Proton-proton scattering from AdS/QCD

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Outline

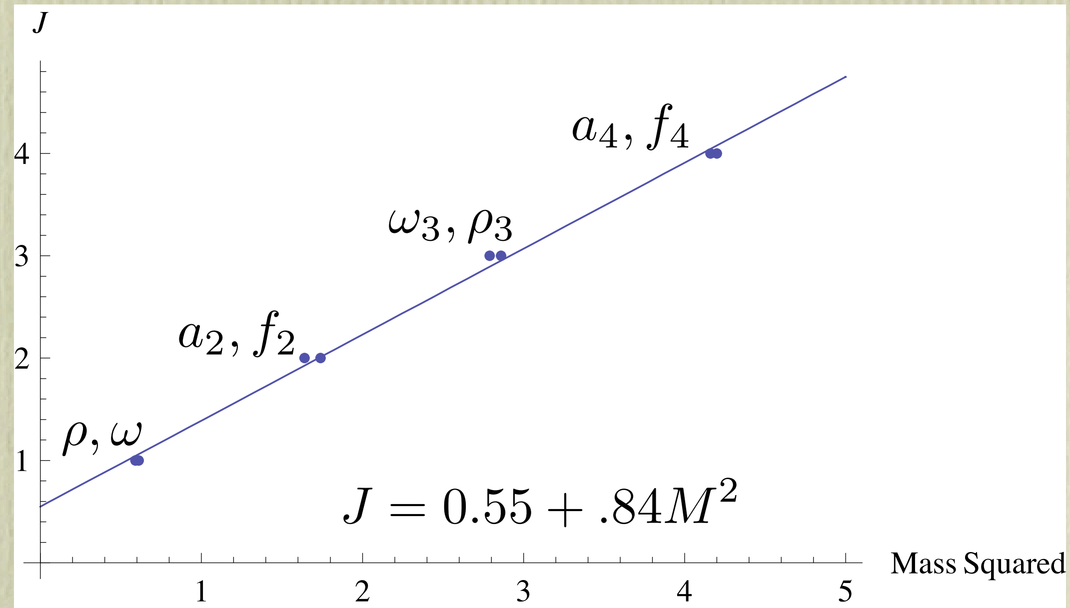
1. Introduction and motivation
2. Review of Regge theory and the Pomeron
3. String dual model for Pomeron exchange
4. Fits to pp and pp-bar differential and total cross-sections
    - Regime of validity for the model in s and t
    - Fitted values of parameters
5. Computation of parameters in the Sakai-Sugimoto model

A quick review of Reggeons and Pomerons

Regge Trajectories

- The hadron spectrum falls into linear "Regge trajectories":

$$J = \alpha_0 + \alpha' m^2$$



- Linearity of Regge trajectories (with same slope!) for large s and small (negative) t scattering processes.

Scattering in the Regge Regime

- Perturbative QCD is not valid for large s, small |t| scattering -- how to compute such amplitudes (and total cross section)?

- Sum over amplitudes for exchange of a mesons of spin J, mass m in the t-channel:

$$\mathcal{A}(s,t) \propto \frac{s^J}{t - m^2}$$

- As $s \to \infty$, summing over all such t-channel poles violates Froissart bound!

(Regge theory continued...)

- Instead, analytically continue to complex J, with $t \ll s$:

$$\mathcal{A}(s,t) \propto (\alpha' s)^{\alpha(t)}$$

and differential cross section

$$\frac{d\sigma}{dt} = \beta(t)(\alpha' s)^{2\alpha(t)-2}$$

(Regge theory continued...)

- This works remarkably well for processes where only one Regge trajectory exchange is possible, e.g. $\pi^- p \to \pi^0 n$, where only the ϱ is exchanged.

- Diff. cross section given by

$$\frac{d\sigma}{dt} = \beta(t)(\alpha' s)^{2\alpha(t)-2}$$

- Same α(t) as at positive t!

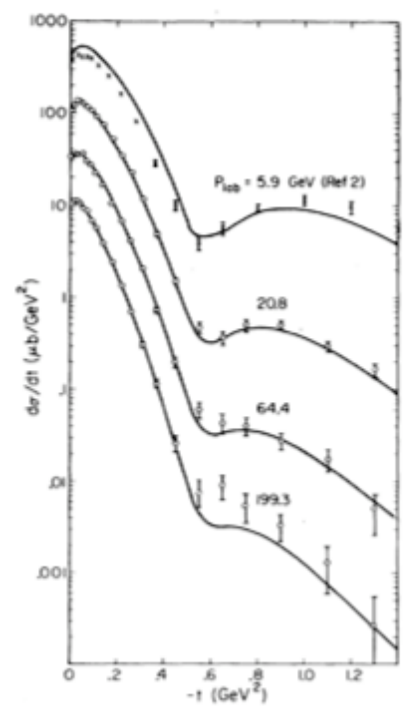


FIG. 1. Differential cross sections at 20.8, 64.4, and 199.3 GeV from this experiment, and at 5.9 GeV from Ref. 2. The curves are the result of a fit described in the text.

The Pomeron

- Regge theory is great! But Reggeon exchange alone cannot explain the behavior of total cross sections at very large s.

