Thermodynamics of Lattice QCD with Colour-Sextet Quarks

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Introduction

Extensions to the Standard Model with a strongly coupled Higgs sector (composite Higgs) are amenable to Lattice Gauge Theory simulation methods. One class of such models is Technicolor where the Higgs field is replaced with the (techni)pions of a QCD-like gauge theory. Potential phenomenological problems with such theories can be circumvented by choosing the gauge group, number of (techni)quarks N_f and their colour representation such that the running coupling constant evolves very slowly – Walking Technicolor.

The running of the gauge coupling $g(\mu)$, where μ is the momentum scale at which g is measured is described by $\beta(g)$.

$$\beta(g) = \mu \frac{\partial g}{\partial \mu} = -\beta_0 \frac{g^3}{(4\pi)^2} - \beta_1 \frac{g^5}{(4\pi)^4} \ldots$$

where the coefficients depend on the gauge group, the number of fermion flavours and their colour representation. For $N_f = 0$, β_0 and β_1 are both positive, and to the extent that we can neglect higher order terms, the theory is asymptotically free and confining. For each group/representation there is some value of N_f where β_1 changes sign. There is an even larger value of N_f for which β_0 also changes sign, and asymptotic freedom is lost. Between these two values, $\beta(g)$ develops a second zero at $g = g^*$. If this describes the physics, g^* is a fixed point describing the infrared behaviour of the massless theory, which is then a conformally invariant field theory with no particles. If, however, a chiral condensate forms before g^* is reached, $\beta(g)$ starts to decrease again, and the IR fixed point is avoided. Close to g^*, g will run very slowly and the theory is a candidate for Walking Technicolor.

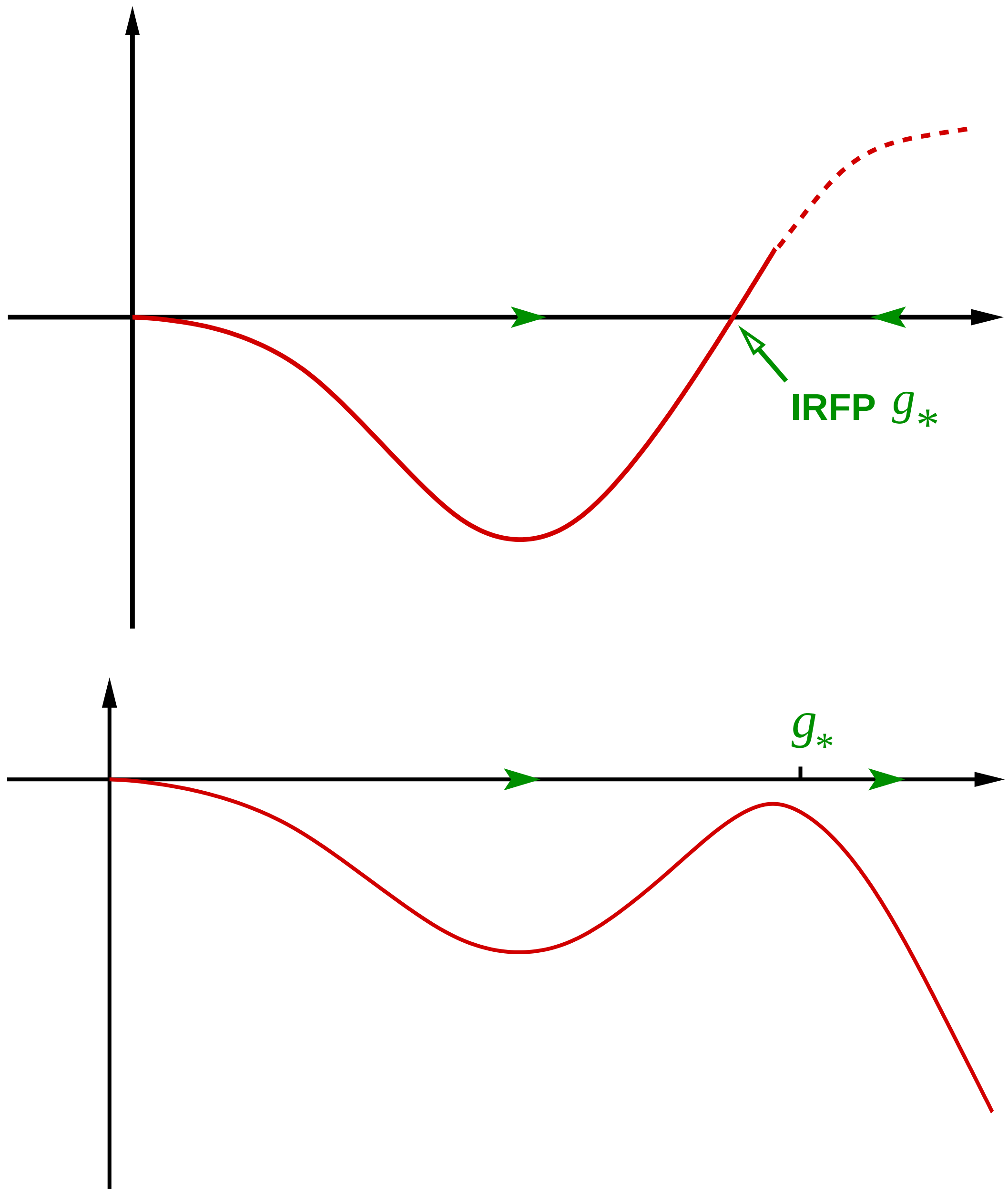

Figure 1: Cartoon of $\beta(g)$ as a function of g a) with an IR fixed point b) where a chiral condensate forms before g^* is reached. [from DeGrand, Shamir and Svetitsky]

For QCD [$SU(3)$ colour] with colour-sextet quarks, the N_f at which asymptotic freedom is lost is $N_f = 3\frac{3}{10}$, while that where the second term in the beta function changes sign is $N_f = 1\frac{28}{125}$. A rainbow graph estimate of that N_f below which a chiral condensate forms is $N_f = 2\frac{163}{325}$. This suggests $N_f = 2$ as a candidate for a Walking gauge theory. A non-perturbative calculation is needed to test this. Preliminary lattice work of DeGrand, Shamir and Svetitsky using Wilson quarks suggests that this $N_f = 2$ theory does have an IR (conformal) fixed point. The same authors have made a preliminary study of the thermodynamics of this theory.

We are simulating lattice QCD with $N_f = 2$ staggered sextet quarks at finite temperature to examine its phase structure and determine the relevant scales to compare with this pioneering work.

Note, since DeGrand, Shamir and Svetitsky's and our thermodynamics simulations study deconfinement and chiral-symmetry restoration transitions then, if the theory is indeed conformal, this regime is not connected to the continuum limit. A corollary is that we will not necessarily observe the same physics. In addition, lattice simulations work with massive quarks, which could enable us to avoid the IR fixed point, even if it is there for massless quarks, making attempts to extrapolate to zero quark mass difficult if not impossible.

Staggered quarks have the advantage over Wilson quarks in that they have a vestige of chiral symmetry, and a well-defined chiral order parameter.

