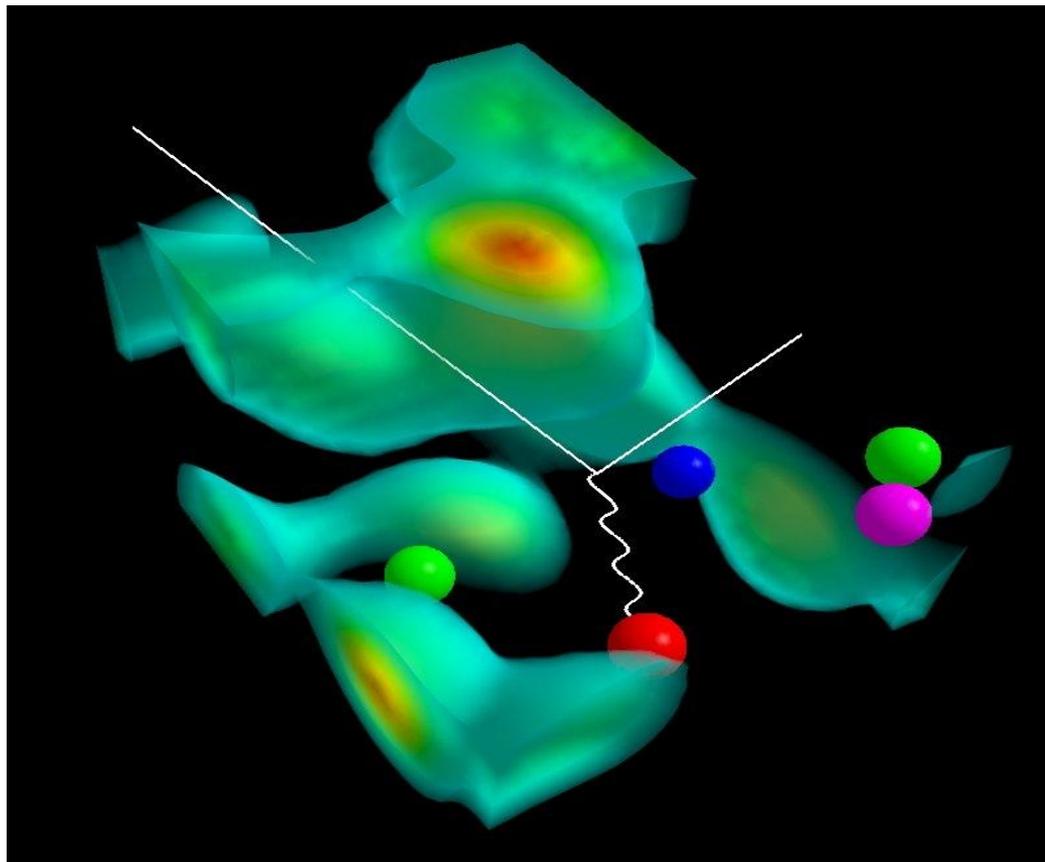


Issues Concerning the Spin of the Proton



Anthony W. Thomas
Workshop on QCD Bound States
Argonne National Lab : June 15th 2009

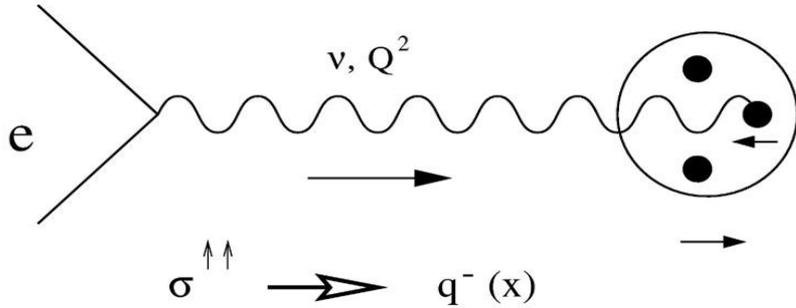


Outline

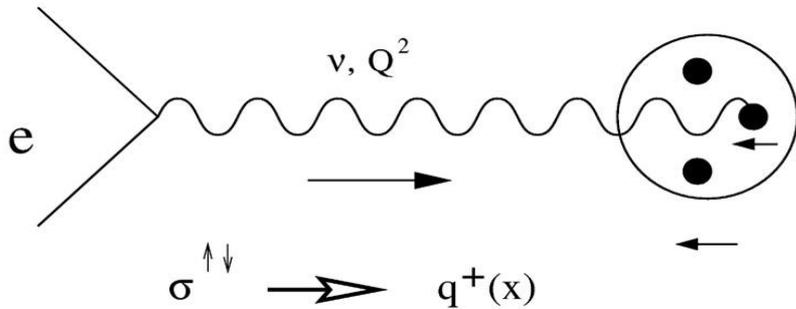
- **A reminder: the proton spin crisis**
- **Progress over the last 20 years**
- **The resolution of the problem**
 - one-gluon-exchange
 - the pion cloud
 - input from lattice QCD
- **Lattice QCD**
- **GPDs at JLab**
 - at 12 GeV
 - recent results



Spin Structure Function $g_1(x)$



$$A_{||} = \frac{N^{\uparrow\uparrow} - N^{\uparrow\downarrow}}{N^{\uparrow\uparrow} + N^{\uparrow\downarrow}}$$



$$x = Q^2 / 2 M_N v$$

= fraction of proton momentum carried by the quark

$$g_1(x) = \frac{1}{2} \sum_q e_q^2 [q^+(x) - q^-(x)] \approx \frac{1}{2} \sum_q e_q^2 \Delta q(x)$$

N.B. At Q^2 sufficiently high ($>2 \text{ GeV}^2$) the dependence on Q^2 is logarithmic and described by perturbative QCD (scaling)

The EMC “Spin Crisis”

Up to standard pQCD coefficients (series in $\alpha_s(Q^2)$):

$$\int_0^1 dx g_1^p(x) = \frac{(\Delta u - \Delta d)}{12} + \frac{(\Delta u + \Delta d - 2\Delta s)}{36}$$

$$+ \frac{(\Delta u + \Delta d + \Delta s)}{9}$$

(up to QCD radiative corrections)

g_A^3 : from β decay of n

naively fraction of proton ‘spin’ carried by its quarks

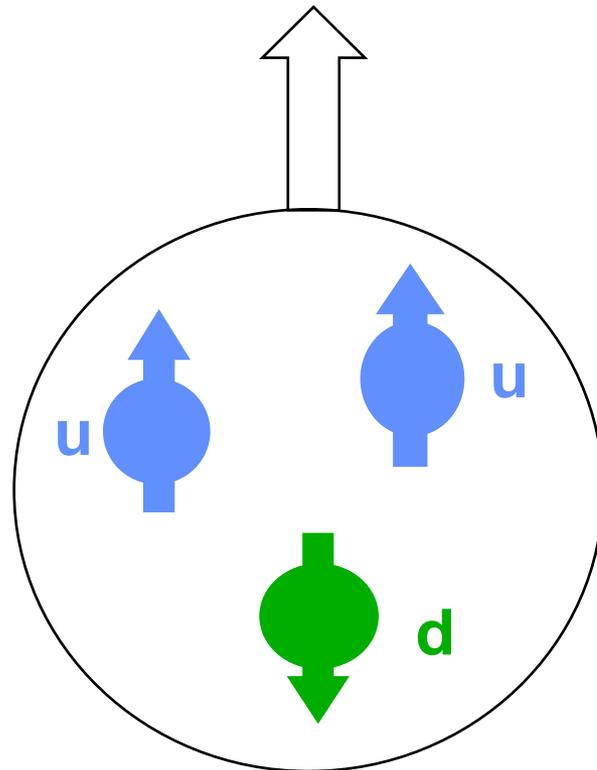
g_A^8 : hyperon β decay

$$\Sigma_{inv} \equiv \Sigma(Q^2 = \infty)$$

$\Delta u \equiv$ fraction of proton spin carried by u and anti-u quarks, etc..

What do we expect ?

Most quark models start with 3 quarks in the 1s-state of a confining potential: proton spin is ALL carried by its quarks $\Rightarrow \Sigma = 100\%$



N.B. Given low values of $m_{u,d}$ the quark motion is relativistic and lower Dirac components have spin down $\Rightarrow \Sigma \sim 65\%$

Ancient History of the Spin Crisis

- **EMC Spin Paper:** 22 Dec 87 - 19 May 88
- **Brodsky et al. Skyrme:** 22 Feb 88 - 19 May 88
- **Schreiber-Thomas CBM:** 17 May 88 - 8 Dec 88
- **Myhrer-Thomas OGE:** 13 June 88 - 1 Sept 88
(neither paper could explain reduction to only 14%!)
- **Efremov-Teryaev Anomaly:** 25 May 88
- **Altarelli-Ross Anomaly:** 29 June 88 - 29 Sept 88



A MEASUREMENT OF THE SPIN ASYMMETRY AND DETERMINATION OF THE STRUCTURE FUNCTION g_1 IN DEEP INELASTIC MUON-PROTON SCATTERING

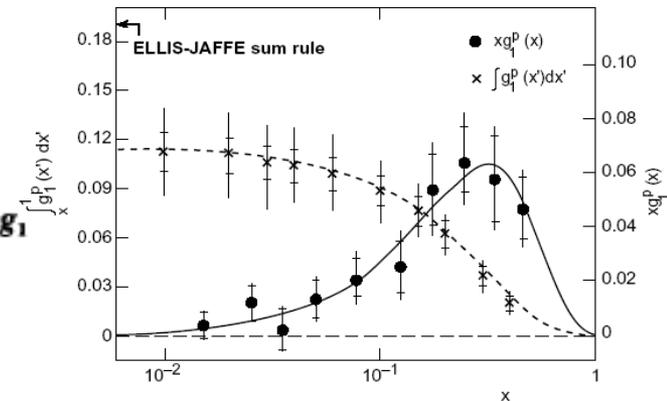
European Muon Collaboration

Aachen, CERN, Freiburg, Heidelberg, Lancaster, LAPP (Annecy), Liverpool, Marseille, Mons, Oxford,
Rutherford, Sheffield, Turin, Uppsala, Warsaw, Wuppertal, Yale

J. ASHMAN ^a, B. BADELEK ^{b,1}, G. BAUM ^{c,2}, J. BEAUFAYS ^d, C.P. BEE ^e, C BENCHOUK ^f,

(93 authors)

The spin asymmetry in deep inelastic scattering of longitudinally polarised muons by longitudinally polarised protons has been measured over a large x range ($0.01 < x < 0.7$). The spin-dependent structure function $g_1(x)$ for the proton has been determined and its integral over x found to be $0.114 \pm 0.012 \pm 0.026$, in disagreement with the Ellis–Jaffe sum rule. Assuming the validity of the Bjorken sum rule, this result implies a significant negative value for the integral of g_1 for the neutron. These values for the integrals of g_1 lead to the conclusion that the total quark spin constitutes a rather small fraction of the spin of the nucleon.



$$\Sigma = 14 \pm 3 \pm 10 \% :$$

i.e. 86% of spin of p NOT carried by its quarks

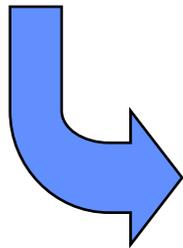
Ancient History of the Spin Crisis

- **EMC Spin Paper:** 22 Dec 87 - 19 May 88
- **Brodsky et al. Skyrme:** 22 Feb 88 - 19 May 88
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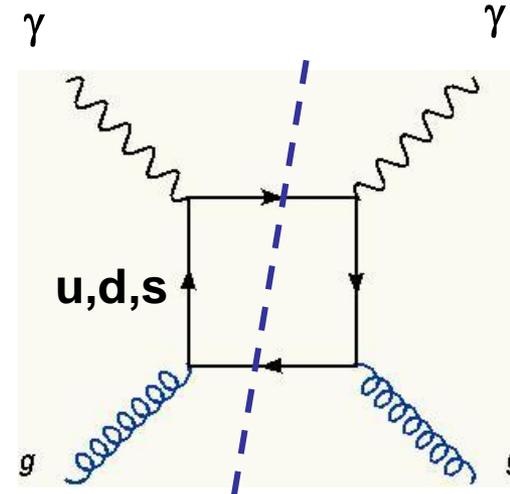
ОБЪЕДИНЕННЫЙ
ИНСТИТУТ
ЯДЕРНЫХ
ИССЛЕДОВАНИЙ
ДУБНА

E2-88-287

A.V.Efremov, O.V.Teryaev*

**SPIN STRUCTURE OF THE NUCLEON
AND TRIANGLE ANOMALY**

Submitted to "Nuclear Physics"



25 May 1988

THE ANOMALOUS GLUON CONTRIBUTION TO POLARIZED LEPTOPRODUCTION

G. ALTARELLI and G.G. ROSS ¹

CERN, CH-1211 Geneva 23, Switzerland

Received 29 June 1988

We show that, due to the anomaly, the gluon contribution to the first moment of the polarized proton structure function, as measured in deep inelastic scattering, is not suppressed by a power of the strong coupling evaluated at a large scale. As a result, the EMC result for the first moment of polarized proton electroproduction is consistent with a large quark spin component.

$$\Sigma_{\text{naive}} \rightarrow \Sigma_{\text{naive}} - \frac{N_f \alpha_s(Q^2) \Delta G(Q^2)}{2\pi}$$

and

QCD evolution $\Rightarrow \alpha_s(Q^2) \Delta G(Q^2)$ does not vanish as $Q^2 \rightarrow \infty$

and polarized gluons would resolve crisis

HOW MUCH?

