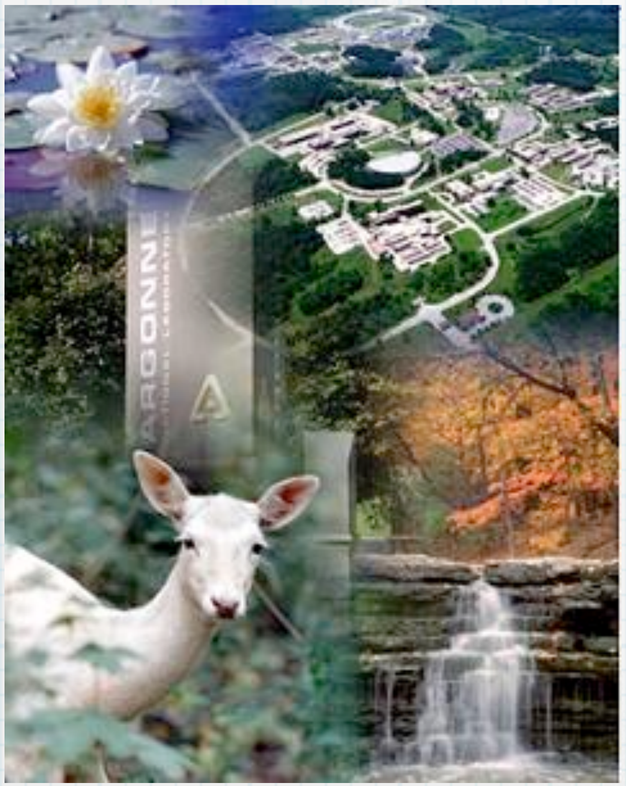
Gravitational and Electromagnetic Form Factors in AdS/QCD



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Dynamics of Symmetry Breaking, ANL, April 13-17, 2009

AdS/CFT correspondence: a link between gauge theories like QCD (in ordinary flat 4D space) and gravitational theories (in a curved 5D space, AdS_5) which may make it possible to carry out accurate calculations in the strong coupling limit of gauge theories.

Outline

* Introduction

  Establish viewpoint

  We do "AdS/QCD" or "bottom-up" approach

* Applications to vector mesons

  Goal is form factors but need masses and w.f. also

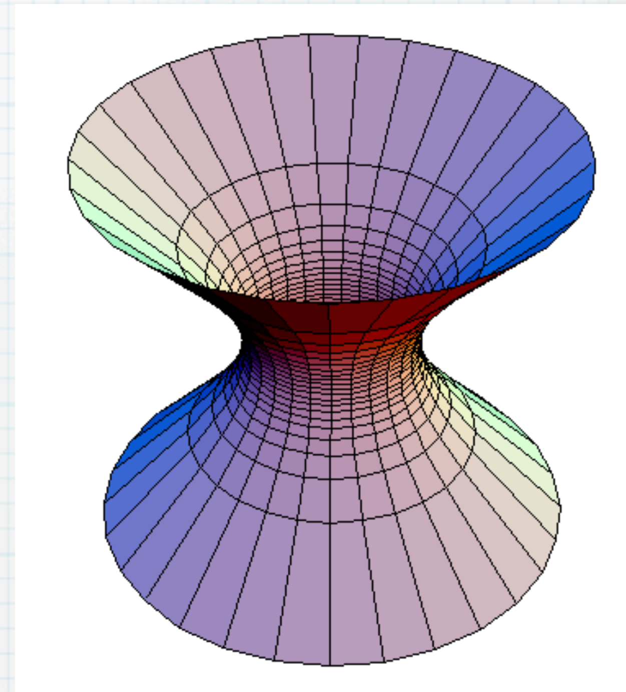
* Applications to baryons

* End!

AdS

One start: AdS$_5$ is 5D hyperboloid embedded in 6D space,

$$\underbrace{t^2 - y_1^2 - y_2^2 - y_3^2}_{\text{ordinary dimensions}} - \underbrace{y_5^2 + y_6^2}_{\text{extra dim.}} = L^2 \quad (= 1)$$



with metric

$$ds^2 = dt^2 - dy_1^2 - dy_2^2 - dy_3^2 - dy_5^2 + dy_6^2$$

- Constant negative curvature
- Invariant under SO(4,2) transformations
- Solves Einstein Eq. for negative cosmological constant Λ
- Usually change variables so that ($0 < z < \infty$)

$$ds^2 = \frac{L^2}{z^2}\left(dx_\mu dx^\mu - dz^2\right) = \frac{L^2}{z^2}\left(dt^2 - d\vec{x}^2 - dz^2\right)$$

4D Conformal Field Theory

Usual symmetries: translation and Lorentz invariance

$$
\begin{aligned}
x_\mu &\rightarrow x'_\mu = x_\mu + a_\mu \\
x_\mu &\rightarrow x'_\mu = \Lambda_\mu^{\ \nu} x_\nu
\end{aligned}
$$

also dilations

$$x_\mu \rightarrow x'_\mu = \lambda\, x_\mu$$

- Commute generators:
  get 4 new operators called special conformal transformations
- commutation relations then close
- 15 generators

"Conformal group" algebra same as SO(4,2)

Original AdS/CFT correspondence

* 4D CFT: SYM (Supersymmetric Yang-Mills), $N_{super} = 4$, N_c colors

$\Leftrightarrow$

  * type IIB string theory, in 10D

* More useful: for CFT take limit
  * $N_c \to \infty$
  * $g_{YM}^2 N_c$ fixed but large

$\Leftrightarrow$

"Real" AdS/CFT correspondence (cont.)

$\Leftrightarrow$

- "low energy" limit of string
  - string looks pointlike
  - has particle-like excitations
  - still supersymmetric: supergravity
  - correspondence has $L^4 = 4\pi g_{YM}^2 N_C / l_s^2$ large (l_s is string length); classical string
  - 10D splits into $AdS_5 \otimes S^5$
- Still too hard

AdS/QCD or “bottom-up”

- Start in the middle: focus on non-supersymmetric particles
- Anticipate 5D AdS_5 action in terms of fields with direct correspondence to recognizable 4D operators
- Choose terms in 5D Lagrangian based on

  simplicity

  symmetries

  relevance to problem at hand

AdS/QCD

E.g., say we are interested in vector mesons

For 5D theory also need gravity. Use action:

$$S_{5D} = \int d^5x \sqrt{g} \left\{ \mathcal{R} + 12 - \frac{1}{2g_5^2} \mathrm{Tr}(F_V^2) \right\}$$

* R = scalar curvature
* Λ = 12 is cosmological constant giving metric for AdS_5

$$F_V^{MN} = \partial^M V^N - \partial^N V^M - i[V^M, V^N]$$

$$V_M(x,z) = V_M^a(x,z)t^a$$ (t^a are n x n matrices)

AdS/QCD 2

- QCD not conformal. E.g., particles have definite masses
- Get masses in 5D theory by deforming AdS to be not quite conformal by, E.g.,
  - restricting the range of z: $0 < z < z_0$

    with BC at z_0 ("hard-wall model")

    or
  - Multiply Lagrangian by factor $e^{-\Phi} = \exp(-\kappa^2 z^2)$

    ("soft-wall model")

AdS/QCD 3

* Need dictionary matching fields in 5D to operators in 4D
* Respect Lorentz symmetry, isospin, parity, etc.
* for vector meson case:

4D operator -- 5D field

$$\Big(\mathcal{O}(x) \quad \leftrightarrow \quad \phi(x,z)\Big)$$

$$\bar{q}\gamma^\mu t^a q = J^{a\mu}(x) \quad \leftrightarrow \quad V^{a\mu}(x,z)$$

$$T_{\mu\nu}(x) \quad \leftrightarrow \quad g_{\mu\nu}(x,z)$$

AdS/CFT 4 --- Correspondence

- General statement: z = 0 boundary of AdS space is a Minkowski space --- our Minkowski space. Need specific connection.

