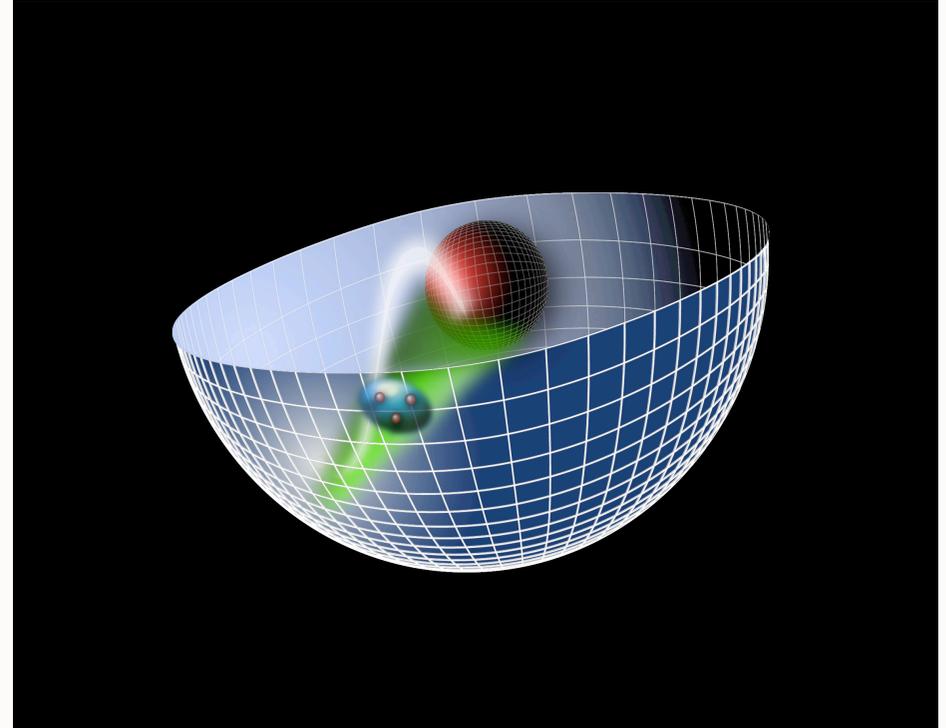
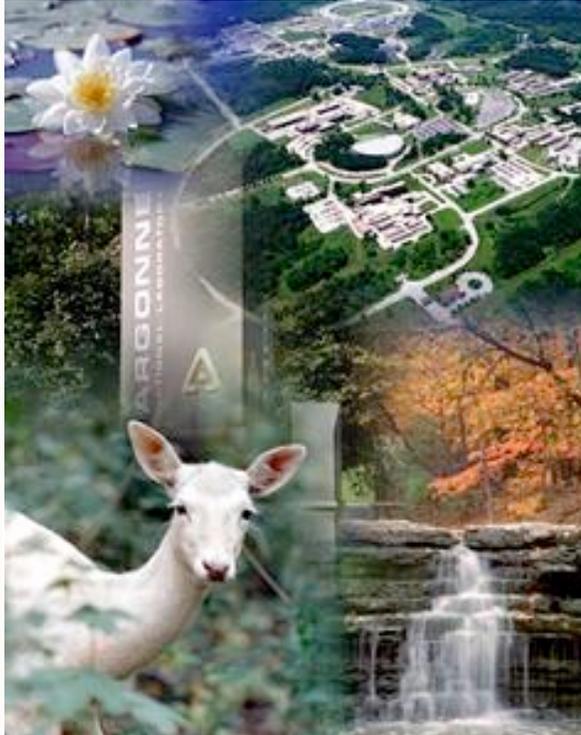


Maximum Wavelength of Confined Quarks and Gluons and Properties of QCD



Stan Brodsky, SLAC

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Argonne
NATIONAL LABORATORY

JTI Workshop on Dynamics of Symmetry Breaking

Argonne National Laboratory, IL

April 13-17, 2009

In Collaboration with Robert Shrock and Guy de Teramond

Maximum Wavelength of Confined Quarks and Gluons and Properties of Quantum Chromodynamics.

[Stanley J. Brodsky](#) ([SLAC](#) & [YITP, Stony Brook](#) & [Durham U.](#)) , [Robert Shrock](#) ([YITP, Stony Brook](#)) . SLAC-PUB-13246, IPPP-08-37, DCPT-08-74, YITP-SB-09-11, Jun 2008. 7pp.

Published in **Phys.Lett.B666:95-99,2008**.

e-Print: **arXiv:0806.1535** [hep-th]

Light-Front Holography and Hadronization at the Amplitude Level.

[Stanley J. Brodsky](#), [Guy de Teramond](#), [Robert Shrock](#) . SLAC-PUB-13306, 51DCPT-08-102, YITP-SB-08-33, Jul 2008. [Temporary entry](#)

Invited talk at 6th International Conference on Perspectives in Hadronic Physics (Hadron 08), Trieste, Italy, 12-16 May 2008.

Published in **AIP Conf.Proc.1056:3-14,2008**.

e-Print: **arXiv:0807.2484** [hep-ph]

AdS/CFT: Anti-de Sitter Space / Conformal Field Theory

Maldacena:

Map $AdS_5 \times S_5$ to conformal $N=4$ SUSY

- **QCD is not conformal**; however, it has manifestations of a scale-invariant theory: Bjorken scaling, dimensional counting for hard exclusive processes
- **Conformal window:** $\alpha_s(Q^2) \simeq \text{const}$ at small Q^2
- **Use mathematical mapping of the conformal group $SO(4,2)$ to AdS_5 space**

Conformal Theories are invariant under the Poincare and conformal transformations with

$$\mathbf{M}^{\mu\nu}, \mathbf{P}^{\mu}, \mathbf{D}, \mathbf{K}^{\mu},$$

the generators of $SO(4,2)$

$SO(4,2)$ has a mathematical representation on AdS_5

Scale Transformations

- Isomorphism of $SO(4, 2)$ of conformal QCD with the group of isometries of AdS space

$$ds^2 = \frac{R^2}{z^2} (\eta_{\mu\nu} dx^\mu dx^\nu - dz^2),$$

invariant measure 

$x^\mu \rightarrow \lambda x^\mu, z \rightarrow \lambda z$, maps scale transformations into the holographic coordinate z .

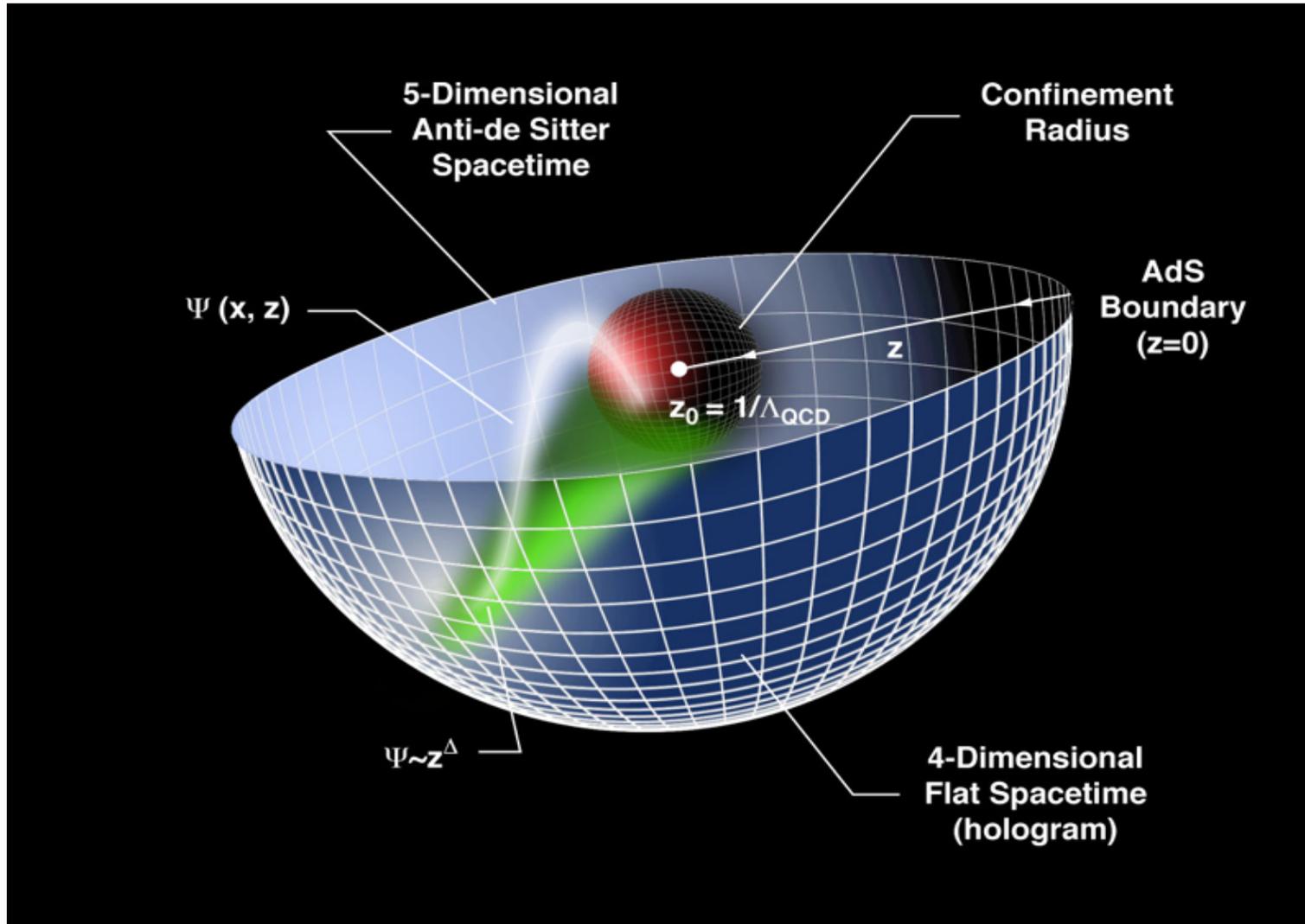
- AdS mode in z is the extension of the hadron wf into the fifth dimension.
- Different values of z correspond to different scales at which the hadron is examined.

$$x^2 \rightarrow \lambda^2 x^2, \quad z \rightarrow \lambda z.$$

$x^2 = x_\mu x^\mu$: invariant separation between quarks

- The AdS boundary at $z \rightarrow 0$ correspond to the $Q \rightarrow \infty$, UV zero separation limit.

Applications of AdS/CFT to QCD



Changes in physical length scale mapped to evolution in the 5th dimension z

de Teramond, sjb

**JTI Workshop ANL
April 14, 2009**

Maximal Wavelength and QCD Properties

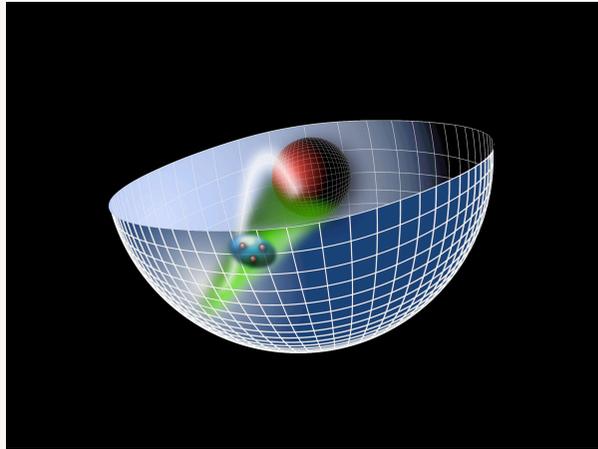
6

**Stan Brodsky
SLAC**

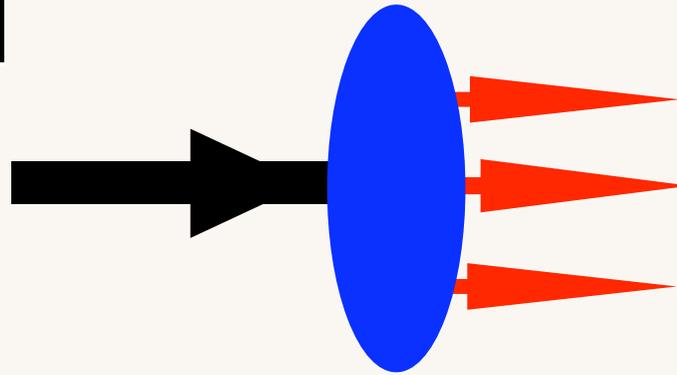
Goal:

- **Use AdS/CFT to provide an approximate, covariant, and analytic model of hadron structure with confinement at large distances, conformal behavior at short distances**
- **Analogous to the Schrodinger Theory for Atomic Physics**
- *AdS/QCD Light-Front Holography*
- *Hadronic Spectra and Light-Front Wavefunctions*

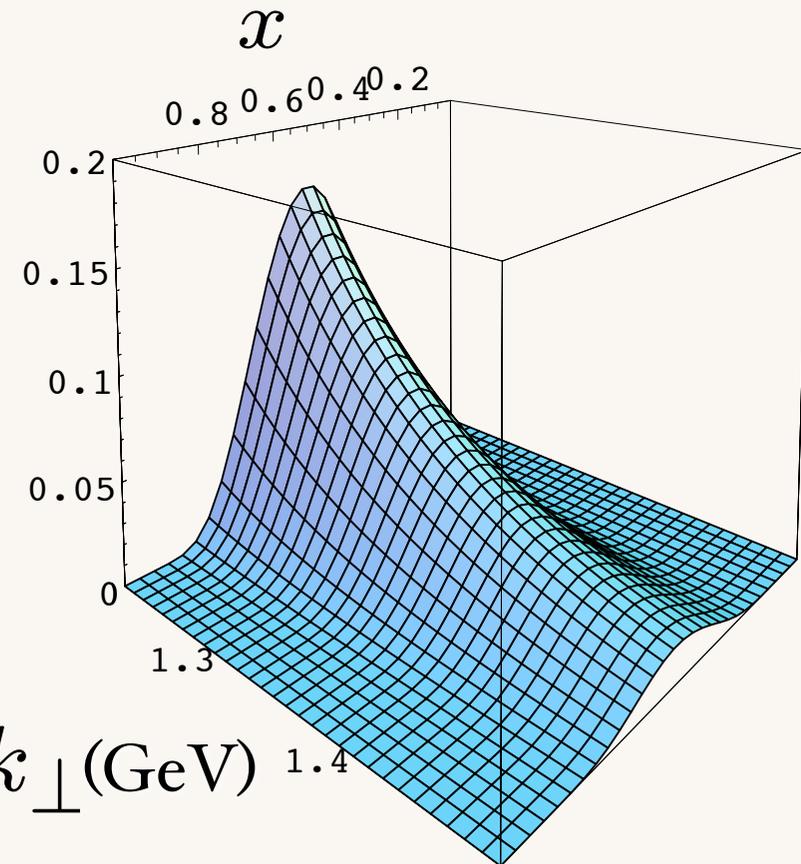
$$\phi(z)$$



- *Light-Front Holography*



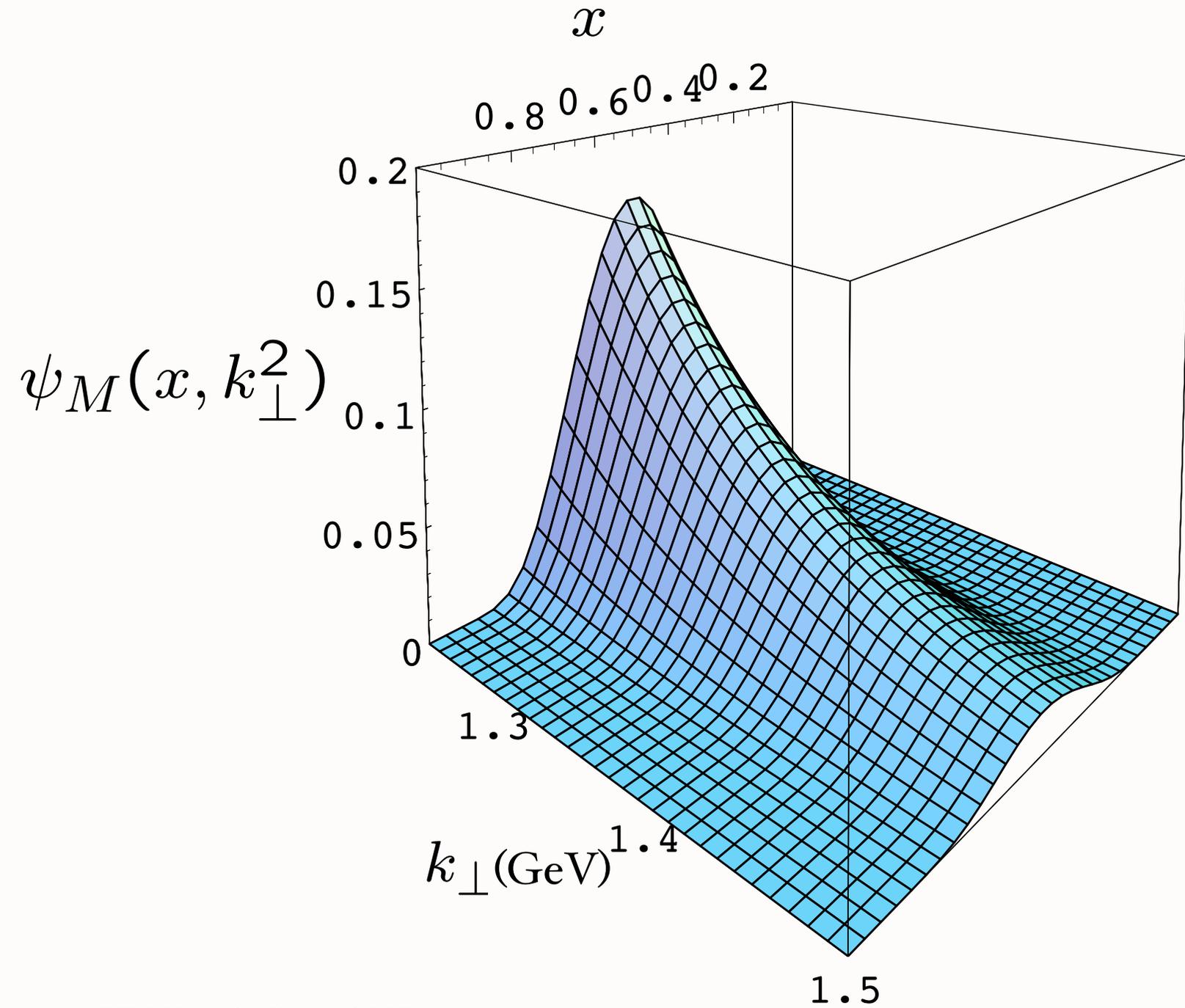
$$\Psi_n(x_i, \vec{k}_{\perp i}, \lambda_i)$$



- *Light Front Wavefunctions:*

Schrödinger Wavefunctions
of Hadron Physics

Prediction from AdS/QCD: Meson LFWF



**“Soft Wall”
model**

de Teramond, sjb

String Theory



AdS/CFT

Mapping of Poincare' and Conformal $SO(4,2)$ symmetries of 3+1 space to AdS5 space

Goal: First Approximant to QCD

Counting rules for Hard Exclusive Scattering
Regge Trajectories
QCD at the Amplitude Level



AdS/QCD

Conformal Invariance + Confinement at large distances

Semi-Classical QCD / Wave Equations



Boost Invariant 3+1 Light-Front Wave Equations

Light Front Holography

$J=0, 1, 1/2, 3/2$ plus L

Integrable!