Our preliminary indications are that QCD with 2 flavours of colour-sextet quarks shows well-separated deconfinement and chiral-symmetry restoration transitions at finite temperature. We have seen evidence that the deconfinement transition moves to weaker coupling as the number of lattice sites in the time direction is increased. This is consistent with this being a finite temperature transition for an asymptotically free field theory (rather than a bulk transition). More simulations will be necessary to draw conclusions as to the nature of the chiral transition. In addition we have observed 2 different 'deconfined' phases – a standard deconfined phase, and phase in which charge conjugation is spontaneously broken – the vestige of the Z_3 centre symmetry of the pure gauge theory.

Simulations of QCD with $N_f = 2$ sextet quarks at finite temperature

We simulate lattice QCD with the standard Wilson gauge action and (unimproved) staggered quarks. The exact RHMC algorithm is used to tune the number of flavours to 2.

Finite temperature T is achieved by simulating on a lattice of temporal extent $N_t = 1/T$ with periodic/antiperiodic boundary conditions in the time direction, and spatial extent $N_s >> N_t$.

Our current simulations are on $8^3 \times 4$, $12^3 \times 4$ and $12^3 \times 6$ lattices, with quark masses $m = 0.02$, $m = 0.01$ and $m = 0.005$. For each m and $\beta = 6/g^2$ we run for 10,000 – 100,000 length-1 trajectories.

To determine the position of the deconfinement transition we use the Wilson Lines (Polyakov Loops) in both the triplet and sextet representations. The chiral transition is determined from measuring the chiral condensate $\langle\bar{\psi}\psi\rangle$.

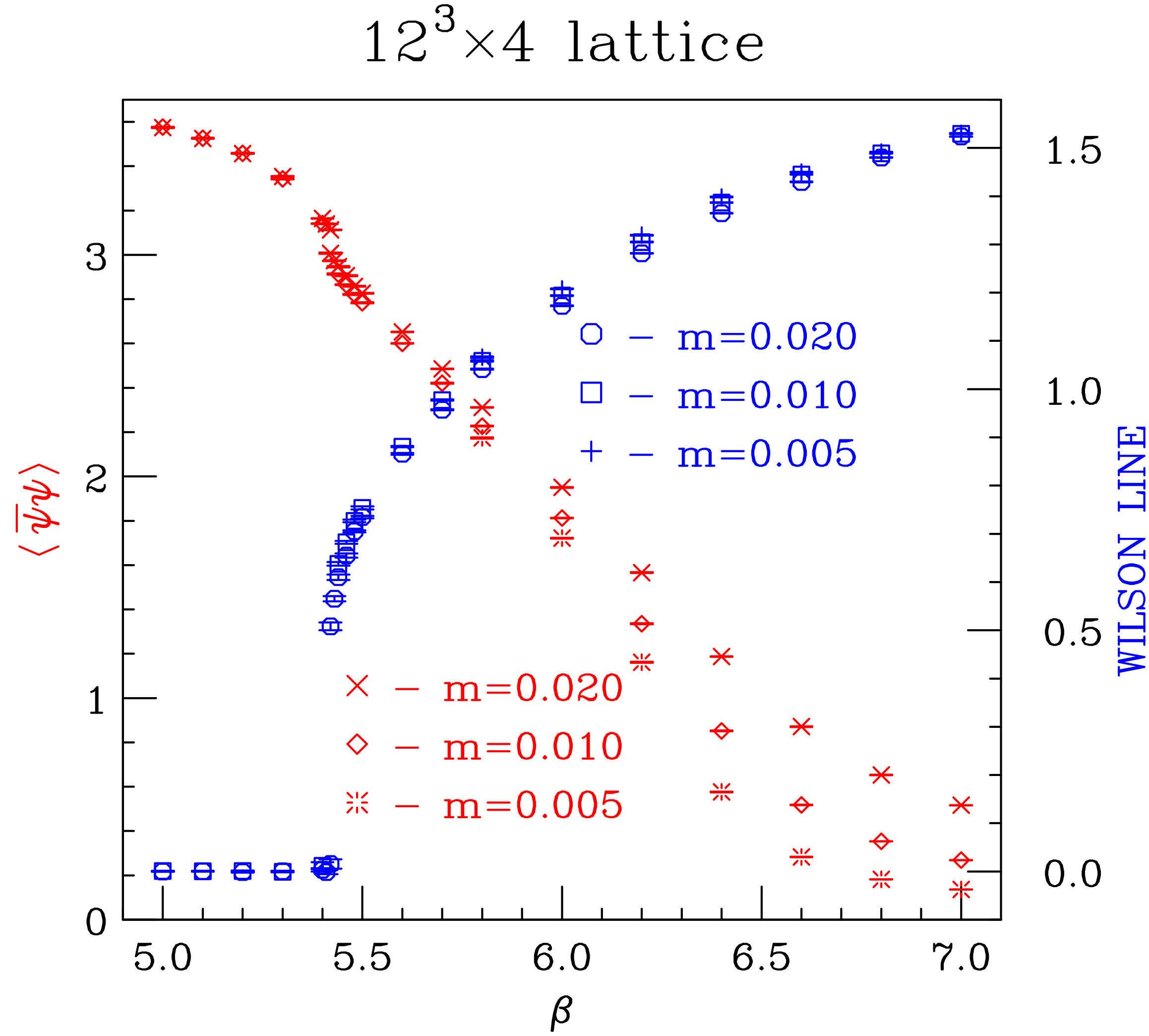

Figure 2: Wilson Line and chiral condensate as functions of $\beta = 1/g^2$ on a $12^3 \times 4$ lattice

The deconfinement transition occurs where the Wilson Line shows a jump at $\beta = 5.420(5)$. The time history at $\beta = 5.42$ suggests that this is a first-order phase transition.

