Nucleon-to-Delta Transition in Holographic QCD

Hovhannes R. Grigoryan (*ANL*)

in collaboration with

Harry Lee (*ANL*) & Ho-Ung Yee (*ICTP, Trieste*)

Joint Theory Institute

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Overview

- We study EM $N \to \Delta$ transition FF in holographic QCD
- N – topological soliton of the 5D gauge theory
- 5D g.t. describes a tower of mesons ($\rho,\ \rho',\ a_1,\ \ldots$)
- External EM field – nonnormalizable mode of 5D g.t.
- Interaction to baryons is completely VM dominated
- Using non-relativistic $N\Delta v^{(n)}$ interaction (Park & Yi '08), we propose its relativistic generalization that allows consistent treatment of FF
- We compare the results from hQCD with experimental data

General Introduction

- $\Delta(1232)$ resonance is the 1st excited state of N
- Δ's are produced in scattering π or e off a nucleon target
- Δ has $S = I = 3/2$ (Δ^{++}, Δ^{+}, Δ^{0}, Δ^{-}) with $\approx$ same mass and width
- It decays via $\Delta \to N\pi$ with 99% branching ratio and only less than 1% to the total decay width is coming from the EM channel ($\Delta \to N\gamma$)
- This $\Delta \to N\gamma$ transition is dominantly of the $M1$ type
- $\gamma^* N \to \Delta$ has 3 FF related to $M1$, $E2$ and $C2$ types of transitions
- $\gamma N \Delta$ was measured in π photoproduction near Δ-resonance region

Introduction

- $E2$ and $C2$ were found to be small at moderate Q^2, with $R_{EM} = E2/M1$ and $R_{SM} = C2/M1 \sim -2 \div 3\%$
- The smallness of these ratios seems to have nonperturbative origin
- pQCD studies (Carlson '85) predict $E2 \sim M1$ at large Q^2
- Experimentally $E2/M1 < 0$ and ≈ 0 for $Q^2 \leq 4$ GeV^2 (Beck '97)
- In quark model $N \to \Delta$ is described by spin flip of quark in s-wave state, leading to $M1$ transition

Introduction

- d-wave admixture in w.f. of Δ will produce $E2$ and $C2$ $\Rightarrow$
- Measuring these, one can observe d-wave components and quantify how N or Δ w.f. deviate from spherical shape
- In large N_c QCD $E2/M1 \sim \mathcal{O}(1/N_c^2)$ (Jenkins et al. '02)
- Lattice calculations (Alexandrou et al. '04) of $N\Delta$ transition FF up to 1.5 GeV^2 give small negative values for the ratio $E2/M1$
- This suggests that smallness of $E2/M1$ has non-perturbative origin

Model of Sakai and Sugimoto (PTP113&114, 2005)

- This model is constructed by placing N_f $D8$- and $\overline{D8}$- branes in S^1 compactified N_c $D4$-brane background of Type IIA s.t.
- SUSY on $D4$ is broken by anti-periodic BC on fermions along S^1 $\Rightarrow$
- At low energies we are left with the 4D pure $SU(N_c)$ g.t.
- When $N_f/N_c \ll 1$, we ignore backreaction of $D8$- and $\overline{D8}$-branes
- $q_{L,R}$ appear from $D8 - D4$ and $\overline{D8} - D4$ strings
- SS model aims to reproduce large N_c QCD with N_f massless quarks.

Model of Sakai and Sugimoto

- χ symmetry of QCD $\Leftarrow$ $U(N_f)_L \times U(N_f)_R$ g.t. living on $D8$ and $\overline{D8}$
- ChSB to $U(N_f)_V$ $\Leftarrow$ "merging" of $D8$ and $\overline{D8}$ in background of $D4$
- Resulting configuration is a U-shaped stack of N_f $D8$-branes
- NG arise from Wilson line connecting asmpt. separated regions on $D8$
- Other modes on $D8$ correspond to tower of V and $A - V$ mesons
- Resulting 5D theory on $D8$-branes $\rightarrow$ $U(N_f)$ YM theory + CS term
- In addition to x^μ, there is a "holographic dimension" $Z \in (-\infty, \infty)$

Model of Sakai and Sugimoto

$$S_{D8} = \frac{\pi}{4}\left(\frac{f_\pi}{M_{KK}}\right)^2 \int d^4x\, dZ\ \mathrm{Tr}\left(-\frac{1}{2}\left(1+Z^2\right)^{-\frac{1}{3}} F_{\mu\nu}F^{\mu\nu} + M_{KK}^2\left(1+Z^2\right) F_{\mu Z}F^{\mu}{}_{Z}\right)$$

$$S_{CS} = \frac{N_c}{24\pi^2}\int_{R^4\times Z} \mathrm{Tr}\left(AF^2 + \frac{i}{2}A^3F - \frac{1}{10}A^5\right)$$

