Chiral crystalline phases in phenomenological models of QCD

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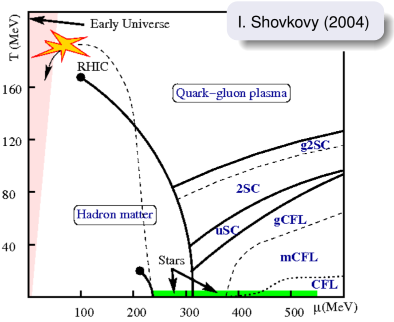
Dynamics of Symmetry Breaking
April 2009, Argonne

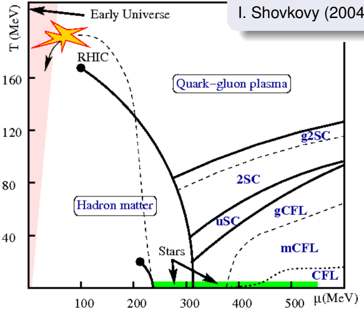
schematic phase diagram of QCD



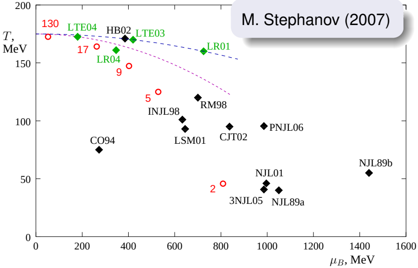
introduction: QCD phase diagram and critical point

schematic phase diagram of QCD

I. Shovkovy (2004)



M. Stephanov (2007)

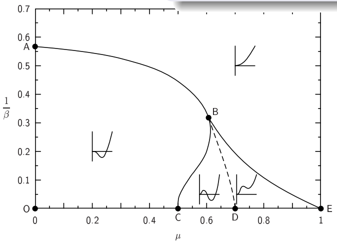


moderate densities and $N_c = 3$
$\longrightarrow$ typically investigated in NJL-type models

won't try to add an additional point, instead ...

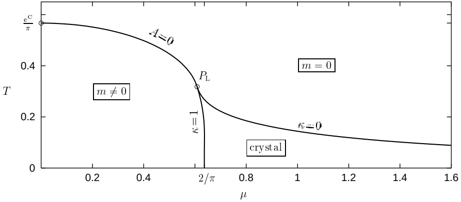
motivation: (massless) Gross-Neveu model on mean-field (i.e. large N)

A. Barducci et al. (1995)



$\longrightarrow$

O. Schnetz *et al.* (2004)



inhomogeneous phases

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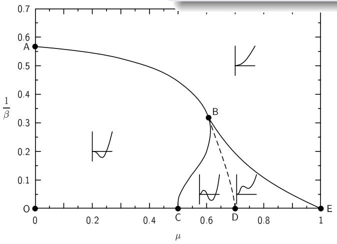

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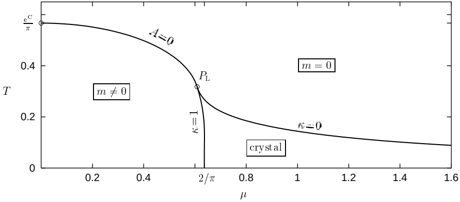

analogues in QCD/NJL:

- large N analysis — D. Deryagin *et al.* (1992)
- plane-wave — E. Nakano *et al.* (2005)
- Ginzburg-Landau expansion at $T = 0$ — R. Rapp *et al.* (2001)

outline

1. Ginzburg-Landau analysis at chiral critical point
2. one-dimensional modulations in the NJL model
3. one-dimensional modulations in the quark-meson model
4. outlook and summary

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NJL model in mean-field approximation

- NJL Lagrangian (1+1D $\rightarrow$ chiral Gross-Neveu)

$$\mathcal{L}_{NJL} = \bar{\psi}\left(i\gamma^\mu\partial_\mu - m_0\right)\psi + \frac{G}{2}\left((\bar{\psi}\psi)^2 + (\bar{\psi}i\gamma^5\tau^a\psi)^2\right)$$

- expand around $\langle\bar{\psi}\psi\rangle = \frac{m_0 - M(x)}{G}$ and $\langle\bar{\psi}i\gamma^5\tau^a\psi\rangle = 0$

$$\mathcal{L}_{NJL}^{mean-field} = \bar{\psi}\left(i\gamma^\mu\partial_\mu - M(x)\right)\psi - \frac{1}{4G}\left(M(x) - m_0\right)^2$$