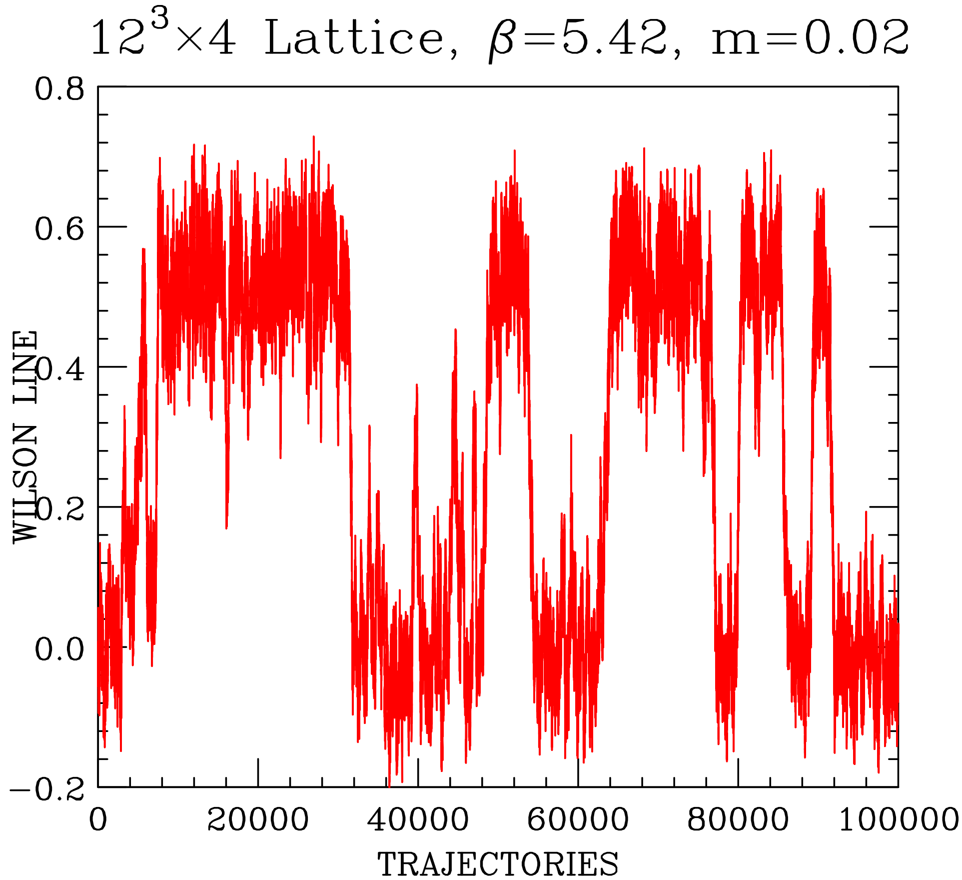

Figure 3: Time history of the triplet Wilson Line(Polyakov Loop) at $\beta = 5.42$ on a $12^3 \times 4$ lattice

Below this transition, the triplet Wilson Line is consistent with zero, rather than just being small as would be expected since the Z_3 centre symmetry is explicitly broken by the quark fields. However, the sextet Wilson Line, although it also shows the transition, is small but finite in this region. This suggests that the reason the triplet Wilson Line is so small, is because in QCD with sextet quarks the triplet string can only break by producing a $q\bar{q}$ pair *and* excited glue, or 3 $q\bar{q}$ pairs, which requires extra energy. The sextet string can break by producing a $q\bar{q}$ pair. This suggests that the triplet Wilson Line might not be zero in the confined regime for larger lattices.

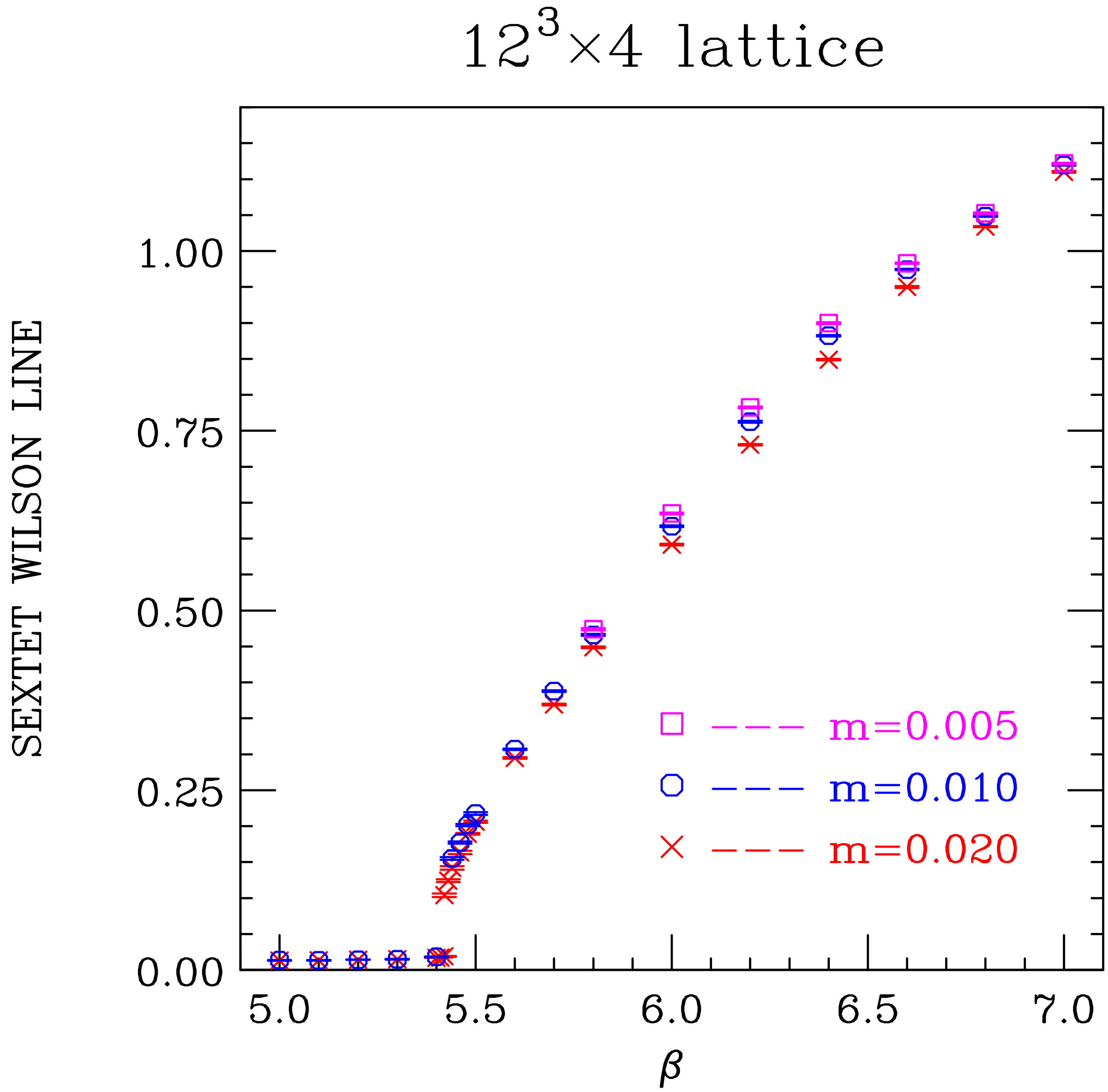

Figure 4: Sextet Wilson Line(Polyakov Loop) as functions of $\beta = 6/g^2$ on a $12^3 \times 4$ lattice

With masses as large as those simulated here, it is difficult to determine the position of the chiral transition with great precision. Our best guestimate would be $\beta \approx 6.5$. Hence the $N_t = 4$ chiral transition appears to occur at a considerably higher β than the deconfinement transition. Such a separation of these two transitions has been observed by others for QCD with 2 or 3 flavours of adjoint quarks, and by us for QCD with fundamental quarks and strong enough extra 4-fermion interactions. For fundamental quarks without these additional interactions, these two transitions appear to be coincident.

We have also identified another deconfined phase. For $\beta \geq 6.0$ this phase has a negative Triplet Wilson Line with magnitude roughly $1/3$ of that in the normal phase. For $\beta \leq 5.8$ the Wilson Line is complex, approximately aligning itself with one of the non-trivial cube roots of unity. Although the negative Wilson Line regime could represent a separate phase where the gauge group breaks $SU(3) \rightarrow SU(2) \times U(1)_Y$, the large fluctuations of its imaginary part makes us suspect that this is the same as the complex Wilson Line phase, except that it is distorted by the small spatial lattice. At lower β this phase undergoes a transition to the confined phase at a β close to that for the standard deconfined phase. There must be 2 such states corresponding to the 2 non-trivial cube roots of unity and connected by charge conjugation. Hence this phase is characterised by spontaneous breakdown of charge conjugation.

