Responses of Holographic Matter

Andrei Parnachev

C.N.Yang ITP, Stony Brook, USA

Preview:

Will consider brane setup which in field theoretic regime gives rise to fermion matter. In the strong coupling regime can be analyzed by gauge/string duality techniques. Will hunt for signatures of the fermion matter at strong coupling. Will study responses to small perturbations from the ground state such as small temperature and small oscillations.

Mostly based on recent work with M. Kulaxizi.

Outline

- Introduction. $D4 - D8 - \overline{D8}$ model of QCD
- Fermi liquid: Heat capacity, zero sound
- Linear heat capacity and zero sound from $D4 - D8 - \overline{D8}$
- $D3 - D7$ system
- Comments/Conclusions
- Open questions

Introduction

Gauge/string duality allows computations in field theories at strong coupling. QCD is an important real-life example, but not the only one! QCD applications of string theory are mostly qualitative [because of large N and separation of Λ_{QCD} from $(\alpha')^{-1/2}$]

Condensed matter physics provides a variety of real-life strongly coupled systems. Perhaps string theory can give quantitative predictions, especially for universal quantities.

Introduction

Not easy to find examples with non-abelian gauge interactions. Constructing holographic models respecting relevant symmetries (a-la AdS/QCD) might help. One can also search for qualitative/universal physics independent of the details of the interactions.

We consider two examples which have known lagrangian description is some region of the parameter space. $D4 - D8 - \overline{D8}$ system is often used to model QCD. $D3 - D7$ system is another interesting example.

Introduction

Consider QCD at finite temperature T and chemical potentials μ_i. In this example high energy theory meets condensed matter. At large μ, due to asymptotic freedom, dynamics of quarks near the Fermi surface determines the physics.

We can compare with holographic model. We can also ask more general questions about the behavior of strongly coupled systems at finite T and μ, like critical behavior. Here $D3 - D7$ system will be useful.

$D4 - D8 - \overline{D8}$

Brane construction: N $D4$-branes spanning $x^0 \ldots x^4$ $N_f \ll N_c$ $D8 - \bar{D}8$ pairs spanning every coordinate but $\tau = x^4 \in [0, 2\pi R_4)$. The x^4 circle has antiperiodic boundary conditions for fermions.

Theory at energies much smaller then $1/l_s$: large N 5d Yang-Mills with N_f 4d quarks. Theory at energies much smaller then $1/R_4$: large N QCD with quarks. Can also take $R_4 \to \infty$. (This is NJL limit at weak coupling).

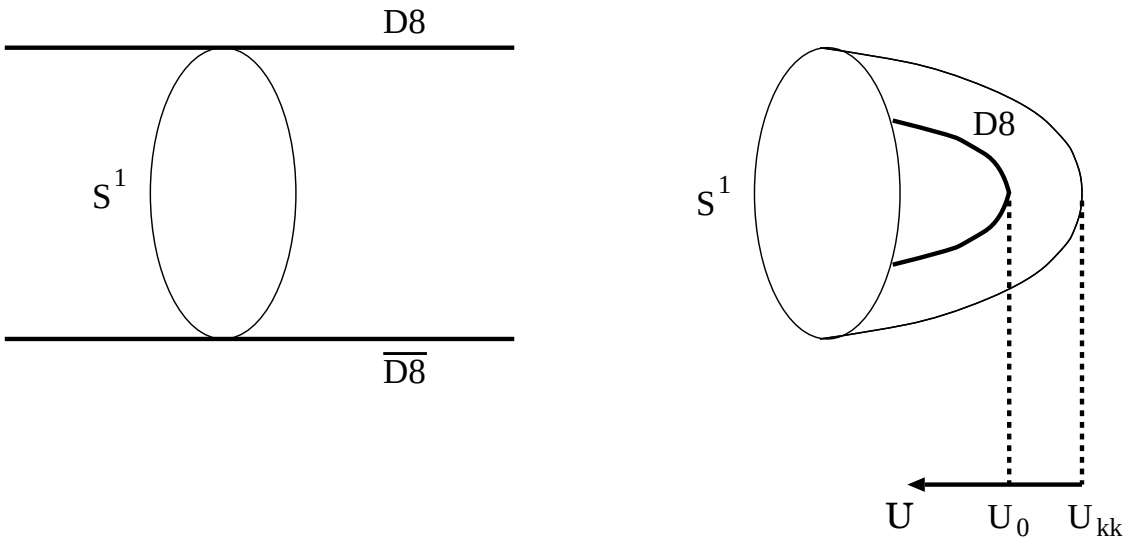
$D4 - D8 - \overline{D8}$

Parameters: $\lambda = g_{YM}^2 N = g_s l_s$; 4-dimensional t'Hooft coupling at scale $1/R_4$ $\lambda_4 = \lambda/R_4$; brane-antibrane asymptotic separation L.