- 4D generating function

$$Z_{4D}[\phi^0] = \left\langle \exp\left(iS_{4D} + i\int d^4x\, \mathcal{O}(x)\phi^0(x) \right) \right\rangle$$

- Used for expectation values

$$\langle \mathcal{O}(x) \ldots \rangle = -i\frac{\delta Z_{4D}}{\delta\phi^0(x)\ldots} = -i\frac{\delta\left(iS_{5D}\right)}{\delta\phi^0(x)\ldots}$$

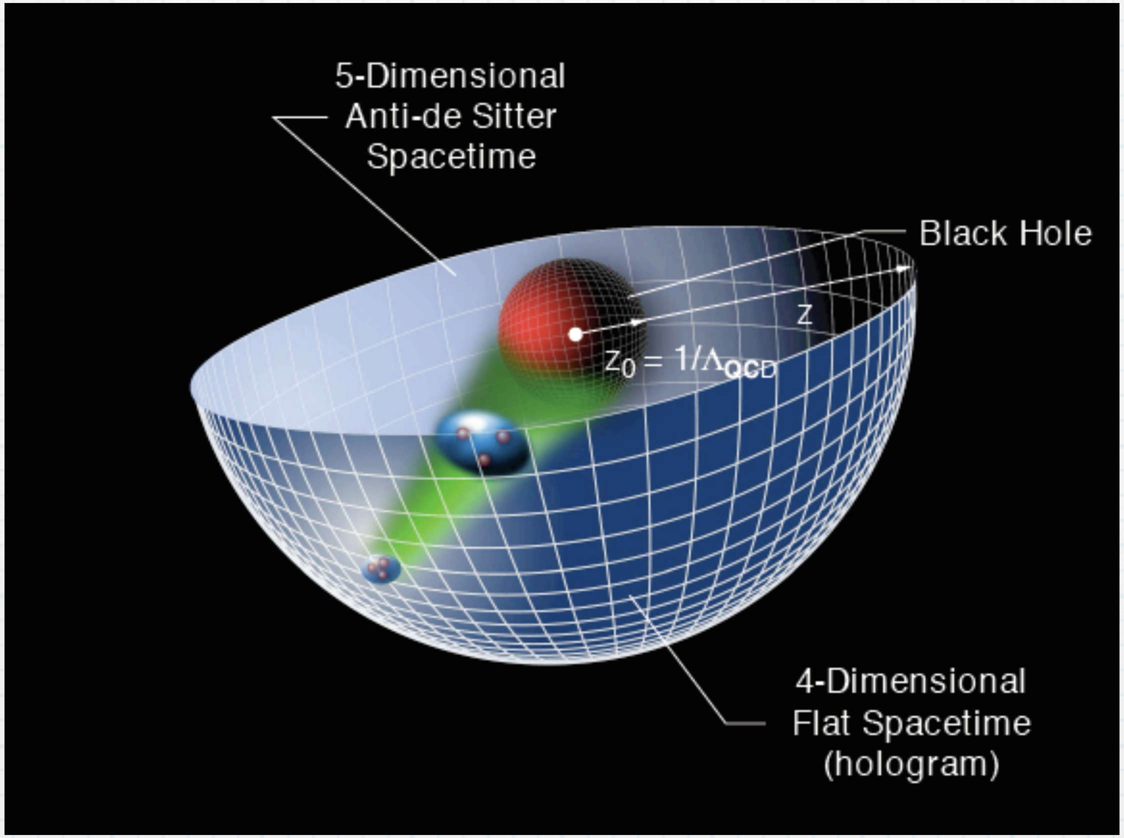
- Correspondence

$$Z_{4D}[\phi^0] = e^{iS_{5D}[\phi_{cl}]}$$

5D action evaluated for classical solutions with boundary condition

$$\lim_{z\to 0}\phi_{cl}(x,z) = \phi^0(x)$$

Understand Stanley's picture



picture borrowed from Stan Brodsky

Some references for this talk

AdS/CFT now extensive field---apologies for all omitted references
Original 1997 Maldacena paper has 6016 citations

Calculations of form factors: "fancy"
Start from string theory, develop QCD analogs on lower dimensional branes — Sakai & Sugimoto

"Bottom-up"
Anticipate what 5D Lagrangian must be (guess), directly involving desired rho, pi, a1, ... fields and connect to matching QCD structures — Erlich et al.; Da Rold & Pomarol

EM form factors in "bottom-up" approach — Brodsky & de Teramond; Radyushkin & Grigoryan

Gravitational form factors in bottom-up approach — Zainul Abidin & me

Soft-wall — Karch, Katz, Son, and Stephanov; Batell, Gherghetta, and Sword

Real calculation: vector fields

Step 1: need masses and w.f. Get from 2-point function

Action

$$S_V = \int d^5x \sqrt{g}\, \mathrm{Tr}\{-\frac{1}{2g_5^2} F_V^2\}$$

$$= -\frac{1}{4g_5^2} \int d^4x \int_\epsilon^{z_0} dz \, \sqrt{g}\, g^{KL} g^{MN}\, V^a_{KM} V^a_{LN}$$

$$g \equiv \det g_{MN} = \frac{L^{10}}{z^{10}}; \quad V^a_{KM} = \partial_K V^a_M - \partial_M V^a_K$$

EoM

$$\left(z\, \partial_z \left(\frac{1}{z} \partial_z V^a_\mu(x,z) \right) - \partial_\nu \partial^\nu V^a_\mu(x,z) \right)_\perp = 0$$

Two-point function for vector fields

Switch to momentum space for x (derivative -> iq)

$$\left(z\,\partial_z \left(\frac{1}{z} \partial_z V_\mu^a(q,z) \right) + q^2 V_\mu^a(q,z) \right)_\perp = 0$$

- q^2 is 4D momentum squared
- Eqn. same for all μ

$$\therefore \qquad V_{\perp\mu}(q,z) = V(q,z) V_\mu^0(q)$$

(V(q,z) = "Profile function")

BC $$V(q,\epsilon) = 1\,; \;\; \partial_z V(q,z_0) = 0$$

"Anyone can see" solutions are Bessel fcns., $z\,J_1(qz)$ & $z\,Y_1(qz)$

w/ BC $$V(q,z) = \frac{\pi}{2} zq \left(\frac{Y_0(qz_0)}{J_0(qz_0)} J_1(qz) - Y_1(qz) \right)$$