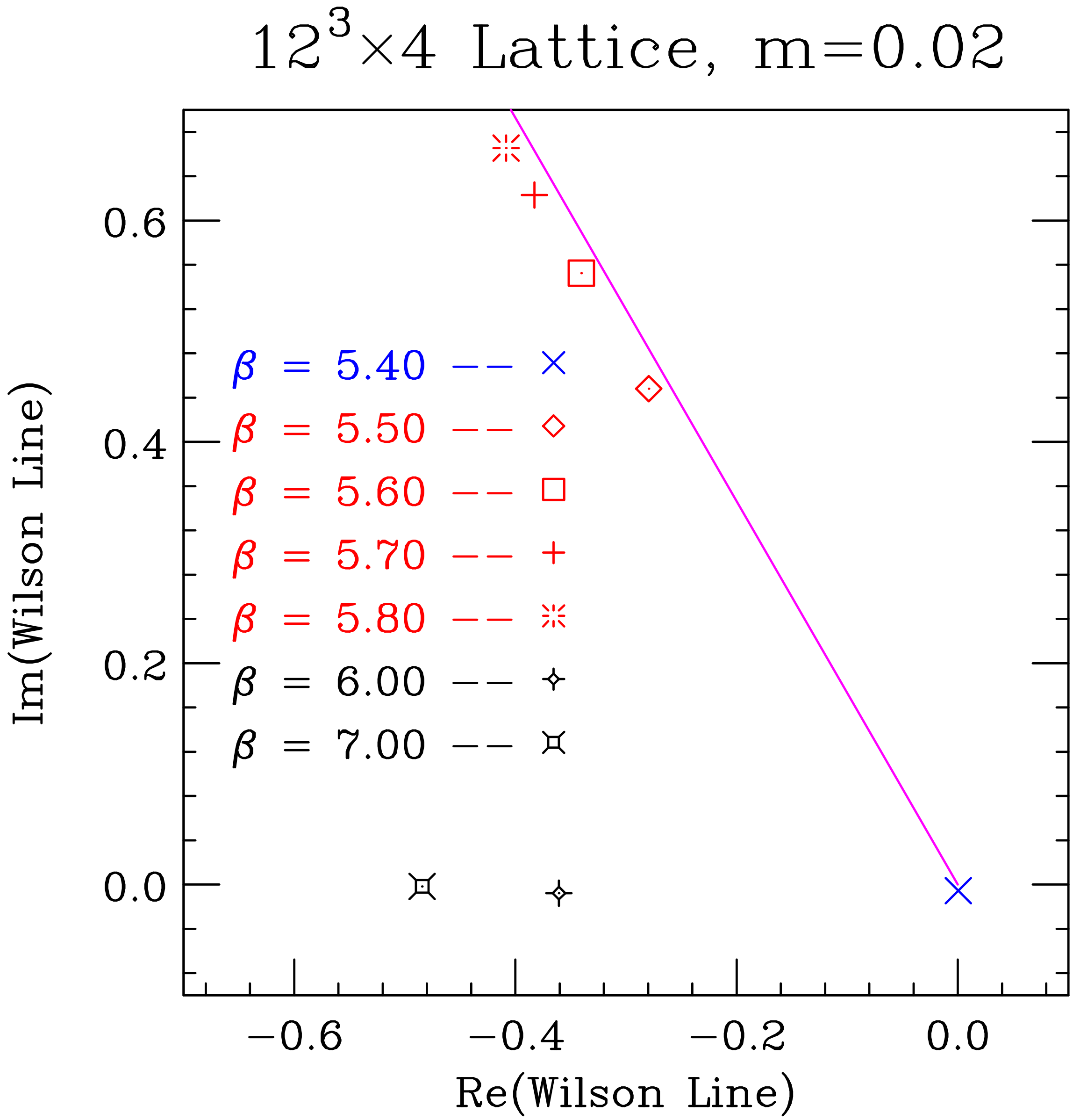

Figure 5: Values of the complex Wilson Line for the charge-conjugation violating phase. Error bars have been suppressed for clarity.

So far $12^3 \times 6$ simulations have been limited to $m = 0.02$, so it is impossible to estimate the position of the chiral transition. Preliminary indications are that the deconfinement transition is at $\beta \approx 5.6$ – significantly larger than the $\beta = 5.420(5)$ at $N_t = 4$. Such an increase is what one would expect for a finite temperature transition in an asymptotically free theory.

Discussion

Our simulations on $N_t = 4$ lattices show well separated deconfinement and chiral transitions. For $m = 0.02$ the deconfinement transition is at $\beta = 6/g^2 = 5.420(5)$ and for $m = 0.01$ it is at $5.41 \leq \beta \leq 5.42$. We estimate that the chiral-symmetry restoration occurs at $\beta \approx 6.5$. On $N_t = 6$ lattices the deconfinement transition occurs at $\beta \approx 5.6$. This suggests that the deconfinement transition is a finite temperature transition, and that the coupling is increasing at longer distances. These results are consistent with what would happen if a chiral condensate forms before the theory can encounter the infrared fixed point predicted by perturbation theory. Of course, the finite quark mass and lattice artifacts could also be responsible for the avoidance of the IR fixed point. Simulations at larger N_t (and N_s), including zero temperature simulations, as well as simulations at smaller masses are needed to resolve this. Calculation of 'hadron' spectra, f_π, etc. would help, as would a serious renormalization-group analysis.

Our observation of both a normal deconfined phase and a charge conjugation violating deconfined phase is of interest. Larger lattices will be needed to tell if the charge-conjugation violating phase really does transition to a phase where the gauge group is partially broken, or whether this is a finite size effect. We have yet to study chiral symmetry breaking in these exotic phases. Simulations in the confined and normal deconfined phase on both $8^3 \times 4$ and $12^3 \times 4$ lattices, indicate that finite size effects are small. We have yet to perform simulations in the charge conjugation violating phase on the $8^3 \times 4$ lattice.

The phase diagram we observe is very similar to that for QCD with $N_f = 2$ fundamental quarks and strong 4-fermion interactions. Physics at scales much longer than the chiral-symmetry breaking scale is apparently insensitive to the detailed nature of the chiral symmetry breaking.

Our results are rather different from those obtained by DeGrand, Shamir and Svetitsky. They find the deconfinement and chiral-symmetry restoring transitions to be coincident. It will be interesting to see if these transitions become closer together as we increase N_t in our simulations. They see the charge conjugation violating but not the normal deconfined phase. Their discrete renormalization group analysis suggests that QCD with $N_f = 2$ massless sextet quarks is conformal. We hope to clarify this.

Conclusions

- QCD with 2 massless flavours of colour-sextet quarks is a potential candidate for Walking Technicolor.
- It remains an open question as to whether it 'walks' or is a conformal 'unparticle' theory.
- In either case, its behaviour is qualitatively different than standard QCD which makes it interesting to study as a field theory.
- At finite temperatures, it exhibits both a deconfinement and a chiral-symmetry restoration transition. These appear well separated, at least on coarse lattices.
- After the current set of runs, we need larger lattices and smaller quark masses. Zero temperature runs are also needed.

