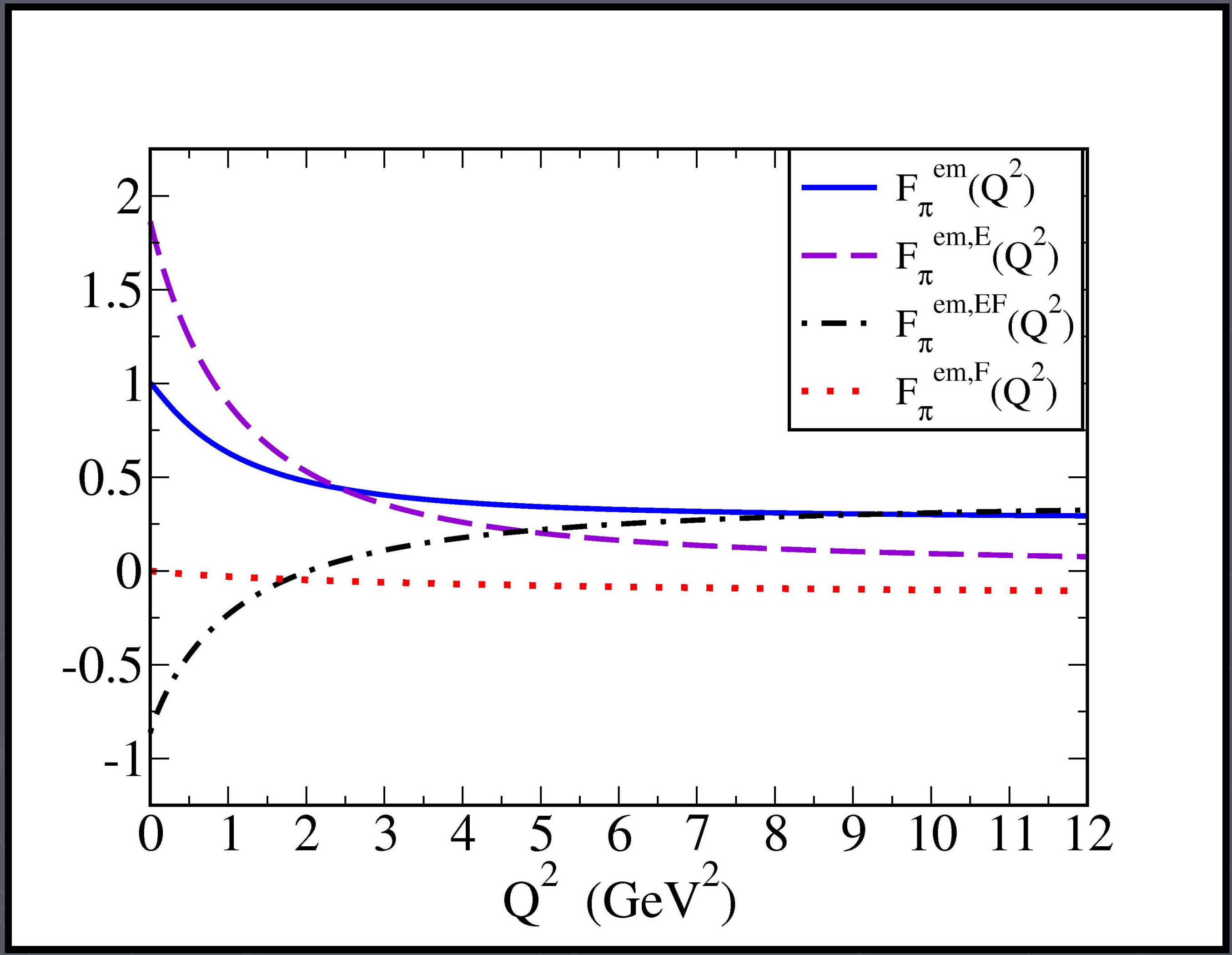
$$2K_\mu F_\pi^{\rm em} = \frac{3}{2\pi^2} \int_t^\Lambda {\rm Tr_D} \left[i\Gamma_\pi^C(-p_2)\, S(t + p_2)\, i\gamma_\mu\, S(t + p_1)\, i\Gamma_\pi^C(p_1)\, S(t) \right]$$