Alternative expansion for profile function

Normalizable solutions for expansions. Sturm-Liouville

$$z\,\partial_z\left(\frac{1}{z}\partial_z\psi_n(z)\right)+m_n^2\,\psi_n(z)=0$$

with BC $$\psi_n(0)=0\ \ \&\ \ \partial_z\psi_n(z_0)=0$$

solutions $$\psi_n(z)=const.\times z\,J_1(m_n z)$$

and $$m_n \quad s.t. \quad J_0(m_n z_0)=0$$

Expand $$V(q,z)=-g_5\sum_n\frac{F_n\psi_n(z)}{q^2-m_n^2}$$

two-point function for vector fields

AdS/CFT

$$\langle 0|\,\mathcal{T}\,J^{a\mu}(x)J^{b\nu}(y)\,|0\rangle = -i\frac{\delta^2 S_V}{\delta V^{a0}_{\mu}(x)\delta V^{b0}_{\nu}(y)}$$

5D

Putting solutions in action, get only surface term:

$$S_V = \int \frac{d^4q}{(2\pi)^4} V^{0\mu}(q){V^0}_{\mu}(q)\left(-\frac{\partial_z V(q,z)}{2g_5^2 z}\right)_{z=\epsilon}$$

$\therefore$

$$\boxed{i\int d^4x\, e^{iqx}\,\langle 0|\,\mathcal{T}\,J^a_{\mu}(x)J^b_{\nu}(0)\,|0\rangle = \sum_n \frac{F_n^2\,\delta^{ab}}{q^2 - m_n^2 + i\varepsilon}\left(\eta_{\mu\nu} - \frac{q_\mu q_\nu}{q^2}\right)}$$

Poles in 4D q² (masses) fixed by e'values of 5D equation

Two-point function for vector fields

Alternative: insert complete set, with

$$< 0|J^a_\mu(0)|\rho^b_n(p) >= F_n \delta^{ab} \varepsilon_\mu(p)$$

Get same result, i.e., same F_n.

Using rho mass ➜ $J_0(m_\rho z_0) = 0$ ➜ $\frac{1}{z_0} \equiv \Lambda_{QCD} \approx 0.31 \text{ GeV}$

Form factors

Step 2 (of 2): get form factors from 3-point functions

- EM FF defined from matrix elements of an EM current,

$$\left\langle \rho_n^a(p_2,\lambda_2) \middle| J^\mu(0) \middle| \rho_n^b(p_1,\lambda_1) \right\rangle$$

- gravitational FF using the stress [energy-momentum] tensor

$$\left\langle \rho_n^a(p_2,\lambda_2) \middle| \hat{T}^{\mu\nu}(0) \middle| \rho_n^b(p_1,\lambda_1) \right\rangle$$

- illustrate with latter

Three point functions: form factors

Start with $$\langle 0 | \mathcal{T}\left(J_a{}^{\alpha}(x) T^{\mu\nu}(y) J_b{}^{\beta}(w)\right) |0\rangle$$

Insert complete sum (twice) $$\sum_n \int \frac{d^3 p}{(2\pi)^3 2p^0} |\rho_n^a(p)\rangle\langle\rho_n^a(p)| = 1$$

Isolate $$\left\langle \rho_n^a(p_2,\lambda_2) \middle| \hat{T}^{\mu\nu}(0) \middle| \rho_n^b(p_1,\lambda_1) \right\rangle$$

$$= \lim_{p_1^2, p_2^2 \to m_n^2} \varepsilon_\alpha^*(p_2,\lambda_2)\varepsilon_\beta(p_1,\lambda_1)\left(p_1^2 - m_n^2\right)\left(p_2^2 - m_n^2\right)$$

$$\times \frac{1}{F_n^2} \langle 0 | \mathcal{T}\left(J_a{}^{\alpha}(p_2) \hat{T}^{\mu\nu}(0) J_b{}^{\beta}(p_1)\right) |0\rangle$$

Three point functions: form factors

AdS/CFT $$T_{\mu\nu}(x) \quad \leftrightarrow \quad h_{\mu\nu}(x,z)$$

for $$g_{\mu\nu}(x,z) = \frac{1}{z^2}\left(\eta_{\mu\nu} + h_{\mu\nu}(x,z)\right)$$

Can also do 2-point function calculation for $h_{\mu\nu}$,

and get 5D $h_{\mu\nu}$ as boundary term $\times$ profile function,

$$h_{\mu\nu}(q,z) = h^0_{\mu\nu}(q)\mathcal{H}(q,z)$$

AdS/CFT,

$$\langle 0|\mathcal{T} J^\alpha(x)\hat{T}^{\mu\nu}(y)J^\beta(w)|0\rangle = \frac{-2\,\delta^3 S_{5D}}{\delta V^0_\alpha(x)\delta h^0_{\mu\nu}(y)\delta V^0_\beta(w)}$$

Three point functions: form factors

Relevant part of S_{5D}

$$S_{5D} \;\overrightarrow{\supset}\; -\frac{1}{4g_5^2}\int d^5x\sqrt{g}\,g^{lm}g^{pn}V^a_{mn}V^a_{lp}$$

$$\overrightarrow{\supset}\; \frac{1}{2g_5^2}\int \frac{d^5x}{z}\left(\eta^{\rho\gamma}\eta^{\sigma\delta}h_{\gamma\delta}\left[-V_{\sigma z}V_{\rho z}+\eta^{\alpha\beta}V_{\sigma\alpha}V_{\rho\beta}\right]\right)$$

Proceed, and define form factors

$$\left\langle \rho^a_n(p_2,\lambda_2)\middle|T^{\mu\nu}(0)\middle|\rho^b_n(p_1,\lambda_1)\right\rangle = \varepsilon^*_{2\alpha}\varepsilon_{1\beta}$$

$$\times\Big\{-2A(q^2)\eta^{\alpha\beta}p^\mu p^\nu$$

$$-4\left(A(q^2)+B(q^2)\right)q^{[\alpha}\eta^{\beta](\mu}p^{\nu)}+\ 4\text{ more}\Big\}$$