$$\Lambda_{QCD} = \frac{1}{R_4} e^{-\frac{1}{\lambda_4}}$$

so we have good approximation to QCD in the regime $\lambda_4 \ll 1$. Holographic (stringy) dual is solvable in the opposite regime $\lambda_4 \gg 1$. Yang-Mills=gravity; quarks=Dirac-Born-Infield (DBI) action for D8 branes

$D4 - D8 - \overline{D8}$



Gravity dual has a form $R^{3,1} \times S^1_{x^4} \times R_{+(U)} \times S^4$. In the $U - x^4$ plane geometry has a cigar-like form. $D8 - \bar{D}8$ branes connect in the IR (small U), hence chiral symmetry is broken.

$D4 - D8 - \overline{D8}$

Metric:

$$\begin{aligned} ds^2 &= \left(\frac{U}{R}\right)^{\frac{3}{2}} \left((dx_\mu)^2 + f(U)(dx^4)^2\right) \\ &\quad + \left(\frac{U}{R}\right)^{-\frac{3}{2}} \left(\frac{dU^2}{f(U)} + U^2 d\Omega_4^2\right) \end{aligned}$$

where $f(U) = 1 - U_K^3/U^3$.
$U = U_K$ bounds U from below; determined via
$2\pi R_4 = 4\pi R^{3/2}/3U_K^{1/2}$

Dilaton is $e^\Phi = \left(\frac{U}{R}\right)^{\frac{3}{4}}$
Similarly to AdS_5, $R^3 = \pi\lambda_5$

$D4 - D8 - \overline{D8}$

Fundamental matter comes from N_f $D8 - \bar{D}8$ pairs with asymptotic separation L. Their shape is determined by solving DBI equations of motion.

Results: branes are connected at $U = U_0$; chiral symmetry is broken; pion lagrangian can be derived. U_0 and L are related: $L = R^{3/2}/U_0^{1/2}$

Fluctuations of the flavor branes give rise to scalar and vector mesons with masses $\sim 1/L$. Baryons can be represented as (small) instantons on the worldvolume of $D8$.

Finite temperature; Yang-Mills

Compactify euclidean time t_E with asymptotic circumference $\beta = 1/T$. There are 2 geometries which asymptote to this:

- (1) $(x^4, U > U_K)_{cigar} \times t_E$
- (2) $(t_E, U > U_T)_{cigar} \times x^4$

The Lorenzian version of (2) is a black hole (BH); it has a horizon. The geometries really are the same; smaller cigar dominates partition function.

For $T < 1/2\pi R_4$ (1) dominates; gluon confining state
For $T > 1/2\pi R_4$ (2) (BH) dominates; gluon deconfinement

Finite temperature; matter

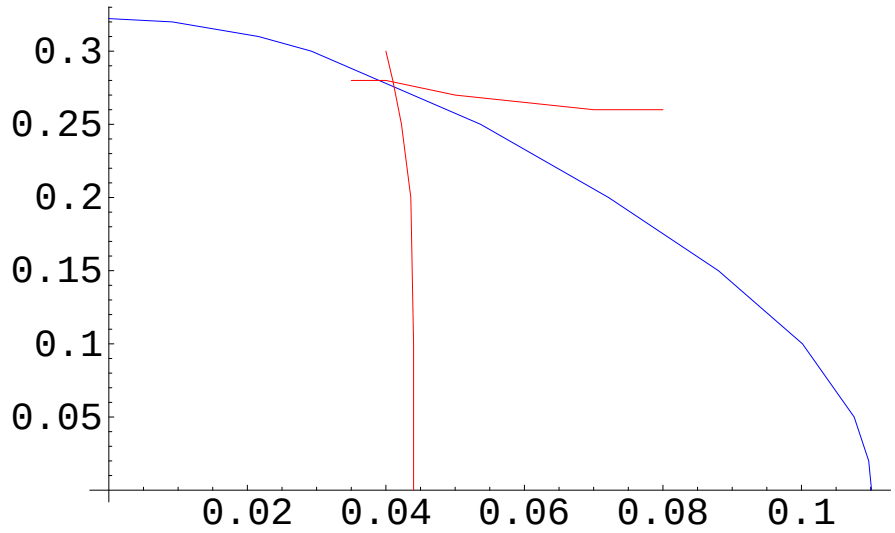
In the following we consider gluon-deconfining phase where black hole appears. There are two possibilities:

