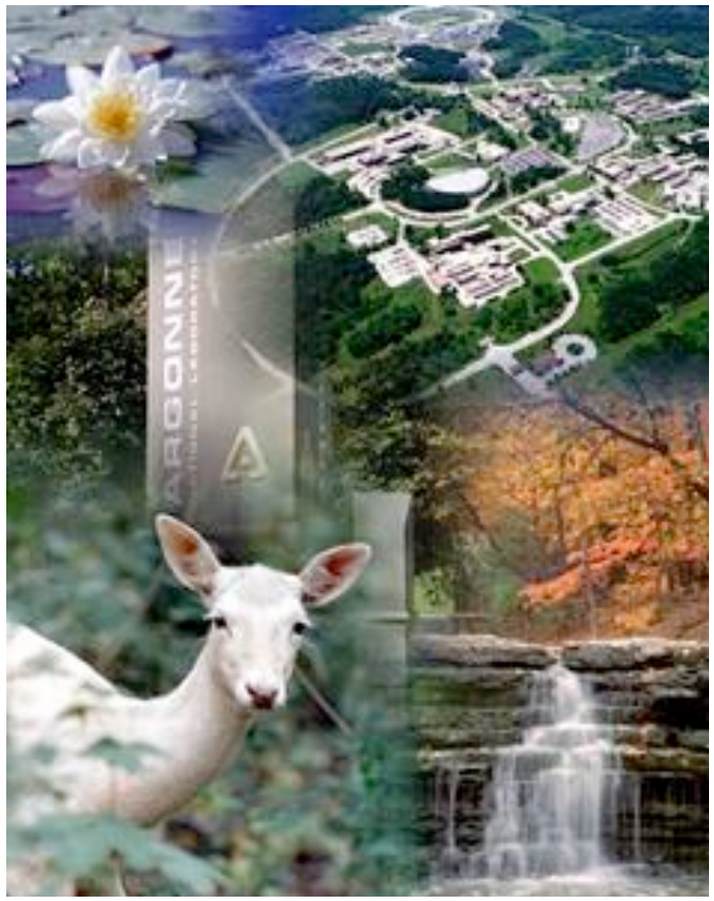
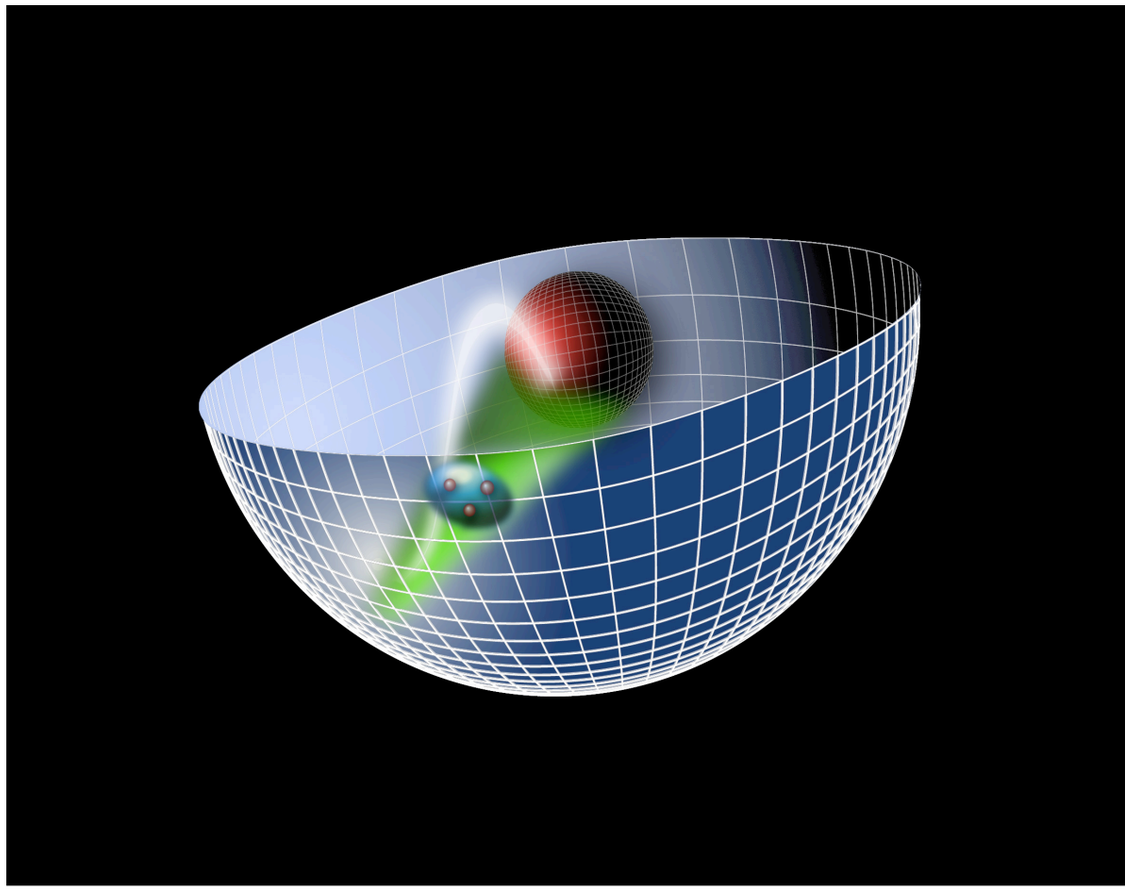
AdS/QCD and Light-Front Holography