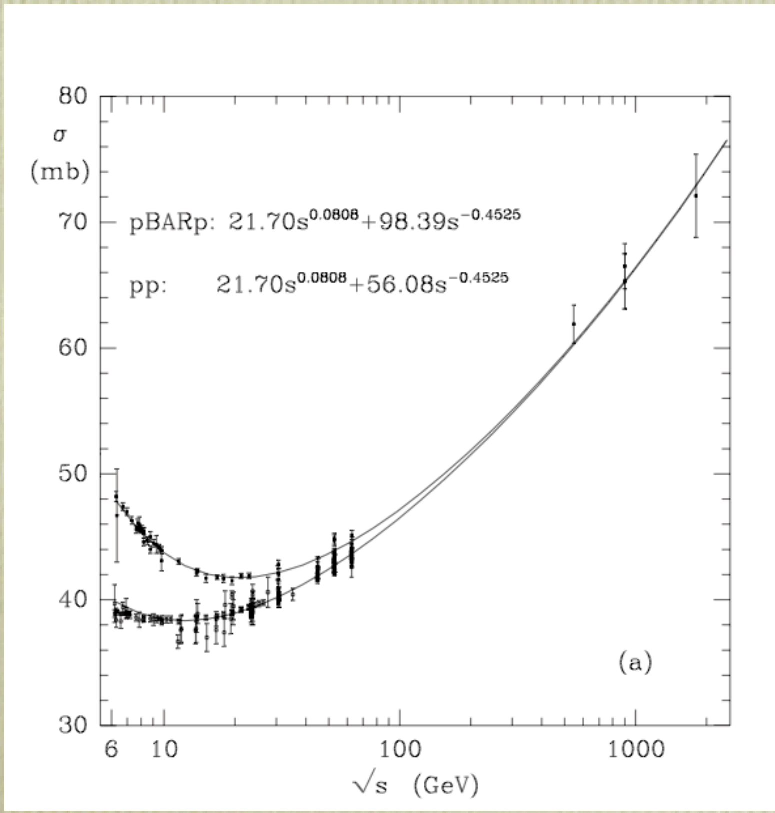
$$\sigma_{tot} \propto (\alpha' s)^{\alpha(0)-1}$$

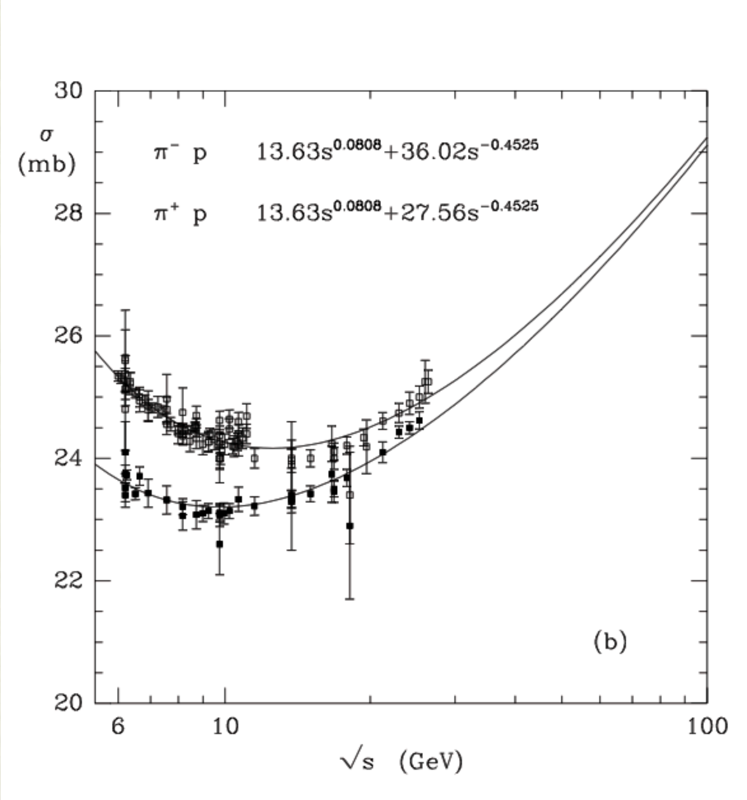
For leading Reggeon $(\alpha_0 - 1) \approx -0.45$ the cross section should decrease as s increases: inconsistent with experiment!

- Chew and Fratschi introduce the "Pomeron" in 1961 (named after Pomeranchuk): trajectory with vacuum quantum numbers and intercept $\alpha_0 > 1$.
    - Leading mode on the trajectory is a 2^{++} state, possibly 2^{++} glueball.

The Pomeron ... continued

- Fits to total cross section including Reggeon and Pomeron exchange (Donnachie and Landshoff)





The Pomeron Reborn

- In AdS/QCD models, natural dual to the 2^{++} glueball is the graviton.
- Summing over the entire trajectory is difficult (need to use string theory, not just SUGRA limit)! Some impressive work by Brower, Polchinski, Strassler, Tan (BPST) on relating hard (large t) and soft Pomerons.
- We will use the ideas of Regge theory and dual theories to treat pp and ppbar scattering.

A holographic model for proton-proton scattering

Proton-proton scattering in AdS/QCD

- Follow similar procedure to "Reggeization" of meson exchange, but from a more stringy perspective:

1. Find the coupling of the lowest state on the Pomeron trajectory to the proton from a dual model, and compute the amplitude.

2. Take the large s, small |t| limit of the amplitude by an appropriate generalization of the amplitude for closed string scattering (where the full trajectory is exchanged).

3. Compare with data (and fit any parameters you can't compute!)

2^{++} glueball - proton coupling

- In holographic theories, the 2^{++} glueball in 4d is dual to the tower of states generated by the 5d (or 10d) graviton -- i.e. a fluctuation around the 10d background metric.

- The graviton couples (by definition) to the 5d energy-momentum tensor.

    - To good approximation, the 2^{++} piece couples to the energy momentum tensor of the proton in 4d:

$$h_{\mu\nu} T^{\mu\nu}$$

    - Coupling determined by wavefunction overlaps (more on this later)

2^{++} glueball - proton coupling

- Decompose matrix elements of the energy-momentum tensor into form factor (as with the EM current):

$$\langle p's' \,|T_{\mu\nu}|\, p, s\rangle = \overline{u}(p's') \left[A(t)\gamma_{(\mu}P_{\nu)} + B(t)\frac{iP_{(\mu}\sigma_{\nu)\rho}q^{\rho}}{2m_p} + C(t)\frac{q_\mu q_\nu - \eta_{\mu\nu}q^2}{m_p} \right] u(p,s)$$

where $P = \dfrac{p+p'}{2}$ and $q \equiv p - p'$.

