Holographic Hydrodynamics of QGP with Multiple/non-Abelian Symmetries

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Reference : arXiv:0903.4894 [hep-th]
with Mahdi Torabian

My naive understanding of RHIC...

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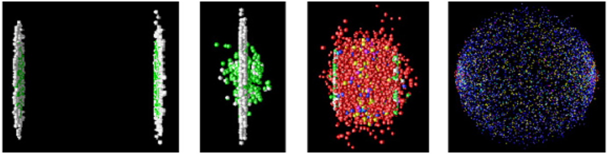

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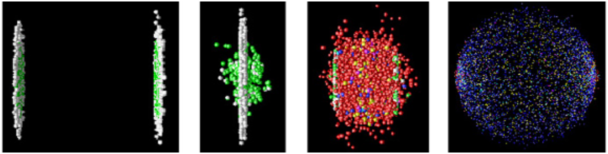
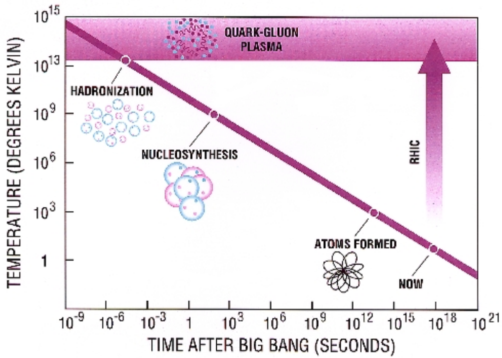


- Gold-Gold collision with several hundreds of GeV's
- Plasma lasts less than 10^{-23} seconds
- Temperature can reach 10^{12} K

What I have heard about...



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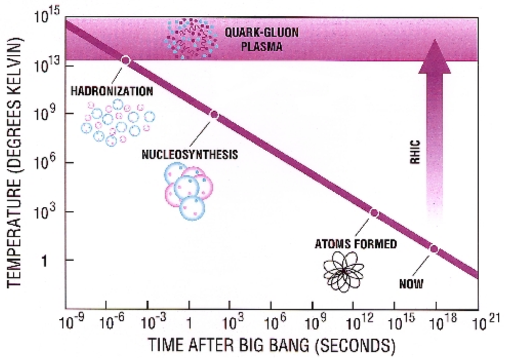

- Some kind of plasma of quarks and gluons
- Naive perturbative QCD seems to fail in describing it, eg. jet quenching
- More like a strongly coupled hydrodynamic liquid

Can AdS/CFT help?

OBJECTIVE : **How to study strongly coupled quark-gluon plasma at finite temperature ?**

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- Supersymmetric conformal field theory
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- In most cases, no quarks in fundamental representation

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- In most cases, no quarks in fundamental representation

No obvious reason to expect any usefulness for real QCD

Claim : Some universality ?

Only common features between RHIC plasma and finite temperature AdS/CFT

- Finite temperature gauge theory
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CLAIM : These are enough for some things to be universal

QUESTION : Precisely what can be universal and why ?

What is hydrodynamics ?

- We look at long wave-length, slowly varying fluctuations. Local thermalization is assumed at each point.
- Wait ! RHIC is a very violent collision ...
- But, we have to measure slowliness compared to the thermalization and relaxation time scales of the system. RHIC is claimed to be quickly thermalized at least approximately
- Local thermal equilibrium implies that only local thermodynamic variables are important in the description, such as T, u^μ, μ. etc
- Dynamics is simply dictated by conservation laws of energy-momentum and global symmetries such as baryon number

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More about hydrodynamics

- Given local thermodynamic variables, T, u^μ, μ, one needs constitutive relations to write down energy-momentum tensor $T^{\mu\nu}$ and the global symmetry current J^μ in terms of them
- The dynamics is then completely determined by conservation laws

$$\partial_\mu T^{\mu\nu} = 0 \quad , \quad \partial_\mu J^\mu = 0$$

- Note that specifics of detailed microscopic theory enter in the constitutive relations
- In general, derivative expansion is implemented for slowly-varying fluctuations compared to the time scale of the plasma

$$T^{\mu\nu} = (p+\epsilon)u^\mu u^\nu + p\eta^{\mu\nu} - \eta P^\mu_\alpha P^\nu_\beta \left(\partial_\alpha u_\beta + \partial_\beta u_\alpha - \frac{2}{3}\eta_{\alpha\beta}\left(\partial_\gamma u^\gamma\right)\right) + \cdots$$

