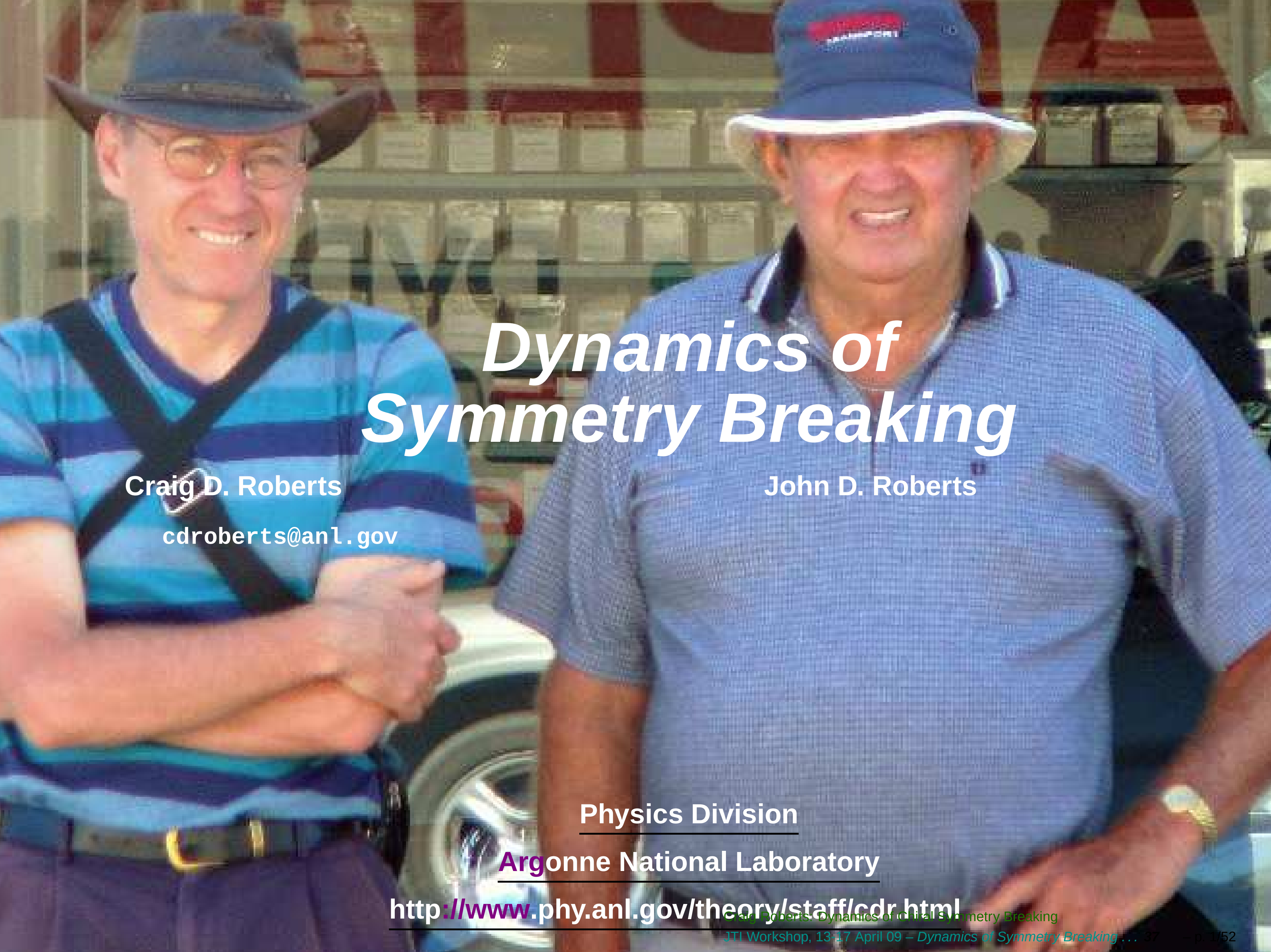

Dynamics of Symmetry Breaking

Craig D. Roberts

John D. Roberts

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Physics Division

Argonne National Laboratory

http://www.phy.anl.gov/theory/staff/cdr.html

Universal Truths

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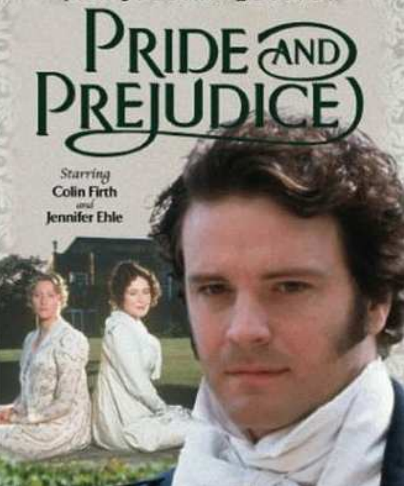

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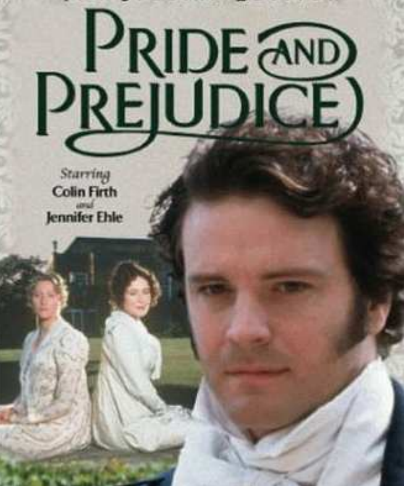

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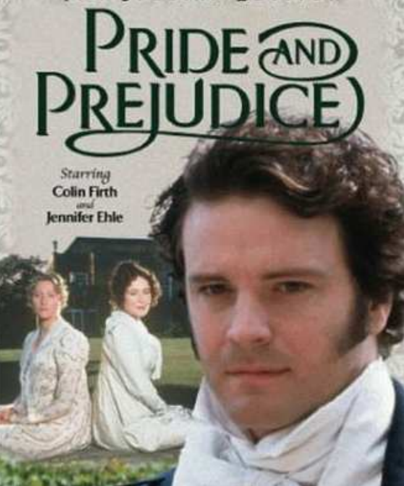

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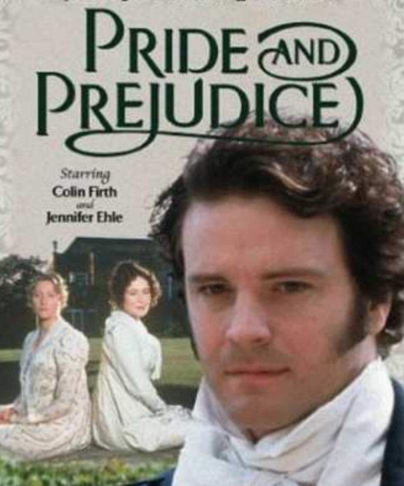

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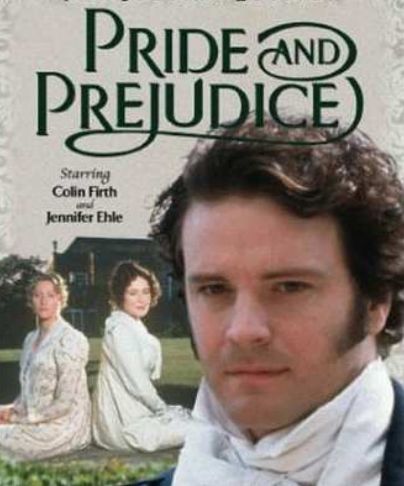

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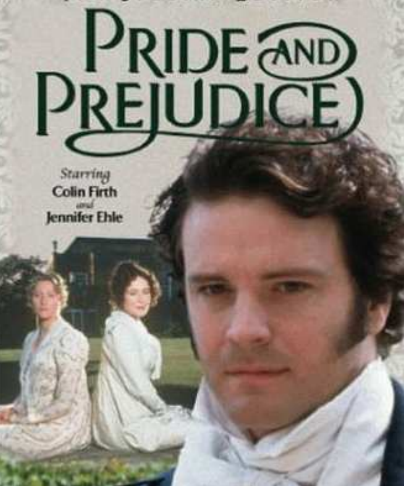

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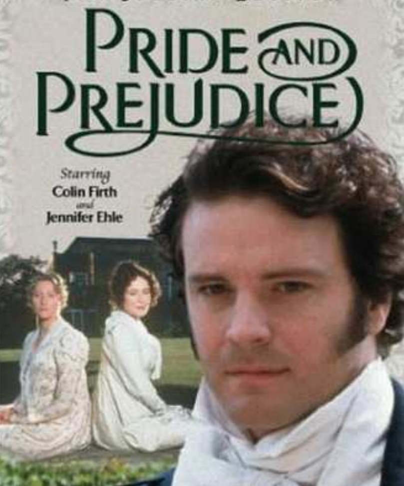

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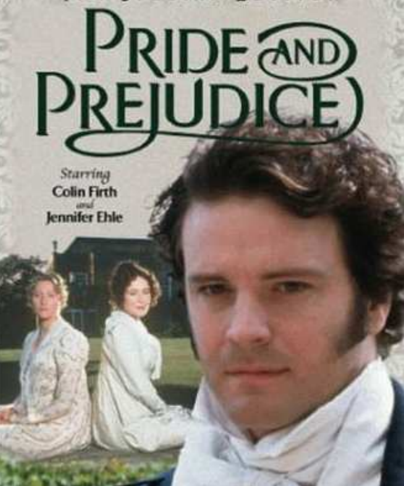

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- Challenge: understand relationship between parton properties on the light-front and rest frame structure of hadrons. Problem because, e.g., DCSB - an established keystone of low-energy QCD and the origin of constituent-quark masses - has not been realised in the light-front formulation.

QCD's Challenges

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- Quark and Gluon Confinement
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Understand Emergent Phenomena

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- QCD – Complex behaviour

  arises from apparently simple rules

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Dichotomy of Pion
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  - Interaction between quarks – the Interquark "Potential" – unknown throughout $>$ 98% of a hadron's volume

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Intranucleon Interaction

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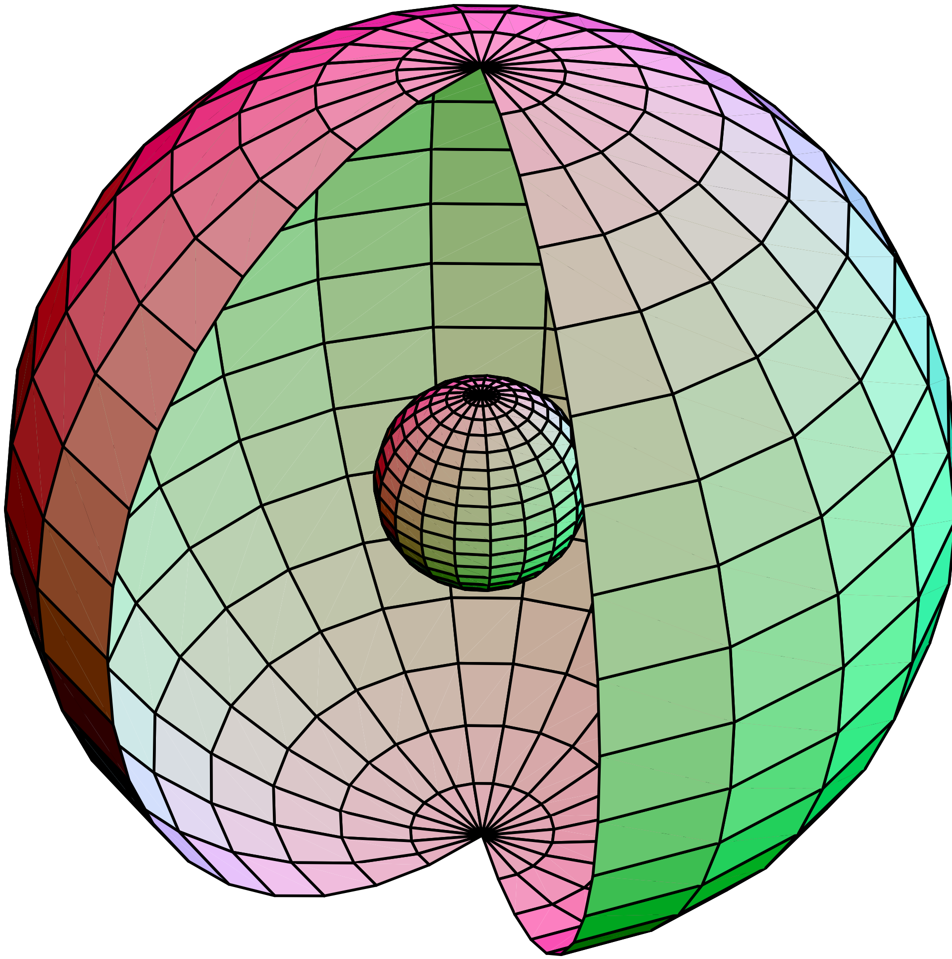

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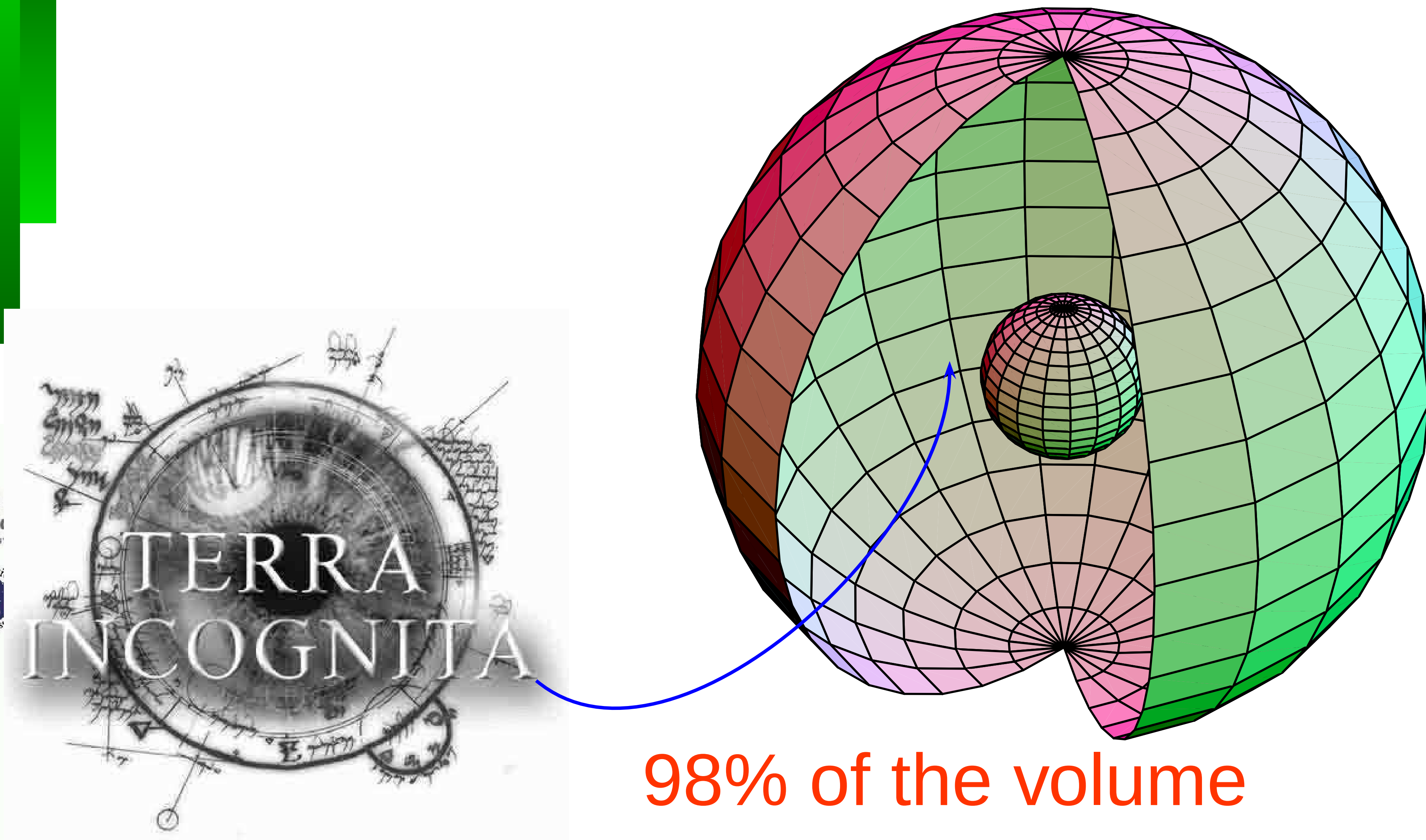
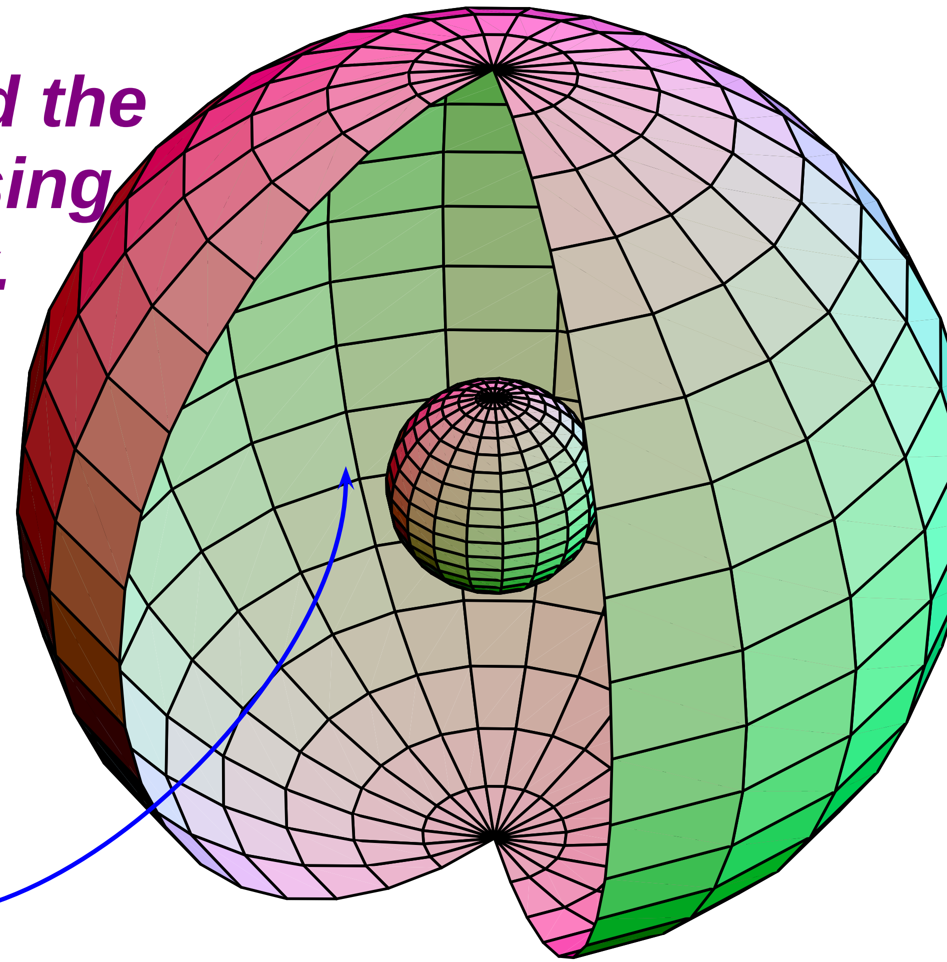


98% of the volume

What is the Intranucleon Interaction?

The question must be rigorously defined, and the answer mapped out using experiment and theory.



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Confinement

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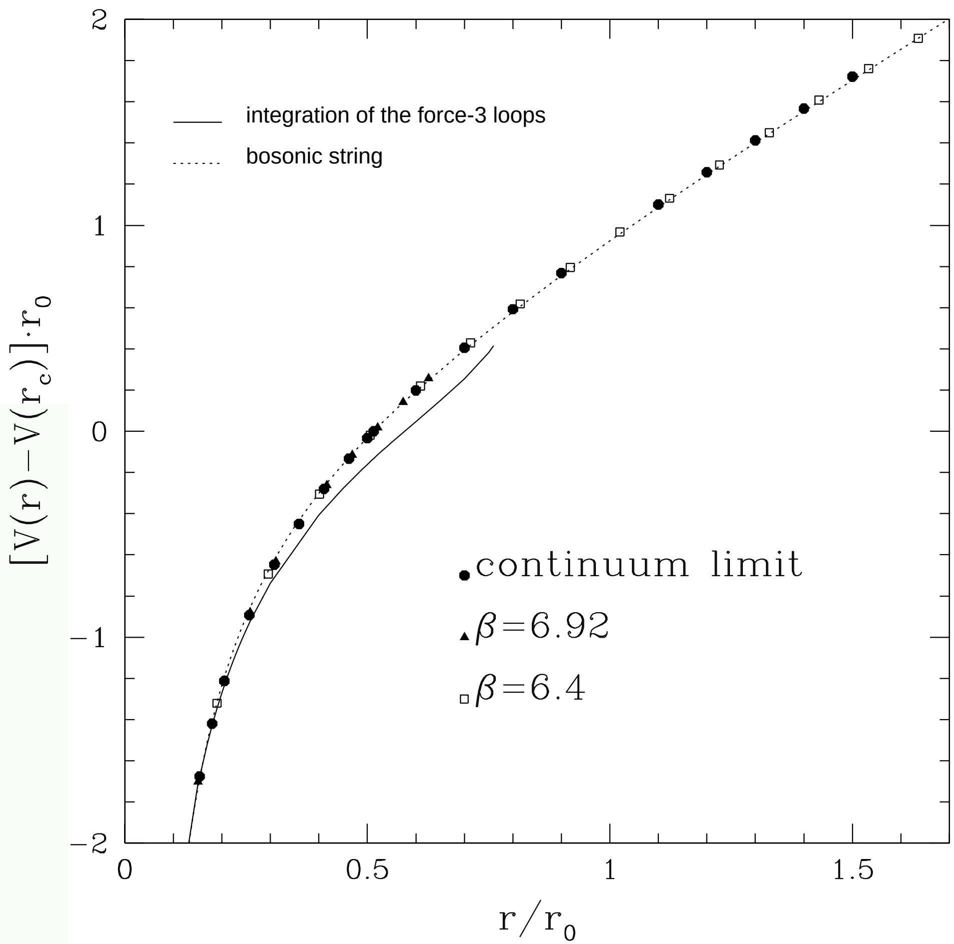
Confinement

- Infinitely Heavy Quarks . . . Picture in Quantum Mechanics



$$V(r) = \sigma\, r - \frac{\pi}{12}\,\frac{1}{r}$$

$\sigma \sim 470\,$MeV

Necco & Sommer
he-la/0108008

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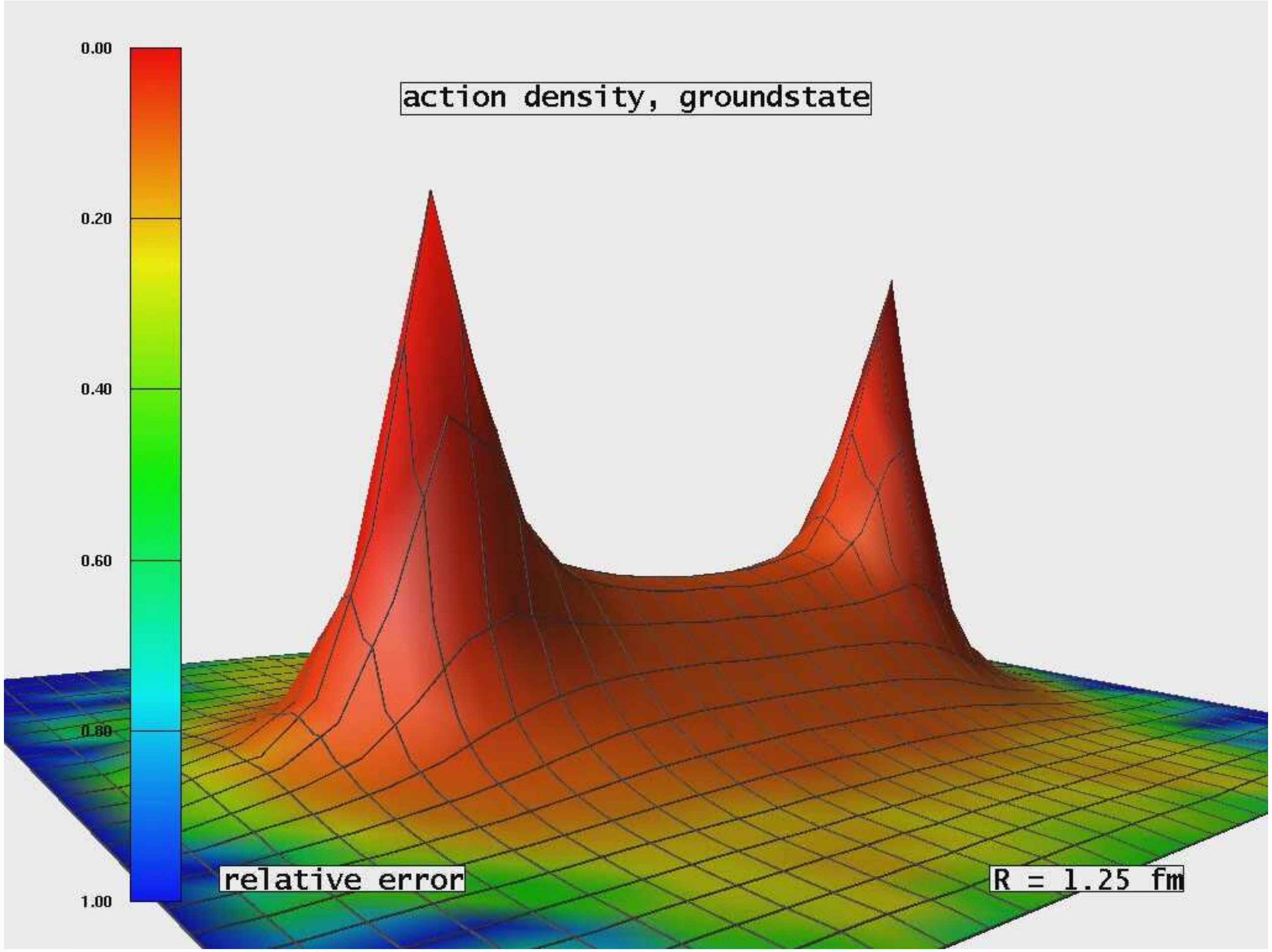
Confinement

- Illustrate this in terms of the action density ... analogous to plotting the Force $= F_{\bar{Q}Q}(r) = \sigma + \frac{\pi}{12}\frac{1}{r^2}$



Bali, *et al*.
he-la/0512018

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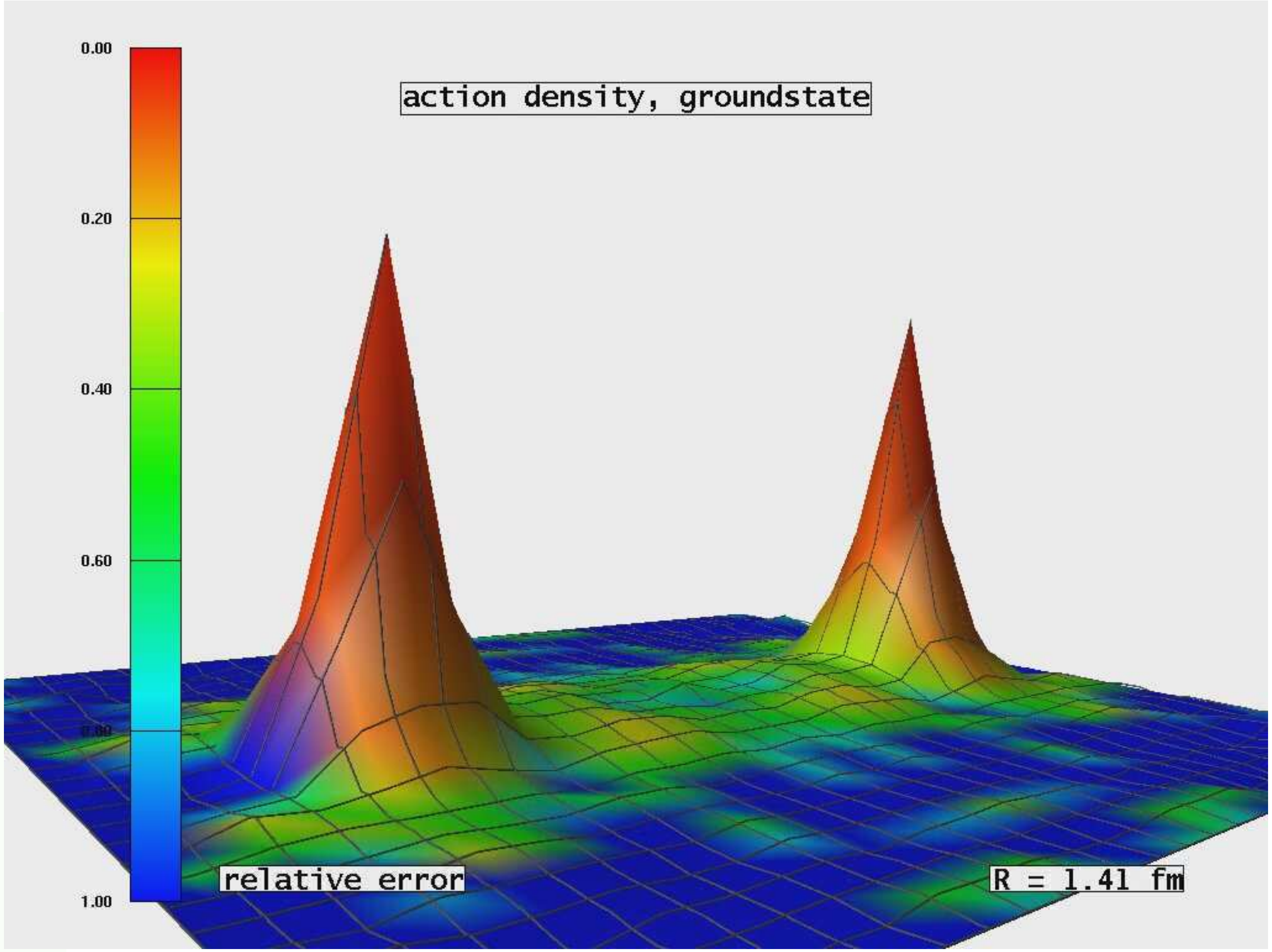