- They connect as before (curved)
- They go straight into the BH horizon (straight)

curved branes= chiral symmetry is broken;
straight branes=chiral symmetry is restored.

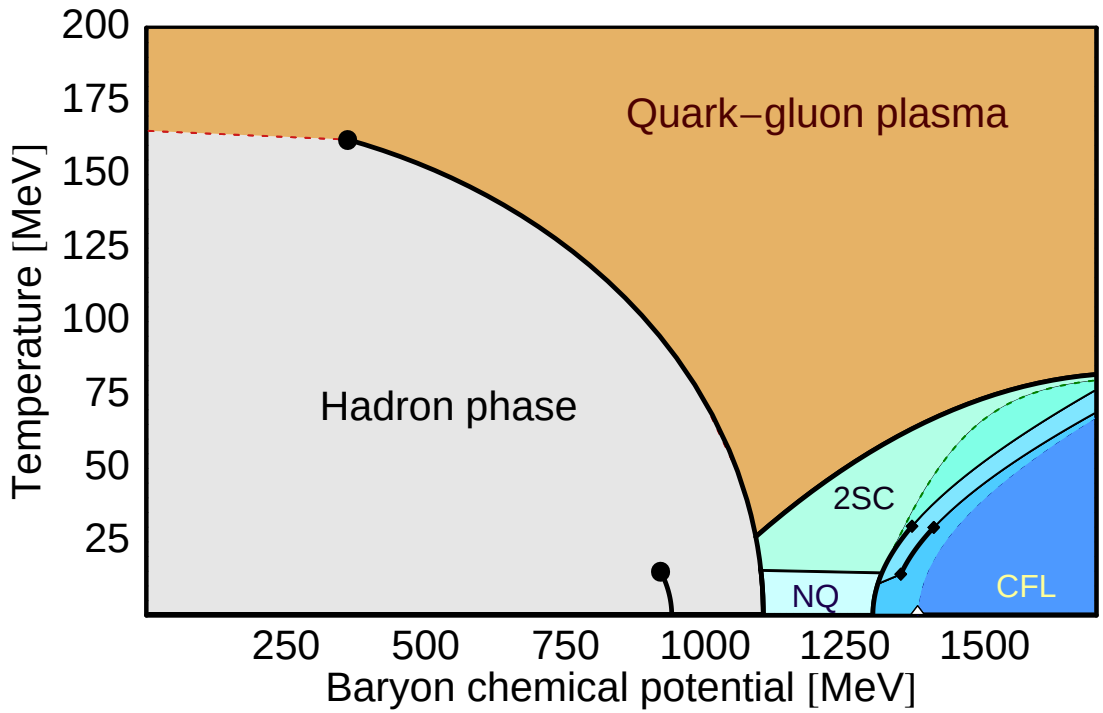
Analyse DBI action to find thermodynamically preferred configuration. 1st order phase transition occurs at $T = 1/L$.

Finite T and μ_B



LT (vertical) vs. $\mu L^2/\lambda$ (horizontal). Baryon density corresponds to electric flux on the branes. Both quarks and baryons can be the sources. There are vacuum, quark and baryon matter phases at finite T and μ_B

Phase diagram of QCD



One version of phase diagram [hep-ph/0503184]. $\mu_B = 0$ line is accessible on the lattice. large μ_B regime is accessible in perturbation theory. Intermediate values of μ_B are hard to analyze.

Is there Fermi liquid at strong coupling?

For large μ_B, small T, the physics of the holographic model is not obviously the same as physics of perturbative QCD.

Leaving QCD aside, we can ask more general question: is there Fermi liquid at strong coupling? Will consider the limit $R_4 \to \infty$; Yang-Mills will provide strong coupling but won't confine.