The 5D gauge field A_M can be mode-expanded in the $A_Z = 0$ gauge as

$$A_\mu(x,Z) = -\frac{2}{\pi f_\pi}\ \partial_\mu\pi(x)\ \arctan(Z) + \sum_{n\geq 1} B_\mu^{(n)}(x)\psi_n(Z) + \cdots$$

Here $B_\mu^{(2n-1)} \equiv v_\mu^{(n)}$ describe vector mesons, while $B_\mu^{(2n)} \equiv a_\mu^{(n)}$ – axial-vector mesons

$$(1+Z^2)^{1/3}\partial_Z\left((1+Z^2)\partial_Z\psi_n\right) = -\frac{m_n^2}{M_{KK}^2}\psi_n\ , \quad \psi_n(Z\to\pm\infty) = 0$$

$$\frac{\pi}{4}\left(\frac{f_\pi}{M_{KK}}\right)^2 \int_{-\infty}^{+\infty} dZ\ (1+Z^2)^{-1/3}\psi_n\psi_m = \delta_{mn}$$

Holographic Baryons

- 5D YM theory has topological solitons along 4D $(\vec{x}, Z)$ s.t.

$$B = \frac{1}{8\pi^2} \int_{R^3 \times Z} \mathrm{Tr}\,(F \wedge F)$$

- B reduces to Skyrmion number upon 4D interpretation $\Rightarrow B$ – baryon #
- To minimize energy baryon localizes at $Z = 0$ and tends to shrink
- From $S_{D8} \Rightarrow$ the soliton mass M_N with $B = 1$ is

$$M_B^0 = 4\pi^2 \, \frac{\pi}{2} \left(\frac{f_\pi}{M_{KK}} \right)^2 M_{KK}$$

- If soliton size is $\rho \Rightarrow \delta M_N \sim \rho^2 \Rightarrow$ shrinking
- 5D CS term gives $1/\rho^2$ contribution to M_N preventing shrinking
- As a result the baryon acquires a stable and finite size

Holographic Baryons

- Soliton solutions have moduli space along which one should quantize
- Moduli space: $SO(4)$ spatial rotations interlocked with the iso-spin $SU(2)_I$ + translational moduli.
- Size is not a moduli, but one can quantize along it (Hata et al. '07)
- Remember, YM instanton has purely non-Abelian nature with profile

$$A^a_m \sim -\rho^2 \bar{\eta}^a_{mn} \partial_n \frac{1}{r^2} \ , \quad r \to \infty$$

- This gives information about coupling of baryons with $A_M(x, Z = 0)$
- The logic is to replace soliton with a point like field that has a coupling to mesons with the right strength to source the asymptotic meson tail of the original soliton solution

Non-relativistic Treatment (Park & Yi '08)

- A convenient *non-relativistic* notation for spin, iso-spin $1/2$ states is

$$\mathcal{U}^{\epsilon}_{\alpha}$$

  $\alpha = \{+, -\}$ is spinor index and $\epsilon = \{1, 2\}$ is iso-spin index

- Similarly for spin, iso-spin $3/2$ states

$$\mathcal{U}^{\epsilon_1\epsilon_2\epsilon_3}_{\alpha_1\alpha_2\alpha_3} \quad ,$$

  where α_i's and ϵ_i's must be totally symmetric to be in the $S = I = 3/2$ reps. under spatial rotation and iso-spin

- These *non-relativistic* fields are presumed to be localized at $Z = 0$
- Quantum spread along $Z \neq 0$ is a $\mathcal{O}(1/N_c)$ effect

Non-relativistic Treatment

- The relevant *non-relativistic* $N-\Delta$ coupling A_M is

$$S^{5D}_{N\Delta} = -k_{\frac{1}{2}} \left(\frac{1}{2} F^a_{ij} \epsilon_{ijk} \left(\mathcal{U}^{\epsilon}_{\alpha}\right)^* (\sigma_2\sigma_k)^{\beta\beta'} (\tau_2\tau_a)^{ee'} \mathcal{U}^{ee'\epsilon}_{\beta\beta'\alpha} + F^a_{Zi} \left(\mathcal{U}^{\epsilon}_{\alpha}\right)^* (\sigma_2\sigma_i)^{\beta\beta'} (\tau_2\tau_a)^{ee'} \mathcal{U}^{ee'\epsilon}_{\beta\beta'\alpha} \right) \Big|_{Z=0}$$

where $k_{\frac{1}{2}} = \frac{\sqrt{15}}{4} M^{-1}_{KK}$

- Relevant to EM FF, we only need vector mesons $B^{(2n-1)}_{\mu} = v^{(n)}_{\mu}$
- Since $\psi_{(2n-1)}(Z)$ is even under $Z \to -Z \Rightarrow$

$$S_{N\Delta v} = -\frac{1}{2} k_{\frac{1}{2}} \sum_{n\geq 1} \psi_{(2n-1)}(0) \left(\partial_i v^{(n)}_j - \partial_j v^{(n)}_i\right)^a \epsilon_{ijk} \left(\mathcal{U}^{\epsilon}_{\alpha}\right)^* (\sigma_2\sigma_k)^{\beta\beta'} (\tau_2\tau_a)^{ee'} \mathcal{U}^{ee'\epsilon}_{\beta\beta'\alpha}$$