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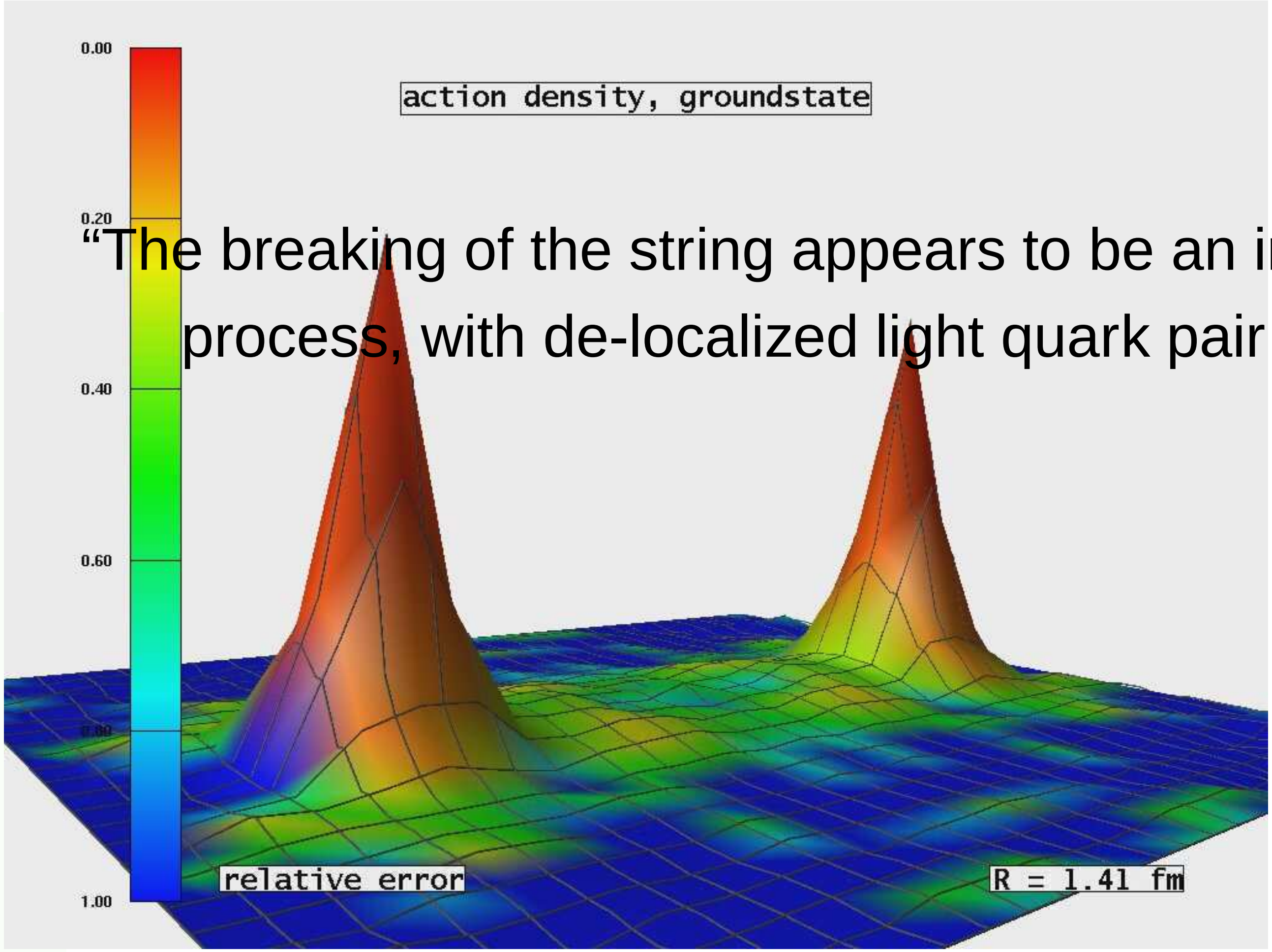

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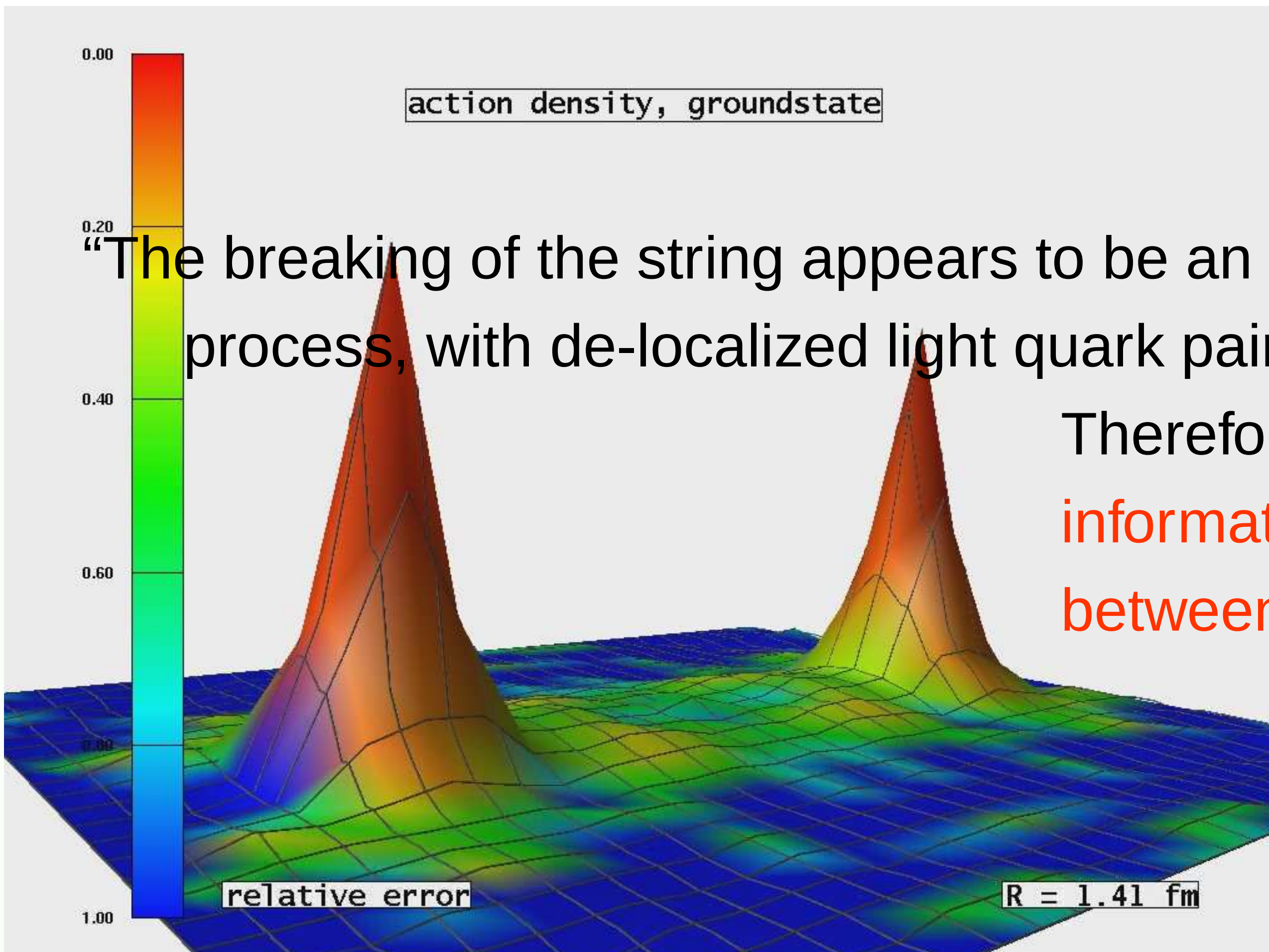

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Therefore . . . No information on *potential* between light-quarks.

What is the light-quark Long-Range Potential?

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Potential between static (infinitely heavy) quarks measured in simulations of lattice-QCD ***is not related*** in any simple way to the light-quark interaction.

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- This is a well-posed problem whose solution is an elemental goal of modern hadron physics.

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- DSEs connect β-function to experimental observables. Hence, comparison between computations and observations of, e.g., hadron mass spectrum can be used to chart β-function's long-range behaviour
- To realise this goal, a nonperturbative symmetry-preserving DSE truncation is necessary
    - On other hand, at present significant qualitative advances possible with symmetry-preserving kernel *Ansätze* that express important additional nonperturbative effects, difficult to capture in any finite sum of contributions

Dyson-Schwinger Equations

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Cross-Sections built from Schwinger Functions

Perturbative Dressed-quark Propagator

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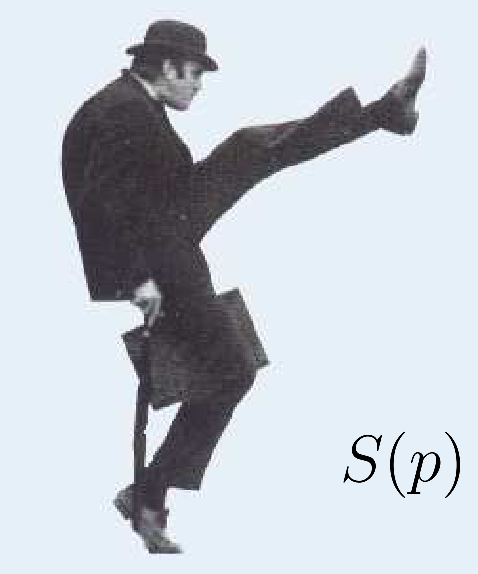
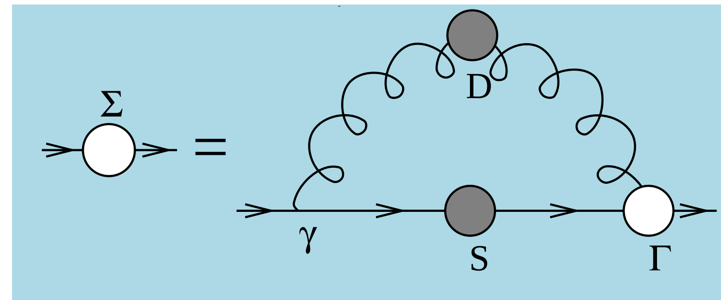


$$S(p) = \frac{Z(p^2)}{i\gamma \cdot p + M(p^2)}$$



Gap Equation

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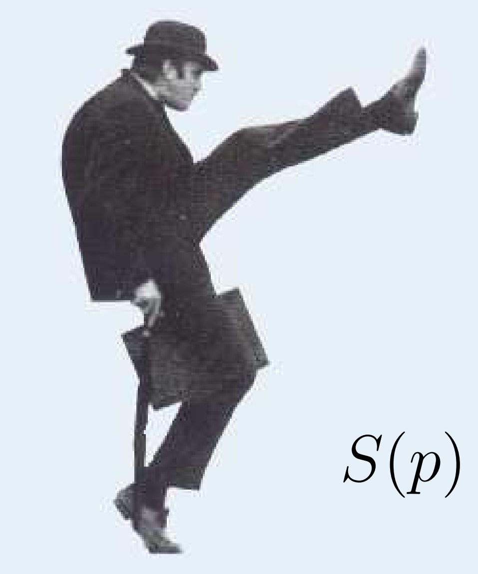

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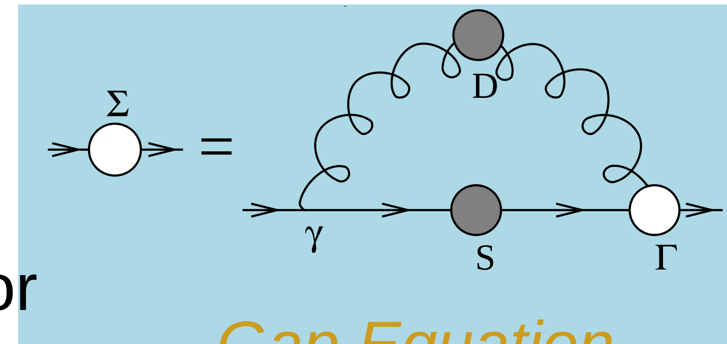

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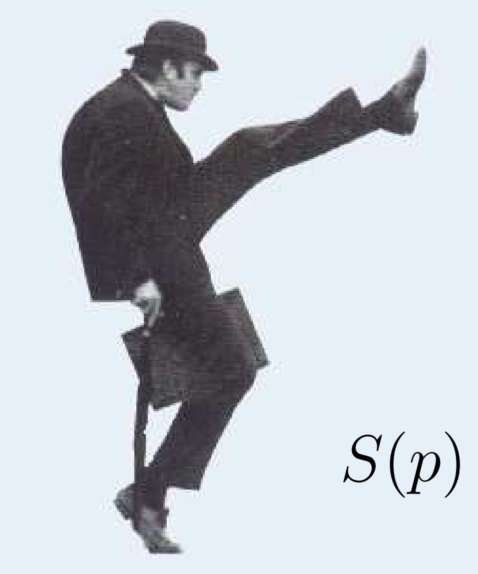

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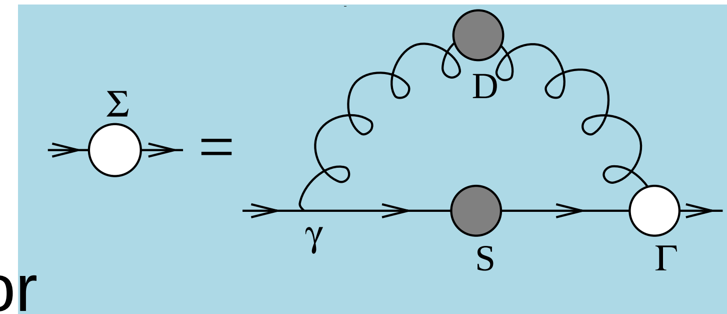

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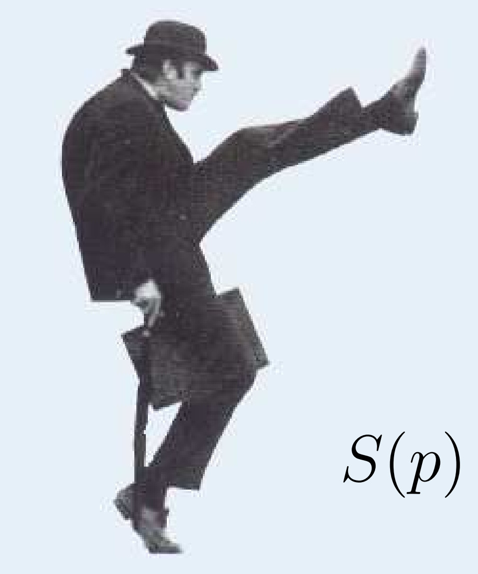

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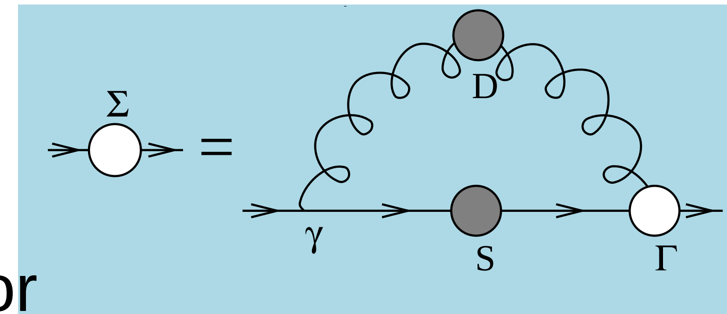

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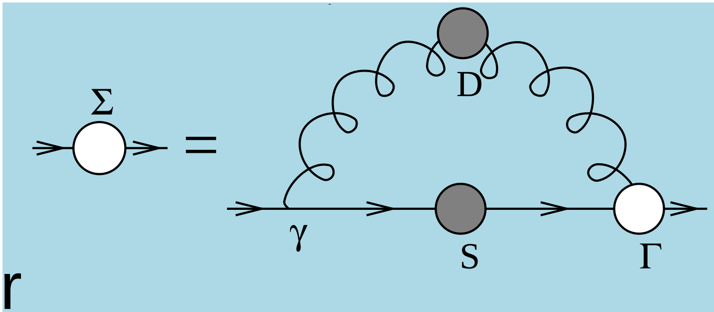

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No DCSB Here!

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Frontiers of Nuclear Science: A Long Range Plan (2007)

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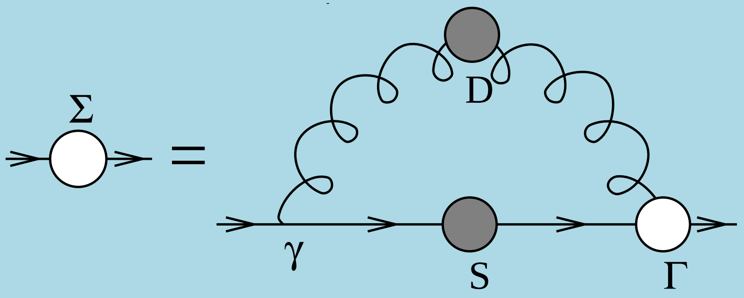
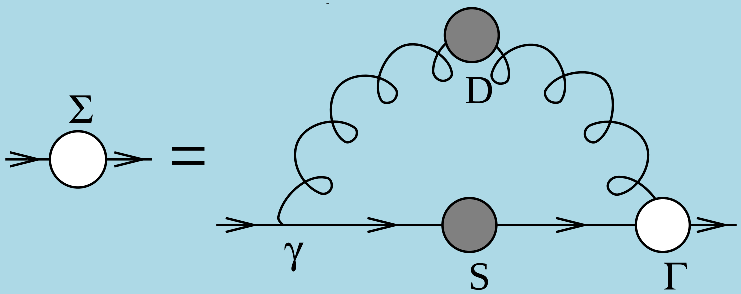
Frontiers of Nuclear Science: Theoretical Advances



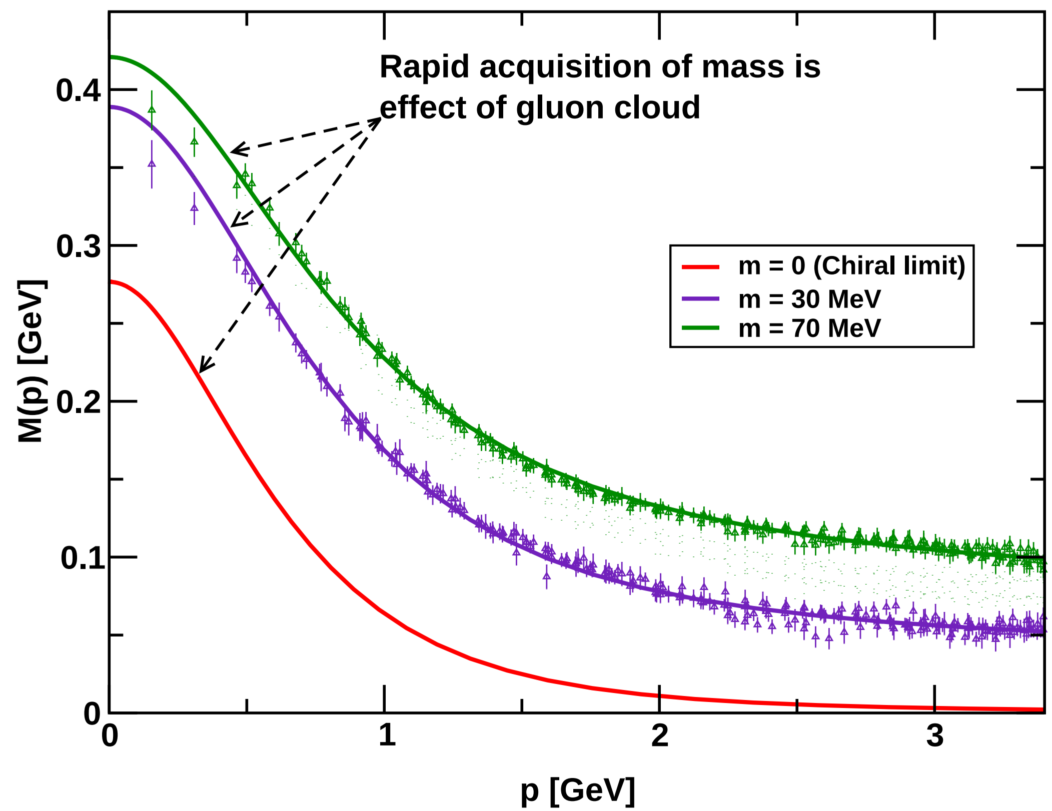
Gap Equation

Frontiers of Nuclear Science: Theoretical Advances



Gap Equation

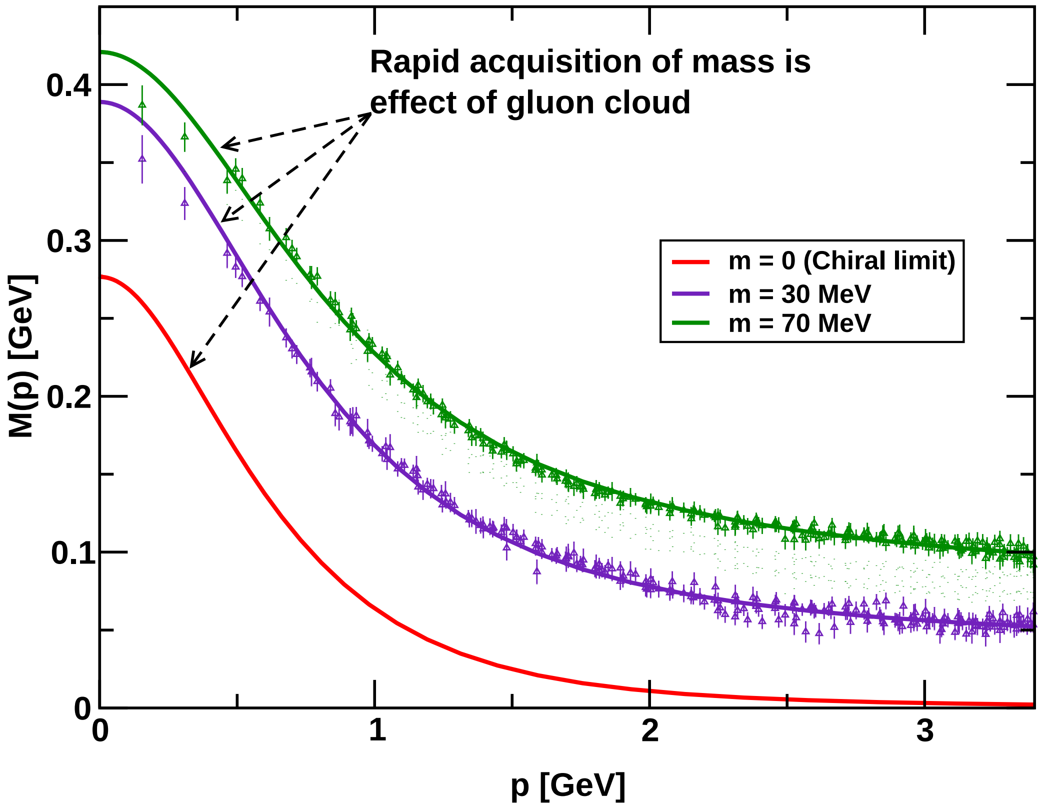
$$S(p) = \frac{Z(p^2)}{i\gamma \cdot p + M(p^2)}$$



Mass from nothing.

$$S(p) = \frac{Z(p^2)}{i\gamma \cdot p + M(p^2)}$$

In QCD a quark's effective mass depends on its momentum. The function describing this can be calculated and is depicted here. Numerical simulations of lattice QCD (data, at two different bare masses) have confirmed model predictions (solid curves) that the vast bulk of the constituent mass of a light quark comes from a cloud of gluons that are dragged along by the quark as it propagates. In this way, a quark that appears to be absolutely massless at high energies ($m = 0$, red curve) acquires a large constituent mass at low energies.



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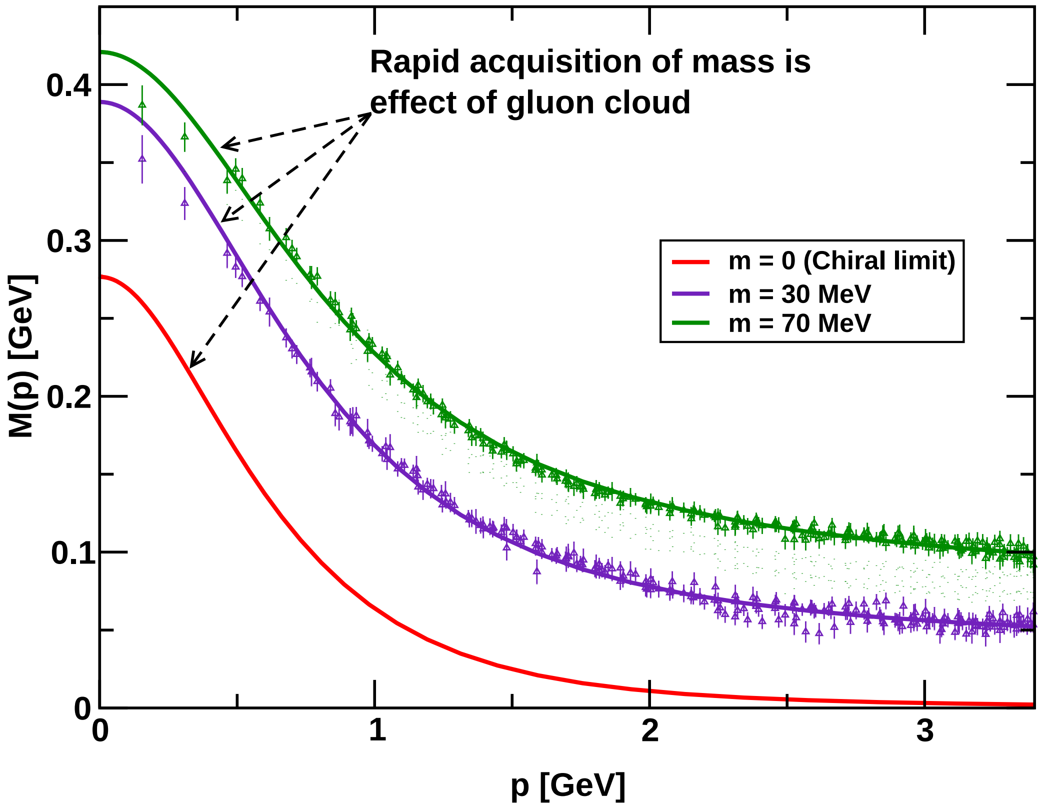

In QCD

a quark's mass must depend on

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俳句


ALWAYS LOOK
ON THE
BRIGHT SIDE

- Established understanding of two- and three-point functions

Hadrons

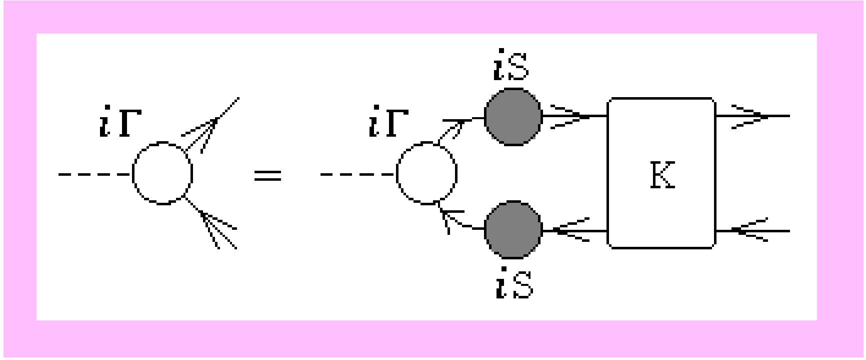
- Established understanding of two- and three-point functions
- What about bound states?

- Without bound states, Comparison with experiment is impossible

Hadrons

- Without bound states, Comparison with experiment is impossible
- They appear as pole contributions to $n \geq 3$-point colour-singlet Schwinger functions

- Without bound states, Comparison with experiment is impossible
- Bethe-Salpeter Equation



QFT Generalisation of Lippmann-Schwinger Equation.

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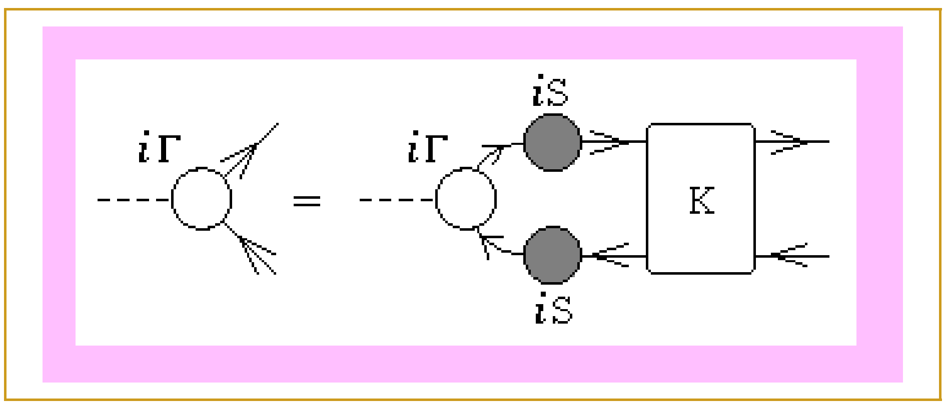

QFT Generalisation of Lippmann-Schwinger Equation.

- What is the kernel, K?

or What is the long-range potential in QCD?