Hadron Spectra, Wavefunctions, Dynamics

Verification of Asymptotic Freedom

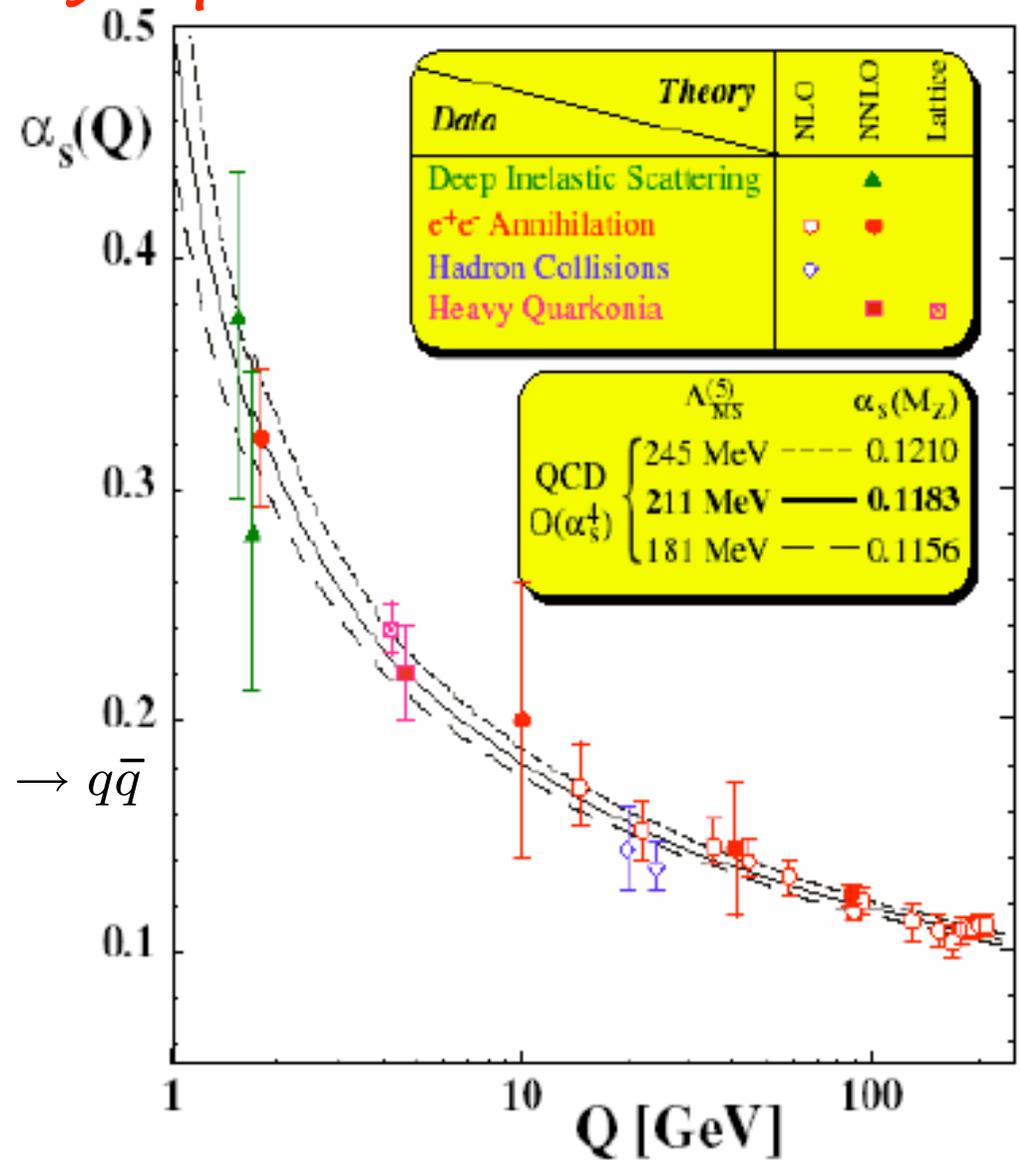
Gross, Wilczek, Politzer
Khriplovich, 't Hooft

$$\frac{\sigma(e^+e^- \rightarrow \text{three jets})}{\sigma(e^+e^- \rightarrow \text{two jets})}$$

Ratio of rate for $e^+e^- \rightarrow q\bar{q}g$ to $e^+e^- \rightarrow q\bar{q}$
at $Q = E_{CM} = E_{e^-} + E_{e^+}$

proportional to $\alpha_s(Q)$

$$\alpha(Q^2) \simeq \frac{4\pi}{\beta_0} \frac{1}{\log Q^2 / \Lambda_{QCD}^2}$$



JTI Workshop ANL
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Maximal Wavelength and QCD Properties

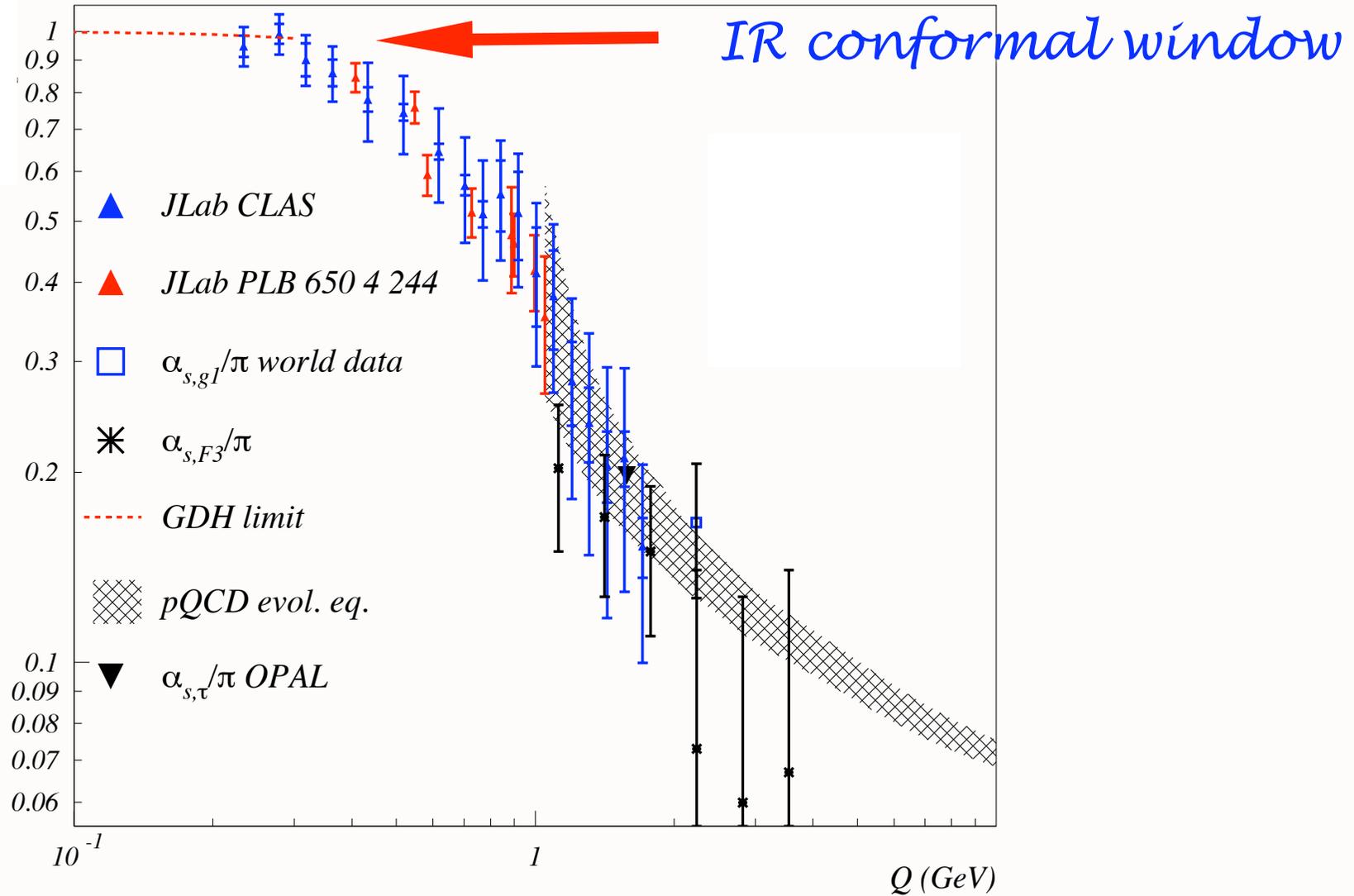
Stan Brodsky
SLAC

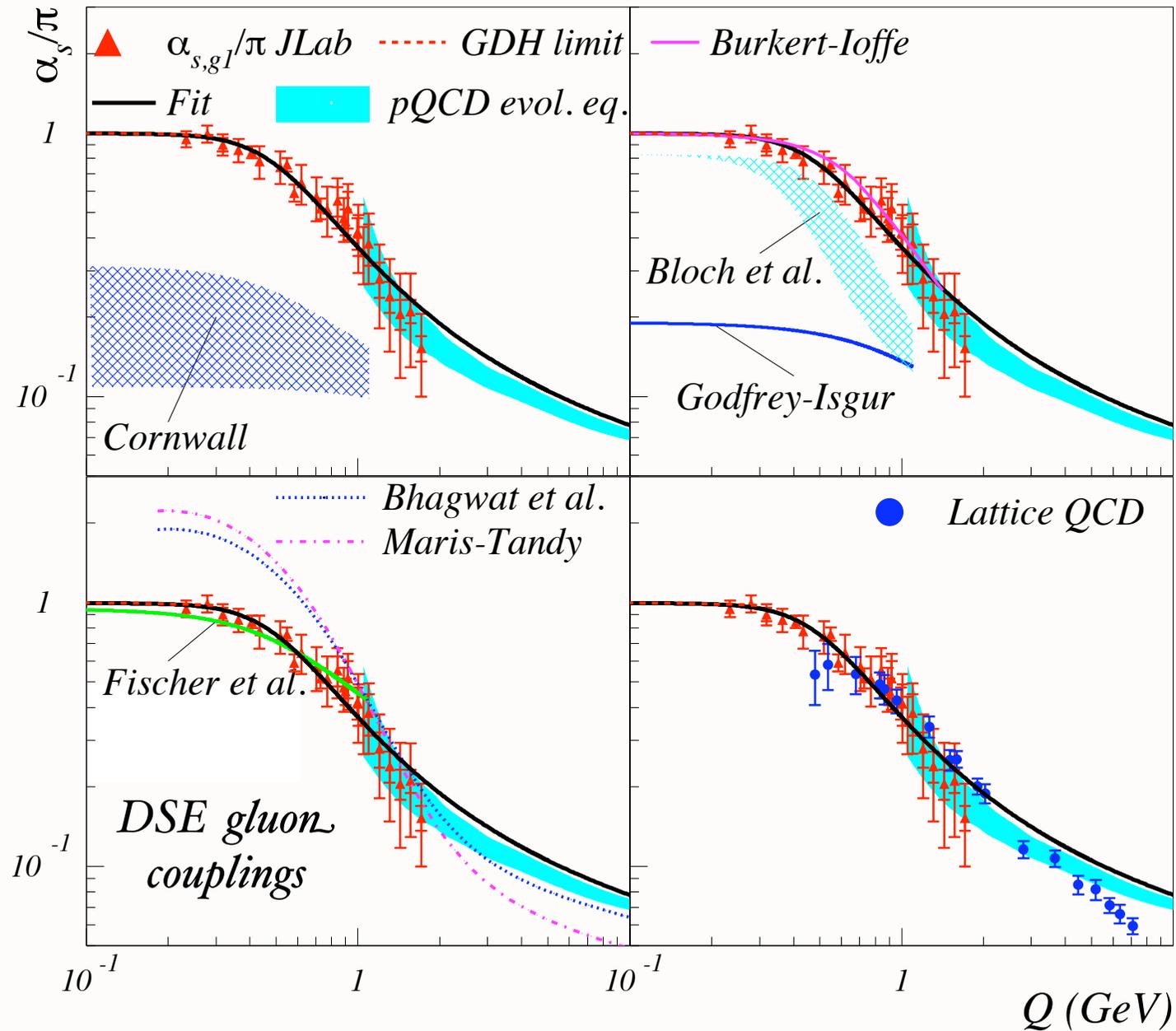
Deur, Korsch, et al: Effective Charge from Bjorken Sum Rule

$$\Gamma_{bj}^{p-n}(Q^2) \equiv \frac{g_A}{6} \left[1 - \frac{\alpha_s^{g_1}(Q^2)}{\pi} \right]$$

GDH constraint →

$$\frac{\alpha_s^{g_1}(Q^2)}{\pi}$$





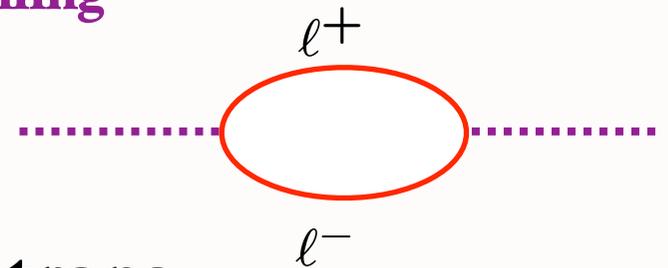
IR Conformal Window for QCD

- *Dyson-Schwinger Analysis:* **QCD gluon coupling has IR Fixed Point**
- *Evidence from Lattice Gauge Theory*
- Define coupling from observable: **indications of IR fixed point for QCD effective charges**
- Confined gluons and quarks have maximum wavelength: **Decoupling of QCD vacuum polarization at small Q^2**

Shrock, de Teramond, sjb

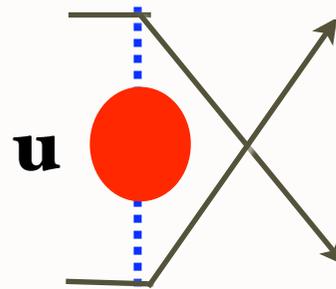
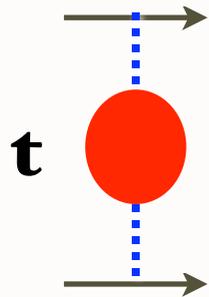
Serber-Uehling

$$\Pi(Q^2) \rightarrow \frac{\alpha}{15\pi} \frac{Q^2}{m^2} \quad Q^2 \ll 4m^2$$



- **Justifies application of AdS/CFT in strong-coupling conformal window**

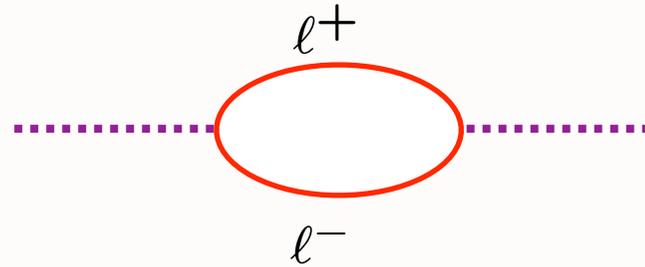
$$\mathcal{M}_{ee \rightarrow ee}(++;++) = \frac{8\pi s}{t} \alpha(t) + \frac{8\pi s}{u} \alpha(u)$$



$$\alpha(t) = \frac{\alpha(0)}{1 - \Pi(t)}$$

Gell Mann-Low Effective Charge for QED

QED One-Loop Vacuum Polarization



$$t = -Q^2 < 0$$

(t spacelike)

$$\Pi(Q^2) = \frac{\alpha(0)}{3\pi} \left[\frac{5}{3} - \frac{4m^2}{Q^2} - \left(1 - \frac{2m^2}{Q^2}\right) \sqrt{1 + \frac{4m^2}{Q^2}} \log \frac{1 + \sqrt{1 + \frac{4m^2}{Q^2}}}{|1 - \sqrt{1 + \frac{4m^2}{Q^2}}|} \right]$$

$$\Pi(Q^2) \simeq \frac{\alpha(0)}{3\pi} \log \frac{Q^2}{m^2} \quad Q^2 \gg 4m^2$$

$$\beta = \frac{d\left(\frac{\alpha}{4\pi}\right)}{d \log Q^2} = \frac{4}{3} \left(\frac{\alpha}{4\pi}\right)^2 n_\ell > 0$$

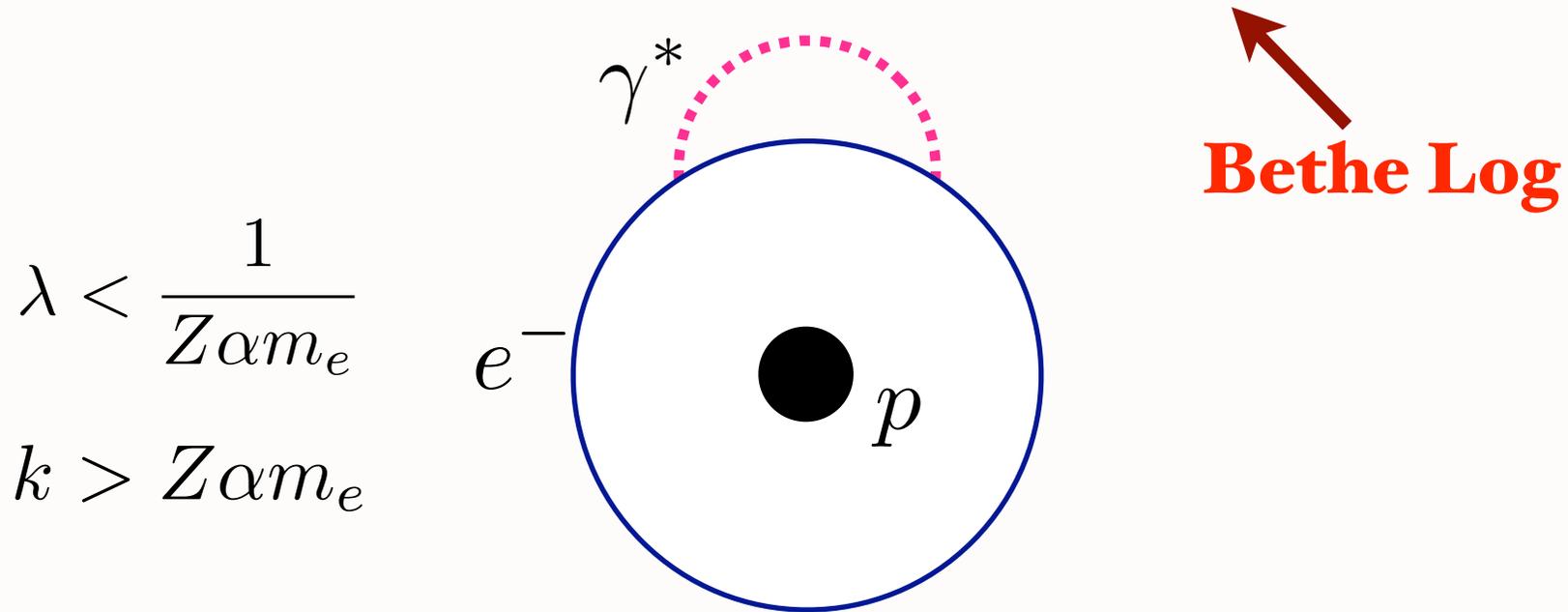
$$\Pi(Q^2) = \frac{\alpha(0)}{15\pi} \frac{Q^2}{m^2} \quad Q^2 \ll 4m^2 \quad \text{Serber-Uehling}$$

$$\beta \propto \frac{Q^2}{m^2} \quad \text{vanishes at small momentum transfer}$$

Lesson from QED:

Lamb Shift in Hydrogen

$$\Delta E \sim \alpha (Z\alpha)^4 \ln (Z\alpha)^2 m_e$$



Maximum wavelength of bound electron

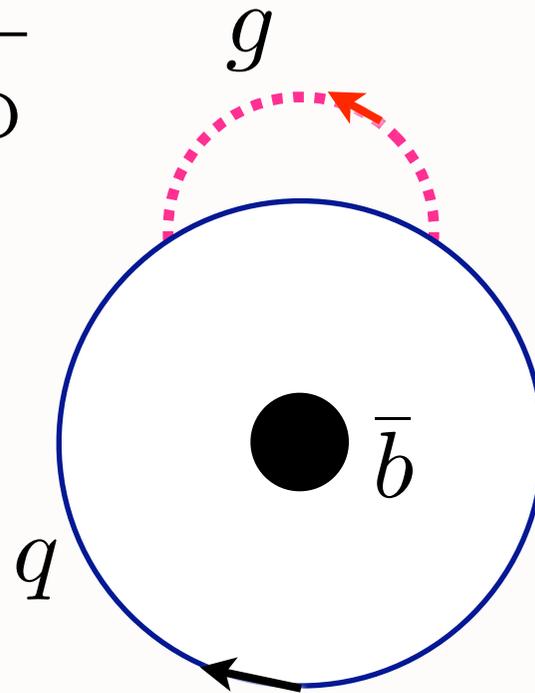
Infrared divergence of free electron propagator removed because of atomic binding

Lesson from QED and Lamb Shift:

maximum wavelength of bound quarks and gluons

$$k > \frac{1}{\Lambda_{\text{QCD}}}$$

$$\lambda < \Lambda_{\text{QCD}}$$



B-Meson

Shrock, sjb

*gluon and quark propagators cutoff in IR
because of color confinement*

Maximal Wavelength of Confined Fields

- **Colored fields confined to finite domain** $(x - y)^2 < \Lambda_{QCD}^{-2}$
- **All perturbative calculations regulated in IR**
- **High momentum calculations unaffected**
- **Bound-state Dyson-Schwinger Equation**
- **Analogous to Bethe's Lamb Shift Calculation**

*Quark and Gluon vacuum polarization insertions
decouple: IR fixed Point*

J. D. Bjorken,
SLAC-PUB 1053
Cargese Lectures 1989

A strictly-perturbative space-time region can be defined as one which has the property that any straight-line segment lying entirely within the region has an invariant length small compared to the confinement scale Λ_{QCD}^{-1} (whether or not the segment is spacelike or timelike).