- Better actions are needed to reduce lattice artifacts near the IR fixed point. Overlap fermions would be best, but highly improved staggered actions might provide a cheaper alternative.
- We should consider studying QCD with 3 massless flavours of colour-sextet quarks which is almost certainly conformal.
- When we have exhausted this model, we should apply these methods to other candidate theories.
- With such conformal or quasi-conformal gauge theories it would be interesting to know how much of the information we want can be obtained using AdS/CFT.
- Our simulations use Franklin at NERSC and Abe at NCSA.

Appendix

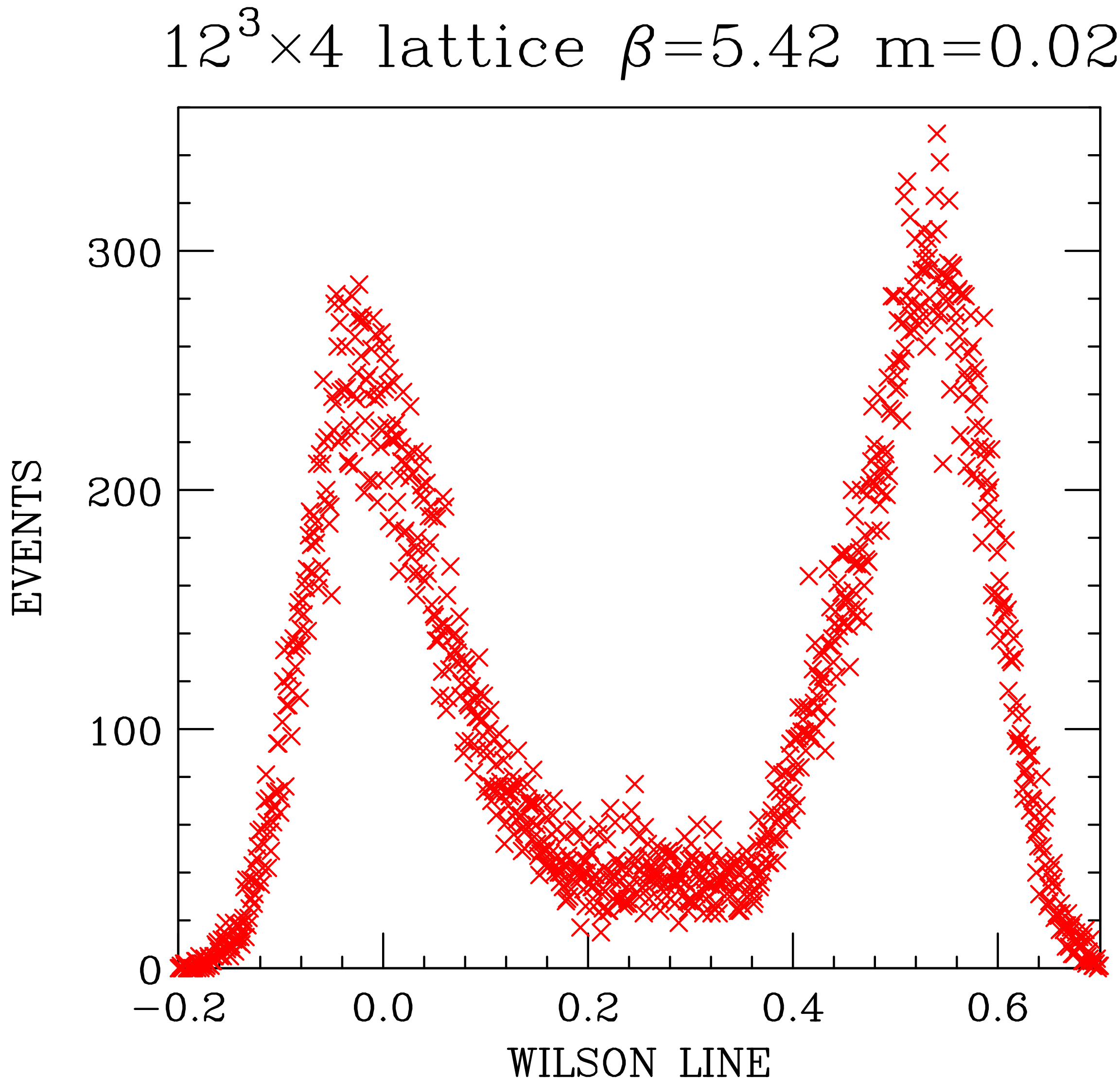

Figure 6: Histogram of Re(Wilson Line), $12^3 \times 4$ lattice, $m = 0.02$, $\beta = 5.42$.