- This *non-relativistic* result will be the starting point of our analysis

Vector Dominance in Holographic QCD

- SS model can also include external vector potentials as its nonnormalizable modes near $Z \to \pm\infty$ boundaries

$$A_\mu(x,Z) = \left(-\frac{1}{f_\pi}\partial_\mu \pi(x) + \mathcal{A}_\mu(x)\right)\psi_0(Z) + \mathcal{V}_\mu(x) + \sum_{n\geq 1} B_\mu^{(n)}(x)\psi_{(n)}(Z)$$

- In terms of EM vector potential A_μ^{EM}

$$\mathcal{V}_\mu = e \cdot \begin{pmatrix} \frac{2}{3} & 0 \\ 0 & -\frac{1}{3} \end{pmatrix} A_\mu^{EM} \quad , \quad \mathcal{A}_\mu = 0$$

- As a result the effective 4D theory will be

$$S_{4D} = \int d^4x \left[\mathrm{Tr}\left(-\mathcal{F}^{\mathcal{V}}_{\mu\nu} \sum_{n\geq 1} a_{\mathcal{V}v^n} F^{(n)\mu\nu}\right) + \sum_{n\geq 1} \mathrm{Tr}\left(-\frac{1}{2}F^{(n)}_{\mu\nu}F^{(n)\mu\nu} + m^2_{v^n} v^{(n)}_\mu v^{(n)\mu}\right)\right]$$

$$a_{\mathcal{V}v^n} = \frac{\pi}{4}\left(\frac{f_\pi}{M_{KK}}\right)^2 \int_{-\infty}^{+\infty} dZ \left(1+Z^2\right)^{-\frac{1}{3}} \psi_{(2n-1)}(Z)$$

with $\mathcal{F}^{\mathcal{V}}_{\mu\nu} = \partial_\mu \mathcal{V}_\nu - \partial_\nu \mathcal{V}_\mu$ and $F^{(n)}_{\mu\nu} = \partial_\mu v^{(n)}_\nu - \partial_\nu v^{(n)}_\mu$

Vector Dominance in Holographic QCD

It is more convenient to work in basis where $v_\mu^{(n)} = \tilde{v}_\mu^{(n)} - a_{\mathcal{V}_{v^n}} \mathcal{V}_\mu$

$$S_{4D} = \int d^4x \sum_{n\geq 1} \mathrm{Tr}\left(-\frac{1}{2}\tilde{F}_{\mu\nu}^{(n)}\tilde{F}^{(n)\mu\nu} + m_{v^n}^2 \tilde{v}_\mu^{(n)}\tilde{v}^{(n)\mu} - 2m_{v^n}^2 a_{\mathcal{V}_{v^n}} \tilde{v}_\mu^{(n)} \mathcal{V}^\mu \right)$$

In this basis $A_\mu(x,Z) = \sum_{n\geq 1} \tilde{v}_\mu^{(n)}(x)\psi_{(2n-1)}(Z)$

Any interaction of the system with the external vector potential is completely mediated by tree-level exchanges of the massive vector mesons $\tilde{v}_\mu^{(n)}$

Finally, in this basis:

$$S_{N\Delta\tilde{v}} = -\frac{1}{2}k_{\frac{1}{2}} \sum_{n\geq 1} \psi_{(2n-1)}(0) \left(\partial_i \tilde{v}_j^{(n)} - \partial_j \tilde{v}_i^{(n)}\right)^a \epsilon_{ijk} \left(\mathcal{U}_\alpha^\epsilon\right)^* (\sigma_2\sigma_k)^{\beta\beta'} (\tau_2\tau_a)^{ee'} \mathcal{U}_{\beta\beta'\alpha}^{ee'\epsilon}$$

Other Approaches

- Other approaches treat *non-relativistic* baryon w.f. along Z differently
- Above w.f. was taken to be $\delta(Z)$-function leading to $\psi_{(2n-1)}(0)$ factor
- Other works also included some of the $\mathcal{O}(1/N_c)$ effects
- This modification of w.f. affects only $\psi_{(2n-1)}(0)$
- E.g. in (Hata et al. '07; Hashimoto et al. '08) motion along Z was approximated by harmonic oscillator
- For our purposes, it is sufficient to take the ground state w.f.