Single-spin asymmetries

Leading Twist Sivers Effect

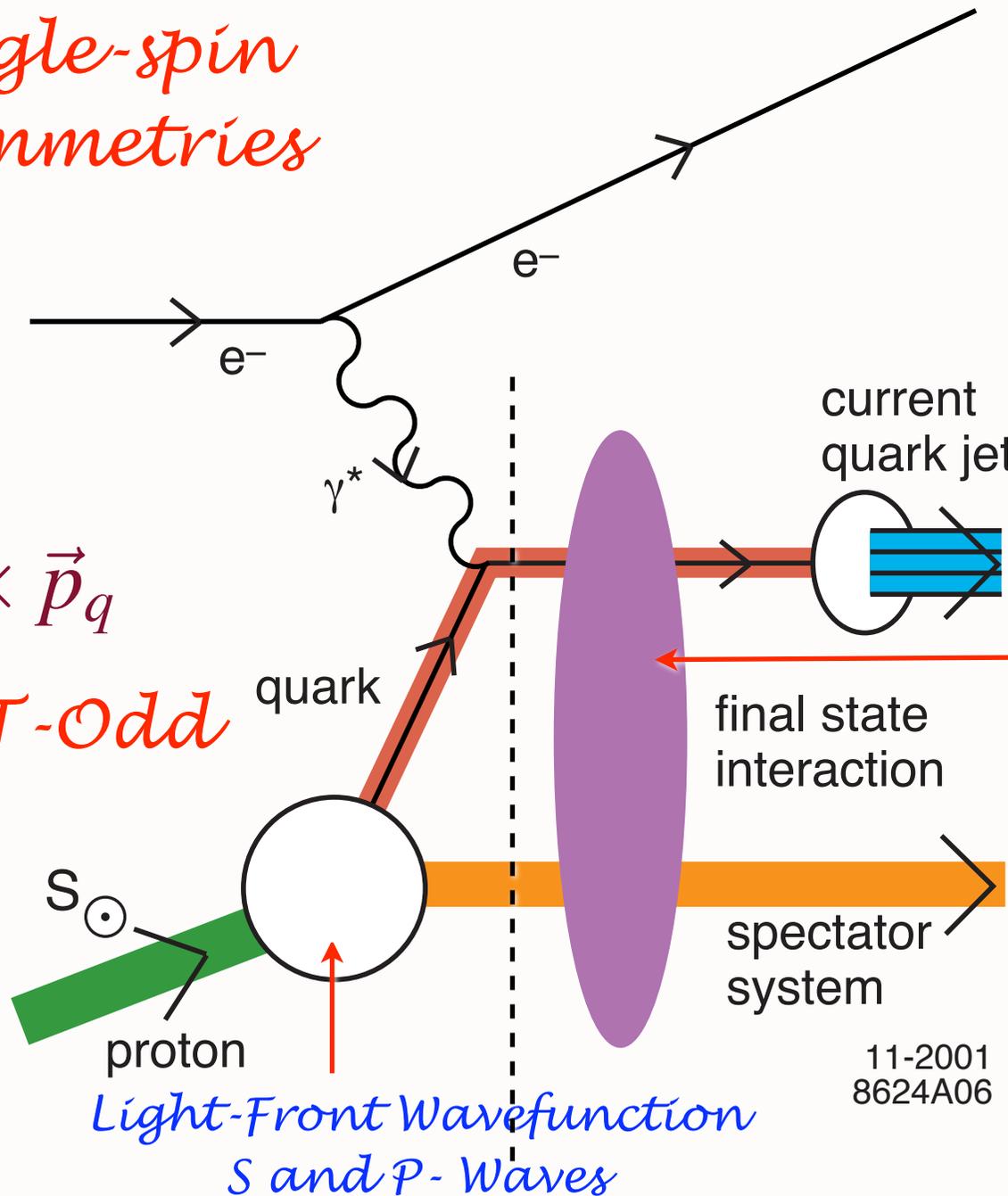
Hwang,
Schmidt, sjb

Collins, Burkardt
Ji, Yuan

*QCD S- and P-
Coulomb Phases
--Wilson Line*

$$i \vec{S}_p \cdot \vec{q} \times \vec{p}_q$$

Pseudo-T-Odd



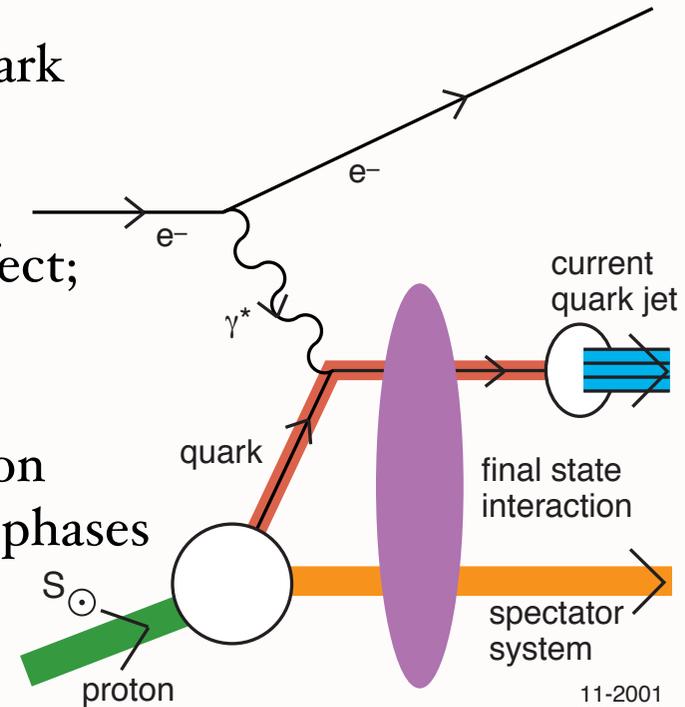
11-2001
8624A06

Final-State Interactions Produce Pseudo-T-Odd (Sivers Effect)

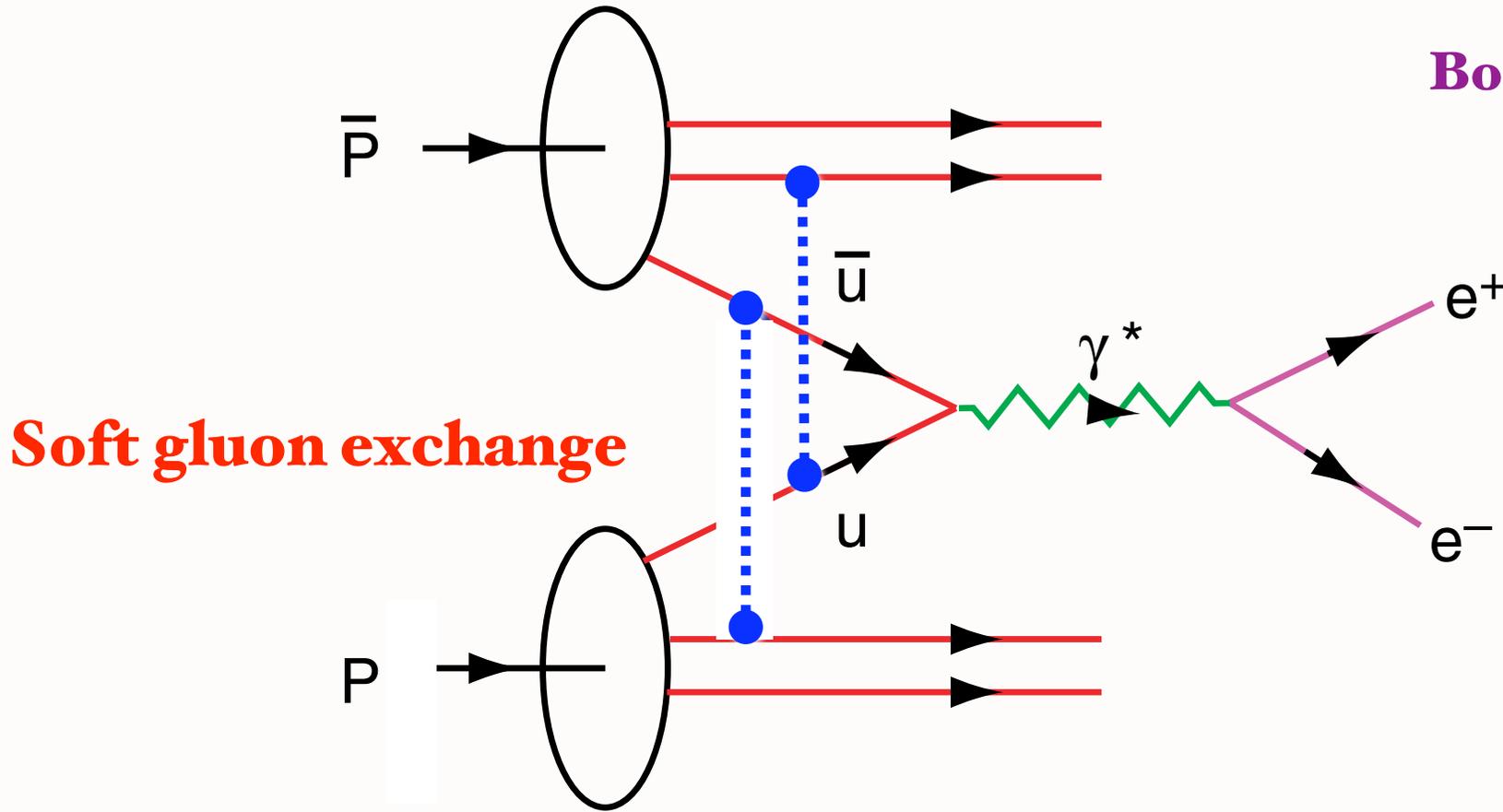
Hwang,
Schmidt, sjb

- Leading-Twist Bjorken Scaling!
- Requires nonzero orbital angular momentum of quark
- Arises from the interference of Final-State QCD Coulomb phases in S- and P- waves; Wilson line effect; gauge independent
- Relate to the quark contribution to the target proton anomalous magnetic moment and final-state QCD phases
- QCD phase at soft scale: IR Fixed Point!
- New window to QCD coupling and running gluon mass in the IR
- QED S and P Coulomb phases infinite -- difference of phases finite

$$i \vec{S} \cdot \vec{p}_{jet} \times \vec{q}$$



11-2001
8624A06



$DY \cos 2\phi$ correlation at leading twist from double ISI

Product of Boer - Mulders Functions

$$h_1^\perp(x_1, \mathbf{p}_\perp^2) \times \bar{h}_1^\perp(x_2, \mathbf{k}_\perp^2)$$

Double Initial-State Interactions

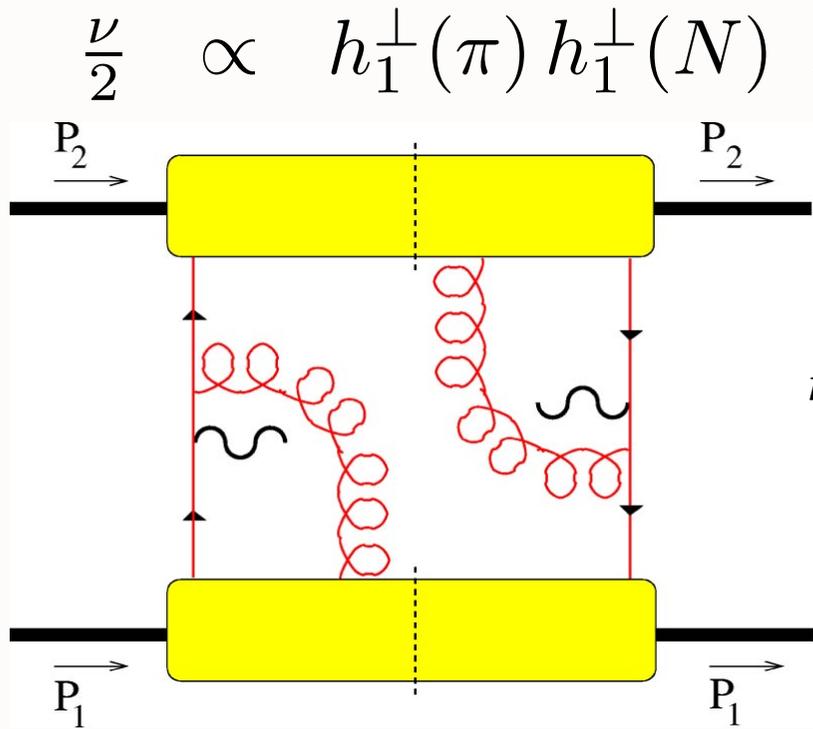
generate anomalous $\cos 2\phi$:

Boer, Hwang, sjb

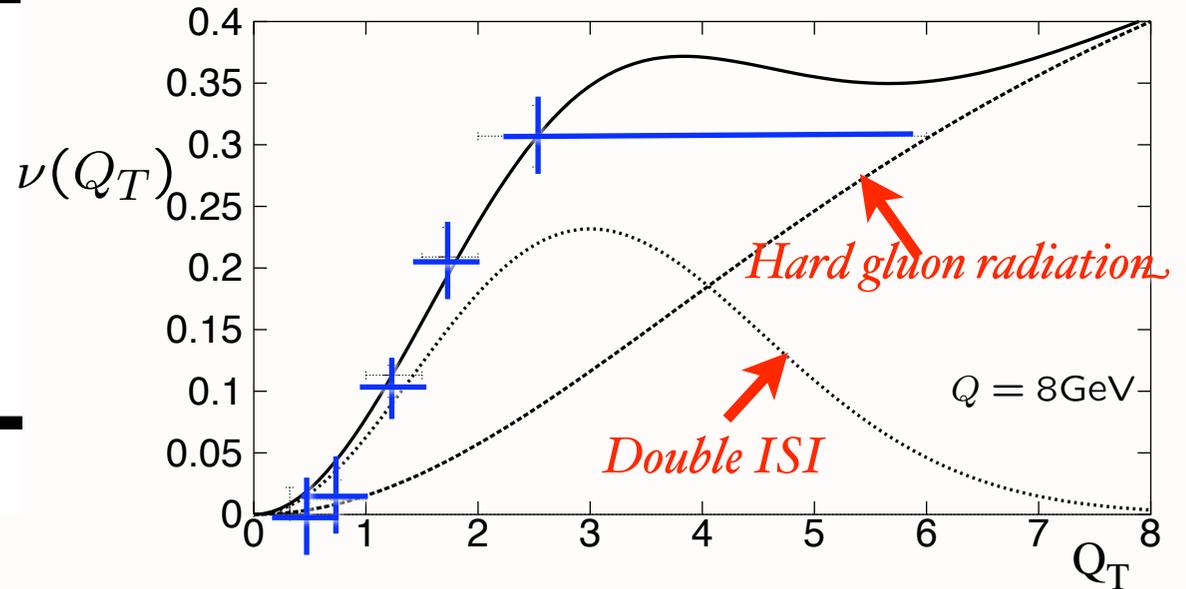
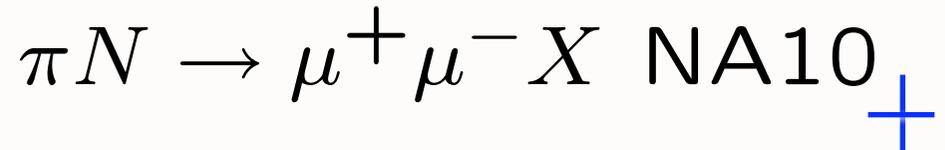
Drell-Yan planar correlations

$$\frac{1}{\sigma} \frac{d\sigma}{d\Omega} \propto \left(1 + \lambda \cos^2 \theta + \mu \sin 2\theta \cos \phi + \frac{\nu}{2} \sin^2 \theta \cos 2\phi \right)$$

PQCD Factorization (Lam Tung): $1 - \lambda - 2\nu = 0$



Violates Lam-Tung relation!

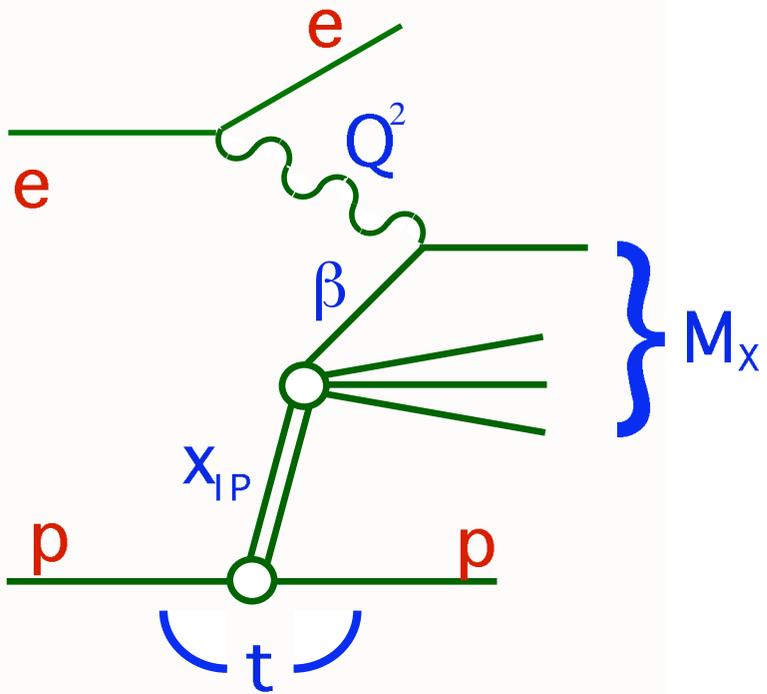


Model: Boer,

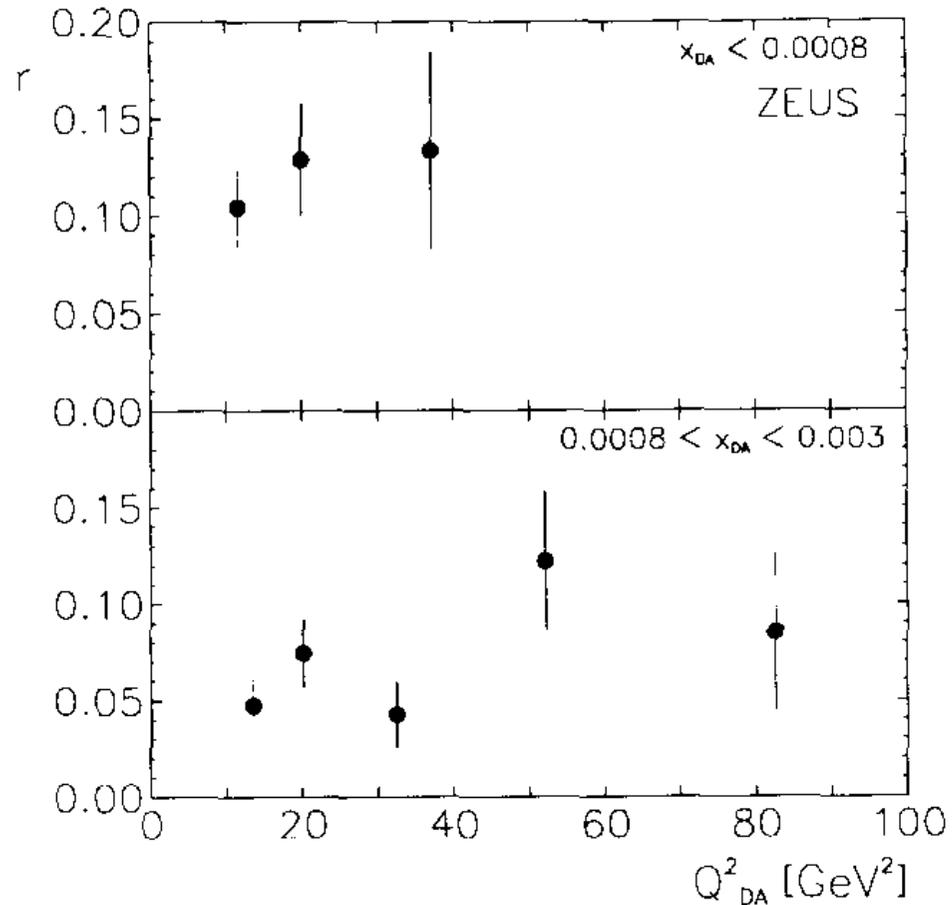
Stan Brodsky

SLAC

Remarkable observation at HERA



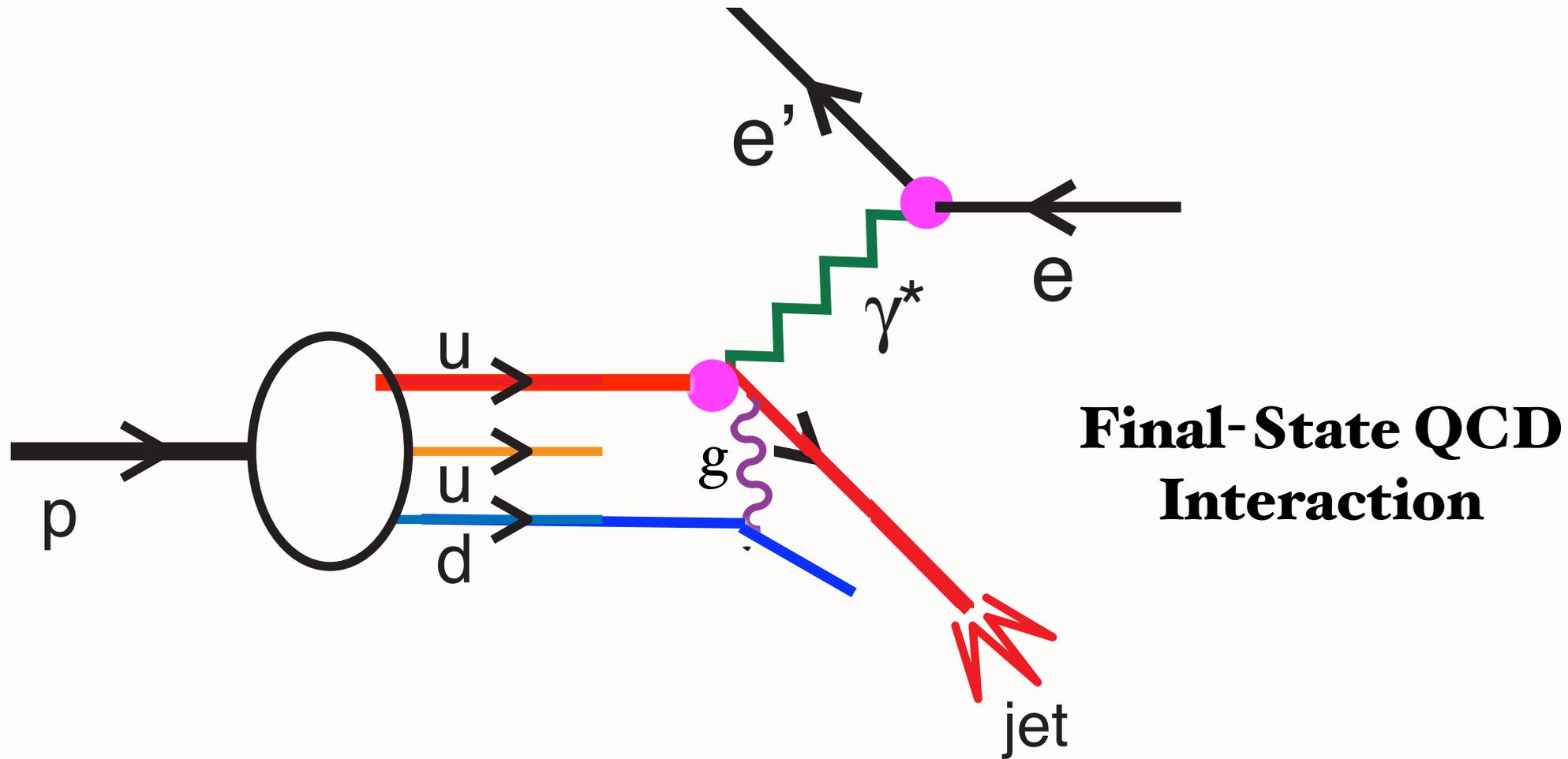
10% to 15%
of DIS events
are
diffractive!



Fraction r of events with a large rapidity gap, $\eta_{\max} < 1.5$, as a function of Q^2_{DA} for two ranges of x_{DA} . No acceptance corrections have been applied.

