Centre vortices and the quark propagator

PATRICK BOWMAN

Centre for Theoretical Chemistry and Physics

and

Institute for Natural Sciences

Massey University, Albany

Introduction

- The strong force exhibits two striking phenomena:
  - Confinement
  - Dynamical Chiral Symmetry Breaking
- Can these phenomena be attributed to a single mechanism?
  - Deconfinement and chiral symmetry restoration at coincident temperatures
- Centre Vortex Model
  - Many nice features
  - SU(2): String tension and chiral condensate (de Forcrand and D'Elia: PRL 82, 4582 (1999) Höllwieser *et al.*: PRD 78, 054508 (2009)).
  - SU(3): Most of the string tension (Langfeld: PRD 69, 014503 (2004)).

Testing the Model

- Landau gauge Green's functions have been used to great effect to study the properties of QCD.
- The quark propagator displays DχSB
  - Is it sensitive to centre vortices?
- Calculation
  - SU(2) and SU(3) quenched gauge configurations.

  1. Identify and isolate centre vortices
  2. Rotate to Landau gauge
  3. Calculate propagator with, only with and without vortices
  4. Contrast and compare.

QCD

Generalisation of Quantum Electrodynamics where colour is the strong force's version of charge: Quantum Chromodynamics.

$$\mathcal{L} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \sum_f \overline{\psi}_f(i\gamma^\mu\partial_\mu - m_f + g\gamma^\mu A_\mu)\psi_f$$

$$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu + g[A_\mu, A_\nu]$$

Gauge invariant: $D_\mu(x) \longrightarrow G(x)D_\mu(x)G^\dagger(x) \qquad G(x) = e^{i\omega(x)\cdot T}$

c.f. $A_\mu(x) \longrightarrow A_\mu(x) - \partial_\mu\omega(x)$

- Gluons and Quarks.
- Vector potentials, A_μ are (3x3) matrices: Self interactions.
- g, quark masses only parameters.
- Where does the mass come from?

Euclidean Space

Generating functional:

$$Z[J,\eta,\overline{\eta}] = \int \mathcal{D}\overline{\psi}\mathcal{D}\psi\mathcal{D}A\, e^{i\int d^3\mathbf{x}dt\mathcal{L}(\mathbf{x},t) + \textit{sources}}$$

with source terms:

$$\text{“}\textit{sources}\text{”} \equiv J_\mu A^\mu + \overline{\eta}\psi + \eta\overline{\psi}$$

Rotation to imaginary time (Wick rotation): $t \to -it$.

Partition function:

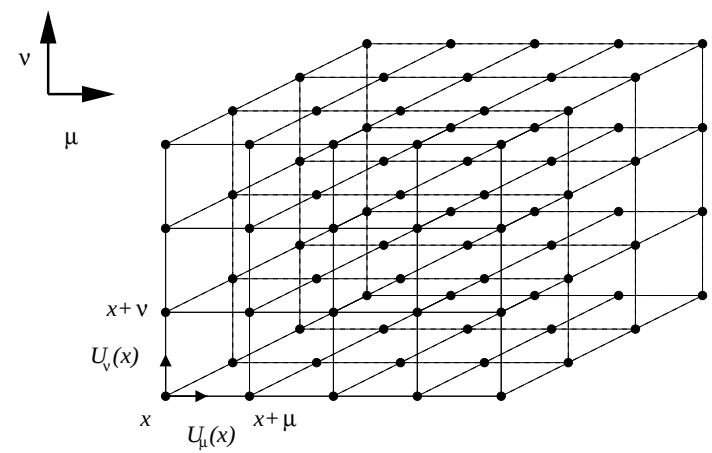
$$Z[J,\eta,\overline{\eta}] = \int \mathcal{D}\overline{\psi}\mathcal{D}\psi\mathcal{D}A\, e^{-\int d^4x\mathcal{L}(x) + \textit{sources}}$$

Discretisation

The gauge fields, A_μ, are replaced by parallel transport operators or *links*

$$U_\mu(x) = \mathcal{P}\exp\left\{iag\int_0^1 A_\mu(x+at\hat{\mu})dt\right\} \in \text{SU(3)}$$
$$\simeq e^{iagA_\mu(x)}$$

$U_\mu(x)$ connects lattice sites x and $x+a\hat{\mu}$. Taking the inverse of a link reverses its direction, and since they are unitary: $U_\mu^\dagger(x) = U_{-\mu}(x+a\hat{\mu})$.



Gauge transformation law

$$U_\mu^G(x) = G(x)U_\mu(x)G_\mu^\dagger(x+\hat{\mu})$$
$$G(x) \in \text{SU(3)}$$

The lattice gauge action is built of closed loops of links (which are gauge invariant) called *Wilson* loops, the simplest of which is the *plaquette*

$$P_{\mu\nu}(x) = U_\mu(x) U_\nu(x+\hat{\mu}) U_\mu^\dagger(x+\hat{\nu}) U_\nu^\dagger(x)$$

$$\begin{aligned} S_G^W[U] &= \frac{\beta}{N} \sum_{\text{plaquettes}} \mathrm{Tr}\Big\{1 - \frac{1}{2}\big(P_{\mu\nu} + P_{\mu\nu}^\dagger\big)\Big\} \\ &= S_G + \mathcal{O}(a^2). \end{aligned}$$

The coupling constant of the theory has been absorbed into the parameter,

$$\beta = \frac{2N}{g^2}.$$

Or Symanzik improved action:

$$S_G^S[U] = \frac{5\beta}{3N_c}\sum_{\text{pl}} \text{Tr}\Big\{1 - \frac{1}{2}\big(P_{\mu\nu} + P_{\mu\nu}^\dagger\big)\Big\} - \frac{\beta}{12N_c u_0^2}\sum_{\text{rect}} \text{Tr}\Big\{1 - \frac{1}{2}\big(R_{\mu\nu} + R_{\mu\nu}^\dagger\big)\Big\}$$

$$= S_G + \mathcal{O}(a^4) + \mathcal{O}(a^2 g^2),$$

Discretisation of quarks:

$$S_F[\overline{\psi}, \psi, U] = \sum_{xy} \overline{\psi}(x) M(x, y; U) \psi(y)$$

Partition function:

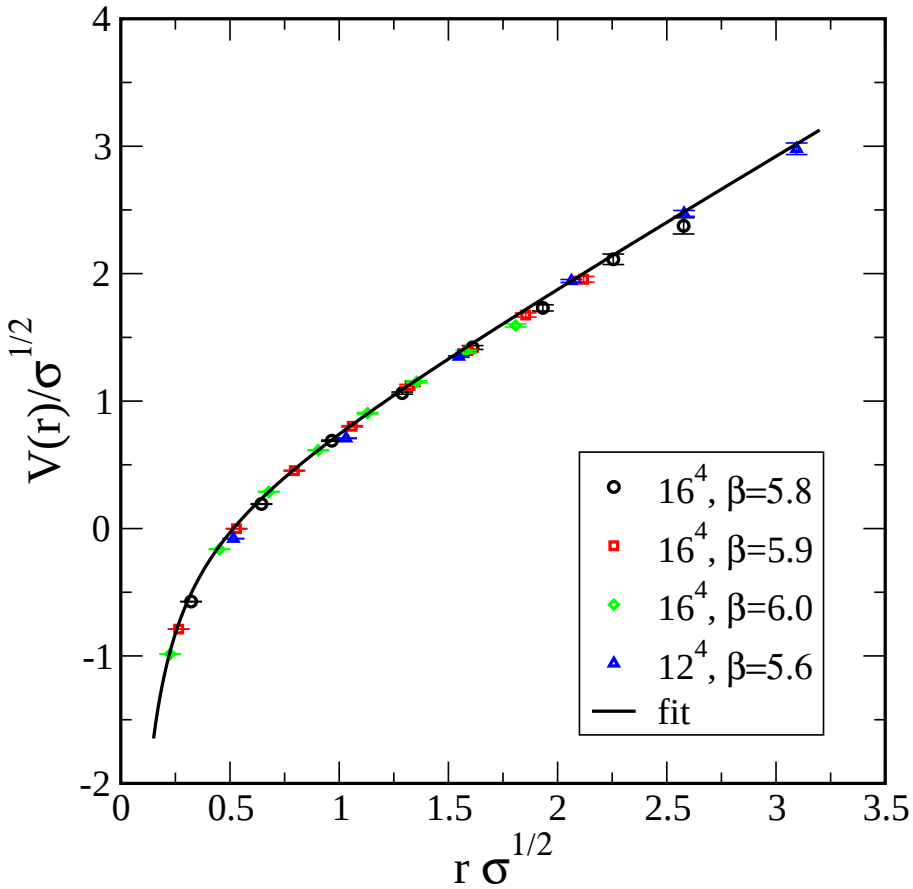
$$Z[0] = \int \mathcal{D}\overline{\psi}\mathcal{D}\psi\mathcal{D}U\, e^{-S_G[U] - S_F[\overline{\psi},\psi,U]}$$

$$= \int \mathcal{D}U \det(M[U])\, e^{-S_G[U]}$$

"Quenched approximation": $\det(M[U]) = 1$

Partition function well defined even without gauge fixing.

Static quark potential



K. Langfeld Phys.Rev.D69:014503,2004

Landau Gauge Fixing

Landau gauge fixing is performed by enforcing the Lorenz gauge condition, $\sum_\mu \partial_\mu A_\mu(x) = 0$ on a configuration by configuration basis. This can be achieved in the continuum through the minimisation of the L_2 norm

$$\mathcal{F}^G[A] = \int d^4x \mathsf{Tr} \sum_\mu \left(A_\mu^G(x)\right)^2.$$

On the lattice this may be formulated as the maximisation of a functional such as,

$$\mathcal{F}^G[U] = \frac{1}{2} \sum_{x,\mu} \mathsf{Tr}\left\{U_\mu^G(x) + U_\mu^{G\dagger}(x)\right\},$$

which is equivalent to the continuum condition up to $\mathcal{O}(a^2)$.

- Note that finding a maximum of the above functional also ensures that the Faddeev-Popov operator is positive: the configuration is inside the Gribov region.
- For any given configuration there are, in general, many local extrema of the gauge fixing functional; each one corresponds to a *Gribov copy*.