- thermodynamic potential (V $\equiv$ Wigner-Seitz cell)

$$\Omega_{NJL}^{mean-field}[T,\mu;M(x)] = -\frac{T}{V}\mathrm{Tr}\,\mathrm{Log}\left(i\partial_t - \underbrace{\gamma^0(-i\gamma^i\partial_i + M(x))}_{H_{NJL}} + \mu\right) + \frac{1}{V}\int_V \frac{1}{4G}\left(M(x) - m_0\right)^2$$

- gap-equation from $\frac{\delta\Omega}{\delta M(x)} = 0$

complexity of mean-field approximation

- evaluation of mean-field thermodynamic potential

$$\frac{T}{V}\mathrm{Tr}\,\mathrm{Log}\Big(i\partial_t - H + \mu\Big) \quad \propto \quad \sum_n \Big(E_n + 2T\,\ln(1 + e^{-(E_n-\mu)/T})\Big)$$

$\rightarrow$ quasiparticle energy spectrum

- gap-equation

$$M(x) \quad = \quad m_0 + 2GN_c \sum_n \tanh\left(\frac{\mu - E_n}{T}\right) \psi^\dagger_{E_n} \frac{\partial H_{NJL}}{\partial M(x)} \psi_{E_n}$$

$\rightarrow$ self-consistency condition on eigensystem of Hamiltonian

homogeneous phases	$\rightarrow$	one-parameter problem
inhomogeneous phases	$\rightarrow$	?

go one step back

$$\Omega_{NJL}^{mean-field}[T,\mu;M(x)] = -\frac{T}{V}\mathrm{Tr}\,\mathrm{Log}\Big(i\gamma^\mu\partial_\mu + \mu\gamma^0 - M(x)\Big) + \frac{1}{V}\int_V \frac{1}{4G}\left(M(x)-m_0\right)^2$$

and expand in $M(x)$

$$\Delta\Omega_{NJL}^{mean-field}[T,\mu;M(x)] = -\frac{T}{V}\sum_{n>0}\frac{1}{n}\mathrm{Tr}_{D,c,f,V}\left(S_0(x_i,x_{i+1})M(x_{i+1})\right)^n + \frac{1}{V}\int_V \frac{1}{4G}\left(M(x)-m_0\right)^2$$

finally Taylor expanding $M(x_i)$ and ordering gives ...

generalized Ginzburg-Landau expansion II

$$
\begin{aligned}
\Delta\Omega_{NJL}^{mean-field}[T,\mu;M(x)] \quad = \quad & -\frac{1}{2G}m_0 M(x) \\
& +\frac{\alpha_2}{2}M(x)^2 \\
& +\frac{\alpha_4}{4}\left(M(x)^4+(\nabla M(x))^2\right) \\
& +\frac{\alpha_6}{6}\left(M(x)^6+5M(x)^2(\nabla M(x))^2+\frac{1}{2}(\Delta M(x))^2\right) \\
& +\dots
\end{aligned}
$$

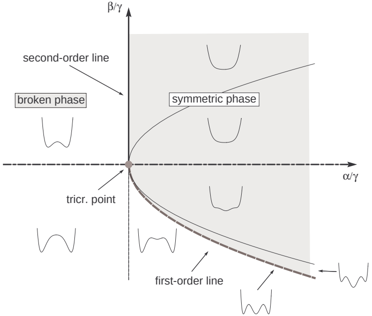
where

$$
\alpha_n \quad = \quad (-1)^{n/2}4N_fN_cT\sum_n\int_{reg.}\frac{d^3p}{(2\pi)^2}\frac{1}{((\omega_n+i\mu)+p^2)^{n/2}}+\frac{\delta_{2n}}{2G}
$$

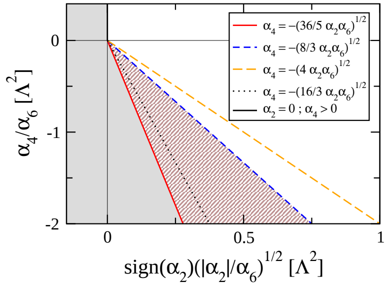
remarks:

- similar form as in Gross-Neveu model - not coincidental
- inhomogeneous phases go up to chiral critical point ($\alpha_2=\alpha_4=0$)!
- assumption on regularization!

for simplicity chiral limit ($m_0 = 0$)



$\longrightarrow$



$$\begin{aligned}\Delta\Omega &= \frac{\alpha}{2}M^2 + \frac{\beta}{2}M^4 + \frac{\gamma}{6}M^6 \\ &\rightarrow \frac{\alpha_2}{2}M^2 + \frac{\alpha_4}{2}(M^4 + M'^2) + \frac{\alpha_6}{6}(M^6 + 5M^2M'^2 + \frac{1}{2}M''^2)\end{aligned}$$

no first order transition!