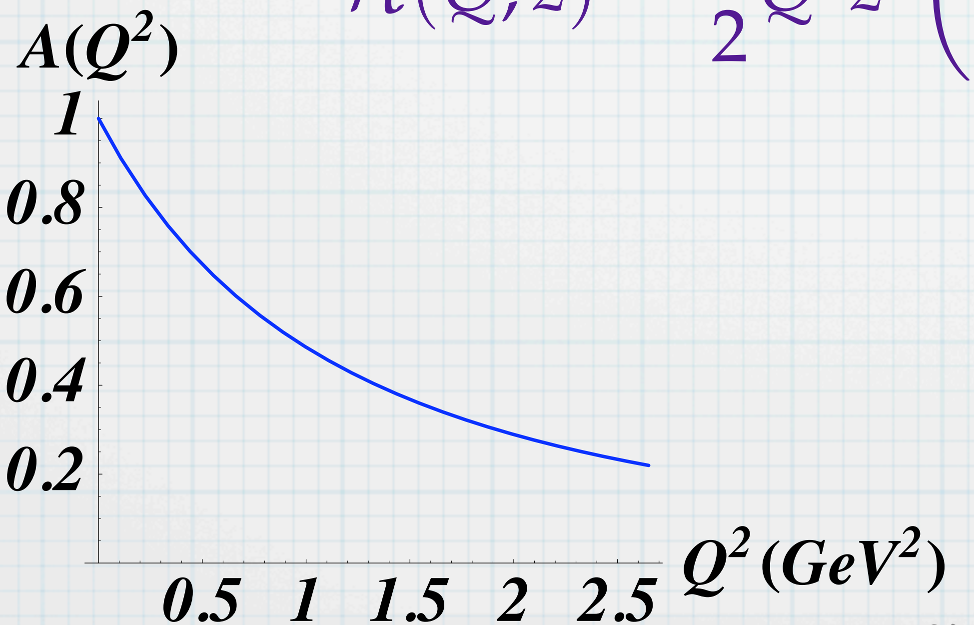
Three point functions: form factors

Obtain

$$
\begin{aligned}
A(q^2) &= \int_0^{z_0} \frac{dz}{z} \mathcal{H}(Q,z)\psi_n(z)\psi_n(z) \\
B(q^2) &= 0 \\
&\dots
\end{aligned}
$$

ψ_n **already known,**

$$\mathcal{H}(Q,z) = \frac{1}{2} Q^2 z^2 \left(\frac{K_1(Qz_0)}{I_1(Qz_0)} I_2(Qz) + K_2(Qz) \right)$$



Three point functions: form factors

Gravitational radius

$$\left\langle r^2 \right\rangle_{\text{grav}} = -6 \left. \frac{\partial A}{\partial Q^2} \right|_{Q^2=0} = \frac{3.24}{m_\rho^2} = 0.21 \text{ fm}^2$$

- Can do same for EM form factors

$$\left\langle r^2 \right\rangle_{\text{C}} = -6 \left. \frac{\partial G_C}{\partial Q^2} \right|_{Q^2=0} = 0.53 \text{ fm}^2$$

- Momentum density in rho more concentrated than charge

AdS form factors: asymptotic limits

More commentary on EM form factors

- $\exists$ three EM form factors for spin-1 particles, G_C, G_M & G_Q
- Old high Q^2 PQCD result $G_C = (Q^2/6m_n^2)G_Q$

  satisfied by AdS/CFT rho meson results

- Stronger results (Brodsky-Hiller) follow if corrections to dominant PQCD are neglected,

$$G_C : G_M : G_Q = (1 - \frac{Q^2}{6M^2}) : 2 : -1$$

exactly satisfied by AdS/CFT results

AdS form factors: why gravitational?

Related to integrals of generalized parton distributions (spin-1)

$$\int_{-1}^{1} x dx \, H_1(x,\xi,t) = A(t) - \xi^2 C(t) + \frac{t}{6m_n^2} D(t)$$

$$\int_{-1}^{1} x dx \, H_2(x,\xi,t) = 2\left(A(t) + B(t)\right)$$

Relations between the stress tensor and the momentum and angular momentum operators lead to

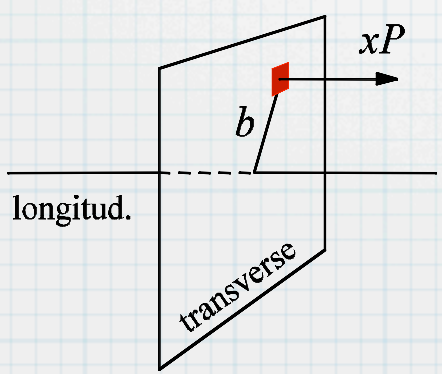
$$A(0) = 1$$

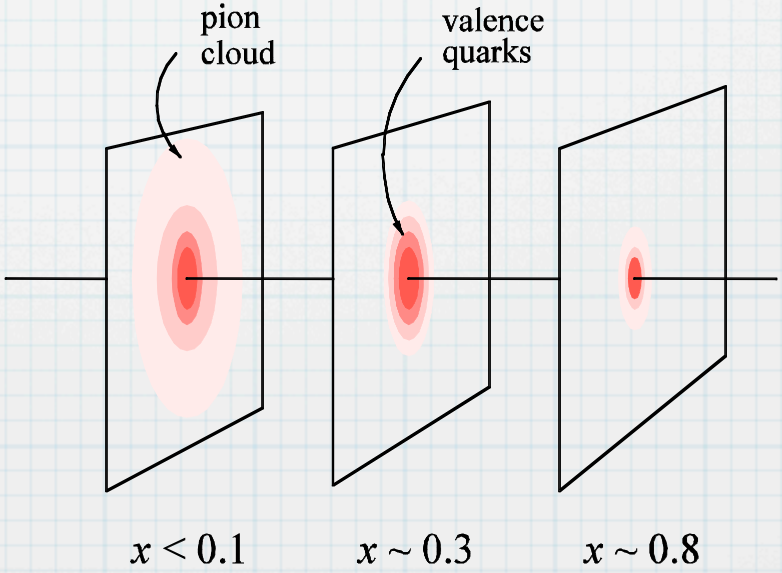
$$A(0) + B(0) = (J_z)_{\max} = 1$$

Quark flavor specific version of latter exploited by X. Ji in his "killer application" for the GPDs.

Concentration of momentum density

- A(Q2) measuring distribution of momentum density; EM form factors measure distribution of charge. Relating to integrals over GPDs, EM form factors are moments without x factor (i.e., x^0).
- Known picture: higher x quarks have more compact distribution in transverse directions. Hence x weighted integral will have smaller average transverse size.





Baryon states

* Page of credits:

  Henningson and Sfetsos; Mueck and Viswanathan

  Contino and Pomarol

  Hong, Inami, and Yee; Hong, Rho, Yee, and Yi

  Brodsky and de Téramond

  Hata, Sakai, Sugimoto, and Yamato

  Pomarol and Wulzer

Baryon states

- Important: work only with independent degrees of freedom. 4-spinor fermions have redundant components.

$$\Psi_{R,L} = \frac{1}{2}(1 \pm \gamma^5)\Psi$$

- Pick one, say Ψ_L. Determine other from Eq. of motion from

$$\Psi_{R,L}(p,z) = z^{\Delta} f_{R,L}(p,z)\; \Psi^0(p)_{R,L}$$

4d boundary field

profile function

and

$$f_R = \frac{1}{p}\left(\partial_z - \frac{d/2 - M - \Delta}{z}\right) f_L$$

Baryon states

- Version: have fundamental fermion fields in 5D interacting with AdS gravitational background
- (Alternative not pursued: treat the fermions as Skyrmions within the 5D model)

$$
\begin{aligned}
S_F &= \int d^{d+1}x\sqrt{g}\,e^{-\Phi(z)}\Big(\frac{i}{2}\bar{\Psi}e^N_A\Gamma^A D_N\Psi \\
&\quad -\frac{i}{2}(D_N\Psi)^\dagger\Gamma^0 e^N_A\Gamma^A\Psi - (M+\Phi(z))\bar{\Psi}\Psi\Big)
\end{aligned}
$$

- Soft wall $\Phi = \kappa^2 z^2$ or hard wall has BC and $\Phi = 0$.

$$
\begin{aligned}
e^N_A &= z\delta^N_A \quad \text{(inverse vielbein)}; \quad \Gamma^A = (\gamma^\mu, -i\gamma^5) \\
D_N &= \partial_N + \frac{1}{8}\omega_{NAB}[\Gamma^A,\Gamma^B] - iV_N
\end{aligned}
$$

Baryon states

* Same procedure: find profile functions, then get 3-point functions ($\rightarrow$ form factors).