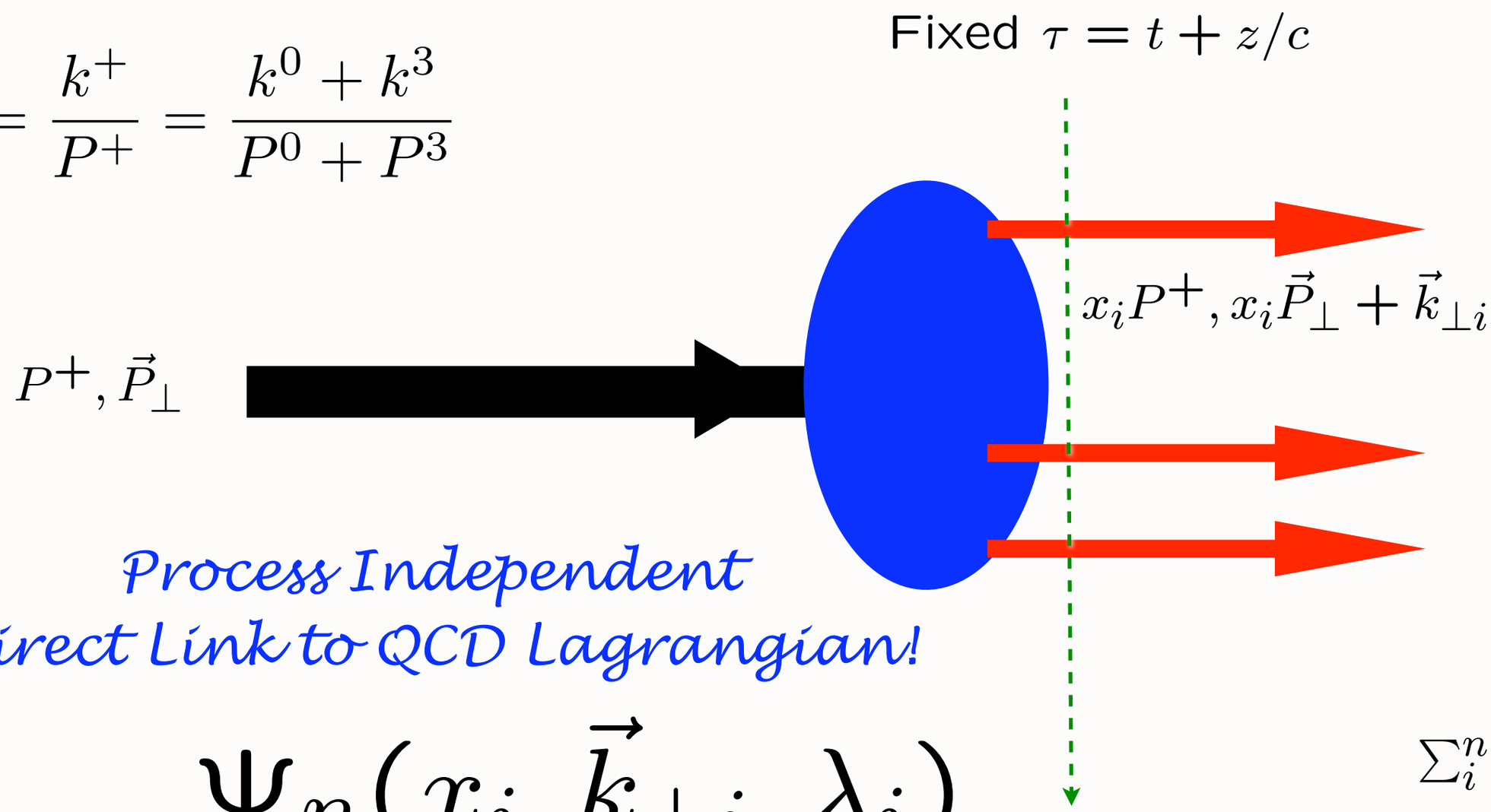
Stan Brodsky, SLAC

JTI Workshop on Dynamics of Symmetry Breaking

Argonne National Laboratory, IL *April 13-17, 2009*

Light-Front Wavefunctions: rigorous representation of composite systems in quantum field theory

$$x = \frac{k^+}{P^+} = \frac{k^0 + k^3}{P^0 + P^3}$$



Process Independent
Direct Link to QCD Lagrangian!

$$\Psi_n(x_i, \vec{k}_{\perp i}, \lambda_i)$$

$$\sum_i^n x_i = 1$$

$$\sum_i^n \vec{k}_{\perp i} = \vec{0}_\perp$$