Review: Fermi liquid

Fermi statistics of quasiparticle excitations. Change in energy due to the change in distribution function δn

$$\delta E = \int \epsilon(k)\delta n(\mathbf{k})d\tau + \int f(\mathbf{k},\mathbf{k}')\delta n(\mathbf{k})\delta n(\mathbf{k}')d\tau d\tau'$$

$\epsilon(k)$ defines dispersion relation and Fermi velocity $\upsilon_F = \frac{\partial\epsilon(k)}{\partial k}|_{k=k_F}$; effective mass $m^* = \frac{k_F}{\upsilon_F}$; and interaction $f(\mathbf{k},\mathbf{k}') = \frac{k_F m^*}{\pi^2\hbar^3}F(\vartheta)$ Convenient to expand: $F(\vartheta) = \sum(2l+1)F_l P_l(\vartheta)$

Review: Fermi liquid

The most important thermodynamical fact about near degenerate Fermi liquid is heat capacity which vanishes linearly with temperature. This is because quasiparticles are excited in the narrow region of momenta $\sim T$ with an average energy $\sim T$.

$$C_v = T\left(\frac{\partial S}{\partial T}\right)_V = aT$$

where $a = m_* q_F/3$.

Review: Fermi liquid

The key dynamical property is the existence of a sound (gapless) excitation at zero temperature. The speed of zero sound u_0 is determined by

$$(s - \cos\theta)\nu(\theta, \varphi) = \cos\theta \int F(\vartheta')\nu(\theta', \varphi')\frac{d\Omega'}{2\pi}$$

where $s = u_0/v_F$ and $\nu(\theta, \varphi)$ parametrizes deformations of spherical Fermi surface; θ denotes polar angle with respect to the direction of sound propagation

Back to branes

In the $R_4 \to \infty$ regime any small temperature leads to the appearance of the black hole horizon. (No confinement.) Will focus on the phase where branes fall into the horizon. Chiral symmetry is restored. Easy to analyze, because brane embedding is simple ($X^4 = const.$)

Thermodynamics

$$S_{D8} \sim \int_{U_T}^{\infty} dU U^{\frac{5}{2}} \left(\sqrt{1 - (\partial_U A_0)^2} - 1 \right) \sim d^{\frac{7}{5}}$$

where $U_T \sim \lambda T^2$ and $d = \partial \mathcal{L} / \partial(\partial_U A_0)$ is proportional to the charge density. The value of chemical potential

$$\mu = \frac{1}{2\pi} \int_{U_T}^{\infty} \frac{d dU}{\sqrt{d^2 + U^5}} \simeq c_1 d^{\frac{2}{5}} + c_2 U_T$$

This implies
$(\partial d / \partial T)_\mu = -(\partial \mu / \partial T)_d / (\partial \mu / \partial d)_T \sim T;$

Thermodynamics

We can use this to compute $S = -(\partial\Omega/\partial T)_\mu$ via $S = (\partial\Omega/\partial T)_d + (\partial\Omega/\partial d)_T(\partial d/\partial T)_\mu \sim T$. This leads to

$$C_v = aT + \mathcal{O}(T^{11})$$

where $a = 16\pi^2\lambda\rho V_3/9$ where ρ is the charge density. Note that this is completely different from the $D3 - D7$ system where $C_v \sim T^6$ [Karch, Son, Starinets]

Thermodynamics

At small temperature we can derive equation of state. $\Omega = -2/7\mathcal{N}\gamma d^{\frac{7}{5}}$. Density is $\rho = 2\pi\mathcal{N}d$; Energy density

$$\epsilon = \Omega + \mu\rho = \frac{5}{7}\mathcal{N}\gamma d^{\frac{5}{7}} = \frac{5}{2}P$$

From this equation we deduce the speed of (normal) sound in the hydrodynamic ($\omega, q \ll T$) regime: $u^2 = 2/5$

Small fluctuations. Zero sound

What about zero sound? (massless excitation in the collisionless regime, $\omega, q \gg T$) Consider gauge field fluctuations on the $D8$ branes at finite μ.