- In the limit $s \to \infty$ and $t \ll s$, only the first term contributes to leading order. From now on consider only the term containing $A(t)$.

- Specifics of $A(t)$ are model-dependent, but can usually approximate with dipole form:

$$A(t) = \frac{A(0)}{(t - M_d^2)^2}$$

2^{++} glueball - proton coupling

- The differential cross-section for exchange of a single 2^{++} glueball (summed over proton spins) is

$$\frac{d\sigma}{dt} = \underbrace{\frac{\lambda^4 A^4(t)}{\pi}}_{\text{vertex factor}} s^2 \underbrace{\left(\frac{1}{t - m_g^2}\right)^2}_{\text{propagator}}$$

coupling

...now we need to sum over the whole glueball trajectory...

“Reggeizing” the propagator

- Consider features of 4 → 4 flat space, closed string scattering amplitude (bosonic or superstring):

$$\mathcal{A}(s,t) = \frac{\Gamma[-\alpha(s)]\Gamma[-\alpha(t)]\Gamma[-\alpha(u)]}{\Gamma[-\alpha(t)-\alpha(s)]\Gamma[-\alpha(t)-\alpha(u)]\Gamma[-\alpha(u)-\alpha(s)]} K_{1,2,3,4}$$

- $K_{1,2,3,4}$ is a function of polarizations, spins, momenta of in- and outgoing particles

- $\alpha(x) = a_0 + a'x$. Mass of lightest state on the trajectory:

  $$m^2 = -\frac{a_0}{a'}$$ (e.g. $m^2=0$ for NS-NS bosons)

- Residue at n^{th} pole $\sim\ (a's)^{2n}$

"Reggeizing" the propagator

Assume the basic characteristics hold for curved space:

- Linear trajectories: $\alpha(x) = a_0 + a' x$
- Mass of lowest mode relates slope and intercept: $m_g^2 = -a_0/a'$
- Mass-shell condition:

$$\chi \equiv \alpha(s) + \alpha(t) + \alpha(u) = a' \left(4m_p^2 - 3m_g^2\right)$$

- $K_{1,2,3,4} \sim s^2$. The pole with residue $\sim s^2$ relates exchanged angular momentum to $\alpha(t)$ as $\alpha_0 = 2a_0 + 2$ where $\alpha' = 2a'$ and $J = \alpha_0 + \alpha' t$.

"Reggeizing" the propagator

Applying these rules to the single graviton propagator:

$$\frac{1}{t-m_g^2} \quad \rightarrow \quad - \, \frac{a'\Gamma[-\chi]\Gamma[-\alpha(t)]}{\Gamma[\alpha(t)-\chi]} \, (a's)^{2\alpha(t)}$$

Gives:

- Infinite sequence of poles at $\alpha(t) = n$ with residues appropriate to exchange of spin $J = 2n + 2$ massive states.

Ta-da!

Combining the Reggeized propagator with single-glueball-exchange amplitude, we have the differential cross-section for pp and pp-bar scattering in the Regge limit:

$$\frac{d\sigma}{dt} = \frac{\lambda^4 A(t)^4}{\pi} \left(\frac{\Gamma[-\chi]\Gamma[-\alpha(t)]}{\Gamma[\alpha(t)-\chi]} \right)^2 (a's)^{4\alpha(t)+2}$$

Undetermined parameters: $a_0,\ a',\ \lambda,\ M_d$

... get them from a dual model, or data fitting.

Regime of Validity and Fits to Data

Regime of Validity

- Fit to data for pp and pp-bar scattering (from Durham database) as a function of s and t

- Constrain the range of s and t by (mostly) excluding effects our model does not take into account:

  1. Reggeon Contamination (restricts s from below)

  2. Multi-pomeron exchange, AdS curvature effects (restrict s from above)

  3. Coulomb interaction (restricts |t| from below)

  4. Perturbative QCD effects (restrict |t| from above

  5. t/s corrections

Reggeon contamination

- Pomerons do not differentiate between pp and pp-bar, but Reggeons do! We *should* fit σ as

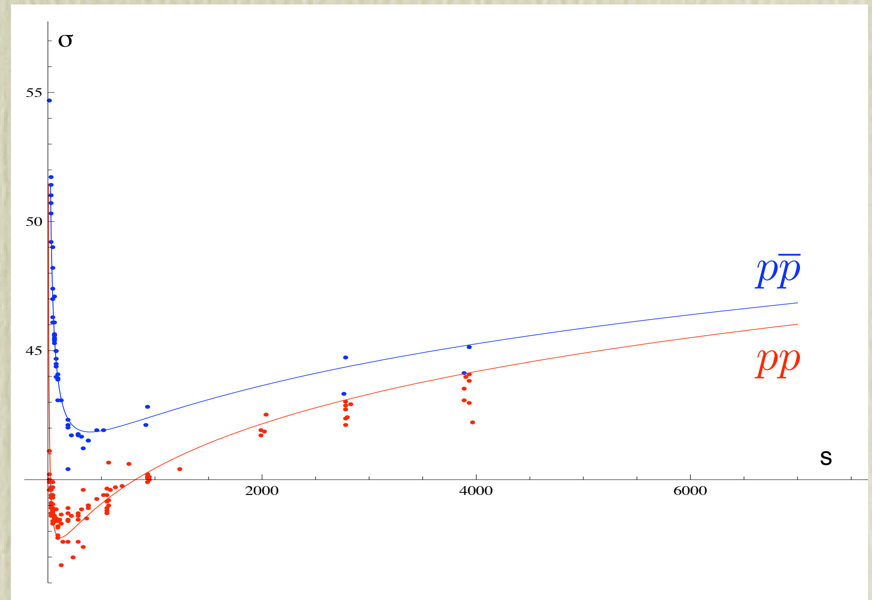
$$\sigma = A_P s^{a-1} + B_{\pm} s^{b-1}$$