Gluon Propagator

Landau gauge: $\sum_\mu \partial_\mu A_\mu(x) = 0 \,\forall\, x$

$$\langle A_\mu(x) A_\nu(y) \rangle = \frac{1}{Z[0]} \int \mathcal{D}\overline{\psi}\mathcal{D}\psi\mathcal{D}U \; A_\mu[U](x) A_\nu[U](y) \, e^{-S_{\mathrm{QCD}}[\overline{\psi},\psi,U]}$$

In Landau gauge, in the continuum, the gluon propagator has the tensor structure

$$D_{\mu\nu}(q^2) = \left(\delta_{\mu\nu} - \frac{q_\mu q_\nu}{q^2}\right) D(q^2).$$

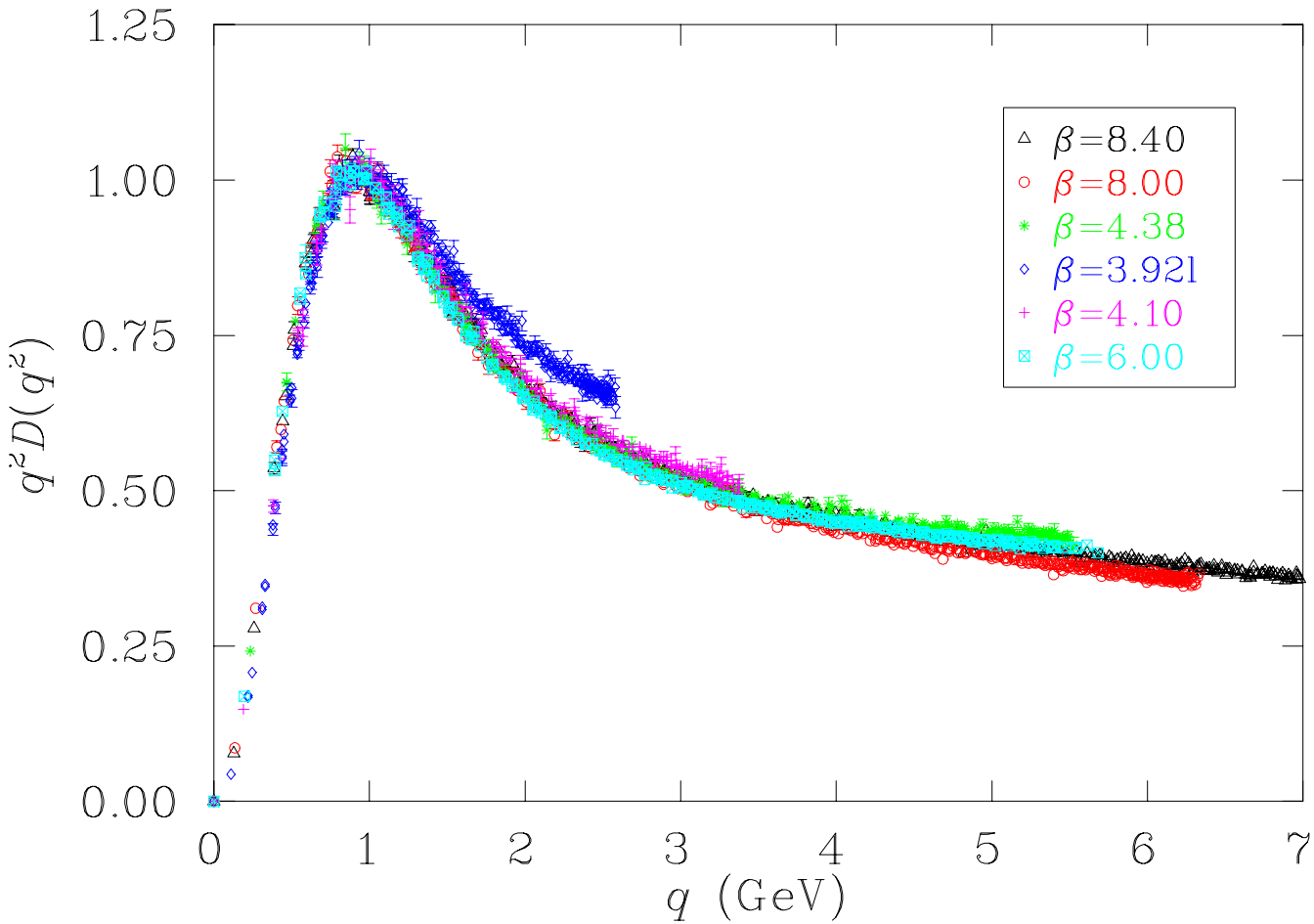
At tree-level

$$D(q^2) = \frac{1}{q^2}.$$

Three conjectures:

- Infrared enhanced: $D \sim q^{-4}$ Confinement by OGE.
- Infrared finite: $D \sim 1$
- Infrared vanishing: $D \sim \sqrt{q}$ Violates positivity.

Gluon Dressing Function

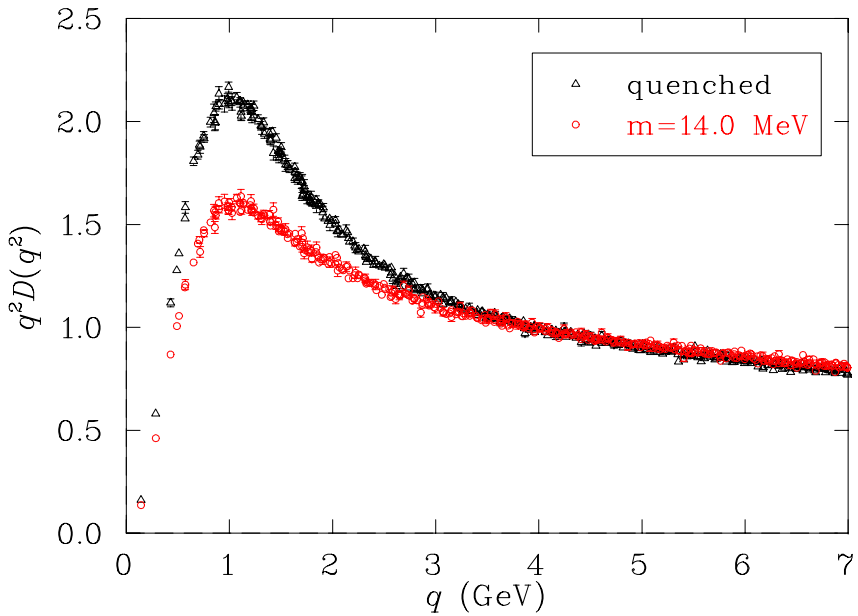


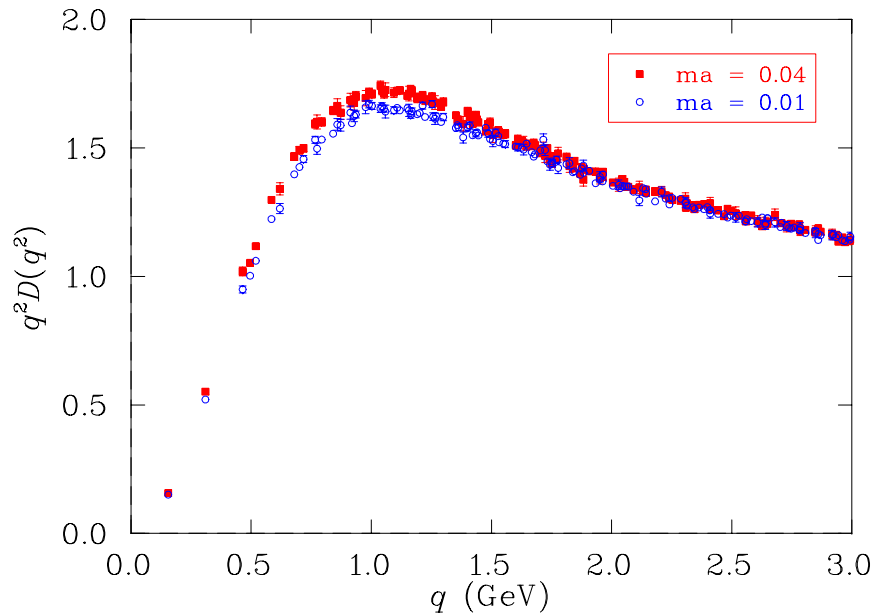
Quenched gluon dressing function on a variety of lattices.

F.D.R. Bonnet, POB, D.B. Leinweber, A.G. Williams, J.M. Zanotti, Phys.Rev.D64:034501,2001

POB, U.M. Heller, D.B. Leinweber, M.B. Parappilly, A. Sternbeck, L. von Smekal, A.G. Williams, J.B. Zhang, Phys.Rev.D76:094505,2007

Quark loops



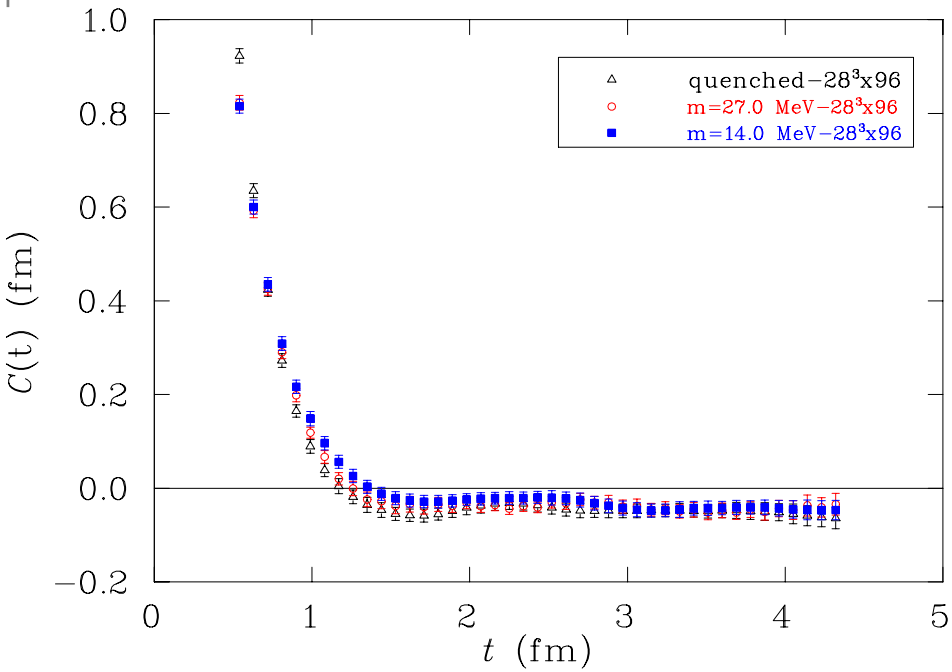


Sea quark mass dependence of the gluon dressing function.