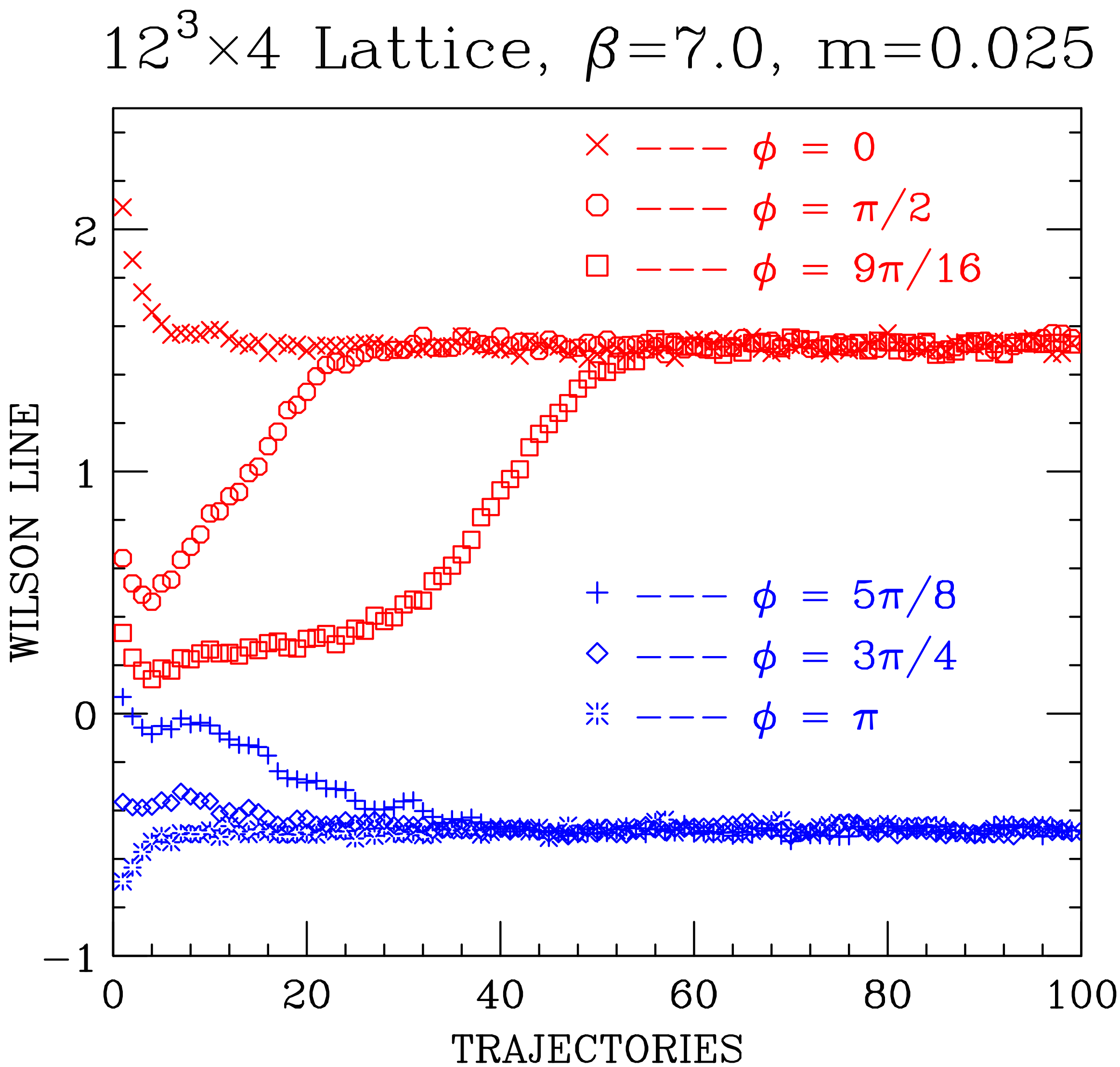

Figure 7: Time evolution of Wilson Line from start with $U_t(N_t = 1) = \text{diag}(e^{i\phi}, e^{-i\phi}, 1)$, all other U's set to identity, $12^3 \times 4$ lattice, $m = 0.02$, $\beta = 7$.

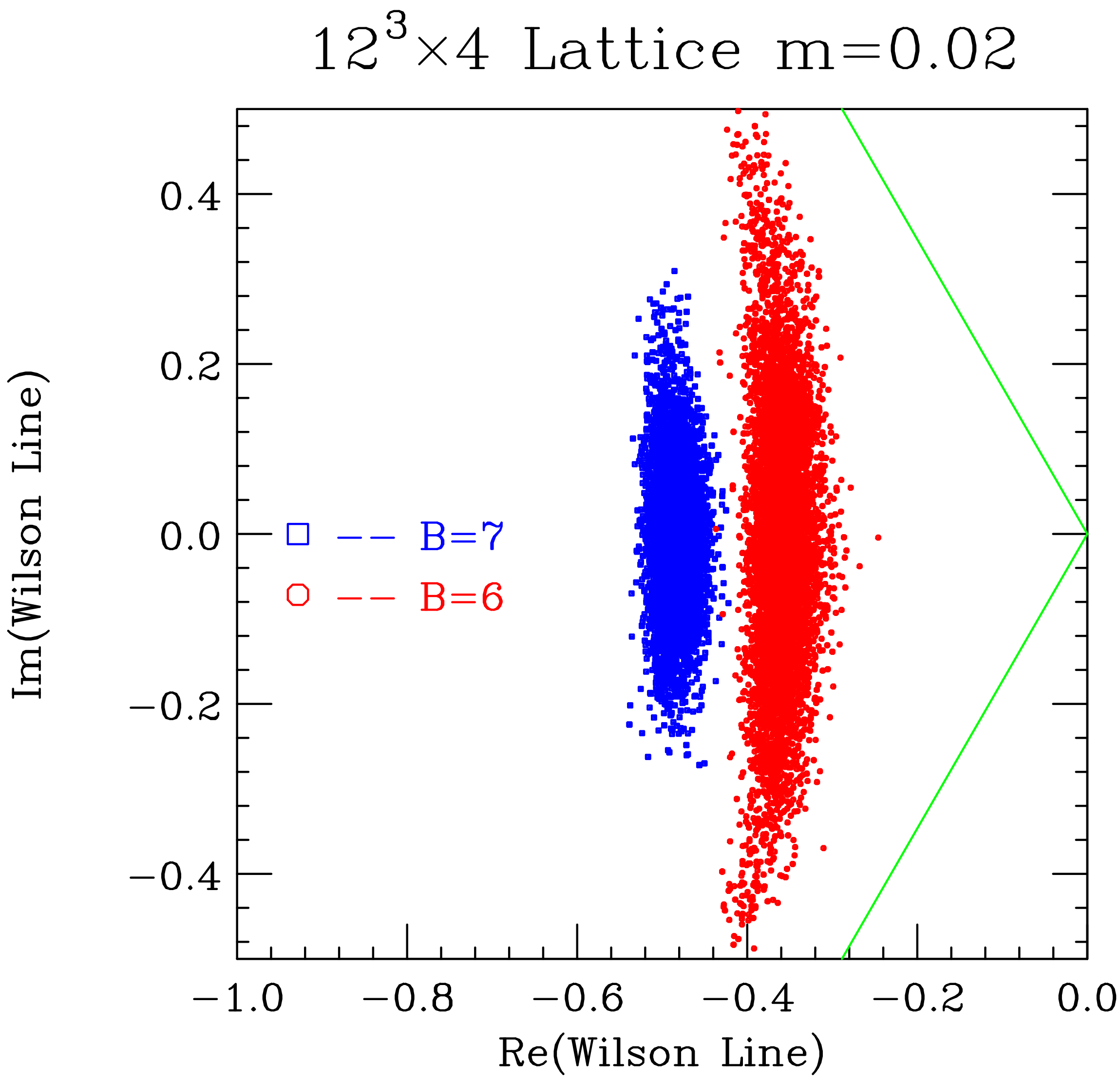

Figure 8: Scatterplot of Wilson Lines $12^3 \times 4$ lattice, $m = 0.02$, $\beta = 6$ and $\beta = 7$, exotic phase.

