Dynamical Symmetry Breaking in Holographic QCD with Orientifolds

Shigeki Sugimoto (IPMU)

with T. Imoto and T. Sakai (Nagoya Univ.)

(Work in progress)

Joint Theory Institute workshop on "Dynamics of Symmetry Breaking" @ Argonne National Laboratory, 4/13-17 (4/15)

1 Introduction

- **Chiral symmetry breaking in QCD**

quark: $\psi = \begin{pmatrix} \psi_L \\ \psi_R \end{pmatrix}$ $\quad \psi_{L\,\alpha}^{\,ai} \quad \psi_{R\,\dot\alpha}^{\,ai'}$

$a = 1, 2, \cdots, N_c$ color

$i, i' = 1, 2, \cdots, N_f$ flavor

$\alpha, \dot\alpha = 1, 2$ Lorentz

$$U(N_f)_L \times U(N_f)_R \longrightarrow U(N_f)_V$$

$$\uparrow$$

$$\langle (\psi_R^*)^{\alpha}_{ai'} \psi_{L\alpha}^{ai} \rangle = c\, \delta^i_{i'} \qquad (c \neq 0)$$

- **Chiral symmetry breaking in holographic QCD**

(wait for the explanation)

$$U(N_f)_L \times U(N_f)_R \longrightarrow U(N_f)_V$$

Geometric realization of the chiral sym breaking !

To gain more insight,
consider the cases with G=O(Nc), USp(Nc)

gauge group

- $G = O(N_c)$ quark: ψ_α^{ai} flavor sym: $U(N_f)$

$$\langle \epsilon^{\alpha\beta} \delta_{ab} \psi_\alpha^{ai} \psi_\beta^{bj} \rangle = c\, \delta^{ij}$$

$$\boxed{U(N_f) \longrightarrow O(N_f)}$$

- $G = USp(N_c) \equiv \{ g \in U(N_c) \,|\, gJg^T = J \}$ $J = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}$

quark: ψ_α^{ai} flavor sym: $U(N_f)$ (N_c and N_f should be even)

$$\langle \epsilon^{\alpha\beta} J_{ab} \psi_\alpha^{ai} \psi_\beta^{bj} \rangle = c\, J^{ij}$$

$$\boxed{U(N_f) \longrightarrow USp(N_f)}$$

Question:

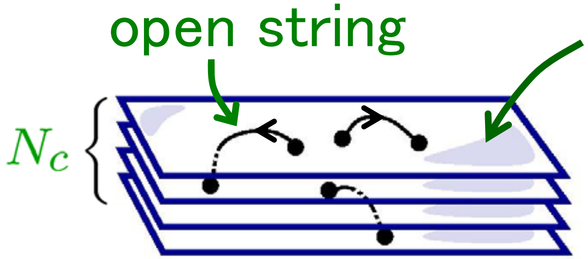
Can we obtain these results from String theory?

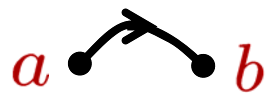
Plan of Talk

✓ 1. Introduction
2. D-branes and Orientifolds
3. Holographic QCD
4. Holographic Symmetry Breaking
5. Baryons
6. Outlook

2 D-branes and Orientifolds

D-brane and Gauge theory





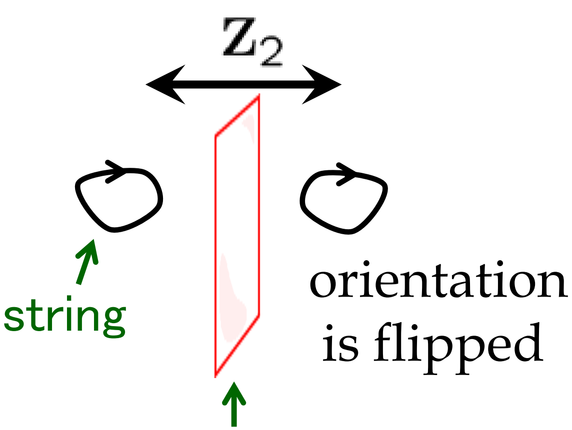
$a, b = 1 \sim N_c$

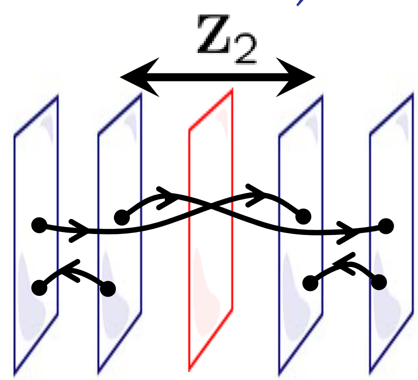
→ $(A_\mu)^a{}_b$ etc.