Pion Electromagnetic Form Factor $F_\pi^{em}(Q^2)$



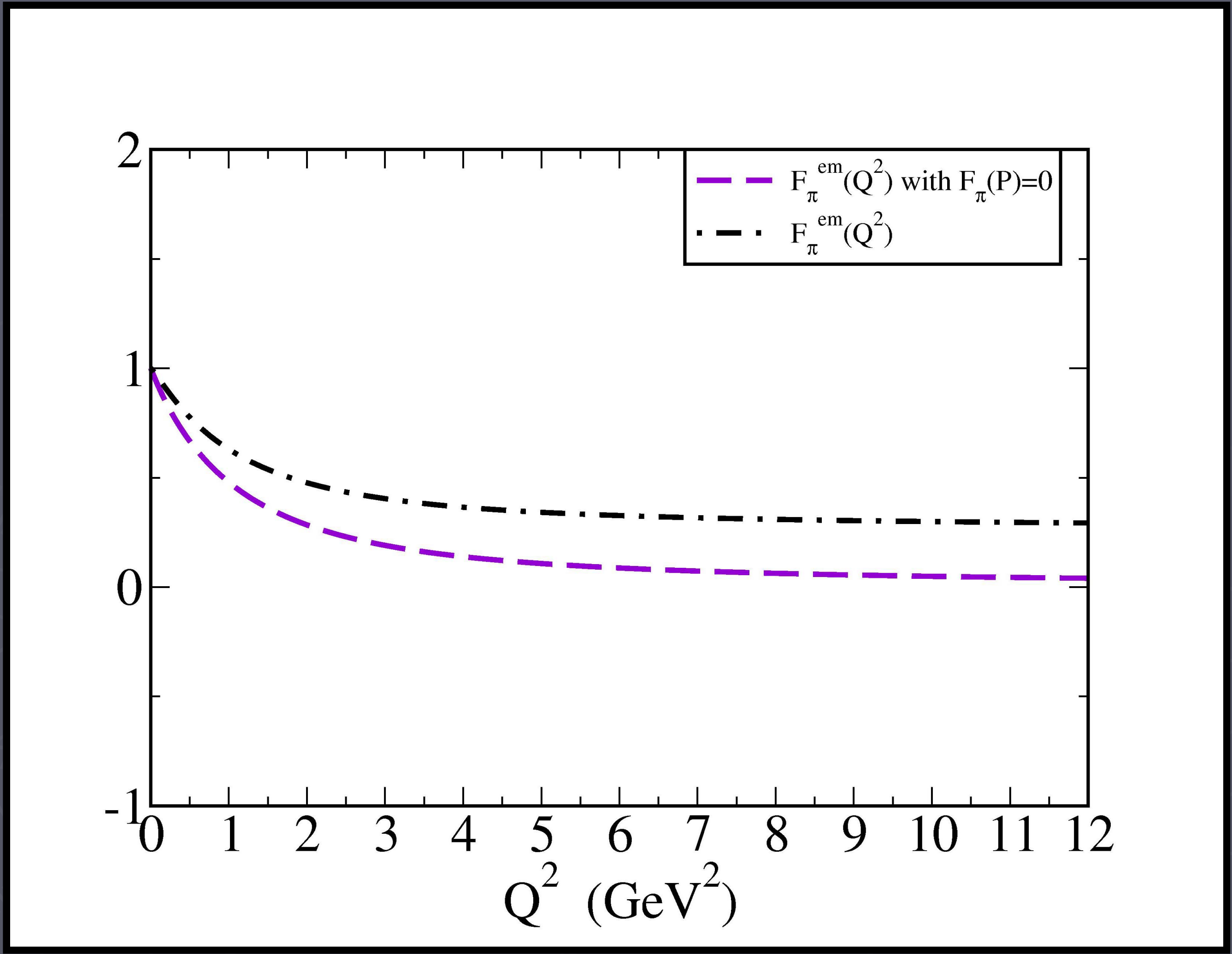
Without the pseudo vector component.