Pomeron

Reggeons
("+" for p and "-" for pbar)

Fit Parameters:

$$a = 1.0847,\ b = 0.546$$
$$A_P = 21.325$$
$$B_+ = 53.169$$
$$B_- = 103.556$$



Reggeon Contamination

- Optical theorem relates $\sigma \sim s^{-1}\mathrm{Im}\mathcal{A}(t=0)$, so approximately

$$\frac{d\sigma}{dt} \sim s^{-2} \cdot A^2 s^{2a} \left(1 + \frac{2B_\pm}{A} s^{b-a} + \ldots\right)$$

- Estimate error due to Reggeon contamination as ratio of first two terms. For ppbar:

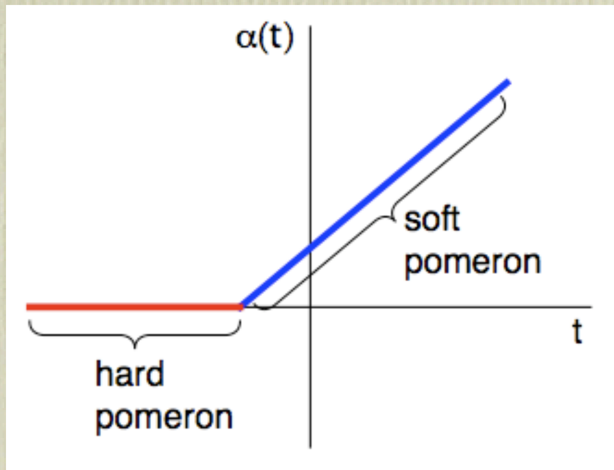
$$\begin{aligned} \sqrt{s} &= 31 \text{ GeV}: \ 22\% \\ \sqrt{s} &= 1800 \text{ GeV}: \ 0.3\% \end{aligned}$$

Include by adding in quadrature to experimental errors.

Perturbative QCD effects

- For large enough **t**, pQCD takes over
    - gluons, not glueballs
    - hard (BFKL) Pomeron, not soft Pomeron
- Hard Pomeron has similar behavior, but different $\alpha(t)$
- Consistent with strings

  in AdS (BPST)



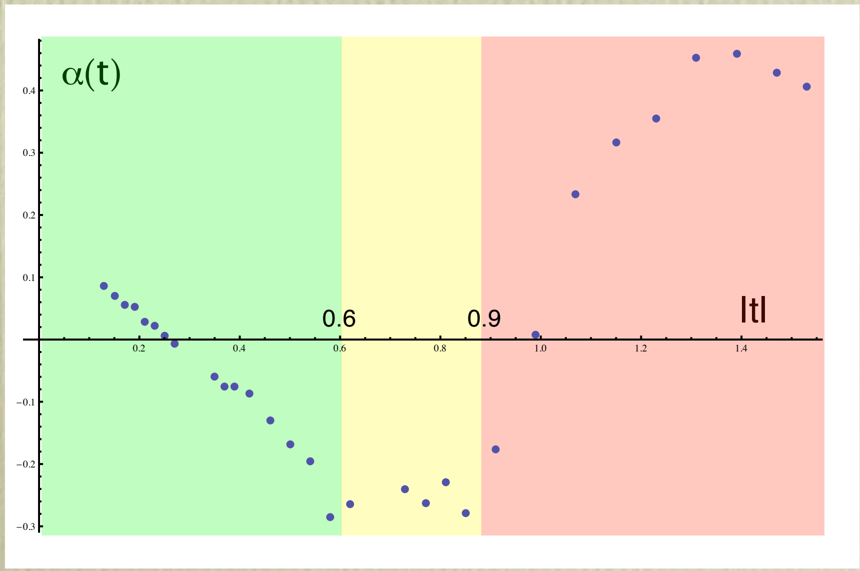
pQCD continued

- Consider

$$\ln\left(\frac{d\sigma}{dt}\right) = \alpha(t)\ln s + (\text{some function of t})$$

- Fit at various fixed t as a function of $\ln s$

- Plot $\alpha(t)$ versus t, and identify turnover value of t.