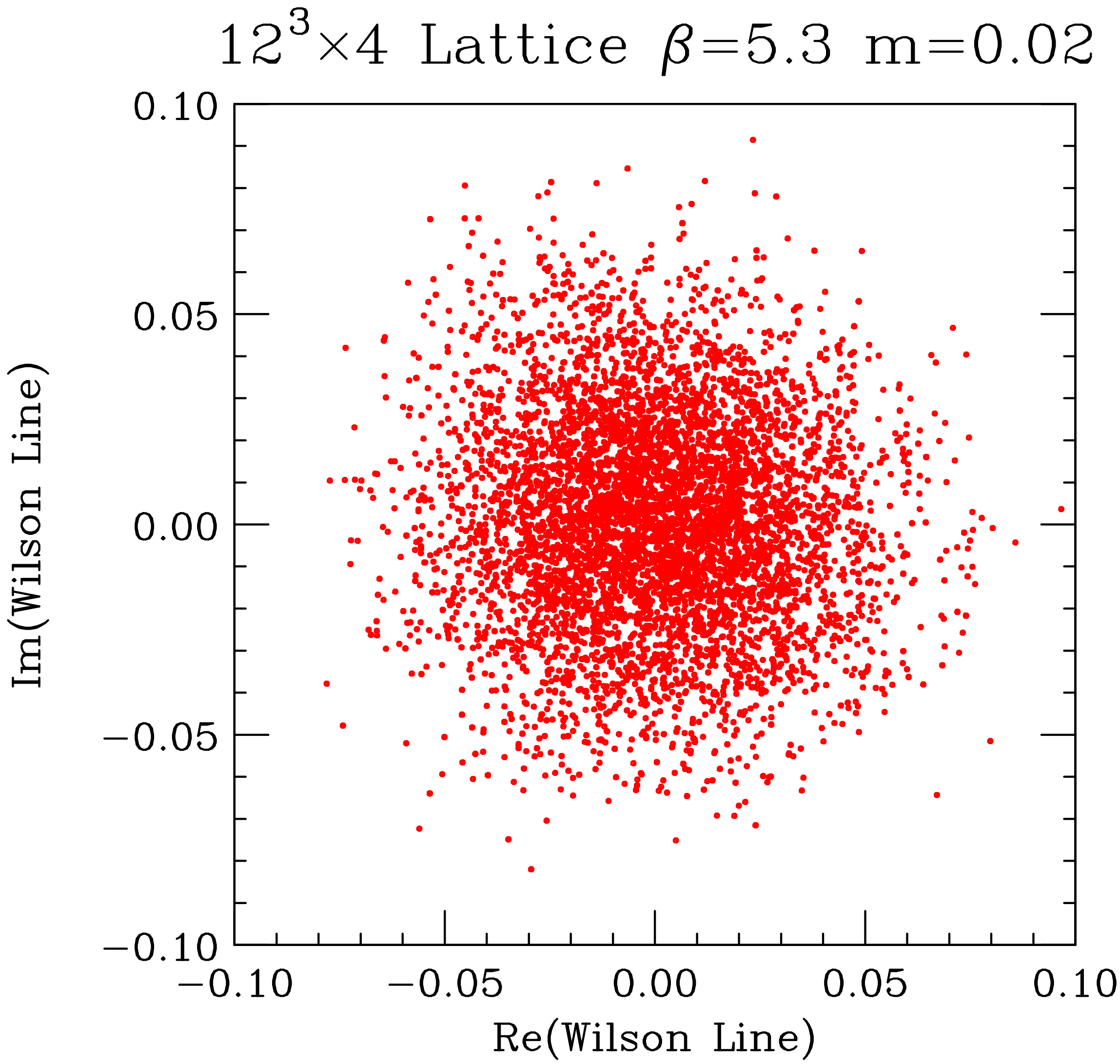

Figure 9: Scatterplot of Wilson Lines $12^3 \times 4$ lattice, $m = 0.02$, $\beta = 5.3$

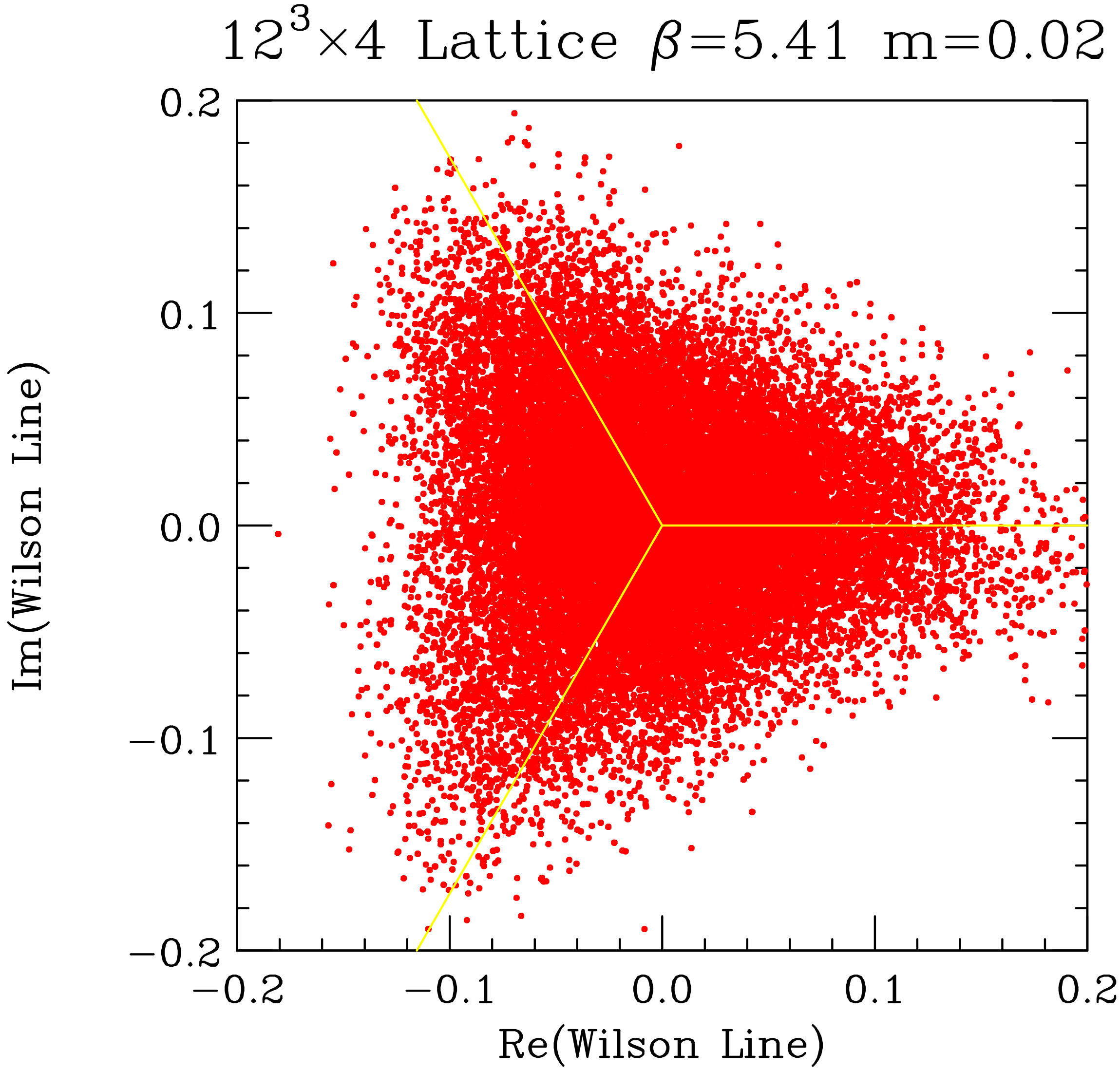

Figure 10: Scatterplot of Wilson Lines $12^3 \times 4$ lattice, $m = 0.02$, $\beta = 5.41$