Pion Electromagnetic Form Factor $F_\pi^{em}(Q^2)$



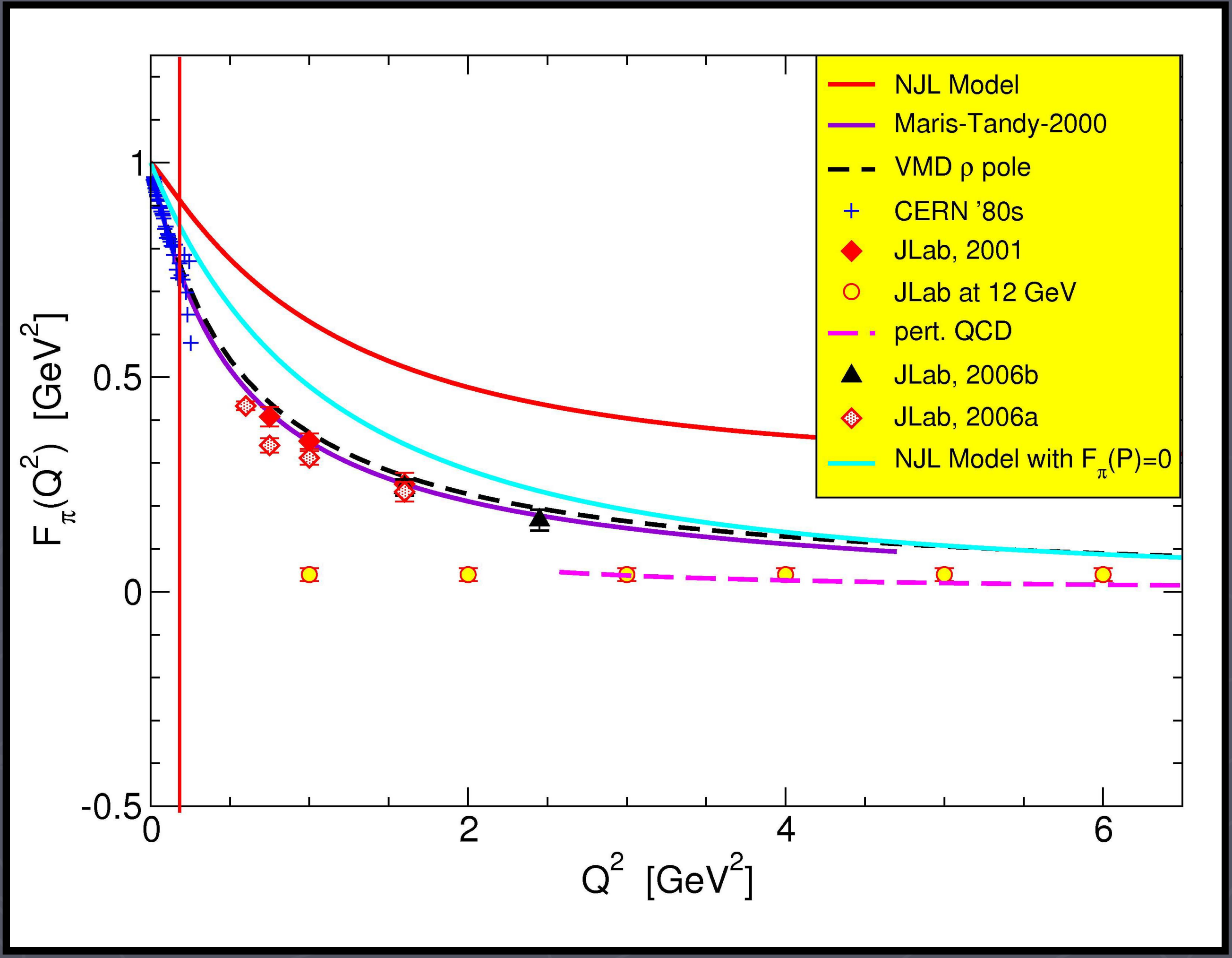
Interplay of pseudo scalar and pseudo vector components.