Bethe-Salpeter Kernel

Bethe-Salpeter Kernel

- Axial-vector Ward-Takahashi identity

$$P_\mu \; \Gamma^l_{5\mu}(k;P) \quad = \quad \mathcal{S}^{-1}(k_+)\,\frac{1}{2}\lambda^l_f i\gamma_5 + \frac{1}{2}\lambda^l_f i\gamma_5 \;\; \mathcal{S}^{-1}(k_-)$$

$$-M_\zeta\, i\Gamma^l_5(k;P) - i\Gamma^l_5(k;P)\, M_\zeta$$

QFT Statement of Chiral Symmetry

Bethe-Salpeter Kernel

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Satisfies BSE

Satisfies DSE

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- Relation must be preserved by truncation
- Failure $\Rightarrow$ Explicit Violation of QCD's Chiral Symmetry

Persistent Challenge

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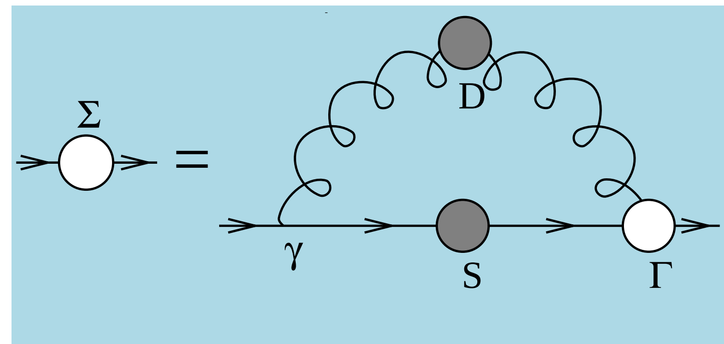
Persistent Challenge

- Infinitely Many Coupled Equations



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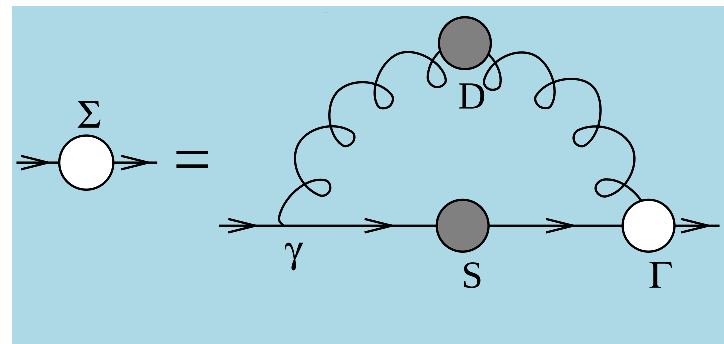

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Persistent Challenge

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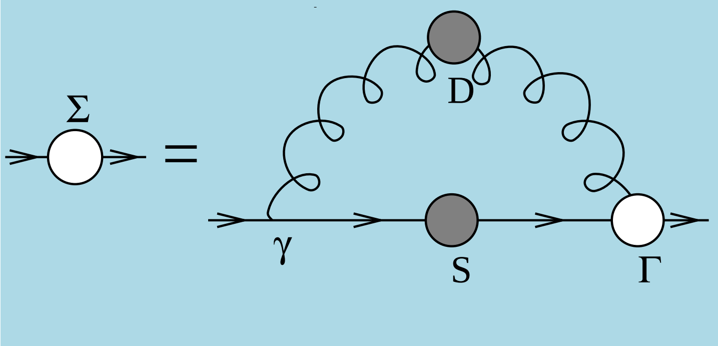

- Coupling between equations necessitates truncation
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U.S. DEPARTMENT OF ENERGY
Office of Nuclear Physics
Argonne NATIONAL LABORATORY

Persistent Challenge

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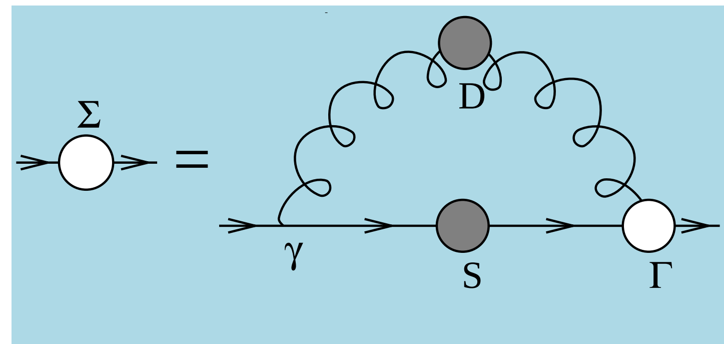

- Coupling between equations necessitates truncation
  - Weak coupling expansion $\Rightarrow$ Perturbation Theory
    Not useful for the nonperturbative problems
    in which we're interested

Persistent Challenge

- Infinitely Many Coupled Equations
- There is at least one systematic nonperturbative, symmetry-preserving truncation scheme
  H.J. Munczek Phys. Rev. D **52** (1995) 4736
  Dynamical chiral symmetry breaking, Goldstone's theorem and the consistency of the Schwinger-Dyson and Bethe-Salpeter Equations
  A. Bender, C. D. Roberts and L. von Smekal, Phys. Lett. B **380** (1996) 7
  Goldstone Theorem and Diquark Confinement Beyond Rainbow Ladder Approximation

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Radial Excitations
& Chiral Symmetry

(Maris, Roberts, Tandy
nu-th/9707003)

$$f_H \;\; m_H^2 = - \;\; \rho_\zeta^H \;\; \mathcal{M}_H$$

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Radial Excitations & Chiral Symmetry

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- Mass^2 of pseudoscalar hadron

First Contents Back Conclusion

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$$f_H \; m_H^2 = - \; \rho_\zeta^H \; \mathcal{M}_H$$

$$\mathcal{M}_H := \mathrm{tr}_{\mathrm{flavour}} \left[M_{(\mu)} \left\{ T^H, \left(T^H\right)^{\mathrm{t}} \right\} \right] = m_{q_1} + m_{q_2}$$

- Sum of constituents' current-quark masses
- e.g., $T^{K^+} = \frac{1}{2}\left(\lambda^4 + i\lambda^5\right)$

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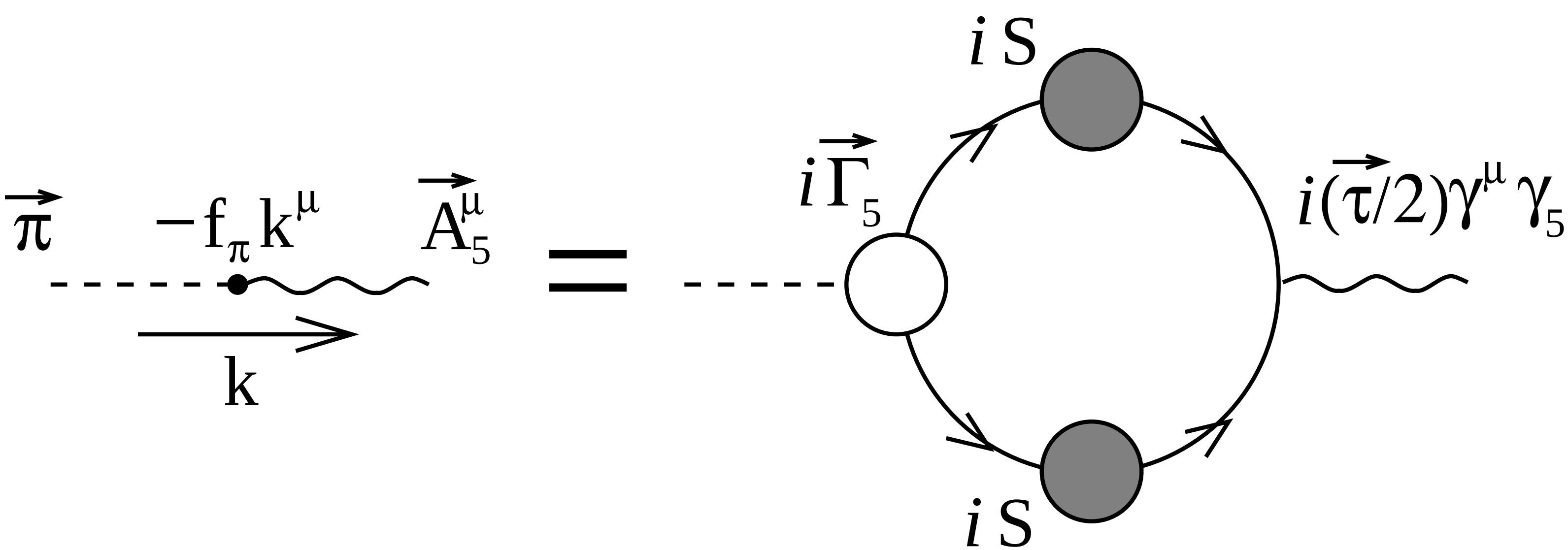
- Pseudovector projection of BS wave function at $x = 0$
- Pseudoscalar meson's leptonic decay constant



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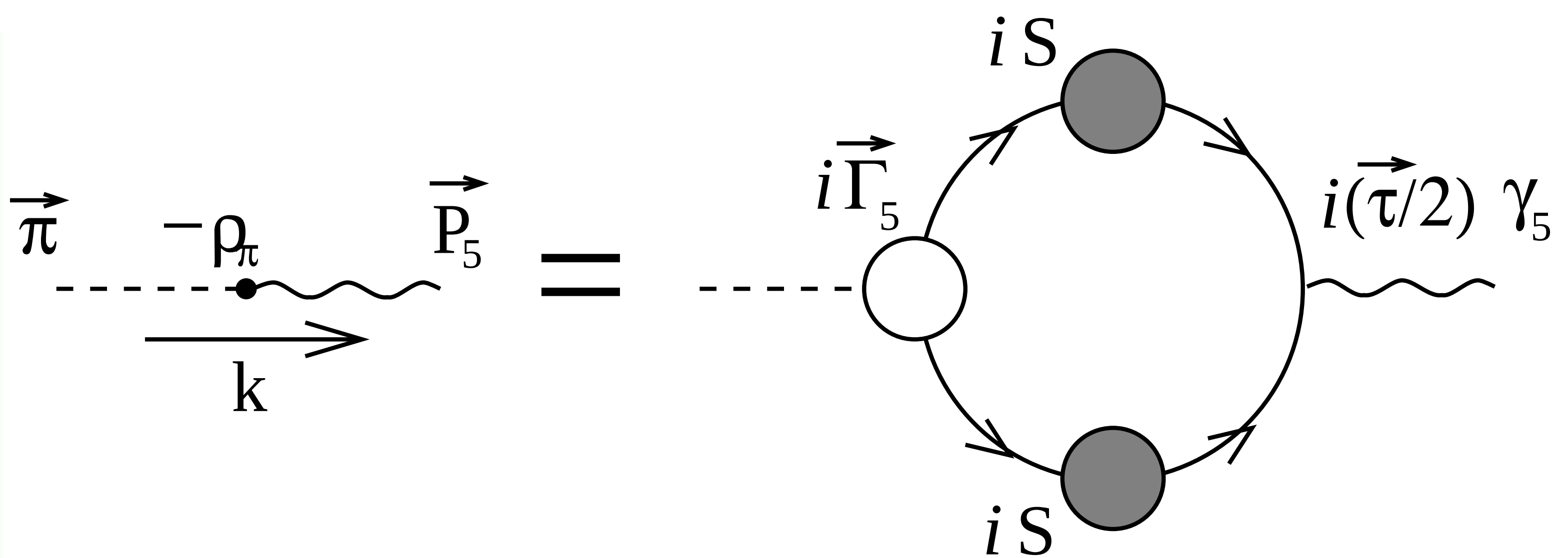

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- Heavy-quark + light-quark

$\Rightarrow f_H \propto \dfrac{1}{\sqrt{m_H}}$ and $\rho_\zeta^H \propto \sqrt{m_H}$

Hence, $\boxed{m_H \propto m_q}$

. . . QCD Proof of Potential Model result

Höll, Krassnigg, Roberts
nu-th/0406030

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- Dynamical Chiral Symmetry Breaking

  – Goldstone's Theorem –

  impacts upon *every* pseudoscalar meson

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Radial Excitations & Lattice-QCD

McNeile and Michael
he-la/0607032

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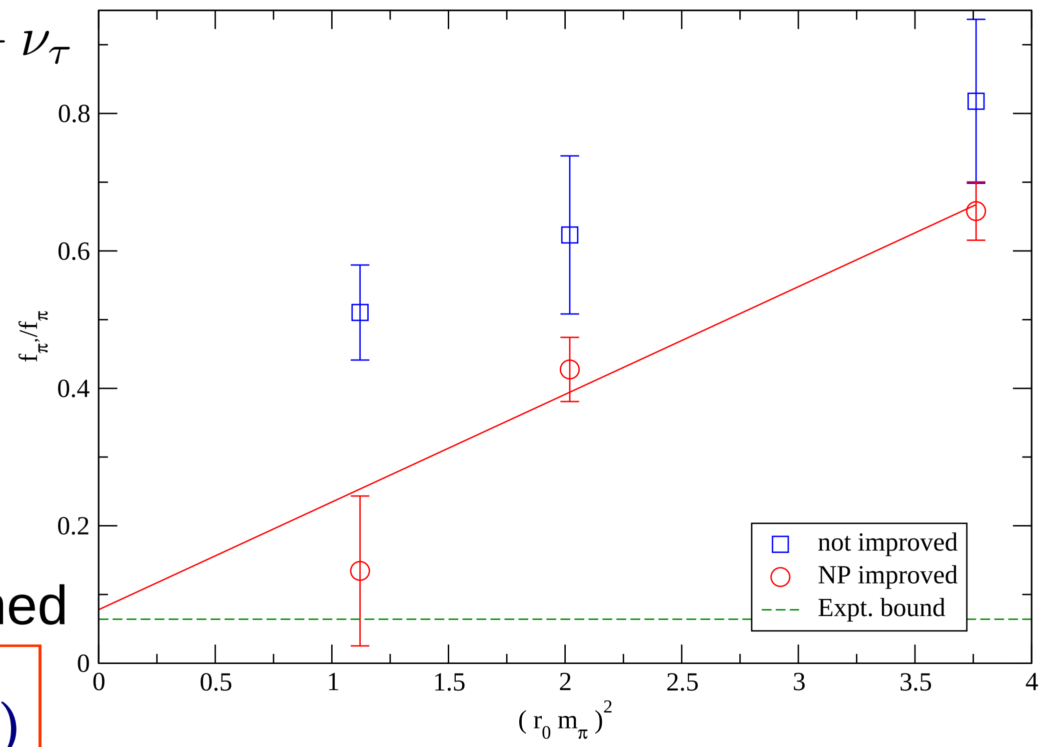

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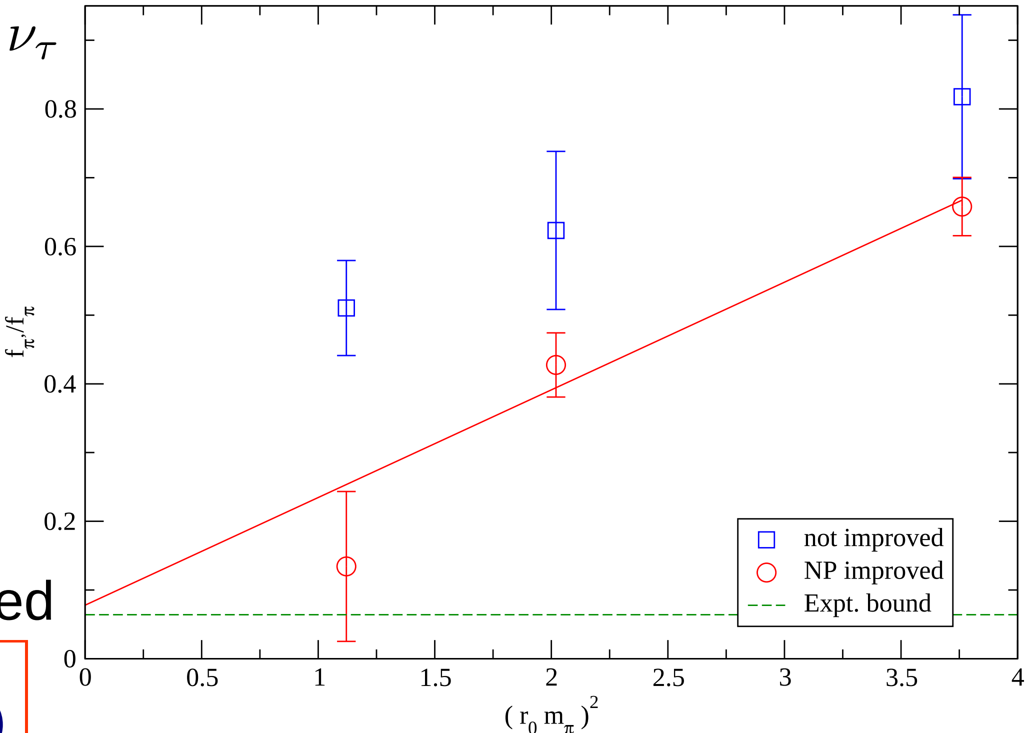

- Full ALPHA formulation is required to see suppression, because PCAC relation is at the heart of the conditions imposed for improvement (determining coefficients of irrelevant operators)

Radial Excitations & Lattice-QCD

McNeile and Michael
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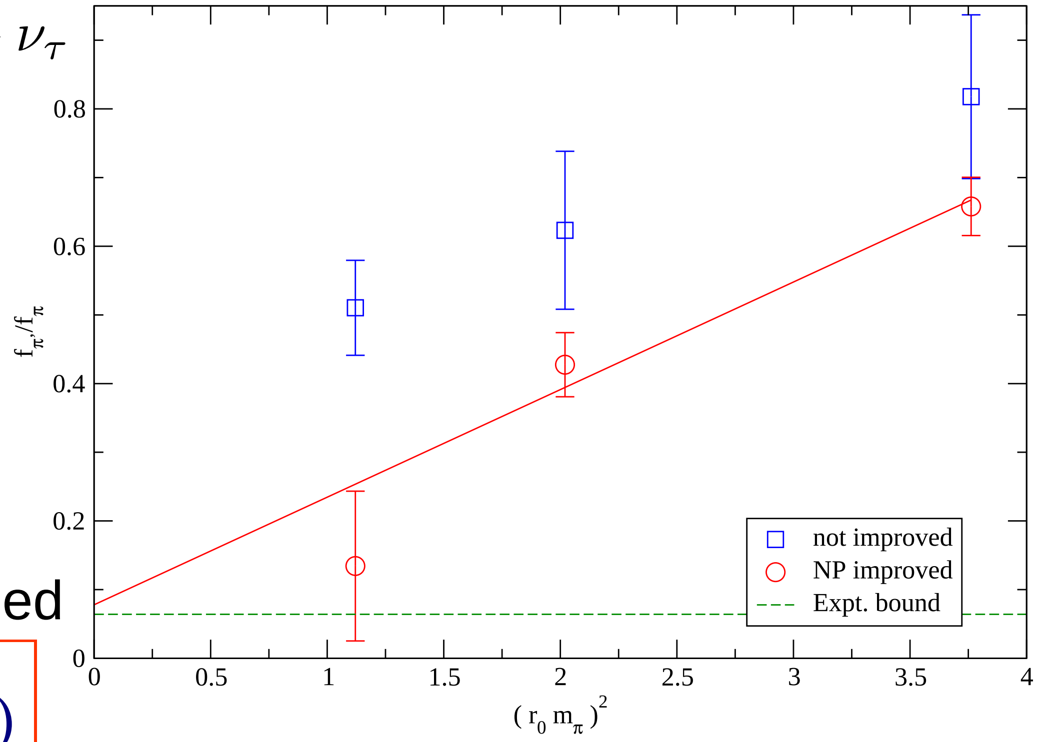

- *The suppression of f_{π_1} is a useful benchmark that can be used to tune and validate lattice QCD techniques that try to determine the properties of excited states mesons.*

Pion Form Factor

Procedure Now Straightforward

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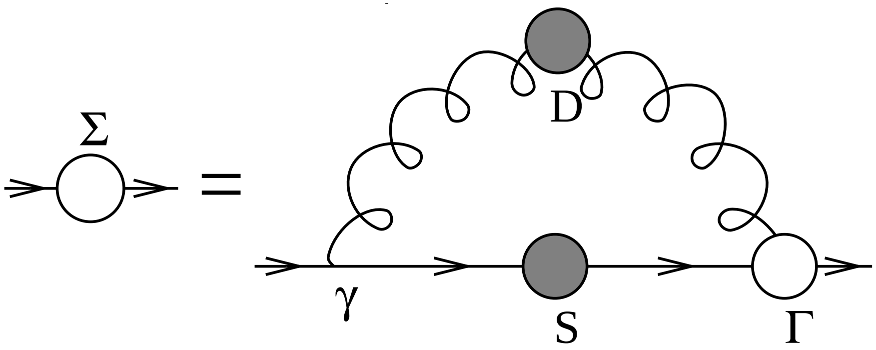
Pion Form Factor

- Solve Gap Equation

  $\Rightarrow$ Dressed-Quark Propagator, $S(p)$



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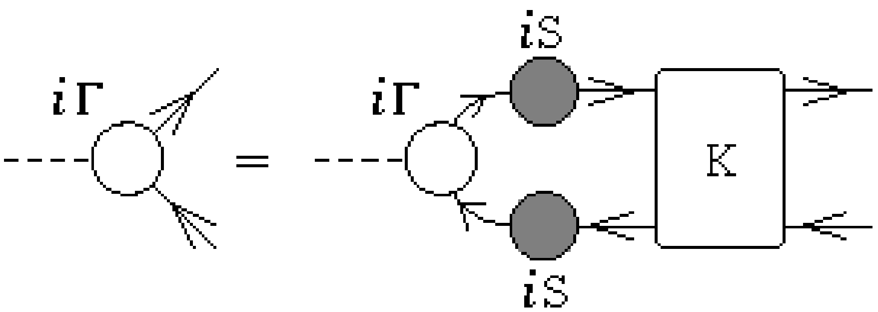
Pion Form Factor

- Use that to Complete Bethe Salpeter Kernel, K
- Solve Homogeneous Bethe-Salpeter Equation for Pion Bethe-Salpeter Amplitude, Γ_π



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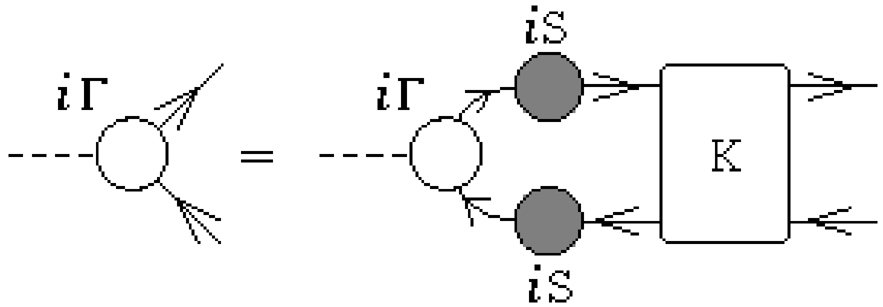
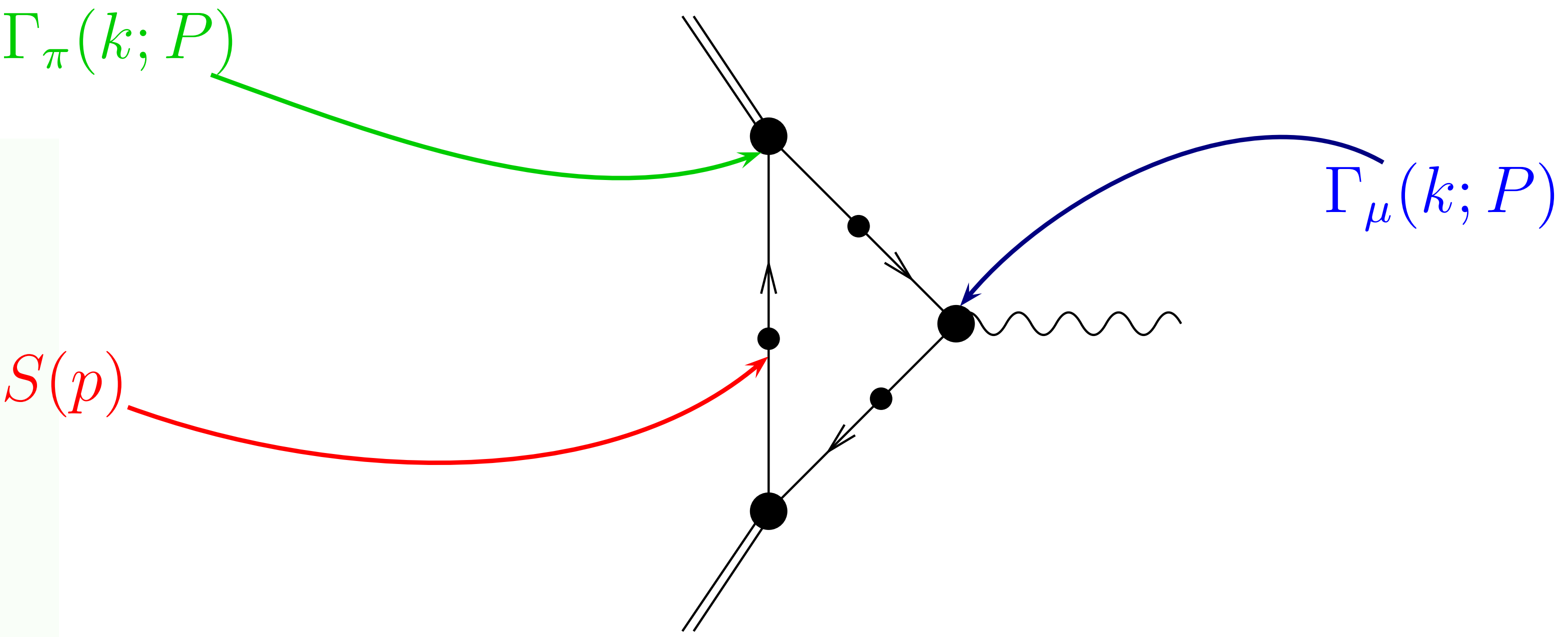


- Solve Inhomogeneous Bethe-Salpeter Equation for Dressed-Quark-Photon Vertex, Γ_μ

Pion Form Factor

- Now have all elements for Impulse Approximation to Electromagnetic Pion Form factor



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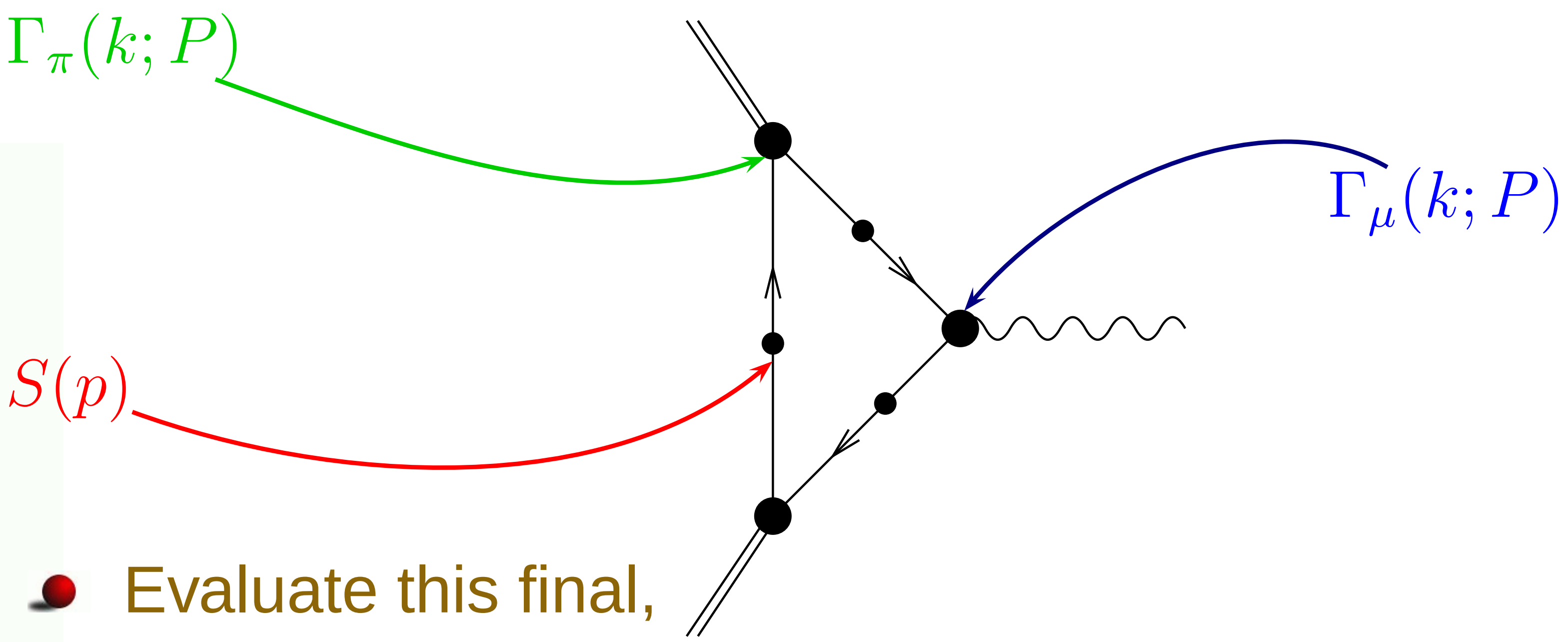
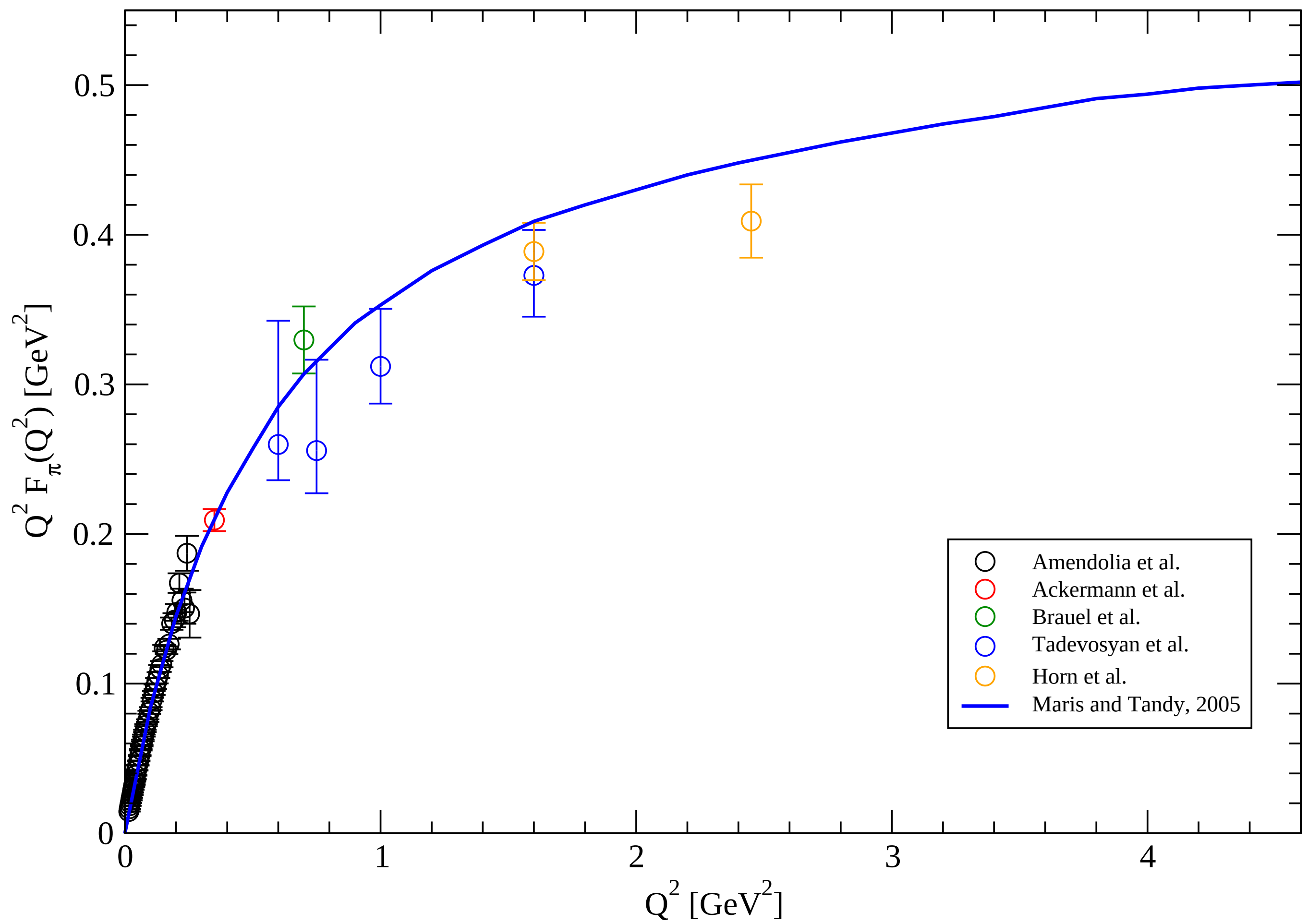


- Evaluate this final, three-dimensional integral

Calculated Pion Form Factor

Calculation first published in 1999; No Parameters Varied

Numerical method improved in 2005



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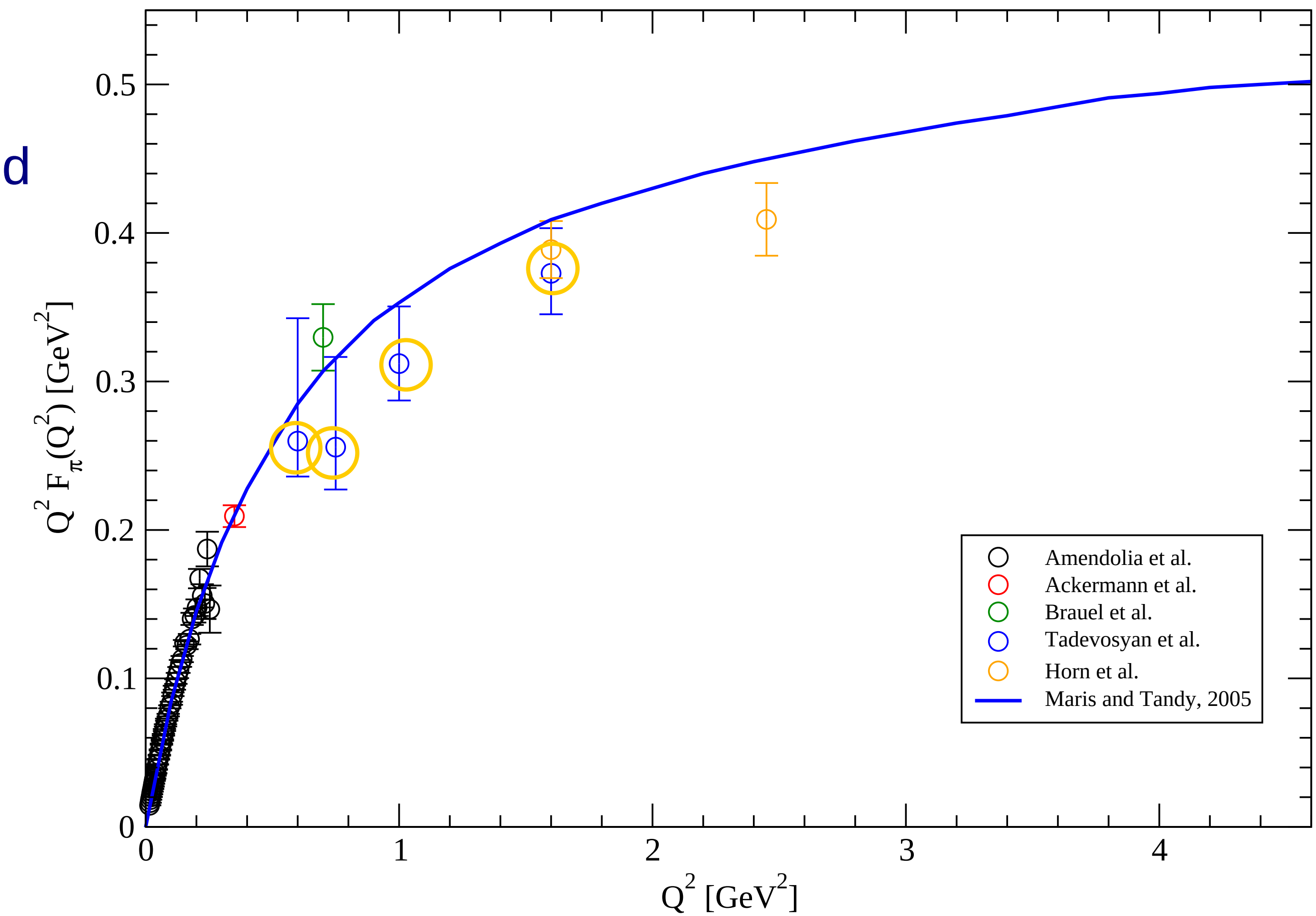