$U(N_c)$ gauge field

➡ (p+1) dim $U(N_c)$ gauge theory

Orientifold (space-time / Z_2)





related by $\mathbf{Z}_2$

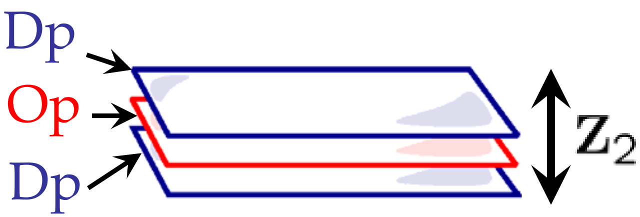
$$A_\mu \sim A_\mu^T$$

2 consistent choices

$Op^- : \quad A_\mu = -A_\mu^T$ → $O(N_c)$

$Op^+ : \quad A_\mu = -J A_\mu^T J^{-1}$ → $USp(N_c)$

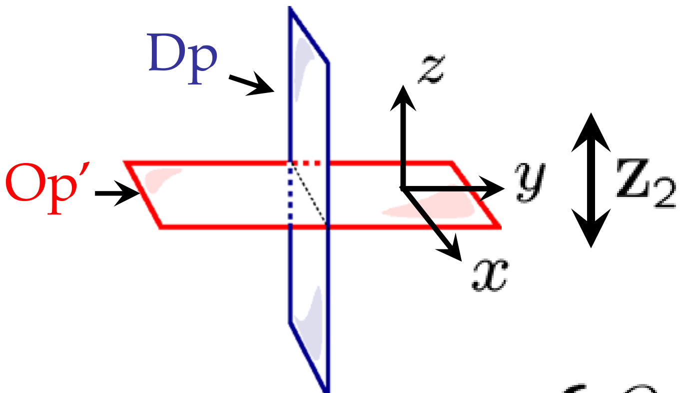
More on Orientifold planes



$Op^- : \quad A_\mu = -A_\mu^T \quad \rightarrow \quad O(N_c)$

$Op^+ : \quad A_\mu = -J A_\mu^T J^{-1} \quad \rightarrow \quad USp(N_c)$

More generally,



$D_{\rm int} \equiv \dim\{(y,z)\}$

=# intersecting directions

$$D_{\rm int} = 0, 8 \quad \rightarrow \quad \begin{cases} Op^- : A_\mu(x,z) = -A_\mu^T(x,-z) \\ Op^+ : A_\mu(x,z) = -J A_\mu^T(x,-z) J^{-1} \end{cases}$$

$$D_{\rm int} = 4 \quad \rightarrow \quad \begin{cases} Op^- : A_\mu(x,z) = -J A_\mu^T(x,-z) J^{-1} \\ Op^+ : A_\mu(x,z) = -A_\mu^T(x,-z) \end{cases}$$

$D_{\rm int} = 2, 6 \quad \rightarrow$ Dp−brane $\overset{\mathbf{Z}_2}{\longleftrightarrow}$ $\overline{\text{Dp}}$−brane

Dp-brane with opposite orientation

③ Holographic QCD

★ Gauge / String duality

AdS/CFT [Maldacena 1997]

D3-brane → $\mathcal{N}$=4 Super Yang-Mills
D3-brane → String theory in $AdS_5 \times S^5$

dual

Note: SUSY, conformal sym are not essential in this idea

holographic QCD

$\boxed{?}$ → QCD
$\boxed{?}$ → $\boxed{?}$

Some brane config.

dual

String theory in some curved b.g.