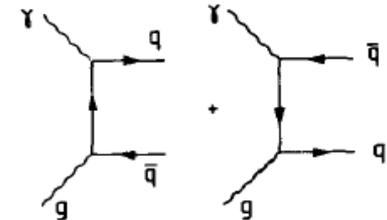


Fig. 1. Diagrams contributing to a finite mixing of order α_s between g_1^+ and the polarized gluon parton density.

Scale of the Gluon Contribution

At 3 GeV² $\alpha_s \sim 0.3$

and $N_f = 3$, so IF all of the

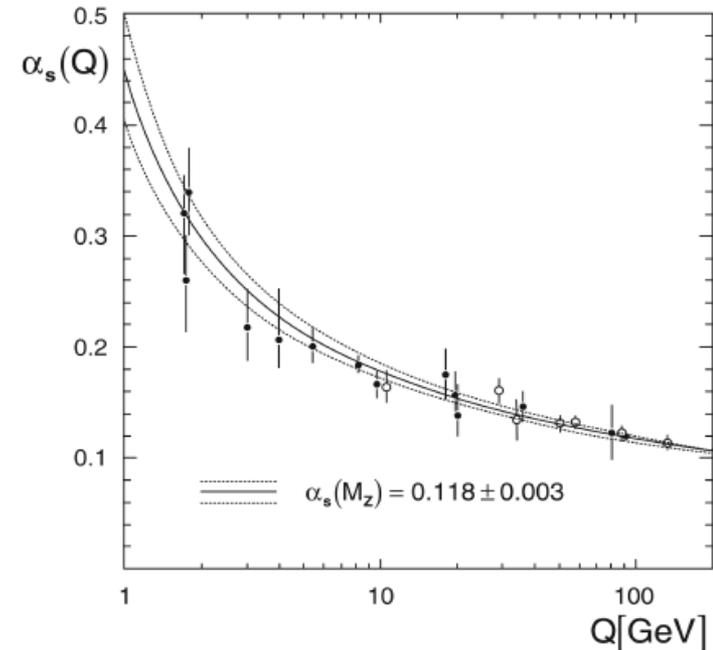
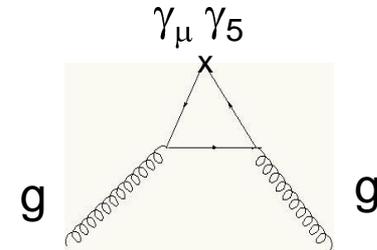
N spin carried by quarks is

cancelled by gluons:

$$\Delta G = + \frac{2 * \pi * 1}{3 * 0.3} \sim + 6$$

...actually $\Delta G \sim + 4$ better

- a truly remarkable result

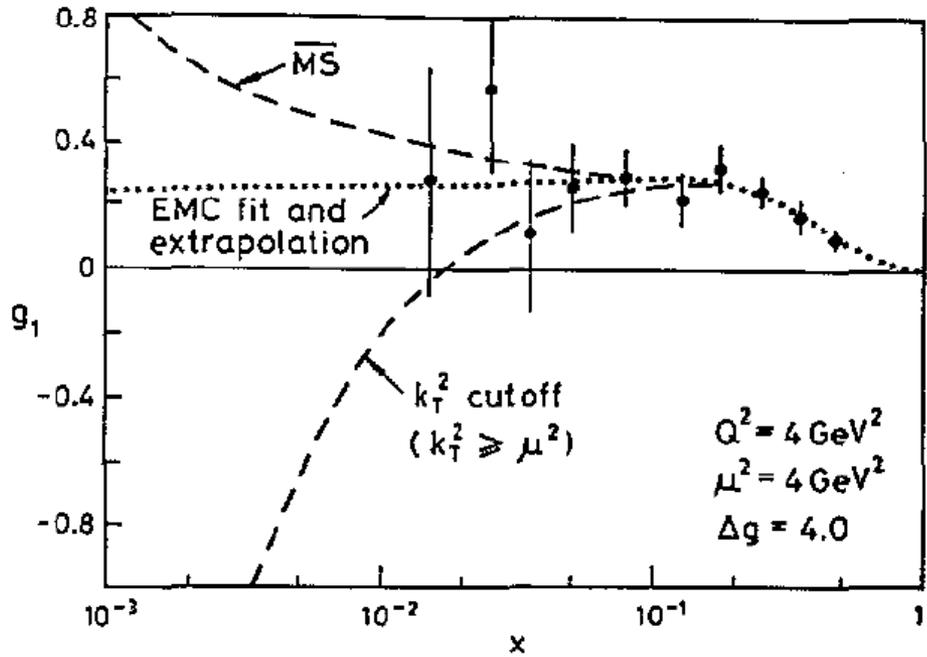


for which **no physical explanation was ever offered**

This spurred a tremendous experimental effort

- **DIS measurements of spin structure functions of polarized p, d, ^3He (and ^6Li) at SLAC, CERN, Hermes, JLab**
- **Direct search for high- p_T hadrons at Hermes, COMPASS, RHIC to directly search for effects of polarized glue in the p**
- **This effort has lasted the past 20 years, with great success**

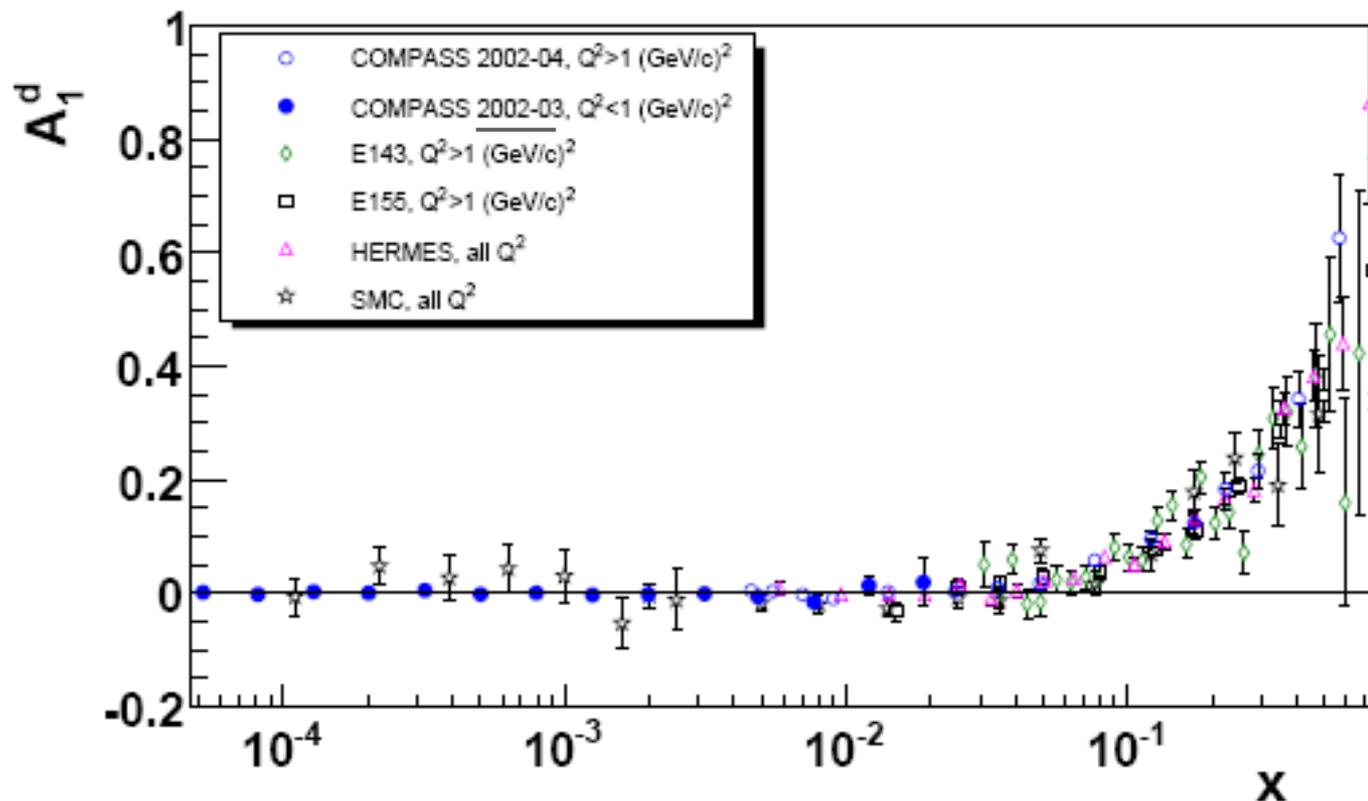
Effect of Photon-Gluon Fusion – with axial anomaly



Bass and Thomas,
 J. Phys. G19 (1993) 925

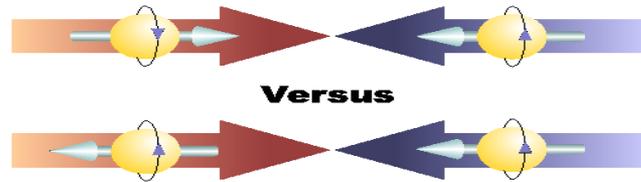
**COMPASS: at $x \sim 3 \times 10^{-3}$: $|x g_1^d| < 0.001$
 and hence $|g_1^d| < 0.3$, c.f. >1.0 with $\Delta G = 4$
 and data at lower x makes it much worse**

COMPASS compared with earlier data

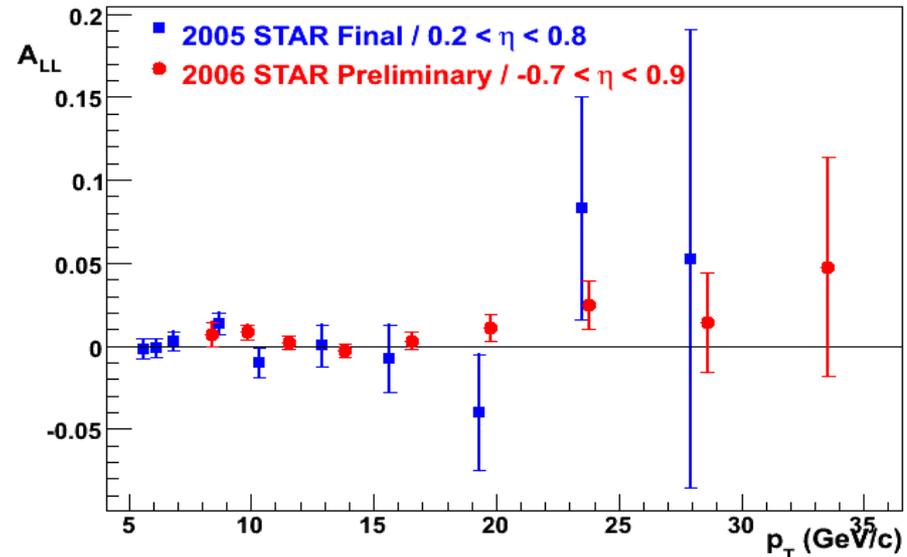
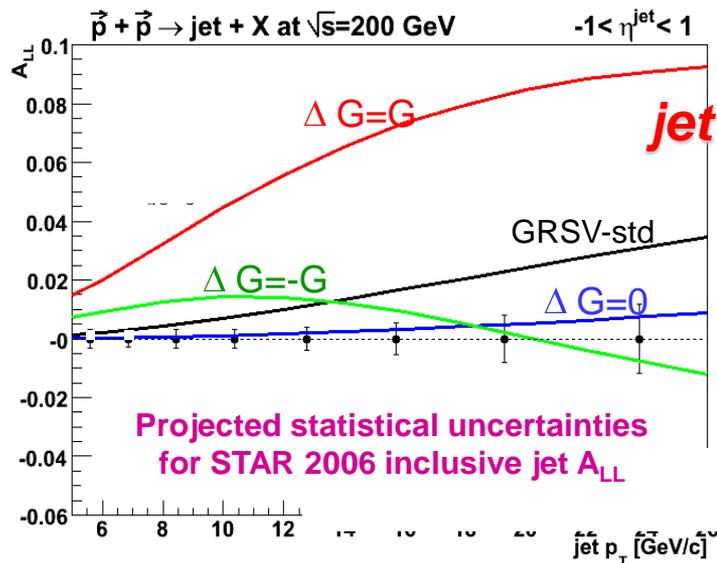


- very good agreement with SMC (the only other experiment at low x)
- factor 10–20 improvement of statistical errors compared to SMC