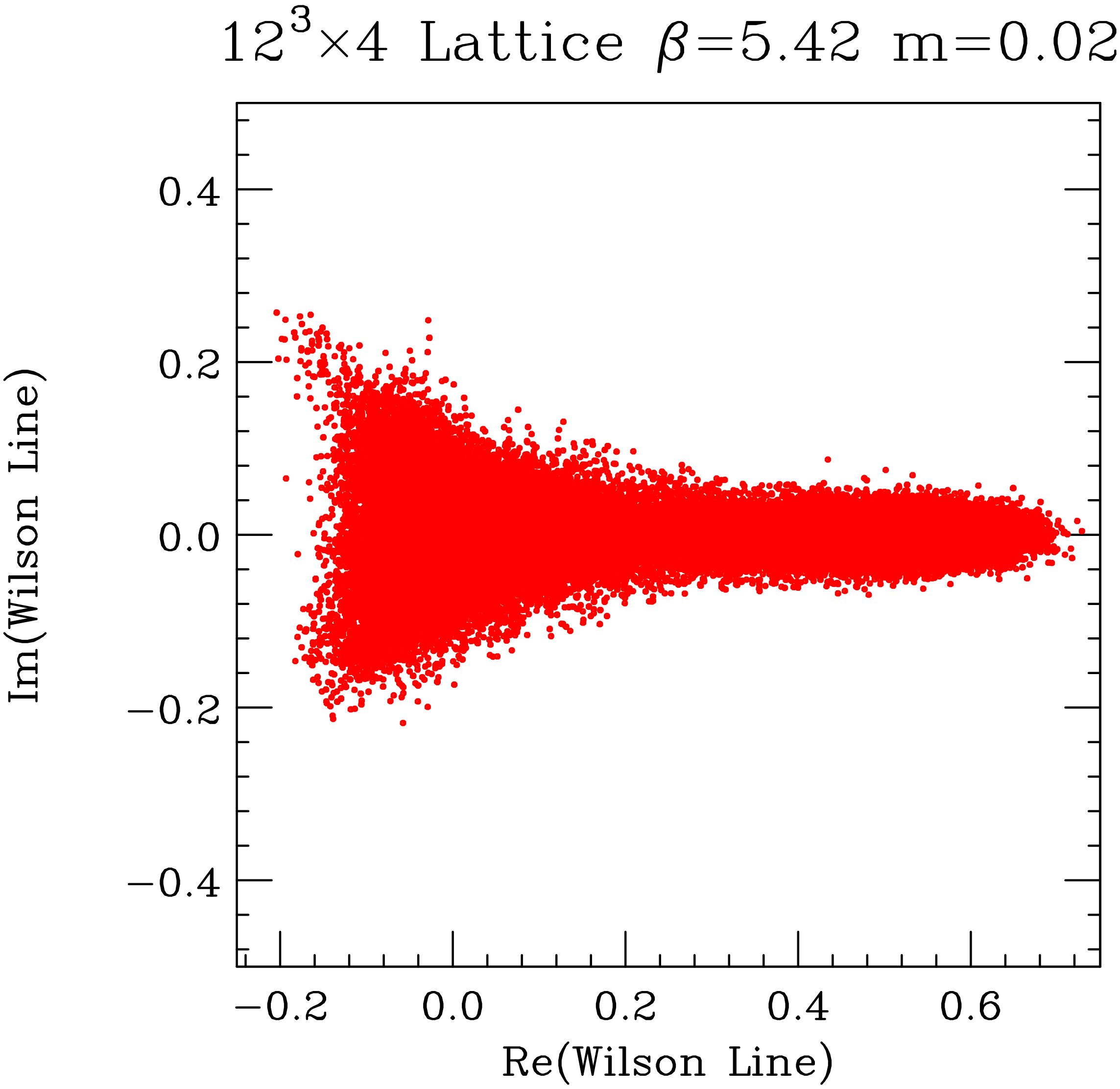

Figure 11: Scatterplot of Wilson Lines $12^3 \times 4$ lattice, $m = 0.02$, $\beta = 5.42$

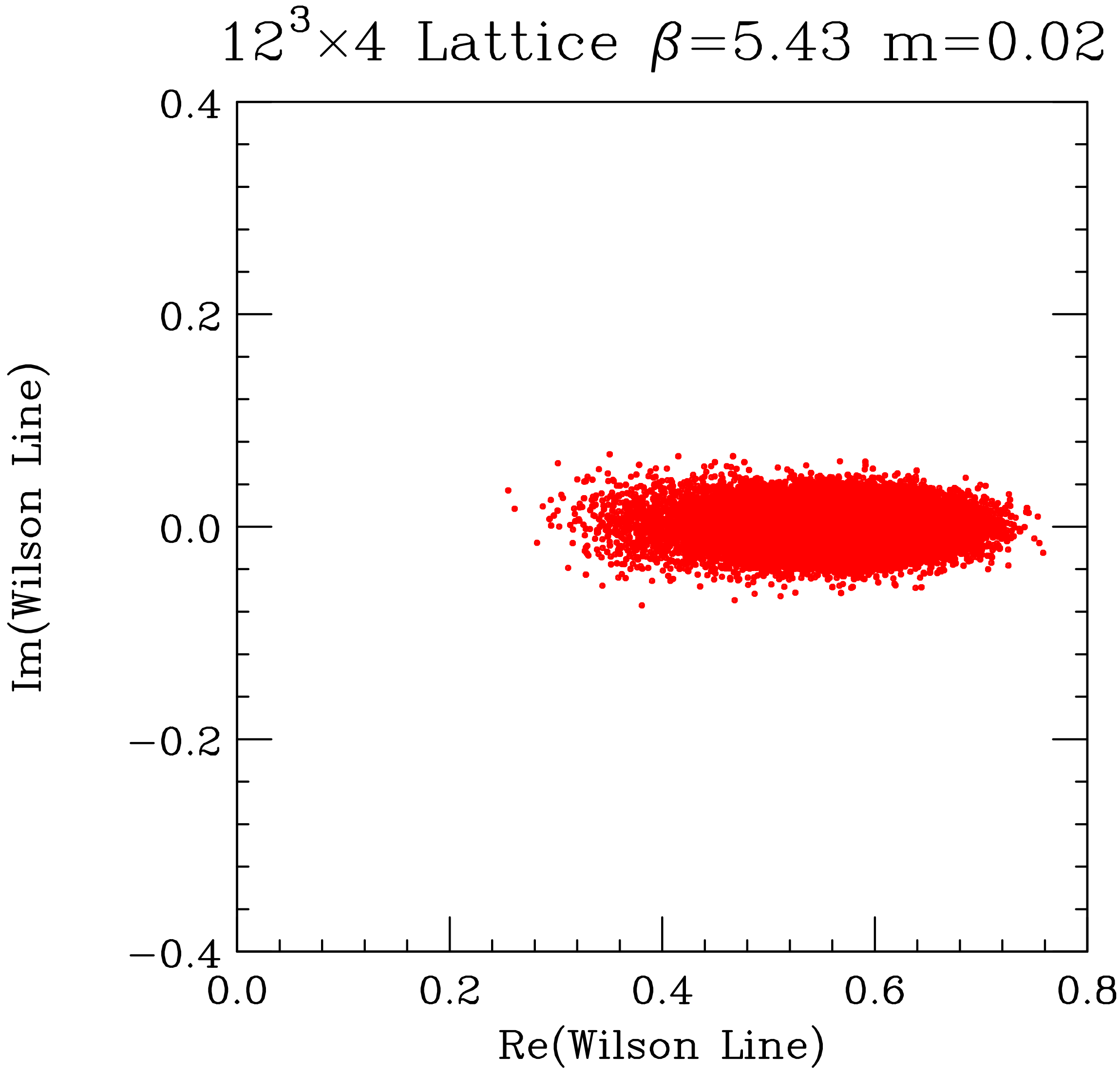

Figure 12: Scatterplot of Wilson Lines $12^3 \times 4$ lattice, $m = 0.02$, $\beta = 5.43$