➡ "holographic QCD"

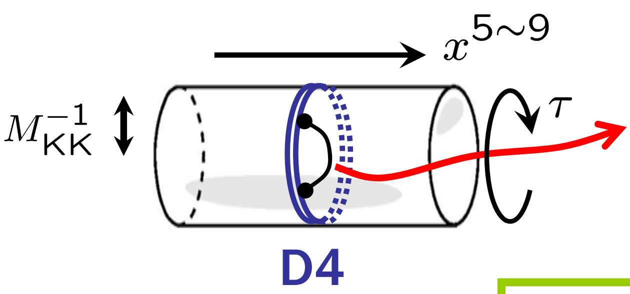
★ Brane configuration

[Witten 1998]

- D4-brane $\times N_c$ on S^1 with $\psi(x^\mu, \tau + 2\pi M_{\rm KK}^{-1}) = -\,\psi(x^\mu, \tau)$

$\mu = 0 \sim 3$

fermion



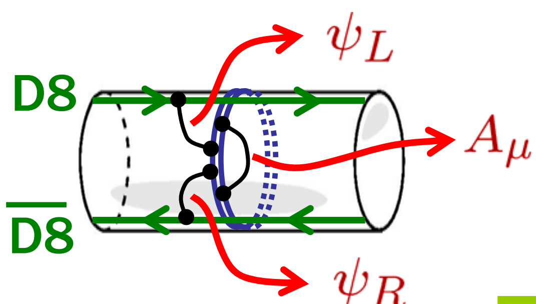
$A_\mu, \ \underbrace{A_\tau, \ \phi \times 5, \ \psi \times 4}_{\text{acquire mass}} \sim M_{\rm KK}$ $\quad$ ~~SUSY~~

$\boxed{\text{4 dim } U(N_c) \text{ pure Yang-Mills}}$ (at low energy)

- To add quarks, we add D8-$\overline{\text{D8}}$ pairs $\times N_f$

[Sakai-S.S. 2004]

		x^0	x^1	x^2	x^3	τ	x^5	x^6	x^7	x^8	x^9
D4	$\times N_c$	○	○	○	○	○	–	–	–	–	–
D8-$\overline{\text{D8}}$	$\times N_f$	○	○	○	○	–	○	○	○	○	○



	D4 $U(N_c)$	D8 $U(N_f)_L$	$\overline{\text{D8}}$ $U(N_f)_R$
A_μ	adjoint	1	1
ψ_L	N_c	N_f	1
ψ_R	N_c	1	N_f

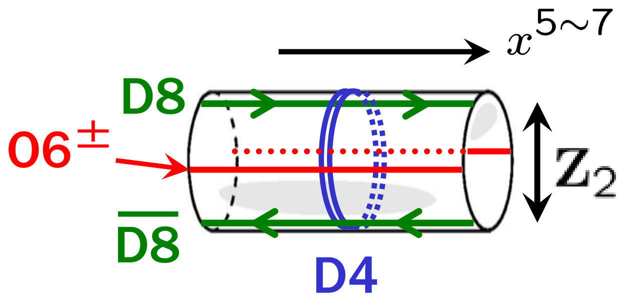
$\boxed{\text{4dim } U(N_c) \text{ QCD with } N_f \text{ massless quarks}}$

★ $G = O(N_c)$, $USp(N_c)$ cases

	x^0	x^1	x^2	x^3	τ	x^5	x^6	x^7	x^8	x^9
D4 $\times N_c$	○	○	○	○	○	–	–	–	–	–
D8-$\overline{\text{D8}}$ $\times N_f$	○	○	○	○	–	○	○	○	○	○
O6$^\pm$	○	○	○	○	–	○	○	○	–	–

D4 vs O6 : $D_{\text{int}} = 4$

D8 vs O6 : $D_{\text{int}} = 2$



O6$^+$ → $O(N_c)$ QCD

O6$^-$ → $USp(N_c)$ QCD

quark: $\psi_L^i \overset{\mathbf{Z}_2}{\longleftrightarrow} \psi_R^i$ → N_f Weyl fermions

flavor sym: $\underbrace{U(N_f)_L \times U(N_f)_R}_{\mathbf{Z}_2}$ → $U(N_f)$ flavor sym

as desired !