turnover at

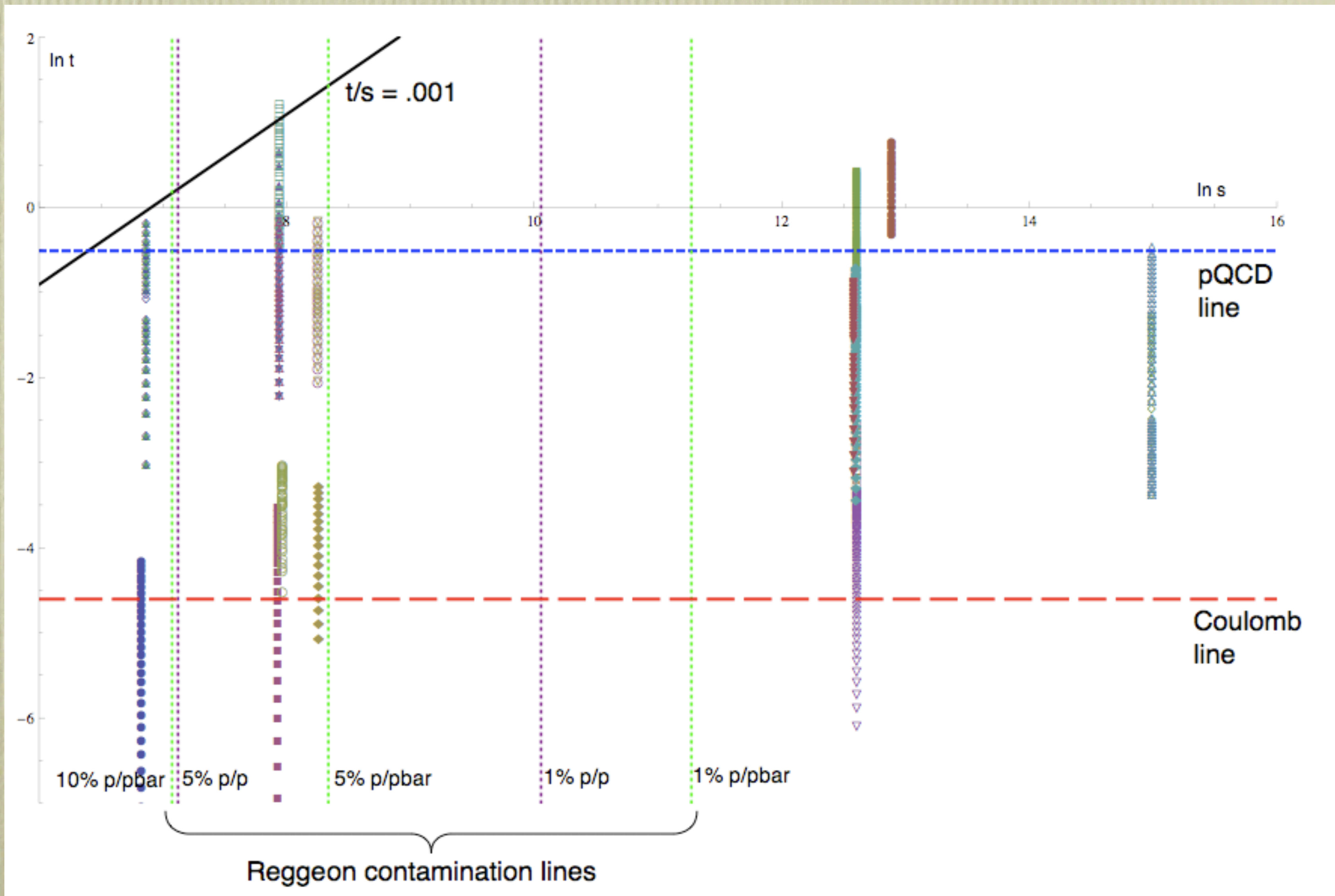
$t > 0.6\ \text{GeV}^2$

Summary of data and restrictions

- Data sets ranging from $\sqrt{s} = 31\ \text{GeV}$ to $\sqrt{s} = 1800\ \text{GeV}$
- Restrict range of t: $0.01 < t < 0.6$
- Include Reggeon contamination by adding approximate contamination error in quadrature to experimental error bars.

COMPETE collaboration uses similar range

Summary of Data Sets and Restrictions



Comparison to DL model

- Industry standard for treating Pomeron exchange is the Donnachie-Landshoff model
- Assumes photon-like coupling to proton; replaces our (model-dependent) form factor with the EM form factor

$$\left.\frac{d\sigma}{dt}\right|_{DL} = \frac{(3\beta F_1(t))^4}{4\pi}\left(\frac{s}{s_0}\right)^{2\alpha(t)-2}$$

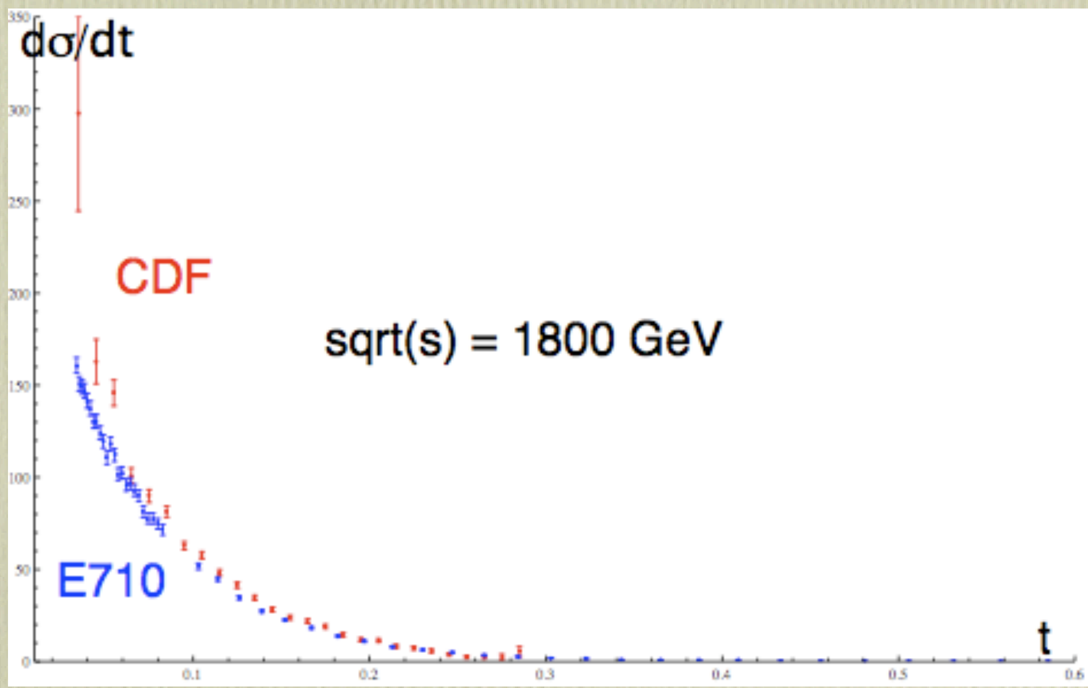
where

$$F_1(t) = \frac{4m_p^2 - 2.79t}{4m_p^2 - t}\frac{1}{(1-t/0.71)^2}$$

- For comparison, also fit using DL model, varying $\beta,\ a_0,\ a'$

The 1800 controversy

- Some controversy regarding $\sqrt{s} = 1800$ GeV data sets from E710 and CDF.



- Our fits use both, then just CDF, then just E710.