Pion Electromagnetic Form Factor $F_\pi^{em}(Q^2)$



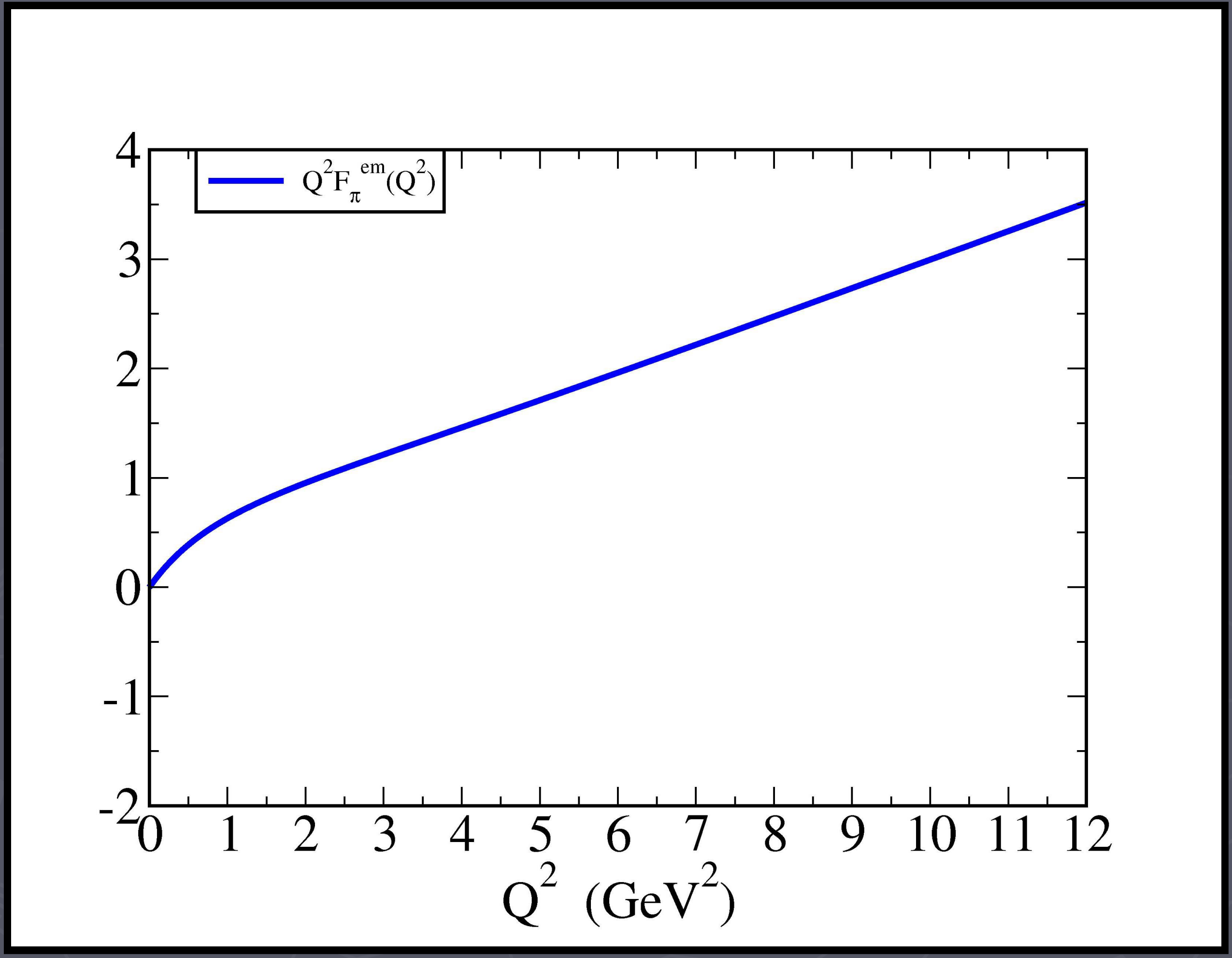
With and without the pseudo vector component.