$$\Psi_B^{n_Z=0}(Z) = \left(\frac{2a}{\pi}\right)^{\frac{1}{4}} e^{-aZ^2}\,, \quad a = \frac{2\pi^3}{\sqrt{6}}\left(\frac{f_\pi}{M_{KK}}\right)^2 \approx 0.240$$

$$\psi_{(2n-1)}(0) \;\rightarrow\; \langle\psi_{(2n-1)}(Z)\rangle = \frac{\int_{-\infty}^{+\infty} dZ\; \psi_{(2n-1)}(Z)\, e^{-2aZ^2}}{\int_{-\infty}^{+\infty} dZ\; e^{-2aZ^2}}$$

- In summary,

$$\begin{aligned} \text{Type I} \quad &: \qquad \psi_{(2n-1)}(0) \\ \text{Type II} \quad &: \qquad \psi_{(2n-1)}(0) \to \langle \psi_{(2n-1)}(Z) \rangle \end{aligned}$$

- As a final comment, the three types of approaches we consider share one common feature that seems to be universal.

$$\sum_{n \geq 1} a_{\mathcal{V}_{v^n}} \psi_{(2n-1)}(Z) = 1$$

for *any* Z, one can easily see that the *sum rule*

$$\sum_{n \geq 1} a_{\mathcal{V}_{v^n}} \langle \psi_{(2n-1)}(Z) \rangle = 1$$

holds whatever approximation we use for $\langle \psi_{(2n-1)}(Z) \rangle$

- As a result, we get universal (robust) results for transition FF at $Q^2 = 0$

Relativistic Generalization

- A difficulty for relativistic formulation was the absence of a relativistic 5D formalism of high spin fermions
- Without going into details, the resulting 4D theory, after integrating over Z, must be reduced to a 4D relativistic theory for a spin-$3/2$ field
- The *non-relativistic* limit of this 4D theory should reproduce *non-relativistic* result
- We expect an ambiguity of sub-leading terms of order $\mathcal{O}(1/M_B) \sim \mathcal{O}(1/N_c)$

Rarita-Schwinger Field

- The $\Delta(1232)$ is a spin-$3/2$ resonance and can be described in terms of a Rarita-Schwinger field: $\Psi_\mu^{(\sigma)}$, where μ is the vector and σ the spinor index
- The free and massive RS field obeys

$$(i\partial\!\!\!/ - M)\Psi_\mu = 0\,, \quad \partial^\mu \Psi_\mu = 0\,, \quad \gamma^\mu \Psi_\mu = 0$$

- Constraints ensure that # of independent components of vector-spinor field is reduced to physical # of spin d.o.f.
- In interacting theory, coupling of RS must be compatible with free theory construction, to preserve the physical spin d.o.f., otherwise one ends up with un-physical d.o.f. with negative-norm states or super-luminal modes
- Since we are interested in tree-level Feynman diagrams, we will not discuss issues related to quantization of RS field

Basics of Rarita-Schwinger Spin $\frac{3}{2}$ Field

- Consider specific momentum state: $\partial_\mu = -ip_\mu$, which has following components $p_0 = E = M$ and $p_i = 0$ $(i = 1, 2, 3)$
- Solving EOM with constraints, one finds $\Psi_0 = 0$ and

$$\Psi_i = \begin{pmatrix} \mathcal{U}_i \\ 0 \end{pmatrix} \quad ,$$

with three two-component spinors $\mathcal{U}_i$ $(i = 1, 2, 3)$ satisfying

$$\sum_{i=1}^{3} \sigma_i \mathcal{U}_i = 0 \quad .$$

- $\mathcal{U}_i$ $(i = 1, 2, 3)$ with this constraint is a way of describing usual non-relativistic $s = \frac{3}{2}$ states of $SO(3)$ rotation group
- First note that the independent number of components is indeed $2 \times 3 - 2 = 4$ as in the case of spin $s = 3/2$ representation

Relativistic $N\Delta\tilde{v}^{(n)}$ Vertex

- While we have to generalize the space rotation indices (lower indices) into a relativistic RS field, we should still keep the iso-spin indices (upper indices)
- If we take non-relativistic limit as is done in the previous section, the spinors reduce to non-relativistic two-component spinors as

$$N = \begin{pmatrix} \mathcal{U} \\ 0 \end{pmatrix} \quad , \quad \Psi_i = \begin{pmatrix} \mathcal{U}_i \\ 0 \end{pmatrix} \quad , \quad \Psi_0 = 0 \quad ,$$

One can check that the only relativistic operator that reproduce the non-relativistic result is:

$$S^{\text{rel}}_{N\Delta\tilde{v}} = \frac{\sqrt{30}}{4} M_{KK}^{-1} \sum_{n\geq 1} \psi_{(2n-1)}(0) F^{(n)a}_{\mu\nu} \, (\tau_2\tau_a)^{ee'} \, \bar{N}^{\epsilon} \, \gamma^{\mu}\gamma^{5} \, (\Psi^{\nu})^{ee'\epsilon} + \text{H.c.}$$