DHM versus DL diff. cross section fits

our favorite

DHM fits

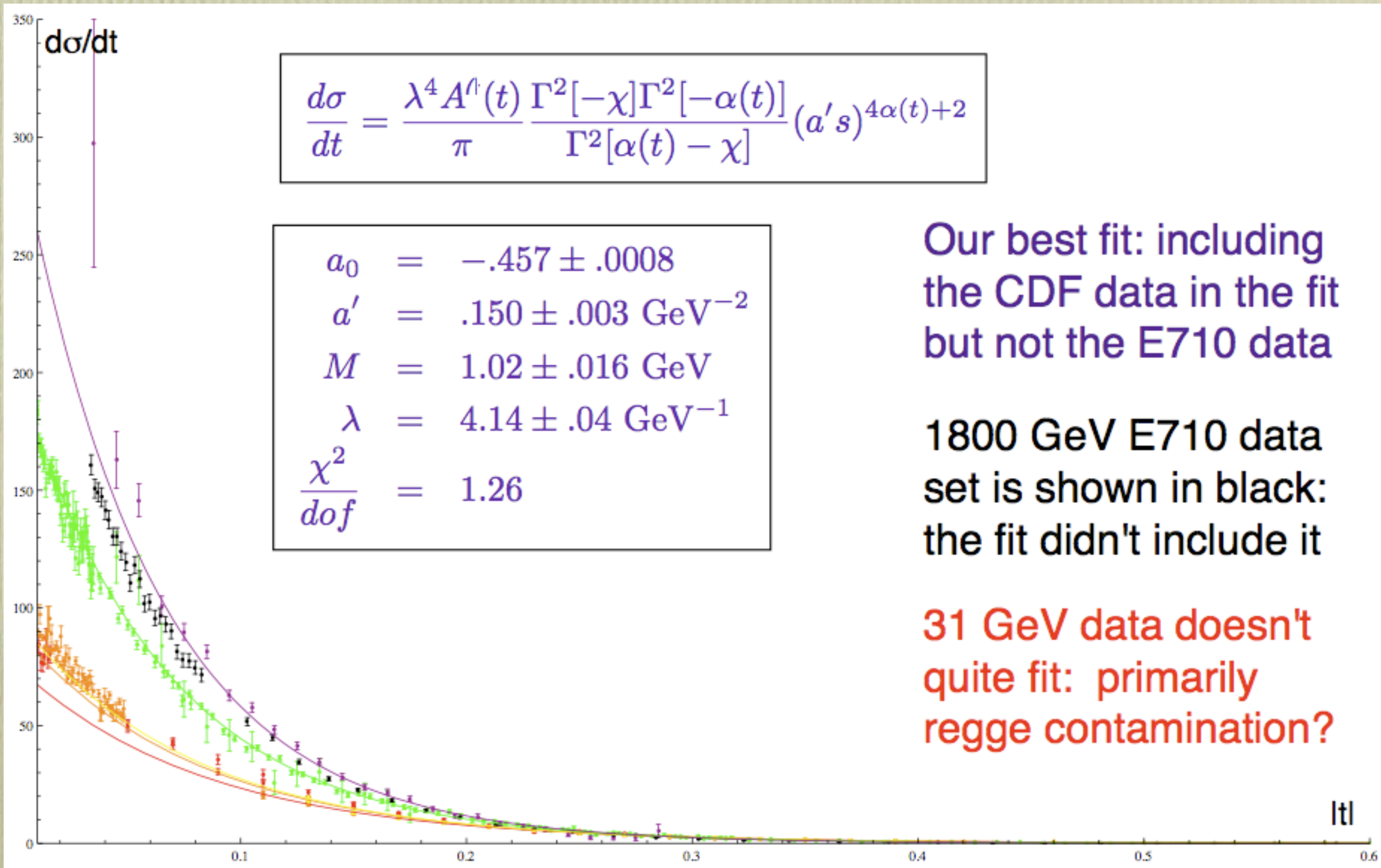
both data sets	just E710	just CDF
$\alpha_0 = 1.076 \pm .0016$	$\alpha_0 = 1.074 \pm .0016$	$\alpha_0 = 1.086 \pm .0016$
$\alpha' = .290 \pm .006\ \mathrm{GeV}^{-2}$	$\alpha' = .286 \pm .006\ \mathrm{GeV}^{-2}$	$\alpha' = .300 \pm .006\ \mathrm{GeV}^{-2}$
$M = .983 \pm .016\ \mathrm{GeV}$	$M = .970 \pm .016\ \mathrm{GeV}$	$M = 1.02 \pm .016\ \mathrm{GeV}$
$\lambda = 4.28 \pm .03\ \mathrm{GeV}^{-1}$	$\lambda = 4.31 \pm .03\ \mathrm{GeV}^{-1}$	$\lambda = 4.14 \pm .03\ \mathrm{GeV}^{-1}$
$\frac{\chi^2}{d.o.f.} = 1.65$	$\frac{\chi^2}{d.o.f} = 1.41$	$\frac{\chi^2}{d.o.f.} = 1.26$

DL fits

both data sets	just E710	just CDF
$\alpha_0 = 1.076 \pm .0013$	$\alpha_0 = 1.075 \pm .0013$	$\alpha_0 = 1.082 \pm .0018$
$\alpha' = .289 \pm .003\ \mathrm{GeV}^{-2}$	$\alpha' = .289 \pm .003\ \mathrm{GeV}^{-2}$	$\alpha' = .289 \pm .003\ \mathrm{GeV}^{-2}$
$\beta = 1.858 \pm .016\ \mathrm{GeV}^{-1}$	$\beta = 1.877 \pm .016\ \mathrm{GeV}^{-1}$	$\beta = 1.801 \pm .020\ \mathrm{GeV}^{-1}$
$\frac{\chi^2}{d.o.f} = 1.97$	$\frac{\chi^2}{d.o.f.} = 1.66$	$\frac{\chi^2}{d.o.f} = 1.79$

...χ^2 are comparable to (or better than) DL...