- Note that pressure p, energy density ϵ, and the shear viscosity η are in general given in terms of local thermodynamic variables T, u^μ, by constitutive relations
- Upshot : Hydrodynamics is just an effective framework

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- Different theories would give us different constitutive relations for the energy-momentum tensor and global symmetry currents in terms of local thermodynamic variables
- The consequent dynamics would then be different. This seems expected and trivial at the end ...
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MAIN POINT : Strong coupling can give us universality for transport coefficients

This is the only justification of the usefulness of AdS/CFT in describing real QCD plasma at RHIC

How would this be possible

Conceptual understanding of how this could be possible

- Transport coefficients are determined by how perturbations propagate into adjacent regions
- There may be some intrinsic quantum mechanical bound on how fast these could happen

$$\tau \geq \frac{h}{k_B T}$$

- Strong coupling dynamics may be extreme in this sense and saturate this bound. The subsequent transport coefficients from it would then be universal, and in fact saturate the bound
- A famous example is the viscosity-entropy ratio

$$\frac{\eta}{s} = \frac{1}{4\pi}$$

- AdS/CFT is just one example of strongly coupled field theory/gravity correspondence, where calculations can be done easily in the gravity side. Just use it to see what we get ! The results might be **universal** and useful for RHIC.

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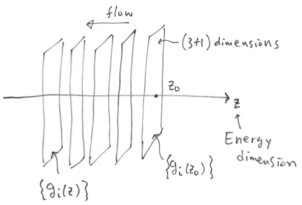
Finite temperature plasma in AdS/CFT



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- Renormalization group survives in the large N limit
- A classical theory with additional energy coordinate is a way to realize these aspects
- RG invariance = general covariance in 5 dimension → **Gravity theory in 5D**

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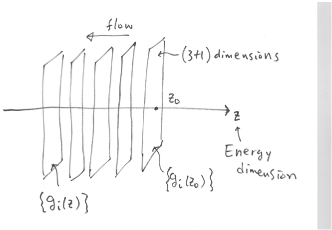

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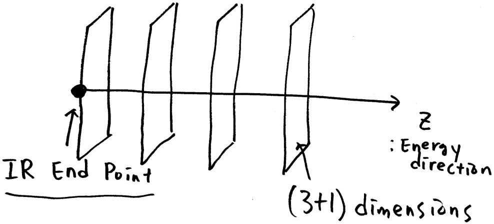
one example that has been tested successfully for 10 years:

N=4 Super Yang-Mills in 4D $\longleftrightarrow$ IIB Super Gravity in $AdS_5 \times S^5$

Finite temperature plasma is described by Black Hole

In the gravity description, finite temperature plasma is described by Black Hole or more precisely, Black Brane



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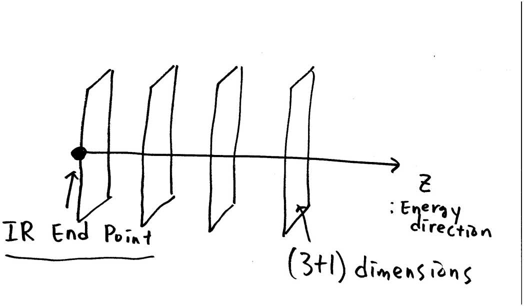

- There is no rigorous understanding of this
- It has been more like **"empirical"**

Why Black Hole ?