outline

inhomogeneous solutions in the GN model

mean-field problem (naive):
guess 'super-potential' $M(x)$

$$\begin{pmatrix} & -id_x - iM(x) \\ -id_x + iM(x) & \end{pmatrix} \begin{pmatrix} \phi_+(x) \\ \phi_-(x) \end{pmatrix} = E \begin{pmatrix} \phi_+(x) \\ \phi_-(x) \end{pmatrix}$$

or better

$$(-d_x^2 + M(x)^2 \pm M'(x))\phi_{\pm,n}(x) = E_n^2 \phi_\pm(x)$$

check self-consistency

$$M(x) = m_0 + G \sum_n \tanh\left(\frac{\mu - E_n}{T}\right) (\phi_{+,n}^*, \phi_{-,n}^*) \begin{pmatrix} & -i \\ i & \end{pmatrix} \begin{pmatrix} \phi_{+,n} \\ \phi_{-,n} \end{pmatrix}$$

all known cases: self-consistency for each orbit separately!

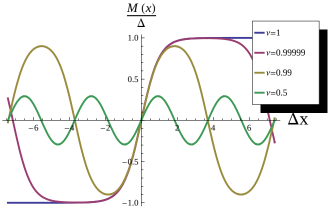
O. Schnetz *et al.* (2004); O. Schnetz *et al.* (2005); G. Basar *et al.* (2008)

the real, one-band solution

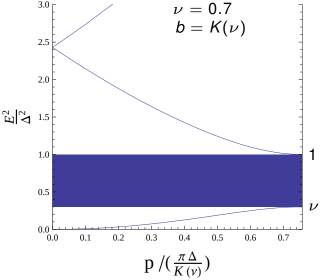
self-consistent solution:

$$\color{blue}{M(x)} = \nu\Delta \mathrm{sn}(b|\nu)\mathrm{sn}(\Delta x|\nu)\mathrm{sn}(\Delta(x+b)|\nu) + \Delta\frac{\mathrm{cn}(b|\nu)\mathrm{dn}(b|\nu)}{\mathrm{sn}(b|\nu)}$$

three parameters: scale Δ, elliptic modulus ν, explicit breaking b



$\updownarrow$ $b \neq K(\nu)$



$\uparrow$ $b \neq K(\nu)$

lower dimensions $\rightarrow$ lower dimensional modulations

- concentrate on one-dimensional modulations: $M(x) \equiv M(z)$
  translation invariance in xy-direction $\rightarrow$ conserved $P_\perp$
- only need eigenvalues of H with $p_\perp = 0$:
  $\psi_{E,p_\perp} = S^{-1}(\Lambda(\lambda, p_\perp))\psi_{\lambda,0}; \qquad E = \sqrt{\lambda^2 + p_\perp^2}$
- Hamiltonian for $p_\perp = 0$

$$H_{NJL} = \begin{pmatrix} H_{GN} & \\ & H_{GN} \end{pmatrix}$$

(general feature for lower dimensional modulations)

NJL model and regularization

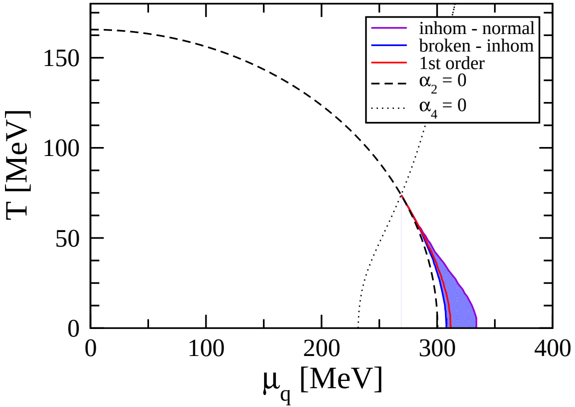
$$\Omega_{NJL}[T,\mu;M(x)] \quad = \quad \underbrace{\Omega_{NJL}[0,0;M(x)]}_{\textit{UV divergent}} + \underbrace{\delta\Omega_{NJL,medium}[T,\mu;M(x)]}_{\textit{finite}}$$

- inhomogeneous phases $\rightarrow$ no momentum cutoff possible (phase diagram phenomenology?!)

$$\mathrm{Tr}\,\mathrm{Log}\,(i\omega_n + H) \quad \rightarrow \quad -\frac{1}{2}\mathrm{Tr}\int_0^\infty \frac{d\tau}{\tau} f(\tau) e^{-\tau(\omega_n^2+H^2)}$$