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Timelike *Pion Form Factor*

Ab initio calculation into timelike region. Deeper than ground-state ρ-meson pole

Timelike Pion Form Factor

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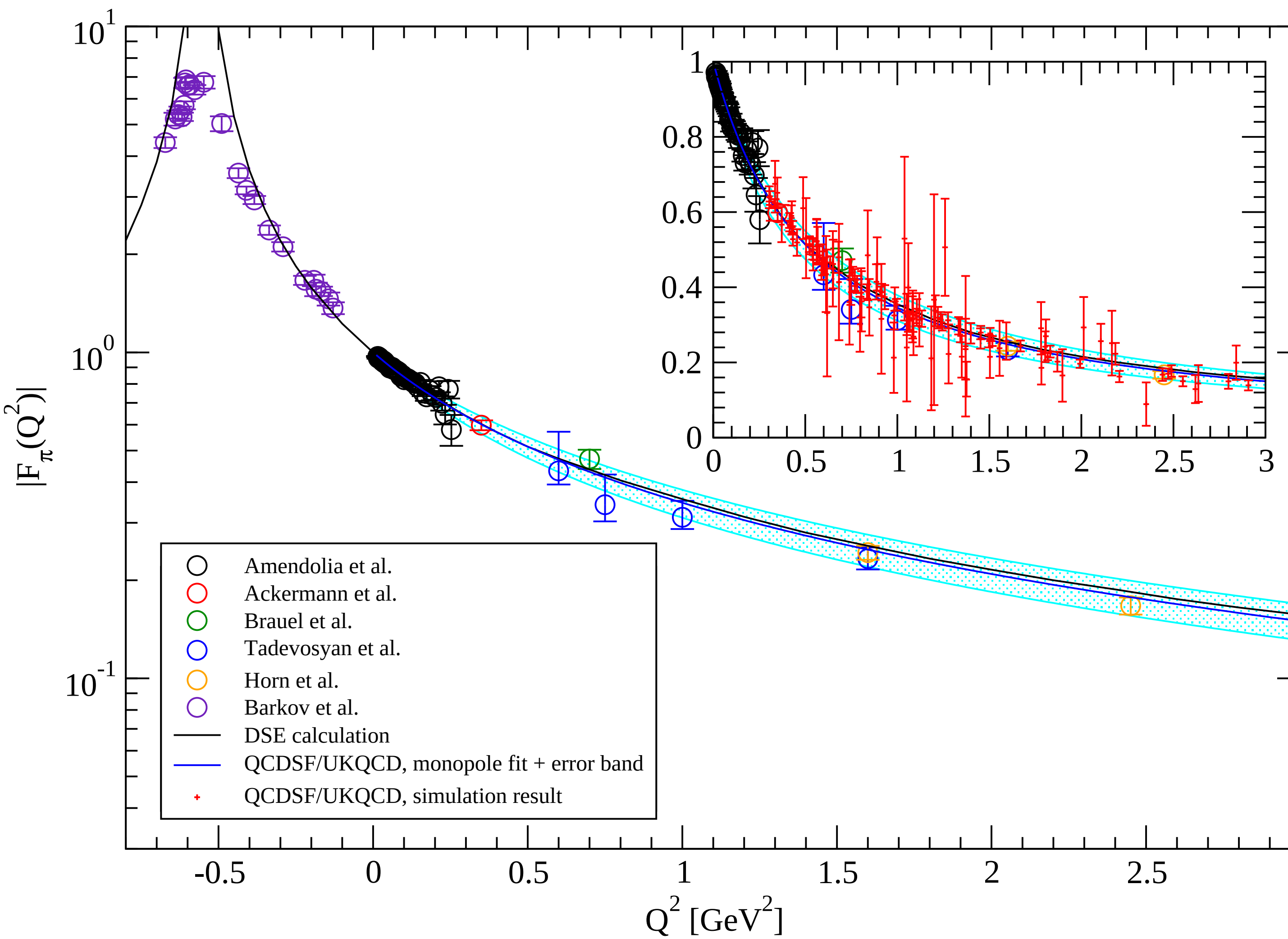
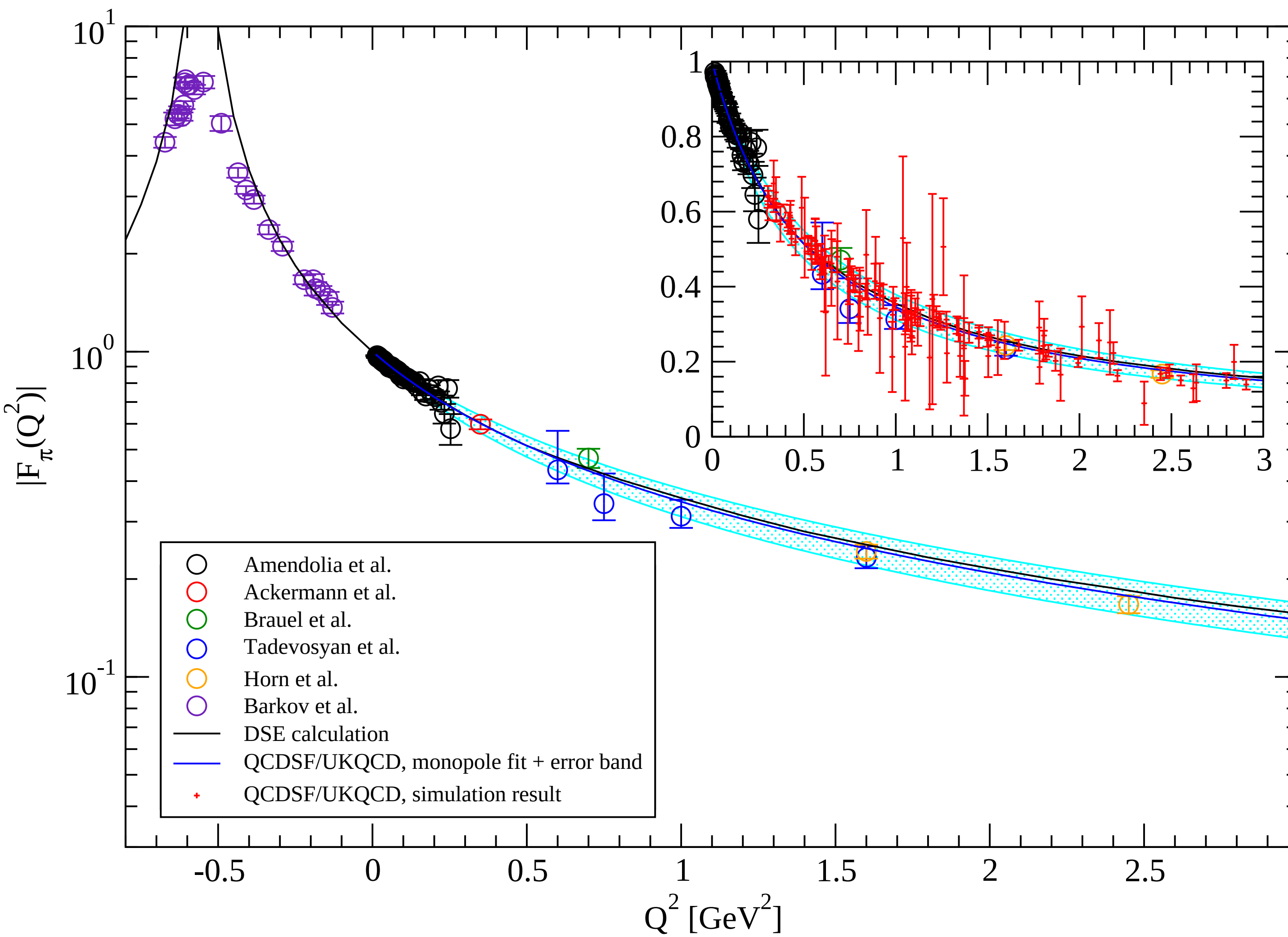
Timelike Pion Form Factor



Ab initio calculation into timelike region. Deeper than ground-state ρ-meson pole

Timelike Pion Form Factor

ρ-meson not put in “by hand” – generated dynamically as a bound-state of dressed-quark and dressed-antiquark



Dimensionless product: $r_\pi \, f_\pi$

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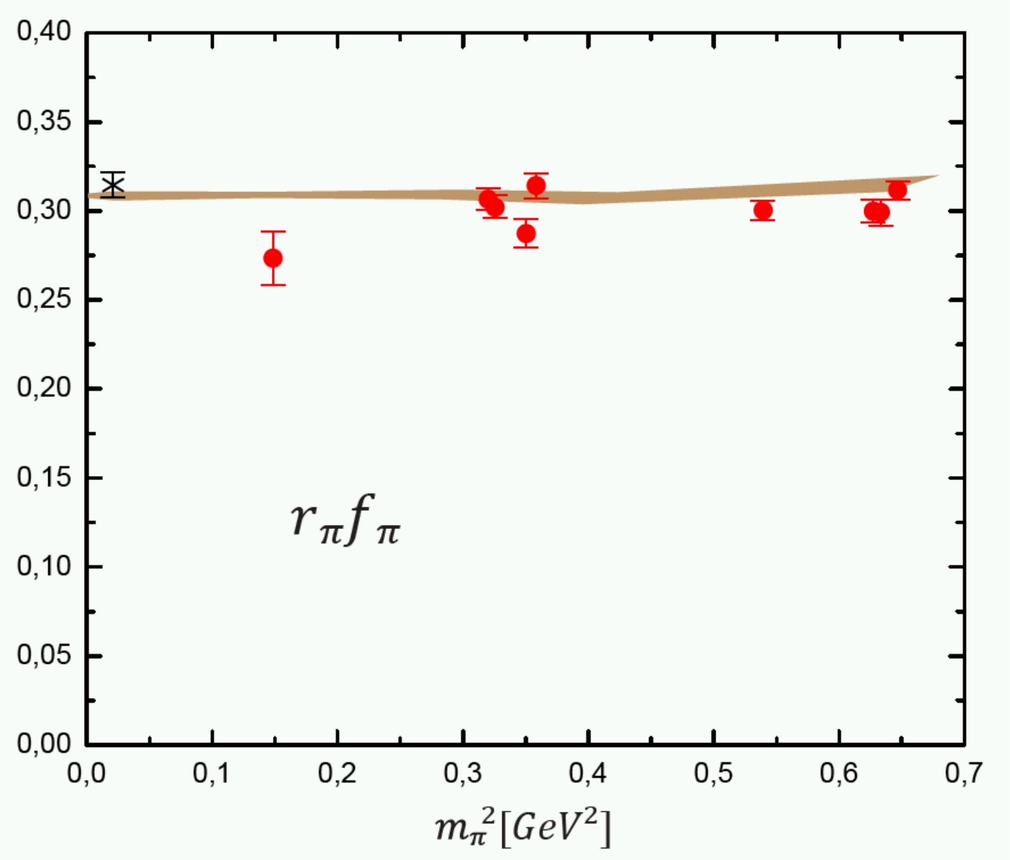
- DSE prediction



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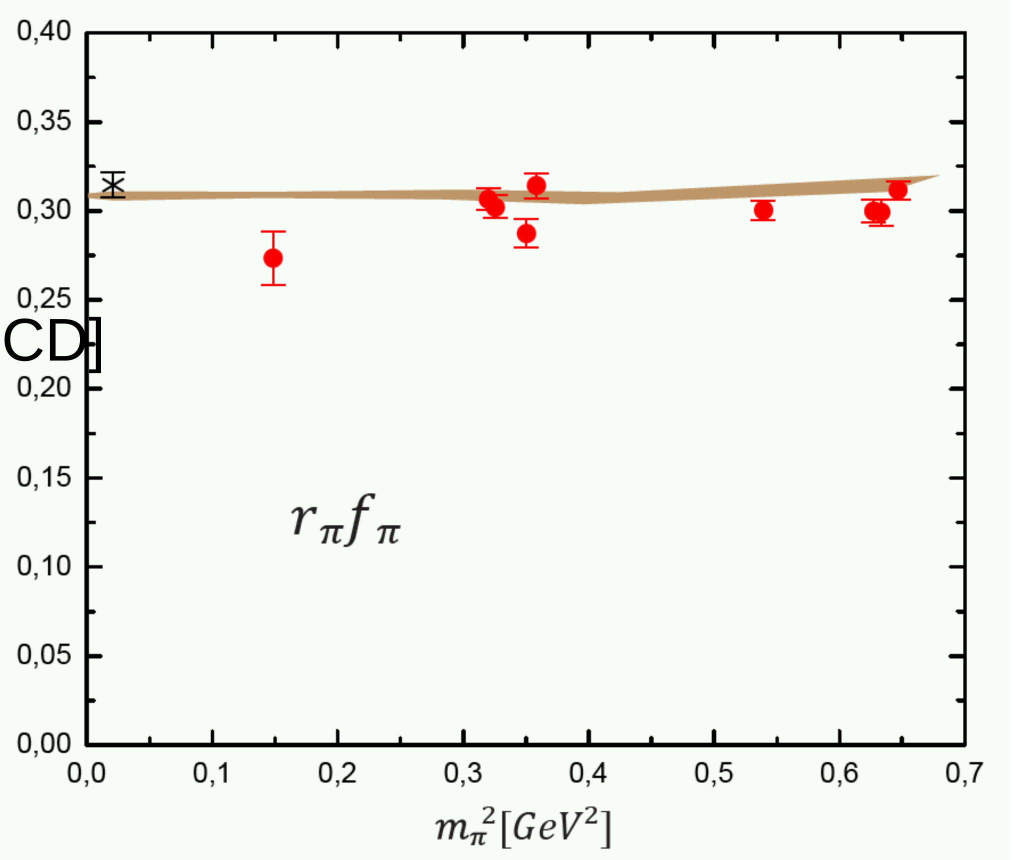

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Dimensionless product: $r_\pi\, f_\pi$

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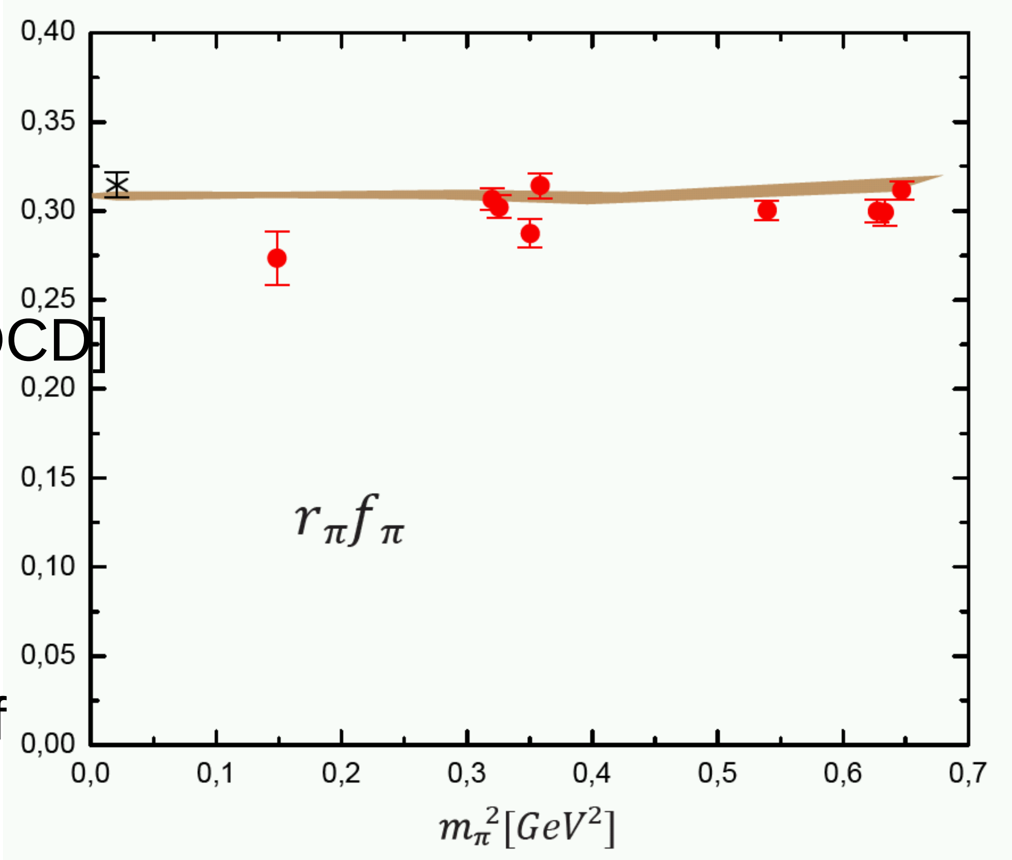

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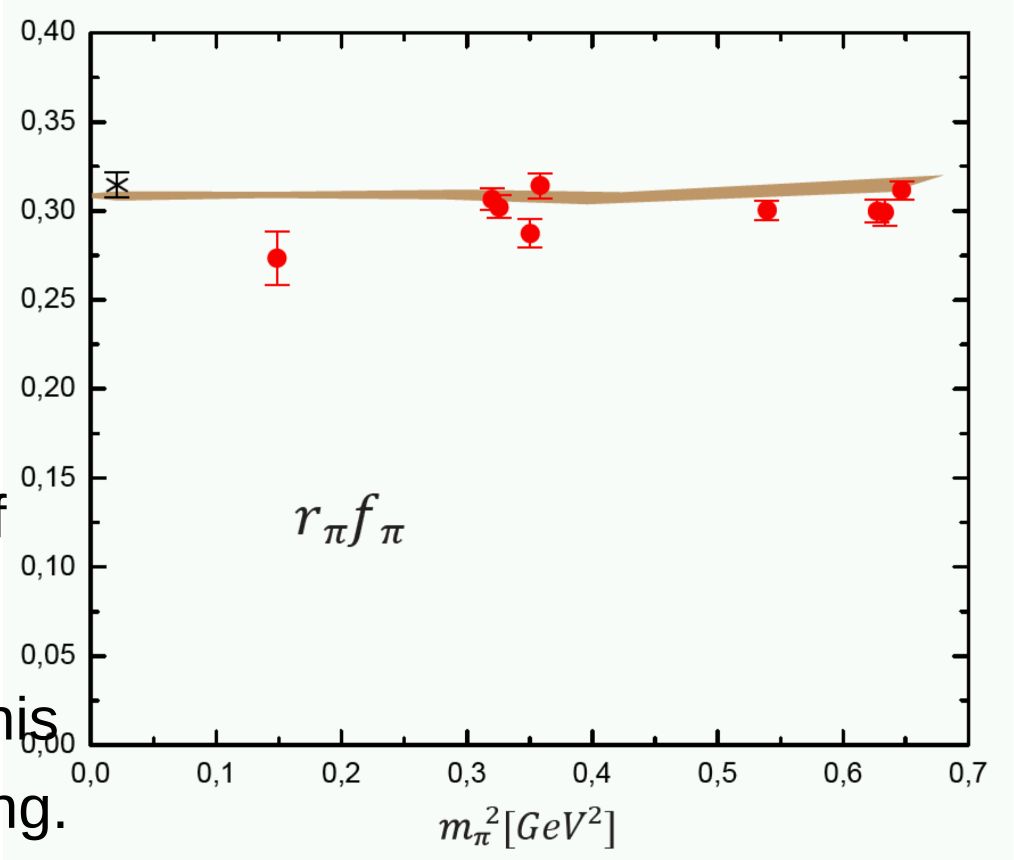

- DSE prediction
- Fascinating result:
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  – Experimental value
  obtains independent of
  current-quark mass.
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  Implications far-reaching.

Maris, Roberts, Tandy
nucl-th/9707003

Goldberger-Treiman for pion

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- Pseudoscalar Bethe-Salpeter amplitude

$$\Gamma_{\pi^j}(k;P) = \tau^{\pi^j}\gamma_5 \Big[iE_\pi(k;P) + \gamma\cdot P F_\pi(k;P) + \gamma\cdot k\, k\cdot P\, G_\pi(k;P) + \sigma_{\mu\nu}\, k_\mu P_\nu\, H_\pi(k;P) \Big]$$

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$\Rightarrow$

$$\begin{aligned} f_\pi E_\pi(k;P=0) &= B(p^2)\\ F_R(k;0)+2\, f_\pi F_\pi(k;0) &= A(k^2)\\ G_R(k;0)+2\, f_\pi G_\pi(k;0) &= 2A'(k^2)\\ H_R(k;0)+2\, f_\pi H_\pi(k;0) &= 0\end{aligned}$$

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Goldberger-Treiman for pion

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$$\Gamma_{\pi^j}(k;P) = \tau^{\pi^j}\gamma_5\left[iE_\pi(k;P)+\gamma\cdot P F_\pi(k;P)\right.$$
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Pseudovector components necessarily nonzero

- Dressed-quark Propagator: $S(p)=\dfrac{1}{i\gamma\cdot p\,A(p^2)+B(p^2)}$
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GT for pion – QCD and $F_\pi^{\rm em}(Q^2)$

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GT for pion – QCD and $F_\pi^{\rm em}(Q^2)$

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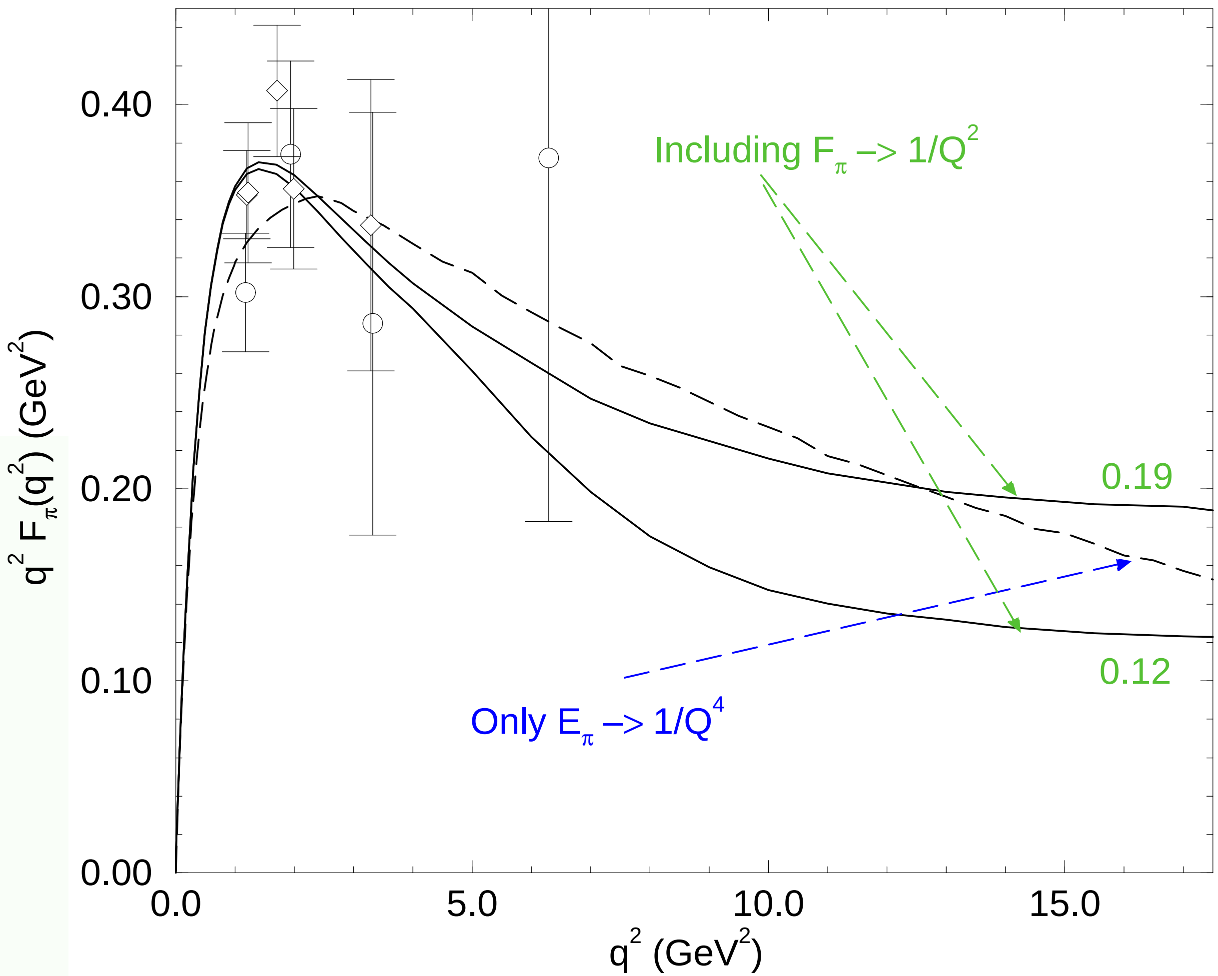

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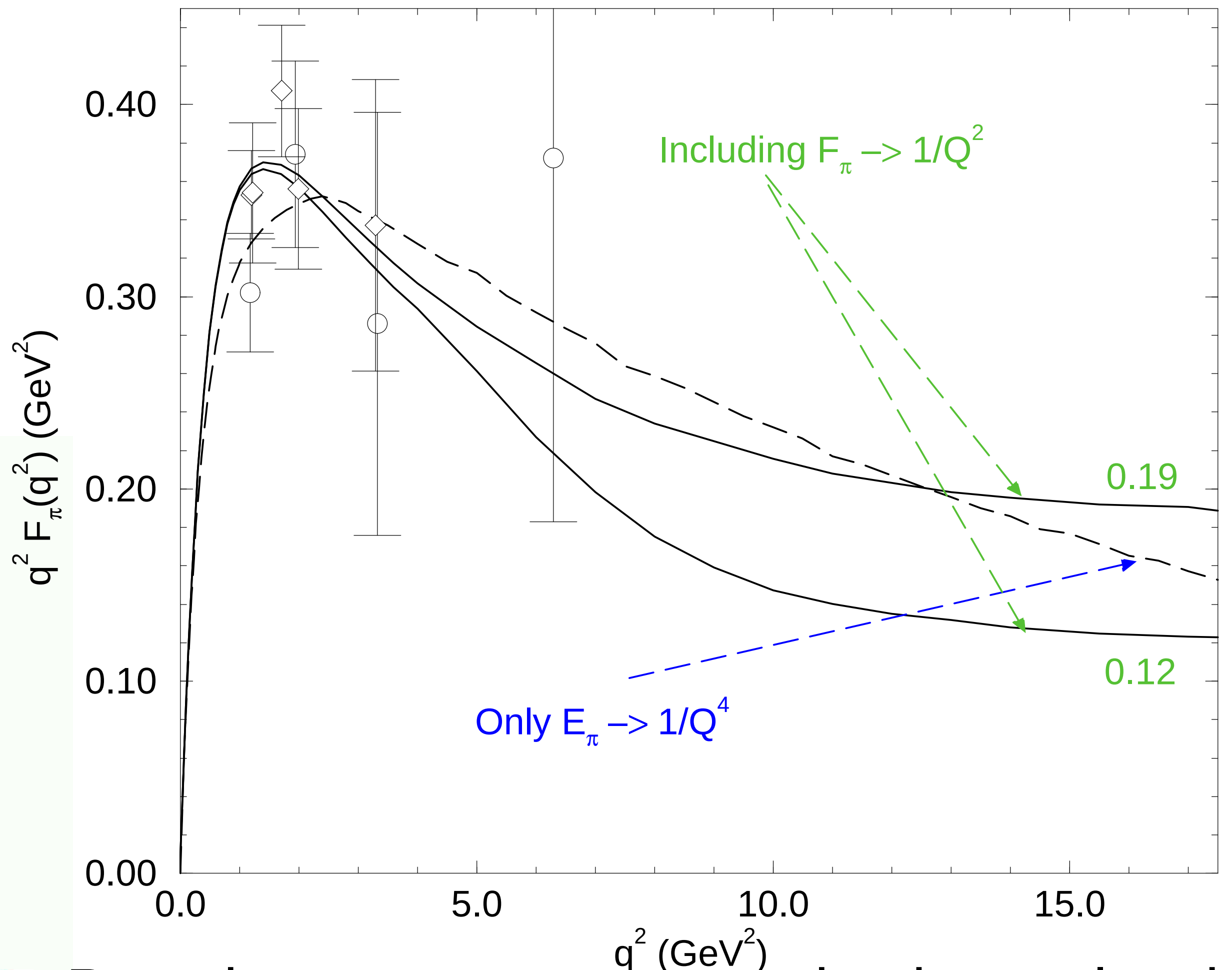

- Pseudovector components dominate ultraviolet behaviour of electromagnetic form factor

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Gap Equation
General Form

- Return to general bound-state problem . . .

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- To study the Poincaré covariant bound-state problem for mesons, one must first solve the gap equation

$$S_f(p)^{-1} = Z_2\,(i\gamma\cdot p + m_f^{\rm bm}) + \Sigma_f(p)\,,$$

$$\Sigma_f(p) = Z_1 \int_q^\Lambda g^2 D_{\mu\nu}(p-q)\frac{\lambda^a}{2}\gamma_\mu S_f(q)\frac{\lambda^a}{2}\Gamma_\nu^f(q,p),$$

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- $D_{\mu\nu}(k)$ is the dressed-gluon propagator;
- $\Gamma_\nu^f(q,p)$ is the dressed-quark-gluon vertex;
- $m^{\rm bm}(\Lambda)$ is the Lagrangian current-quark bare mass;
- $Z_{1,2}(\zeta^2,\Lambda^2)$ are respectively the vertex and quark wave function renormalisation constants, with ζ the renormalisation point.