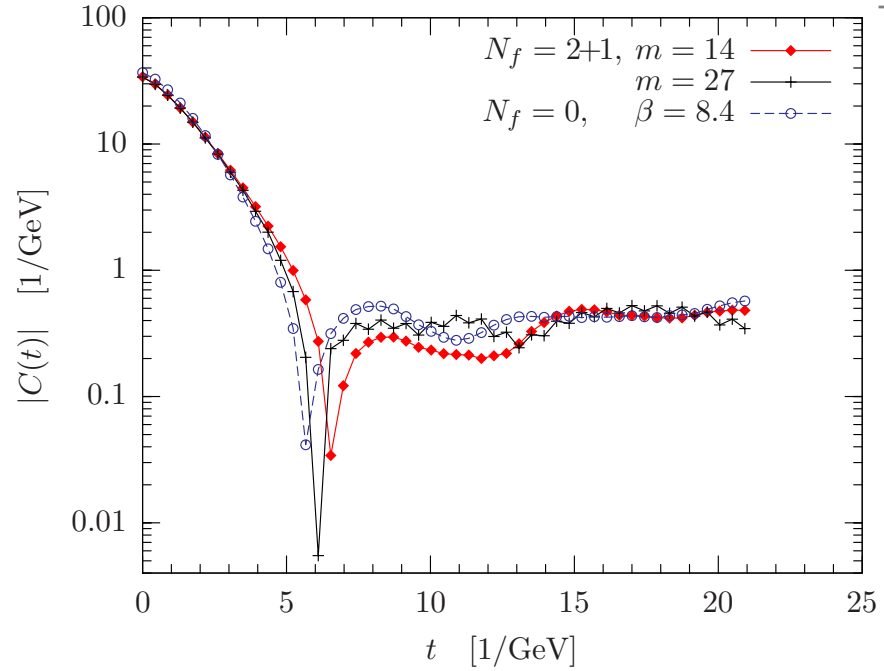
POB, U.M. Heller, D.B. Leinweber, M.B. Parappilly, A.G. Williams Phys.Rev.D71:054607,2005

Positivity violation

The gluon Schwinger function

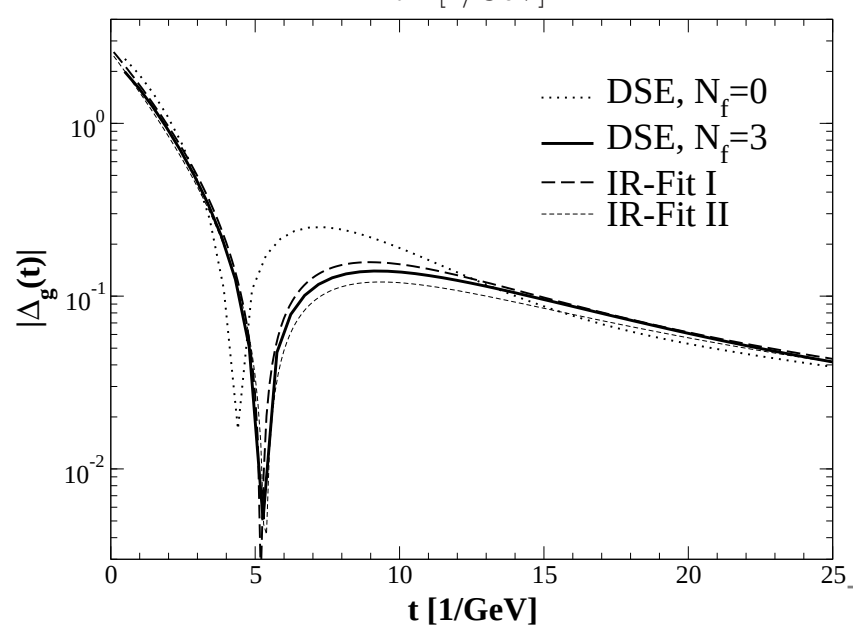


POB *et al.*, Phys.Rev.D76:094505,2007

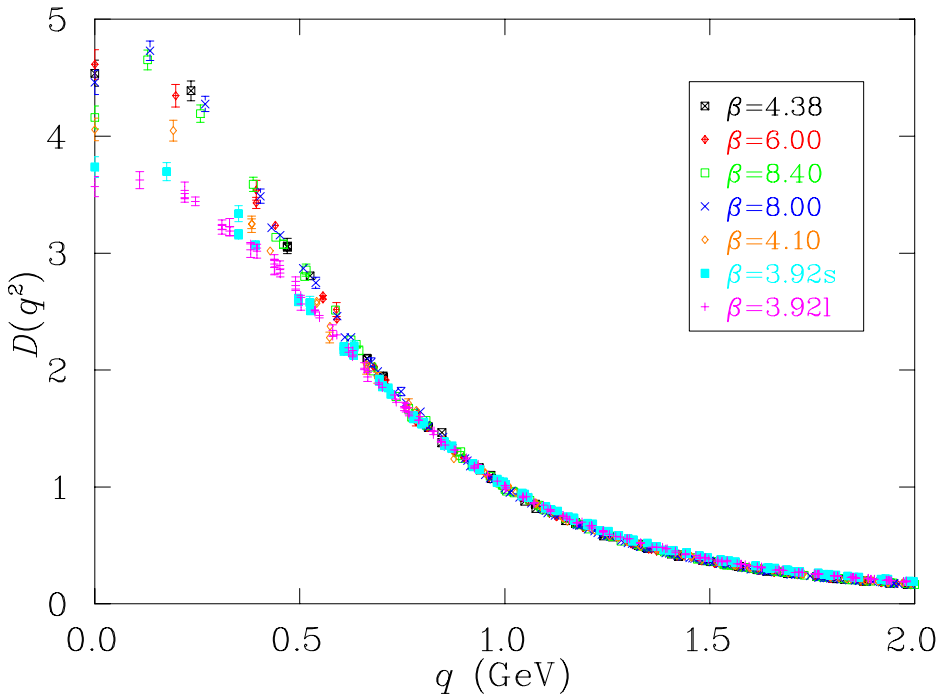


R. Alkofer et al.

PRD 70, 014014 (2004) ⟶



Deep infrared



Quenched gluon propagator on a variety of lattices.

F.D.R. Bonnet *et al.*, Phys.Rev.D64:034501,2001

POB *et al.*, Phys.Rev.D76:094505,2007

Finer lattices

A. Sternbeck, E.-M. Ilgenfritz, M. Mueller-Preussker, A. Schiller, Phys.Rev.D72:014507,2005.

Statistical bounds

A. Cucchieri and T. Mendes, Phys.Rev.Lett.100:241601,2008.

Very large lattices

A. Cucchieri and T. Mendes, PoS LAT2007:297,2007.

Does vanish in 2D

Axel Maas, Phys.Rev.D75:116004,2007.

Quark Propagator

Landau gauge: $\sum_\mu \partial_\mu A_\mu(x) = 0 \,\forall\, x$

$$S(x,y) = \langle \overline{\psi}(x)\psi(y)\rangle = \frac{1}{Z[0]} \int \mathcal{D}U \;\det M[U]\, M^{-1}[U](x,y)\, e^{-S_{\mathsf{YM}}[U]}$$

In the (Euclidean) continuum:

$$S(q^2) = \frac{Z(q^2)}{i\gamma\cdot q + M(q^2)}$$

$$M(q^2) \underset{q^2\to\infty}{=} \frac{c}{q^2}\left[\ln(q^2/\Lambda^2_{\mathsf{QCD}})\right]^{d_M-1} + \frac{\widehat{m}}{\left[\ln(q^2/\Lambda^2_{\mathsf{QCD}})\right]^{d_M}}$$

where $\widehat{m}$ is the RGI (renormalisation group invariant) mass and the anomalous dimension of the quark mass is $d_M = 12/(33-2N_f)$ for N_f quark flavours (zero here).

Lattice Quark Propagator

$\mathcal{O}(a^2)$ improved staggered action (Asqtad):

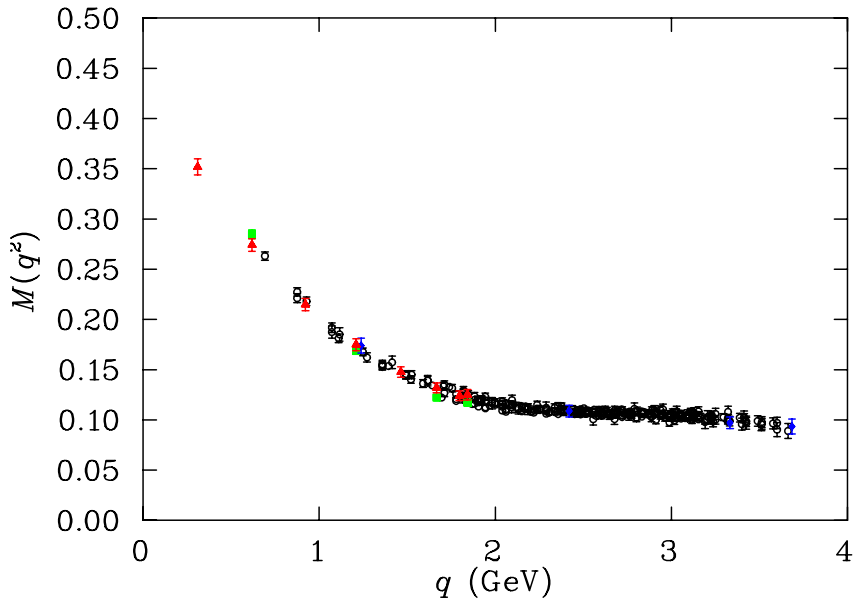
$$S^{\text{tree}}(k_\mu) = \frac{1}{i\overline{\gamma}\cdot q(k_\mu) + m}, \qquad q_\mu \equiv \sin(k_\mu)\big[1 + \frac{1}{6}\sin^2(k_\mu)\big],$$

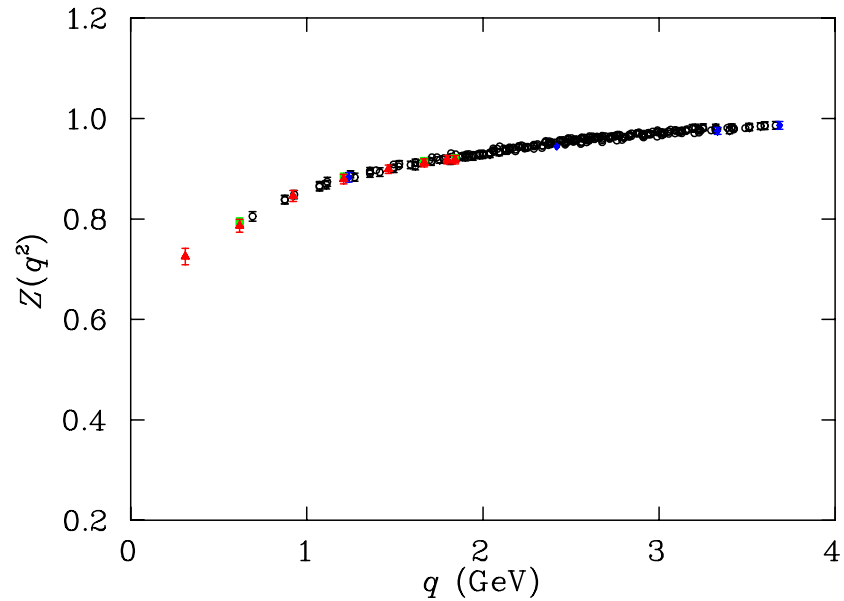
where

$$k_\mu = \frac{2\pi n_\mu}{aL_\mu} \qquad n_\mu \in \Big[\frac{-L_\mu}{4}, \frac{L_\mu}{4}\Big).$$

Note:
Only half the Brilluoin zone.