Technically one needs to solve second order differential equation for fluctuations with incoming boundary conditions at $U = 0$. Looking for quasinormal mode with $\omega, q \to 0$. Define $y = 2R^{\frac{3}{2}}/\sqrt{U}$. The boundary is at $y = 0$ and the horizon is at $y = \infty$. Fluctuations of the gauge field $A_\mu = \tilde{A}_\mu(y)e^{iqz-i\omega t}$ don't mix with the embedding fluctuations.

Small fluctuations. Zero sound

Equation for the gauge invariant quantity $E = q\tilde{A}_0 + \omega\tilde{A}_z$ is

$$\ddot{E} + \left(\frac{3}{y} + \frac{\dot{g}}{g} + \frac{\dot{f}_1}{f_1}\frac{q^2}{q^2 - \omega^2 f_1}\right)\dot{E} + \frac{1}{f_1}(\omega^2 f_1 - q^2)E = 0$$

where $\dot{g}/g = -5/(f_1 y)$, $f_1 = 1 + \kappa^{10} y^{10}$, $\kappa \sim (\mu/\lambda)^{\frac{1}{2}}$. In the two regimes, $\kappa y \gg 1$ and $\omega y \ll 1$ equation simplifies and can be solved analytically. For these two regimes to overlap we need $\omega, q \ll \kappa$, which is the regime where we want to discover a sound mode anyway.

Small Fluctuations.

The result is the sound pole $\omega = u_0 q - iq^3/\kappa^2$ where $u_0^2 = 2/5$. Note that the speed of zero sound if equal to that of normal sound in the hydrodynamic regime.

Another interesting question is $\omega = 0$ limit of the current-current correlator. Setting $\omega = 0$ in the equation for small fluctuations implies dependence on dimensionless parameter q/κ. Taking $\omega \to 0$ limit allows fixing boundary conditions at $y = \infty$. The correlator is structureless.

D3-D7 system

Consider a D7 brane in the background of $N \gg 1$ D3 branes at large 't Hooft coupling λ. Field theory: $\mathcal{N} = 4$ SYM coupled to $\mathcal{N} = 2$ fundamental hypermultiplet

Turn on a chemical potential for $U(1)_V$. As far as fermions are concerned, this is a strongly interacting Fermi system. We are going to analyse it from the holographic point of view.

D3-D7 system

Background metric

$$ds^2 = \left(\frac{\rho}{L}\right)^2 (-dt^2 + dx^i dx_i) + \left(\frac{\rho}{L}\right)^{-2} \left(dr^2 + r^2 d\Omega_3^2 + dR^2 + R^2 d\phi^2\right)$$

where $L^4 = 2\lambda$, $\rho^2 = r^2 + R^2$. D7 brane embedding is specified by $R(r)$ [and $\phi = const$]. DBI action is $S \sim \int dr r^3 \sqrt{1 + R'(r)^2 - A_0'(r)^2}$

D3-D7 system

Physical parameters are $R(r \to \infty) = 2\pi m$ and $A_0(r \to \infty) = 2\pi\mu$. For given m, μ there are two solutions:

- Minkowski embedding, $R = m, A_0 = \mu$. (preferred for $\mu < m$)
- Black hole embedding $R' = \frac{c}{\sqrt{r^6+d^2-c^2}}$, $A_0' = \frac{d}{\sqrt{r^6+d^2-c^2}}$ (preferred for $\mu > m$).

where $c \sim m(\mu^2 - m^2)$, $d \sim \mu(\mu^2 - m^2)$ (density) [A.Karch, A.O'Bannon]

This is $T = 0$ limit of first order phase transition at $T \neq 0$

Zero sound

We focus on the gapless excitation in the collisionless regime ($T = 0$). From the bulk point of view, need to find quasinormal mode with dispersion relation $w = \pm u_0 q$.

$m \neq 0$ case is technically more challenging then $m = 0$ case because gauge field fluctuation couple to the embedding fluctuations. But we can go to the vicinity of the critical point!