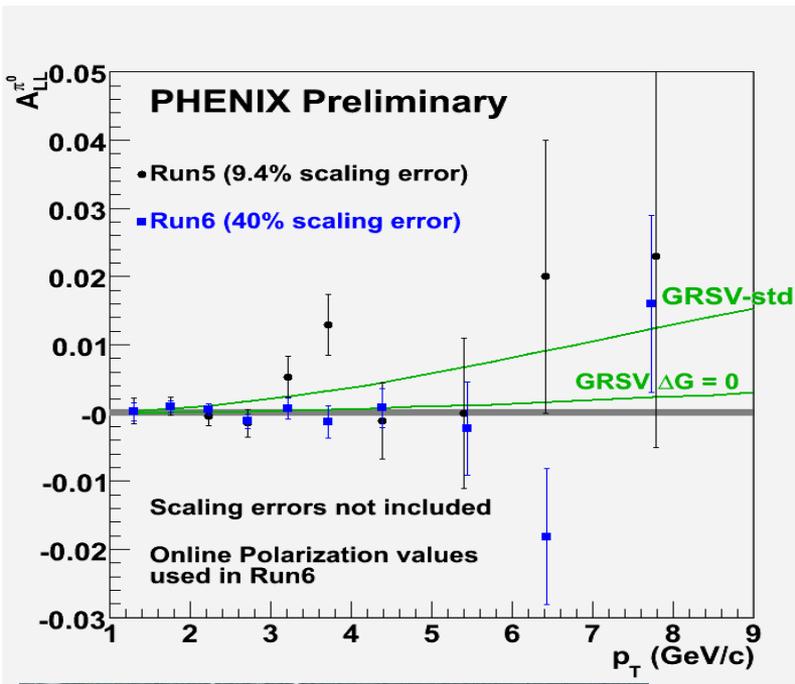
Latest STAR result - Sarsour DNP Oct 07



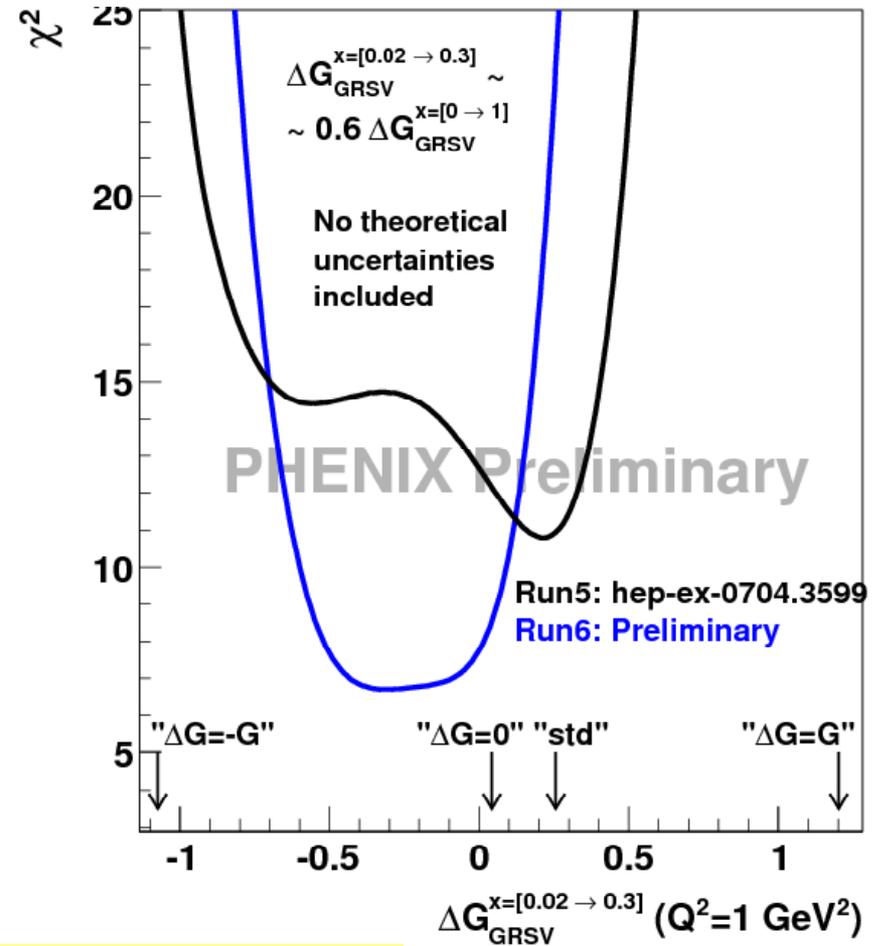
- NLO pQCD describes inclusive jet cross section at RHIC
- **Within GRSV framework, 2005 results constrain ΔG to less than 65% of the proton spin with 90% confidence**
- Significant increase in precision in Run 2006 data provides even stronger constraints on gluon polarization



Latest PHENIX Result: From A_{LL} to ΔG



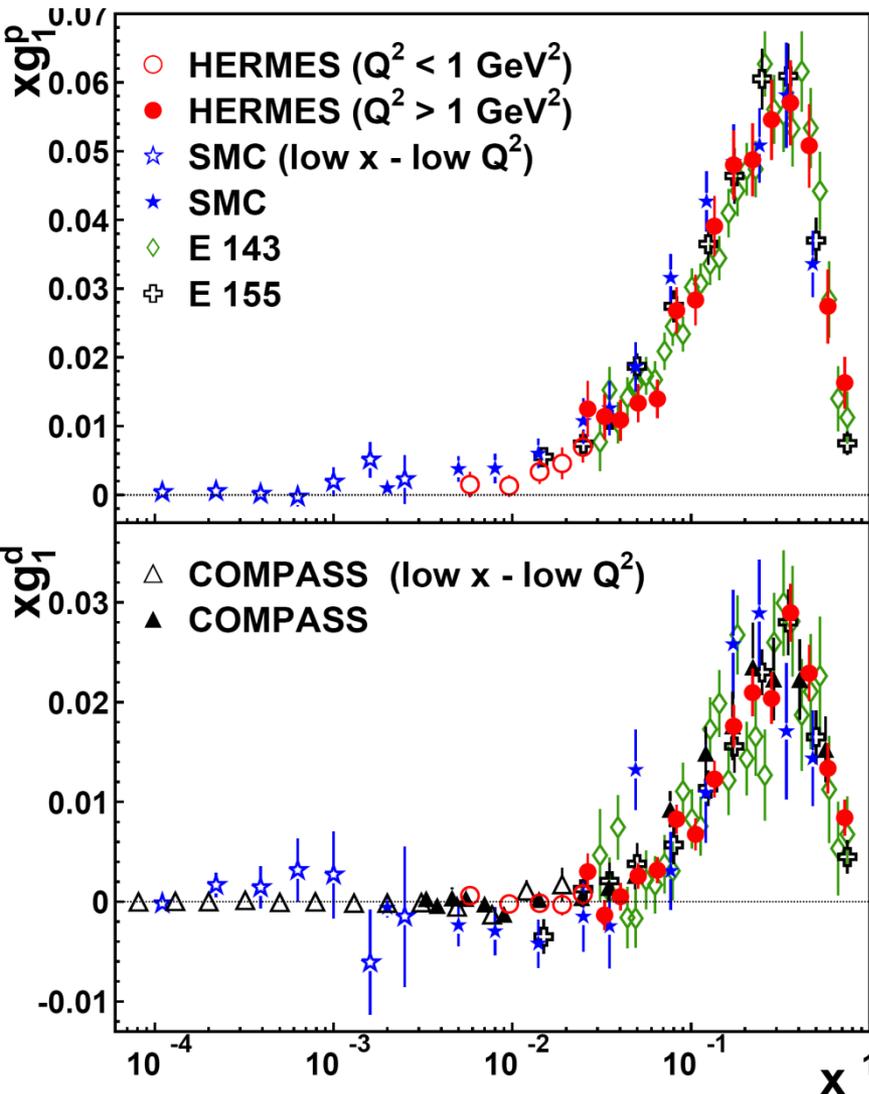
Calc. by W.Vogelsang and M.Stratmann



“std” scenario, $\Delta G(Q^2=1\text{GeV}^2)=0.4$, is excluded by data on >3 sigma level



From HERMES fit: similar results



$$a_0 = \begin{matrix} & \text{(theory)} & \text{(exp)} & \text{(evol)} \\ \mathbf{0.330} & \mathbf{0.011} & \mathbf{0.025} & \mathbf{0.028} \end{matrix}$$

Bradamante Erice 0907



$$a_0 = 0.33 \quad 0.03(\text{stat}) \quad 0.05(\text{sys+evol})$$

$$\Sigma = a_0 \text{ in } \overline{\text{MS}}$$

Where is the Spin of the proton?

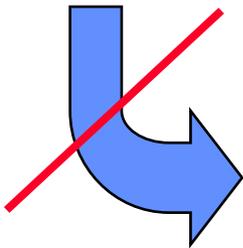


- **Modern data yields:**
 $\Sigma = 0.33 \pm 0.03 \pm 0.05$

(c.f. $0.14 \pm 0.03 \pm 0.10$ originally)
- **In addition, there is little or no polarized glue**
 - COMPASS: $g^D_1 = 0$ to $x = 10^{-4}$
 - A_{LL} (π^0 and jets) at PHENIX & STAR $\rightarrow \Delta G \sim 0$
 - Hermes, COMPASS and JLab: $\Delta G / G$ small
- **Hence: axial anomaly plays at most a small role in explaining the spin crisis**
- **Return to alternate explanation lost in 1988 in rush to explore the anomaly**

Ancient History of the Spin Crisis

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One-Gluon-Exchange Correction

PHYSICAL REVIEW D

VOLUME 38, NUMBER 5

1 SEPTEMBER 1988

Rapid Communications

The Rapid Communications section is intended for the accelerated publication of important new results. Since manuscripts submitted to this section are given priority treatment both in the editorial office and in production, authors should explain in their submittal letter why the work justifies this special handling. A Rapid Communication should be no longer than 3½ printed pages and must be accompanied by an abstract. Page proofs are sent to authors, but, because of the accelerated schedule, publication is not delayed for receipt of corrections unless requested by the author or noted by the editor.

Spin structure functions and gluon exchange

F. Myhrer

Department of Physics and Astronomy, University of South Carolina, Columbia, South Carolina 29208

A. W. Thomas

*Department of Physics and Mathematical Physics, University of Adelaide, Adelaide, South Australia 5000, Australia
and Department of Theoretical Physics, Oxford University, Oxford OX1 3NP, Oxfordshire, England**

(Received 13 June 1988)

Two-quark correlations due to gluon exchange give corrections to both the proton and neutron spin-dependent structure functions in the Bjorken sum rule. They are found to be as large as the pionic corrections in the cloudy bag model of the nucleon. While still not enough to explain the result published recently by the European Muon Collaboration, it is compatible with the reanalysis of the data by Close and Roberts.



SU(6) violations due to one-gluon exchange

H. Høgaasen

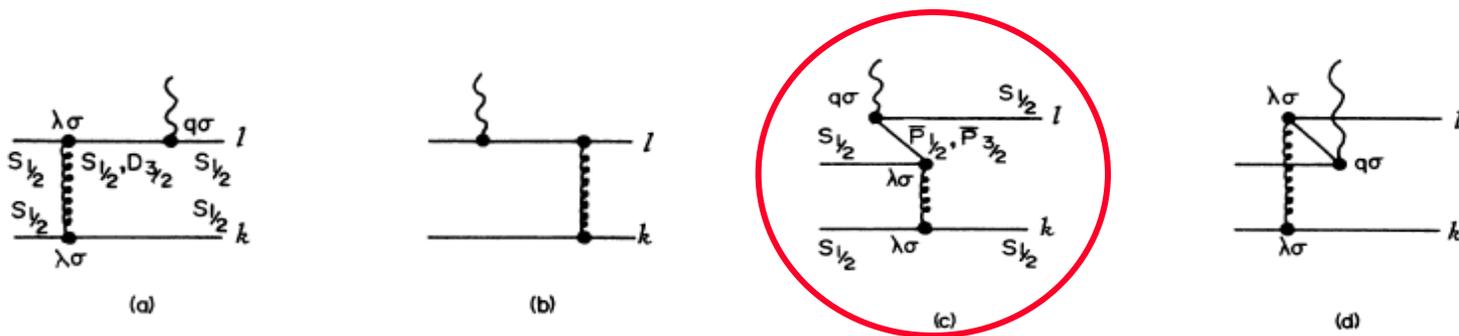
Fysisk Institutt, University of Oslo, Blindern, 0316 Oslo 3, Norway

F. Myhrer

Department of Physics, University of South Carolina, Columbia, South Carolina 29208

(Received 26 October 1987)

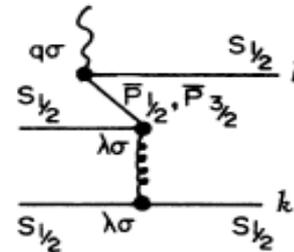
The one-gluon-exchange corrections to the baryon magnetic moments and the weak semileptonic decays are shown to have the correct two-body operator in order to explain recent data. An explicit model calculation using a mode sum for the quark propagator is then performed. In this model calculation the two lowest states dominate the corrections. This value of SU(6) breaking explains the measured ratio $\Sigma^- \rightarrow ne\bar{\nu}/\Lambda \rightarrow pe\bar{\nu}$ as well as why $\mu_{\Xi^-} < \mu_{\Lambda}$ and it restores $\mu_p/\mu_n \approx -\frac{3}{2}$ in chiral bag models.