The "logic" has been following ;

- Accept AdS/CFT
- Consider finite temperature
- Ask for a gravity object in AdS with finite temperature
- The only thing we know is Black Hole (Black Brane) with Hawking temperature and entropy
- Study this object more carefully to find that due to geometric potential wall in AdS, outgoing Hawking radiation returns back and the BH can be in a thermally stable state called Hartle-Hawking state
- It seems to be the correct object for finite temperature plasma for about 10 years

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Reduction to gravity in AdS

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- After this point, all analyses become simply those of general relativity in asymptotic AdS spacetime
- Many previous results in general relativity have found their nice applications and meanings in this context
- Maybe more things out there waiting for application

Two approaches

In studying hydrodynamics, there have been two approaches

- Study linear response of the Black Hole to external perturbations to extract transport coefficients indirectly (Kovtun, Son, Starinets, Policastro, Herzog): Greens function method, Kubo formula
- Harder way: Solve the gravity equation of motion directly for slowly varying parameters, such as temperature and chemical potential to extract hydrodynamics directly (Battacharrya, Hubeny, Rangamani, Minwalla)
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The basic idea is very simple

- Start from the known Black Brane solution of the Eistein equation and its suitable extension

$$ds^2 = -r^2 f(r) u_\mu u_\nu dx^\mu dx^\nu + 2u_\mu dx^\mu dr + r^2 (\eta_{\mu\nu} + u_\mu u_\nu) dx^\mu dx^\nu$$

with $f(r) = 1 - \frac{m}{r^4}$.

- The solution is parameterized by the 4-velocity u^μ of the fluid, and the energy density m which is related to the temperature $m \sim T^4$
- But, what we really want to describe is the situation where these local thermodynamic parameters are slowly varying over the 4D spacetime x^μ
- One simply inserts the slowly varying parameters $u^\mu(x)$ and $m(x)$ into the above solution. Note that we keep the original form of the solution
- This means physically that locally we are in thermal equilibrium so that the solution is still given by the above with local thermal variables, but slowly varying over x^μ
- With slowly varying parameters in the solution, it is no longer a solution of the Einstein equation. We have to add corrections that will be sourced by derivatives of $u^\mu(x)$ and $m(x)$ to satisfy the equation of motion

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- But, what we really want to describe is the situation where these local thermodynamic parameters are slowly varying over the 4D spacetime x^μ
- One simply inserts the slowly varying parameters $u^\mu(x)$ and $m(x)$ into the above solution. Note that we keep the original form of the solution
- This means physically that locally we are in thermal equilibrium so that the solution is still given by the above with local thermal variables, but slowly varying over x^μ
- With slowly varying parameters in the solution, it is no longer a solution of the Einstein equation. We have to add corrections that will be sourced by derivatives of $u^\mu(x)$ and $m(x)$ to satisfy the equation of motion

More about the method..

- In this way, one gets a full solution with slowly varying local thermodynamic parameters in derivative expansion

$$g_{MN}(u^\mu(x), m(x)) + g^{(1)}_{MN} + g^{(2)}_{MN} + \cdots$$

where $g^{(k)}_{MN}$ is proportional to k-th order in derivatives of $u^\mu(x)$ and $m(x)$.

- Recall that in hydrodynamics, we have a similar derivative expansion of energy-momentum tensor

$$T^{\mu\nu} = (p+\epsilon)u^\mu u^\nu + p\eta^{\mu\nu} - 2\eta\sigma^{\mu\nu} + \cdots \quad (1)$$

in terms of derivatives of $u^\mu(x)$ and $T(x)$

- In fact, using the AdS/CFT dictionary between 5D gravity solution and the 4D energy-momentum tensor, what we are doing is precisely obtaining the 4D energy-momentum tensor in derivative expansion of local thermal variables
- Along the way, one can easily obtain the transport coefficients such as viscosity η directly
- In principle, one can go to higher nonlinear orders systematically

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We obtained an interesting non-Abelian structure for the current

$$\begin{aligned} J^a_\mu &= \rho^a u_\mu - \mathcal{D}\left(\frac{\rho\cdot P^\nu_\mu(\partial_\nu\rho)}{\rho\cdot\rho} - u^\nu\partial_\nu u_\mu\right)\rho^a + \mathcal{D}_1\epsilon^{abc}\rho^b P^\nu_\mu(\partial_\nu\rho^c) \\ &+ \mathcal{D}_2 P^\nu_\mu\left(\rho^a(\rho\cdot\partial_\nu\rho) - (\rho\cdot\rho)(\partial_\nu\rho^a)\right) \end{aligned}$$

Thank you very much

Thank you very much for listening