Best fit to differential cross section



$$\frac{d\sigma}{dt} = \frac{\lambda^4 A^4(t)}{\pi} \frac{\Gamma^2[-\chi]\Gamma^2[-\alpha(t)]}{\Gamma^2[\alpha(t)-\chi]} (a' s)^{4\alpha(t)+2}$$

$$\begin{aligned} a_0 &= -.457 \pm .0008 \\ a' &= .150 \pm .003 \text{ GeV}^{-2} \\ M &= 1.02 \pm .016 \text{ GeV} \\ \lambda &= 4.14 \pm .04 \text{ GeV}^{-1} \\ \frac{\chi^2}{dof} &= 1.26 \end{aligned}$$

Our best fit: including the CDF data in the fit but not the E710 data

1800 GeV E710 data set is shown in black: the fit didn't include it

31 GeV data doesn't quite fit: primarily regge contamination?

Total cross section

- The total cross section:

$$\sigma_{tot} = \frac{4\pi\lambda^2\Gamma[-\chi]}{\Gamma[1+a_0]\Gamma[a_0-\chi]}(a's)^{1+2a_0} \equiv Cs^b$$

- Using the best fit values from $\frac{d\sigma}{dt}$ results: $b = .0846, \; C = 21.325$

- Fitting to total cross section data:

both 1800's	$b = .076, \; C = 23.727$
just E710	$b = .074, \; C = 24.427$
just CDF	$b = .086, \; C = 21.097$

Prediction for $\rho = \frac{\mathrm{Re}\mathcal{A}(t=0)}{\mathrm{Im}\mathcal{A}(t=0)}$

- Pomeron exchange alone predicts constant value:

$$\rho = -\cot a_0 \pi = 0.136$$

- Relative phase with Reggeon term introduces some s-dependence

- From fitting to data

both 1800's	$\rho = 0.120$
just E710	$\rho = 0.117$
just CDF	$\rho = 0.136$ $\longleftarrow$

Computation of Parameters in the Sakai-Sugimoto Model

Reminder of Sakai-Sugimoto

- N_c color D4-branes (replaced by SUGRA background)

$$ds^2 = \left(\frac{U}{R}\right)^{3/2}\left(\eta_{\mu\nu}dx^\mu dx^\nu + f(U)\overbrace{d\tau^2}^{\text{period} \propto M_{KK}^{-1}}\right) + \left(\frac{R}{U}\right)^{3/2}\left(\frac{dU^2}{f(U)} + U^2 d\Omega_4^2\right)$$

$$e^\phi = g_s\left(\frac{U}{R}\right)^{3/4}, \quad F_4 = dC_3 = \frac{2\pi N_c}{V_4}\epsilon_4, \quad f(U) = 1 - \frac{U_{KK}^3}{U^3}$$

- N_f flavor D8-branes with $N_f \ll N_c$: assume nontrivial profile in (U,τ). (Geometrical realization of χsb)
- Fix M_{KK}, g_s using measured m_ρ, f_π

(Relevant) 4d Field Content

Bulk

h_{MN}

10d graviton

$h_{\mu\nu}(x,U)$

other stuff

$h^{(n)}_{\mu\nu}(x)T(U)$

KK tower

dual to 2^{++} glueball

Brane

A_M

9d gauge field

$A_\mu(x,U)$

$A_U(x,U)$

other stuff

$A^{(n)}(x)\psi_n(U)$

KK tower

dual to (axial) vector mesons

Gauging away A_U yields Skyrme Lagrangian

$\varphi_0(x)\psi_0(U)$

massless pion

What can we compute?

1. $-\frac{\alpha_0}{\alpha'}$ (from glueball mass)

2. λ (from graviton-Skyrmion coupling)

3. M_d (from Skyrmion energy-momentum tensor)

Glueball Mass

- Perturb around D4-brane background metric with $h_{\mu\nu}(p)T(U)$ to find mass eigenvalue equation for 4d graviton (Brower, Mathur, Tan: 2000)

$$\partial_\mu \left\{ U^4 f(U) \partial_U \left[\left(\frac{R}{U} \right)^{3/2} T(U) \right] \right\} \;=\; - m_g^2 \frac{R^{9/2}}{U^{1/2}} T(U)$$

- Lightest mode is leading 2^{++} glueball (heavier modes are on daughter trajectories)

$$m_g \;=\; 1.567\ M_{KK} \;=\; 1.485\ \text{GeV}$$

- Fit value:

$$m_g|_{fit} \;=\; \sqrt{\frac{-a_0}{a'}} \;=\; 1.745\ \text{GeV}$$

Graviton-Pion Coupling

- Treat protons as Skyrmions (4d Skyrme model arises naturally from DBI action)

- Decompose gauge and graviton fields: $\mathcal{U}(x) = e^{-i\pi(x)/f_\pi}$

$$\begin{aligned} A_\mu(x,U) &= \mathcal{U}^{-1}(x)\partial_\mu \mathcal{U}(x)\psi_+(U) \ + \ \text{meson tower} \\ h_{\mu\nu}(x,U) &= h_{\mu\nu}(x)T(U) \end{aligned}$$

- Expand DBI action to find relevant coupling

$$\begin{aligned} S_{D8} &\propto \int d^9x\ e^{-\phi}\sqrt{g}g^{\mu\alpha}g^{\nu\beta}h_{\alpha\beta}(x,z)\mathrm{Tr}\left\{g^{\gamma\delta}F_{\mu\gamma}F_{\nu\delta} + g^{UU}F_{\mu U}F_{\nu U}\right) + \ldots \\ &\propto \int d^4x\ h_{\alpha\beta}(x)\mathrm{Tr}\left\{A_h(\mathcal{U}^{-1}\partial^\alpha\mathcal{U})(\mathcal{U}^{-1}\partial^\beta\mathcal{U}) + B_h\left[\mathcal{U}^{-1}\partial^\alpha\mathcal{U},\mathcal{U}^{-1}\partial_\rho\mathcal{U}\right]\left[\mathcal{U}^{-1}\partial^\beta\mathcal{U},\mathcal{U}^{-1}\partial^\rho\mathcal{U}\right]\right\} + \ldots \\ &\propto \int d^4x\ \lambda h_{\alpha\beta}T^{\alpha\beta} \ + \ \text{correction} \end{aligned}$$