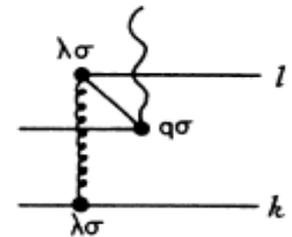
Intermediate quark state contributing <i>M</i>	Intermediate quark energy	Intermediate quark energy		<i>M</i>	Intermediate quark energy	
		$10^4 \Delta\mu$	$10^4 \Delta g_A$		$10^4 \Delta\mu$	$10^4 \Delta g_A$
$S'_{1/2}$	5.40/ <i>R</i>	22	32	8.58/ <i>R</i>	1.0	2.2
$D_{3/2}$	5.12/ <i>R</i>	8	12	8.41/ <i>R</i>	0.4	0.8
$\bar{P}_{1/2}$	3.81/ <i>R</i>	730	-275	7.00/ <i>R</i>	-6.7	7.0
$\bar{P}_{3/2}$	3.20/ <i>R</i>	1349	-332	6.76/ <i>R</i>	-6.1	6.0
Sum		2109	-563		-11.4	16.0

One-Gluon-Exchange Correction

- Further reduces the fraction of spin carried by the quarks in the bag model (naively 0.65)
- $\Sigma \rightarrow \Sigma - 3G$; with $G \sim 0.05$
 $\Sigma \rightarrow 0.65 - 0.15 = 0.5$
- Effect is to transfer quark spin to quark (relativity) and anti-quark (OGE) **orbital angular momentum**



(c)



(d)

The Pion Cloud of the Nucleon

Volume 215, number 1

PHYSICS LETTERS B

8 December 1988

SPIN DEPENDENT STRUCTURE FUNCTIONS IN THE CLOUDY BAG MODEL

A.W. SCHREIBER AND A.W. THOMAS

*Department of Physics and Mathematical Physics, University of Adelaide,
North Terrace, Adelaide, South Australia 5000, Australia*

Received 17 May 1988

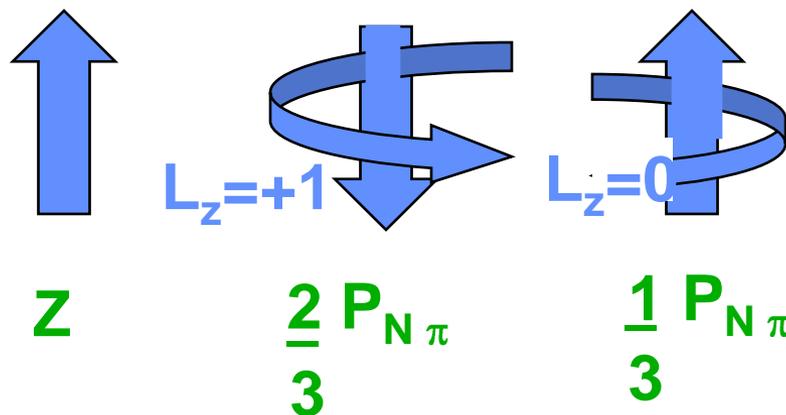
We derive expressions for the integrals of the spin dependent structure functions $g_1(x)$ for the proton and the neutron in the context of the cloudy bag model. We find that the neutron contributes 5–10% to the Bjorken sum rule, while there is a corresponding decrease for the proton's contribution. It is difficult to reconcile these results with those reported in a recent experiment.



Effect of the Pion Cloud

- Probability to find a bare N is $Z \sim 70\%$

- Biggest Fock Component is $N \pi \sim 20-25\%$ and $2/3$ of time N spin points down



- Next biggest is $\Delta \pi \sim 5-10\%$

- To this order (i.e. including terms which yield LNA and NLNA contributions):

- Spin gets renormalized by a factor :

$$Z - \frac{1}{3} P_{N\pi} + \frac{15}{9} P_{\Delta\pi} \sim 0.75 - 0.8$$

$$\Rightarrow \Sigma = 0.65 \rightarrow 0.49 - 0.52$$

Support for Pion Cloud Picture

- Most spectacular example is the prediction* of $\bar{d} > \bar{u}$, because of the pion cloud ($p \rightarrow n \pi^+$)

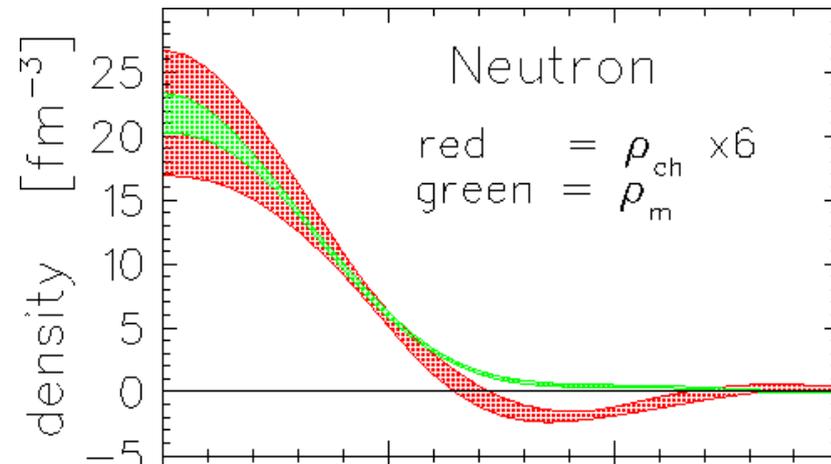
$$\int_0^1 dx [\bar{d} - \bar{u}] = 2 P_{N\pi} / 3 - P_{\Delta\pi} / 3$$
$$\in 0.11 - 0.15$$

(in excellent agreement with latest data)

J.J. Kelly

* Thomas, Phys. Lett. B126 (1983) 97

- Charge distribution of the neutron
- Natural understanding of quark mass dependence of data from lattice QCD (later)



Can one add OGE and Pion Corrections?

Tsushima, Kubodera et al., *assumed* this already in 1989
(23rd Yamada Conference)

- Prime phenomenological need for OGE interaction is the hyperfine splitting of N and Δ masses, Λ and Σ masses, etc. – i.e. hadron spectroscopy
- In early days of chiral models believed some of this hyperfine splitting came from pion self-energy differences
- Maybe double counting to include correction to Σ from both pions and OGE??
- Modern understanding *NO*: from analysis of data in quenched (QQCD) and full QCD, from Lattice QCD - implies 50 MeV (or less) of $m_{\Delta} - m_N$ in this way

Young et al., *Phys. Rev. D*66 (2002) 094507

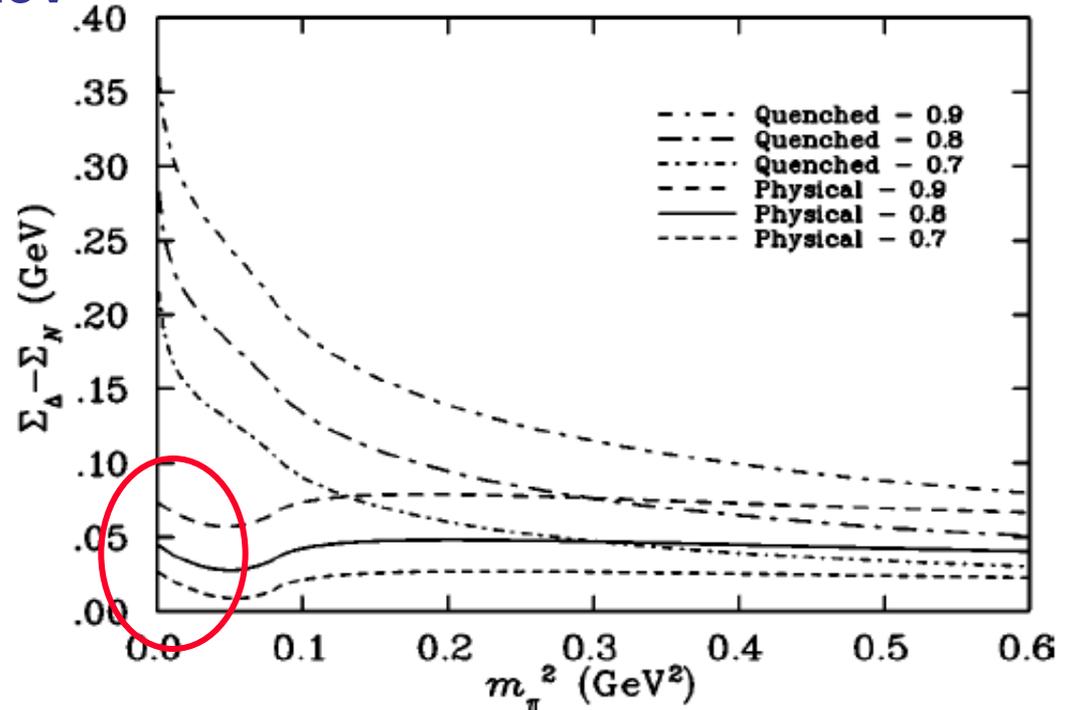


Nucleon - Δ Splitting

Lattice analysis

\Rightarrow pions give 40 ± 20 MeV

PHYSICAL REVIEW D **66**, 094507 (2002)

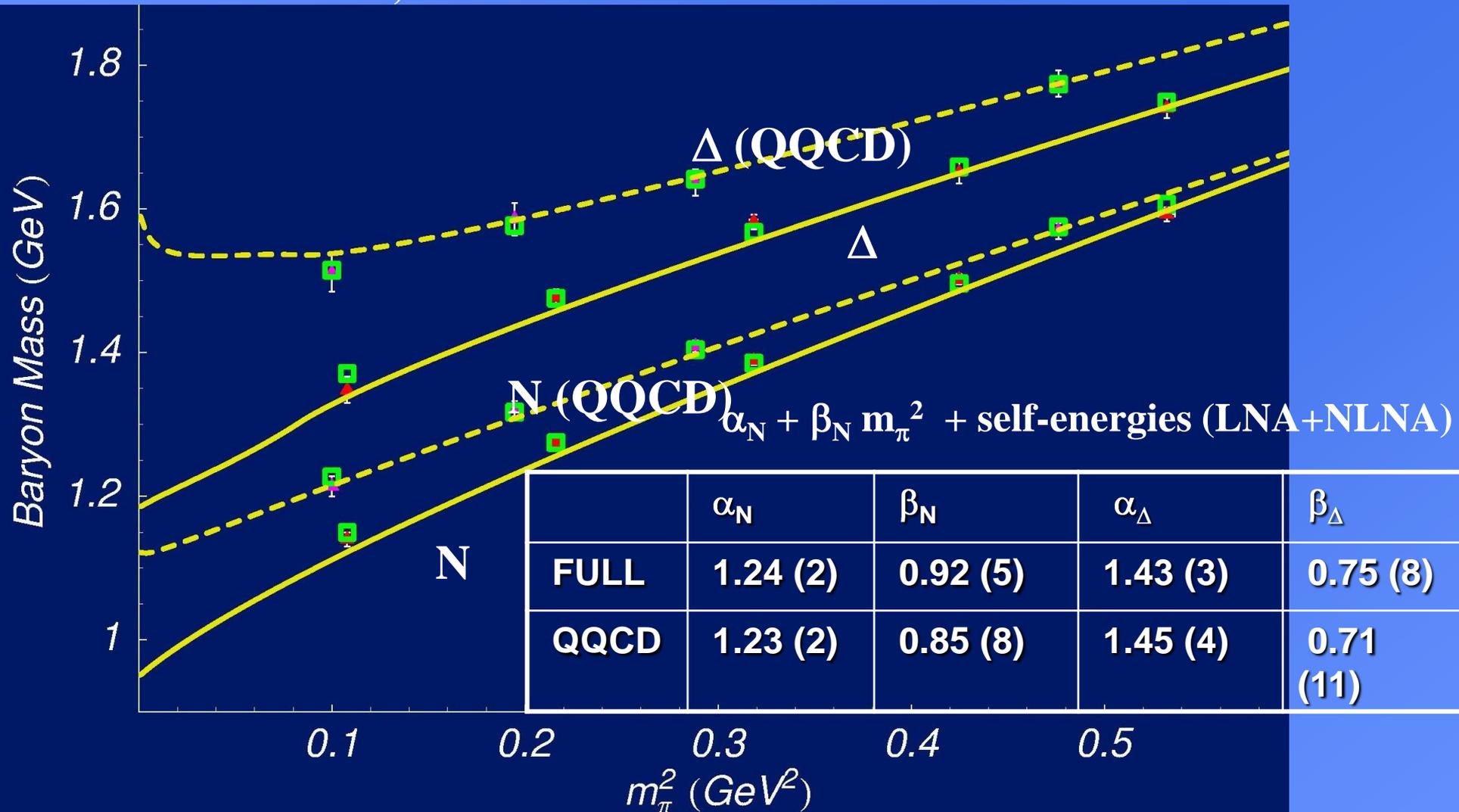


- Hence most of the N- Δ splitting comes from OGE – as in most quark models

- Thus the value of α_s used in the bag model calculation of the exchange current correction is more or less unchanged

- and... one can add the pion and OGE corrections to the spin sum-rule

- Lattice data (from **MILC Collaboration**) : red triangles
- Green boxes: fit evaluating σ 's on same finite grid as lattice
- Lines are exact, continuum results



Young *et al.*, hep-lat/0111041; Phys. Rev. D66 (2002) 094507

Thomas Jefferson National Accelerator Facility



Final Result for Quark Spin

$$\begin{aligned}\Sigma &= (Z - P_{N\pi}/3 + 5 P_{\Delta\pi}/3) \times (0.65 - 3 G) \\ &= (0.7, 0.8) \times (0.65 - 0.15) = (0.35, 0.40)\end{aligned}$$

c.f. Experiment: $0.33 \pm 0.03 \pm 0.05$

- ALL effects, relativity and OGE and the pion cloud
swap quark spin for valence orbital angular momentum
and anti-quark orbital angular momentum

(>60% of the spin of the proton)