Bethe-Salpeter Equation
General Form

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- Pseudoscalar and axial-vector mesons appear as poles in the inhomogeneous Bethe-Salpeter equation.
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$$\Gamma^{fg}_{5\mu}(k;P) = Z_2\gamma_5\gamma_\mu - \int_q g^2 D_{\alpha\beta}(k-q)$$
$$\times \frac{\lambda^a}{2}\gamma_\alpha S_f(q_+)\Gamma^{fg}_{5\mu}(q;P)S_g(q_-)\frac{\lambda^a}{2}\Gamma^g_\beta(q_-,k_-)$$
$$+ \int_q g^2 D_{\alpha\beta}(k-q)\frac{\lambda^a}{2}\gamma_\alpha S_f(q_+)\frac{\lambda^a}{2}\Lambda^{fg}_{5\mu\beta}(k,q;P),$$

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$$+ \int_q g^2 D_{\alpha\beta}(k-q)\frac{\lambda^a}{2}\gamma_\alpha S_f(q_+)\frac{\lambda^a}{2}\Lambda^{fg}_{5\mu\beta}(k,q;P),$$

  - $\Lambda^{fg}_{5\mu\beta}$ is defined completely via the dressed-quark self-energy and, owing to Poincaré covariance, one can employ, e.g., $q_\pm = q \pm P/2$, etc., without loss of generality

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Ward-Takahashi Identity
Bethe-Salpeter Kernel

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Ward-Takahashi Identity
Bethe-Salpeter Kernel

- In any reliable study of light-quark hadrons, axial-vector vertex must satisfy

$$\begin{aligned} P_\mu \Gamma^{fg}_{5\mu}(k;P) &= S_f^{-1}(k_+) i\gamma_5 + i\gamma_5 S_g^{-1}(k_-) \\ &\quad - i\left[m_f(\zeta) + m_g(\zeta)\right] \Gamma_5^{fg}(k;P)\,, \end{aligned}$$

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expresses chiral symmetry & pattern by which it's broken

- The condition ($\Lambda_{5\beta}^{fg}$ pseudoscalar analogue of $\Lambda_{5\mu\beta}^{fg}$)

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to ensure Ward-Takahashi identity satisfied.

- Rainbow-ladder . . .
  - $\Gamma^f_\beta(q,k) = \gamma_\mu$

    $\Rightarrow \Lambda^{fg}_{5\mu\beta}(k,q;P) = 0 = \Lambda^{fg}_{5\beta}(k,q;P)$

Solving the Kernel's Ward-Takahashi Identity

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- Kernel's Ward-Takahashi identity provides means by which to construct a symmetry preserving kernel of the Bethe-Salpeter equation that is matched to any reasonable *Ansatz* for the dressed-quark-gluon vertex
- With this powerful capacity they realise a longstanding goal.

Solving the Kernel's WTI – Illustration

Chang Lei & CDR, arXiv:0903.5461 [nucl-th]

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- Suppose that in the gap equation one employs an *Ansatz* for the dressed-quark-gluon vertex which satisfies

$$P_\mu i\Gamma_\mu^f(k_+, k_-) = \mathcal{B}(P^2)\left[S_f^{-1}(k_+) - S_f^{-1}(k_-)\right] \qquad (*)$$

with $\mathcal{B}$ flavour-independent.

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- NB. While true quark-gluon vertex doesn't satisfy this identity, owing to form of Slavnov-Taylor identity which it does satisfy, it's plausible that solution of Eq. $(*)$ can provide reasonable pointwise approximation to true vertex.

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- Given Eq. $(*)$, then Kernel's WTI entails

$$P_\mu(q-k)_\beta i\Lambda^{fg}_{5\mu\beta}(k,q;P) = P_\mu \mathcal{B}((k-q)^2)\left[\Gamma^{fg}_{5\mu}(q;P) - \Gamma^{fg}_{5\mu}(k;P)\right],$$

$$(q-k)_\beta i\Lambda^{fg}_{5\beta}(k,q;P) = \mathcal{B}((k-q)^2)\left[\Gamma^{fg}_{5}(q;P) - \Gamma^{fg}_{5}(k;P)\right]. \qquad (\#)$$

Solving the Kernel's WTI – Illustration

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- Solution to Eq. (#)

$$\Lambda_{5\beta}^{fg}(k,q;P) \quad := \quad \mathcal{B}((k-q)^2)\,\gamma_5\,\overline{\Lambda}_{\beta}^{fg}(k,q;P)\,,$$

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- with (BC construction)

$$\overline{\Lambda}^{fg}_{\beta}(k,q;P) = 2\ell_\beta\,[i\Delta_{E_5}(q,k;P) + \gamma\cdot P\Delta_{F_5}(q,k;P)]$$
$$+\gamma_\beta\,\Sigma_{G_5}(q,k;P) + 2\ell_\beta\,\gamma\cdot\ell\,\Delta_{G_5}(q,k;P) + [\gamma_\beta,\gamma\cdot P]$$
$$\times\Sigma_{H_5}(q,k;P) + 2\ell_\beta[\gamma\cdot\ell,\gamma\cdot P]\Delta_{H_5}(q,k;P)\,,$$

  - $\ell = (q+k)/2$
  - $\Sigma_\Phi(q,k;P) = [\Phi(q;P) + \Phi(k;P)]/2$
  - $\Delta_\Phi(q,k;P) = [\Phi(q;P) - \Phi(k;P)]/[q^2 - k^2]$

Symmetry-preserving Ansatz

Chang Lei & CDR, arXiv:0903.5461 [nucl-th]

- At this point . . .
  - Began with $\Gamma_\mu(q,p)$, whose diagrammatic content is unknown, but which expresses important additional nonperturbative effects that are difficult to capture in any finite sum of contributions

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  - Given that $\Gamma_\mu(q,p)$ satisfies Eq. ($*$), the equations which follow provide symmetry-preserving closed system whose solution yields predictions for the properties of pseudoscalar mesons
- This system and its predictions can smoothly be connected with those obtained, e.g., in a rainbow-ladder or kindred symmetry-preserving truncation of the DSEs.

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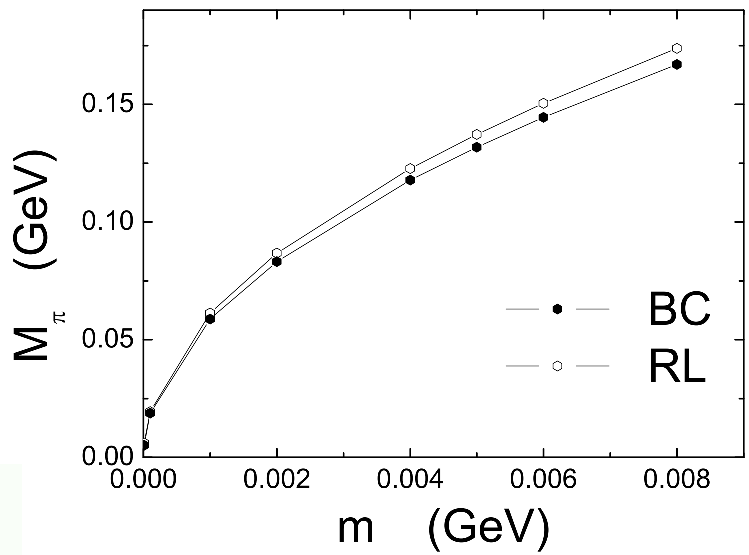
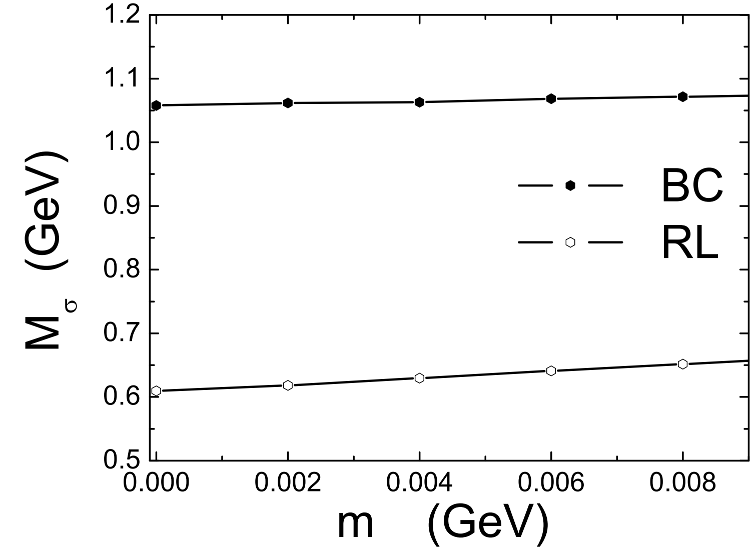
  - Given that $\Gamma_\mu(q,p)$ satisfies Eq. $(*)$, the equations which follow provide symmetry-preserving closed system whose solution yields predictions for the properties of pseudoscalar mesons
- The system can be used to anticipate, elucidate and understand the impact on hadron properties of the rich nonperturbative structure expected of the fully-dressed quark-gluon vertex in QCD

Numerical Illustration

Chang Lei & CDR, arXiv:0903.5461 [nucl-th]





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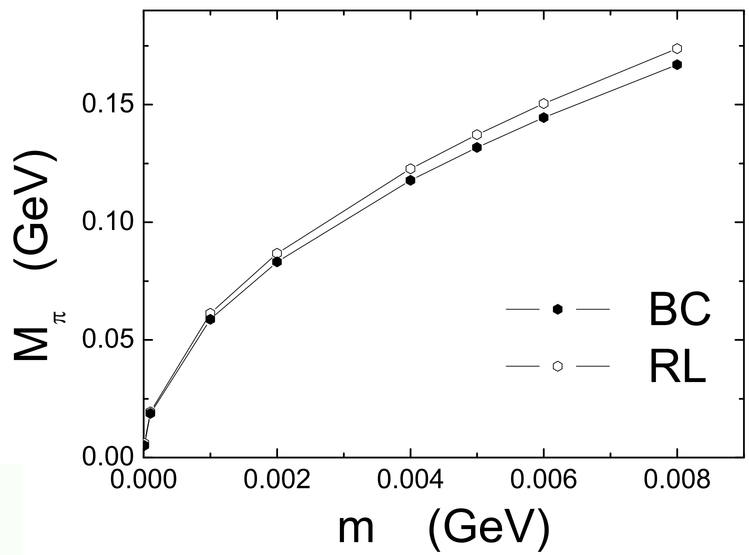

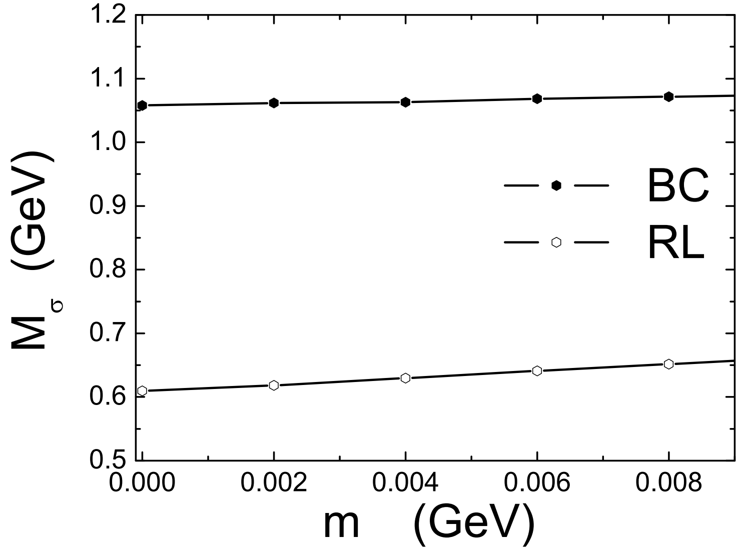

- Single interaction, common mass scale: rainbow-ladder cf. BC-consistent truncation

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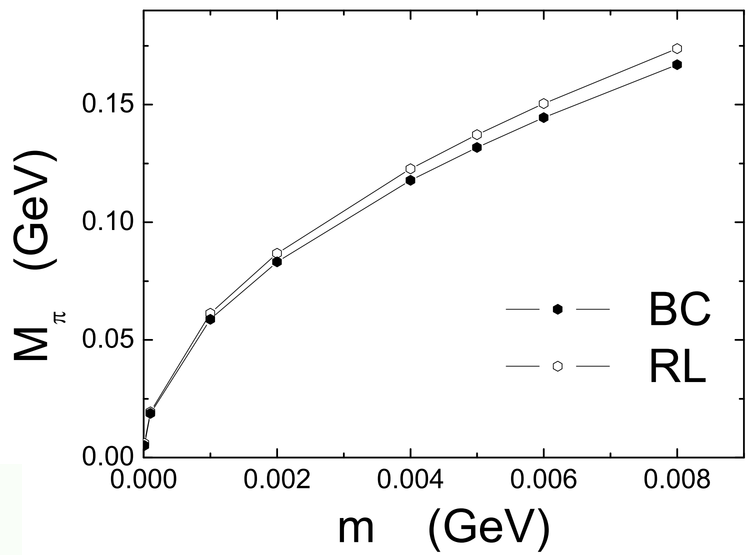

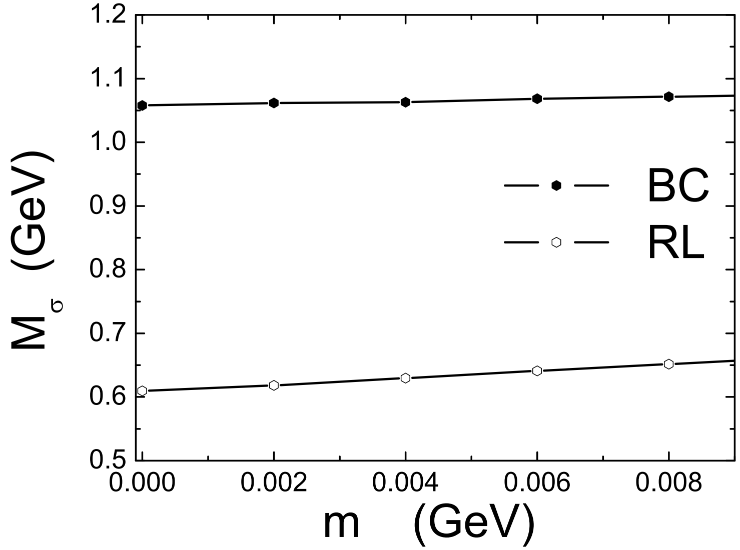

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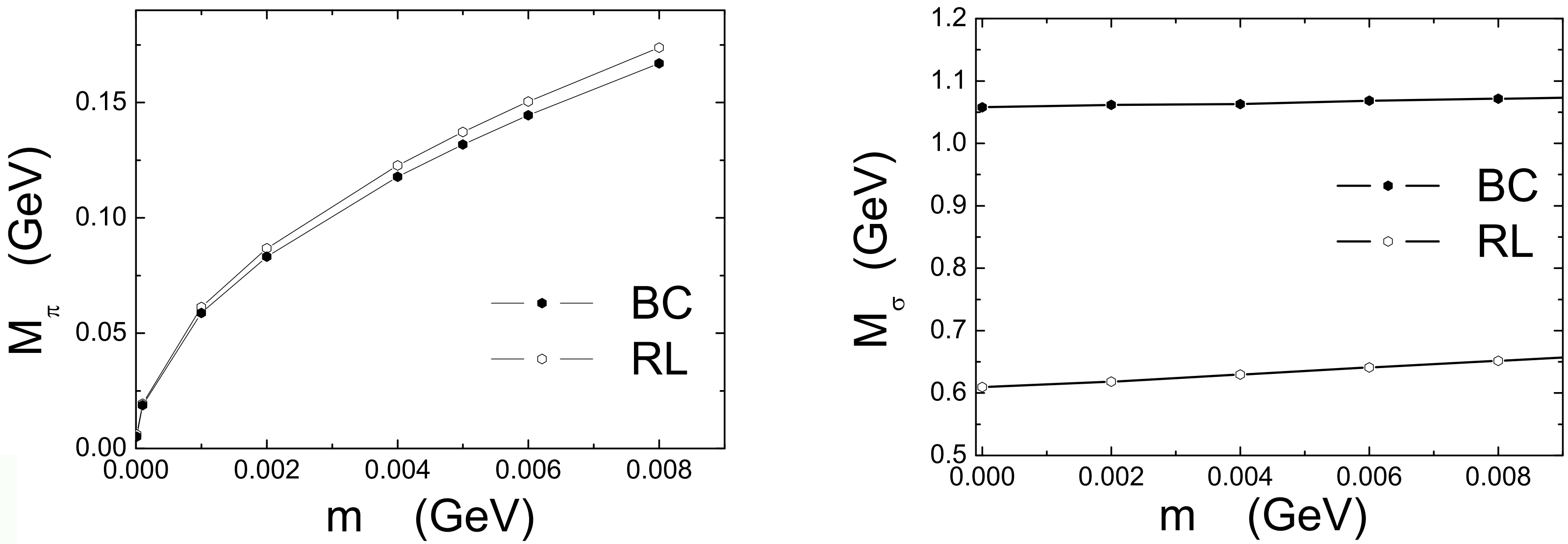

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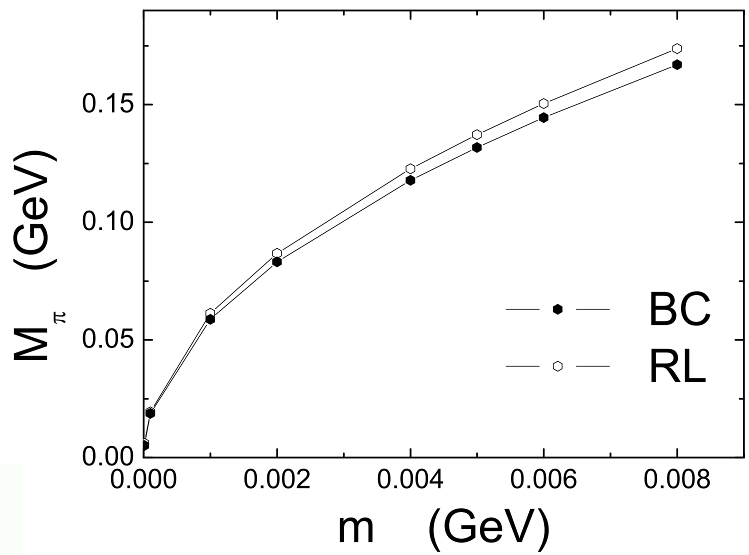

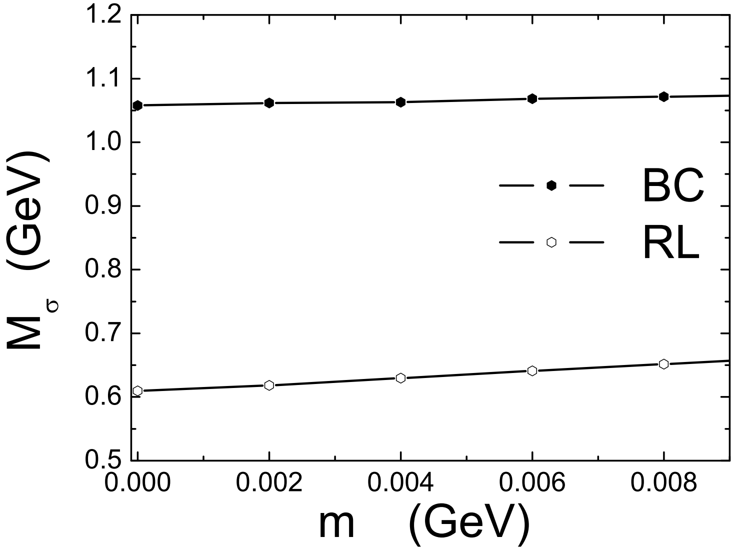

- Single interaction, common mass scale: rainbow-ladder cf. BC-consistent truncation
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  - Added attraction in pseudoscalar channel
  - Added repulsion in scalar channel

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Spin-orbit Interaction

Chang Lei & CDR, arXiv:0903.5461 [nucl-th]

- Rainbow-ladder DSE truncation,
$$\varepsilon_\sigma^{\rm RL} := \left.\frac{2M(0) - m_\sigma}{2M(0)}\right|_{\rm RL} = (0.3 \pm 0.1)\,.$$
- BC-consistent Bethe-Salpeter kernel; viz., $\varepsilon_\sigma^{\rm BC} \lesssim 0.1\,.$

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- From this viewpoint scalar is a spin and orbital excitation of a pseudoscalar meson

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- Extant studies of realistic corrections to the rainbow-ladder truncation show that they reduce hyperfine splitting

Spin-orbit Interaction

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- Scalar mesons $= {}^3P_0$ states: Constituents' spins aligned and one unit of constituent orbital angular momentum
- Clear sign that in a Poincaré covariant treatment the BC-consistent truncation magnifies spin-orbit splitting. Effect owes to influence of quark's dynamically-enhanced scalar self-energy in the Bethe-Salpeter kernel.
  Impossible to demonstrate effect without our new procedure

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Spin-orbit Interaction

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- Scalar mesons $=$ $^3P_0$ states: Constituents' spins aligned and one unit of constituent orbital angular momentum
- Clear sign that in a Poincaré covariant treatment the BC-consistent truncation magnifies spin-orbit splitting.
- Expect this feature to have material impact on mesons with mass greater than 1 GeV.
  prima facie . . . can overcome longstanding shortcoming of RL truncation; viz., splitting between vector & axial-vector mesons is too small

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Spin-orbit Interaction

- Rainbow-ladder DSE truncation,
$$\varepsilon_\sigma^{\mathrm{RL}} := \left.\frac{2M(0) - m_\sigma}{2M(0)}\right|_{\mathrm{RL}} = (0.3 \pm 0.1)\,.$$
- BC-consistent Bethe-Salpeter kernel; viz., $\varepsilon_\sigma^{\mathrm{BC}} \lesssim 0.1\,.$
- Scalar mesons $=\ {}^3P_0$ states: Constituents' spins aligned and one unit of constituent orbital angular momentum
- Clear sign that in a Poincaré covariant treatment the BC-consistent truncation magnifies spin-orbit splitting.
- Expect this feature to have material impact on mesons with mass greater than 1 GeV.
- Promise of realistic meson spectroscopy
  First time, also for mass > 1 GeV

Epilogue

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  - It predicts, amongst other things, that
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    - pseudoscalar mesons are unnaturally light: $m_\rho = 770$ cf. $m_\pi = 140$ MeV
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  - It impacts dramatically upon observables.

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    - Poincaré covariant unification of meson and baryon observables

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  - Ab-initio study of $N \rightarrow \Delta$ transition underway
- Tool enabling insight to be drawn from experiment into long-range piece of interaction between light-quarks

Contents

New Challenges

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- Move on to the problem of a symmetry preserving treatment of hybrids and exotics.

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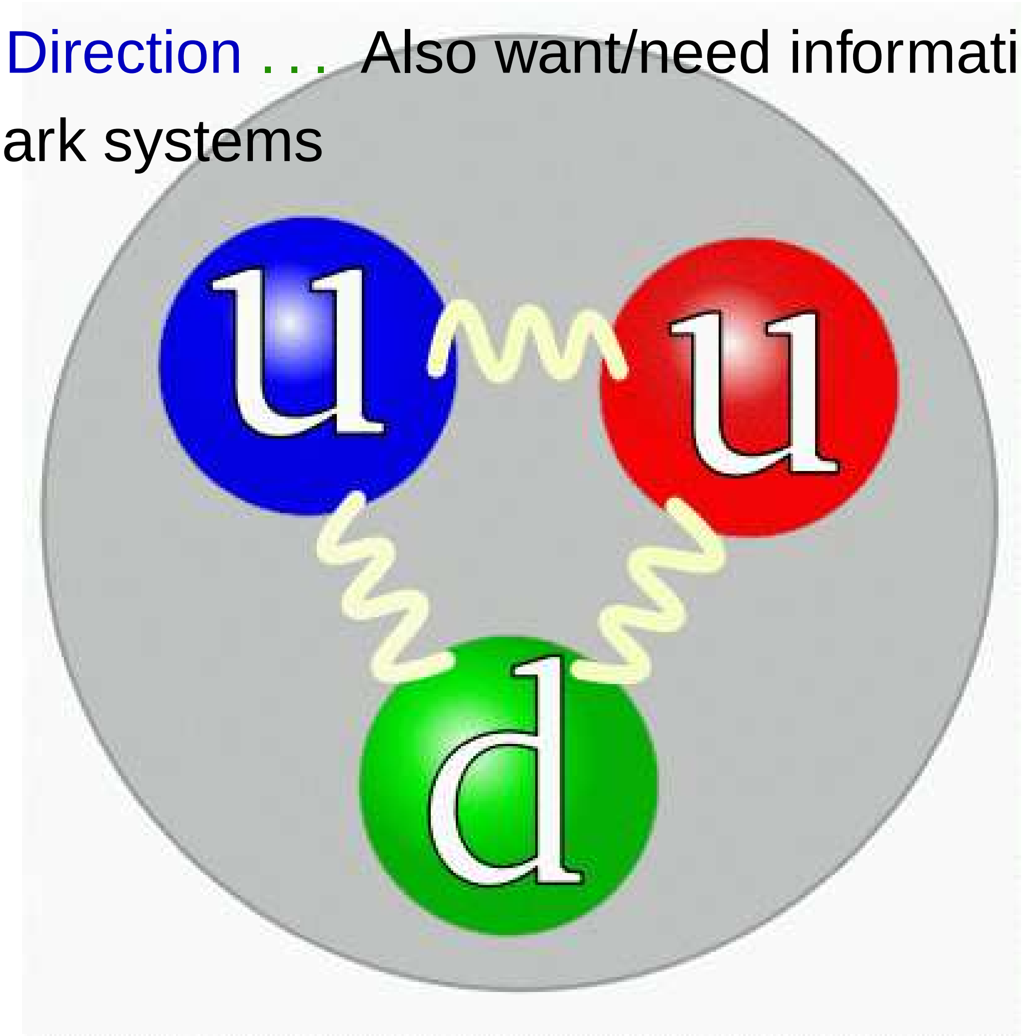
- Another Direction . . . Also want/need information about three-quark systems



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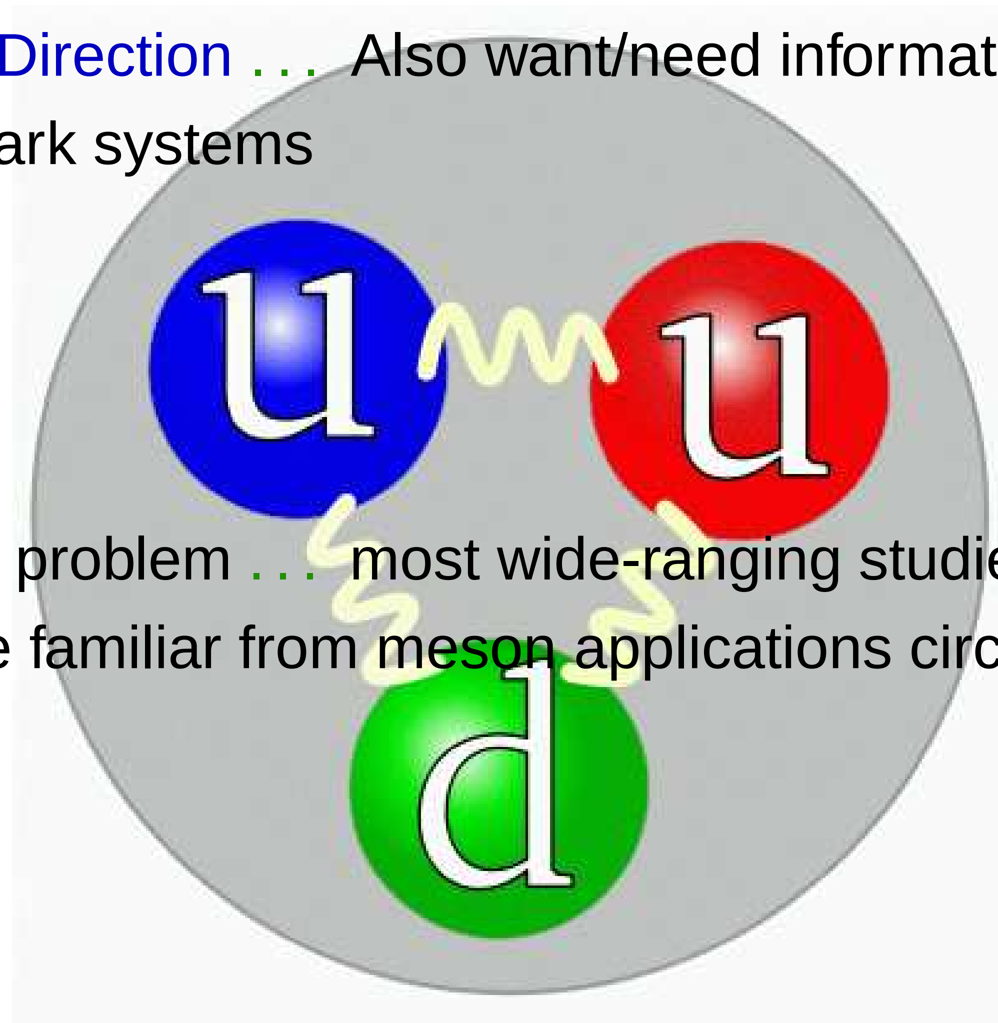

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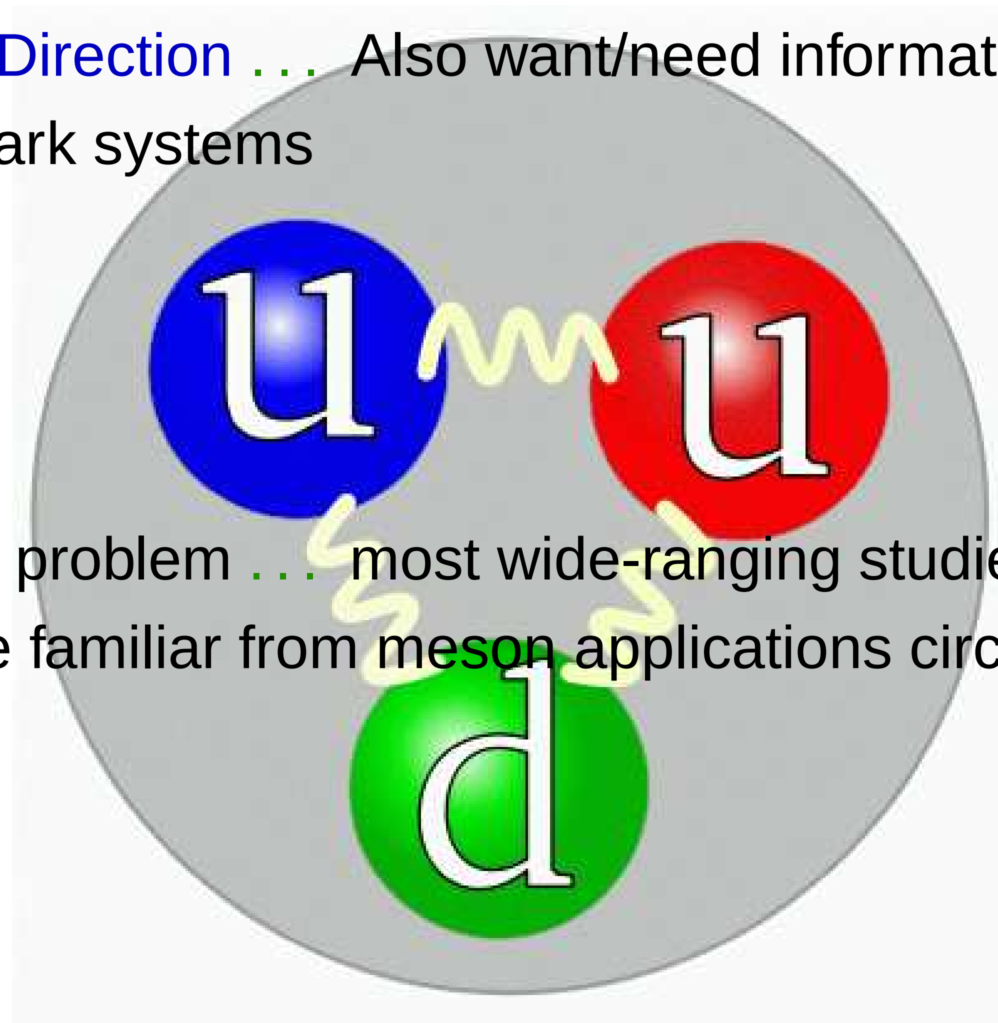

- With this problem . . . most wide-ranging studies employ expertise familiar from meson applications circa ~ 1995.