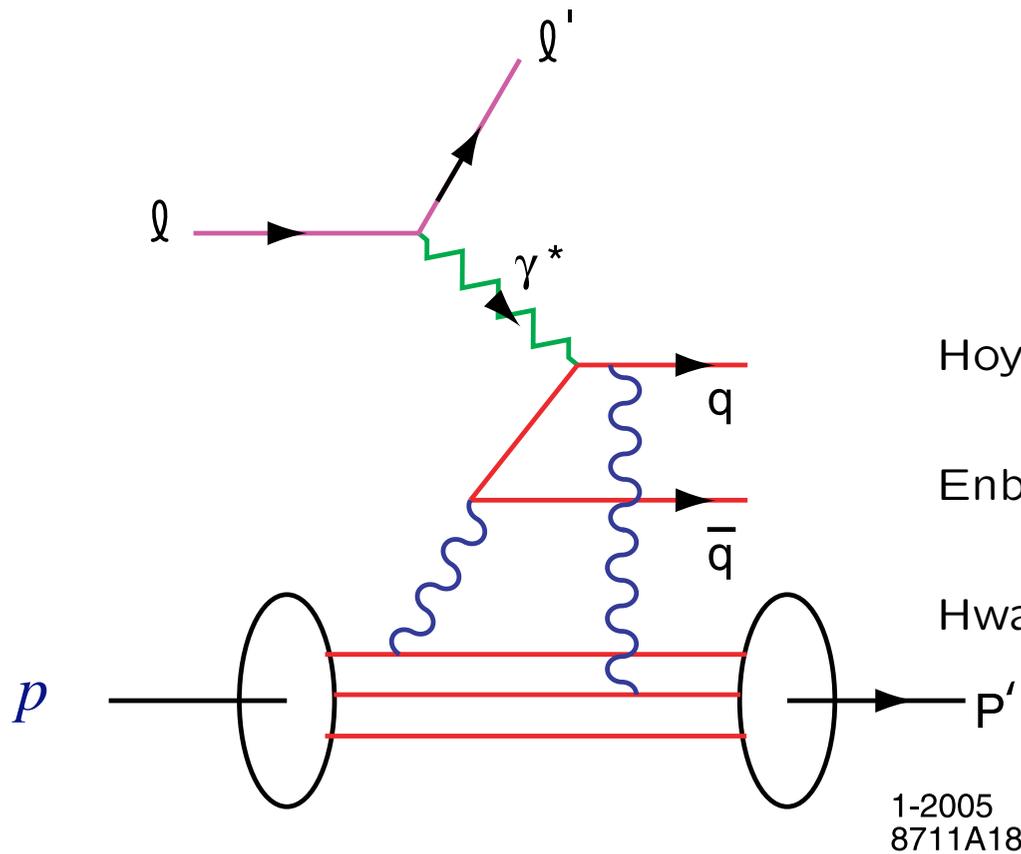
M. Derrick et al. [ZEUS Collaboration], Phys. Lett. B 315, 481 (1993).

Deep Inelastic Electron-Proton Scattering



*Conventional wisdom wrong:
Final-state interactions of struck quark cannot be neglected*

Final-State Interaction Produces Diffractive DIS



Quark Rescattering

Hoyer, Marchal, Peigne, Sannino, SJB (BHM)

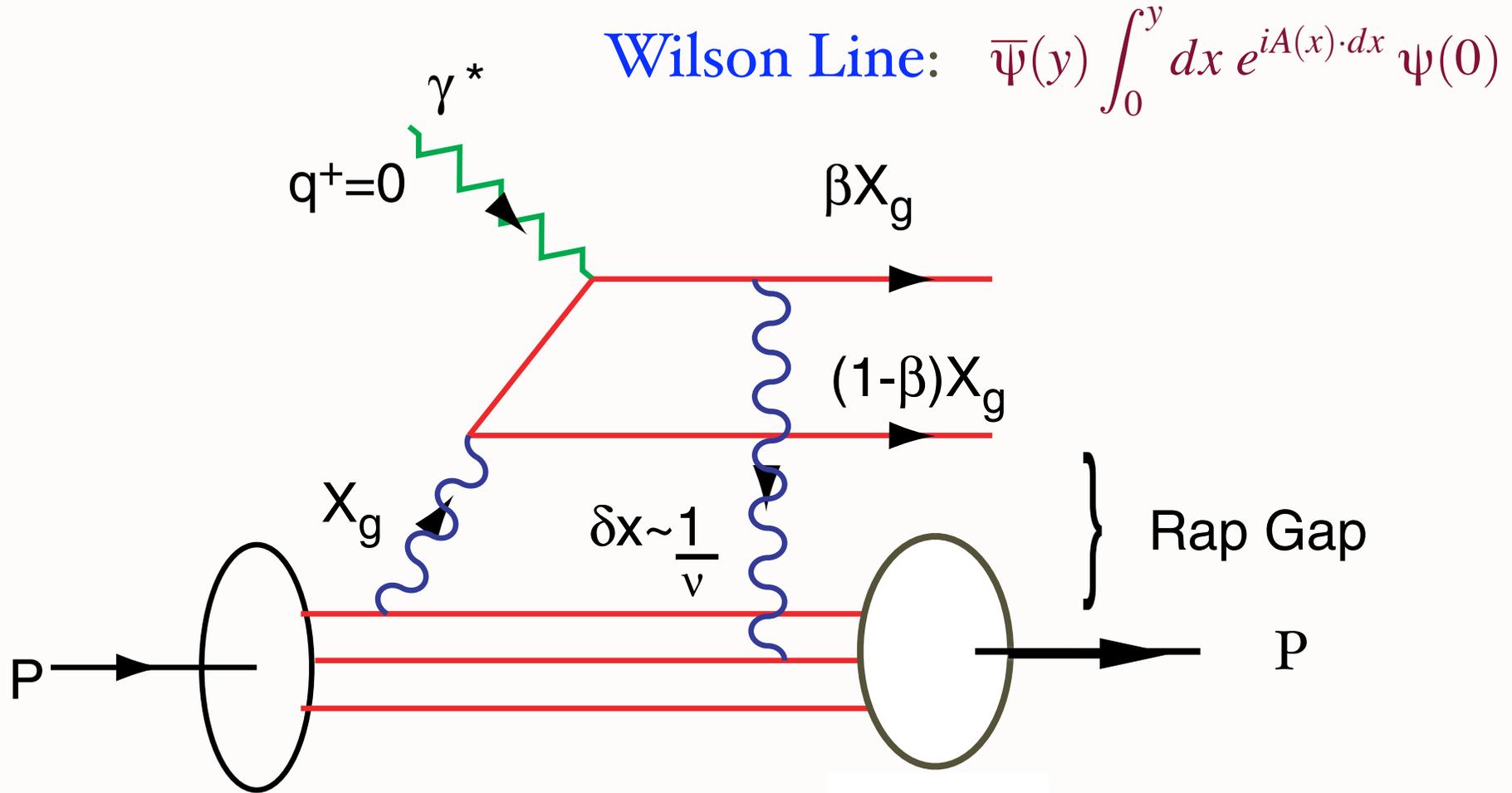
Enberg, Hoyer, Ingelman, SJB

Hwang, Schmidt, SJB

1-2005
8711A18

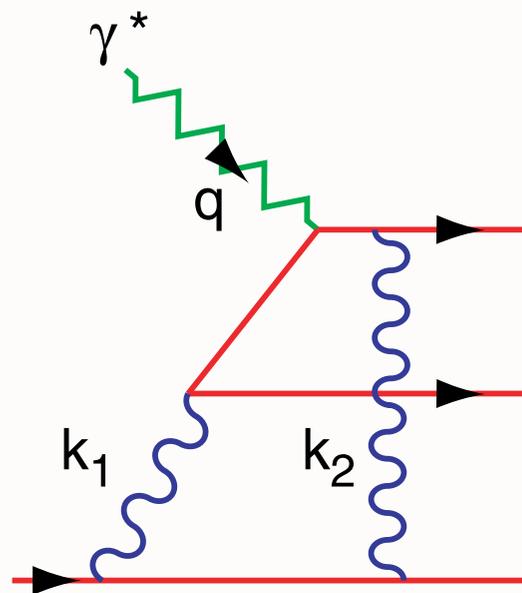
Low-Nussinov model of Pomeron

QCD Mechanism for Rapidity Gaps

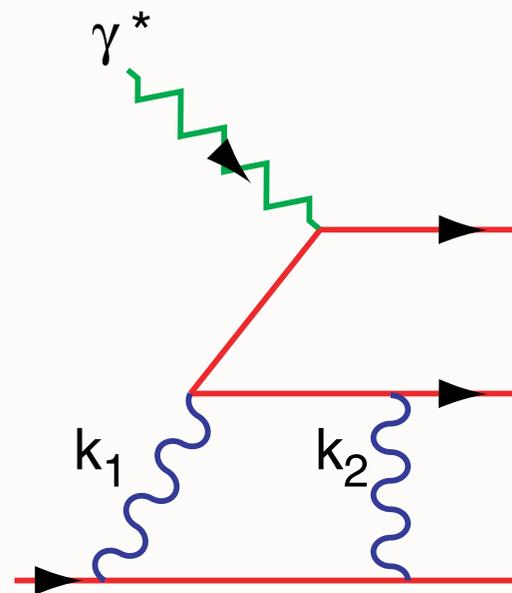


*Origin of Diffractive DIS
Reproduces lab-frame color dipole approach*

Final State Interactions in QCD



Feynman Gauge



Light-Cone Gauge

Result is Gauge Independent

Conformal symmetry: Template for QCD

- **Take conformal symmetry as initial approximation; then correct for non-zero beta function and quark masses**
- **Eigensolutions of ERBL evolution equation for distribution amplitudes** V. Braun et al;
Frishman, Lepage, Sachrajda, sjb
- **Commensurate scale relations: relate observables at corresponding scales:** H-J. Lu, sjb
Generalized Crewther Relation Kataev, Gabadadze, Rathsmann,
Lu, sjb
- **Fix Renormalization Scale (BLM, Effective Charges)** Grunberg
Lepage, Mackenzie, Binger, sjb
- **Use AdS/CFT**

Relate Observables to Each Other

- Eliminate intermediate scheme
- No scale ambiguity
- Transitive!
- Commensurate Scale Relations
- Example: Generalized Crewther Relation

$$R_{e^+e^-}(Q^2) \equiv 3 \sum_{\text{flavors}} e_q^2 \left[1 + \frac{\alpha_R(Q)}{\pi} \right].$$

$$\int_0^1 dx [g_1^{ep}(x, Q^2) - g_1^{en}(x, Q^2)] \equiv \frac{1}{3} \left| \frac{g_A}{g_V} \right| \left[1 - \frac{\alpha_{g_1}(Q)}{\pi} \right]$$

Generalized Crewther Relation

$$\left[1 + \frac{\alpha_R(s^*)}{\pi}\right] \left[1 - \frac{\alpha_{g_1}(q^2)}{\pi}\right] = 1$$

$$\sqrt{s^*} \simeq 0.52Q$$

*Conformal relation true to all orders in
perturbation theory*

No radiative corrections to axial anomaly

Nonconformal terms set relative scales (BLM)

Analytic matching at quark thresholds

No renormalization scale ambiguity!

$$\begin{aligned}
\frac{\alpha_R(Q)}{\pi} = & \frac{\alpha_{\overline{\text{MS}}}(Q)}{\pi} + \left(\frac{\alpha_{\overline{\text{MS}}}(Q)}{\pi} \right)^2 \left[\left(\frac{41}{8} - \frac{11}{3} \zeta_3 \right) C_A - \frac{1}{8} C_F + \left(-\frac{11}{12} + \frac{2}{3} \zeta_3 \right) f \right] \\
& + \left(\frac{\alpha_{\overline{\text{MS}}}(Q)}{\pi} \right)^3 \left\{ \left(\frac{90445}{2592} - \frac{2737}{108} \zeta_3 - \frac{55}{18} \zeta_5 - \frac{121}{432} \pi^2 \right) C_A^2 + \left(-\frac{127}{48} - \frac{143}{12} \zeta_3 + \frac{55}{3} \zeta_5 \right) C_A C_F - \frac{23}{32} C_F^2 \right. \\
& + \left[\left(-\frac{970}{81} + \frac{224}{27} \zeta_3 + \frac{5}{9} \zeta_5 + \frac{11}{108} \pi^2 \right) C_A + \left(-\frac{29}{96} + \frac{19}{6} \zeta_3 - \frac{10}{3} \zeta_5 \right) C_F \right] f \\
& \left. + \left(\frac{151}{162} - \frac{19}{27} \zeta_3 - \frac{1}{108} \pi^2 \right) f^2 + \left(\frac{11}{144} - \frac{1}{6} \zeta_3 \right) \frac{d^{abc} d^{abc}}{C_F d(R)} \frac{\left(\sum_f Q_f \right)^2}{\sum_f Q_f^2} \right\}.
\end{aligned}$$

$$\begin{aligned}
\frac{\alpha_{g_1}(Q)}{\pi} = & \frac{\alpha_{\overline{\text{MS}}}(Q)}{\pi} + \left(\frac{\alpha_{\overline{\text{MS}}}(Q)}{\pi} \right)^2 \left[\frac{23}{12} C_A - \frac{7}{8} C_F - \frac{1}{3} f \right] \\
& + \left(\frac{\alpha_{\overline{\text{MS}}}(Q)}{\pi} \right)^3 \left\{ \left(\frac{5437}{648} - \frac{55}{18} \zeta_5 \right) C_A^2 + \left(-\frac{1241}{432} + \frac{11}{9} \zeta_3 \right) C_A C_F + \frac{1}{32} C_F^2 \right. \\
& \left. + \left[\left(-\frac{3535}{1296} - \frac{1}{2} \zeta_3 + \frac{5}{9} \zeta_5 \right) C_A + \left(\frac{133}{864} + \frac{5}{18} \zeta_3 \right) C_F \right] f + \frac{115}{648} f^2 \right\}.
\end{aligned}$$

**Eliminate MSbar,
Find Amazing Simplification**

$$R_{e^+e^-}(Q^2) \equiv 3 \sum_{\text{flavors}} e_q^2 \left[1 + \frac{\alpha_R(Q)}{\pi} \right].$$

$$\int_0^1 dx \left[g_1^{ep}(x, Q^2) - g_1^{en}(x, Q^2) \right] \equiv \frac{1}{3} \left| \frac{g_A}{g_V} \right| \left[1 - \frac{\alpha_{g_1}(Q)}{\pi} \right]$$

$$\frac{\alpha_{g_1}(Q)}{\pi} = \frac{\alpha_R(Q^*)}{\pi} - \left(\frac{\alpha_R(Q^{**})}{\pi} \right)^2 + \left(\frac{\alpha_R(Q^{***})}{\pi} \right)^3$$

Geometric Series in Conformal QCD

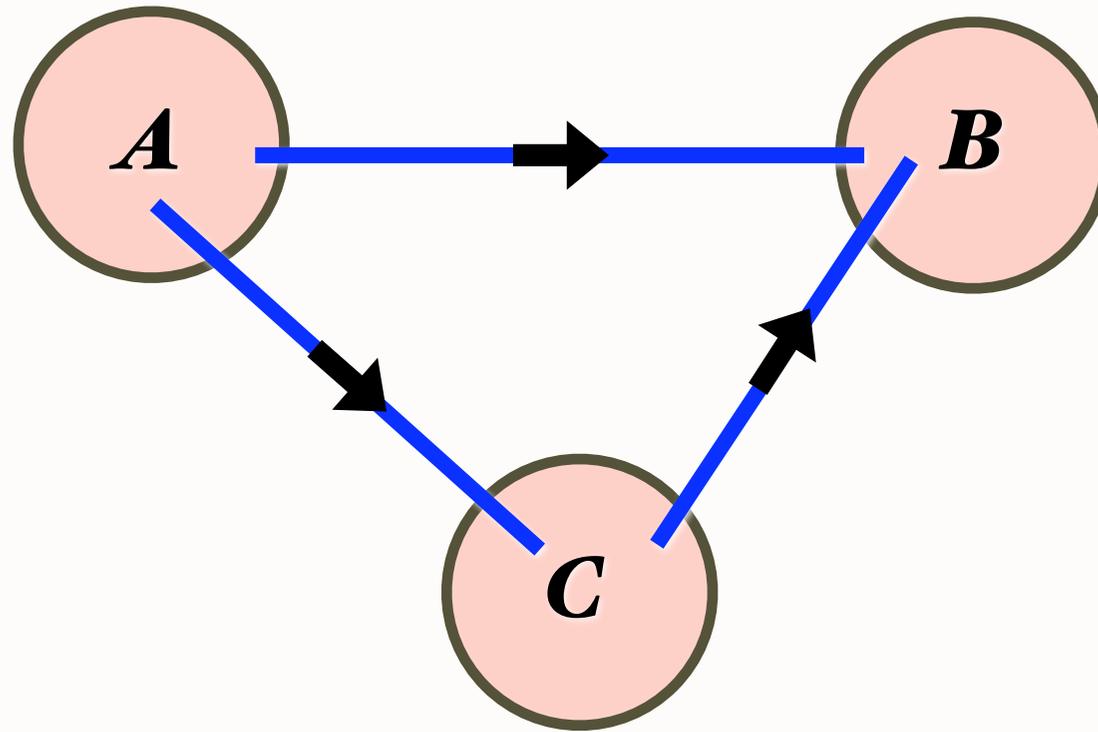
Generalized Crewther Relation

Lu, Kataev, Gabadadze, Sjb

Relate Observables to Each Other

- Eliminate intermediate scheme
- No scale ambiguity
- Transitive!
- Commensurate Scale Relations
- Example: Generalized Crewther Relation

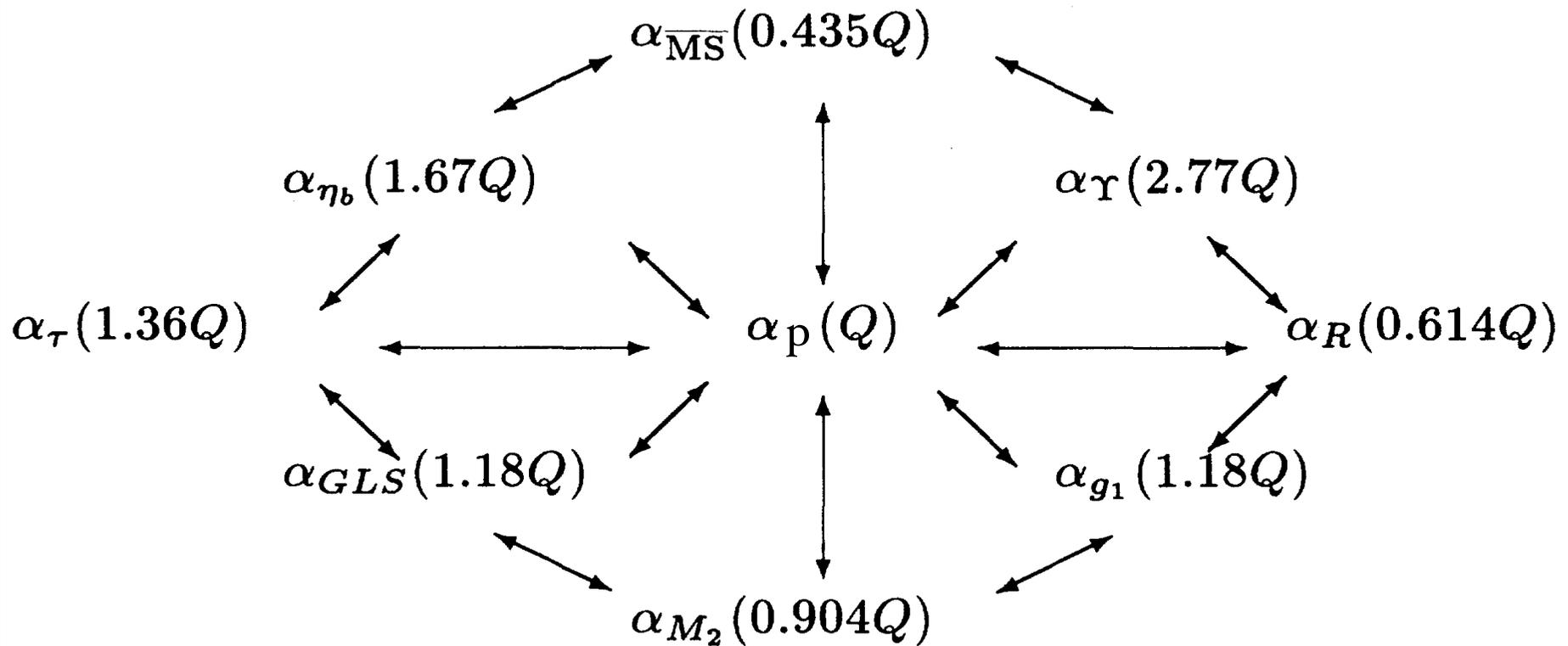
Transitivity Property of Renormalization Group



$$**A \rightarrow C \quad C \rightarrow B \quad \textit{identical to} \quad A \rightarrow B**$$