Invariant under boosts! Independent of P^μ

Light-Front QCD

Heisenberg Matrix Formulation

Physical gauge: $A^+ = 0$

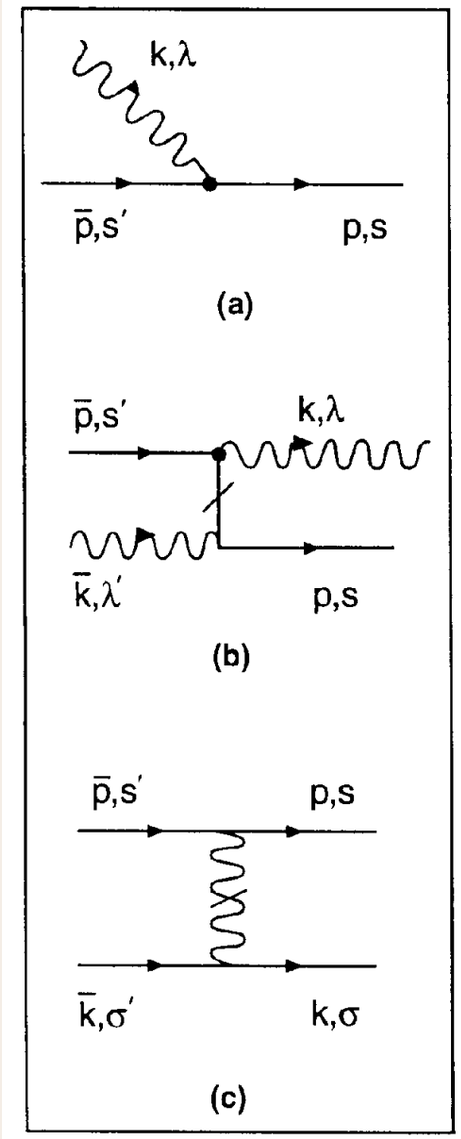
$$L^{QCD} \rightarrow H_{LF}^{QCD}$$

$$H_{LF}^{QCD} = \sum_i [\frac{m^2 + k_\perp^2}{x}]_i + H_{LF}^{int}$$

H_{LF}^{int}: Matrix in Fock Space

$$H_{LF}^{QCD} |\Psi_h >= \mathcal{M}_h^2 |\Psi_h >$$

Eigenvalues and Eigensolutions give Hadron Spectrum and Light-Front wavefunctions