* For variation, give results for soft wall model

$$f_L(p,z) = N_L\, \xi^{\alpha} U\left(\alpha - \frac{p^2}{4\kappa^2}, \alpha+1; \xi\right)$$

(Kummer function of 2nd kind) $\qquad \alpha = M + \frac{1}{2}$

* or expanding in terms of normalizable solutions,

$$f_L(p,z) \quad = \quad \sum_{n=0}^{\infty} \frac{f_n m_n\, \psi_L^{(n)}(z)}{p^2 - m_n^2}$$

where we have decay constants and masses from solving Eq. of motion, $\qquad m_n^2 = 4\kappa^2(n+\alpha)$

Three point functions

- For fermion fields, generating function uses $\Psi_L^{(0)}$ as source of 4D operator O_R.

$$\mathcal{Z}_{CFT}[\Psi_L^0, \bar{\Psi}_L^0, \ldots] = \left\langle e^{i\int d^dx\left(\bar{\mathcal{O}}_R(x)\Psi_L^0(x)+h.c.+\ldots\right)}\right\rangle = e^{iS_{AdS}(\Psi_L^{cl},\ldots)}$$

- Derivatives give VEVs, and from VEVs project matrix elements that give form factors,

$$\lim_{p^0_{1,2}\to E^n_{1,2}} (p_1^2 - m_n^2)(p_2^2 - m_n^2)\int d^4x d^4y$$

$$\times e^{i(p_2x - qy - p_1w)}\left\langle 0|\mathcal{T}\mathcal{O}_R^i(x)J^{a\mu}(y)\bar{\mathcal{O}}_R^j(w)|0\right\rangle$$

$$= f_n^2\, u(p_2,s_2)\left\langle p_2,s_2|J^{a\mu}(0)|p_1,s_1\right\rangle \bar{u}(p_1,s_1)\ \times\delta^{(4)}(p_2-p_1-q)$$

Three point functions

* For the record,

$$\langle p_2, s_2 | J^\mu(0) | p_1, s_1 \rangle = u(p_2, s_2) \left(F_1(Q^2)\gamma^\mu + F_2(Q^2) \frac{i\sigma^{\mu\nu} q_\nu}{2m_n} \right) u(p_1, s_1)$$

* Part of action giving 3-point function,

$$S_F = \int d^5x \sqrt{g} e^{-\Phi} \bar{\Psi} e^M_A \Gamma^A V_M \Psi$$

* Gives o.k. F_{1p}, but no F_2 and zero F_{1n}.

Three point functions

- **Inlcude further isoscalar and isovector terms,**

$$S_{\eta_{S,V}} = \eta_{S,V} \int d^5x \, \sqrt{g} e^{-\Phi} \, \frac{i}{2} \bar{\Psi} \, e^M_A \, e^N_B \, [\Gamma^A, \Gamma^B] \, F^{(S,V)}_{MN} \, \Psi$$

- **Result (algebraic)**

$$C_1(Q) = \int \tfrac{dz}{2z^{2M}} \, e^{-\Phi} \, V(Q,z) \left({\psi_L}^2(z) + {\psi_R}^2(z) \right) ,$$

$$C_2(Q) = \int \tfrac{dz}{2z^{2M-1}} \, e^{-\Phi} \, \partial_z V(Q,z) \left({\psi_L}^2(z) - {\psi_R}^2(z) \right) ,$$

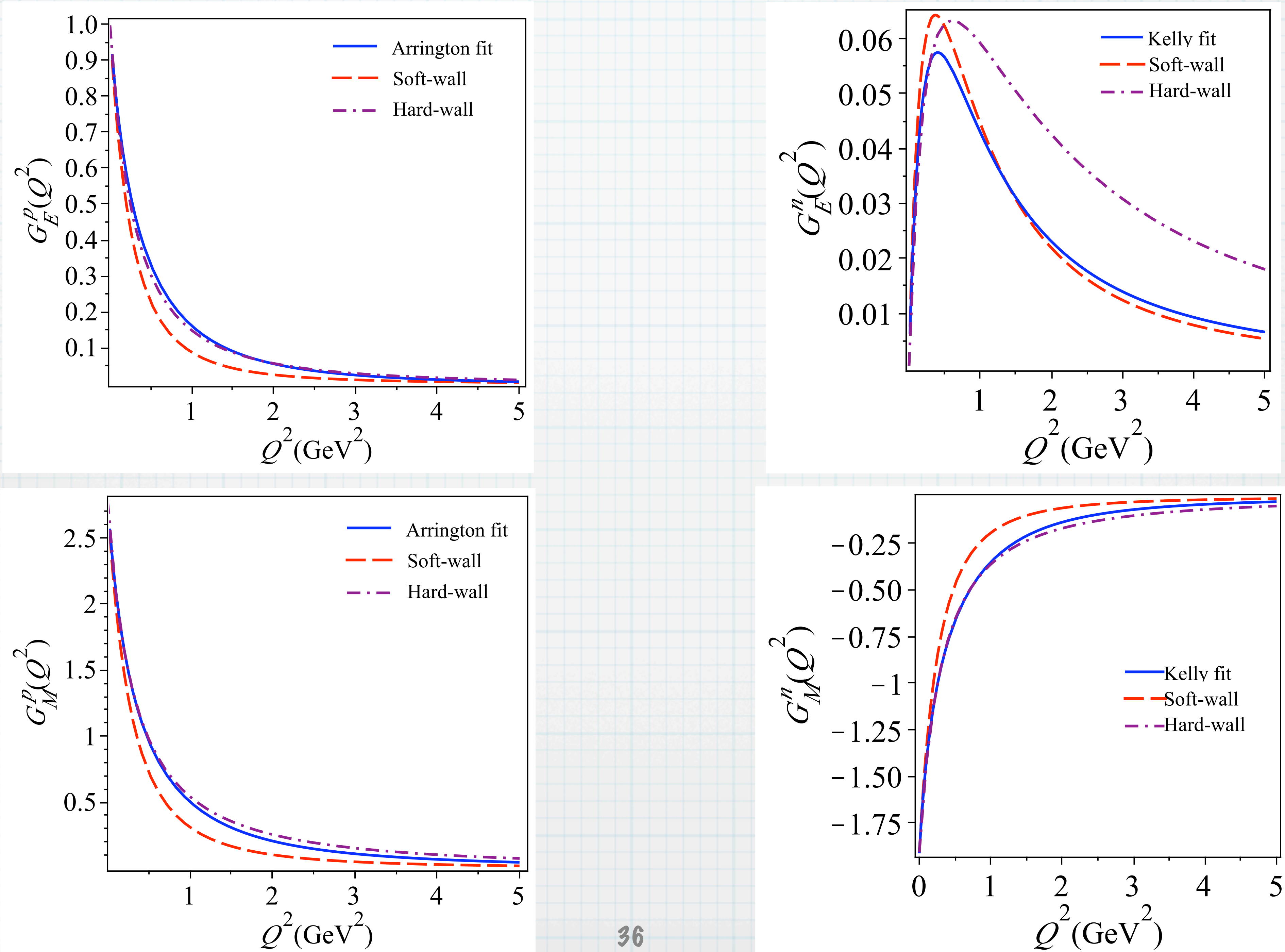
$$C_3(Q) = \int \tfrac{dz}{z^{2M-1}} \, e^{-\Phi} \, 2m_n V(Q,z) \psi_L(z)$$