Relation of observables independent of intermediate scheme C

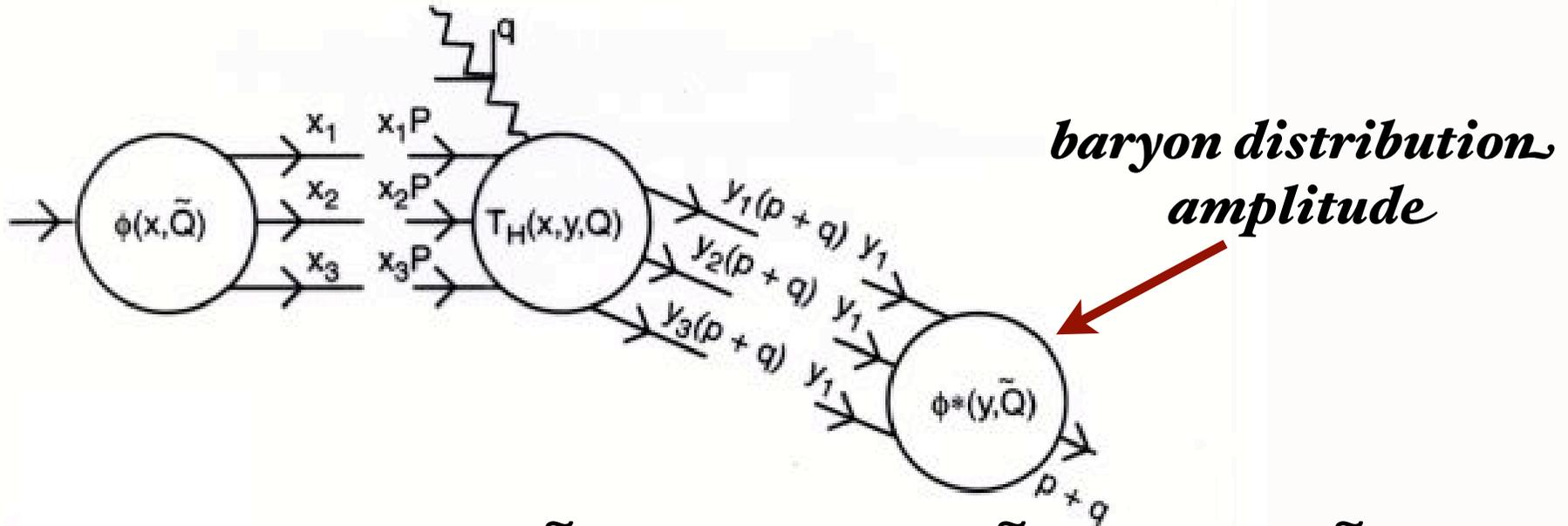
Leading Order Commensurate Scales



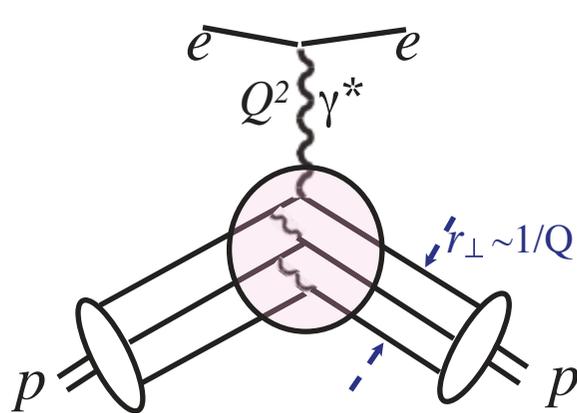
Translation between schemes at LO

Leading-Twist PQCD Factorization for form factors, exclusive amplitudes

Lepage, sjb



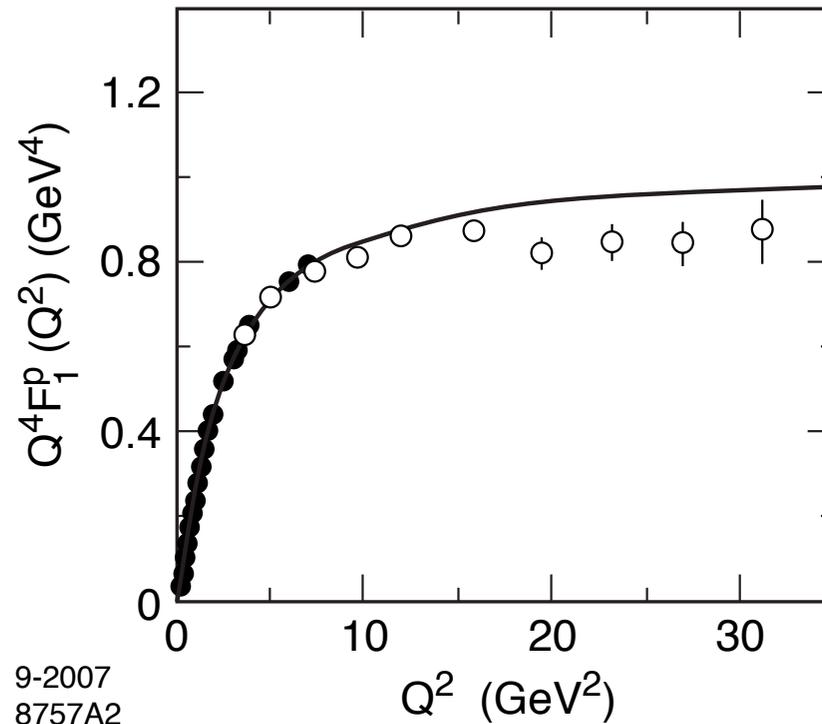
$$M = \int \prod dx_i dy_i \phi_F(x_i, \tilde{Q}) \times T_H(x_i, y_i, \tilde{Q}) \times \phi_I(y_i, \tilde{Q})$$



If $\alpha_s(\tilde{Q}^2) \simeq \text{constant}$

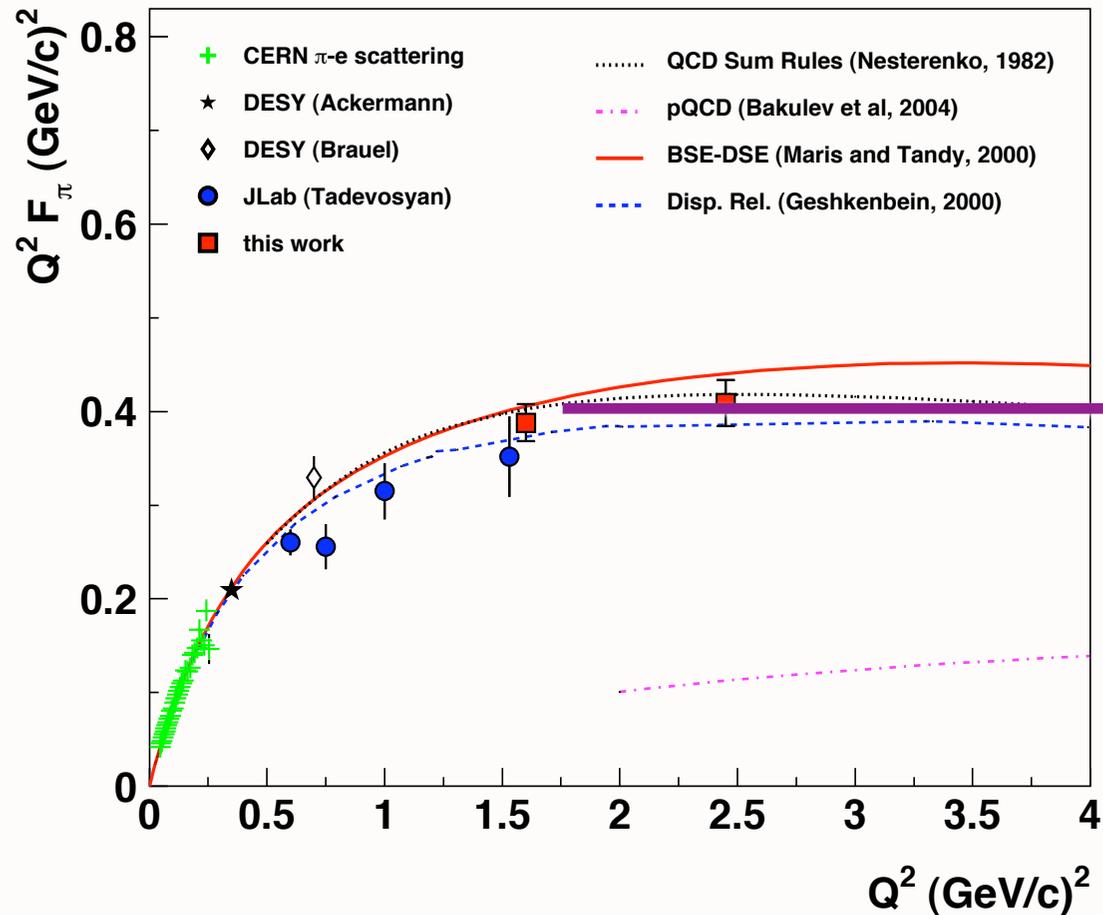
$$Q^4 F_1(Q^2) \simeq \text{constant}$$

- Scaling behavior for large Q^2 : $Q^4 F_1^p(Q^2) \rightarrow \text{constant}$ Proton $\tau = 3$



SW model predictions for $\kappa = 0.424$ GeV. Data analysis from: M. Diehl *et al.* Eur. Phys. J. C **39**, 1 (2005).

Conformal behavior: $Q^2 F_\pi(Q^2) \rightarrow \text{const}$



Determination of the Charged Pion Form Factor at $Q^2=1.60$ and 2.45 $(\text{GeV}/c)^2$.
 By Fpi2 Collaboration ([T. Horn et al.](#)). Jul 2006. 4pp.
 e-Print Archive: [nucl-ex/0607005](#)

Consequences of Maximum Quark and Gluon Wavelength

- Infrared integrations regulated by confinement
- Infrared fixed point of QCD coupling

$$\alpha_s(Q^2) \text{ finite, } \beta \rightarrow 0 \text{ at small } Q^2$$

- Bound state quark and gluon Dyson-Schwinger Equation
- Quark and Gluon Condensates exist within hadrons

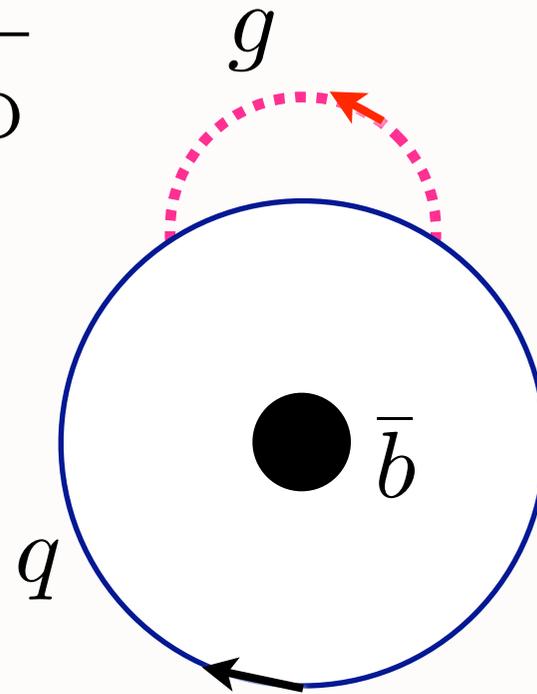
Casher, Susskind

Shrock, sjb

Maximum wavelength of bound quarks and gluons

$$k > \frac{1}{\Lambda_{\text{QCD}}}$$

$$\lambda < \Lambda_{\text{QCD}}$$



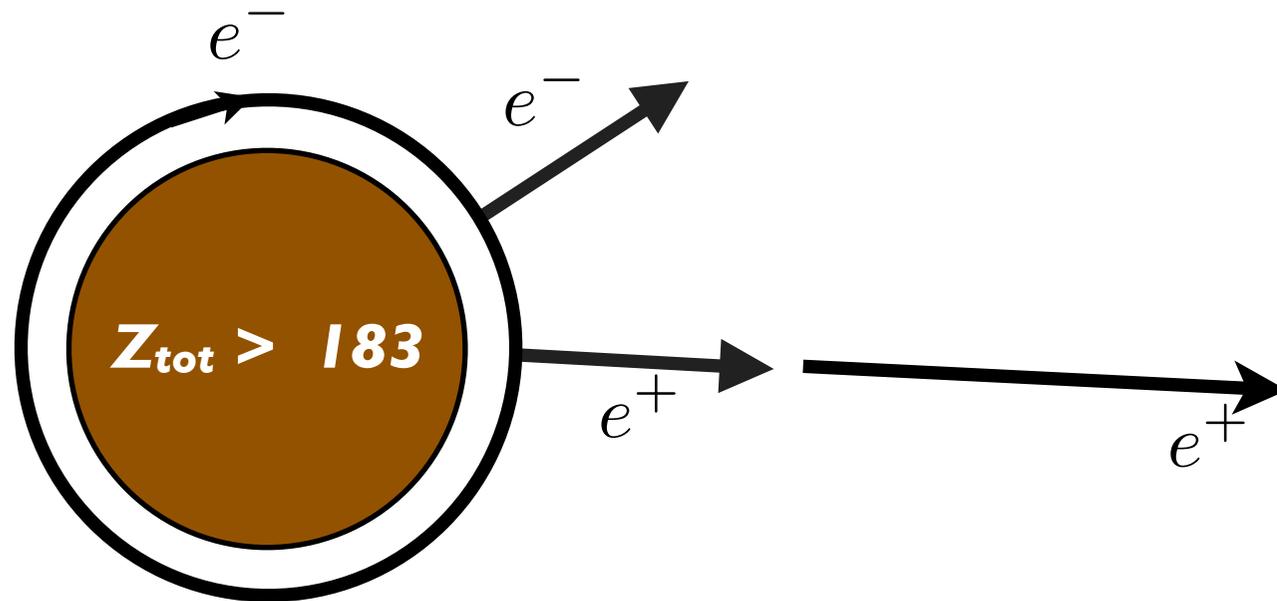
B-Meson

Shrock, sjb

Use Dyson-Schwinger Equation for bound-state quark propagator: find confined condensate

$$\langle \bar{b} | \bar{q} q | \bar{b} \rangle \text{ not } \langle 0 | \bar{q} q | 0 \rangle$$

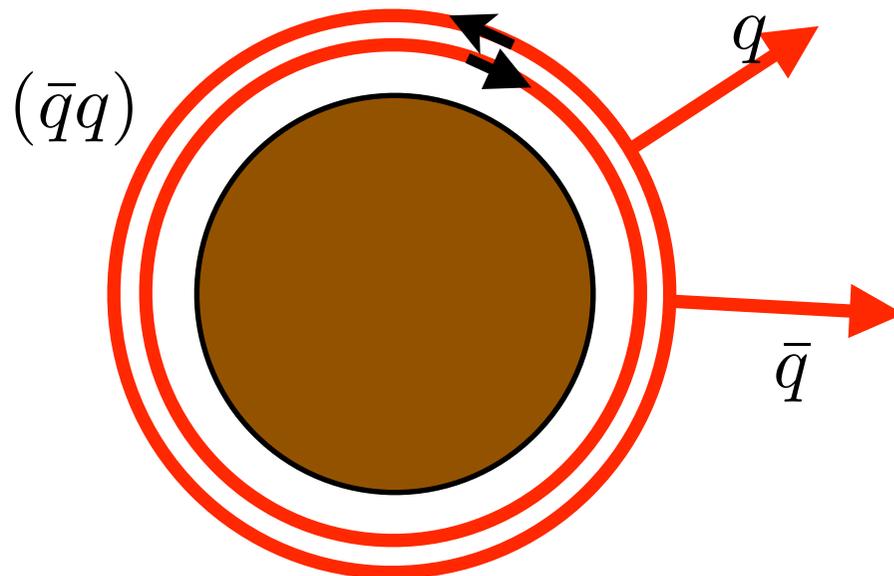
Spontaneous Production in QED: Adiabatic Collision of Heavy Ions (GSI)



$$Z_1 + Z_2 \rightarrow Z_{tot} + e^+ e^- \rightarrow [Z_{tot} e^-] + e^+$$

Vacuum charge changed by +1

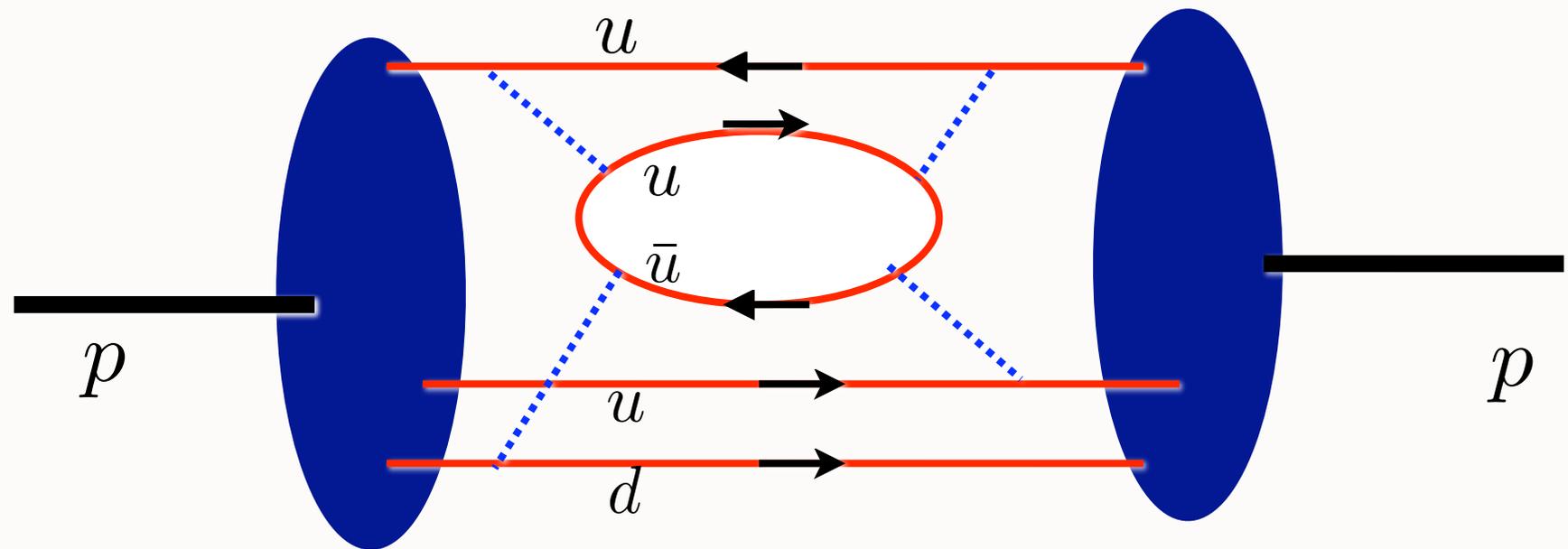
Spontaneous Production in Supercritical QCD: Color Confinement: Enriched Fock state



$$\alpha_s \rightarrow \alpha_s > \alpha_s^{critical}$$

$$|uud\rangle \rightarrow \bar{\psi}\psi|uud\rangle = |uud(q\bar{q})\rangle$$

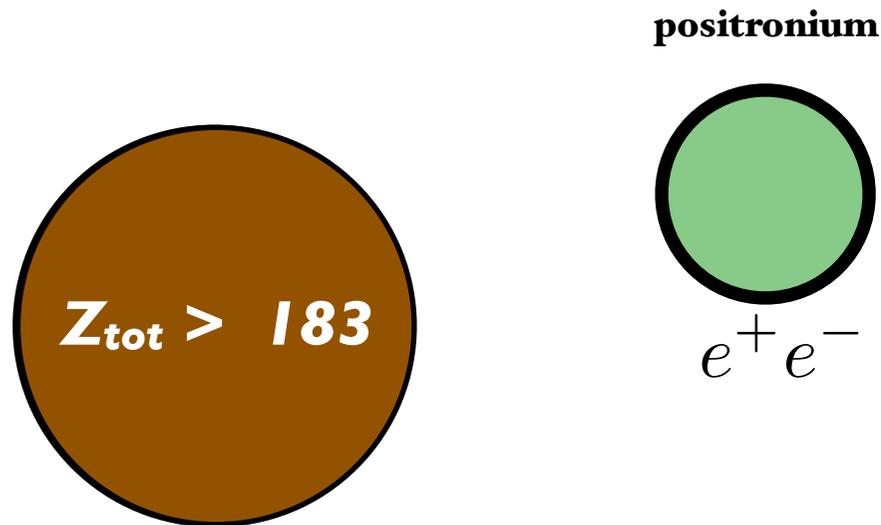
Supercritical QCD produces an extra quark pair



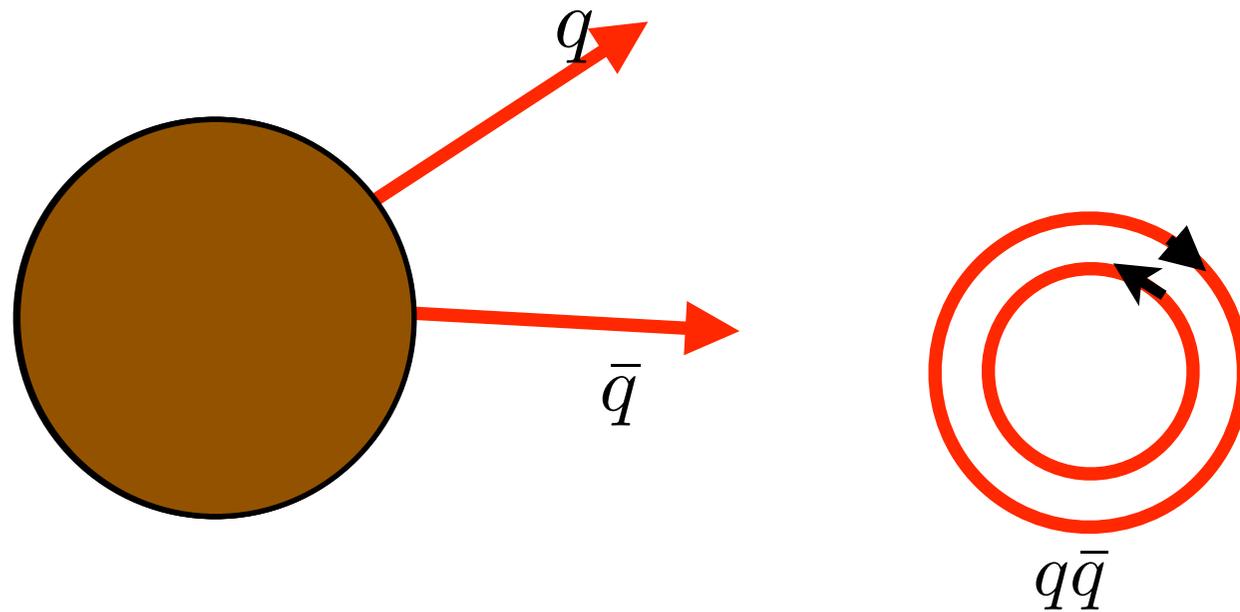
$|uud\bar{u}u\rangle$ **Fock state**

arrows indicate quark LF chirality

*Spontaneous Positronium Production in
QED: Adiabatic Collision of Heavy Ions
(GSI)*



Spontaneous Production in QCD: Color Confinement: Pion production



$$\alpha_s \rightarrow \alpha_s > \alpha_s^{critical}$$

$$|uud\rangle \rightarrow \bar{\psi}\psi|uud\rangle = |udd\rangle + |ud\bar{u}\rangle$$

$$|uud\rangle \rightarrow |dd\bar{u}ud\rangle \rightarrow n + \pi^-$$

*Quark and Gluon condensates
reside within hadrons, not vacuum*
Shrock, sjb

- **Bound-State Dyson-Schwinger Equations**
- **LF vacuum trivial up to $k^+ = 0$ zero modes**
- **Analogous to finite size superconductor**
- **Usual picture for $m_\pi \rightarrow 0$**
- **Implications for cosmological constant --
reduction by 45 orders of magnitude!**

Chiral magnetism (or magnetohadronics)

Aharon Casher and Leonard Susskind

Tel Aviv University Ramat Aviv, Tel-Aviv, Israel

(Received 20 March 1973)

I. INTRODUCTION

The spontaneous breakdown of chiral symmetry in hadron dynamics is generally studied as a vacuum phenomenon.¹ Because of an instability of the chirally invariant vacuum, the real vacuum is “aligned” into a chirally asymmetric configuration.