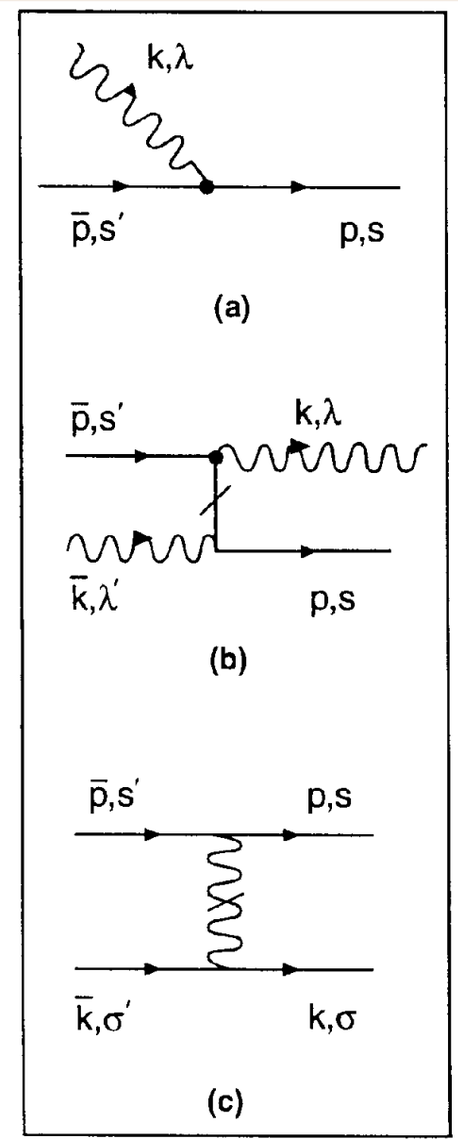
Light-Front QCD

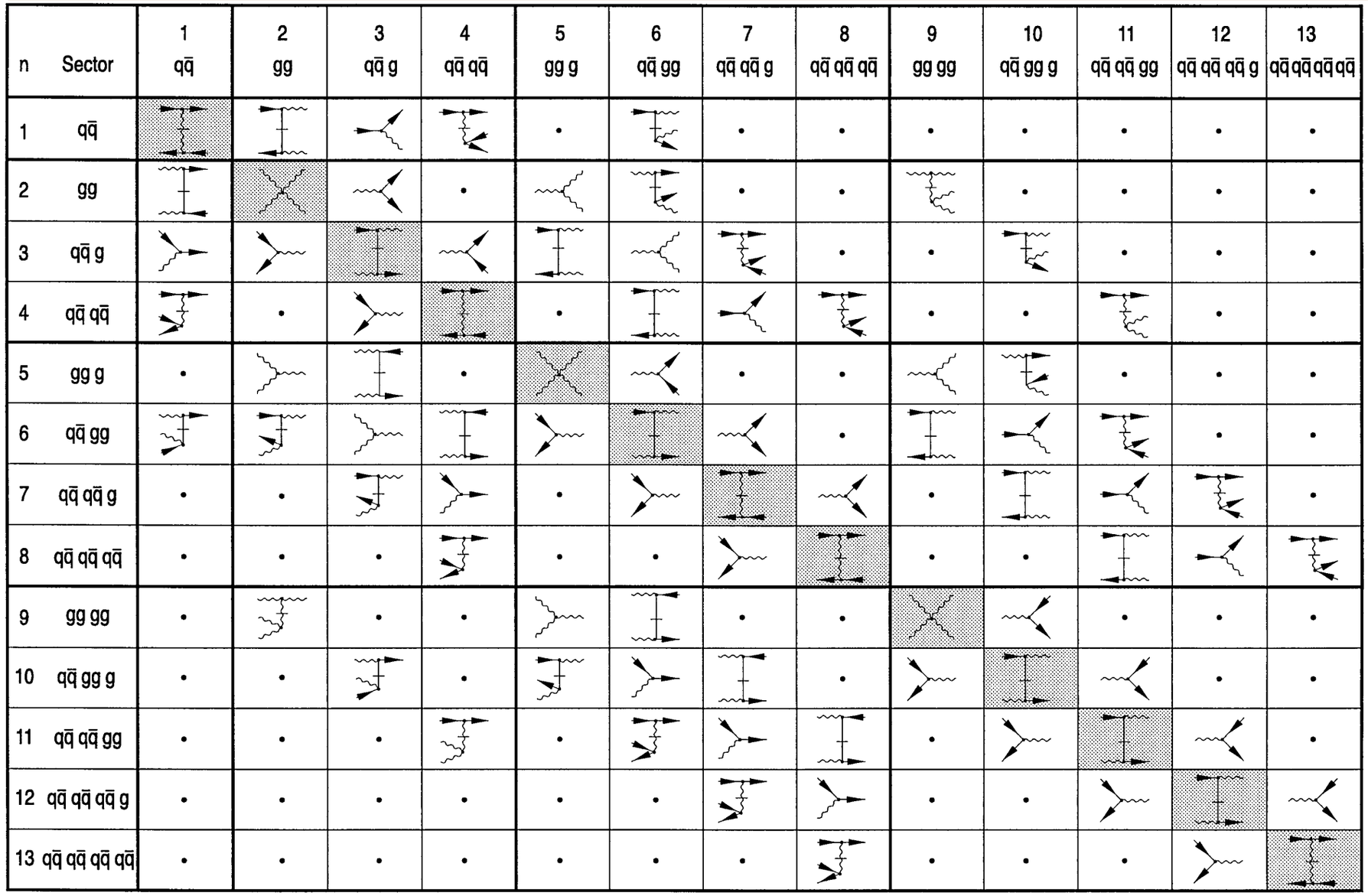
Heisenberg Matrix Formulation

$$H_{LF}^{QCD}|\Psi_h>=\mathcal{M}_h^2|\Psi_h>$$

H.C. Pauli & sjb

Discretized Light-Cone Quantization





Eigenvalues and Eigensolutions give Hadron Spectrum and Light-Front wavefunctions

DLCQ: Frame-independent, No fermion doubling; Minkowski Space