with

$$F_1^{(P)}(Q) \quad = \quad C_1(Q) + \eta_P C_2(Q), \quad F_2^{(P)}(Q) = \eta_P C_3(Q)$$

and similarly for neutron

Nucleon form factors in pictures

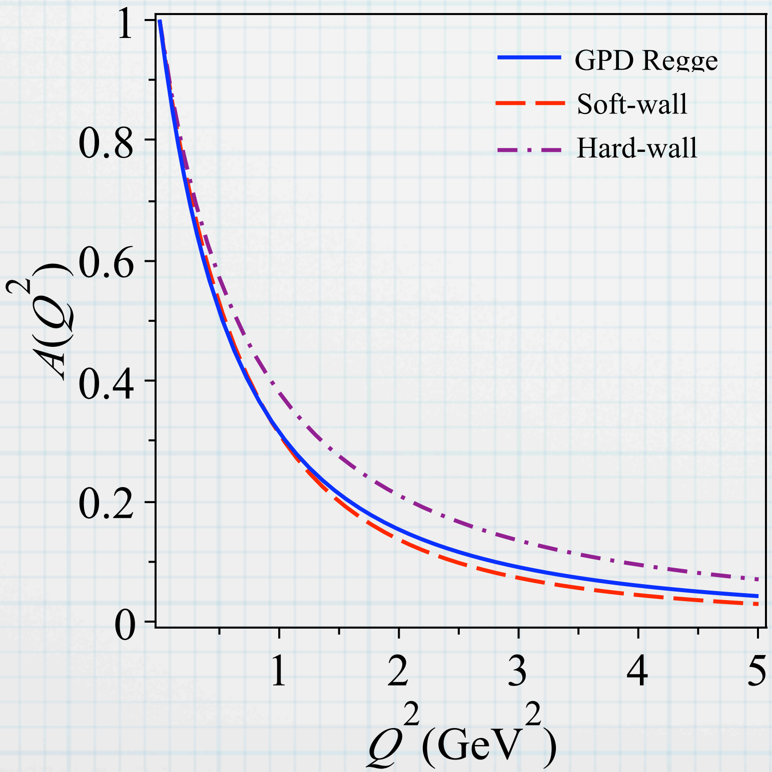


Form factors

- **Same for gravitational form factors**

$$\langle p_2, s_2 | T^{\mu\nu}(0) | p_1, s_1 \rangle = u(p_2, s_2) \Big(A(Q) \gamma^{(\mu} p^{\nu)}$$

$$+ B(Q) \frac{i p^{(\mu} \sigma^{\nu)\alpha} q_\alpha}{2 m_n} + C(Q) \frac{q^\mu q^\nu - q^2 \eta^{\mu\nu}}{m} \Big) u(p_1, s_1)$$



Charge and gravitational radii

* Proton charge radius

$$\left\langle r_C^2 \right\rangle_p = -\frac{6}{G_E(0)} \frac{dG_E(0)}{dQ^2} = (0.961\,\mathrm{fm})^2 \qquad [\mathrm{data},\ 0.877\,\mathrm{fm}]$$

* Neutron charge radius

$$\left\langle r_C^2 \right\rangle_n = -6 \frac{dG_{En}(0)}{dQ^2} = (-0.136\,\mathrm{fm}^2) \qquad [\mathrm{data},\ -0.112\,\mathrm{fm}^2]$$

* Gravitational (momentum density) charge radius

$$\left\langle r_A^2 \right\rangle = -\frac{6}{A(0)} \frac{dA(0)}{dQ^2} = (0.575\,\mathrm{fm})^2 \qquad [\mathrm{GPD\ model},\ 0.608\,\mathrm{fm}]$$

* Same feature of smaller gravitational radius

$A(Q^2)$ measured?

* Yesterday: report of Domokos et al. fitting pp cross sections using pomeron trajectory with vertex function given in terms of gravitational form factor $A(Q^2)$.

  Reverse: they parameterize

  $$A(Q^2) = \frac{1}{(1 + Q^2/M^2)^2}$$

  and determine that **M = 1.02 GeV.**

* Thus obtain form factor stiffer than EM.

* Result here (converting gravitational radius):

$$M = \begin{cases} 1.19\ \text{GeV} & \text{AdS/QCD} \\ 1.12\ \text{GeV} & \text{GPD model} \end{cases}$$

Summary/Conclusions

- Connection between 5D theories with gravitational interactions and 4D conformal or QCD-like theories.
- Did less-fancy AdS/QCD starting with idea of what 5D action must be. Sometimes called “bottom up” approach.
- Calculate two-point functions---propagators with interactions---to obtain masses and decay constants.
- Main target was form factors, obtained from three-point (vertex) functions Results for both EM and gravitational FF, and for both mesons (including scalar and axial mesons) and baryons.
- Particles appear smaller viewed gravitationally than electromagnetically. Momentum density is more concentrated than charge.