Quark Propagator



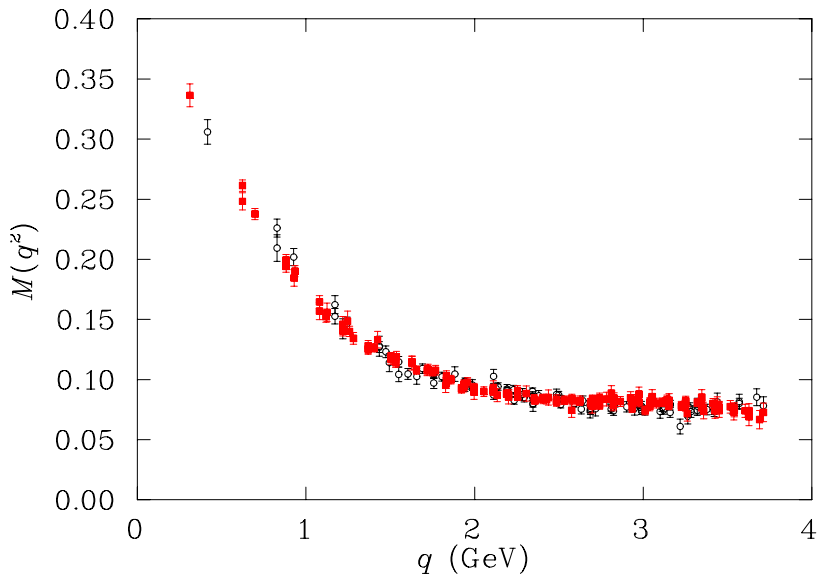


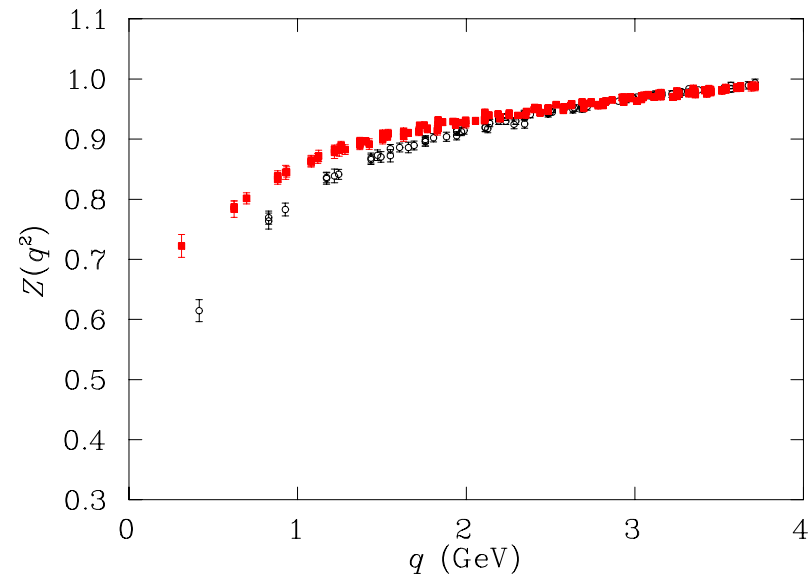
Lattice Landau gauge quark mass function (top) and wavefunction renormalisaton function (bottom) for a single bare quark mass ($ma = 0.048$).

POB, U.M. Heller, D.B. Leinweber, A.G. Williams, Nucl.Phys.Proc.Suppl.119:323-325,2003

POB, U.M. Heller, D.B. Leinweber, A.G. Williams, J.B. Zhang, Lect.Notes Phys.663:17-63,2005

Finite volume effects

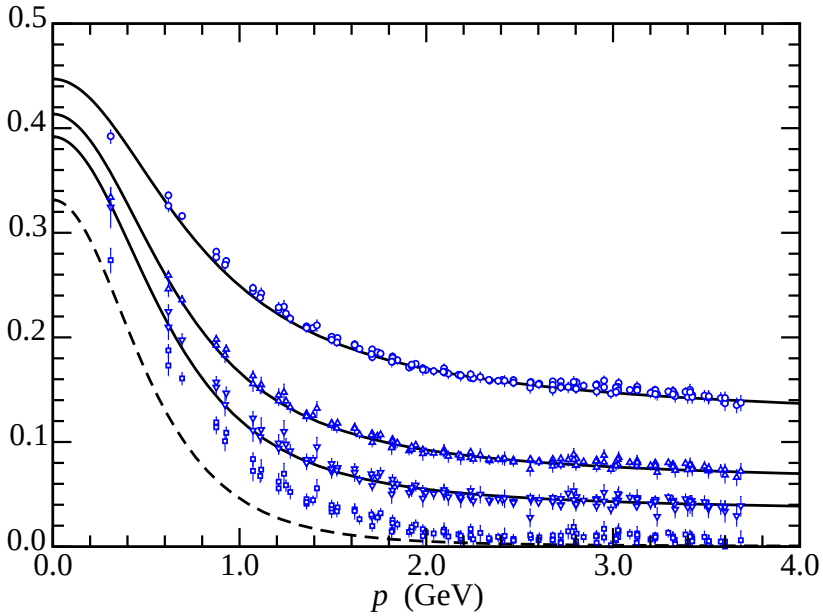


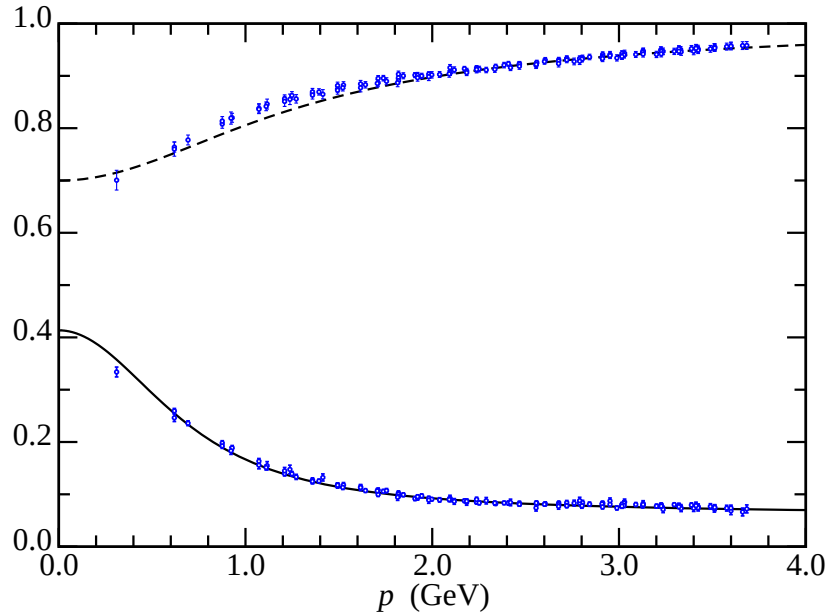


Finite volume effects: Asqtad quark mass function (top) and Z function (bottom) for mass $m \simeq 57$ MeV at $\beta = 4.60$. Comparison on $12^3 \times 24$ lattice (open circles) and 16×32 lattice (solid squares).

POB, U.M. Heller, D.B. Leinweber, A.G. Williams, J.B. Zhang Nucl.Phys.Proc.Suppl.128:23-29 (2004)

Comparison with DSE

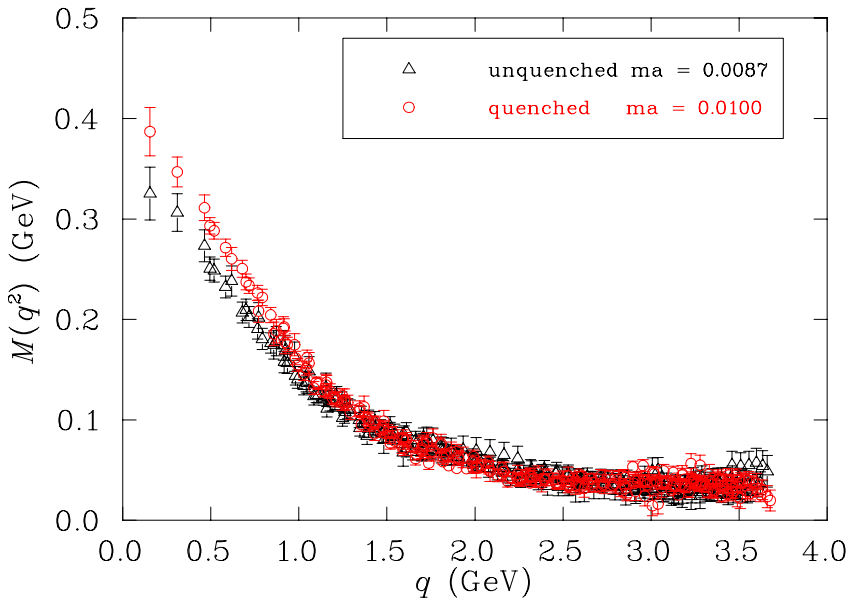


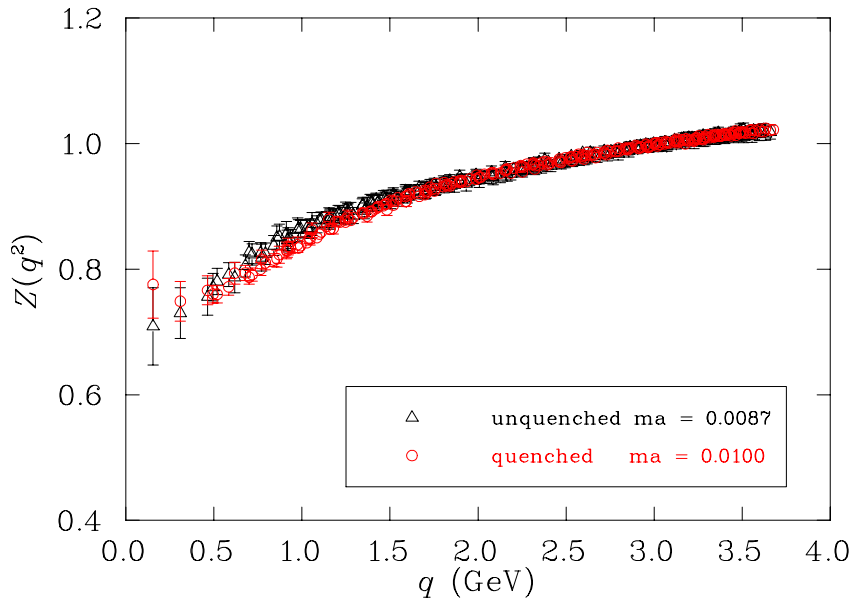


Comparison with Dyson-Schwinger Equation model

Bhagwat *et al.* Phys.Rev.C68:015203 (2003)

Quark loops





Lattice Landau gauge quark mass function (left) and wavefunction renormalisation function (right) comparing quenched and unquenched values for matched running masses.