DLCQ: Periodic BC in x^-. Discrete k^+; frame-independent truncation

$$|p, S_z> = \sum_{n=3} \Psi_n(x_i, \vec{k}_{\perp i}, \lambda_i) |n; \vec{k}_{\perp_i}, \lambda_i >$$

sum over states with n=3, 4, ...constituents

The Light Front Fock State Wavefunctions

$$\Psi_n(x_i, \vec{k}_{\perp i}, \lambda_i)$$

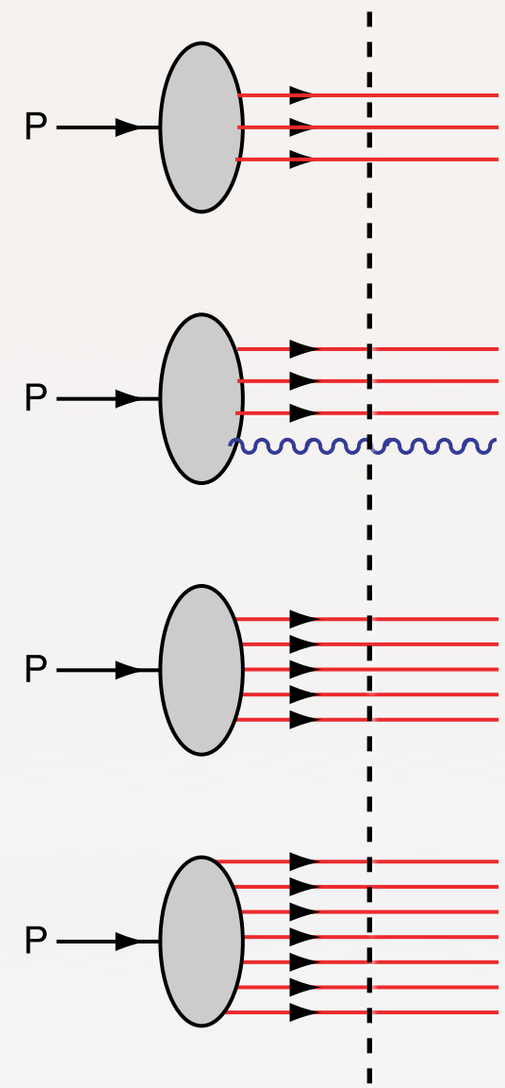
are boost invariant; they are independent of the hadron's energy and momentum P^μ.