Pion Electromagnetic Form Factor $F_\pi^{em}(Q^2)$



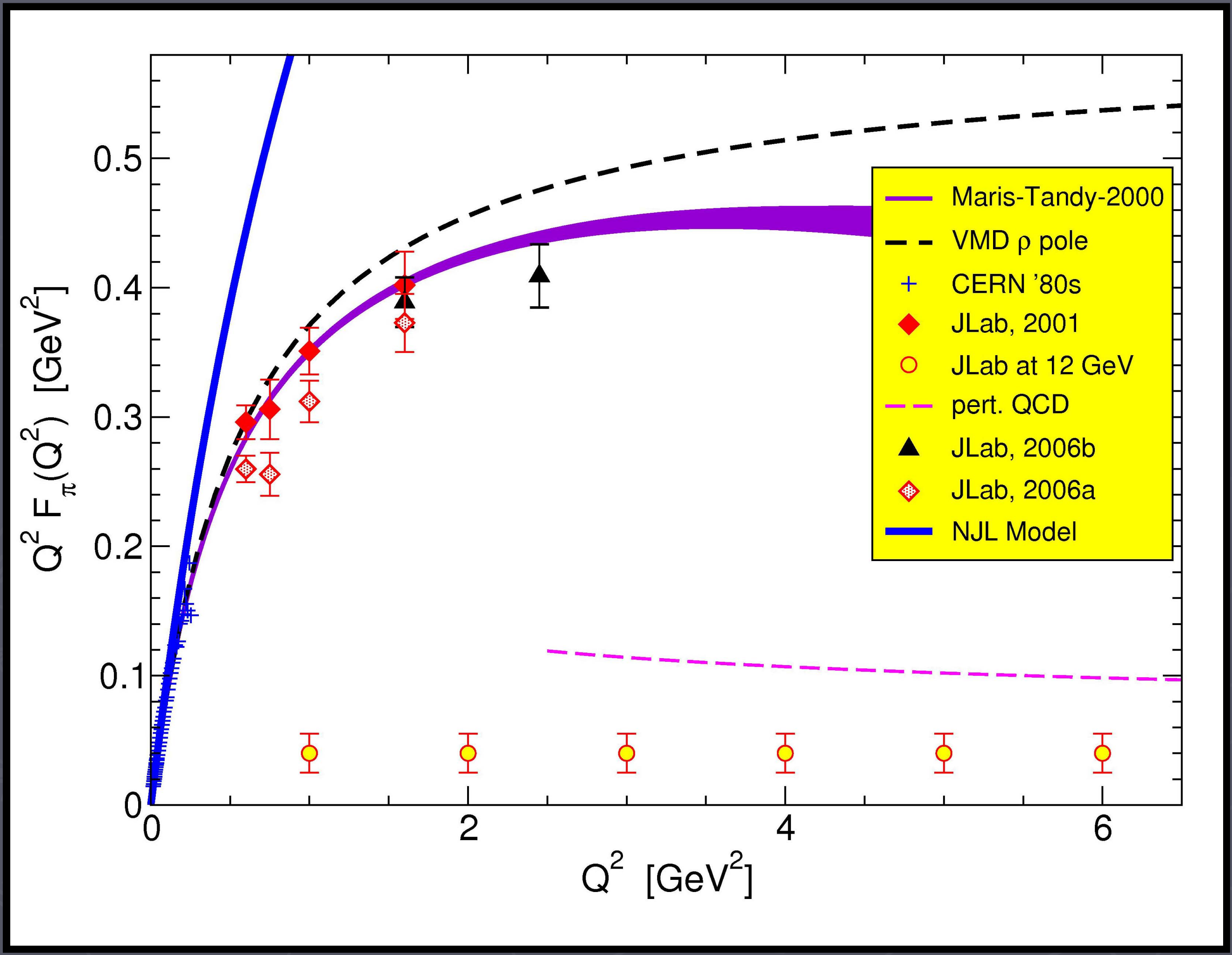
NJL model deviates 20% from MT-curve for Q^2=0.18 GeV^2.