POB, U.M. Heller, D.B. Leinweber, M.B. Parappilly, A.G. Williams and J.B. Zhang, Phys.Rev.D71:054507,2005

Asymptotic Behaviour

In the Landau gauge, in the chiral limit ($\widehat{m} = 0$), we have

$$c \simeq -\frac{4\pi^2 d_M}{3} \frac{\langle \overline{\psi}\psi \rangle}{[\ln(\mu^2/\Lambda^2_{\text{QCD}})]^{d_M}},$$

where μ^2 is a choice of renormalisation point. The RGI quark mass in the top equation can be replaced by the running quark mass,

$$m(\mu^2)[\ln(\mu^2/\Lambda^2_{\text{QCD}})]^{d_M}.$$

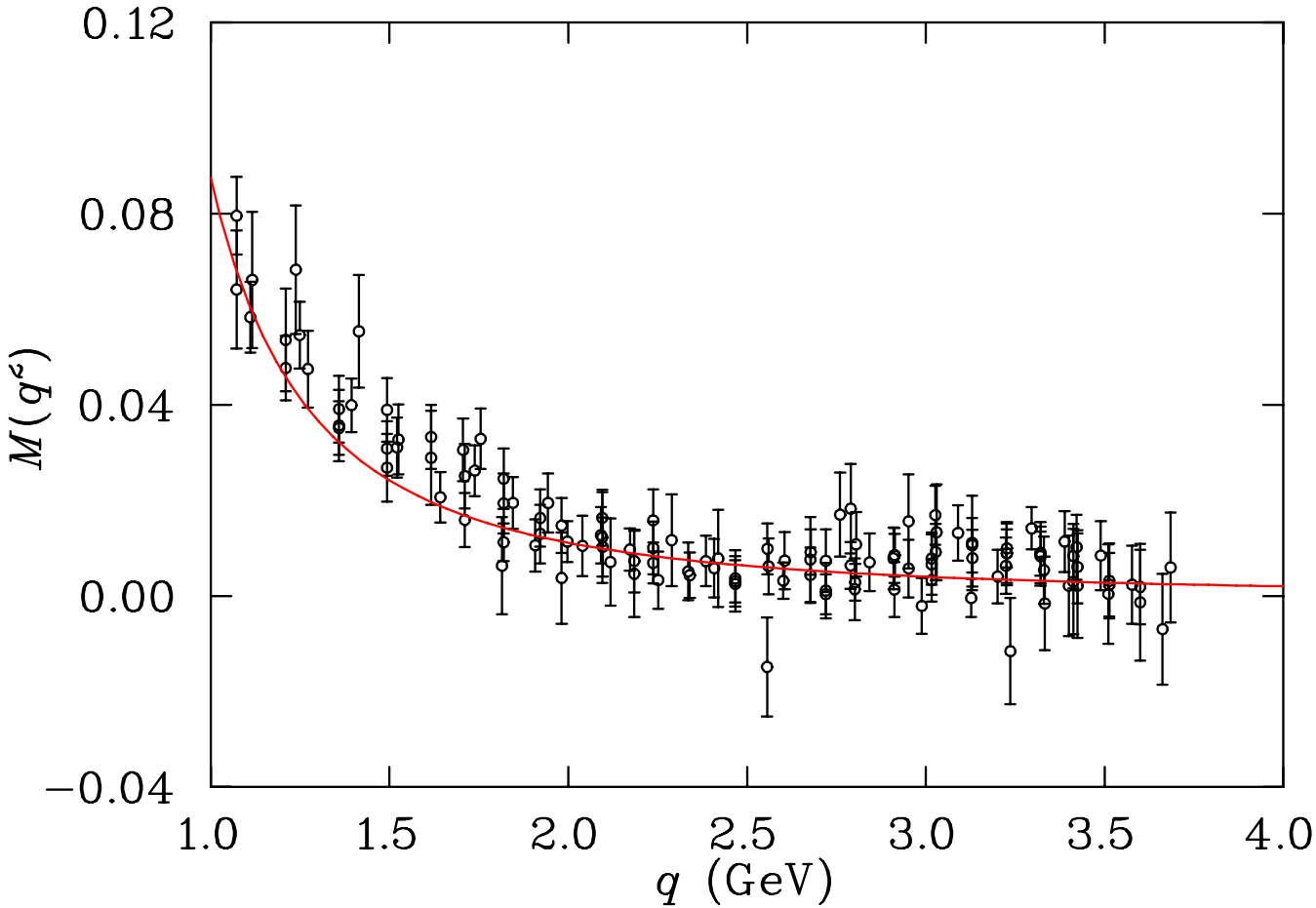
A fit of this asymptotic form to the mass function between 1.9 and 2.9 GeV (51 data points) produces a value for the condensate of

$$(-\langle \overline{q}q \rangle)^{1/3} = 276(24) \text{ MeV}.$$

From $Z(p^2)$:

$$\alpha_s(\mu^2)\langle A^2 \rangle_\mu = 0.39(9) \text{ GeV}^2$$

E. Ruiz Arriola, POB and W. Broniowski, Phys.Rev.D70:097505,2004



Chiral Limit

POB, U.M. Heller, D.B. Leinweber, A.G. Williams, J.B. Zhang, Lect. Notes Phys.663:17-63,2005

Quark Masses

We calculate the pion mass using the Asqtad quark action on this set of lattices and fit the five lowest masses to the form

$$am_\pi = aB\sqrt{m_0}$$

to find the bare mass corresponding to the physical point. We find $am_0 = 0.00215(1)$. Then we fit the lowest RGI masses to the form

$$a\widehat{m} = Aam_0$$

and find $A = 1.57(16)$. Putting these together we get

$$\widehat{m} = 5.3(6) \text{ MeV}.$$

Thus by choosing the renormalisation point $\mu = 2$ GeV and inserting $\Lambda_{\text{QCD}}^{\overline{\text{MS}}} = 239$ MeV, we get

$$m(\mu^2)^{\overline{\text{MS}}} = 3.1(4) \text{ MeV}.$$

The strange quark mass is determined from the mass of the ϕ and a straight line interpolation between the relevant quark masses. The RGI mass is

$$\widehat{m}_s = 142(10) \text{ MeV}.$$

which corresponds to

$$m_s(\mu^2)^{\overline{\text{MS}}} = 84(4) \text{ MeV}.$$

Centre Vortex Mechanism

- Regge theory –> Nambu strings
- Vortices –> Strings (Nielsen & Olesen)

Why look to the Centre?

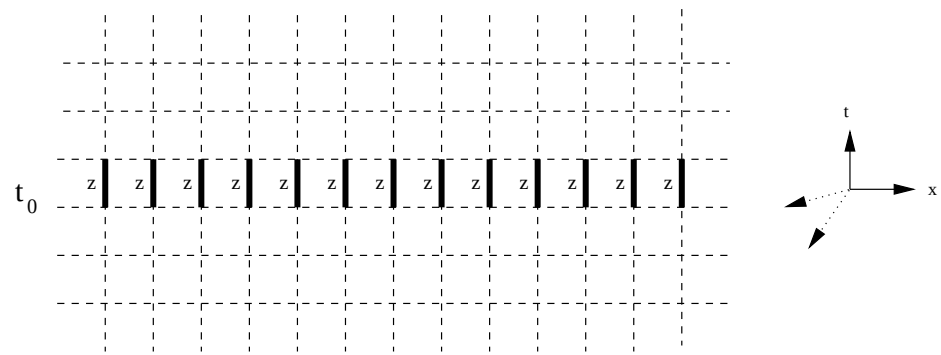
Polyakov loop:

$$P(\vec{x}) = \mathrm{Tr}\, U_4(\vec{x},1)U_4(\vec{x},2).....U_4(\vec{x},T)$$

$$\langle P(\vec{x})\rangle = e^{-FT}$$

Consider transformation

$$U_4(\vec{x},t_0) \to zU_4(\vec{x},t_0), \quad z \in Z_N$$



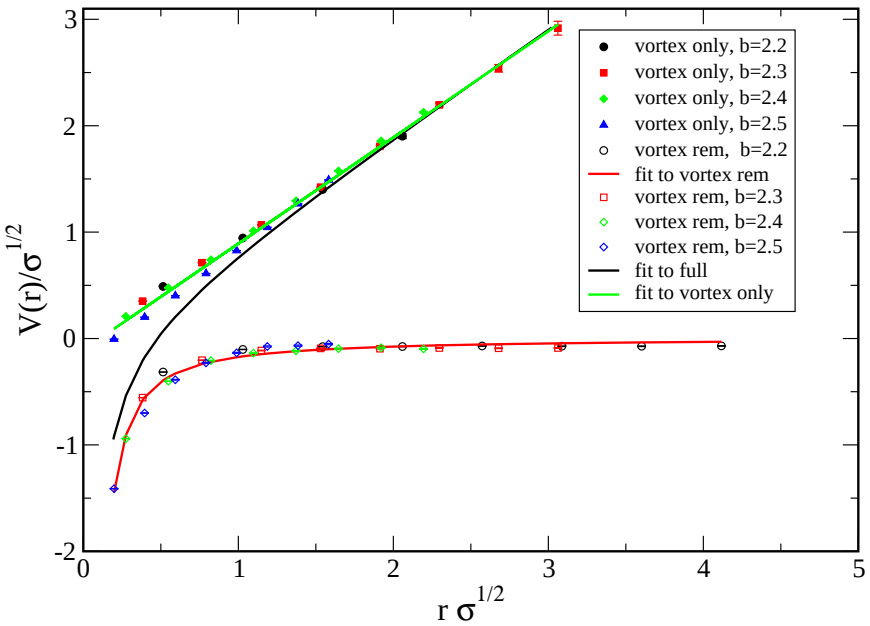
for some t_0 and all $\vec{x}$.

$$\langle P(\vec{x})\rangle \to z\langle P(\vec{x})\rangle$$

$$\langle P(\vec{x})\rangle = \begin{cases} 0 & \text{unbroken } Z_N \text{ symmetry} \quad \text{confined} \\ \text{non-zero} & \text{broken } Z_N \text{ symmetry} \quad \text{deconfined} \end{cases}$$