Myhrer & Thomas, hep-ph/0709.4067

The Balance Sheet – fraction of total spin

	$L_{u+u\bar{}}$	$L_{d+d\bar{}}$	Σ
Non-relativistic			1.0
Relativity (e.g. Bag)	0.46	-0.11	0.65
Plus OGE	0.52	-0.02	0.50
Plus pion	0.50	0.12	0.38

At model scale: $L_u + S_u = 0.25 + 0.42 = 0.67 = J_u$
 $: L_d + S_d = 0.06 - 0.22 = -0.16 = J_d$

LHPC Lattice Results

- At first glance shocking : $L^u \sim -0.1$ and $L^d \sim +0.1$
(c.f. $+0.25$ and $+0.06$ in our “resolution”)
- N.B. Disconnected terms missing \rightarrow no anomaly, sea wrong

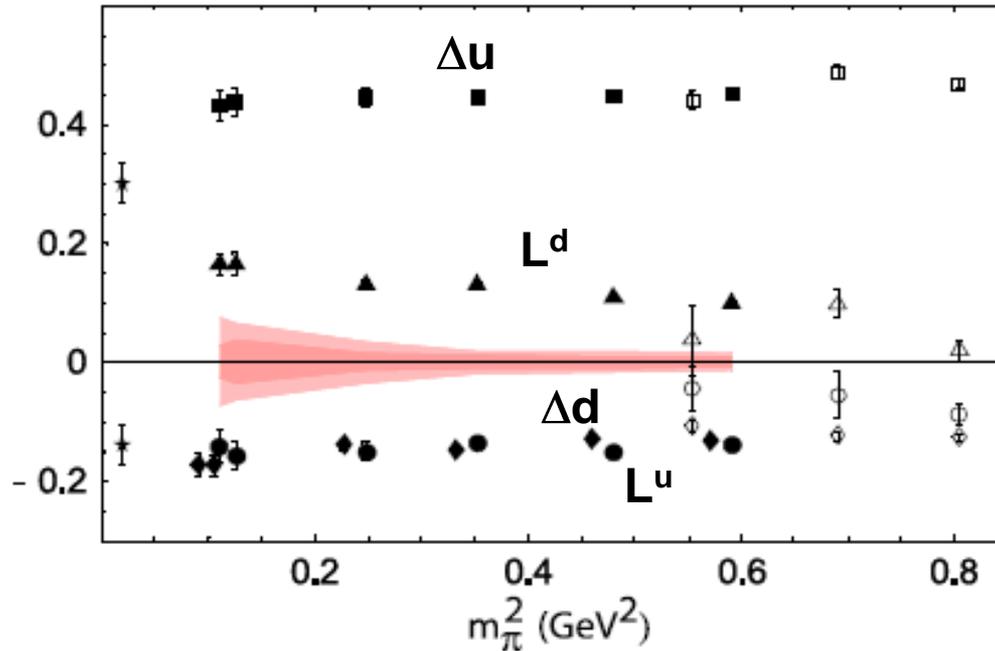


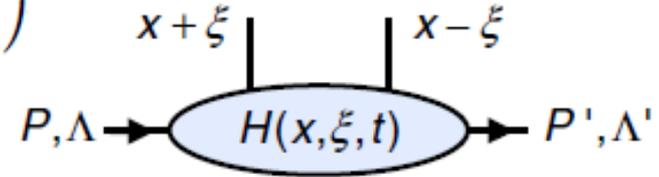
Figure 16: Nucleon spin decomposition by flavor. Squares denote $\Delta\Sigma^u/2$, diamonds denote $\Delta\Sigma^d/2$, triangles denote L^u , and circles denote L^d .

LHPC: [hep-lat/0610007](https://arxiv.org/abs/hep-lat/0610007)

Calculation: Hägler et al. (LHPC) PR D77 094502 (2008)

$$\mathcal{O}_\Gamma(x) = \int \frac{d\lambda}{4\pi} e^{i\lambda x} \bar{q}\left(\frac{-\lambda n}{2}\right) \Gamma \mathcal{P} e^{-ig \int_{-\lambda/2}^{\lambda/2} d\alpha n \cdot A(\alpha n)} q\left(\frac{\lambda n}{2}\right)$$

$$\langle P', \Lambda' | \mathcal{O}_\mu(x) | P, \Lambda \rangle = \langle \langle \mu \rangle \rangle H(x, \xi, t) + \frac{n_\mu \Delta_\nu}{2m} \times \langle \langle i\sigma^{\mu\nu} \rangle \rangle E(x, \xi, t),$$



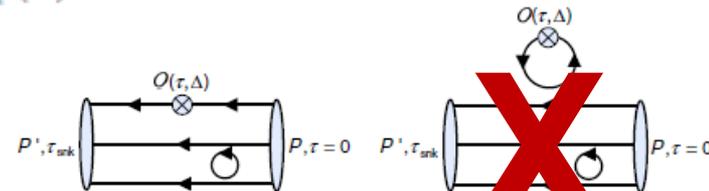
OPE \Rightarrow

$$\langle P' | \mathcal{O}^{\mu_1} | P \rangle = \langle \langle \gamma^{\mu_1} \rangle \rangle A_{10}(t) + \frac{i}{2m} \langle \langle \sigma^{\mu_1 \alpha} \rangle \rangle \Delta_\alpha B_{10}(t),$$

$$\langle P' | \mathcal{O}^{\{\mu_1 \mu_2\}} | P \rangle = \bar{P}^{\{\mu_1} \langle \langle \gamma^{\mu_2\} \rangle \rangle A_{20}(t) + \frac{i}{2m} \bar{P}^{\{\mu_1} \langle \langle \sigma^{\mu_2\} \alpha \rangle \rangle \Delta_\alpha B_{20}(t) + \frac{1}{m} \Delta^{\{\mu_1} \Delta^{\mu_2\}} C_{20}(t),$$

where: $\mathcal{O}_{[\gamma_5]}^{\{\mu_1 \dots \mu_n\}} = \bar{q}(0) \gamma^{\{\mu_1} [\gamma_5] i \vec{D}^{\mu_2} \dots i \vec{D}^{\mu_n\} q(0)$

$$\mathbf{J}^q = [\mathbf{A}_{20}(0) + \mathbf{B}_{20}(0)] / 2$$

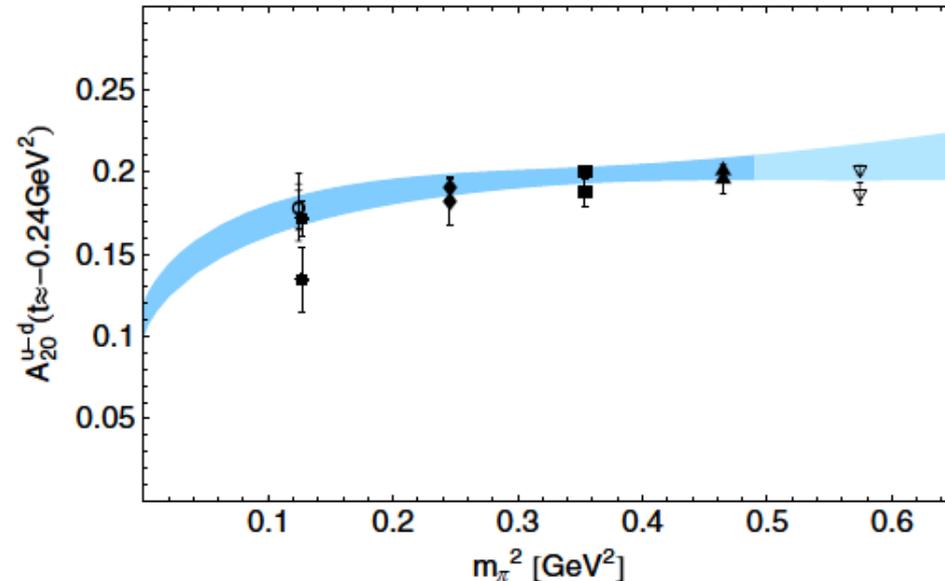
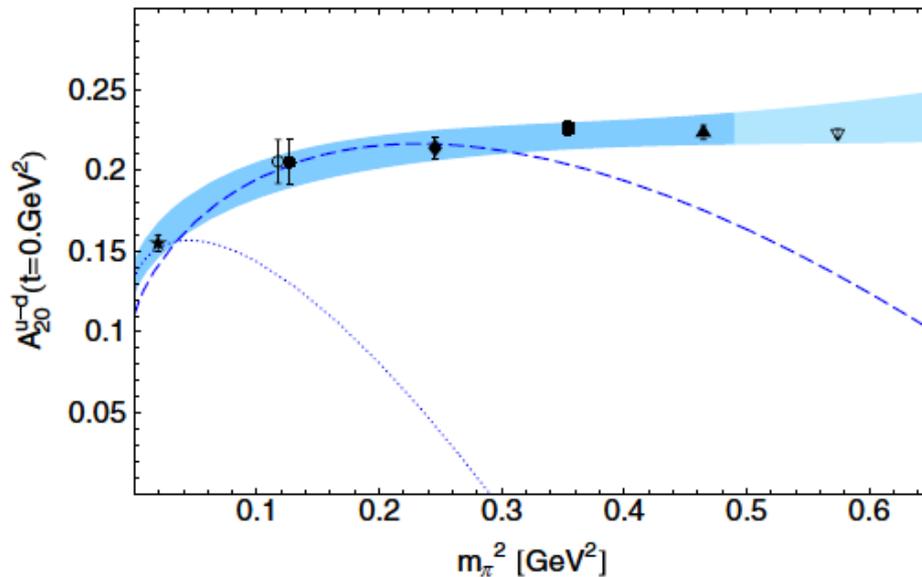


Need Chiral Extrapolation and $t \rightarrow 0$

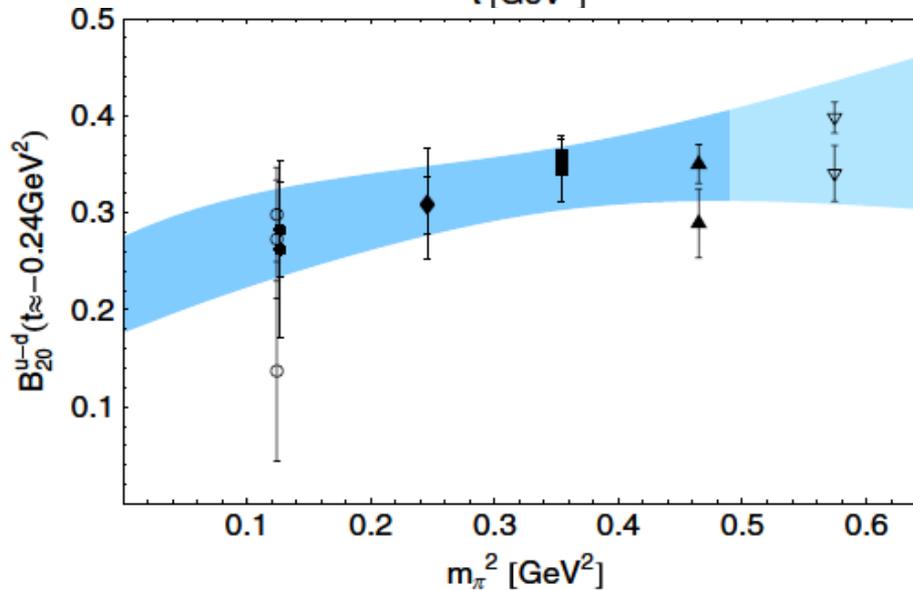
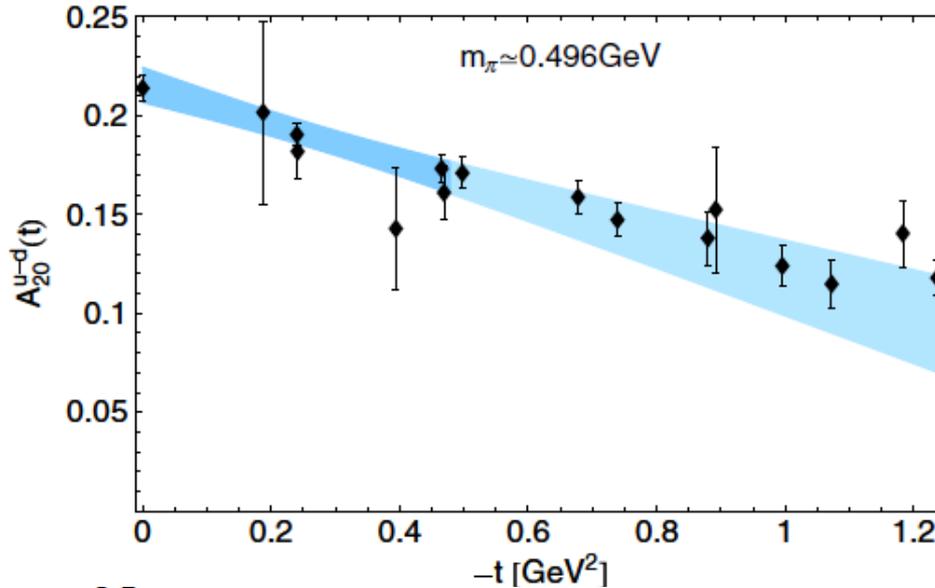
Hägler et al. use dim-reg over large range of m_π^2

— we know this is beyond range of convergence and therefore suspect (prefer FRR)

— also extrapolate $B_{20}(t)$ *linearly* in t over $(0, 1.2)$ GeV^2



LHPC Results cont'.