The light-cone momentum fraction

$$x_i = \frac{k_i^+}{p^+} = \frac{k_i^0 + k_i^z}{P^0 + P^z}$$

are boost invariant.

$$\sum_i^n k_i^+ = P^+, \ \sum_i^n x_i = 1, \ \sum_i^n \vec{k}_i^\perp = \vec{0}^\perp.$$



Fixed LF time

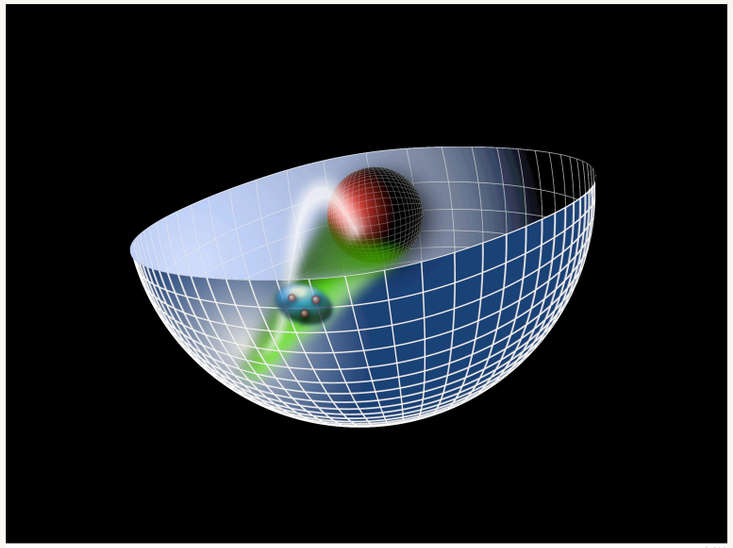
Intrinsic heavy quarks, $\bar{s}(x) \neq s(x)$

$\bar{u}(x) \neq \bar{d}(x)$

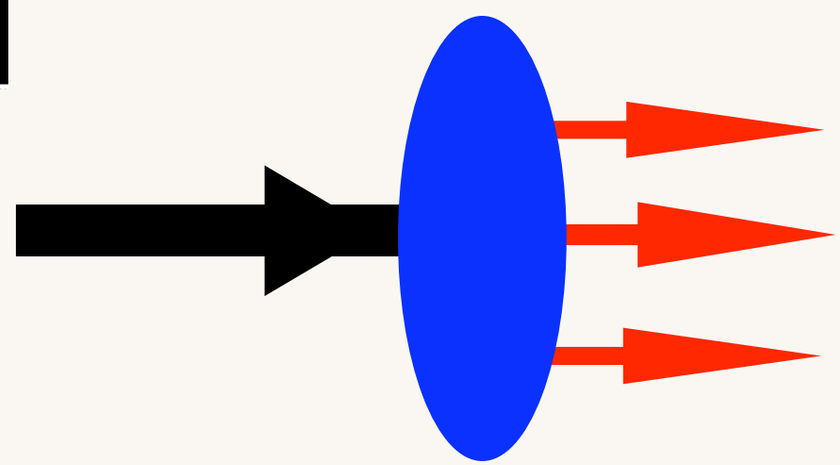
Light-Front QCD Phenomenology

- Hidden color, Intrinsic glue, sea, Color Transparency
- Physics of spin, orbital angular momentum
- Near Conformal Behavior of LFWFs at Short Distances; PQCD constraints
- Vanishing anomalous gravitomagnetic moment
- Relation between edm and anomalous magnetic moment
- Cluster Decomposition Theorem for relativistic systems
- OPE: DGLAP, ERBL evolution; invariant mass scheme