★ holographic description of YM

[Witten 1998]

D4-brane on S^1

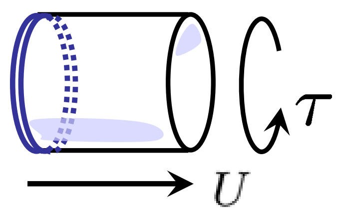
(with $\psi(x^\mu, \tau + 2\pi) = -\psi(x^\mu, \tau)$)

the corresponding SUGRA solution

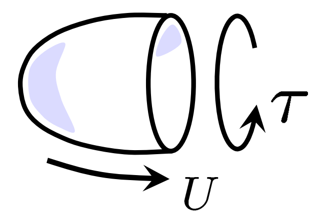
$\sim \mathbf{R}^{1,3} \times \underline{\mathbf{R}^2} \times S^4$ (topologically)

angular directions in $x^{5\sim 9}$

D4



(radial direction in $x^{5\sim 9}$)



4 dim pure Yang-Mills

(at low energy)

dual

String theory in this background

★ Adding quarks

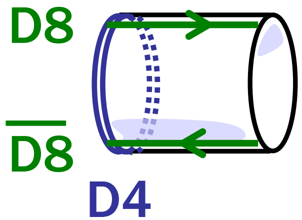
- Here we assume $N_c \gg N_f$ and use "probe approximation".

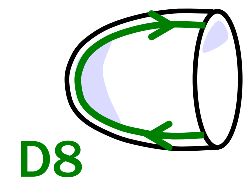
[Karch-Katz 2002]

- D4-branes are replaced with the corresponding background.
- D8-$\overline{\text{D8}}$ pairs are treated as probes.

D4-brane on S^1
+ N_f D8-$\overline{\text{D8}}$ pairs

String theory in the D4 background
+ N_f D8-branes





dual

QCD with N_f massless quarks (at low energy)

④ Holographic Symmetry Breaking

★ Chiral symmetry breaking for U(N_c) QCD

[Sakai-S.S. 2004]

We replace D4 with the corresponding SUGRA sol.

D8 and $\overline{\text{D8}}$ must be smoothly connected in the D4 background

➡ interpreted as the chiral symmetry breaking !

$$U(N_f)_L \times U(N_f)_R \longrightarrow U(N_f)_V$$

$\updownarrow$ D8 $\updownarrow$ $\overline{\text{D8}}$ $\updownarrow$ connected D8

★ G=O(Nc) , USp(Nc) cases

$$\mathbf{R}^{1,3}\times(S^1\times\mathbf{R}^5)/\mathbf{Z}_2$$

$$(x^4,x^8,x^9)\to(-x^4,-x^8,-x^9)$$

$$\sim\mathbf{R}^{1,3}\times(\mathbf{R}^2\times S^4)/\mathbf{Z}_2$$

$$\text{D8}:\quad\mathbf{R}^{1,3}\times(\mathbf{R}\times S^4)/\mathbf{Z}_2$$

$$\text{O6}^\pm:\quad\mathbf{R}^{1,3}\times\mathbf{R}\times S^2$$

$$(D_{\text{int}}=4)$$

Unbroken global gauge symmetry on D8-brane

$\text{O6}^+ \to$ *O(Nf)* $\quad\text{O6}^- \to$ *USp(Nf)*

$$U(N_f)\longrightarrow\begin{cases}O(N_f) & \\ USp(N_f) & \end{cases}$$

for *O(Nc)* QCD

for *USp(Nc)* QCD

$\updownarrow$ **D8** $\quad\updownarrow$ **D8 with O6$^\pm$**

as expected !