- Namely . . . Model-building and Phenomenology, constrained by the DSE results outlined already.

New Challenges

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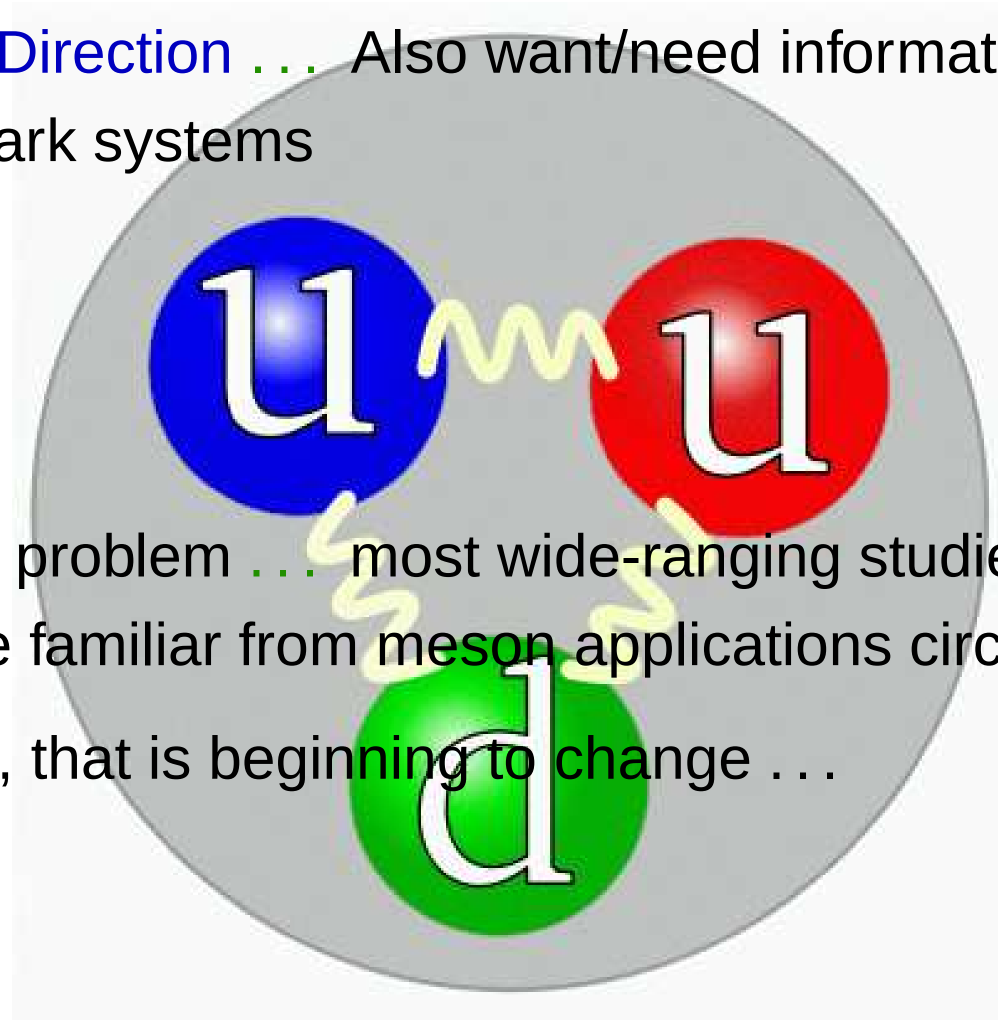
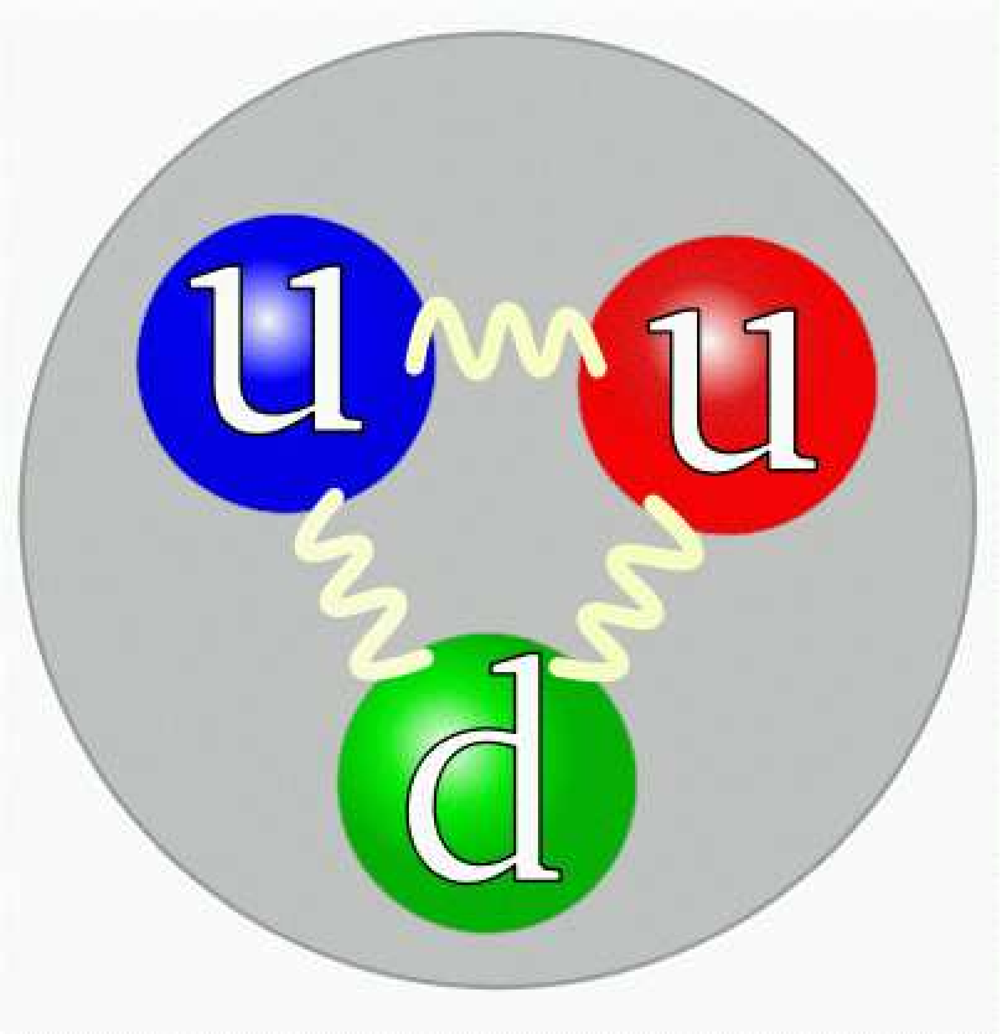


- With this problem . . . most wide-ranging studies employ expertise familiar from meson applications circa ~ 1995.
- However, that is beginning to change . . .

Nucleon . . .

Three-body Problem?



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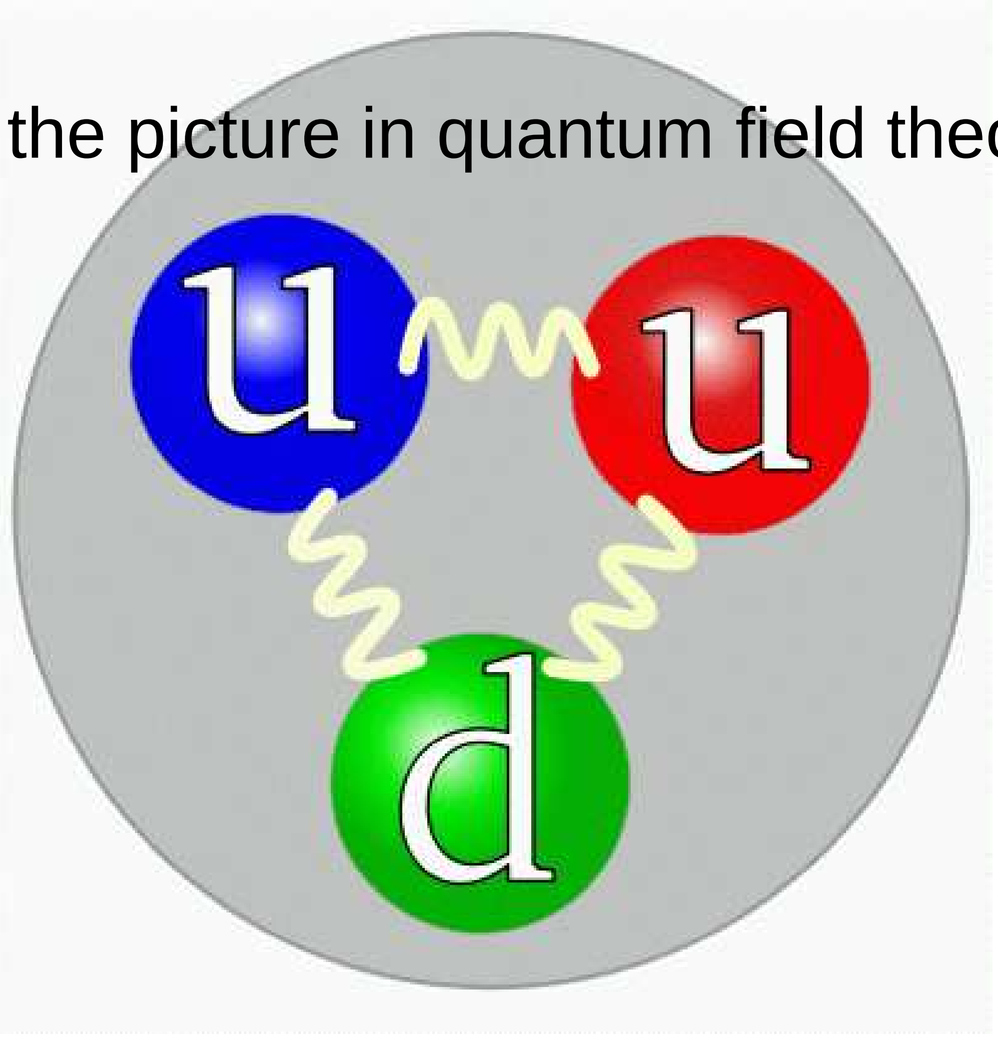
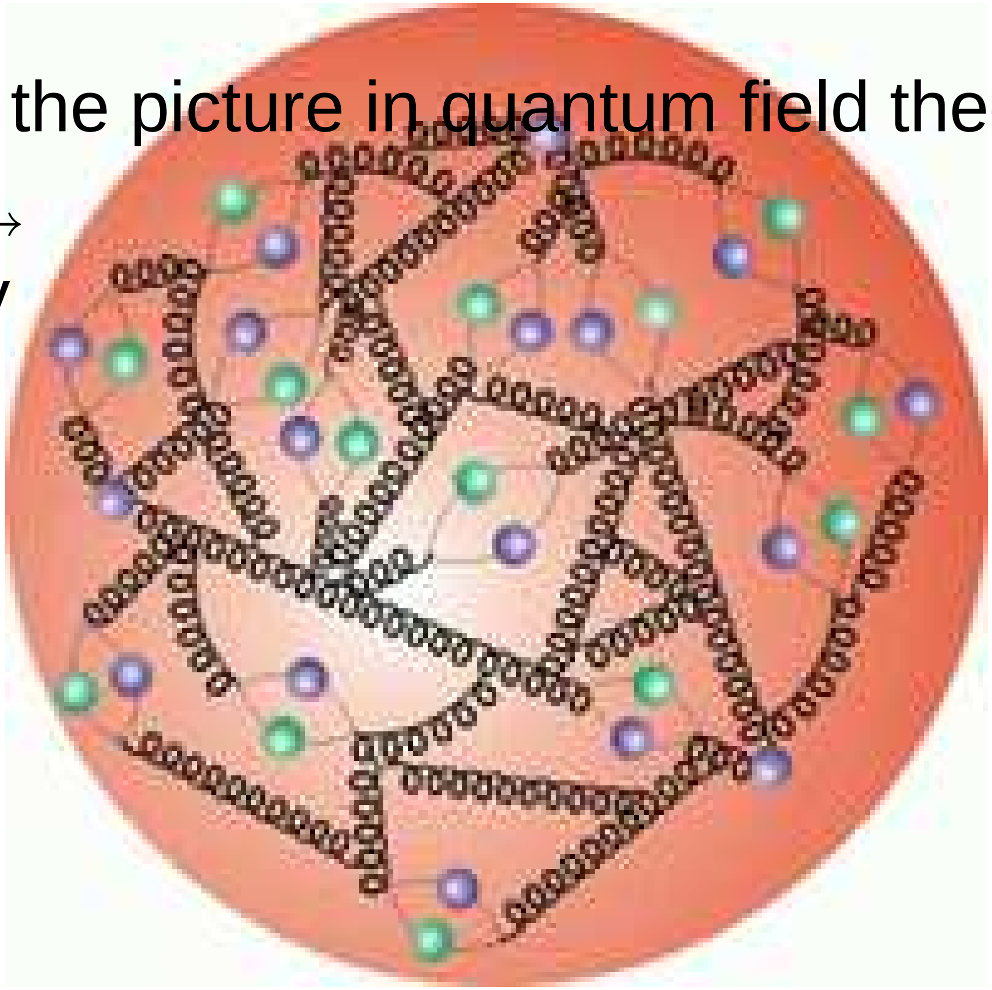
Three-body Problem?

- What is the picture in quantum field theory?



Three-body Problem?

- What is the picture in quantum field theory?
- Three → infinitely many!



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Unifying Study
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  - Poincaré covariant Faddeev equation sums all possible exchanges and interactions that can take place between three dressed-quarks
  - Tractable equation is founded on observation that an interaction which describes colour-singlet mesons also generates quark-quark (diquark) correlations in the colour-$\bar{3}$ (antitriplet) channel

Faddeev equation

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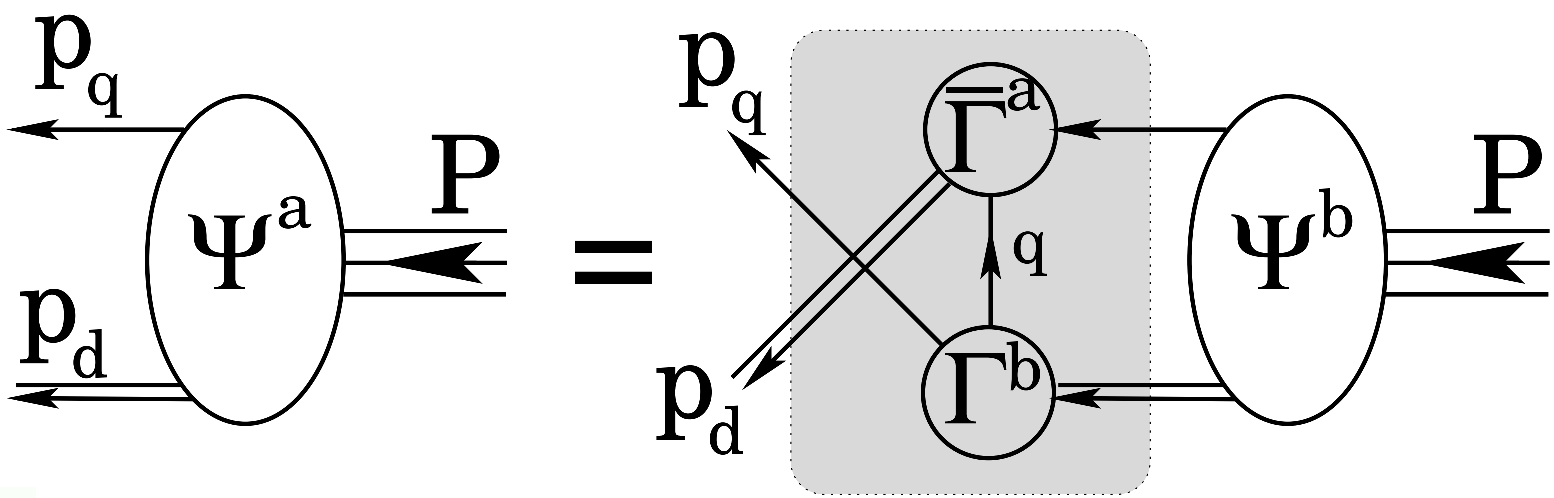

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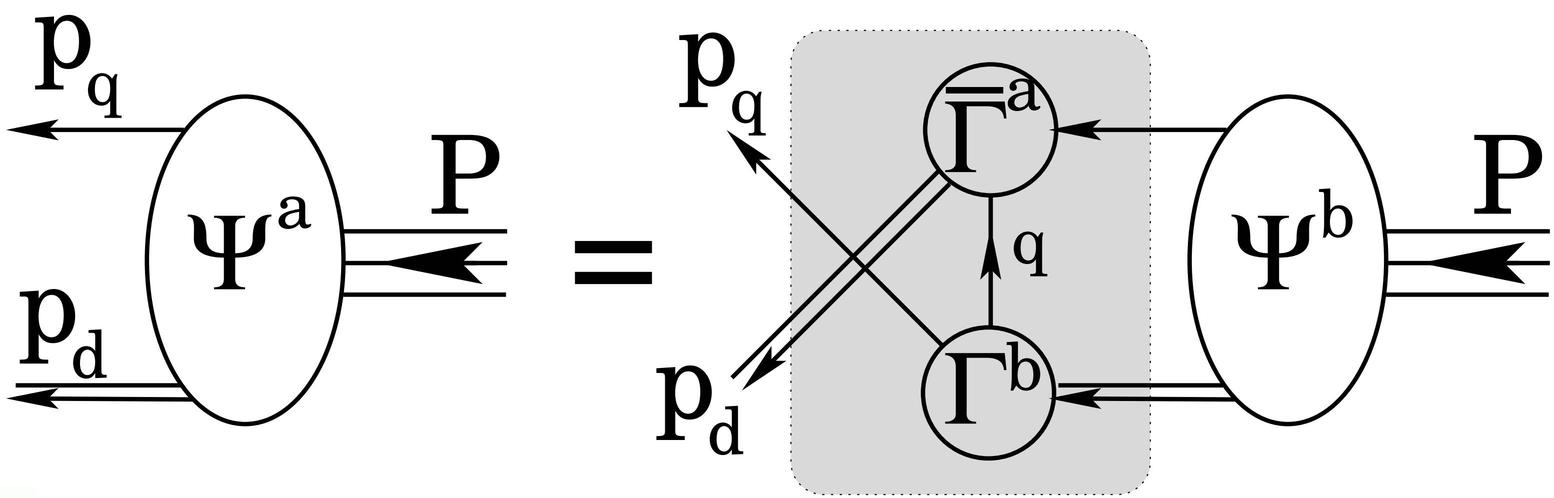
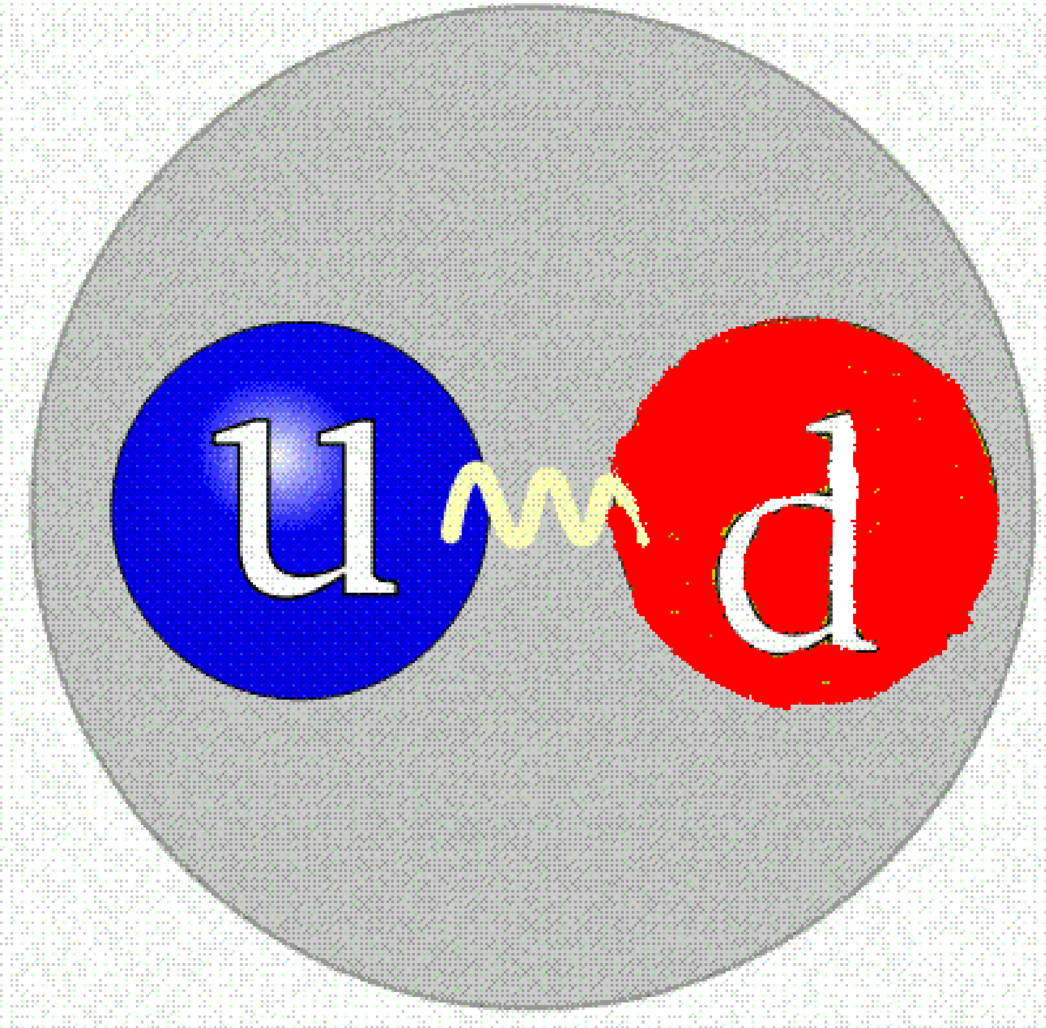


- Linear, Homogeneous Matrix equation
  - Yields *wave function* (Poincaré Covariant Faddeev Amplitude) that describes quark-diquark relative motion within the nucleon
- Scalar and Axial-Vector Diquarks ... In Nucleon's Rest Frame Amplitude has ... $s-$, $p-$ & $d-$wave correlations

Diquark correlations



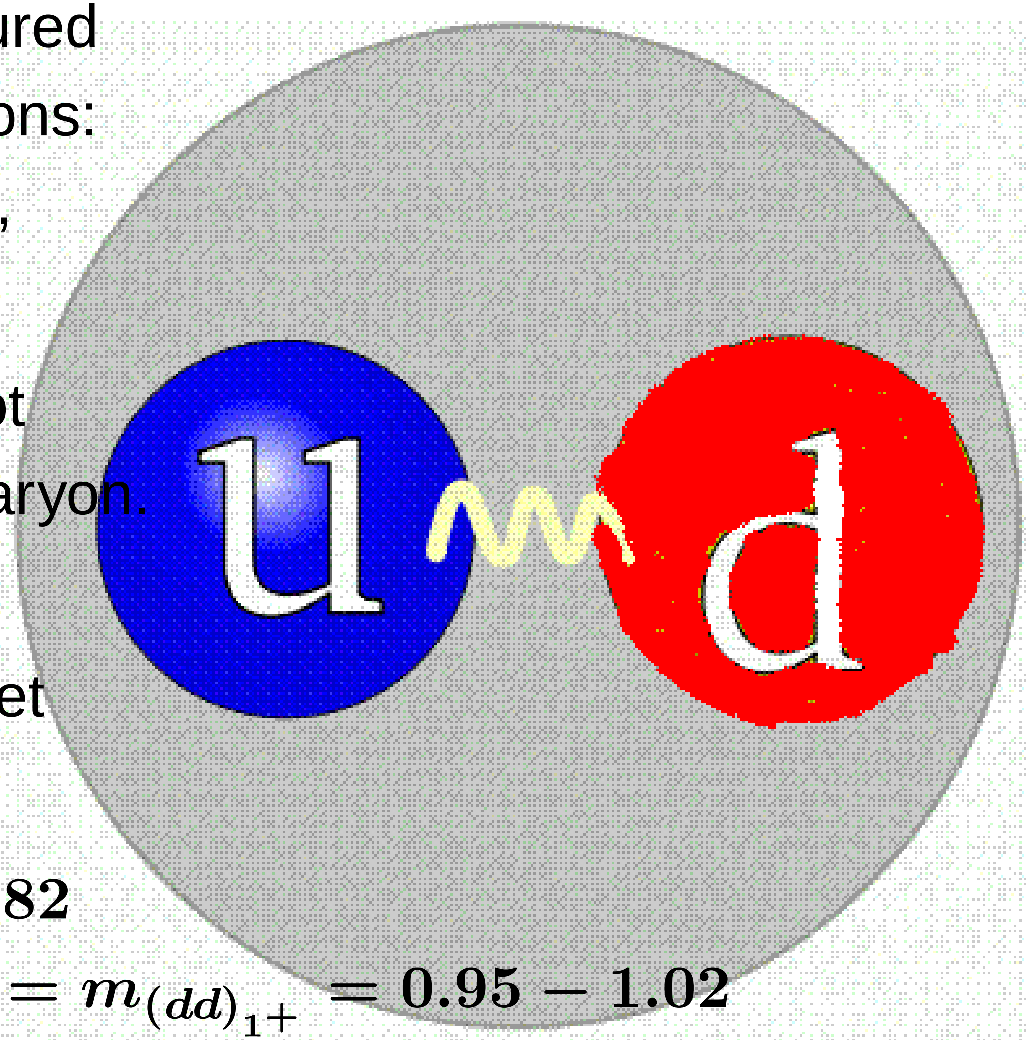
QUARK-QUARK

Diquark correlations

- Same interaction that describes mesons also generates three coloured quark-quark correlations: blue–red, blue–green, green–red
- Confined . . . Does not escape from within baryon.
- Scalar is isosinglet, Axial-vector is isotriplet
- DSE and lattice-QCD

$$m_{[ud]_{0^+}} = 0.74 - 0.82$$

$$m_{(uu)_{1^+}} = m_{(ud)_{1^+}} = m_{(dd)_{1^+}} = 0.95 - 1.02$$



QUARK-QUARK

Ab-initio study of mesons & nucleons

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Ab-initio study of mesons & nucleons

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Ab-initio study of mesons & nucleons

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  - Use this knowledge to constrain interaction in infrared
  - Interaction in ultraviolet predicted by perturbative expansion of DSEs

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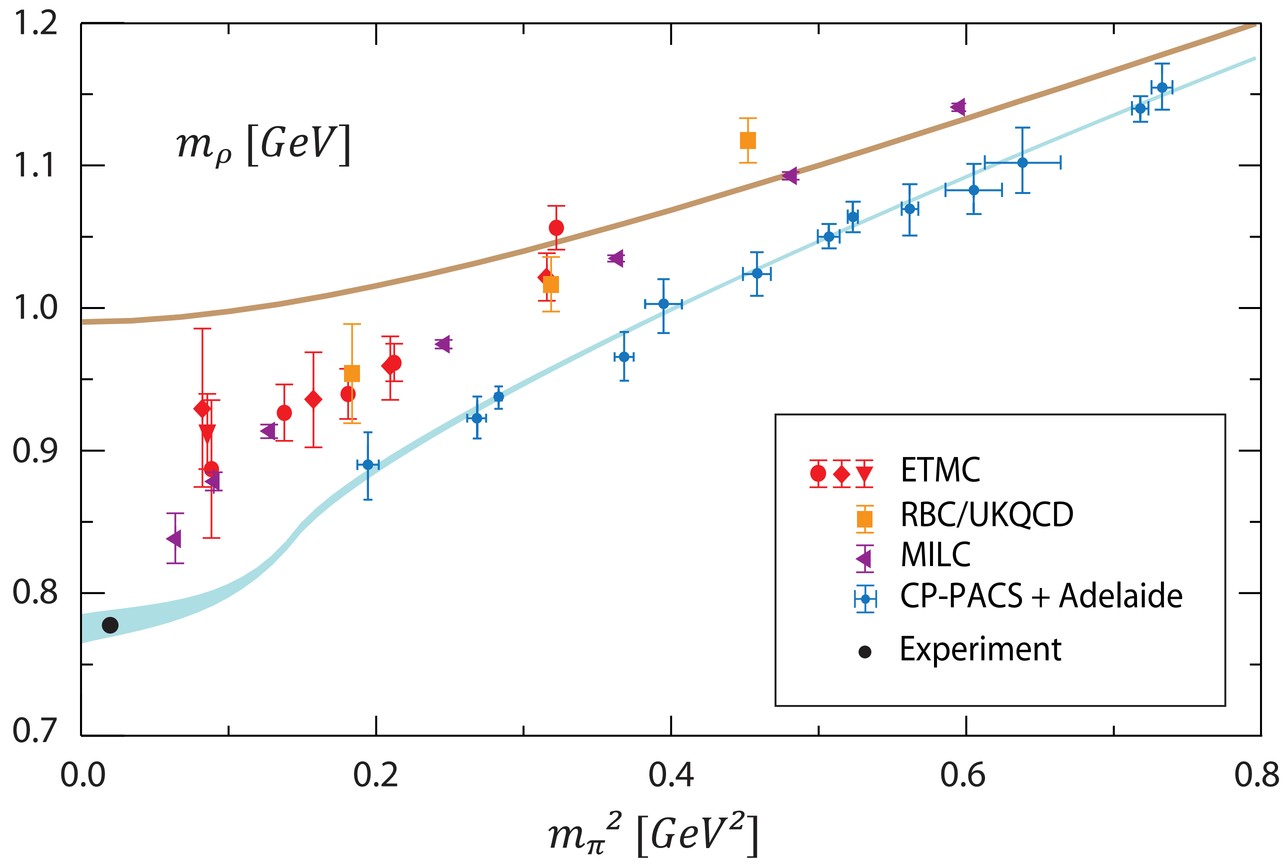