Claim:

$$J^u - J^d = + 0.21 \pm 0.04$$

$$L^u - L^d = - 0.42 \pm 0.04$$

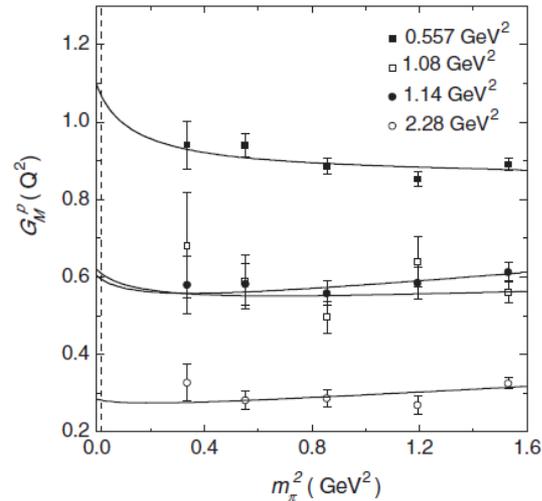
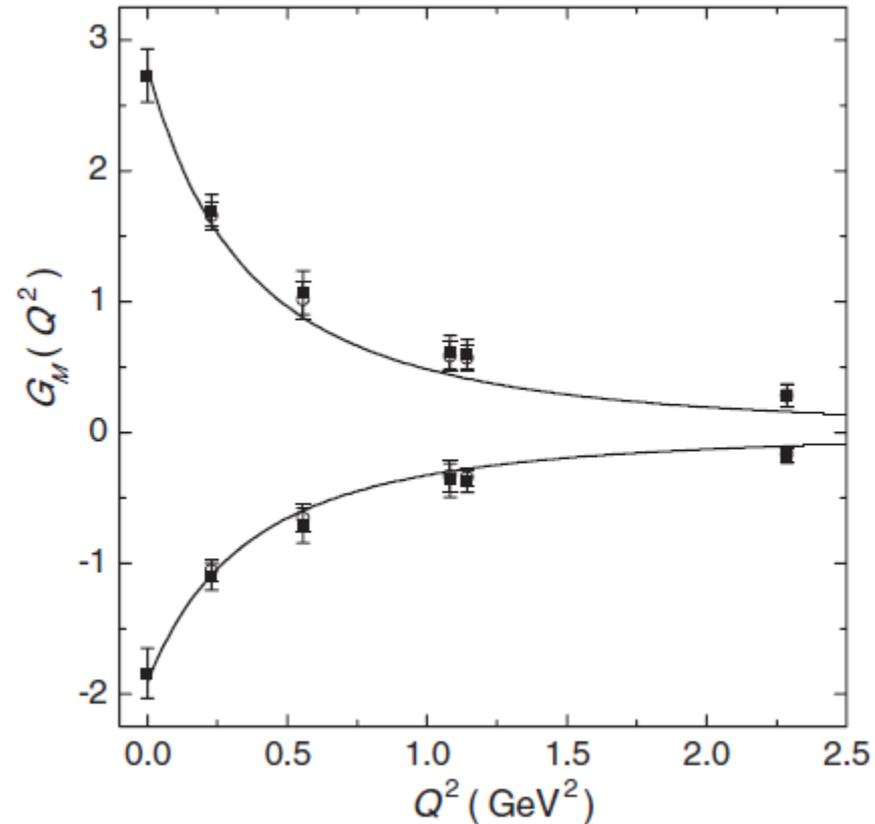
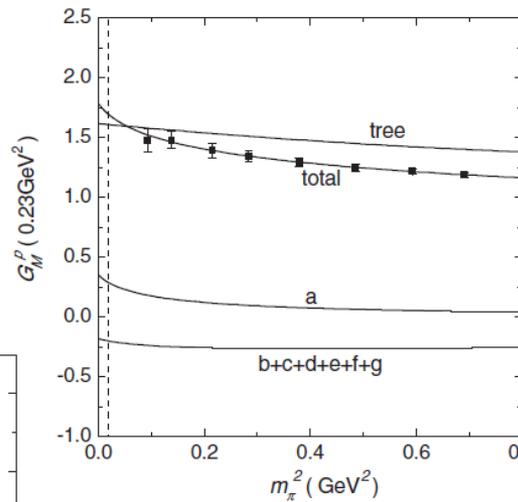
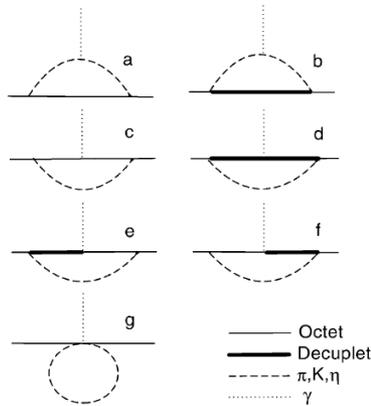
$$L^u = - 0.19 \pm 0.02$$

$$L^d = + 0.023 \pm 0.02$$

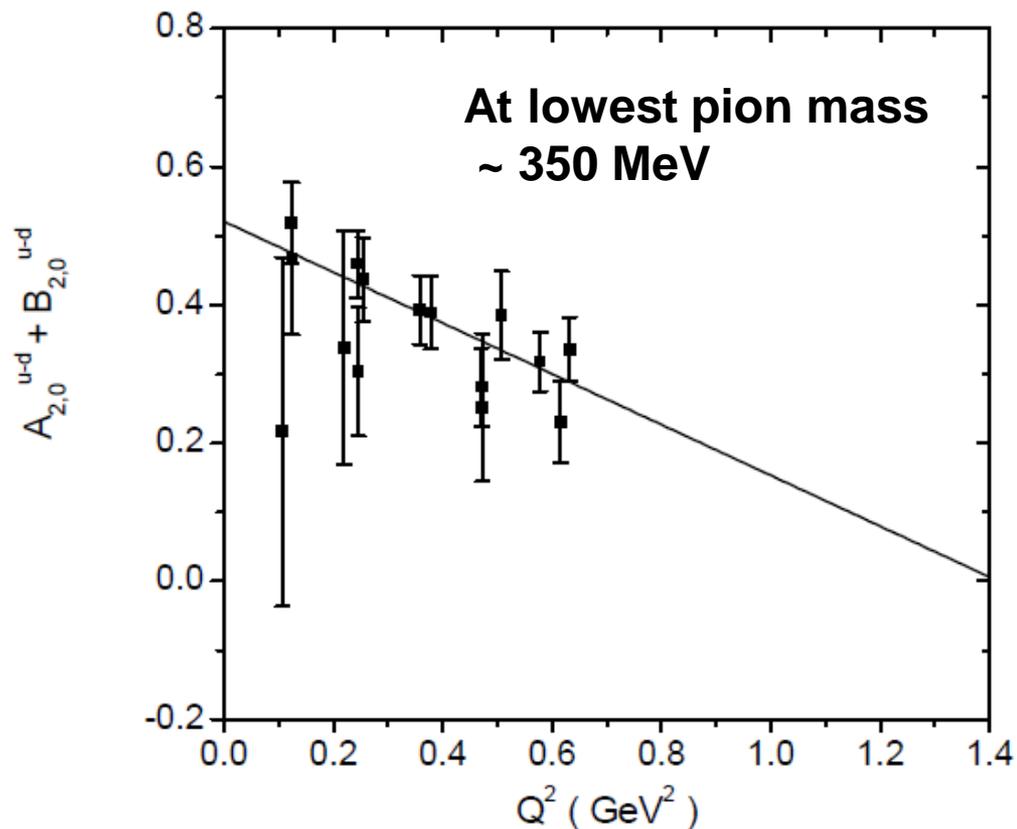
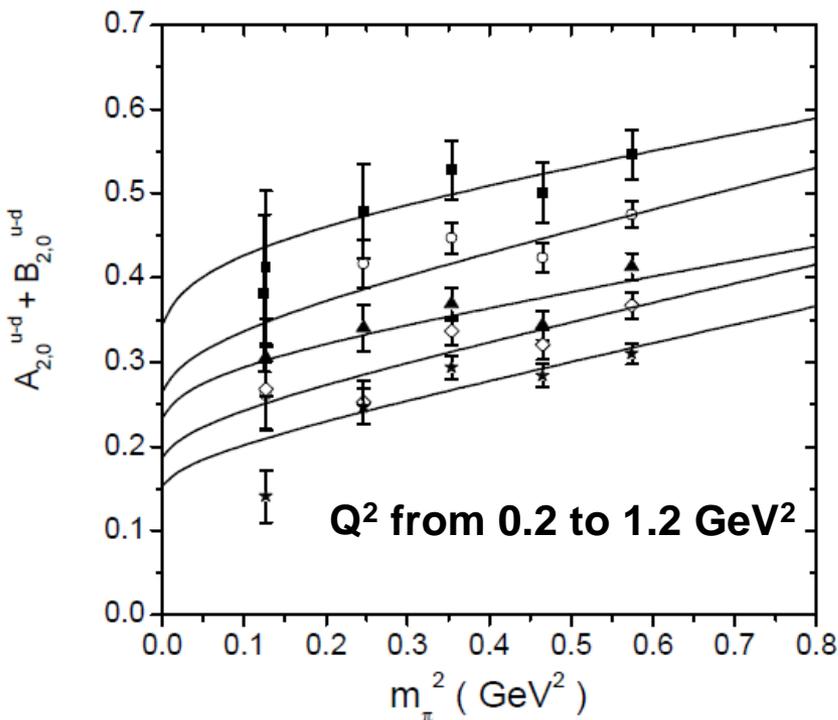
(modulo disconnected terms)

Check Using FRR

As in $G_M^s(Q^2)$ study of Wang et al., PR D75 (2007) 073012



FRR Treatment of GPD moments*



Seems similar to LHPC analysis

$$J^u - J^d \sim 0.2 \pm 0.05$$

BUT

errors may be much bigger
than suggested!

*Wang & Thomas, in preparation

Thomas Jefferson National Accelerator Facility

Page 37

Indeed L_z is not scale invariant – what scale?

- Known since mid-70s (Le Yaouanc et al., Parisi, etc.) that connection between quark models and QCD must be at low- Q^2
- This is because momentum fraction carried by quarks is monotonically decreasing with $Q^2 \uparrow$ and in models quarks carry nearly all the momentum (used by Glück-Reya to model HERA data to very low x - $\mu^2 = 0.23 \text{ GeV}^2$ at LO – Phys Lett 359, 205 (1995))

e.g. Schreiber et al., PR D42, 2226 (1990) : $\mu = 0.5 \text{ GeV}$

(N.B. Using LO rather than NLO QCD changes μ not the results at 5-10 GeV^2)