We expand DBI in fluctuations around classical solution: $A_i = \tilde{A}_i(r) e^{iqz - i\omega t}$, $R = R_0(r) + \tilde{\xi}(r) e^{iqz - i\omega t}$; define $z = 1/r$, $\tilde{q} = \sqrt{2\lambda} q$, $\tilde{\omega} = \sqrt{2\lambda}\omega$

Zero sound

Equations for $E = \tilde{q}\tilde{A}_0 + \tilde{\omega}\tilde{A}_z$ and $\tilde{\xi}$ can be solved in the near horizon regime $z \gg d^{-1/3}$

$$\ddot{E} + \frac{2}{z}\dot{E} + \Omega^2 E = 0$$

Same equation holds for ξ. $\Omega^2 \equiv \tilde{\omega}^2 \frac{d^2 - c^2}{d^2}$ Solutions with incoming boundary conditions at $z = \infty$ are $E(z) = A\frac{e^{+i\Omega z}}{z} \approx i\Omega A + \frac{A}{z}$, $\tilde{\xi}(z) = B\frac{e^{+i\Omega z}}{z} \approx i\Omega A + \frac{A}{z}$ (for $\Omega z \ll 1$).

Zero sound

At the same time, full equations can be solved also in the $\Omega z \ll 1$ regime:

$$\begin{aligned}
\dot{E} &= -\frac{z}{(1+(d^2-c^2)z^6)^{\frac{3}{2}}}[C_1[(1-c^2z^6)\tilde{q}^2 \\
&\quad -(1+(d^2-c^2)z^6)\tilde{\omega}^2]+C_2cdz^6\tilde{q}^2] \\
\dot{\tilde{\xi}} &= \frac{qz}{(1+(d^2-c^2)z^6)^{\frac{3}{2}}}\left[C_1cdz^6 - C_2(1+d^2z^6)\right]
\end{aligned}$$

where C_1, C_2 are arbitrary constants. Integration gives two more constants. The result can be matched to the one for $z \gg d^{-1/3}$. For the two regions to overlap we require $\tilde{\omega}/d^{1/3} \ll 1$.

Zero sound

Requiring $E(z=0) = \tilde{\xi}(z=0) = 0$ (quasinormal mode) we have, at large z,

$$\begin{aligned} E &= b_1 C_1 + b_2 C_2 + \frac{a_1 C_1}{z} + \frac{a_2 C_2}{z} \\ \tilde{\xi} &= \tilde{b}_1 C_1 + \tilde{b}_2 C_2 + \frac{\tilde{a}_1 C_1}{z} + \frac{\tilde{a}_2 C_2}{z} \end{aligned}$$

where b's and a's are proportional to ω^2, q^2. To the leading order in ω, q we have

$b_1 C_1 + b_2 C_2 = 0, \quad \tilde{b}_1 C_1 + \tilde{b}_2 C_2 = 0.$

Zero sound

We end up with two linear equations for C_1, C_2. They admit nontrivial solutions whenever the determinant of the relevant matrix vanishes. Substituting the expressions for b's we arrive at

$$\tilde{\omega}^2 = u_0^2 \tilde{q}^2, \quad u_0^2 = \frac{d^2 - c^2}{3d^2 - c^2}$$

In terms of physical parameters

$$u_0^2 = \frac{\mu^2 - m^2}{3\mu^2 - m^2}$$

This reduces to $1/3$ in the limit $\mu \gg m$ [Karch, Son, Starinets]

Zero sound

The speed vanishes at the critical point, $u_0 \sim (\mu - m)^{\frac{1}{2}}$. From Fermi liquid we expect the same critical behavior, $u_0 \sim k_F \sim (\mu - m)^{\frac{1}{2}}$

This is not consistent with the charge density $d \sim (\mu - m)^{\frac{1}{2}}$. (from Fermi liquid we expect $n \sim k_F^3$.) There is bosonic component in the charge density...

The imaginary part of the dispersion relation behaves like $-iq^2$, which lead to identification of the sound mode with zero sound.

Comments/Conclusions

- Thermodynamics of $D4 - D8$ is crucially different from that of $D3 - D7$! Heat capacity is $\sim T$ for the former but $\sim T^6$ for the latter.
- Zero sound mode exists for both systems. Dissipation is different: $\omega \approx u_0 q - iq^3$ for $D4 - D8$ while $\omega \approx u_0 q - iq^2$ for $D3 - D7$.

Comments/Conclusions

- Do not observe singular behavior of the two-point function.
- Speed of zero sound is equal to the speed of first sound. Speed vanishes at the critical point $\mu = m$ as $(\mu - m)^{\frac{1}{2}}$

Questions

- Can we have a phenomenological theory (a-la Landau) consistent with all the data?
- Interpolation between collisionless and hydro regimes.
- Can we find similar models in Nature?