Here $F^{(n)a}_{\mu\nu} \equiv \left(\partial_\mu \tilde{v}^{(n)}_\nu - \partial_\nu \tilde{v}^{(n)}_\mu\right)^a$ and the upper indices a, e, e' and ϵ represent iso-spin indices

Relativistic $N\Delta\tilde{v}^{(n)}$ Vertex

The final result in terms of the conventional notation of Δ-baryons ($\Delta^{++}, \Delta^{+}, \Delta^{0}, \Delta^{-}$) and the nucleons (p, n)

$$
\begin{aligned}
S^{\text{rel}}_{N\Delta\tilde{v}} &= i\frac{\sqrt{30}}{4}M_{KK}^{-1}\sum_{n\geq 1}\psi_{(2n-1)}(0)F^{(n)+}_{\mu\nu}\left(\sqrt{\frac{2}{3}}\,\bar{p}\,\gamma^{\mu}\gamma^{5}\left(\Delta^{0}\right)^{\nu}+\sqrt{2}\,\bar{n}\,\gamma^{\mu}\gamma^{5}\left(\Delta^{-}\right)^{\nu}\right)\\
&- i\frac{\sqrt{30}}{4}M_{KK}^{-1}\sum_{n\geq 1}\psi_{(2n-1)}(0)F^{(n)-}_{\mu\nu}\left(\sqrt{2}\,\bar{p}\,\gamma^{\mu}\gamma^{5}\left(\Delta^{++}\right)^{\nu}+\sqrt{\frac{2}{3}}\,\bar{n}\,\gamma^{\mu}\gamma^{5}\left(\Delta^{+}\right)^{\nu}\right)\\
&+ i\frac{\sqrt{30}}{4}M_{KK}^{-1}\sum_{n\geq 1}\psi_{(2n-1)}(0)F^{(n)0}_{\mu\nu}\left(\sqrt{\frac{4}{3}}\,\bar{p}\,\gamma^{\mu}\gamma^{5}\left(\Delta^{+}\right)^{\nu}+\sqrt{\frac{4}{3}}\,\bar{n}\,\gamma^{\mu}\gamma^{5}\left(\Delta^{0}\right)^{\nu}\right)+\text{H.c.}
\end{aligned}
$$

where

$$
F^{(n)\pm}_{\mu\nu}=\frac{1}{\sqrt{2}}\left(F^{(n)1}_{\mu\nu}\mp iF^{(n)2}_{\mu\nu}\right)\quad,\quad F^{(n)0}_{\mu\nu}=F^{(n)3}_{\mu\nu}
$$

are the n'th vector meson fields in the EM charge basis, and Δ_{μ} are the Rarita-Schwinger fields for the Δ-baryons

$N\Delta\gamma$ Transition Form Factors: Definition

$$\langle \Delta(p') \mid J_\mu^{\mathrm{EM}}(0) \mid N(p) \rangle = e\, \overline{\Psi}_\beta(p') \Gamma_{\beta\mu} \gamma_5 N(p)$$

$$\Gamma_{\beta\mu} = G_1(Q^2)\left[q_\beta \gamma_\mu - \not{q} g_{\beta\mu}\right] + G_2(Q^2)\left[q_\beta P_\mu - (qP)\, g_{\beta\mu}\right] + G_3(Q^2)\left[q_\beta q_\mu - q^2 g_{\beta\mu}\right]$$

Magnetic dipole G_M, electric quadrupole G_E, Coulomb quadrupole G_C FF

$$G_M(Q^2) = \frac{m_N}{3(m_N + m_\Delta)}\left[((3m_\Delta + m_N)(m_\Delta + m_N) + Q^2)\frac{G_1(Q^2)}{m_\Delta} + (m_\Delta^2 - m_N^2)G_2(Q^2) - 2Q^2 G_3(Q^2)\right],$$

$$G_E(Q^2) = \frac{m_N}{3(m_N + m_\Delta)}\left[(m_\Delta^2 - m_N^2 - Q^2)\frac{G_1(Q^2)}{m_\Delta} + (m_\Delta^2 - m_N^2)G_2(Q^2) - 2Q^2 G_3(Q^2)\right],$$

$$G_C(Q^2) = \frac{2m_N}{3(m_\Delta + m_N)}\left[2m_\Delta G_1(Q^2) + \frac{1}{2}(3m_\Delta^2 + m_N^2 + Q^2)G_2(Q^2) + (m_\Delta^2 - m_N^2 - Q^2)G_3(Q^2)\right]$$

Also define ratios R_{EM} and R_{SM}, often used in the experimental papers:

$$R_{EM}(Q^2) = \frac{E2(Q^2)}{M1(Q^2)} = -\frac{G_E(Q^2)}{G_M(Q^2)} \quad ,$$

$$R_{SM}(Q^2) = \frac{C2(Q^2)}{M1(Q^2)} = -\sqrt{Q^2 + \frac{(m_\Delta^2 - m_P^2 - Q^2)^2}{4m_\Delta^2}}\,\frac{1}{2m_\Delta}\,\frac{G_C(Q^2)}{G_M(Q^2)}$$