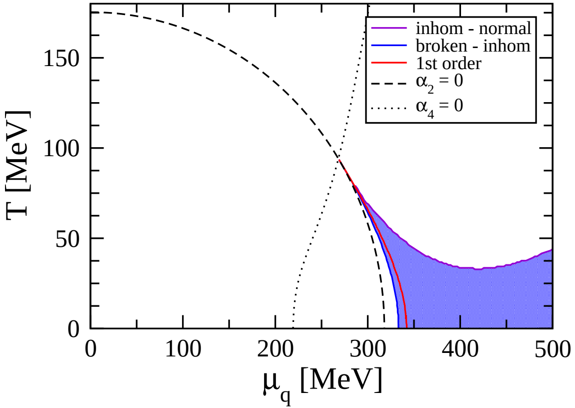
  blocking function $f(\tau) = 1 - 3e^{-\tau\Lambda^2} + 3e^{-\tau 2\Lambda^2} - e^{-\tau 3\Lambda^2}$

- parameters $\{G, \Lambda, m_0 = 0\} \leftrightarrow \{M_q \gtrsim M_N/3, f_\pi = 88 MeV\}$



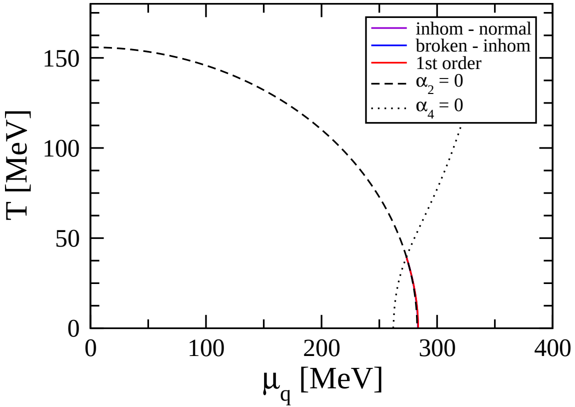
- $M_q = 300\text{MeV} \quad \Rightarrow \quad \langle\bar{\psi}\psi\rangle = (-193 MeV)^3$
- no 1st order phase transition; CP at small T, high μ

phase diagram



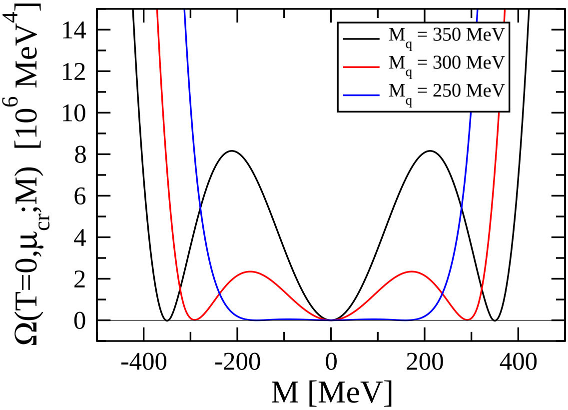
- $M_q = 350\text{MeV} \quad \Rightarrow \quad \langle\bar{\psi}\psi\rangle = (-186MeV)^3$
- regularization artefact?!

phase diagram



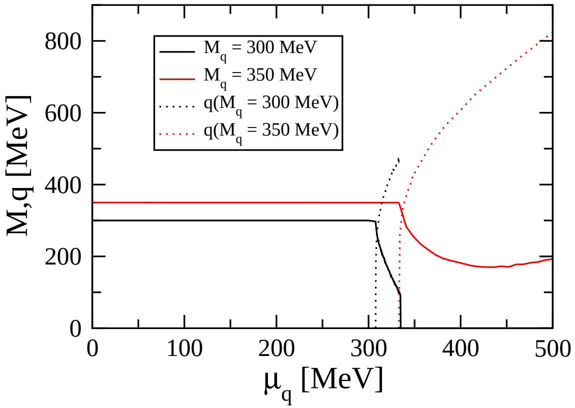
- $M_q = 250\text{MeV} \quad \Rightarrow \quad \langle\bar{\psi}\psi\rangle = (-208 MeV)^3$
- very weak phase transition

the strength of the phase transition



- thermodynamic potential (effective action) at 1^{st} order transition
- region for inhomogeneous phases $\leftrightarrow$ strength of phase transition

zero temperature



- wave-vector q continuously from 0 to $\sim \mu$
- maximum of $M(z) \equiv M$ continuous

outline

the quark-meson model on mean-field

$$\mathcal{L} = \bar{\psi}\left(i\gamma^\mu\partial_\mu - g(\sigma + i\gamma_5\tau^a\pi^a)\right)\psi - U(\sigma,\pi^a)$$

$$U(\sigma,\pi^a) = -\frac{1}{2}\left(\partial_\mu\sigma\partial^\mu\sigma + \partial_\mu\pi^a\partial^\mu\pi^a\right) + \frac{\lambda}{4}\left(\sigma^2 + \pi^a\pi^a - v^2\right)^2 - c\sigma$$

consider chirally broken phase: fix $\{g, v, \lambda, c\}$ through
$\{M_q = 300\mathrm{MeV},\ f_\pi = 93\mathrm{MeV},\ m_\sigma = 600\mathrm{MeV},\ m_\pi\}$