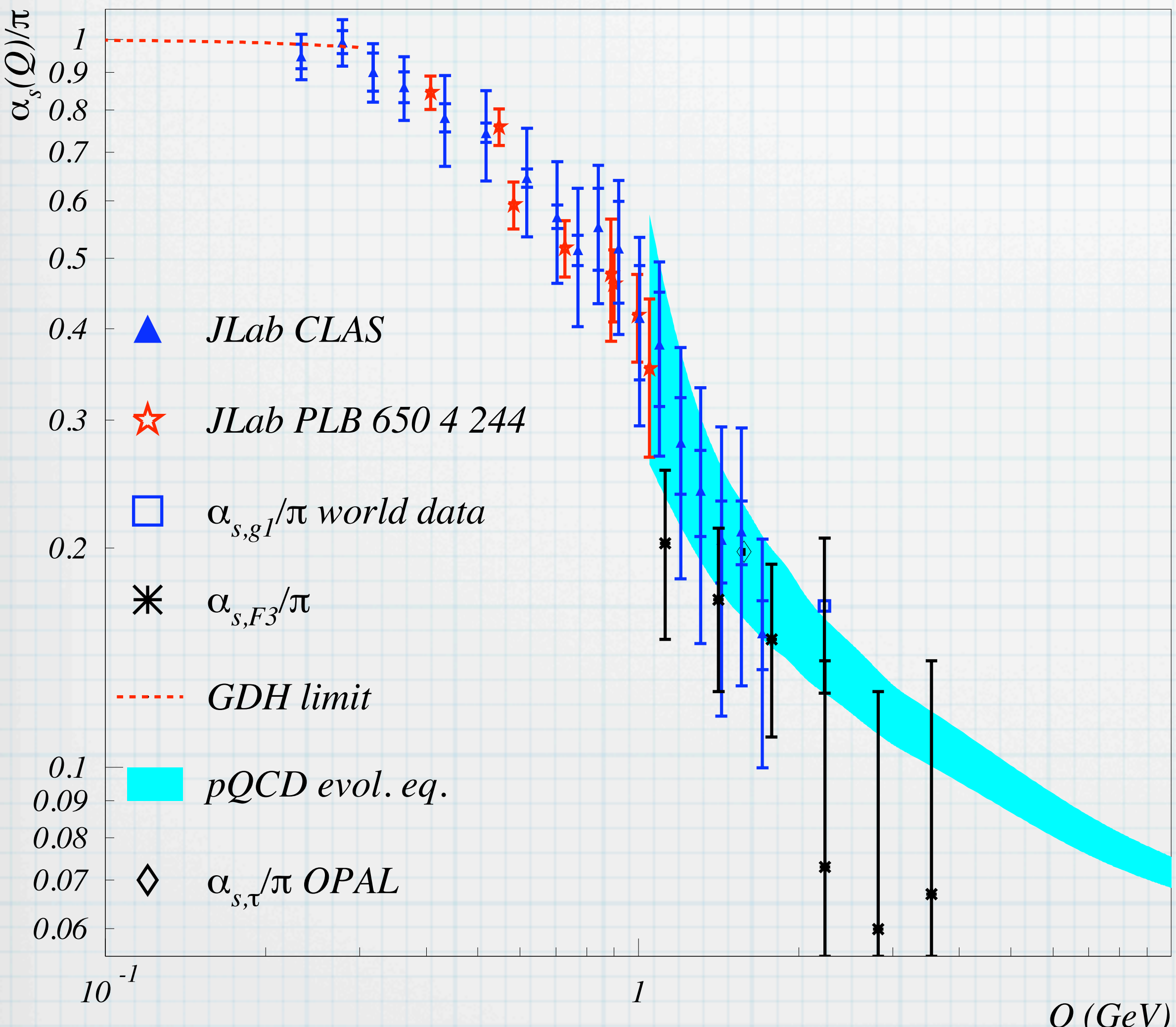
The End

Extra slides

Nearly conformal QCD?

Define α_s from Björkén sum,

$$\Gamma_1^{p-n} \equiv \int_0^1 dx\,\left(g_1^p(x,Q^2) - g_1^n(x,Q^2)\right) = \frac{1}{6} g_A \left(1 - \frac{\alpha_{s,g_1}}{\pi}\right)$$



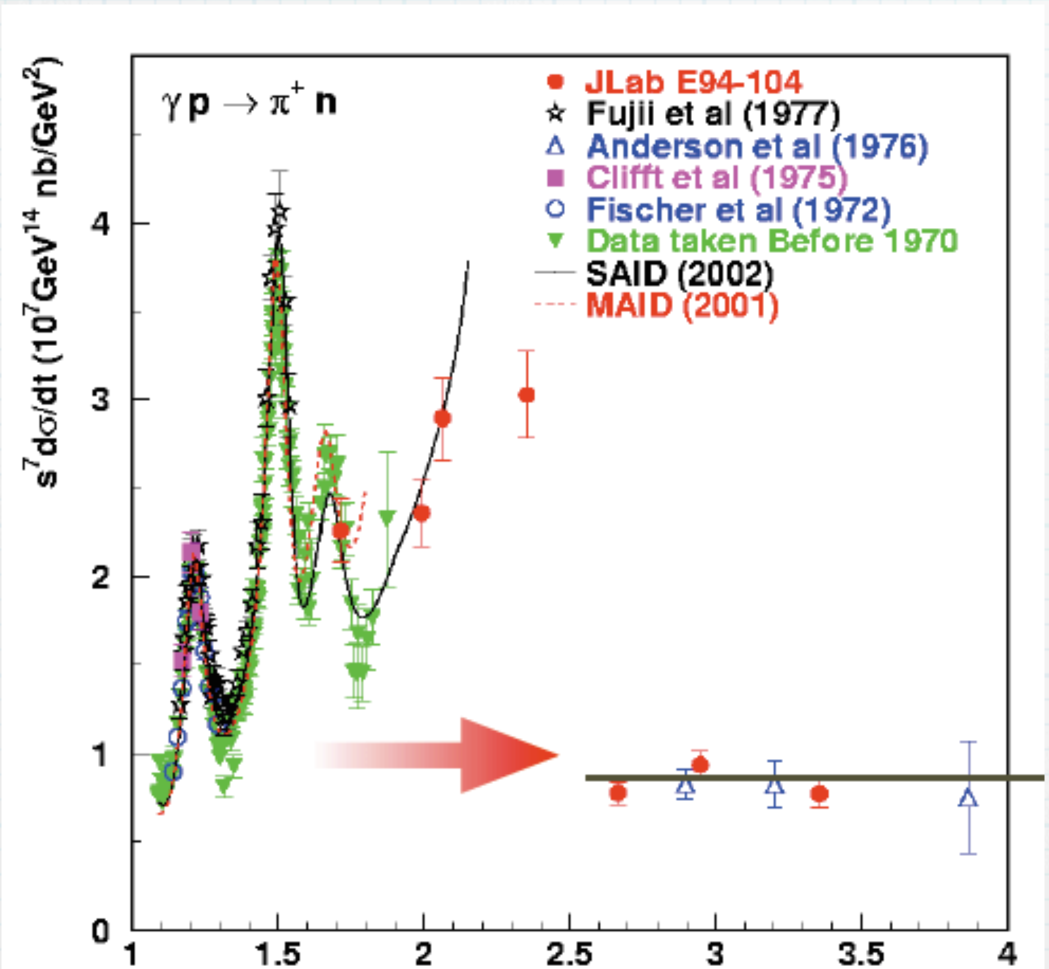
g1 = spin dependent structure function (from inelastic ep scattering

Data from EG1 exp., at JLab CLAS (2008)

α_s runs only modestly at small Q^2

Fig. from 0803.4119, Duer et al.

Nearly conformal QCD?

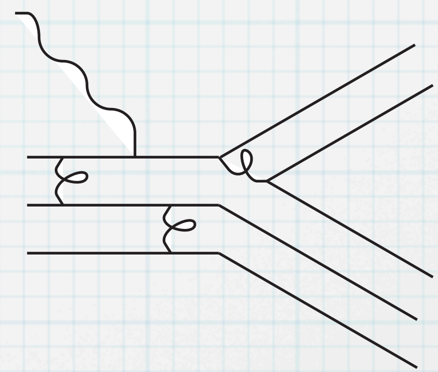


$\sqrt{s}$ (GeV)

PQCD suggests

$$s^7 \frac{d\sigma}{dt}(\gamma p \to \pi^+ n) = const \times \alpha_s^6(t)$$

from diagrams like



running of α_s not seen here;

α_s does not run in conformal thy.