5 Baryons

★ Quark model :

quark

Baryon $\sim \quad \epsilon_{a_1 a_2 \cdots a_{N_c}} q^{a_1} q^{a_2} \cdots \underset{}{q^{a_{N_c}}}$

- $G = O(N_c)$

$$\epsilon_{a_1 a_2 \cdots a_{N_c}} \epsilon_{b_1 b_2 \cdots b_{N_c}} \sim \delta_{a_1 b_1} \delta_{a_2 b_2} \cdots \delta_{a_{N_c} b_{N_c}} \pm \cdots$$

→ 2 baryon can decay to mesons

→ # baryon $\in \mathbf{Z}_2$

- $G = USp(N_c)$

$$\epsilon_{a_1 a_2 \cdots a_{N_c}} \sim J_{a_1 a_2} J_{a_3 a_4} \cdots J_{a_{N_c - 1} a_{N_c}} \pm \cdots$$

→ 1 baryon can decay to mesons

→ # baryon is not conserved

★ Holographic description of baryons

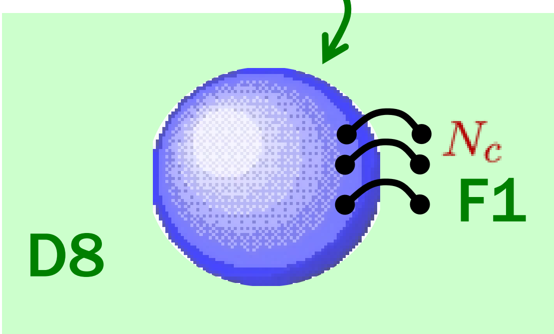
- Baryons in the AdS/CFT context are constructed by D-branes wrapped on non-trivial cycles. [Witten, Gross-Ooguri 1998]

In our case, background $\sim \ \mathrm{R}^{1,3} \times \mathrm{R}^2 \times S^4$

$$\boxed{\text{Baryon} \quad \simeq \quad \text{D4-brane wrapped on the } S^4}$$

- RR flux $\frac{1}{2\pi}\int_{S^4} dC_3 = N_c$ forces N_c F-strings to be attached on it.

D4 on S^4



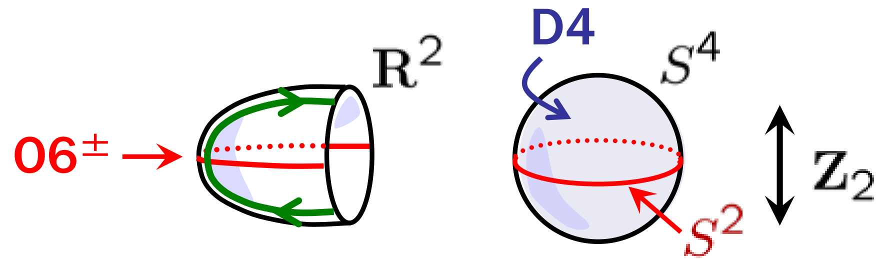
➡ Bound state of N_c quarks

➡ Baryon

Baryon mass ($\propto$ vol. of S^4) is generated by the geometry!

★ $G=O(N_c)$, $USp(N_c)$ cases

In the presence of the O6-plane,



$D_{\mathrm{int}} = 6$ → D4 $\overset{Z_2}{\longleftrightarrow}$ $\overline{\mathrm{D4}}$

$\mathbf{R}^{1,3} \times (\mathbf{R}^2 \times S^4)/\mathbf{Z}_2$

$\mathrm{O6}^{\pm}: \ \mathbf{R}^{1,3} \times \mathbf{R} \times S^2$

Baryon ~ $\underline{\text{D4-}\overline{\text{D4}}}$ pair on S^4

↓ tachyonic

D4-$\overline{\mathrm{D4}}$ $\times\, n$ → **tachyon field T is an n x n matrix**

with $T \overset{Z_2}{\longleftrightarrow} \mp T^T$ **for** $\mathrm{O6}^{\pm}$

$\mathrm{O6}^-$ (***USp(N_c)*** **QCD**) → **tachyon condenses** → **unstable**

$\mathrm{O6}^+$ (***O(N_c)*** **QCD**) → $\left\{ \begin{array}{l} \textbf{stable when } n=1 \\ \textbf{unstable when } n=2 \end{array} \right\}$ → **# baryon** $\in \mathbf{Z}_2$

as expected !

⑥ Outlook

- QCD with $G=O(N_c)$,$USp(N_c)$ is constructed by using D-branes and Orientifold planes
- Flavor symmetry breaking is understood geometrically, and agrees with field theory consideration.
- Some properties of Baryon are also nicely reproduced.

QCD ⟷ (dual !) String theory (in a certain curved background)

Group theory + Dynamics ⟷ Geometry