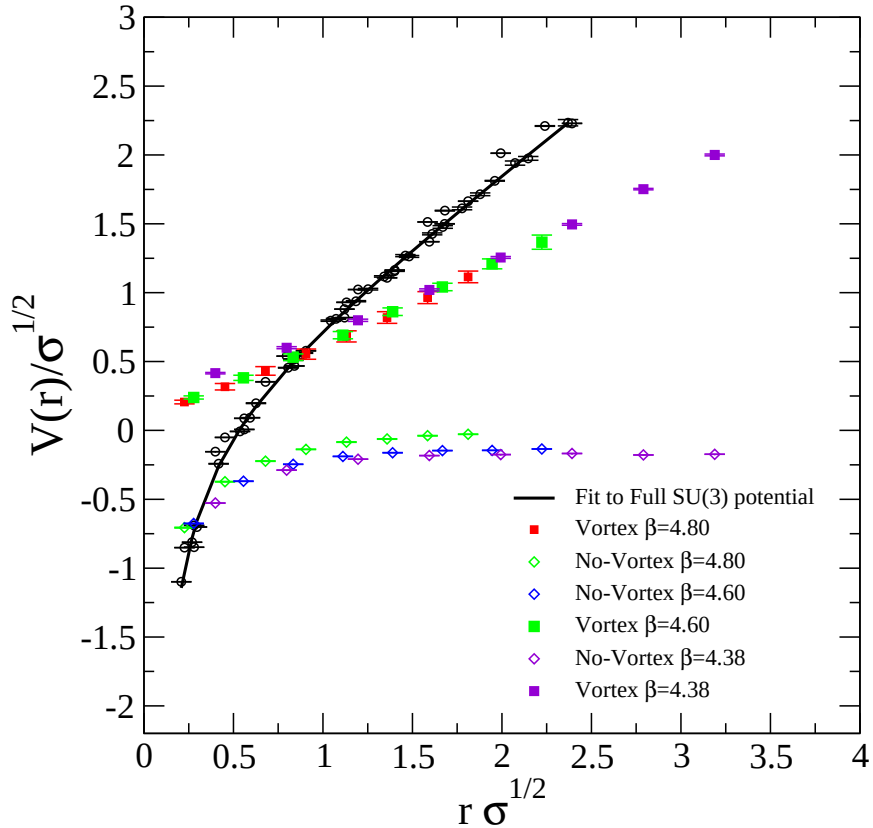
Centre Vortex Confinement

SU(2)



K. Langfeld, Phys. Rev. D69, 014503 (2004)

SU(3)



POB *et al. In preparation*

Hunting Centre Vortices

The centre fluxes through an elementary plaquette are represented by centre link elements $Z_\mu(x)$ which take values in the centre group $Z_3 \in SU(3)$:

$$Z_\mu(x) \;=\; \exp\Big\{ i\frac{2\pi}{3} m_\mu(x) \Big\} \;, \quad m_\mu(x) \in \{0,1,2\} \;.$$

The task is to find a centre projection that is sensible in the continuum limit. The standard solution is

$$\sum_{x,\mu} \Big\| U_\mu^\Omega(x) \;-\; Z_\mu(x) \Big\| \;\stackrel{\Omega, Z_\mu}{\longrightarrow}\; \min.$$

After a suitable gauge transformation $\Omega(x)$, we are looking for those centre links $Z_\mu(x)$ which represent best a given link configuration $U_\mu(x)$.

This implies that the overlap between the gauged links and the centre links is maximized:

$$\sum_{x,\mu} \mathcal{R}e \left[\mathrm{Tr} U_\mu^\Omega(x) \; Z_\mu^\dagger(x) \right] \stackrel{\Omega, Z_\mu}{\longrightarrow} \max. \tag{1}$$

Hence, we will exploit the gauge degrees of freedom to bring $U_\mu^\Omega(x)$ as close as possible to a centre element. Assuming that the deviations of $U_\mu^\Omega(x)$ from a centre element are indeed small, one might approximately solve (1) by setting

$$Z_\mu(x) \approx \frac{1}{3} \mathrm{Tr} U_\mu^\Omega(x) \; , \quad \text{or} \quad Z_\mu(x) \approx \left[\frac{1}{3} \mathrm{Tr} U_\mu^{\dagger\,\Omega}(x) \right]^2 \; .$$

Whence two gauge conditions obtained:

$$R_{\mathrm{mes}} \equiv \sum_{x,\mu} \left| \mathrm{Tr} U_\mu^\Omega(x) \right|^2 \xrightarrow{\Omega} \text{max}, \tag{2}$$

$$R_{\mathrm{bar}} \equiv \sum_{x,\mu} \left[\mathrm{Tr} U_\mu^\Omega(x) \right]^3 \xrightarrow{\Omega} \text{max}. \tag{3}$$

- Both gauge conditions specify a particular Maximal Centre Gauge.
- These two are referred to as the ‘mesonic’ and ‘baryonic’ centre gauge respectively.
- We will only use the mesonic gauge condition

Centre Vortices: The Method

Identify centre vortices with the usual two step procedure.

- Fix to Direct Maximal Centre Gauge:

Maximise (w.r.t. gauge transformations) the functional

$$\mathcal{F}[U] = \frac{1}{V N_c N_D} \sum_{x,\mu} \text{Tr}\, U_\mu(x)^2$$

- Factor:

$$U_\mu(x) = Z_\mu(x) \widetilde{U_\mu}(x)$$

SU(N): $Z_\mu(x) = e^{2\pi i n/N} \qquad n = 0, .., N-1$

Then if

$$P_{\mu\nu}(x) = Z_\mu(x) Z_\nu(x+\hat{\mu}) Z_\mu^\dagger(x+\hat{\nu}) Z_\nu^\dagger(x) \neq 1$$

the plaquette is pierced by a centre vortex.

MC Gauge Fixing Variant

Gauge fixing matrix $g(x) \in SU(2)$

$$g(x) = g_0(x) + i\vec{\tau}\vec{g}(x), \qquad G(x) = \begin{pmatrix} g_0(x) \\ \vec{g}(x) \end{pmatrix},$$

where

$$g_0^2(x) + \vec{g}^2(x) = 1.$$

$\mathcal{F}$ is locally maximised, site by site.

Choosing a single site, x_0, the action of $g(x_0)$ is

$$\mathcal{F}(x_0) \;=\; G^T(x_0)\; M\; G(x_0) \;-\; \lambda \left(G^T(x_0) G(x_0) - 1 \right) ,$$

where M is a real symmetric 4×4 matrix depending on $U_\mu(x_0)$ and $U_\mu(x_0 - \mu)$ and λ is a Lagrange multiplier.

MC Gauge Fixing 2

The eigenvectors and eigenvalues of the matrix M:

$$e_k, \lambda_k, \qquad k = 1 \ldots 4$$

Choosing an eigenvector for the gauge transformation, $G(x_0) = e_k$, the local increase of the gauge fixing functional is:

$$\mathcal{F}(x_0) = \lambda_k$$

The largest eigenvalue gives the greatest change in $\mathcal{F}(x_0)$.

Adopt a “simulated annealing like” procedure:

- Choose $G(x_0) = e_k$
- With a relative probability of $e^{\beta_f \lambda_k}$
- Parameter $\beta_f \to \infty$ recovers standard method.

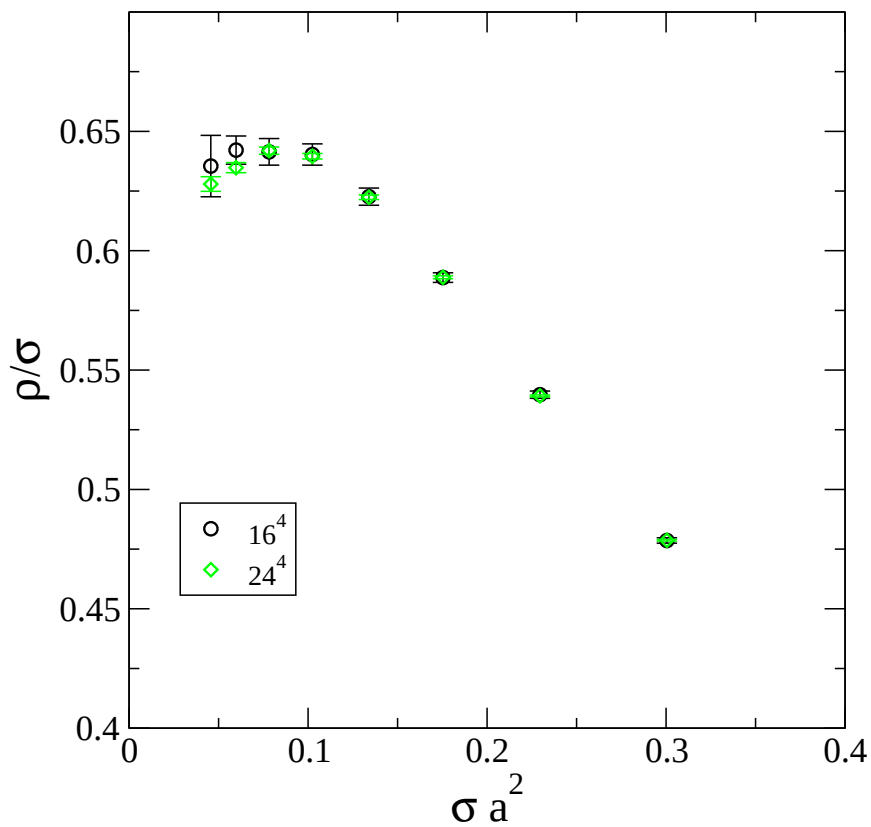
MC Gauge Fixing 3

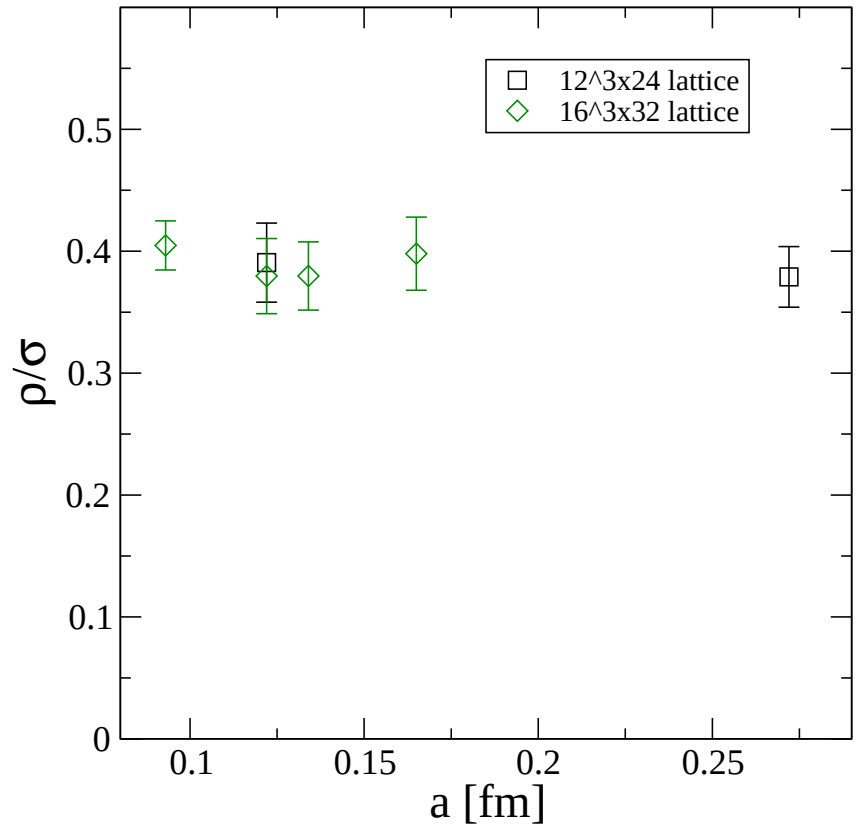
In practice, we started with $\beta_f = 0.02$ and performed 25 sweeps through the lattice until we increased β_f by 0.1. The procedure stopped when no further increase of the gauge fixing functional is achieved.