The axial sector: pions and axial mesons

restart

$$S_{5D} = \int d^5x \sqrt{g} \left\{ \mathcal{R} + 12 + \mathrm{Tr}\left[|DX|^2 + 3|X|^2 - \frac{1}{4g_5^2}(F_L^2 + F_R^2) \right] \right\}$$

with $X(x,z) = \frac{1}{2} v(z) \mathbb{1} \exp(2it^a \pi^a)$; $A = \frac{1}{2}(A_L - A_R)$

To 2nd order in axial fields,

$$S_{5D}^{(A)} = \int d^5x \sqrt{g} \left[\frac{v(z)^2}{2} g^{MN} (\partial_M \pi^a - A_M^a)(\partial_N \pi^a - A_N^a) - \frac{1}{4g_5^2} g^{KL} g^{MN} F^a{}_{KM} F^a_{LN} \right]$$

$$F^a_{KM} = \partial_K A^a_M - \partial_M A^a_K$$

The axial sector: pions and axial mesons

EoM

$$
\begin{aligned}
\partial_z\left(\frac{1}{z}\partial_z A^a_{\nu\perp}\right)+\frac{q^2}{z}A^a_{\nu\perp}-\frac{g_5^2 v^2}{z^3}A^a_{\nu\perp} &= 0\\
\partial_z\left(\frac{1}{z}\partial_z\phi^a\right)+\frac{g_5^2 v^2}{z^3}\left(\pi^a-\phi^a\right) &= 0\\
-q^2\partial_z\phi^a+\frac{g_5^2 v^2}{z^2}\partial_z\pi^a &= 0
\end{aligned}
$$

(Split A into transverse and longitudinal part $A^a_\mu = A^a_{\mu\perp}+\partial_\mu\phi^a$)

For looking at two point functions, $q^2 \to m_\pi^2 \to 0$ (chiral limit) in last equation, and

$$
\partial_z\left(\frac{1}{z}\partial_z\psi^a(q,z)\right)-\frac{g_5^2 v^2}{z^3}\psi^a(q,z)=0
$$

$(\psi^a \equiv \phi^a-\pi^a)$

The axial sector: pions and axial mesons

Choice $$v(z) = \sigma z^3$$

Profile function $$\psi^a(q,z) = \psi^{0a}(q)\Psi(z)$$

Get $$\Psi(z) = z\Gamma[2/3]\left(\frac{\alpha}{2}\right)^{\frac{1}{3}}\left(I_{-\frac{1}{3}}(\alpha z^3) - I_{\frac{1}{3}}(\alpha z^3)\frac{I_{\frac{2}{3}}(\alpha z_0^3)}{I_{-\frac{2}{3}}(\alpha z_0^3)}\right)$$

$$(\text{for } \alpha = g_5\sigma/3)$$

The axial sector: pions and axial mesons

Study two-point functions to find masses and decay constants

Along way, define

$$\left\langle 0 \middle| J^a_{A,\mu}(0)_{||} \middle| \pi^b(p) \right\rangle = i f_\pi p_\mu \delta^{ab}$$

Learn

$$f_\pi^2 = -\left.\frac{\partial_z \Psi(z)}{g_5^2 z}\right|_{z=\epsilon}$$

Use f_π = 92.4 MeV to fix σ.

Find

	AdS/CFT	data
m_{a1}	1376 MeV	1230 MeV
$(F_{a1})^{1/2}$	493 MeV	433 MeV

The axial sector: pions and axial mesons

Get gravitational form factors

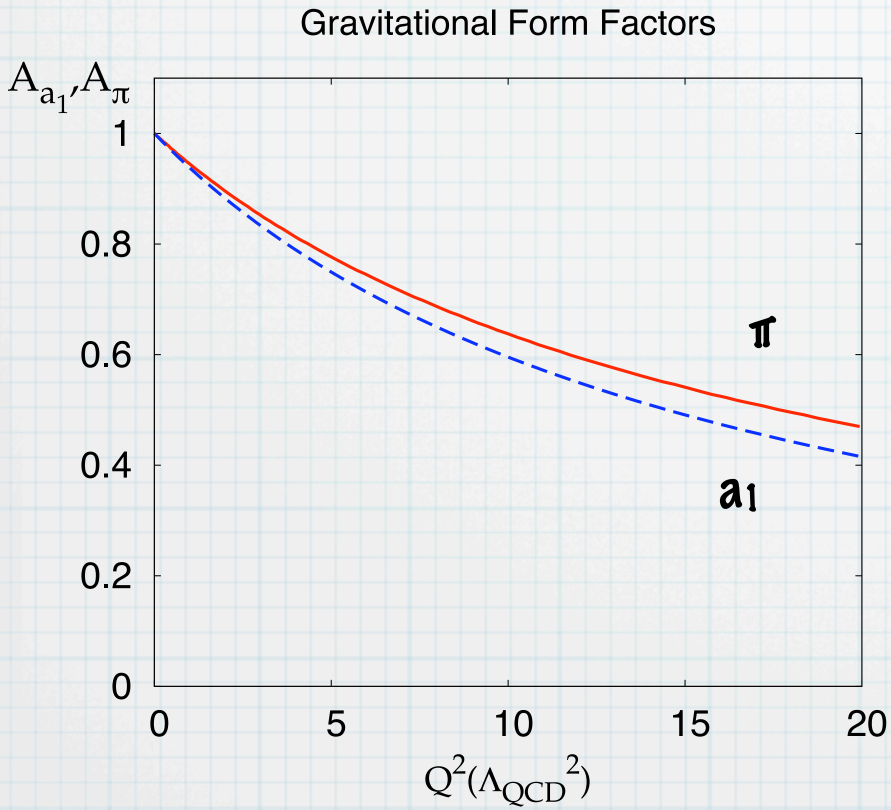
$$\left\langle \pi^a(p_2)|T^{\mu\nu}(0)|\pi^b(p_1)\right\rangle =$$
$$\delta^{ab}\left[2A_\pi(Q^2)p^\mu p^\nu + \frac{1}{2}C_\pi(Q^2)\left(q^2\eta^{\mu\nu} - q^\mu q^\nu\right)\right]$$

from three point function as before:

- find part of S_{5D} linear in $h_{\mu\nu}$ and quadratic in axial fields
- do functional derivative to find three-point function
- etc.

The axial sector: pions and axial mesons

Find $$A_\pi(Q^2) = \int dz\, \mathcal{H}(Q,z) \left(\frac{(\partial_z \Psi(z))^2}{g_5^2 f_\pi^2 z} + \frac{v(z)^2 \Psi(z)^2}{f_\pi^2 z^3} \right)$$



radii

$$\left\langle r_\pi^2 \right\rangle_{\text{grav}} = 0.13\,(\text{fm})^2 = (0.36\,\text{fm})^2$$

Cf.

$$\left\langle r_\pi^2 \right\rangle_{C} = \begin{cases} (0.57\,\text{fm})^2 & \text{AdS/CFT} \\ (0.67\,\text{fm})^2 & \text{data} \end{cases}$$

- Again, energy in constituents more spatially concentrated than charge