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consider chirally broken phase: fix $\{g, v, \lambda, c\}$ through $\{M_q = 300\mathrm{MeV},\ f_\pi = 93\mathrm{MeV},\ m_\sigma = 600\mathrm{MeV},\ m_\pi\}$

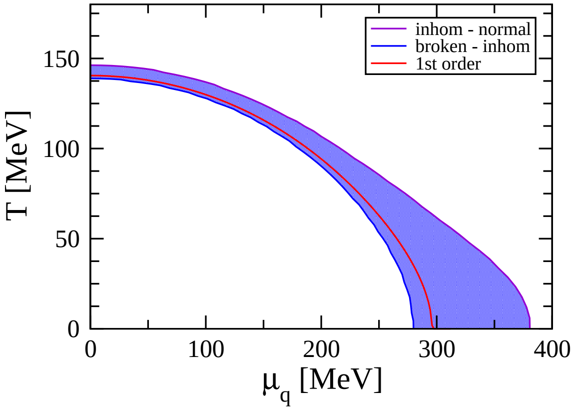
thermodynamic on 'mean-field':

$$\Omega_{QM}[T,\mu; M(x) = g\sigma(x)] = U(\sigma(x),0) + \underbrace{\delta\Omega_{medium}[T,\mu; M(x)]}_{\text{as in NJL}}$$

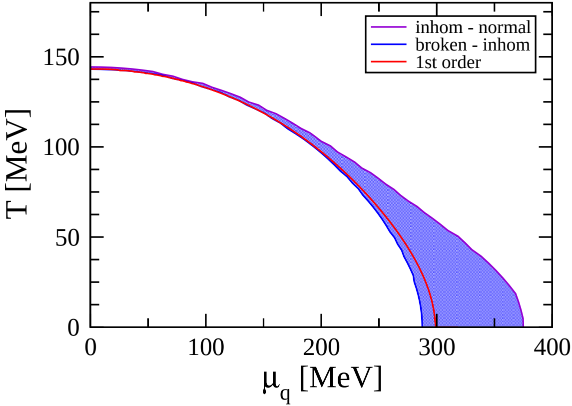
O. Scavenius *et al.* (2000); B. Schaefer *et al.* (2006)

$\rightarrow$ evaluate Ω_{QM} for inhomogeneous $M(x)$ (self-consistent?)

phase diagram

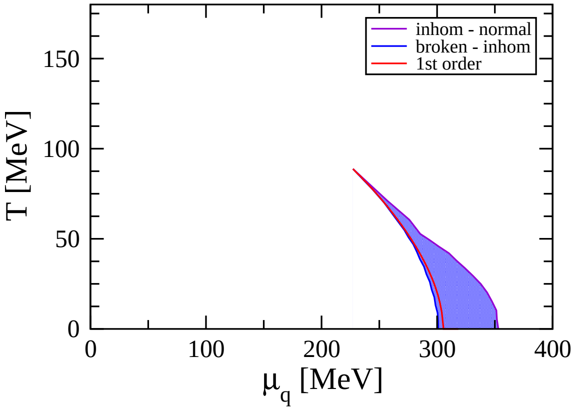


- $M_\pi = 0\text{MeV}$
  $\rightarrow$ no CP, significant domain for inhomogeneous phases



- $M_\pi = 69\text{MeV}$
  $\rightarrow$ still no CP, domain shrinks

phase diagram



- $M_\pi = 138\text{MeV} \quad \rightarrow \quad$ similar scenario as in NJL model before (2nd order transition turn into cross-over for $M_\pi \neq 0$)

outline

summary

inhomogeneous phases in NJL-type models

- generalized Ginzburg-Landau expansion
  $\longrightarrow$ generically reach out to critical point
- similarity to $1+1D$ models
  $\longleftrightarrow$ lower dimensional soltions to lower dimensional modulations
  $\longrightarrow$ no first-order phase transition
- strong first order phase transition
  $\longrightarrow$ significant window in phase diagram

future prospects

- more 'realistic' models
- higher dimensional inhomogeneities
- properties of the inhomogeneous phase
- ...