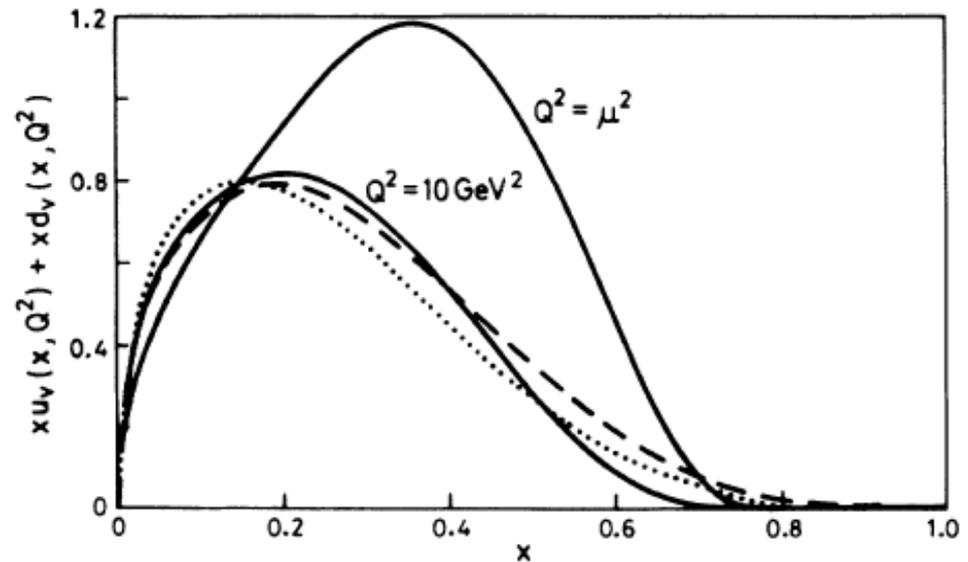
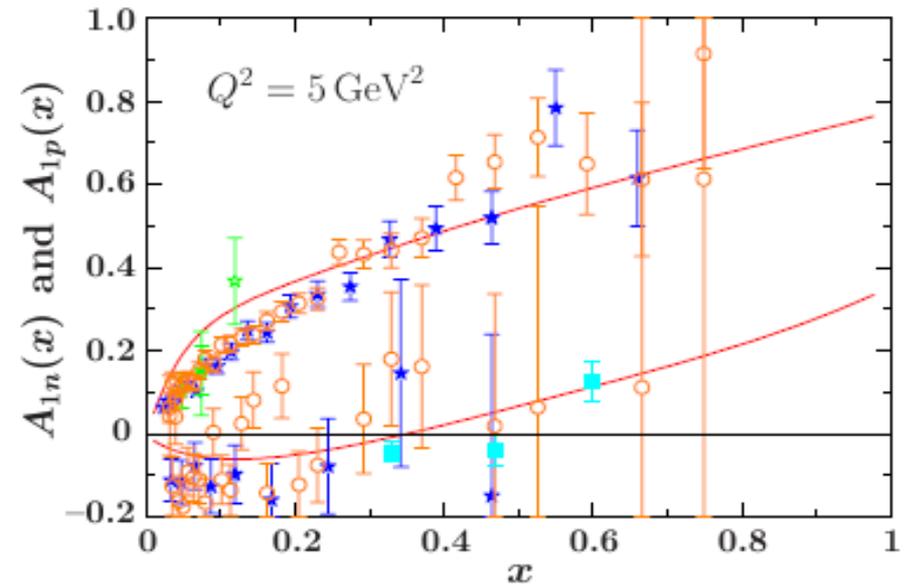
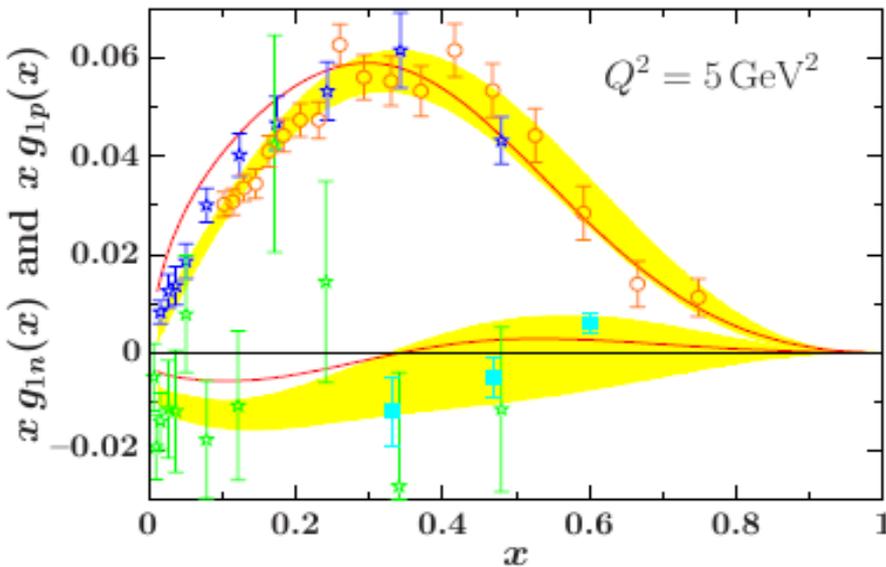
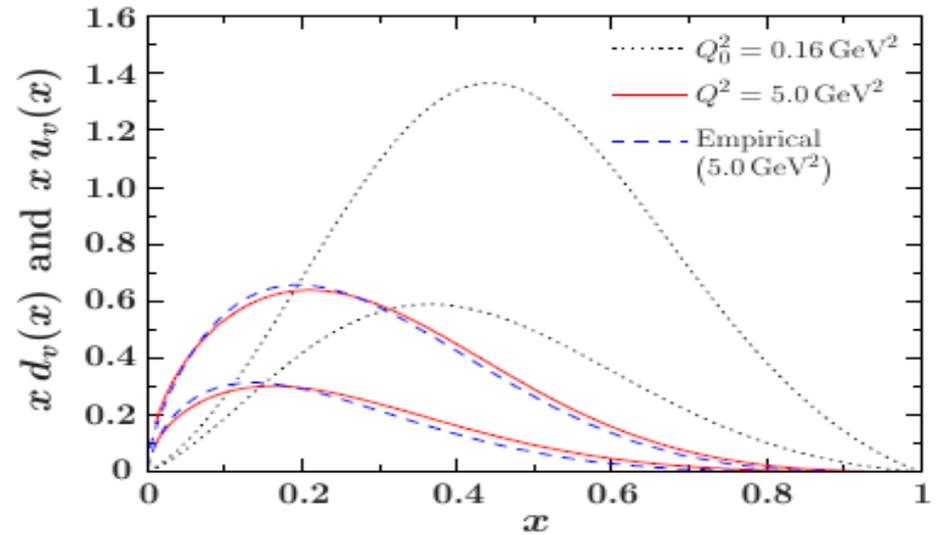


FIG. 1. $xu_v(x, Q^2) + xd_v(x, Q^2)$ at the model scale $Q^2 = \mu^2$ and at $Q^2 = 10 \text{ GeV}^2$ (solid lines). The dashed and dotted lines correspond to the Duke-Owens and Martin-Roberts-Stirling parametrizations of $xu_v(x, Q^2 = 10 \text{ GeV}^2) + xd_v(x, Q^2 = 10 \text{ GeV}^2)$,

More Modern (Confining) NJL Calculations

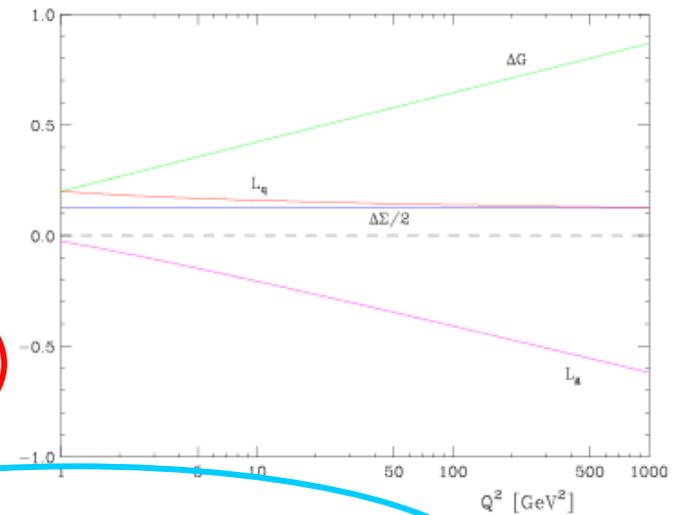
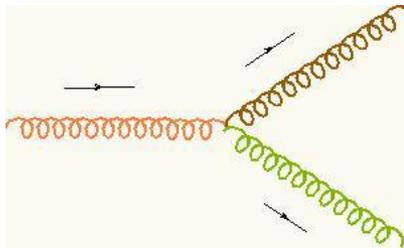
Cloet et al.,
Phys. Lett. B621, 246 (2005)
($\mu = 0.4$ GeV)



Evolution Equations - singlet

$$\Delta \Sigma(t) = \text{const},$$

$$\Delta g(t) = -\frac{4\Delta \Sigma}{\beta_0} + \frac{t}{t_0} \left(\Delta g_0 + \frac{4\Delta \Sigma}{\beta_0} \right)$$



$$L_q(t) = -\frac{1}{2} \Delta \Sigma + \frac{1}{2} \frac{3n_f}{16 + 3n_f} + (t/t_0)^{-2(16+3n_f)/9\beta_0}$$

$$\times \left(L_q(0) + \frac{1}{2} \Delta \Sigma - \frac{1}{2} \frac{3n_f}{16 + 3n_f} \right),$$

$$\frac{d}{dt} \begin{pmatrix} L_q \\ L_g \end{pmatrix} = \frac{\alpha_s(t)}{2\pi} \begin{pmatrix} -\frac{4}{3} C_F & \frac{n_f}{3} \\ \frac{4}{3} C_F & -\frac{n_f}{3} \end{pmatrix} \begin{pmatrix} L_q \\ L_g \end{pmatrix}$$

$$+ \frac{\alpha_s(t)}{2\pi} \begin{pmatrix} -\frac{2}{3} C_F & \frac{n_f}{3} \\ -\frac{5}{6} C_F & -\frac{11}{2} \end{pmatrix} \begin{pmatrix} \Delta \Sigma \\ \Delta g \end{pmatrix}$$

$$L_g(t) = -\Delta g(t) + \frac{1}{2} \frac{16}{16 + 3n_f} + (t/t_0)^{-2(16+3n_f)/9\beta_0}$$

$$\times \left(L_g(0) + \Delta g(0) - \frac{1}{2} \frac{16}{16 + 3n_f} \right).$$

Ji, Tang, Hoodbhoy: PRL 76 (1996) 740
Earlier Ratcliffe, Phys Lett B192 (1987)

Non-singlet Equations for Individual Flavors

$$L^{u-d}(t) + \frac{\Delta u - \Delta d}{2} = \left(\frac{t}{t_0}\right)^{-\frac{32}{9\beta_0}} \left(L^{u-d}(t_0) + \frac{\Delta u - \Delta d}{2} \right)$$

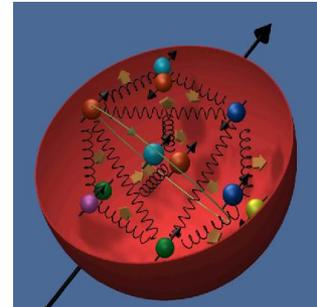
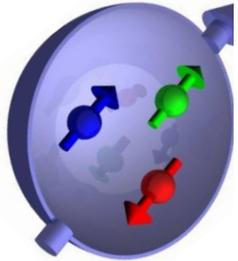
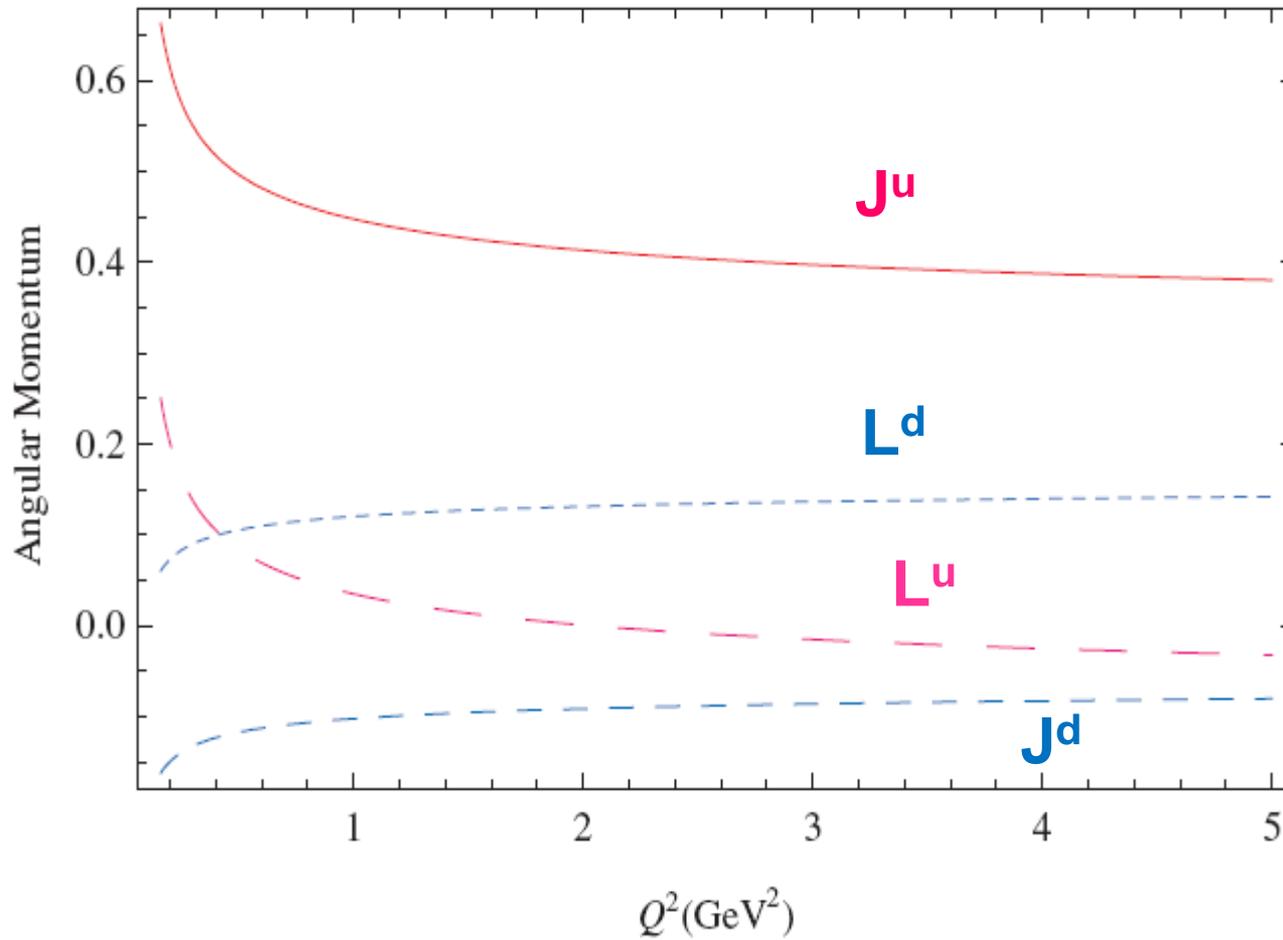
Also solve for non-singlet: $L^{u+d} - 2L^s$

$$\begin{aligned} \Rightarrow L^{u(d)} &= -\frac{\Delta u}{2} \left(-\frac{\Delta d}{2} \right) + 0.06 \\ &+ \frac{1}{3} \left(\frac{t}{t_0}\right)^{-\frac{50}{81}} \left[L^{u+d}(t_0) + \frac{\Sigma}{2} - 0.18 \right] \\ &+ \frac{1}{6} \left(\frac{t}{t_0}\right)^{-\frac{32}{81}} \left[L^{u+d}(t_0) \pm 3L^{u-d}(t_0) \pm g_A^{(3)} + \frac{\Sigma}{2} \right] \end{aligned}$$