It turns out that the vortex matter arising from this procedure has good phenomenological properties such as good scaling properties in the continuum limit.

To test the latter aspect, we calculated the planar vortex area density ρ in units of the (measured) string tension for several values of the lattice spacing a using the standard Wilson action. For sufficiently small values of the lattice spacing, the vortex density becomes independent of the lattice regulator.

P Vortices



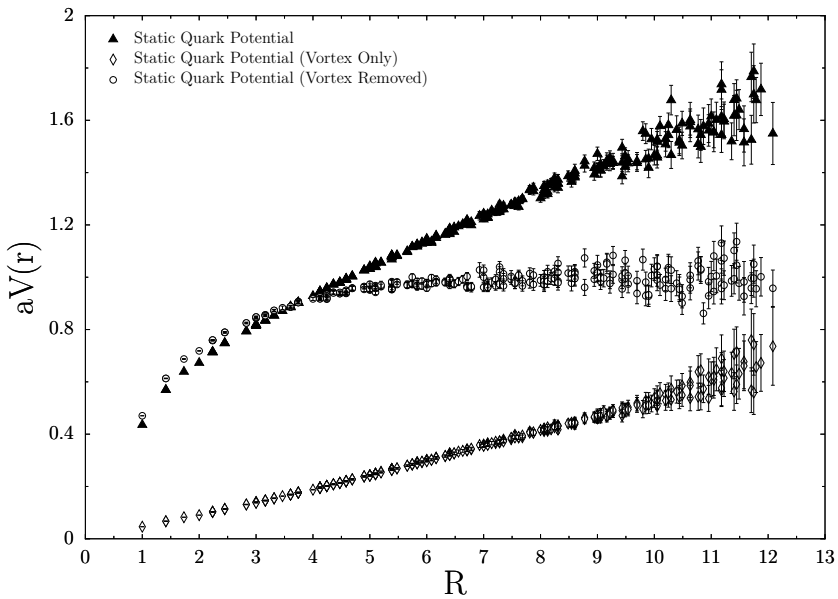


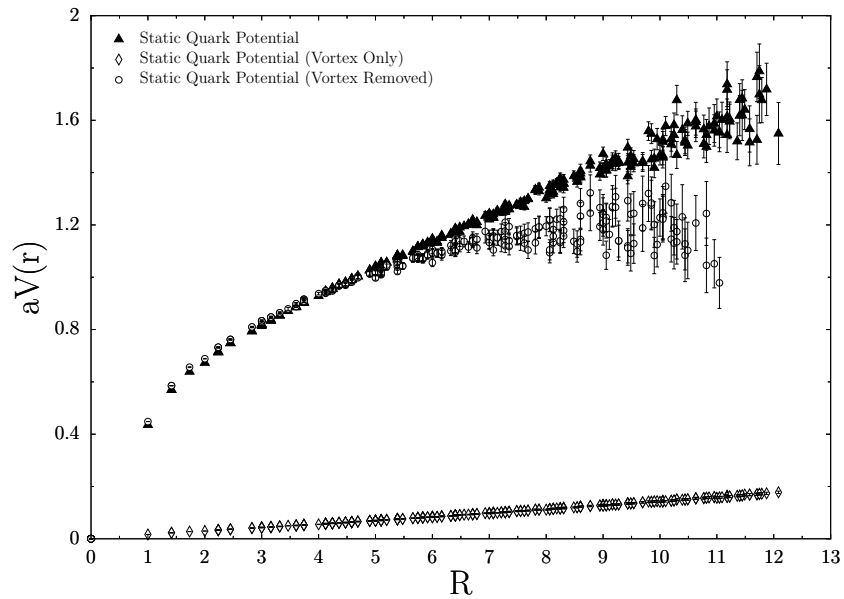
Scaling of the planar vortex area density ρ in units of the string tension σ as function of the lattice spacing a.

POB, K. Langfeld, D.B. Leinweber, A. O'Cais, A. Sternbeck, L. von Smekal and A.G. Williams, PRD 78:054509,2008.

POB *et al. In preparation*

Preconditioned Gaugefixing





Centre vortex potential with smearing preconditioning MCG gaugefixing.

O'Cais *et al.*, arXive:0807.0264

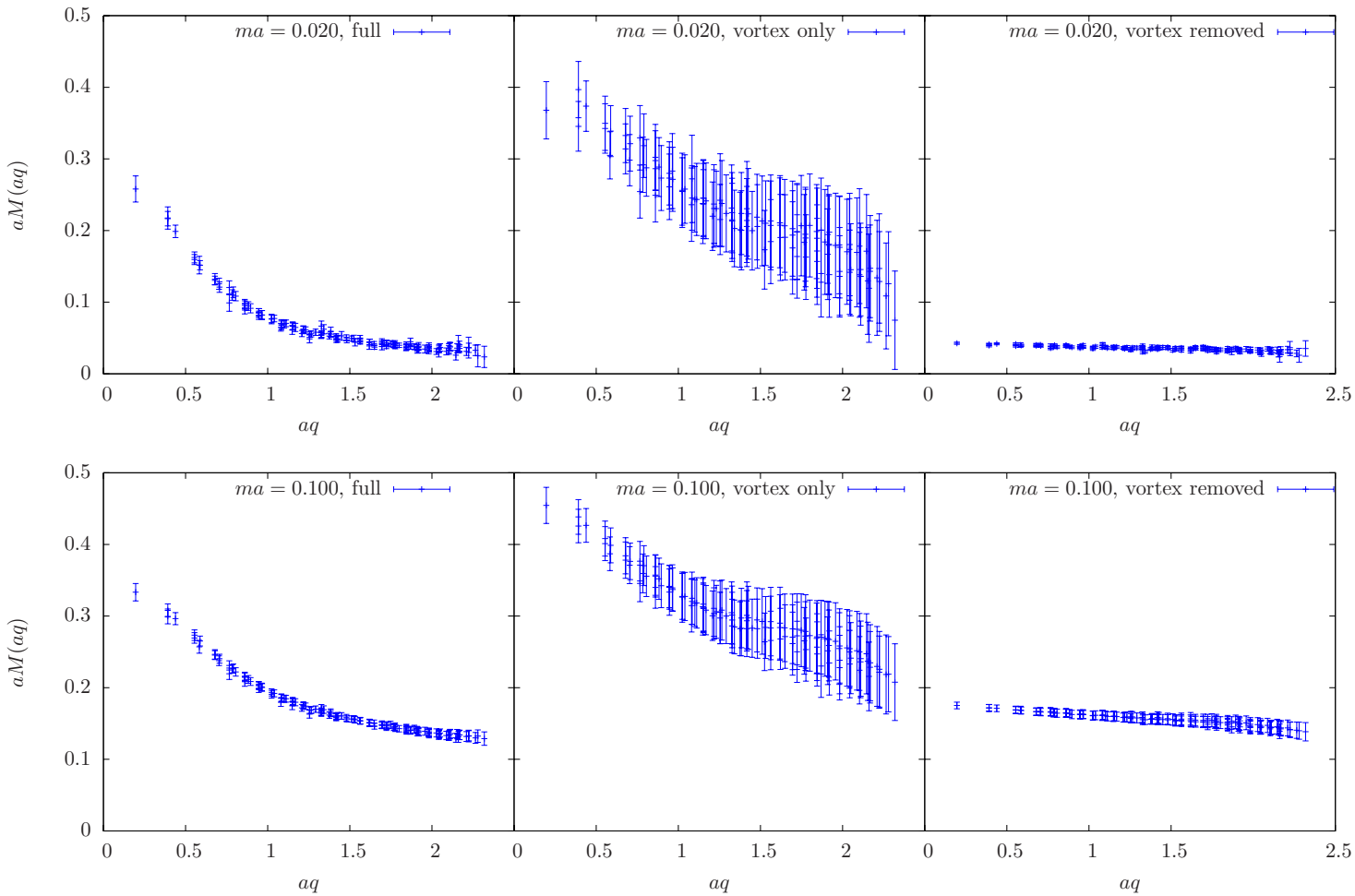
See also: Faber, Greensite and Olejnik, Phys. Rev. D 64, 034511 (2001)

Centre vortex experiment

- Landau gauge quark propagator displays DχSB
  - Is it sensitive to centre vortices?
- Calculation
  - 120 SU(2) Wilson action gauge configurations.
  - $16^3 \times 32$, $\beta = 1.35$.
  - 100 SU(3) Symanzik improved action gauge configurations.
  - $16^3 \times 32$, $\beta = 4.60$.