Predictions from Holographic QCD

Adding Feynman diagrams corresponding to $p \to \Delta^+$ transition:

$$G_1(Q^2) = \sum_{n\geq 1} \frac{g_{\nu^n} g_{N\Delta\nu^n}}{Q^2 + m_{\nu^n}^2} \ , \quad G_2(Q^2) = G_3(Q^2) = 0$$

where $g_{\nu^n} \equiv m_{\nu^n}^2 a_{\mathcal{V}\nu^n}$ and $g_{N\Delta\nu^n} = \sqrt{2}\ \frac{\sqrt{30}}{4M_{KK}} \langle \psi_{(2n-1)}(Z) \rangle$

n	1	2	3	4	5	6	7	8
$m_{\nu^n}^2\,(\mathrm{GeV}^2)$	0.602	2.59	5.94	10.6	16.7	24.0	32.7	42.8
$g_{\nu^n}\,(\mathrm{GeV}^2)$	0.164	-0.707	1.615	-2.884	4.508	-6.484	8.869	-11.58
$g_{N\Delta\nu^n}\,(\mathrm{GeV}^{-1})$	14.12	12.88	12.60	12.51	12.46	12.44	12.43	12.42

$$G_M(Q^2) = \frac{m_N((3m_\Delta + m_N)(m_\Delta + m_N) + Q^2)}{3m_\Delta(m_N + m_\Delta)}\, G_1(Q^2) \ ,$$

$$G_E(Q^2) = \frac{m_N(m_\Delta^2 - m_N^2 - Q^2)}{3m_\Delta(m_N + m_\Delta)}\, G_1(Q^2) \ ,$$

$$G_C(Q^2) = \frac{4m_N m_\Delta}{3(m_\Delta + m_N)}\, G_1(Q^2)$$

Predictions from Holographic QCD

When $Q^2 = 0$ we have

$$G_1(0) = \sqrt{2}\,\frac{\sqrt{30}}{4M_{KK}} \sum_{n\geq 1} a_{\mathcal{V}_{\nu^n}} \langle \psi_{(2n-1)}(Z)\rangle = \frac{\sqrt{15}}{2}\,\frac{1}{M_{KK}}\,,$$

This result is universal for Type I, II models, moreover,

$$G_M(0) = \frac{\sqrt{15}}{2}\,\frac{m_N}{M_{KK}}\,\frac{(3m_\Delta + m_N)}{3m_\Delta} \simeq 2.43\,\frac{m_N}{M_{KK}}\,,$$
$$G_E(0) = \frac{\sqrt{15}}{2}\,\frac{1}{M_{KK}}\,\frac{m_N(m_\Delta - m_N)}{3m_\Delta} \simeq 0.154\,\frac{m_N}{M_{KK}}\,,$$
$$G_C(0) = \frac{\sqrt{15}}{2}\,\frac{1}{M_{KK}}\,\frac{4m_N m_\Delta}{3(m_\Delta + m_N)} \simeq 1.47\,\frac{m_N}{M_{KK}}$$

LO in N_c we have $G_E(0) = 0$,

$$G_M(0) = 2G_C(0) = \frac{4}{3}\,\frac{\sqrt{15}}{2}\,\frac{m_N}{M_{KK}}$$

Numerically: $G_M(0) = 2G_C(0) \simeq 2.58$ and $R_{EM} = R_{SM} = 0$

Predictions from Holographic QCD

- In addition,

$$R_{EM}(0) = R_{SM}(0) = -\frac{m_\Delta - m_N}{3m_\Delta + m_N} \ .$$

- Since $m_{N,\Delta} \sim \mathcal{O}(N_c)$, while $m_\Delta - m_N \sim \mathcal{O}(1/N_c)$

$$R_{EM}(0) = R_{SM}(0) \sim -\mathcal{O}\left(\frac{1}{N_c^2}\right)$$

- R_{EM} was also shown to be $\mathcal{O}(1/N_c^2)$ by (Jenkins et al. '02)
- In the large-N_c limit $R_{EM}(0) = R_{SM}(0)$ was also observed by (Pascalutsa et al. '03)