Solution of the Evolution Equations

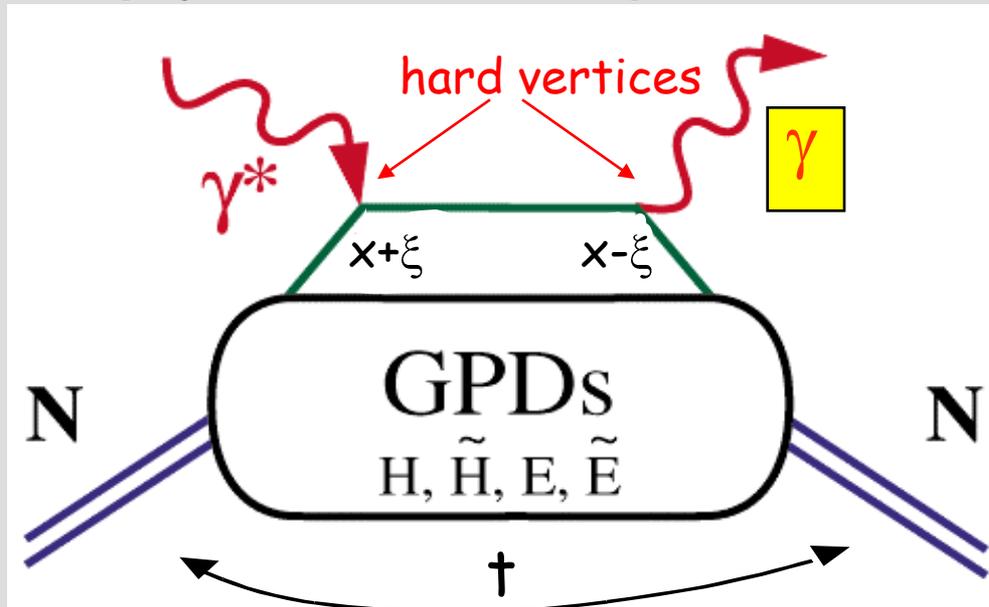
L^u and L^d both small and cross-over rapidly: AWT, PRL 101 (2008) 102003

- model independent !



GPDs & Deeply Virtual Exclusive Processes - New Insight into Nucleon Structure

Deeply Virtual Compton Scattering (DVCS)



x - quark momentum fraction

ξ - longitudinal momentum transfer

$\sqrt{-t}$ - Fourier conjugate to transverse impact parameter



At large Q^2 : QCD factorization theorem \rightarrow hard exclusive process can be described by 4 transitions (Generalized Parton Distributions) :

Vector : $H(x, \xi, t)$

Axial-Vector : $\tilde{H}(x, \xi, t)$

Tensor : $E(x, \xi, t)$

Pseudoscalar : $\tilde{E}(x, \xi, t)$

Relation of Moments to Experiment

$$H^n(\xi, t) \equiv \int_{-1}^1 dx x^{n-1} H(x, \xi, t)$$

$$H^{n=1}(\xi, t) = A_{10}(t), \quad H^{n=2}(\xi, t) = A_{20}(t) + (2\xi)^2 C_{20}(t)$$

$$E^n(\xi, t) \equiv \int_{-1}^1 dx x^{n-1} E(x, \xi, t)$$

$$E^{n=1}(\xi, t) = B_{10}(t), \quad E^{n=2}(\xi, t) = B_{20}(t) - (2\xi)^2 C_{20}(t)$$

$$\langle P', \Lambda' | \mathcal{O}_\#(x) | P, \Lambda \rangle = \langle \langle \# \rangle \rangle H(x, \xi, t) + \frac{n_\mu \Delta_\nu}{2m} \langle \langle i\sigma^{\mu\nu} \rangle \rangle E(x, \xi, t)$$

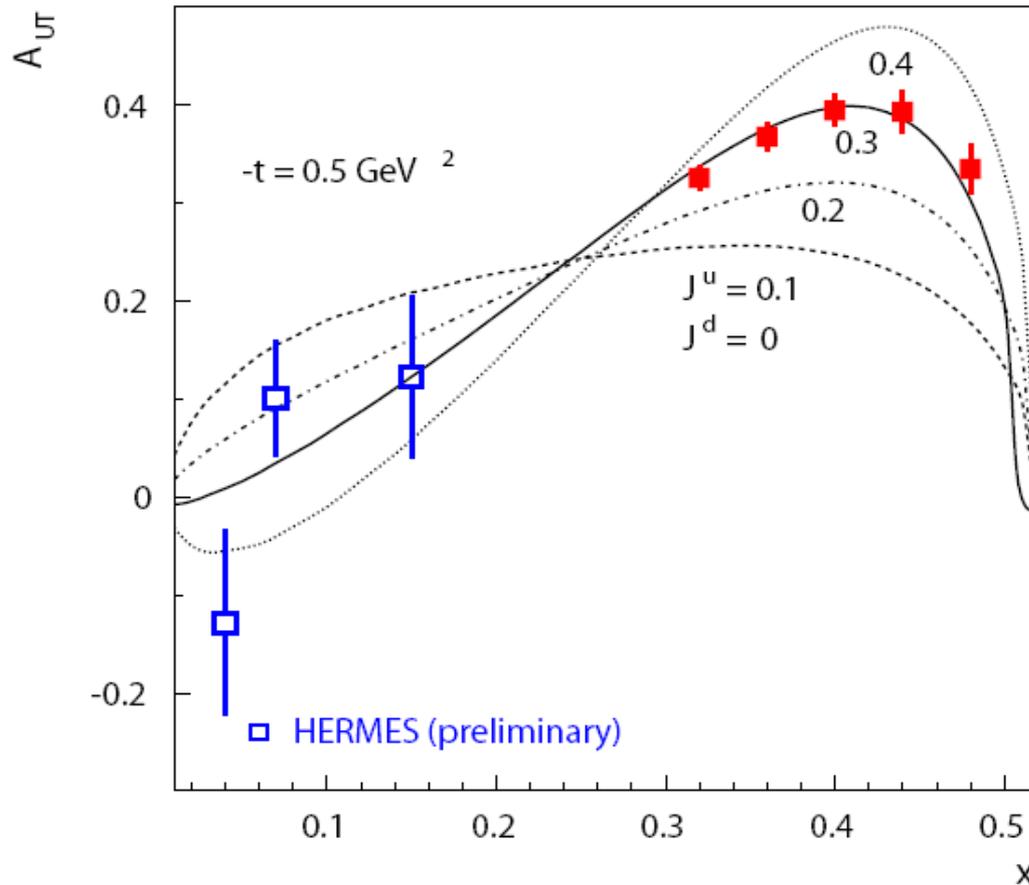
At 12 GeV: e.g. Exclusive ρ^0 with transverse target expect to determine quark orbital angular momentum

$$A_{UT} = - \frac{2\Delta (\text{Im}(AB^*))/\pi}{|A|^2(1-\xi^2) - |B|^2(\xi^2+t/4m^2) - \text{Re}(AB^*)2\xi^2}$$

ρ^0

$$A \sim (2H^u + H^d)$$

$$B \sim (2E^u + E^d)$$



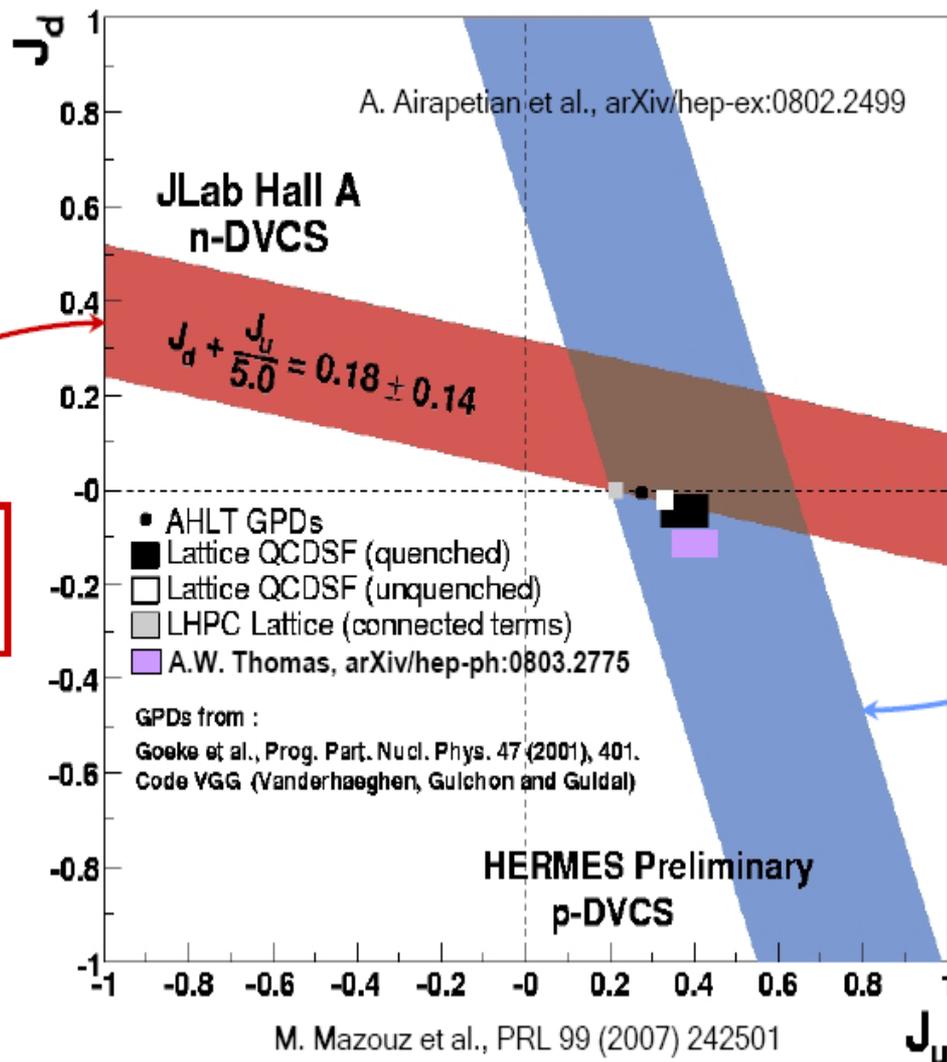
$Q^2 = 5 \text{ GeV}^2$

Asymmetry depends linearly on the GPD E , which enters J_i 's sum rule.

K. Goeke, M.V. Polyakov, M. Vanderhaeghen, 2001

Model Dependent Quark Angular Momenta

$$\chi_{\min}^2 \leq \chi^2(J_u, J_d) = \sum_i \frac{(\Im[C_n^I(\mathcal{F})]^{\text{exp}}(t_i) - \Im[C_n^I(\mathcal{F})]_{VGG}^{J_u, J_d}(t_i))^2}{[\delta_{\text{stat}}^{\text{exp}}(t_i)]^2 + [\delta_{\text{syst}}^{\text{exp}}(t_i)]^2} \leq \chi_{\min}^2 + 1$$



From Eric Voutier
(ECT* June 08)

Measurements off **neutron**
are sensitive to J_d
(**u** quark in the **neutron**)

Measurements off **proton**
are sensitive to J_u
(**u** quark in the **proton**)



Electromagnetic Calorimeter

➤ These experiments show the **complementarity** between **neutron** and **proton** data, and

Additional Observation

- Recall that polarized glue is generated by pQCD evolution

- In fact, at LO:
$$-\frac{N_f \alpha_s(Q^2)}{2\pi} \Delta G(Q^2) \rightarrow -N_f \frac{8\Sigma}{\beta_0^2 t_0}$$

- This yields a correction of order (-0.06,-0.11) from Σ to Σ_{inv}

$\Rightarrow \Sigma_{\text{inv}} \in (0.25, 0.34)$

- Still in excellent agreement with experiment

Polarized strange quarks

- As a corollary: polarized glue $\Rightarrow \Delta s_{inv} = (-0.04, -0.02)$
 - in case where there is no chiral strange contribution
- In practice, $p \rightarrow \Lambda K^+$ gives up to -0.01, so total $\Delta s_{inv} \sim (-0.03, -0.05)$
- This is consistent within large errors with results of neutral current neutrino scattering which yields $\Delta u - \Delta d - \{\Delta s_{inv} + \text{heavy quarks corrections}\}$
- Heavy quark corrections $\sim 1/\ln m_h^2$ calculated to NLO by Bass, Crewther et al. ([Phys. Rev. D66 \(2002\) 031901](#))
- NO physical contradiction with G_M^s as almost all strange content arises through axial anomaly NOT γ^μ

Summary

- Two decades of experiments have given us important new insight into spin structure of the p
 - U(1) axial anomaly appears to play little role in resolving the problem
 - not as severe as in original EMC paper
 - Instead, important details of the non-perturbative structure of the nucleon DO resolve the “crisis”
 - OGE hyperfine interaction
 - chiral symmetry: pion cloud
 - relativistic motion of quarks
- Ingredients of a minimal description of proton structure

Summary

- Important consequence for quark model:
significant orbital angular momentum carried
by valence quarks and anti-quarks in the proton
- SU(3) symmetry broken for a_8 at 15-20% level
- Effect of QCD Evolution is to:
 - flip ordering of L^u and L^d
 - severely reduce the magnitude of orbital angular momentum
 - restore agreement between data, LQCD and Myhrer-Thomas explanation of the spin crisis
- Study of GPDs at JLab provide the primary tool to verify this (maybe transversity too?)