Glueball-proton coupling

- Overlap integral from SS yields

$$\lambda \;=\; 0.389\; f_\pi^{-1} \;=\; 4.18\ \mathrm{GeV}^{-1}$$

- Compare to fit value

$$\lambda|_{fit} \;=\; 4.14 \pm 0.04\ \mathrm{GeV}^{-1}$$

- Correction suppressed by 2 orders of magnitude

M_d from the Skyrme Model

- M_d appears in the proton form factor

- In the Regge limit we approximate

$$\langle p's'|T_{\mu\nu}|p,s\rangle = \overline{u}(p',s')\left[A(t)\gamma_{(\mu}P_{\nu)} \;\;+\;\; \ldots\right]u(p,s)$$

- $A(t)$ is well-approximated by dipole form. Computed in Skyrme model by Cebula, Goeke, Ossmann, Scheitzer

$$A(t) = \frac{A(0)}{(t-M_d^2)^2} \qquad \text{with} \qquad M_d \;=\; 1.17 \text{ GeV}$$

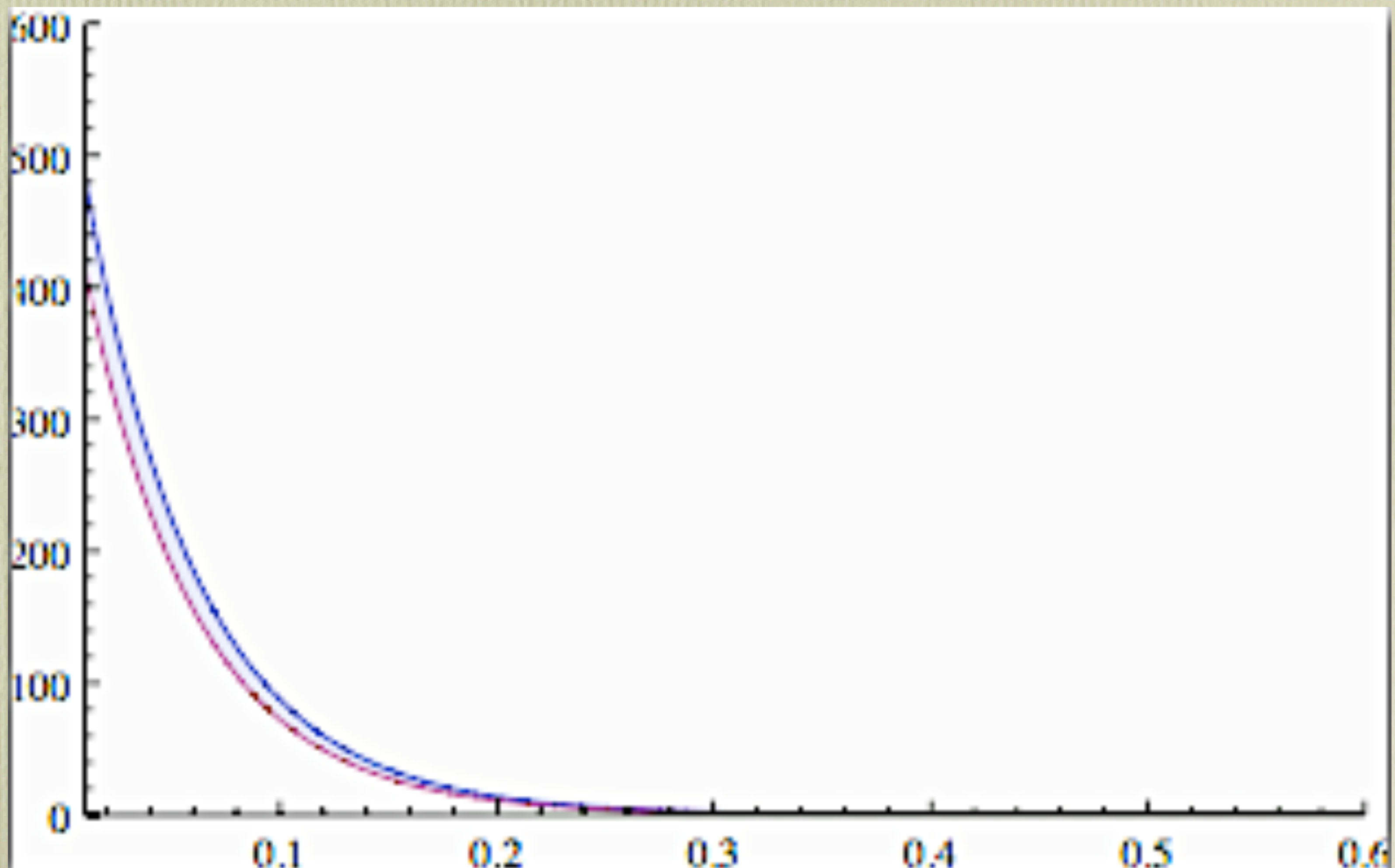
- Compare to fit value $M_d \;=\; 1.02 \pm 0.016 \text{ GeV}$

Conclusions and Future Directions

- Constructed a model for pp and ppbar scattering
  - Used holography to find structure of couplings
  - Used closed string scattering to Reggeize the amplitude
- Fit the model to data: as good or better than DL model
- Computed (some) parameters in SS: reasonable agreement
- Future directions: other scattering processes, inclusion of Reggeons

...and our prediction for the LHC:

$$\frac{d\sigma}{dt}$$



t (GeV)

$$s = 196\ \text{TeV}^2 \quad \sigma_{tot} = 109\ \text{mb}$$