Predictions from Holographic QCD

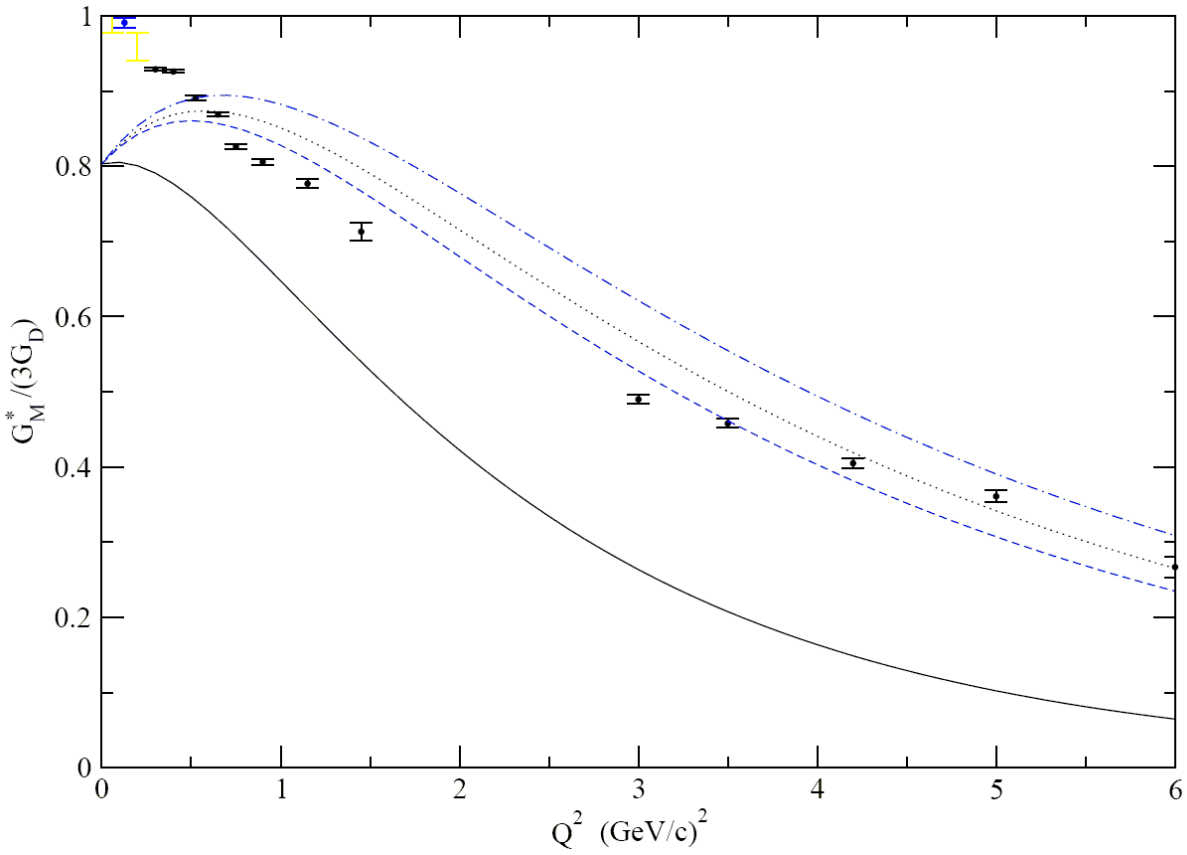

Figure: The plot of the ratio $G_M^*(Q^2)/(3G_D(Q^2))$ as a function of Q^2, where $G_D(Q^2) = 1/(1+Q^2/\Lambda^2)^2$ with $\Lambda^2 = 0.71$ (GeV/c)^2. The solid and dotted lines are the predictions from the holographic Type I and Type II models respectively. The dashed (dash-dotted) curves are from taking the parameter a to be 20% larger (smaller) than the value $a \sim 0.240$. Experimental data points are taken from CLAS Collaboration 2003-06.

Helicity Amplitudes

One can also parametrize $\gamma^* N\Delta$ through the rest frame helicity amplitudes

$$A_{3/2} \equiv -\frac{e}{\sqrt{2q_\Delta}} \frac{1}{(4M_N M_\Delta)^{1/2}} \langle \Delta(\vec{0}, +3/2)|\mathbf{J}\cdot\epsilon_{\lambda=+1}|N(-\vec{q}, +1/2)\rangle \ ,$$
$$A_{1/2} \equiv -\frac{e}{\sqrt{2q_\Delta}} \frac{1}{(4M_N M_\Delta)^{1/2}} \langle \Delta(\vec{0}, +1/2)|\mathbf{J}\cdot\epsilon_{\lambda=+1}|N(-\vec{q}, -1/2)\rangle$$

where $\epsilon_{\lambda=+1} = -1/\sqrt{2}(1, i, 0)$ and

$$q_\Delta \equiv |\mathbf{q}| = \frac{Q_+ Q_-}{2M_\Delta} \ , \quad Q_\pm \equiv \sqrt{(M_\Delta \pm M_N)^2 + Q^2}$$

Helicity amplitudes can be expressed in terms of Jones-Scadron $\gamma^* N\Delta$ FF as

$$A_{3/2} = -\mathcal{N}\frac{\sqrt{3}}{2}(G_M + G_E) = -\mathcal{N}\frac{\sqrt{3}}{2}G_M + \mathcal{O}\left(\frac{1}{N_c^2}\right) \ ,$$
$$A_{1/2} = -\mathcal{N}\frac{1}{2}(G_M - 3G_E) = -\mathcal{N}\frac{1}{2}G_M + \mathcal{O}\left(\frac{1}{N_c^2}\right)$$

$$\mathcal{N} \equiv \frac{e}{2}\left(\frac{Q_+ Q_-}{2M_N^3}\right)^{1/2} \frac{(M_N + M_\Delta)}{Q_+}$$