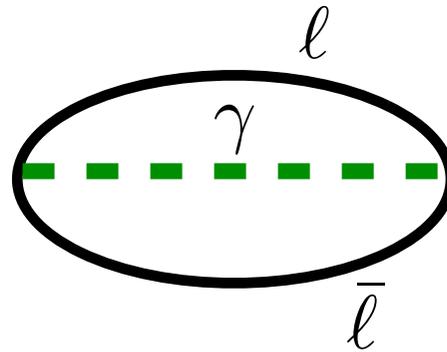
On the other hand an approach to quantum field theory exists in which the properties of the vacuum state are not relevant. This is the parton or constituent approach formulated in the infinite-momentum frame.² A number of investigations have indicated that in this frame the vacuum may be regarded as the structureless Fock-space vacuum. Hadrons may be described as nonrelativistic collections of constituents (partons). In this framework the spontaneous symmetry breakdown must be attributed to the properties of the hadron's wave function and not to the vacuum.³

*Light-Front
(Front Form)
Formalism*

Critical Conflict

*between Standard Model and Phenomenology:
Vacuum Condensates vs Cosmological Constant*

- QCD Condensates: Cosmological Constant 10^{45} larger than measurement
- Higgs potential gives Cosmological Constant 10^{54} larger than measurement
- Equal-Time Vacuum has nonzero vacuum even for QED



- Higgs Theory on LF: Higgs condensate replaced by zero mode;
- LF Vacuum trivial up to zero modes

P. Srivastava, sjb

Determinations of the vacuum Gluon Condensate

$$\langle 0 | \frac{\alpha_s}{\pi} G^2 | 0 \rangle [\text{GeV}^4]$$

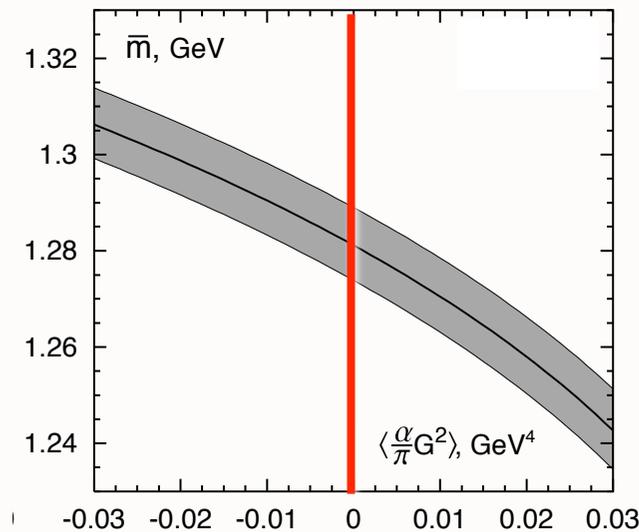
-0.005 ± 0.003 from τ decay.

Davier et al.

$+0.006 \pm 0.012$ from τ decay. Geshkenbein, Ioffe, Zyablyuk

$+0.009 \pm 0.007$ from charmonium sum rules

Ioffe, Zyablyuk



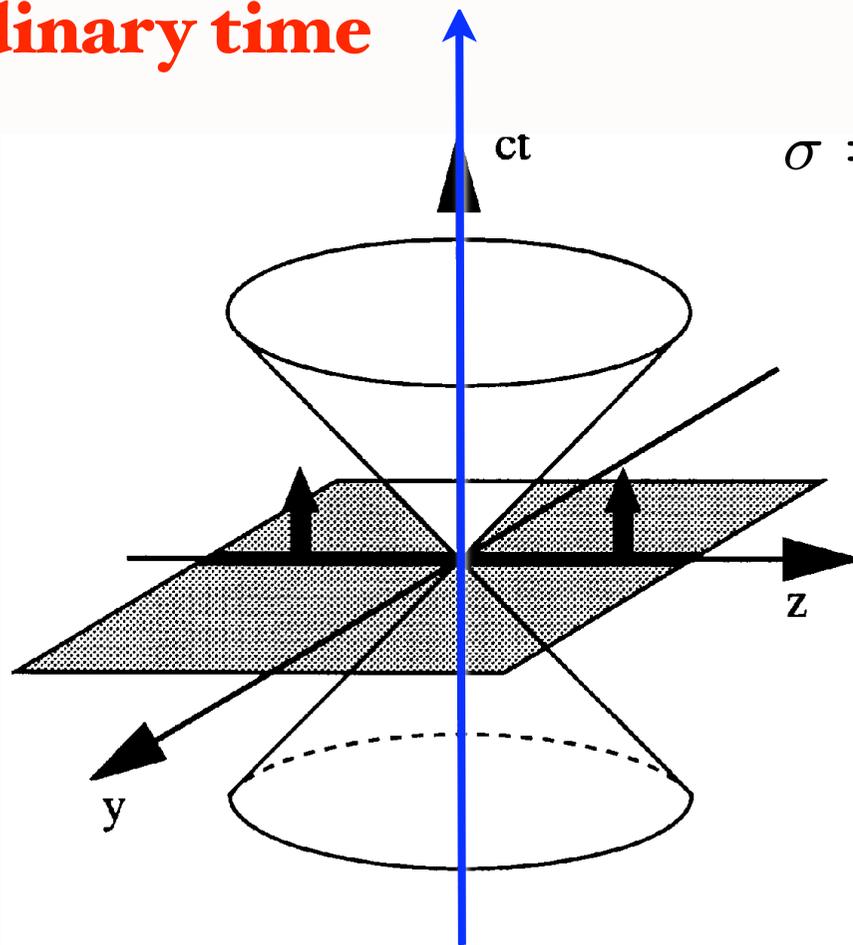
*Consistent with zero
vacuum condensate*

- **Casher & Susskind model shows that spontaneous chiral symmetry breaking can occur in the finite domain of a hadronic LFWF**
- **Infinite number of partons required, but this is a feature of QCD LFWFs --**
- **Regge behavior of DIS due to $x^{-\alpha_R}$ behavior of structure functions (LFWFs squared)**
- **A.H. Mueller: BLKL Pomeron derived from the multi-gluon Fock States of the quarkonium LFWF**
- **F. Antonuccio, S. Dalley, sjb: Construct soft-gluon LFWF via ladder operators**
- **LF Vacuum Trivial up to zero modes**

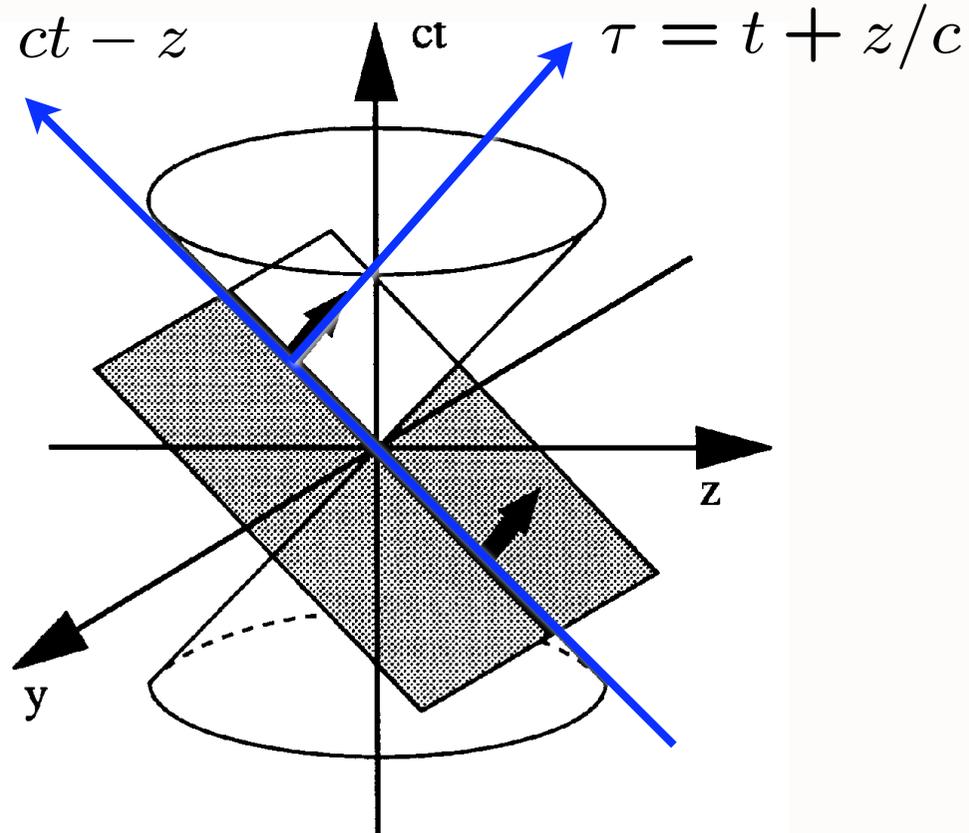
Dirac's Amazing Idea: The Front Form

**Evolve in
ordinary time**

**Evolve in
light-front time!**



$$\sigma = ct - z$$



$$\tau = t + z/c$$

Instant Form

Front Form

Each element of
flash photograph
illuminated
at same LF time

$$\tau = t + z/c$$

Evolve in LF time

$$P^- = i \frac{d}{d\tau}$$

Eigenstate -- independent of τ

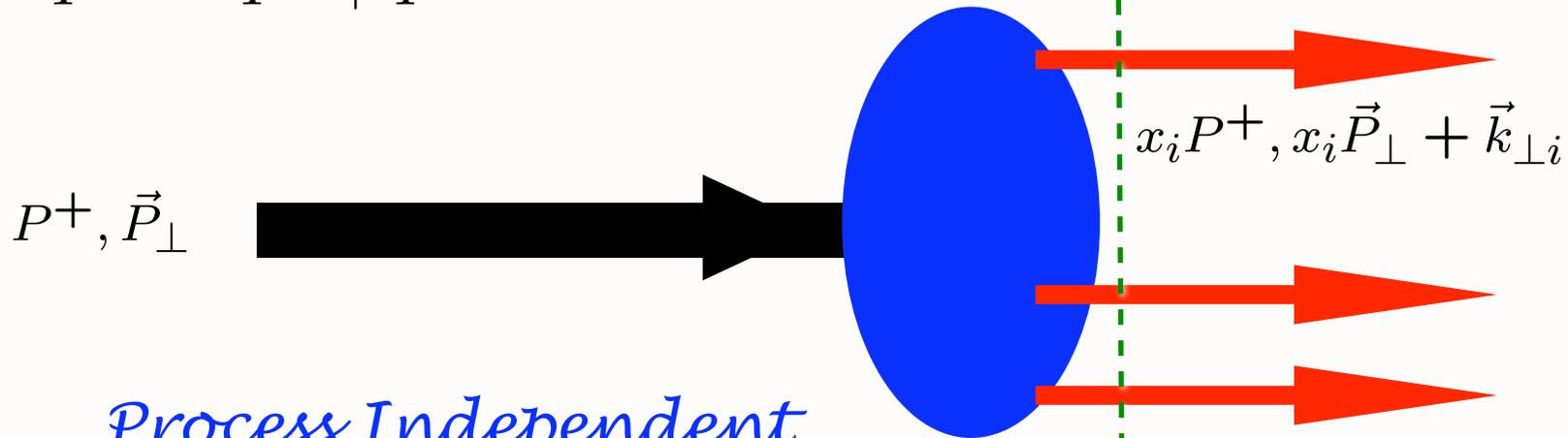


HELEN BRADLEY - PHOTOGRAPHY

Light-Front Wavefunctions: rigorous representation of composite systems in quantum field theory

$$x = \frac{k^+}{P^+} = \frac{k^0 + k^3}{P^0 + P^3}$$

Fixed $\tau = t + z/c$



*Process Independent
Direct Link to QCD Lagrangian!*

$$\Psi_n(x_i, \vec{k}_{\perp i}, \lambda_i)$$

$$\sum_i^n x_i = 1$$

$$\sum_i^n \vec{k}_{\perp i} = \vec{0}_{\perp}$$

Invariant under boosts! Independent of p^μ

Light-Front Wavefunctions

Dirac's Front Form: Fixed $\tau = t + z/c$

$$\Psi_n(x_i, \vec{k}_{\perp i}, \lambda_i) \quad x_i = \frac{k_i^+}{P^+}$$

Invariant under boosts. Independent of P^μ

$$H_{LF}^{QCD} |\psi\rangle = M^2 |\psi\rangle$$

*Remarkable new insights from AdS/CFT,
the duality between conformal field theory
and Anti-de Sitter Space*

$$|p, S_z\rangle = \sum_{n=3} \Psi_n(x_i, \vec{k}_{\perp i}, \lambda_i) |n; \vec{k}_{\perp i}, \lambda_i\rangle$$

sum over states with $n=3, 4, \dots$ constituents

The Light Front Fock State Wavefunctions

$$\Psi_n(x_i, \vec{k}_{\perp i}, \lambda_i)$$

are boost invariant; they are independent of the hadron's energy and momentum P^μ .

The light-cone momentum fraction

$$x_i = \frac{k_i^+}{p^+} = \frac{k_i^0 + k_i^z}{P^0 + P^z}$$

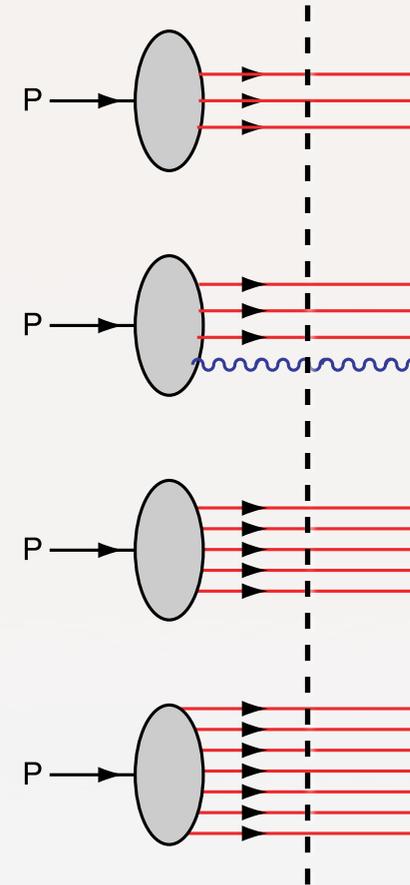
are boost invariant.

$$\sum_i^n k_i^+ = P^+, \quad \sum_i^n x_i = 1, \quad \sum_i^n \vec{k}_i^\perp = \vec{0}^\perp.$$

Intrinsic heavy quarks,

$$\bar{s}(x) \neq s(x)$$

$$\bar{u}(x) \neq \bar{d}(x)$$



Fixed LF time

Angular Momentum on the Light-Front

$A^+=0$ gauge:

No unphysical degrees of freedom

$$J^z = \sum_{i=1}^n s_i^z + \sum_{j=1}^{n-1} l_j^z.$$

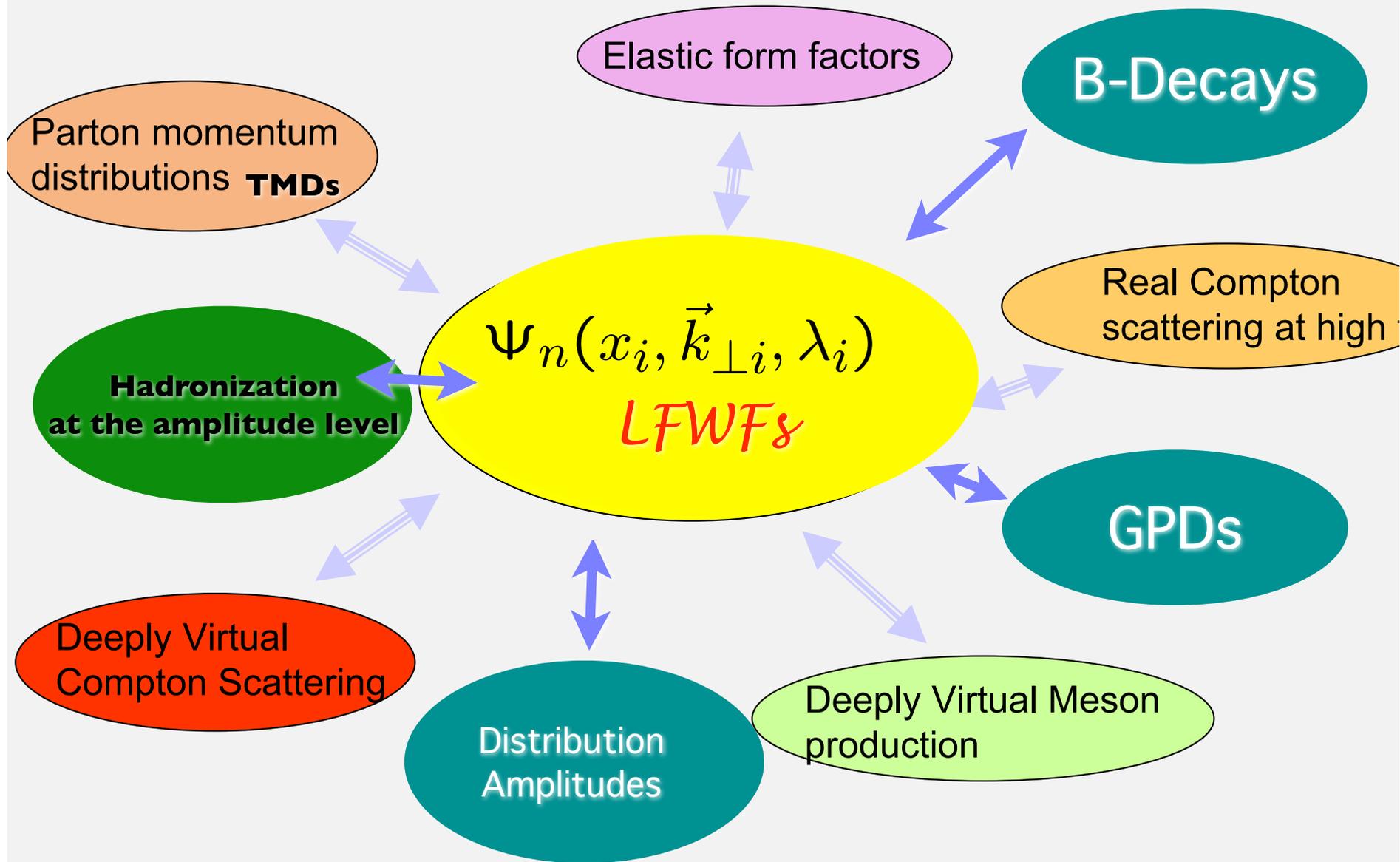
Conserved
LF Fock state by Fock State

$$l_j^z = -i \left(k_j^1 \frac{\partial}{\partial k_j^2} - k_j^2 \frac{\partial}{\partial k_j^1} \right)$$

n-1 orbital angular momenta

*Nonzero Anomalous Moment requires
Nonzero orbital angular momentum.*

A Unified Description of Hadron Structure



Some Applications of Light-Front Wavefunctions

- Exact formulae for form factors, quark and gluon distributions; vanishing anomalous gravitational moment; edm connection to anm
- Deeply Virtual Compton Scattering, generalized parton distributions, angular momentum sum rules
- Exclusive weak decay amplitudes
- Single spin asymmetries: Role of ISI and FSI
- Factorization theorems, DGLAP, BFKL, ERBL Evolution
- Quark interchange amplitude
- Relation of spin, momentum, and other distributions to physics of the hadron itself.