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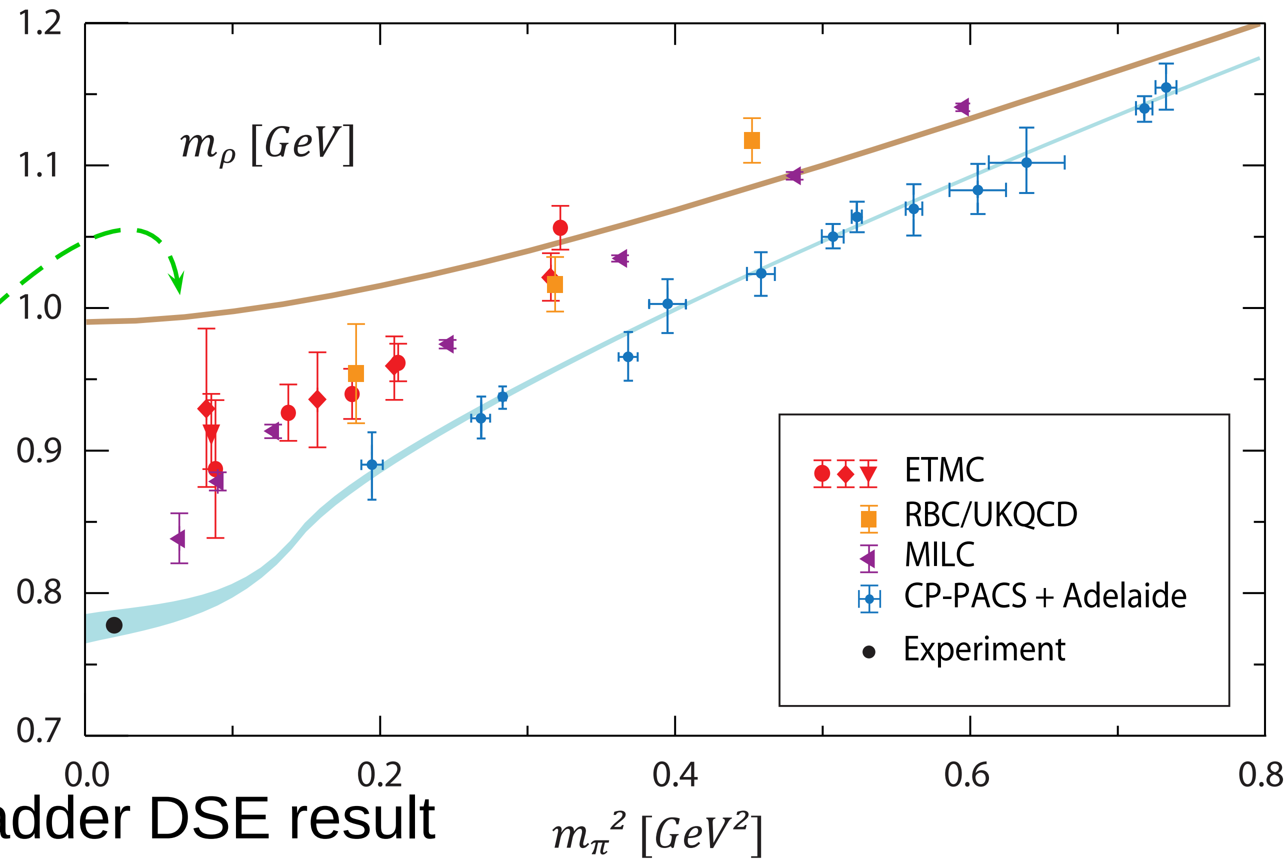

- Rainbow-Ladder DSE result

one parameter for IR – “confinement radius”

Results insensitive to value on material domain

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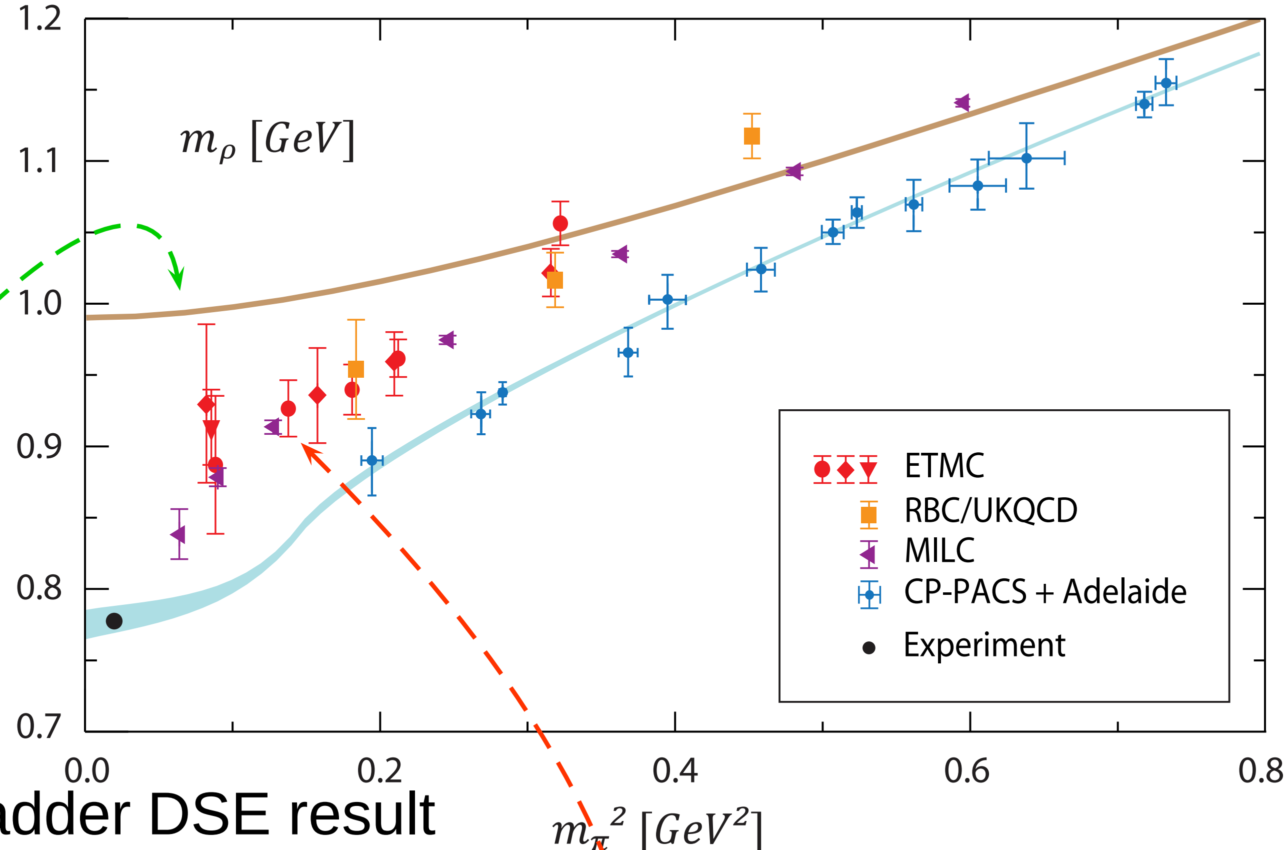

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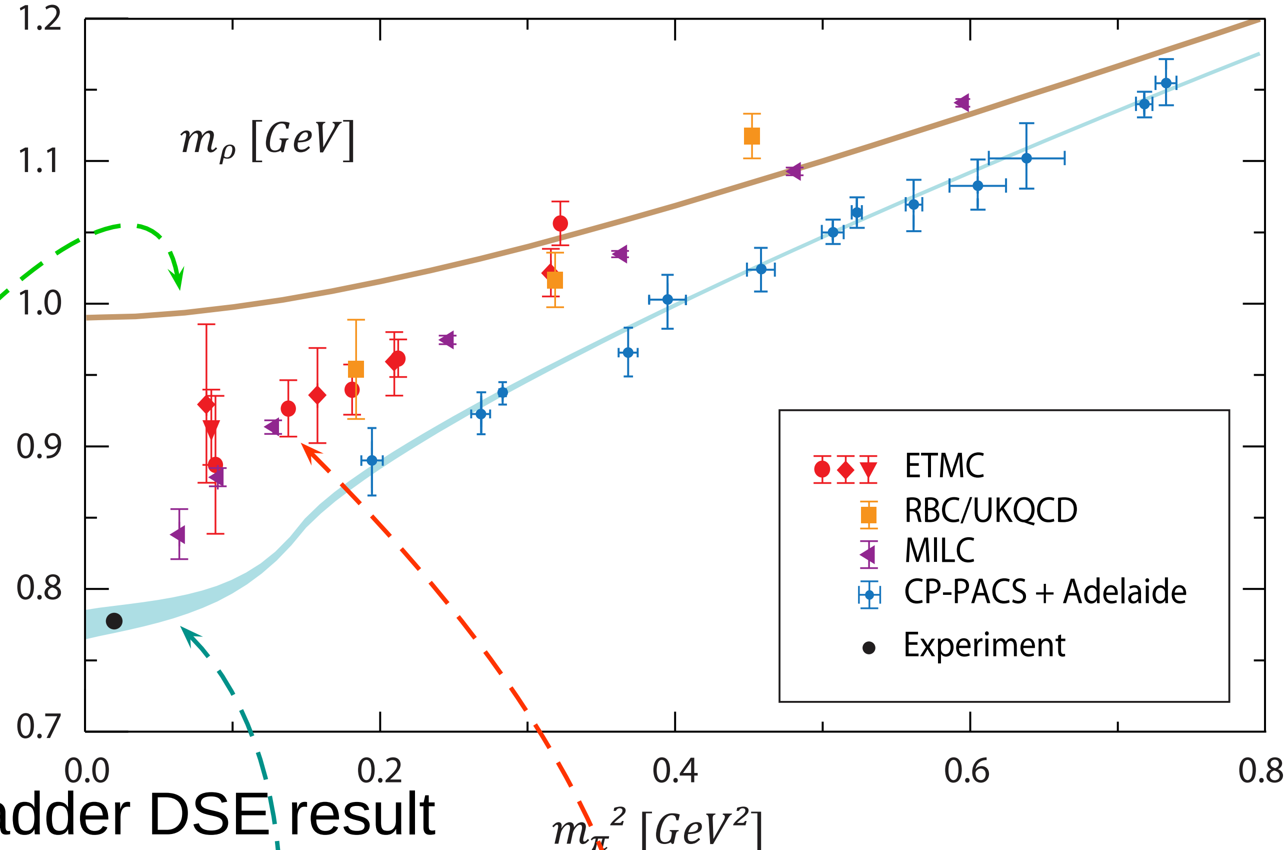

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Ab-initio study of mesons & nucleons

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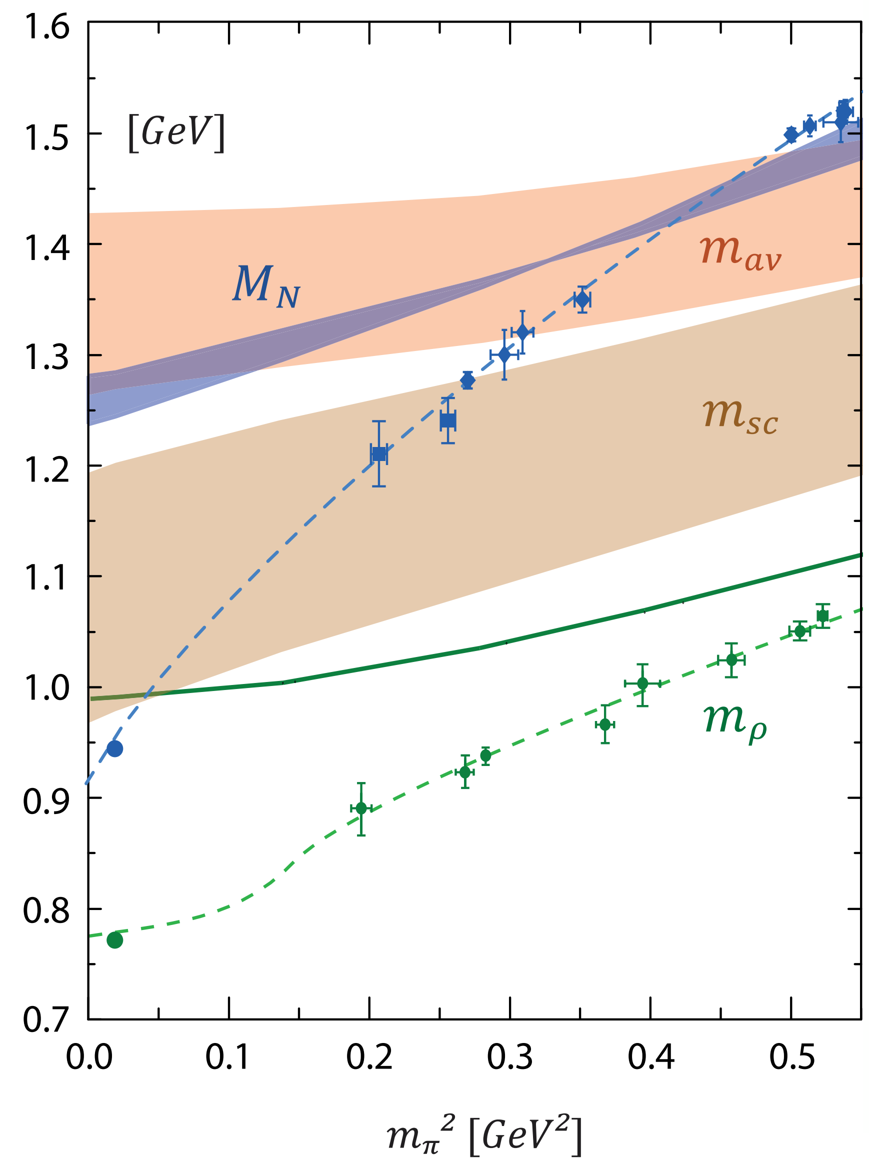

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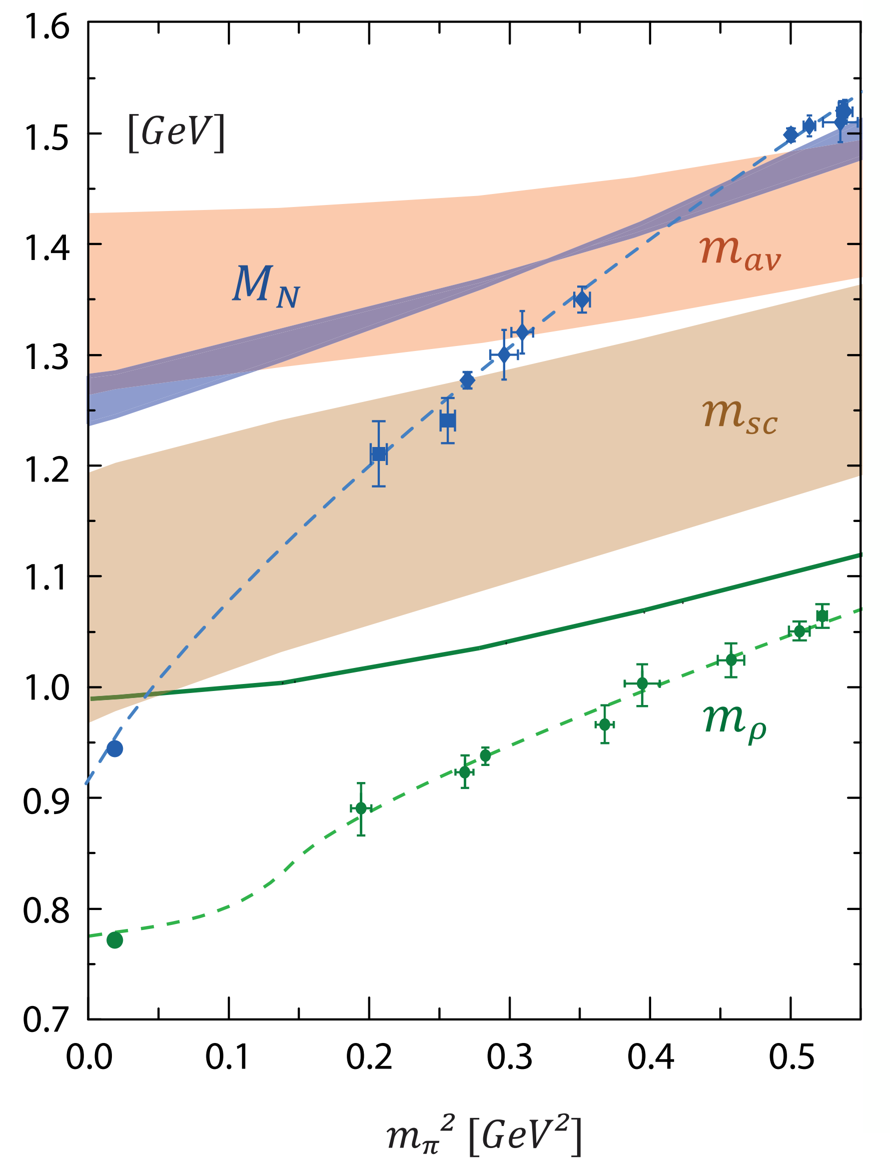

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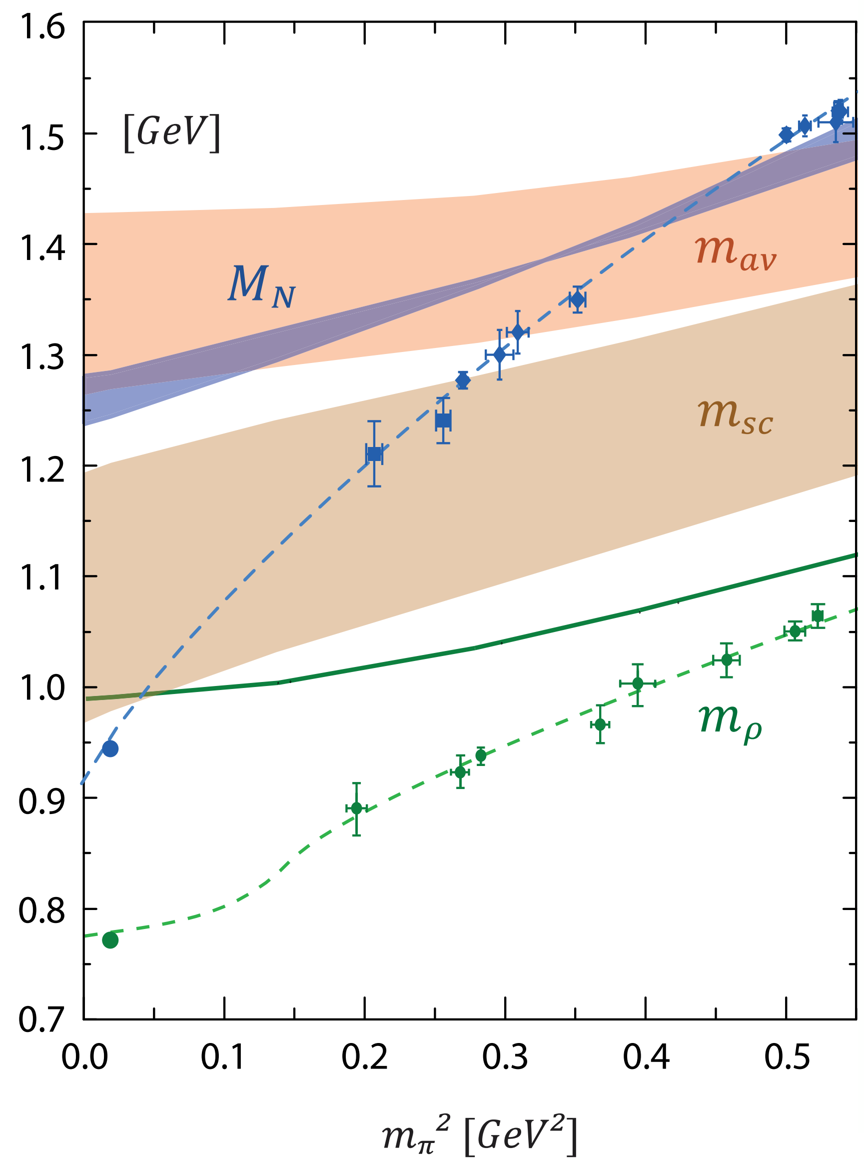
- But ... $[m_{av} - m_{sc}]$, m_ρ & M_N ... are *independent* of that parameter



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Ab-initio study of mesons & nucleons

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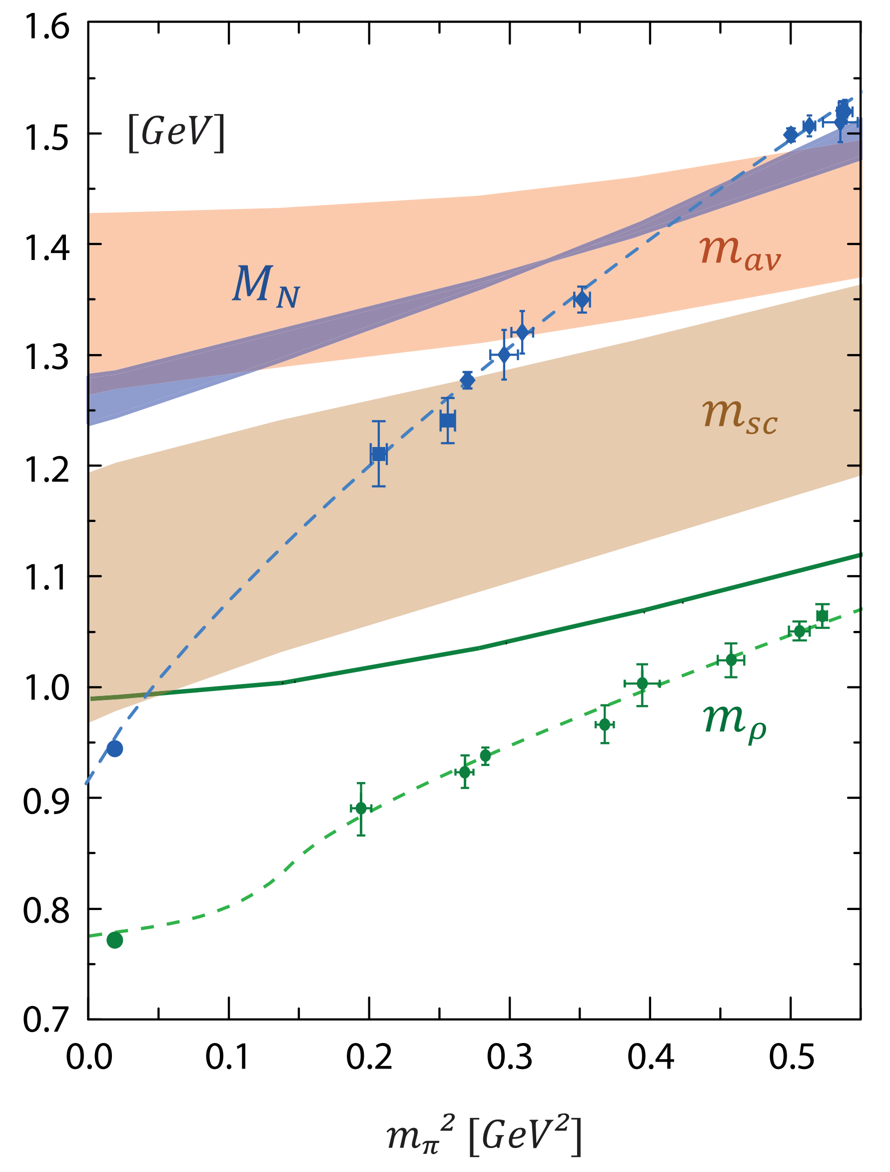

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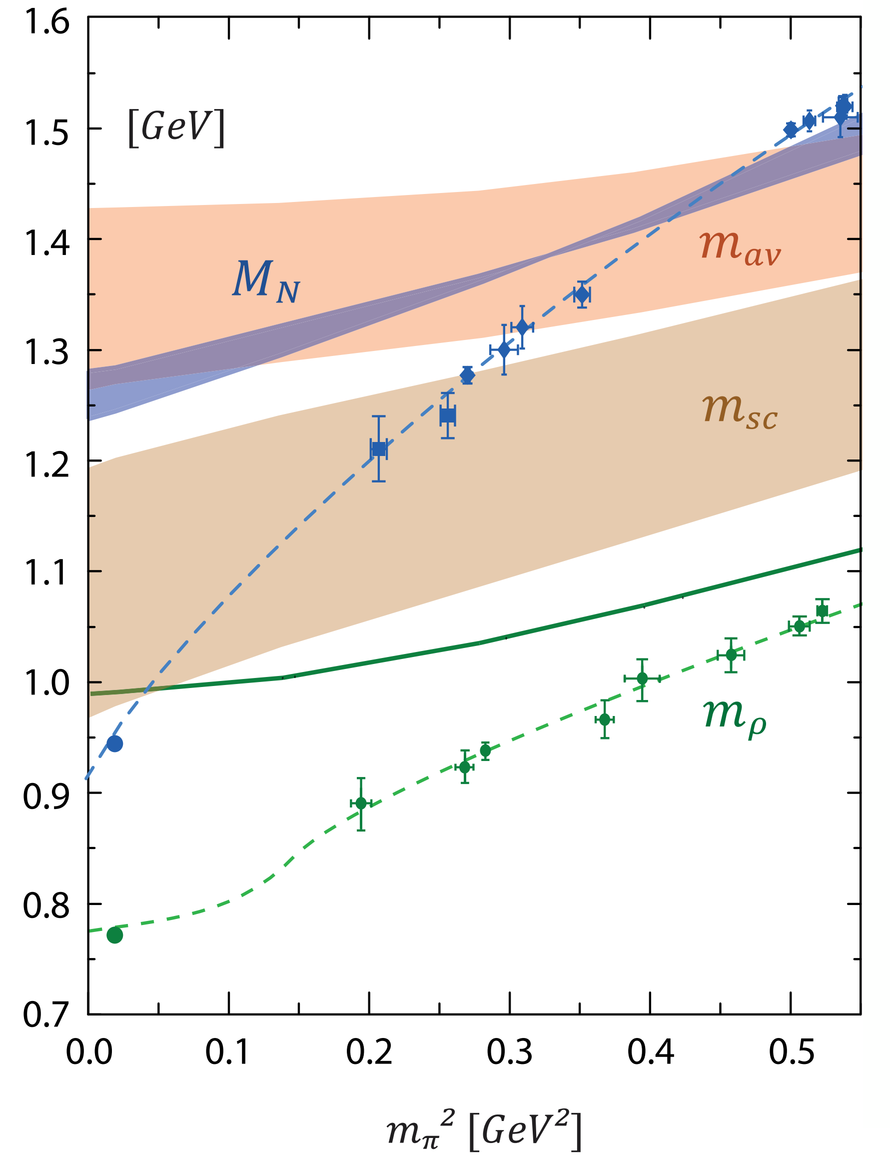

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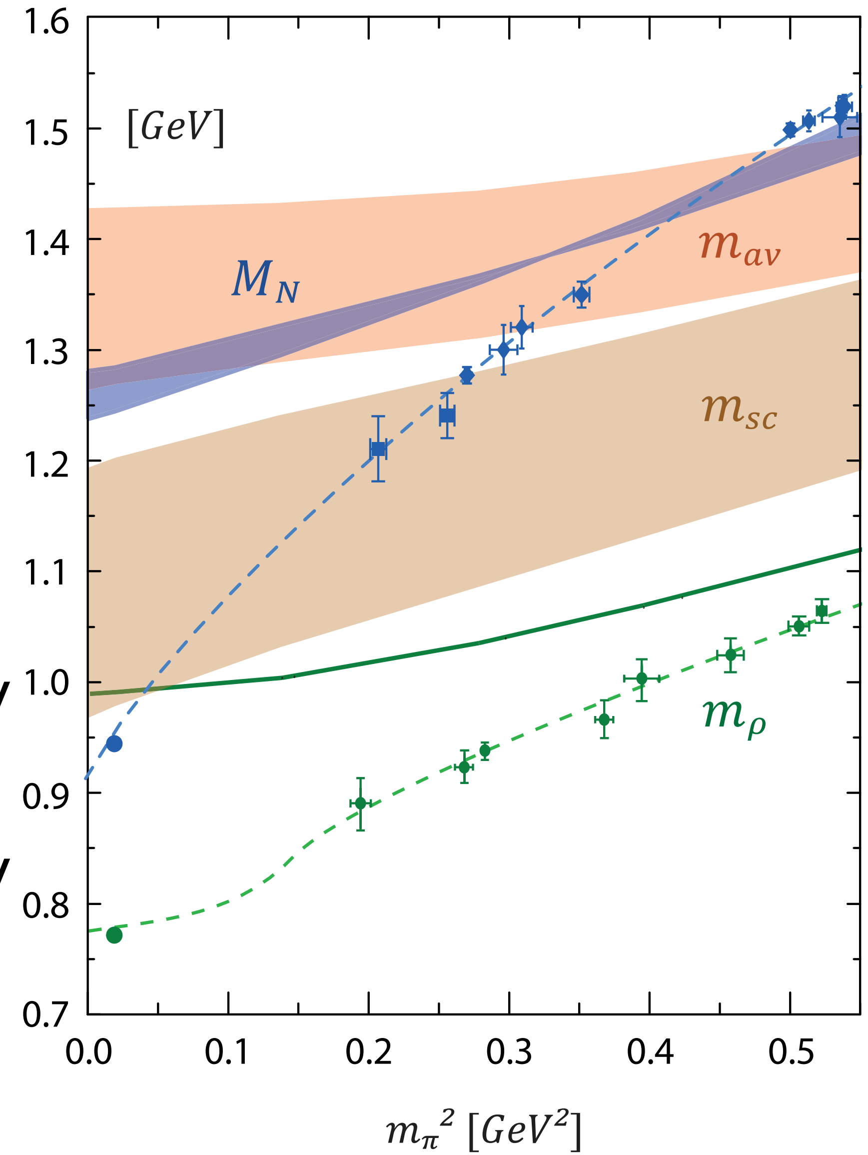
- Prediction: at physical m_π^2, $M_N^{\text{quark-core}} = 1.26(2)$ GeV cf. FRR+lattice-QCD, $M_N^{\text{quark-core}} = 1.27(2)$ GeV

$\Rightarrow$ subleading corrections, including 0^--meson loops, $\delta M_N = -320$ MeV, $\delta m_\rho = -220$ MeV



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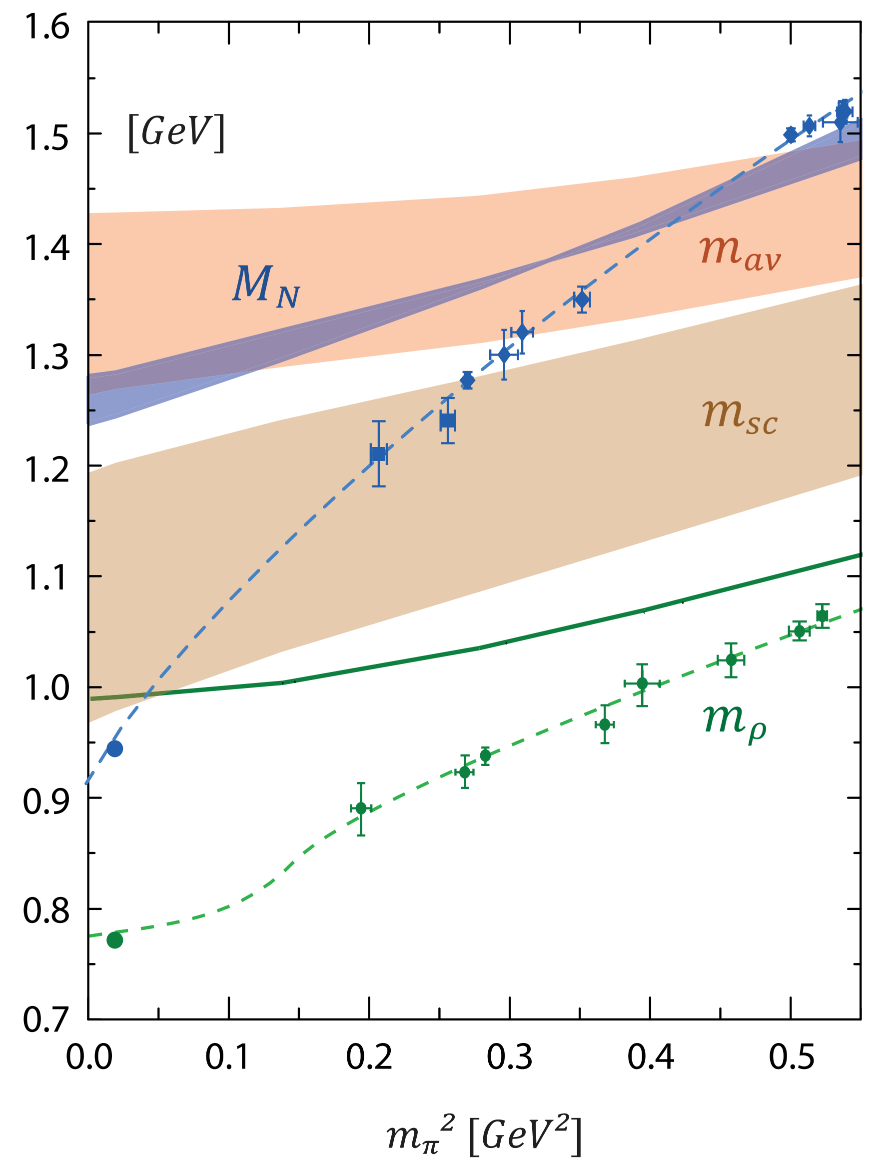