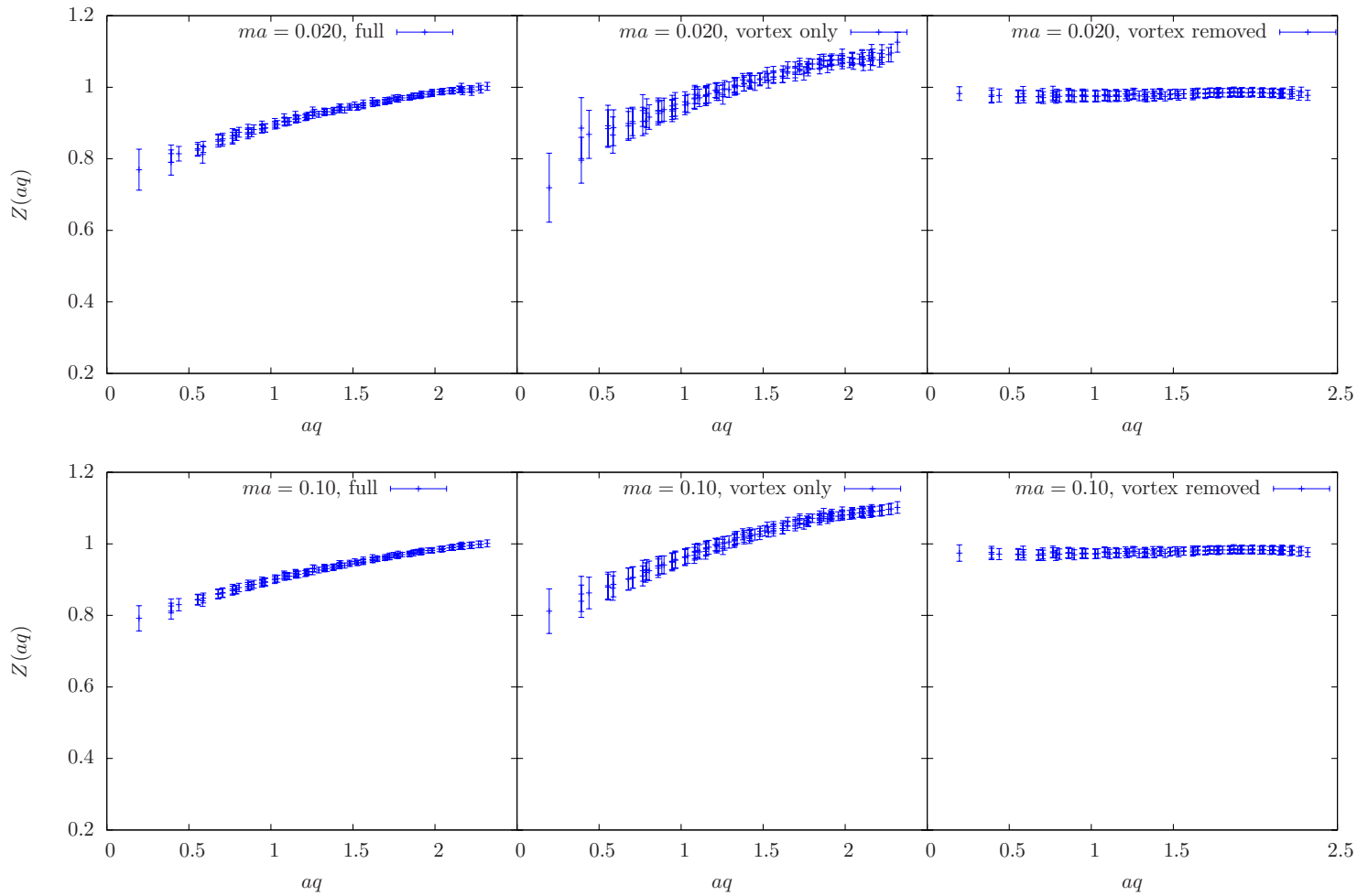
1. Identify and isolate centre vortices
2. Rotate to Landau gauge
3. Calculate propagator with, only with and without vortices
4. Contrast and compare.

SU(2) Quark Mass Function



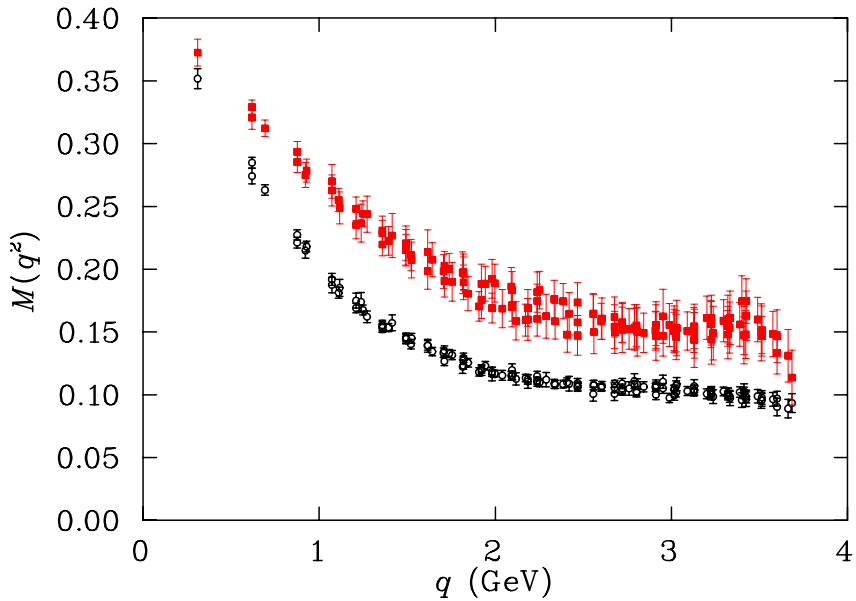
Landau gauge Lattice quark mass function on untouched, vortex removed and vortex only configurations, for two values of the bare quark mass.

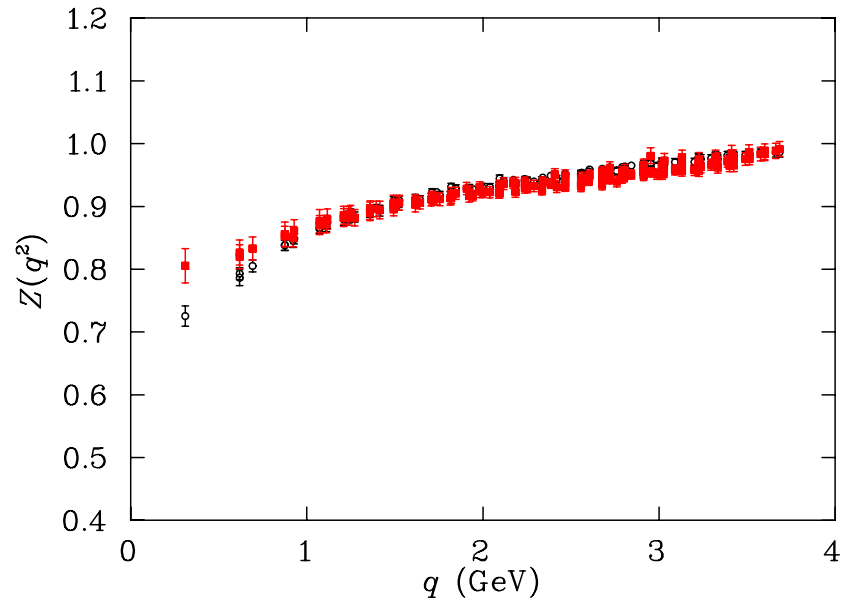
SU(2) Wavefunction Renormalisation



Lattice quark Z function on untouched, vortex removed and vortex only configurations, for two values of the bare quark mass.

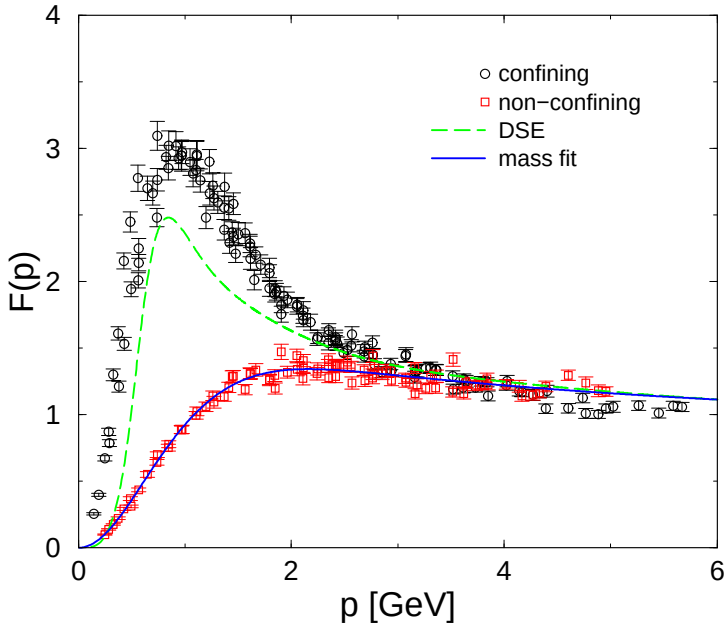
SU(3) Quark Propagator

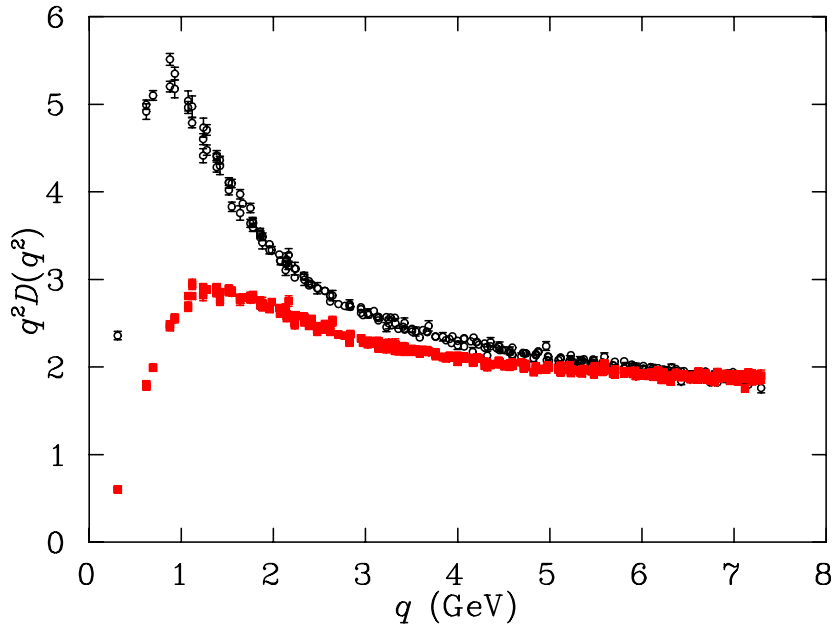




Lattice quark Z and M functions with (open circles) and without (red squares) vortices for a single bare quark mass ($ma = 0.048$).

Gluon Propagator





Langfeld, Reinhardt and Gattnar,
Nucl. Phys. B621, 131 (2002)

The gluon dressing function. Black circles denote results from the original untouched gauge fields, whereas red squares report the dressing function after removing centre vortices.

Conclusions

- Quark and gluon propagators are useful probes of confinement and chiral symmetry breaking.
- Gluon is confined.
  - $q^2 \to 0$?
- Quark propagator displays DχSB.
  - Confinement? Quark-gluon vertex.
- Quenched SU(2) Landau gauge quark propagator similar to the one in SU(3).
- In SU(2), centre vortices do an excellent job of accounting for both DχSB and confinement.
- In SU(3) the vortex identification procedure misses some structure important to DχSB but not confinement.

Questions

1. Why does this method work so well in SU(2) but not SU(3)?
   (a) Is SU(2) a good model for SU(3) in this case?
2. What are we missing in SU(3) that's important for DχSB but not for confinement?
3. What is the relationship between the static quark potential and light quark confinement?
4. What will it take to falsify the centre vortex model?
5. Are we too devoted to the idea of a DχSB / Confinement mechanism?

(Just asking)

Collaborators

Alan O'Cais, Derek Leinweber, Lorentz von Smekal, Andre Sternbeck,
Tony Williams (Adelaide),
Kurt Langfeld (Plymouth)
Urs Heller (APS), Maria Parappilly (Flinders)

And thanks again to the organisers

POB supported by

MARSDEN FUND
TE PŪTEA RANGAHAU A MARSDEN