Light-Front QCD

Heisenberg Matrix Formulation

Physical gauge: $A^+ = 0$

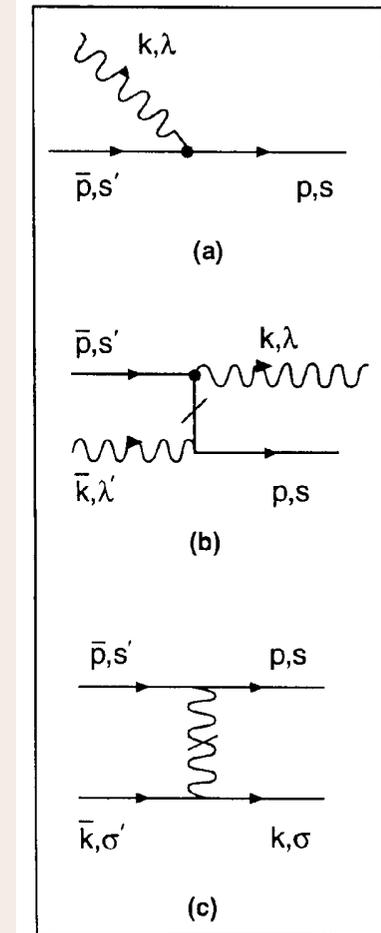
$$L^{QCD} \rightarrow H_{LF}^{QCD}$$

$$H_{LF}^{QCD} = \sum_i \left[\frac{m^2 + k_{\perp}^2}{x} \right]_i + H_{LF}^{int}$$

H_{LF}^{int} : Matrix in Fock Space

$$H_{LF}^{QCD} |\Psi_h\rangle = \mathcal{M}_h^2 |\Psi_h\rangle$$

Eigenvalues and Eigensolutions give Hadron Spectrum and Light-Front wavefunctions



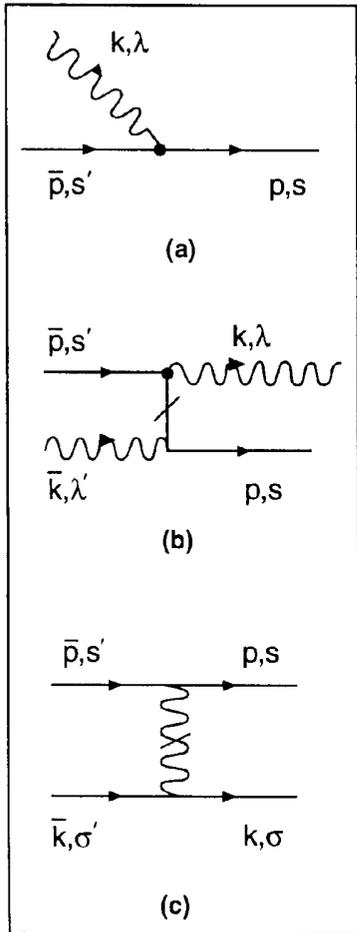
DLCQ: Periodic BC in x^- . Discrete k^+ ; frame-independent truncation

Light-Front QCD

Heisenberg Matrix Formulation

$$H_{LF}^{QCD} |\Psi_h\rangle = \mathcal{M}_h^2 |\Psi_h\rangle$$

H.C. Pauli & sjb
Discretized Light-Cone Quantization



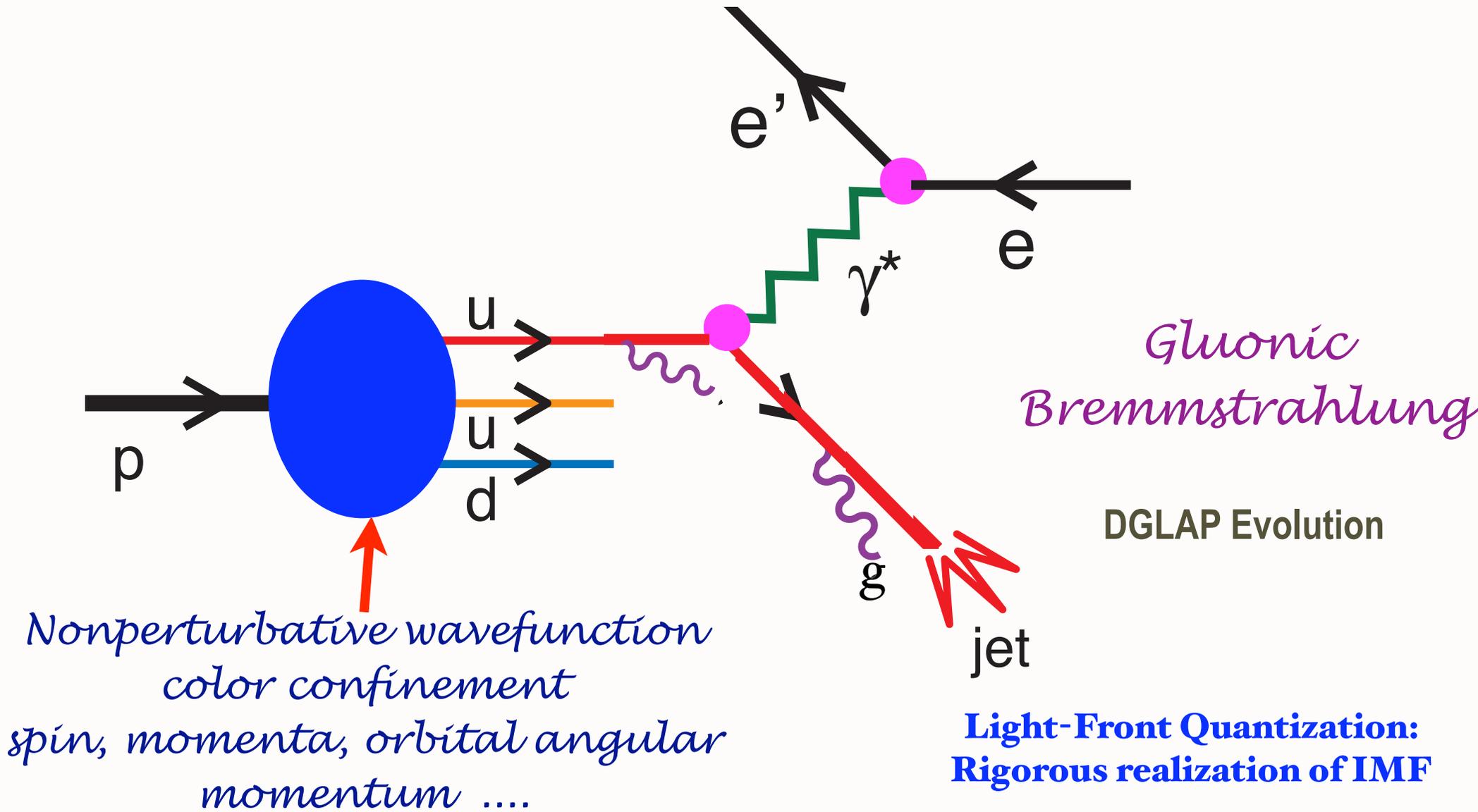
n	Sector	1 q \bar{q}	2 gg	3 q \bar{q} g	4 q \bar{q} q \bar{q}	5 ggg	6 q \bar{q} gg	7 q \bar{q} q \bar{q} g	8 q \bar{q} q \bar{q} q \bar{q}	9 gggg	10 q \bar{q} ggg	11 q \bar{q} q \bar{q} gg	12 q \bar{q} q \bar{q} q \bar{q} g	13 q \bar{q} q \bar{q} q \bar{q} q \bar{q}
1	q \bar{q}	[Diagram]	[Diagram]	[Diagram]	[Diagram]	.	[Diagram]
2	gg	[Diagram]	[Diagram]	[Diagram]	.	[Diagram]	[Diagram]	.	.	[Diagram]
3	q \bar{q} g	[Diagram]	[Diagram]	[Diagram]	[Diagram]	[Diagram]	[Diagram]	.	.	[Diagram]
4	q \bar{q} q \bar{q}	[Diagram]	.	[Diagram]	[Diagram]	.	[Diagram]	[Diagram]	.	.	.	[Diagram]	.	.
5	ggg	.	[Diagram]	[Diagram]	.	[Diagram]	[Diagram]	.	.	[Diagram]	[Diagram]	.	.	.
6	q \bar{q} gg	[Diagram]	[Diagram]	[Diagram]	[Diagram]	[Diagram]	[Diagram]	.	.	[Diagram]	[Diagram]	[Diagram]	.	.
7	q \bar{q} q \bar{q} g	.	.	[Diagram]	[Diagram]	.	[Diagram]	[Diagram]	[Diagram]	.	[Diagram]	[Diagram]	[Diagram]	.
8	q \bar{q} q \bar{q} q \bar{q}	.	.	.	[Diagram]	.	.	[Diagram]	[Diagram]	.	.	[Diagram]	[Diagram]	[Diagram]
9	gggg	.	[Diagram]	.	.	[Diagram]	[Diagram]	.	.	[Diagram]	[Diagram]	[Diagram]	.	.
10	q \bar{q} ggg	.	.	[Diagram]	.	[Diagram]	[Diagram]	.	.	[Diagram]	[Diagram]	[Diagram]	[Diagram]	.
11	q \bar{q} q \bar{q} gg	.	.	.	[Diagram]	.	[Diagram]	[Diagram]	[Diagram]	.	[Diagram]	[Diagram]	[Diagram]	.
12	q \bar{q} q \bar{q} q \bar{q} g	[Diagram]	[Diagram]	.	.	[Diagram]	[Diagram]	[Diagram]
13	q \bar{q} q \bar{q} q \bar{q} q \bar{q}	[Diagram]	.	.	.	[Diagram]	[Diagram]

Eigenvalues and Eigensolutions give Hadron Spectrum and Light-Front wavefunctions

DLCQ: Frame-independent, No fermion doubling; Minkowski Space

DLCQ: Periodic BC in x^- . Discrete k^+ ; frame-independent truncation

Deep Inelastic Electron-Proton Scattering



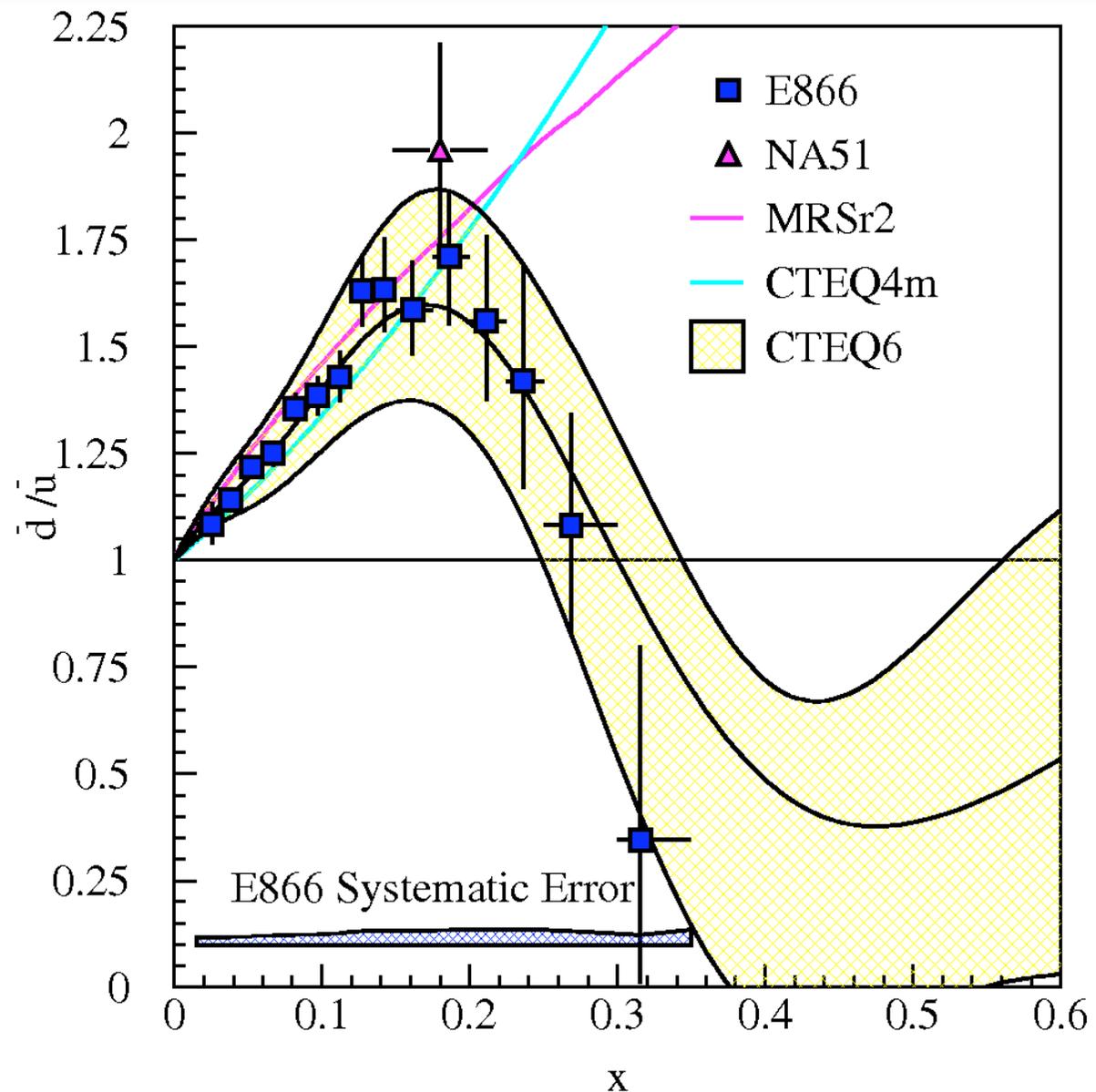
■ E866/NuSea (Drell-Yan)

$$\bar{d}(x) \neq \bar{u}(x)$$

$$s(x) \neq \bar{s}(x)$$

*Intrinsic glue, sea,
heavy quarks*

$\bar{d}(x)/\bar{u}(x)$ for $0.015 \leq x \leq 0.35$



Chiral magnetism (or magnetohadronics)

Aharon Casher and Leonard Susskind

Tel Aviv University Ramat Aviv, Tel-Aviv, Israel

(Received 20 March 1973)

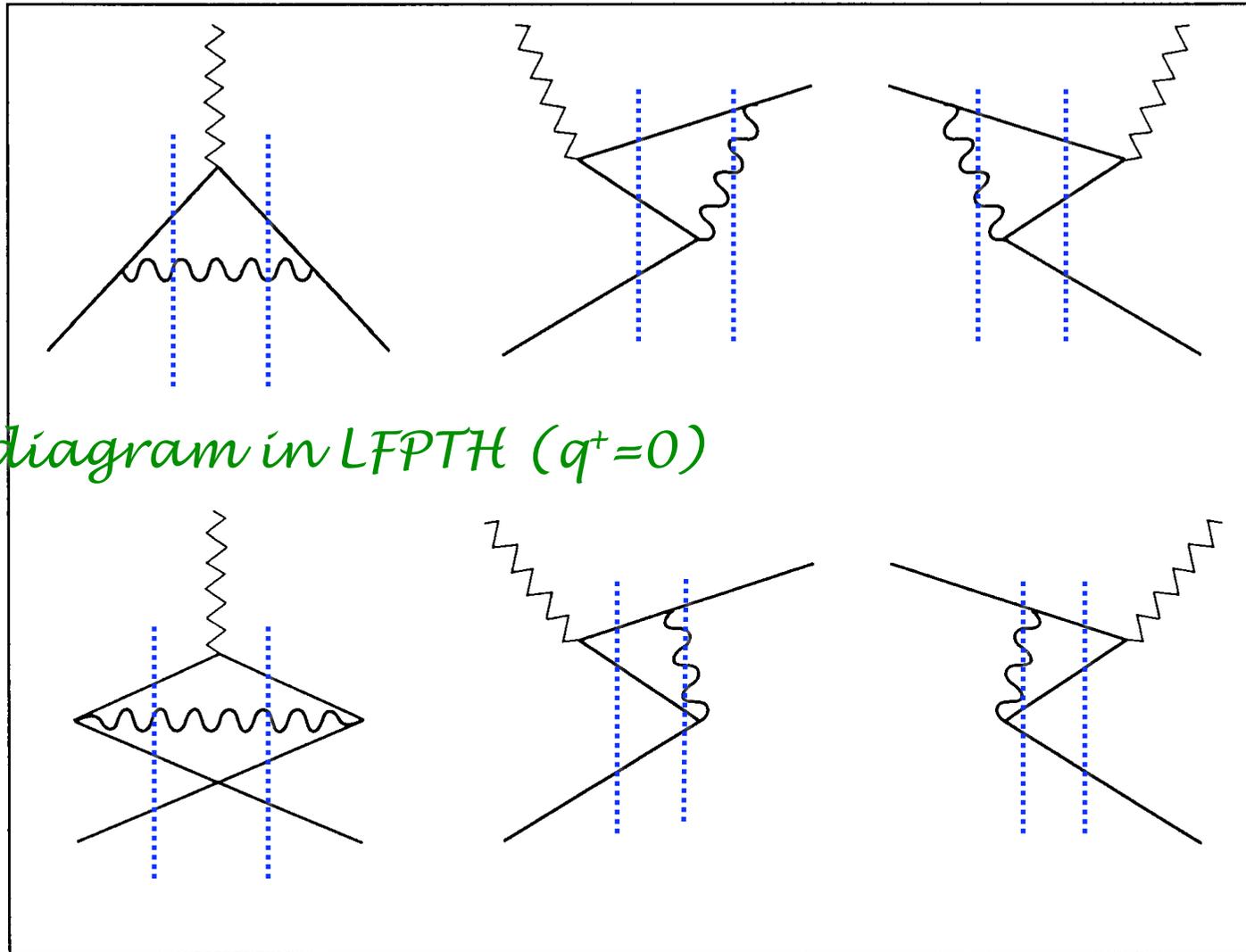
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*Light-Front
(Front Form) Formalism*

Calculation of lepton $g-2$ in TOPTH (Instant form)



Only diagram in LFPTH ($q^+=0$)

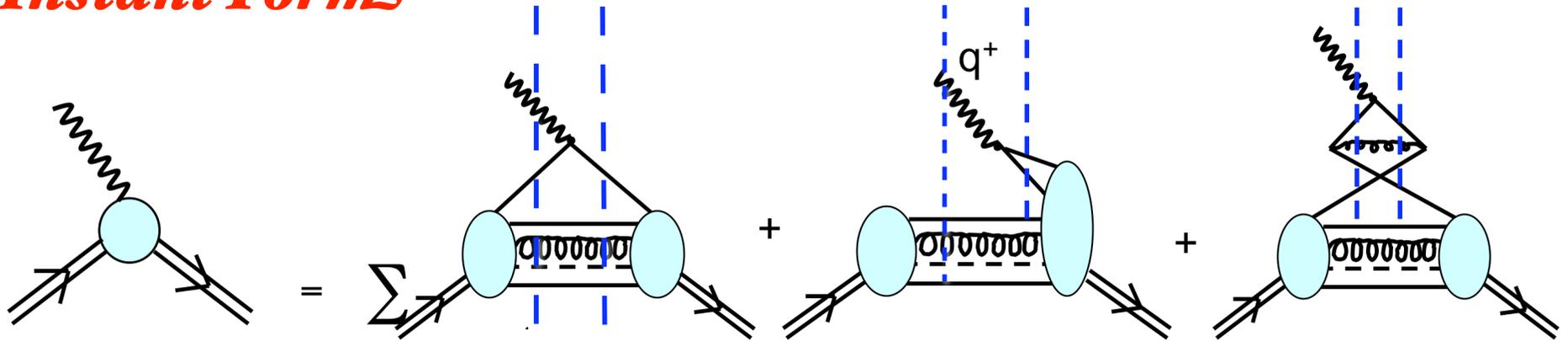
$n!$ diagrams at order e^n

*energy denominators:
frame-dependent and non-analytic*

$$\sqrt{(\vec{p} + \vec{q} - \vec{k})^2 + m^2}$$

Calculation of Form Factors in Equal-Time Theory

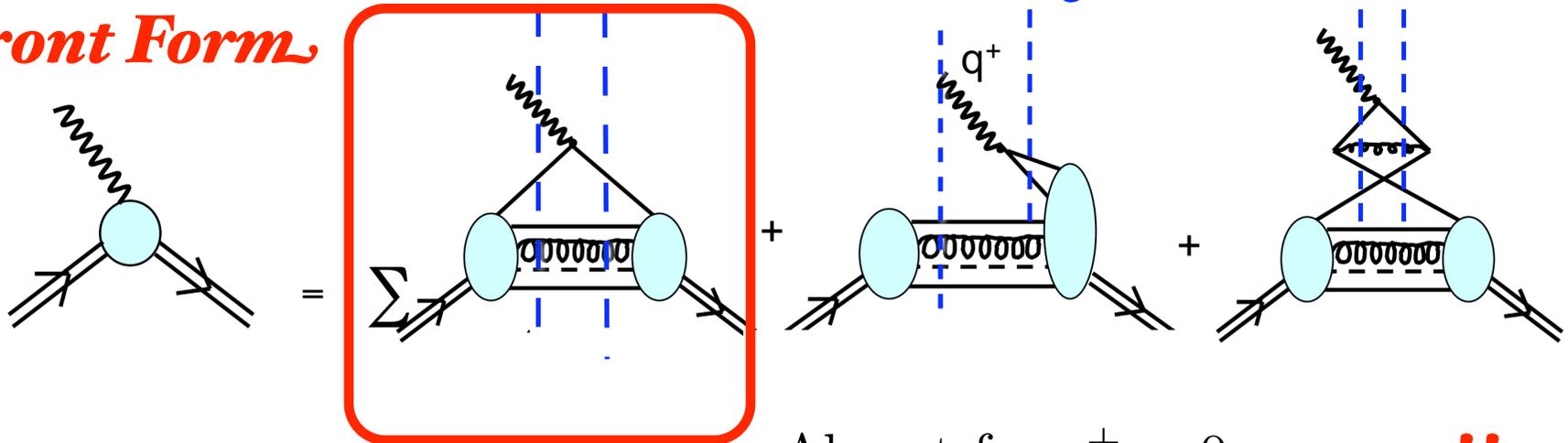
Instant Form



Need vacuum-induced currents

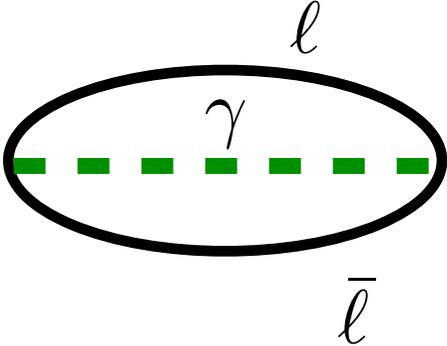
Calculation of Form Factors in Light-Front Theory

Front Form



Absent for $q^+ = 0$ **zero !!**

*Vacuum bubbles vanish
in light-front formalism*


$$k_{\gamma}^{+} + k_{\ell}^{+} + k_{\bar{\ell}}^{+} = 0$$

All $k^{+} \geq 0$

$\sum k_i^{+}$ conserved at every vertex

Drell, sjb

$$\frac{F_2(q^2)}{2M} = \sum_a \int [dx][d^2\mathbf{k}_\perp] \sum_j e_j \frac{1}{2} \times$$

$$\left[-\frac{1}{q^L} \psi_a^{\uparrow*}(x_i, \mathbf{k}'_{\perp i}, \lambda_i) \psi_a^\downarrow(x_i, \mathbf{k}_{\perp i}, \lambda_i) + \frac{1}{q^R} \psi_a^{\downarrow*}(x_i, \mathbf{k}'_{\perp i}, \lambda_i) \psi_a^\uparrow(x_i, \mathbf{k}_{\perp i}, \lambda_i) \right]$$