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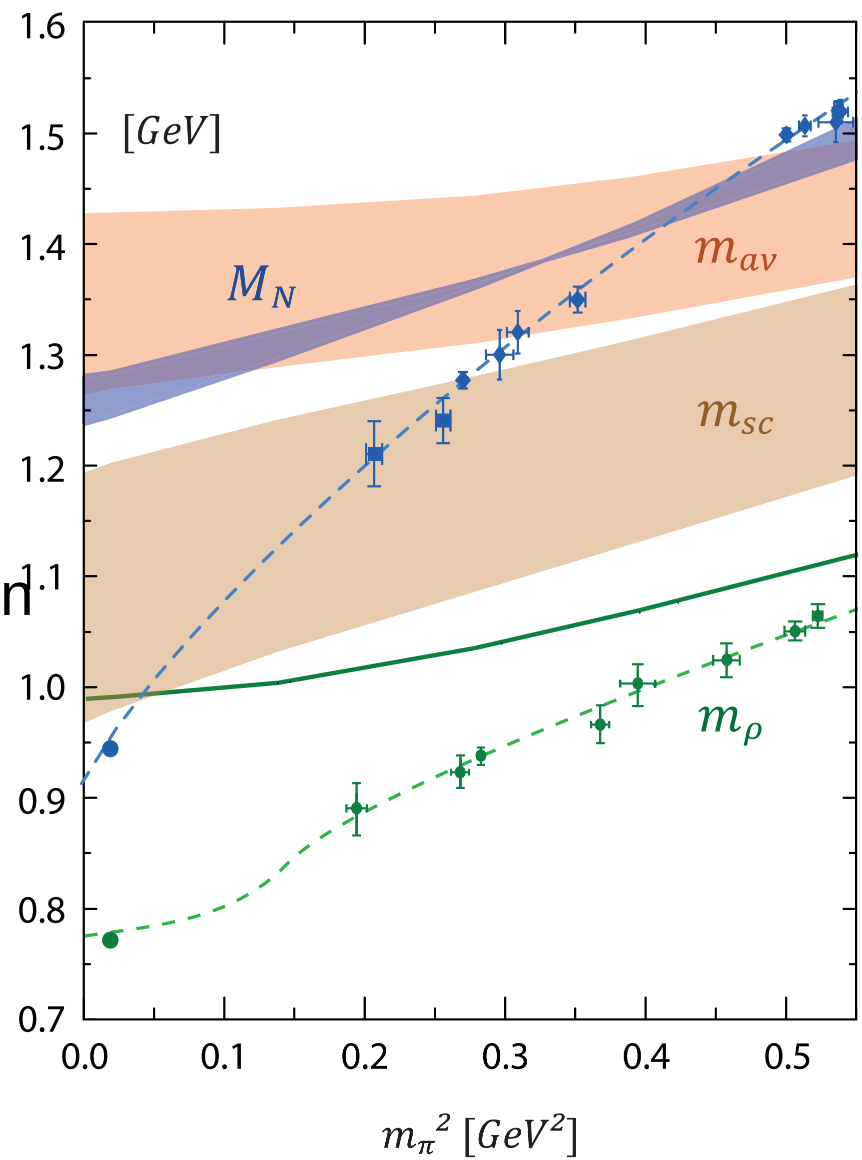

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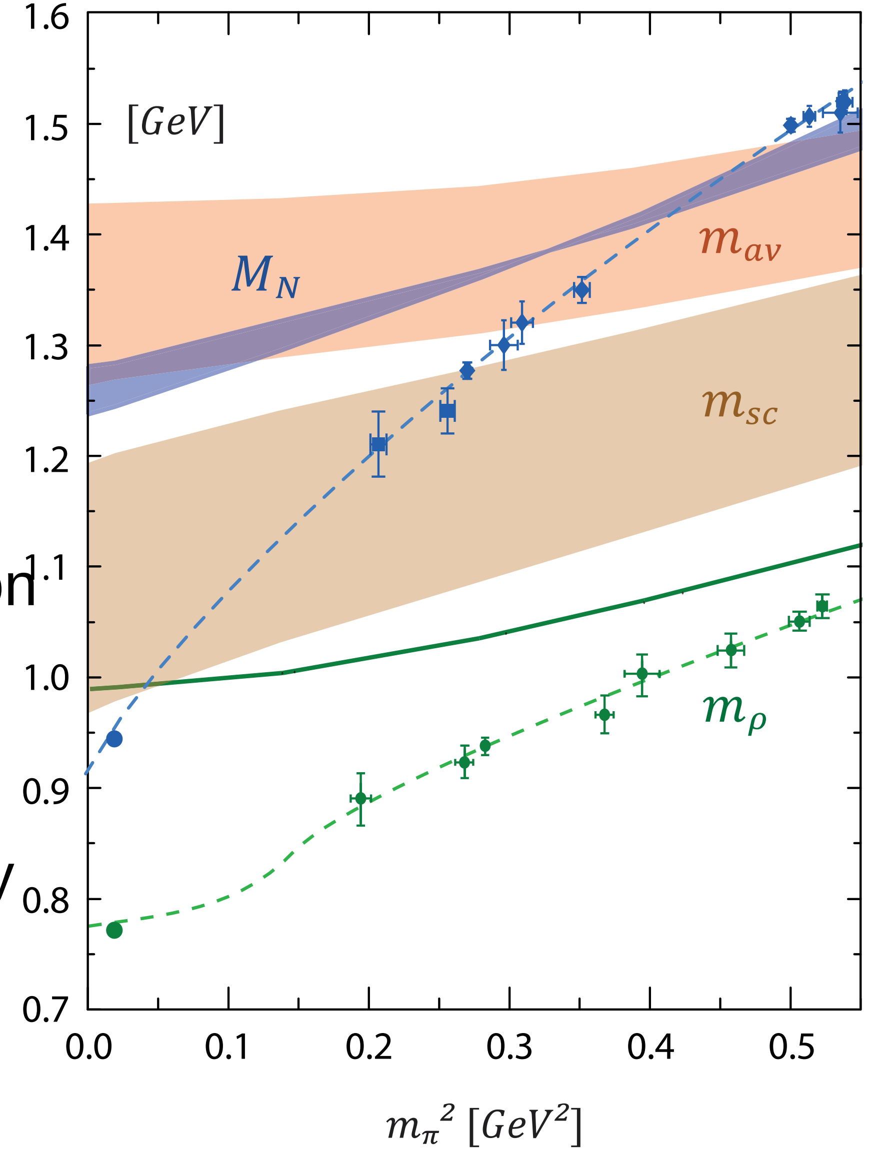

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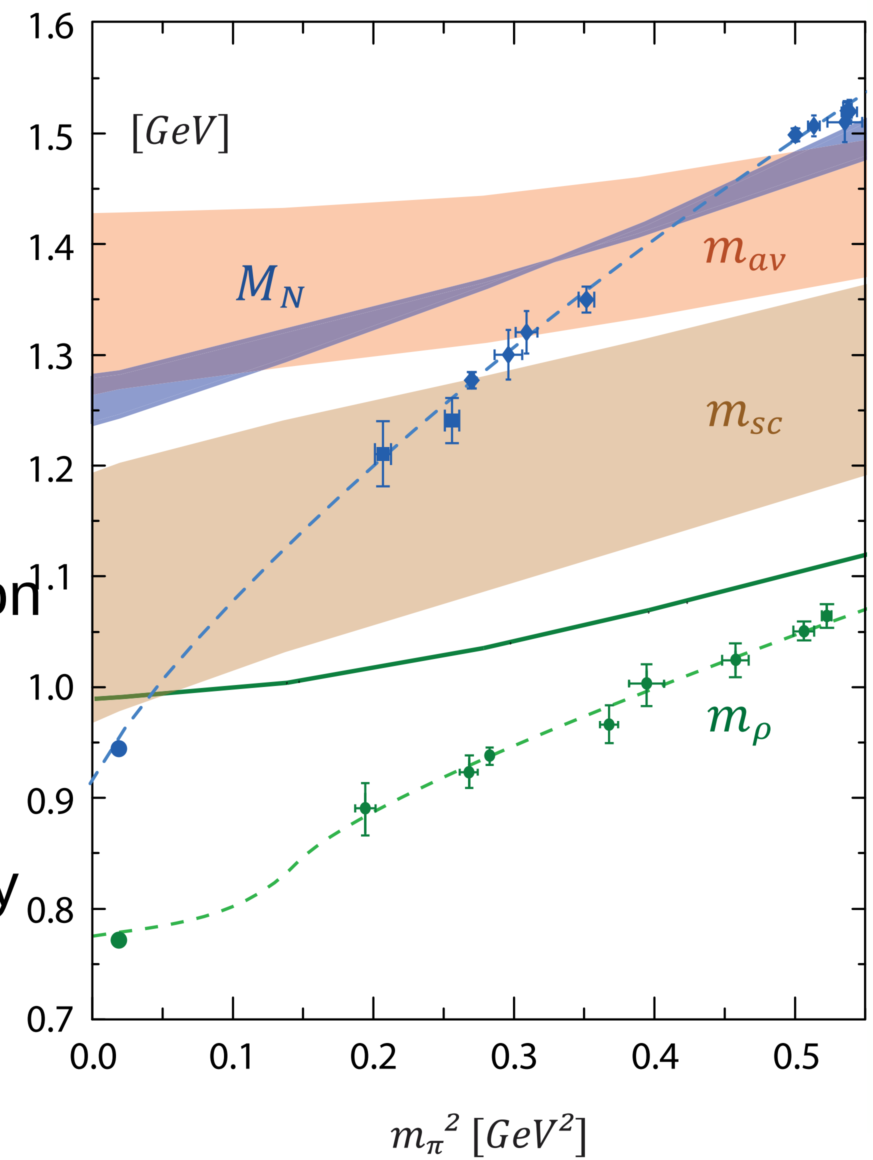
- *Systematically improvable*



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ab initio *Faddeev Equation*

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ab initio

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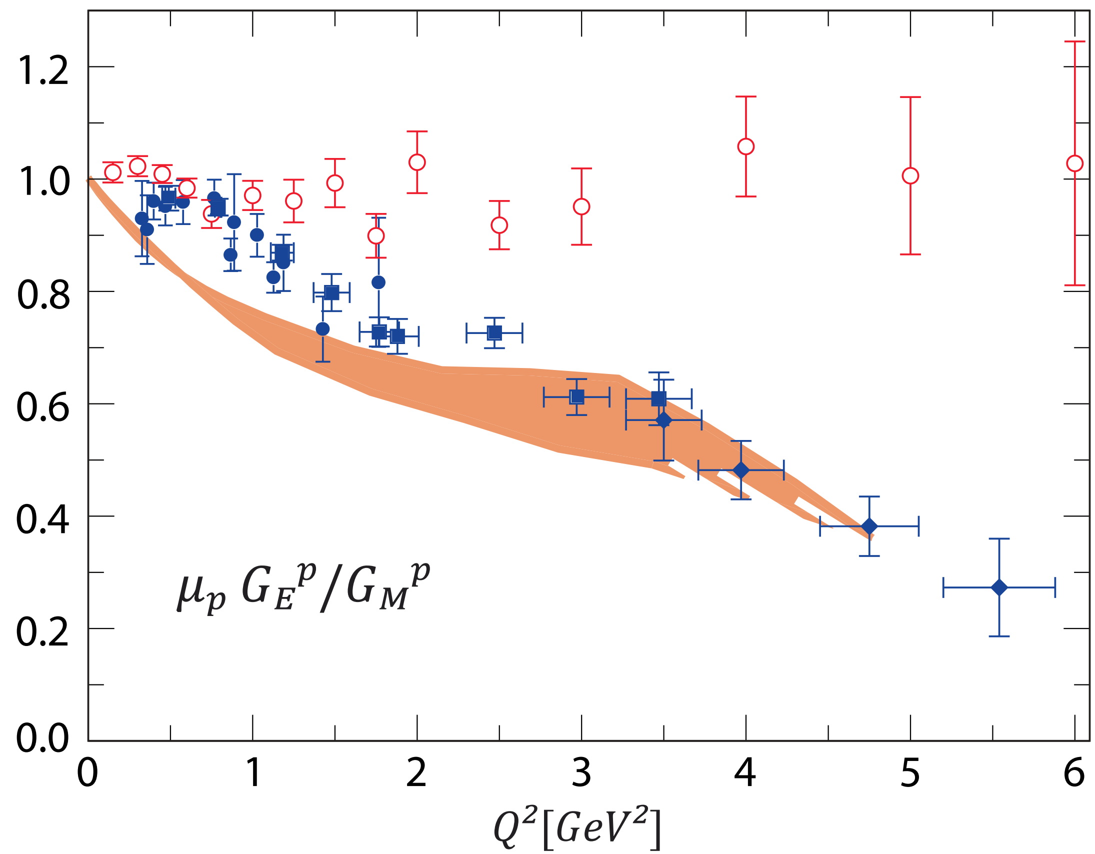

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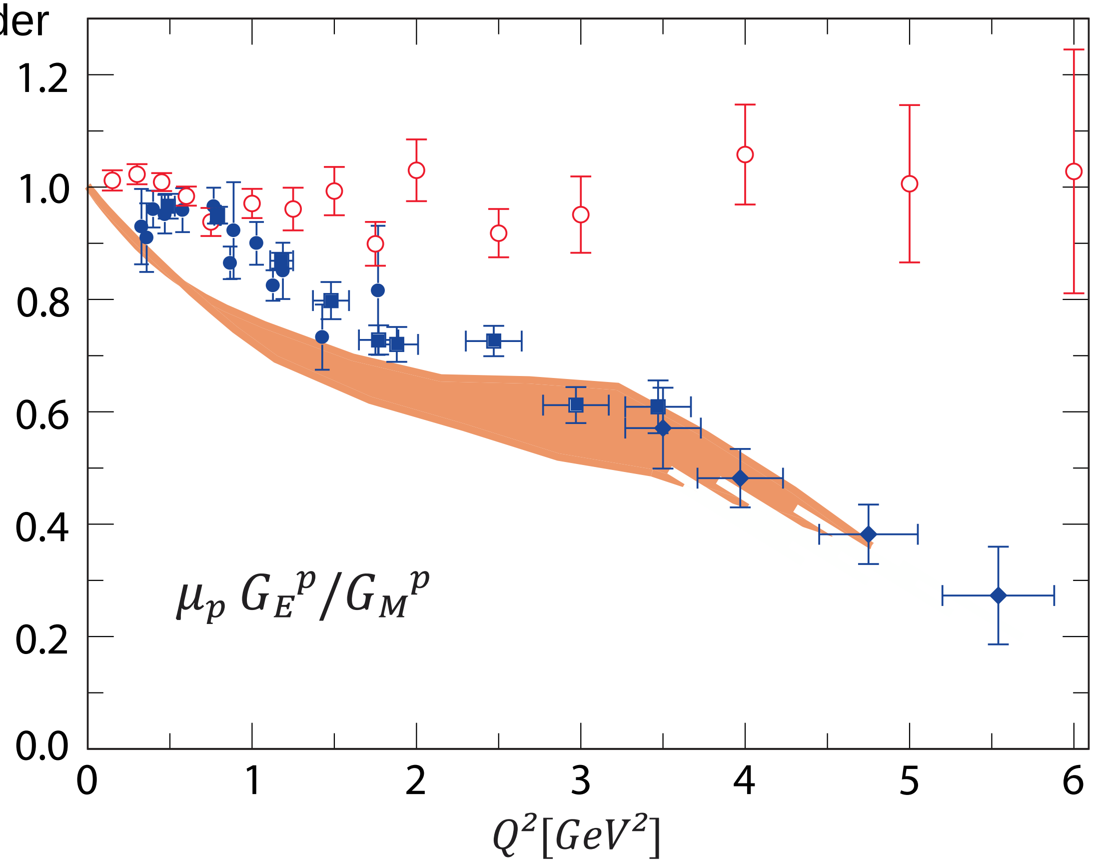

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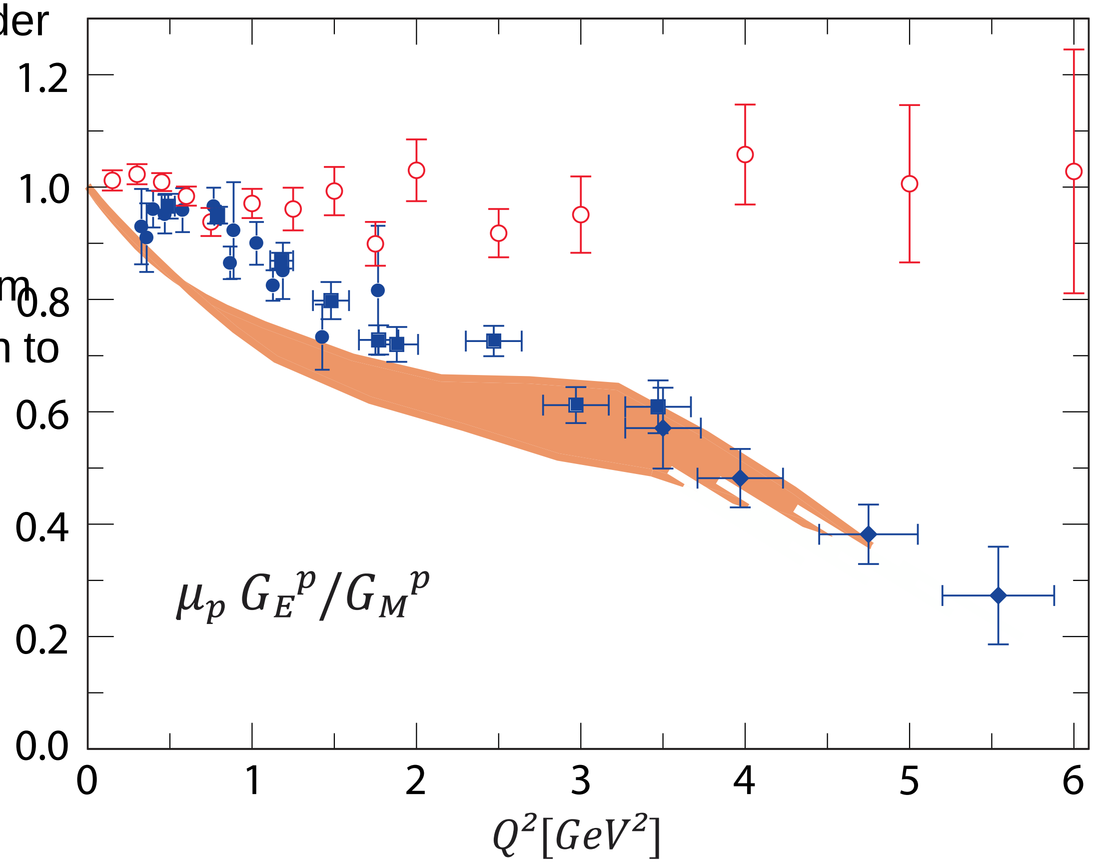

ab initio
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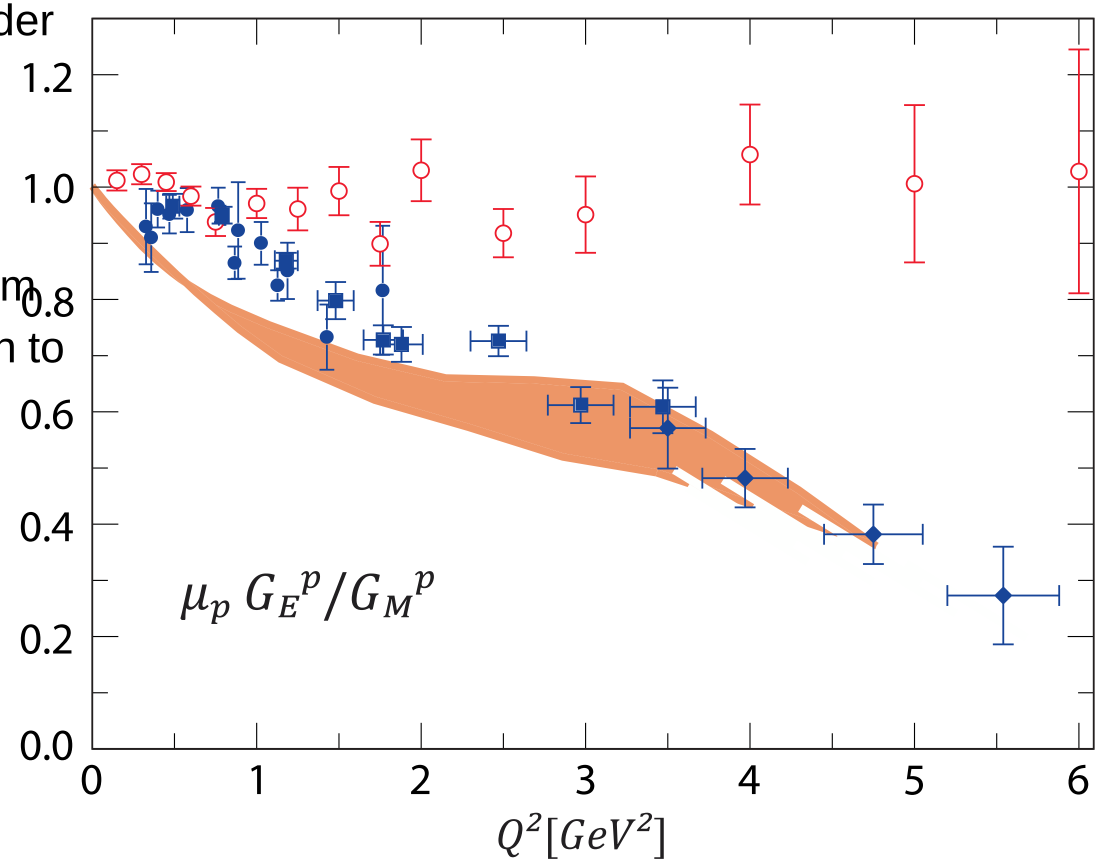


- Calculation unifies π, ρ and nucleon properties – keystone is behaviour of dressed-quark mass function and hence veracious description of QCD's Goldstone mode

Ratio of Neutron Pauli & Dirac Form Factors

- $$\frac{\hat{Q}^2}{(\ln \hat{Q}^2/\hat{\Lambda})^2} \frac{F_2^n(\hat{Q}^2)}{F_1^n(\hat{Q}^2)}$$

  $$\hat{\Lambda} = \Lambda/M_N = 0.44$$

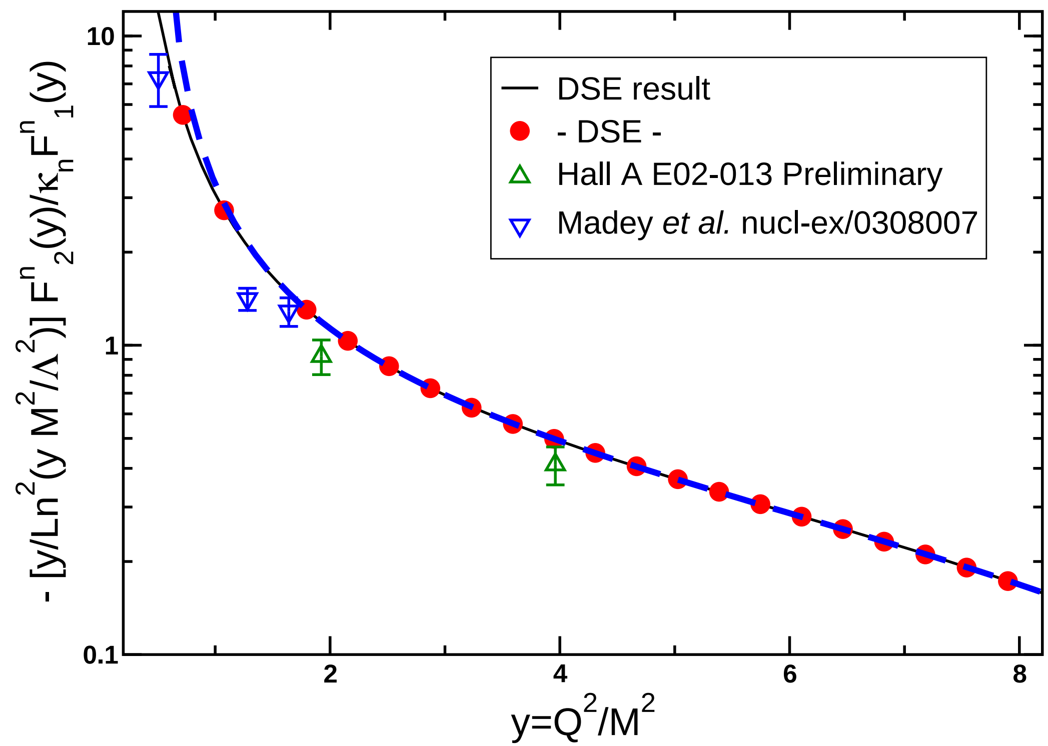
  Ensures proton ratio constant for $\hat{Q}^2 \geq 4$

Ratio of Neutron Pauli & Dirac Form Factors

- $$\frac{\hat{Q}^2}{(\ln \hat{Q}^2/\hat{\Lambda})^2}\,\frac{F_2^n(\hat{Q}^2)}{F_1^n(\hat{Q}^2)}$$

  $$\hat{\Lambda} = \Lambda/M_N = 0.44$$

  Ensures proton ratio constant for $\hat{Q}^2 \geq 4$



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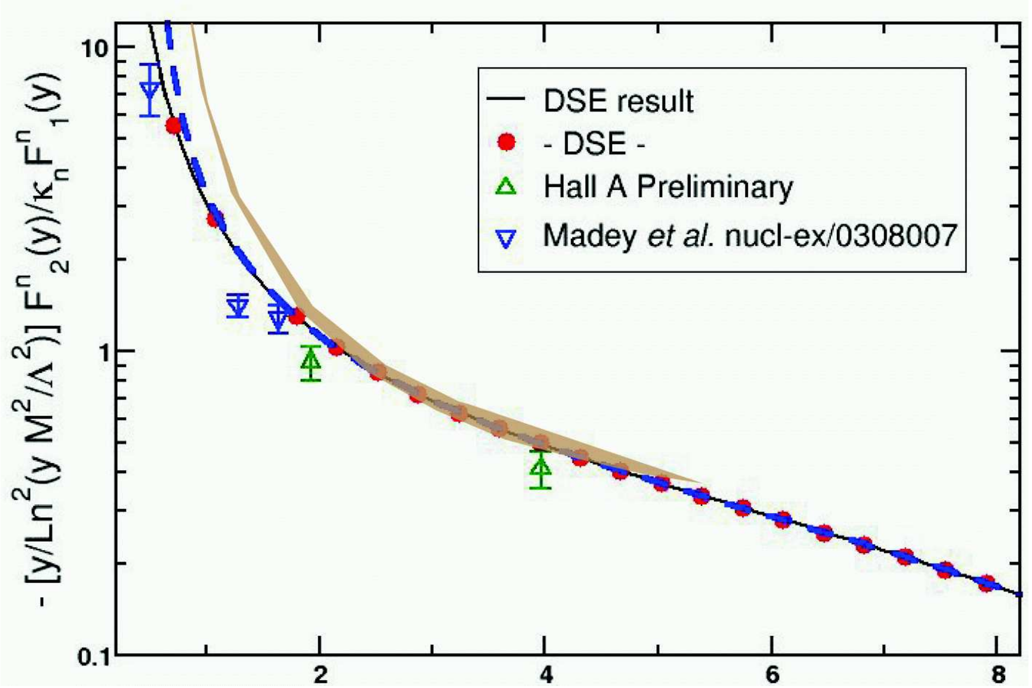
Ratio of Neutron Pauli & Dirac Form Factors

- $$\frac{\hat{Q}^2}{(\ln \hat{Q}^2/\hat{\Lambda})^2} \frac{F_2^n(\hat{Q}^2)}{F_1^n(\hat{Q}^2)}$$

  $$\hat{\Lambda} = \Lambda/M_N = 0.44$$

  Ensures proton ratio constant for $\hat{Q}^2 \geq 4$

- Brown band
  – *ab initio* RL result



Pion Cloud

Pion Cloud
F2 – neutron

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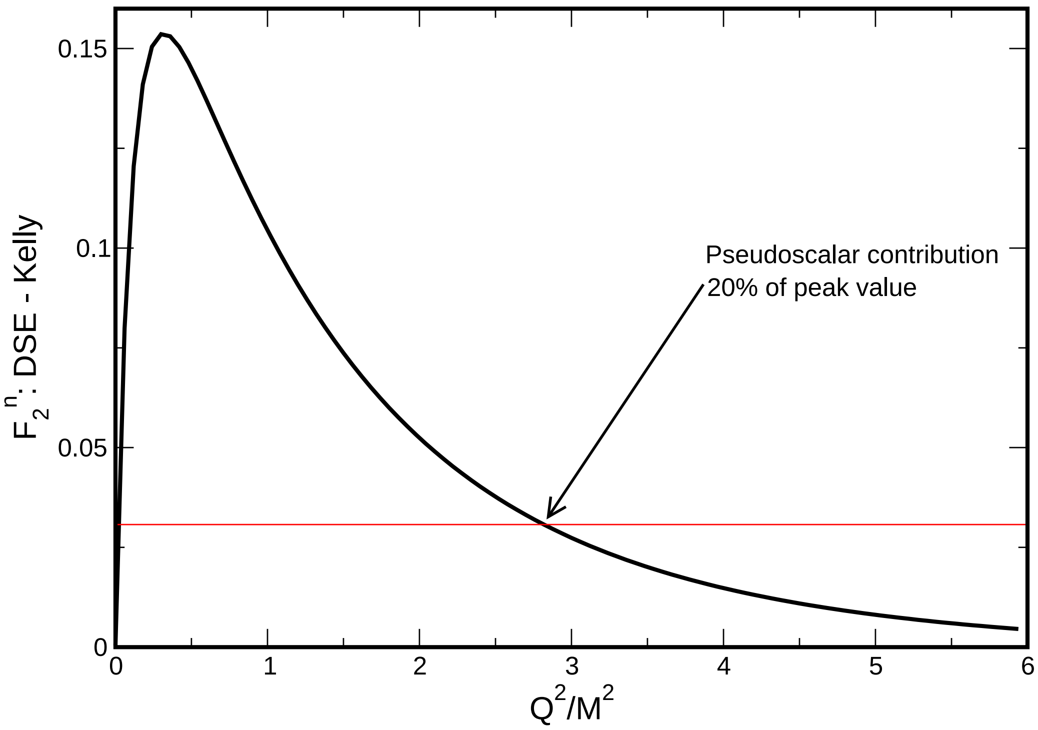
Pion Cloud
F2 – neutron

- Comparison between Faddeev equation result and Kelly's parametrisation
- Faddeev equation set-up to describe dressed-quark core



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Pion Cloud
F2 – neutron

- Comparison between Faddeev equation result and Kelly's parametrisation
- Faddeev equation set-up to describe dressed-quark core
- Pseudoscalar meson cloud (and related effects) significant for $Q^2 \lesssim 3 - 4\, M_N^2$