Helicity Amplitudes

- Helicity amplitudes are expressed in units $\mathrm{GeV}^{-1/2}$, and reduce at $Q^2 = 0$ to the photo-couplings quoted by PDG
- Experimentally, helicity amplitudes are extracted from the $M1$, $E2$, and $C2$ multipoles for the $\gamma^* N \to \pi N$ process at the resonance position, i.e. for πN c.m. energy $W = M_\Delta$.
- In terms of helicity amplitudes,

$$R_{EM} = \frac{A_{1/2} - \frac{1}{\sqrt{3}} A_{3/2}}{A_{1/2} + \sqrt{3} A_{3/2}}$$

- From the results above it follows:

$$\frac{A_{3/2}}{A_{1/2}} = \sqrt{3} + \mathcal{O}\left(\frac{1}{N_c^2}\right)$$

- This was also observed in (Jenkins et al. '02) within the framework of large N_c QCD

Predictions from Holographic QCD

- In the case $Q^2 = 0$, we have $Q_\pm = M_\Delta \pm M_N$, and therefore,

$$\mathcal{N} = \frac{e}{2}\left(\frac{M_\Delta^2 - M_N^2}{2M_N^3}\right)^{1/2} \simeq 0.094\ \mathrm{GeV}^{-1/2}\ .$$

- As a result, we will have from holographic QCD:

$$\begin{aligned} A_{1/2} &\simeq -121\ [10^{-3}\mathrm{GeV}^{-1/2}], \\ A_{3/2} &\simeq -209\ [10^{-3}\mathrm{GeV}^{-1/2}], \\ R_{EM} &\simeq 0\ \% \end{aligned}$$

- Experimental results (MAMI/A2 and LEGS) taken from PDG give:

$$\begin{aligned} A_{1/2} &= -\,(135 \pm 6) \quad [10^{-3}\mathrm{GeV}^{-1/2}], \\ A_{3/2} &= -\,(250 \pm 8) \quad [10^{-3}\mathrm{GeV}^{-1/2}], \\ R_{EM} &= -\,(2.5 \pm 0.5\ \%) \end{aligned}$$

Predictions from Holographic QCD

- From $\gamma^* N\Delta$ FF at $Q^2 = 0$, one can extract more static observables
- From dominant $M1$ transition, one can extract static transition magnetic moment $\mu_{N\to\Delta}$ from the value of $G_M(0)$ (Tiator et al. '03)

$$\mu_{N\to\Delta} = \sqrt{\frac{M_\Delta}{M_N}}\, G_M(0)$$

  expressed in nuclear magnetons $\mu_N \equiv e/(2M_N)$

- One can also extract quadrupole transition moment $Q_{N\to\Delta}$

$$Q_{N\to\Delta} = -6\sqrt{\frac{M_\Delta}{M_N}}\frac{1}{M_N q_\Delta(0)} G_E(0)$$

  where $q_\Delta(0) = (M_\Delta^2 - M_N^2)/2M_\Delta$.

Predictions from Holographic QCD

- Neglecting terms of order $\mathcal{O}(1/N_c)$ we get

$$G_M(0) \simeq 2.58 \ , \quad \mu_{N\to\Delta} \simeq 2.58\mu_N \ , \quad Q_{N\to\Delta} \simeq 0 \ \mathrm{fm}^2$$

- From experiment (Tiator et al. 2000) extracted values

$$\begin{aligned} G_M(0) &= 3.02 \pm 0.03, \\ \mu_{p\to\Delta^+} &= [3.46 \pm 0.03]\, \mu_N \ , \\ Q_{p\to\Delta^+} &= -\,(0.0846 \pm 0.0033)\ \mathrm{fm}^2 \end{aligned}$$

Summary

- Working within the framework of holographic dual model of QCD, proposed by Sakai-Sugimoto, we generalize the non-relativistic $N\Delta v^{(n)}$ vertex to relativistic one
- Using this generalization, we show how to calculate transition FF
- We find that these FF are expressed in terms of only one
- Various relation expected in the large N_c limit are verified
- At $\mathcal{O}(N_c)$, $G_{1,2,3}(Q^2)$ transition FF are expressed in terms of one
- $R_{EM} = R_{SM} \sim \mathcal{O}(1/N_c^2)$ as expected in the large N_c limit

Conclusion

- Taking into account that our results are of only leading order in large N_c, we find about 20% discrepancy with experiments as an indication that the holographic model works consistently
- It is an important open problem to systematically improve the large N_c expansion in the holographic QCD
- We suggest that this approach opens a new possibility for studying other hadronic transitions among the excited states of baryons

THE END