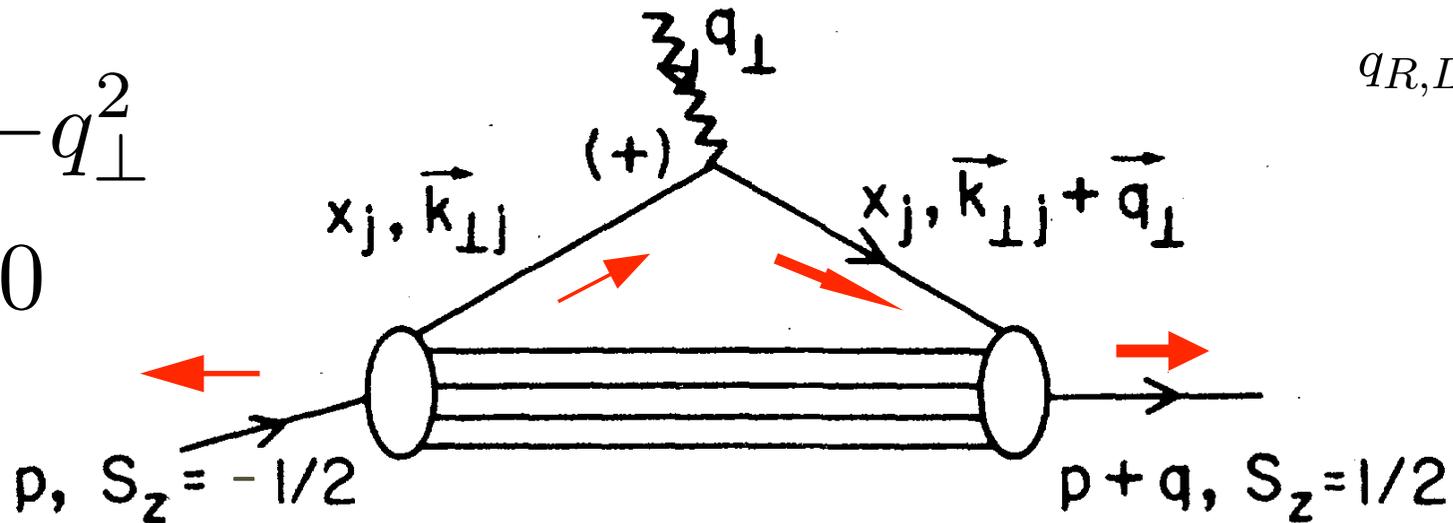
$$\mathbf{k}'_{\perp i} = \mathbf{k}_{\perp i} - x_i \mathbf{q}_\perp$$

$$\mathbf{k}'_{\perp j} = \mathbf{k}_{\perp j} + (1 - x_j) \mathbf{q}_\perp$$

$$q^2 = -q_\perp^2$$

$$q^+ = 0$$

$$q_{R,L} = q^x \pm iq^y$$



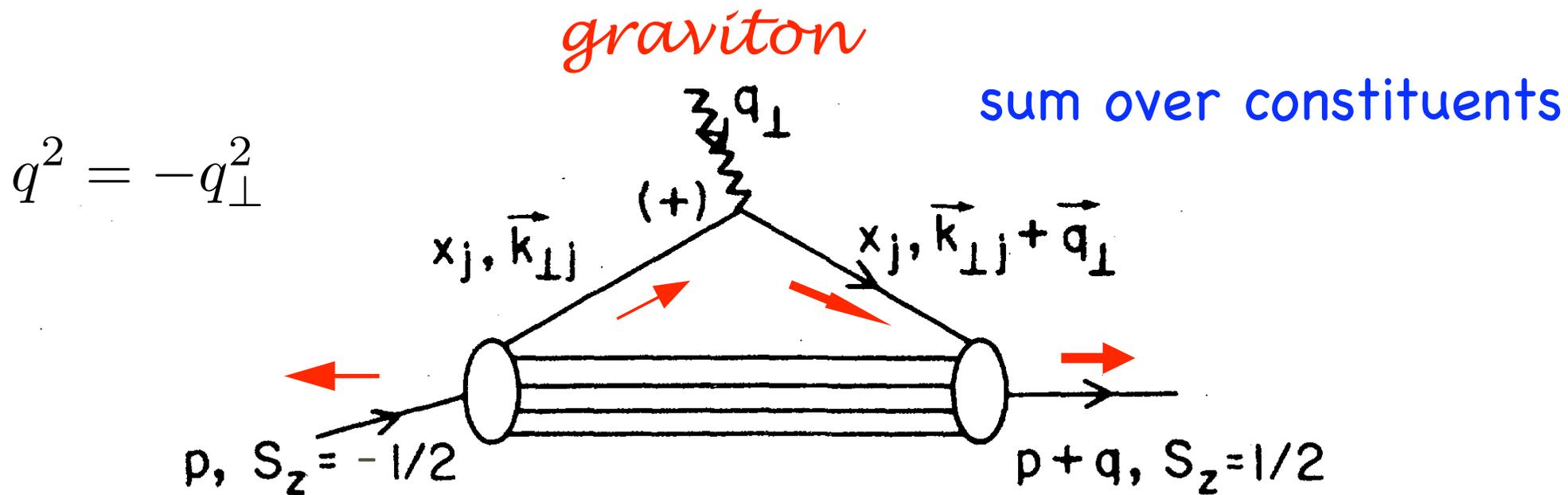
Must have $\Delta l_z = \pm 1$ to have nonzero $F_2(q^2)$

Checked to $\mathcal{O}\alpha^3$ in QED

Roskies, Suaya, sjb

Anomalous gravitomagnetic moment $B(0)$

Okun, Kobzarev, Teryaev: $B(0)$ Must vanish because of Equivalence Theorem



Hwang, Ma, Schmidt,
sjb;
Holstein et al

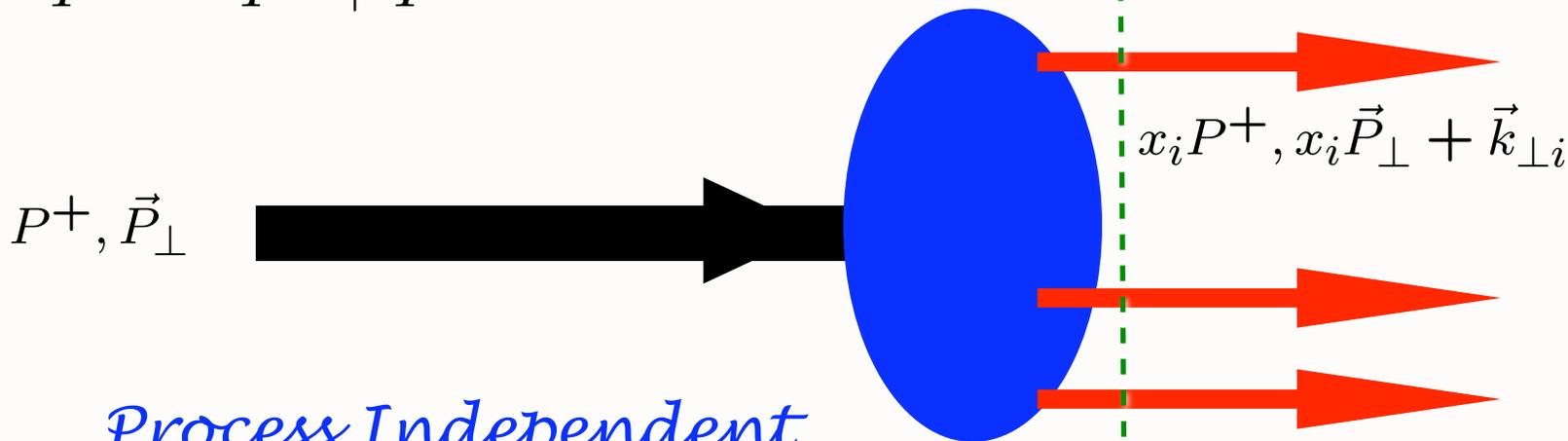
$$B(0) = 0$$

Each Fock State

Light-Front Wavefunctions: rigorous representation of composite systems in quantum field theory

$$x = \frac{k^+}{P^+} = \frac{k^0 + k^3}{P^0 + P^3}$$

Fixed $\tau = t + z/c$



*Process Independent
Direct Link to QCD Lagrangian!*

$$\Psi_n(x_i, \vec{k}_{\perp i}, \lambda_i)$$

$$\sum_i^n x_i = 1$$

$$\sum_i^n \vec{k}_{\perp i} = \vec{0}_{\perp}$$

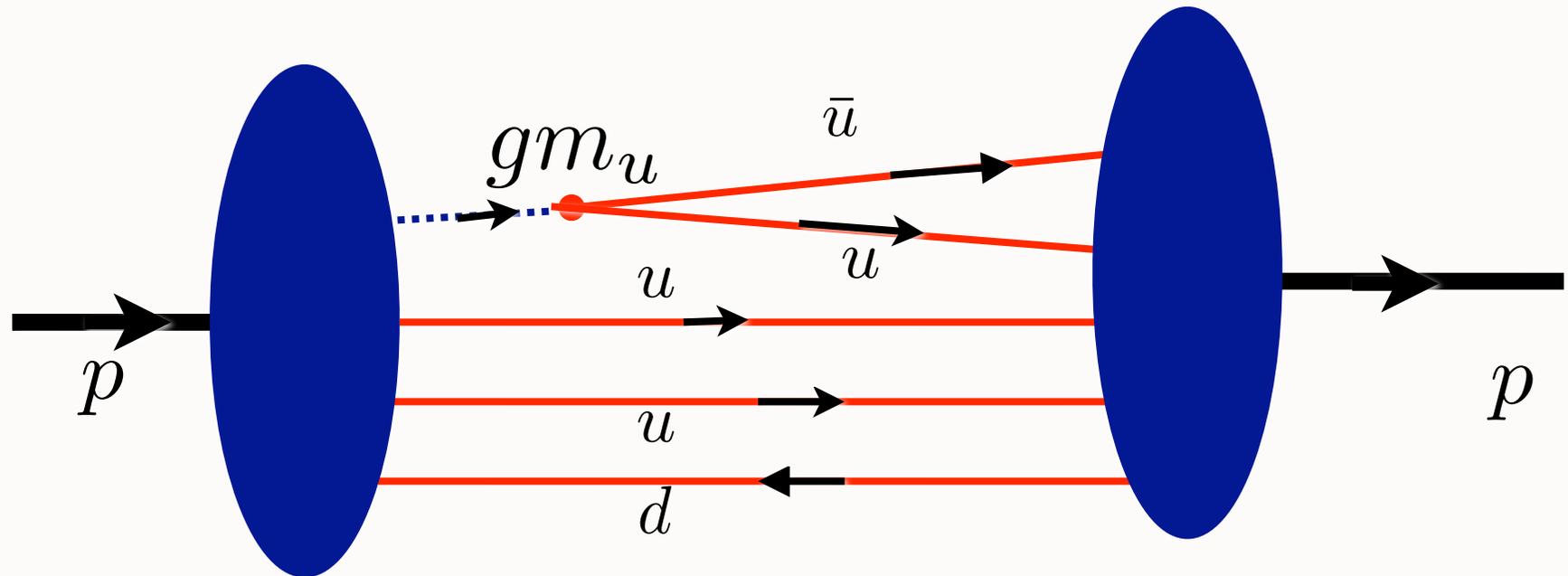
Invariant under boosts! Independent of p^μ

Mass shift in LF QCD

de Teramond, Shrock, sjb
(preliminary)

$$\delta M^2 = \langle p | H_I^{LF} | p \rangle \propto -m_q \langle p | A \bar{\psi} \psi | p \rangle$$

$$H_I^{LF} = g A \bar{\psi} \gamma^\mu \psi \sim -g m_q a_g b_q^\dagger d_q^\dagger$$



$$|p_I \rangle = |u^+ u^+ d^- g^+ \rangle_{L^z = -1} \quad |p_F \rangle = |u^+ u^+ d^- \bar{u}^+ u^+ \rangle_{L^z = -1}$$

*Contribution to the proton mass squared from overlap
of valence and higher Fock states*

The arrows and superscripts indicate LF chirality

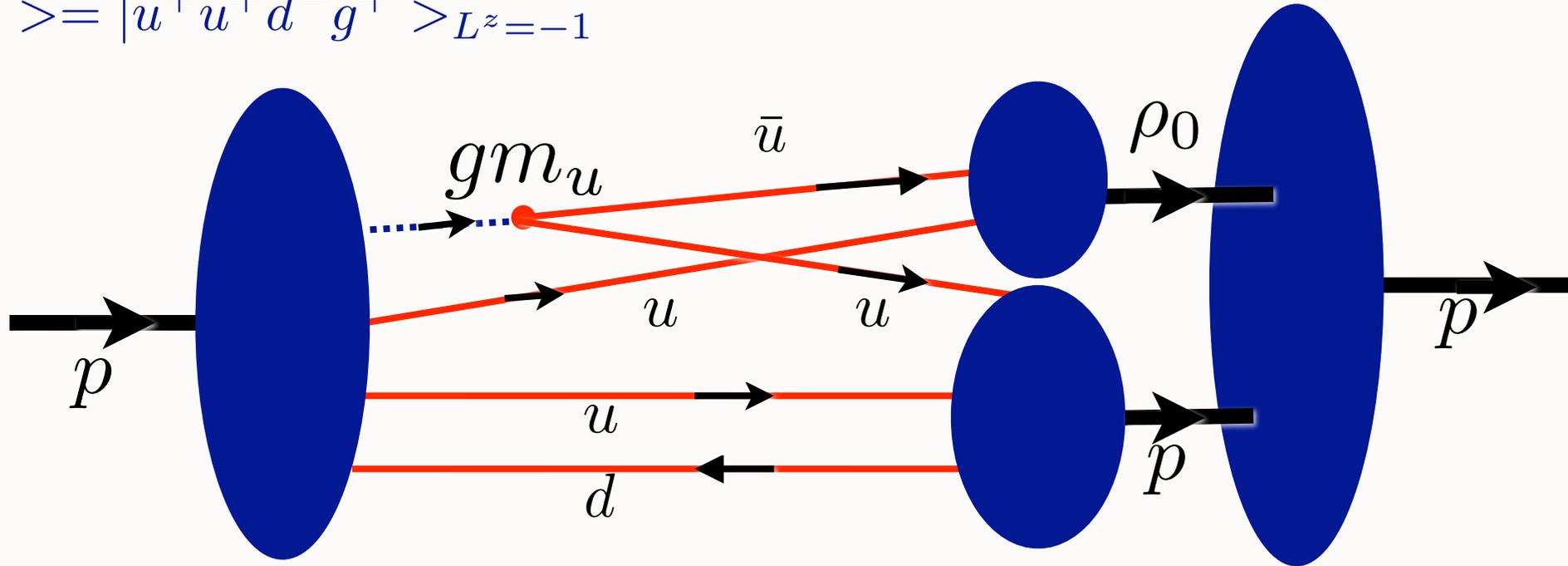
$$\Delta M_p^2 = g \langle p_F | A^\mu \bar{\psi} \gamma_\mu \psi | p_I \rangle$$

$$H_I^{LF} = g A \bar{\psi} \gamma^\mu \psi \sim -g m_q a_g b_q^\dagger d_q^\dagger$$

$$|p_F \rangle = |u^+ u^+ d^- \bar{u}^+ u^+ \rangle_{L_z = -1}$$

$$\simeq |(u^+ u^+ d^-)_p (u^+ \bar{u}^+)_{\rho_0} \rangle_{L_z = -1}$$

$$|p_I \rangle = |u^+ u^+ d^- g^+ \rangle_{L_z = -1}$$



The arrows and superscripts indicate helicity/chirality

Linear quark mass term generated by transition from valence to meson-nucleon LF Fock state

Dynamical chiral symmetry breaking

Chiral Symmetry Breaking in AdS/QCD

We consider the action of the X field which encodes the effects of CSB in AdS/QCD:

$$S_X = \int d^4x dz \sqrt{g} \left(g^{\ell m} \partial_\ell X \partial_m X - \mu_X^2 X^2 \right), \quad (1)$$

with equations of motion

$$z^3 \partial_z \left(\frac{1}{z^3} \partial_z X \right) - \partial_\rho \partial^\rho X - \left(\frac{\mu_X R}{z} \right)^2 X = 0. \quad (2)$$

The zero mode has no variation along Minkowski coordinates

$$\partial_\mu X(x, z) = 0,$$

thus the equation of motion reduces to

$$\left[z^2 \partial_z^2 - 3z \partial_z + 3 \right] X(z) = 0. \quad (3)$$

for $(\mu_X R)^2 = -3$, which corresponds to scaling dimension $\Delta_X = 3$. The solution is

$$X(z) = \langle X \rangle = Az + Bz^3, \quad (4)$$

where A and B are determined by the boundary conditions.

$$A \propto m_q \qquad B \propto \langle \bar{\psi} \psi \rangle$$

Expectation value taken inside hadron

Ehrlich, Katz, Son, Stephanov

**Babington, Erdmenger, Evans,
Kirsch, Guralnik, Thelfall**

**de Teramond, Shrock, sjb
(preliminary)**

In presence of quark masses the Holographic LF wave equation is ($\zeta = z$)

$$\left[-\frac{d^2}{d\zeta^2} + V(\zeta) + \frac{X^2(\zeta)}{\zeta^2} \right] \phi(\zeta) = \mathcal{M}^2 \phi(\zeta), \quad (1)$$

and thus

$$\boxed{\delta M^2 = \left\langle \frac{X^2}{\zeta^2} \right\rangle.} \quad (2)$$

The parameter a is determined by the Weisberger term

$$a = \frac{2}{\sqrt{x}}.$$

Thus

$$\boxed{X(z) = \frac{m}{\sqrt{x}} z - \sqrt{x} \langle \bar{\psi} \psi \rangle z^3,} \quad (3)$$

and

$$\delta M^2 = \sum_i \left\langle \frac{m_i^2}{x_i} \right\rangle - 2 \sum_i m_i \langle \bar{\psi} \psi \rangle \langle z^2 \rangle + \langle \bar{\psi} \psi \rangle^2 \langle z^4 \rangle, \quad (4)$$

where we have used the sum over fractional longitudinal momentum $\sum_i x_i = 1$.

Mass shift from dynamics inside hadronic boundary

Chiral Symmetry Breaking in AdS/QCD

- **Chiral symmetry breaking effect in AdS/QCD depends on weighted z^2 distribution, not constant condensate**

$$\delta M^2 = -2m_q \langle \bar{\psi}\psi \rangle \times \int dz \phi^2(z) z^2$$

- **z^2 weighting consistent with higher Fock states at periphery of hadron wavefunction**
- **AdS/QCD supports confined condensate picture**

de Teramond, Shrock, sjb

“One of the gravest puzzles of theoretical physics”

DARK ENERGY AND THE COSMOLOGICAL CONSTANT PARADOX

A. ZEE

*Department of Physics, University of California, Santa Barbara, CA 93106, USA
Kavil Institute for Theoretical Physics, University of California,
Santa Barbara, CA 93106, USA
zee@kitp.ucsb.edu*

$$(\Omega_{\Lambda})_{QCD} \sim 10^{45}$$

$$\Omega_{\Lambda} = 0.76(\text{expt})$$

$$(\Omega_{\Lambda})_{EW} \sim 10^{56}$$

Quark and Gluon condensates reside within hadrons, not vacuum

- **Bound-State Dyson-Schwinger Equations**
- **Domain becomes infinite at zero pion mass**
- **Finite size phase transition**
- **Analogous to finite-size superconductor!**
- **Phase change observed at RHIC within a single-nucleus-nucleus collisions-- quark gluon plasma!**
- **Implications for cosmological constant -- reduction by 45 orders of magnitude!**

M. Fisher

“Confined QCD Condensates”

Shrock, sjb

Hadron Dynamics at the Amplitude Level

- LFWFS are the universal hadronic amplitudes which underlie structure functions, GPDs, exclusive processes, distribution amplitudes, direct subprocesses, hadronization.
- Relation of spin, momentum, and other distributions to physics of the hadron itself.
- Connections between observables, orbital angular momentum
- Role of FSI and ISIs--Sivers effect
- Higher Fock States give GMOR Relations, Chiral Symmetry Breaking

New Perspectives on QCD Phenomena from AdS/CFT

- **AdS/CFT:** Duality between string theory in Anti-de Sitter Space and Conformal Field Theory
- New Way to Implement Conformal Symmetry
- Holographic Model: Conformal Symmetry at Short Distances, Confinement at large distances
- Remarkable predictions for hadronic spectra, wavefunctions, interactions
- AdS/CFT provides novel insights into the quark structure of hadrons

Features of Soft-Wall AdS/QCD

- Single-variable frame-independent radial Schrodinger equation
- Massless pion ($m_q = 0$)
- Regge Trajectories: universal slope in n and L
- Valid for all integer J & S . Spectrum is independent of S
- Dimensional Counting Rules for Hard Exclusive Processes
- Phenomenology: Space-like and Time-like Form Factors
- LF Holography: LFWFs; broad distribution amplitude
- No large N_c limit
- Add quark masses to LF kinetic energy
- Systematically improvable -- diagonalize H_{LF} on AdS basis

Consequences of Maximum Quark and Gluon Wavelength

- Infrared integrations regulated by confinement
- Infrared fixed point of QCD coupling

$$\alpha_s(Q^2) \text{ finite, } \beta \rightarrow 0 \text{ at small } Q^2$$

- Bound state quark and gluon Dyson-Schwinger Equation
- Quark and Gluon Condensates exist within hadrons

Casher, Susskind

Shrock, sjb

QCD Symmetries

- Color Confinement: Maximum Wavelength of Quark and Gluons
- Conformal symmetry of QCD coupling in IR
- Provides Conformal Template
- Motivation for AdS/QCD
- QCD Condensates inside of hadronic LFWFs
- Technicolor: confined condensates inside of technihadrons -- alternative to Higgs
- Simple physical solution to cosmological constant conflict