Pion Electromagnetic Form Factor $F_\pi^{em}(Q^2)$



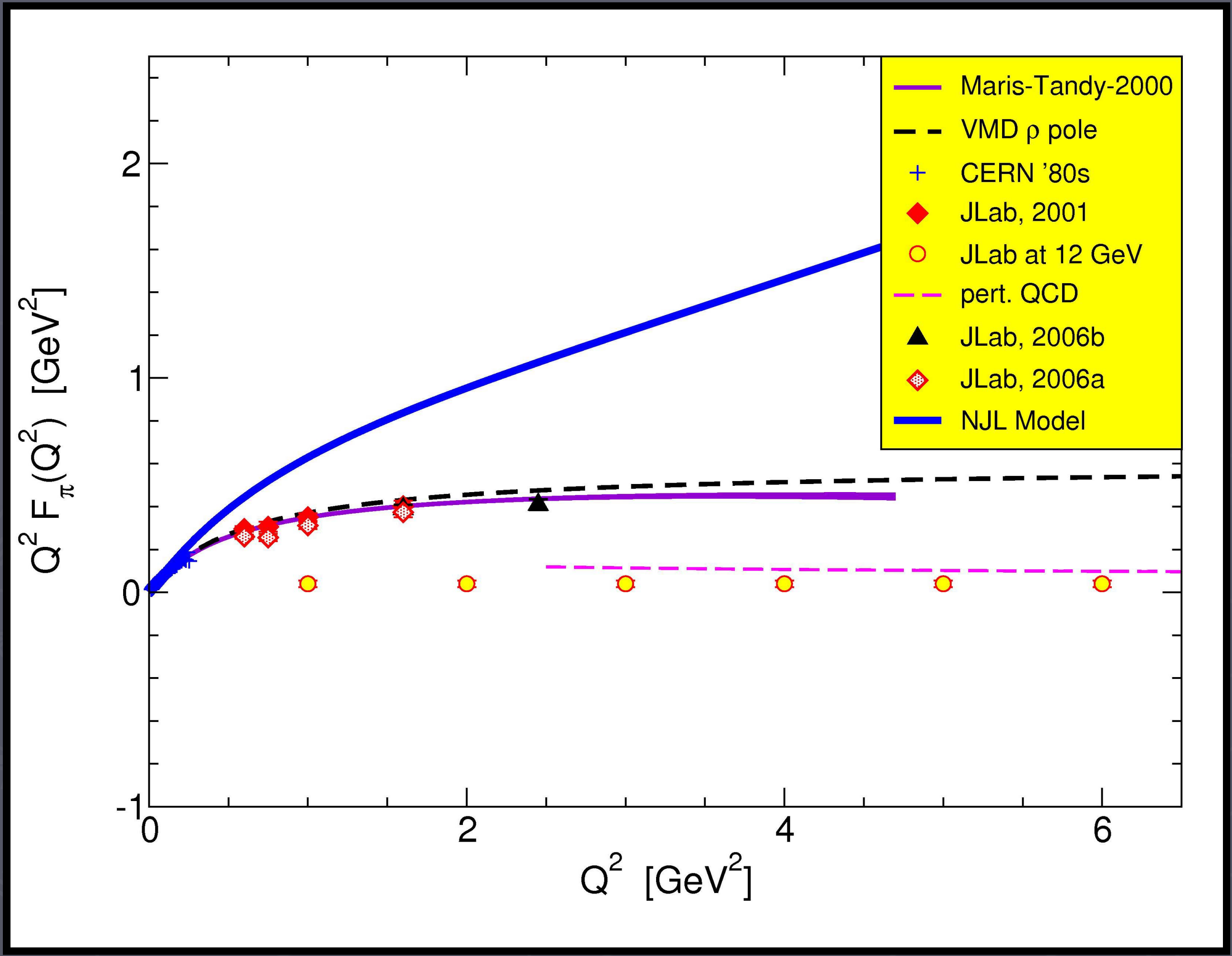
$Q^2 * F_\pi^{em}(Q^2)$ in the NJL Model

Pion Electromagnetic Form Factor $F_\pi^{em}(Q^2)$



Comparison with other models and experiments.

Pion Electromagnetic Form Factor $F_\pi^{em}(Q^2)$



Comparison with other models and experiments.

Conclusions

- We studied the electromagnetic pion form factor in an NJL model.
- We added the pseudo vector component in the description of the pion to study this NJL model.
- We implemented symmetry preserving regularization which ensures AV-WTI is satisfied for arbitrary values of P. Consequently GT-relations are respected and we obtain consistent result for the pion's leptonic decay constant.
- We then used this symmetry preserving NJL model to calculate the pion form factor and compared it with earlier theoretical and experimental results.