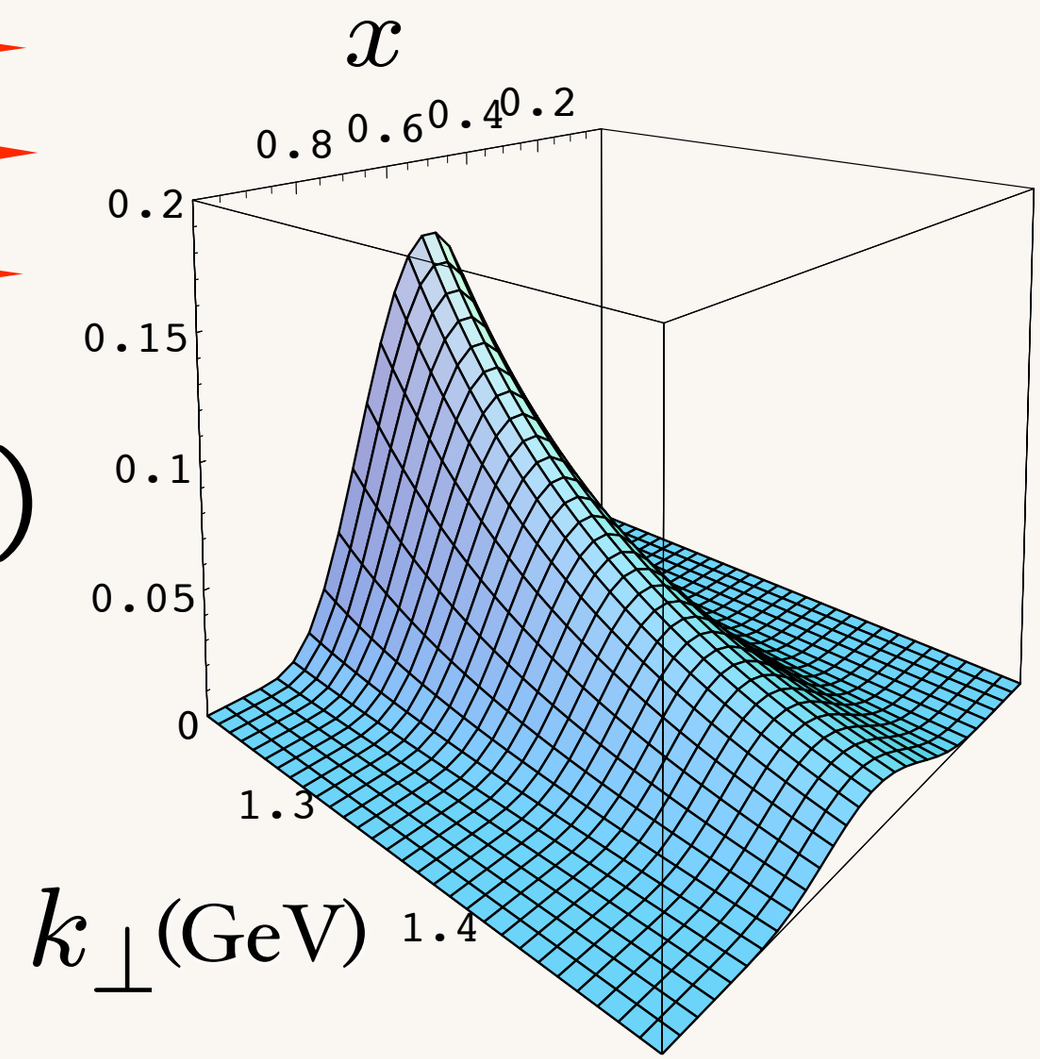
$\phi(z)$



- *Light-Front Holography*



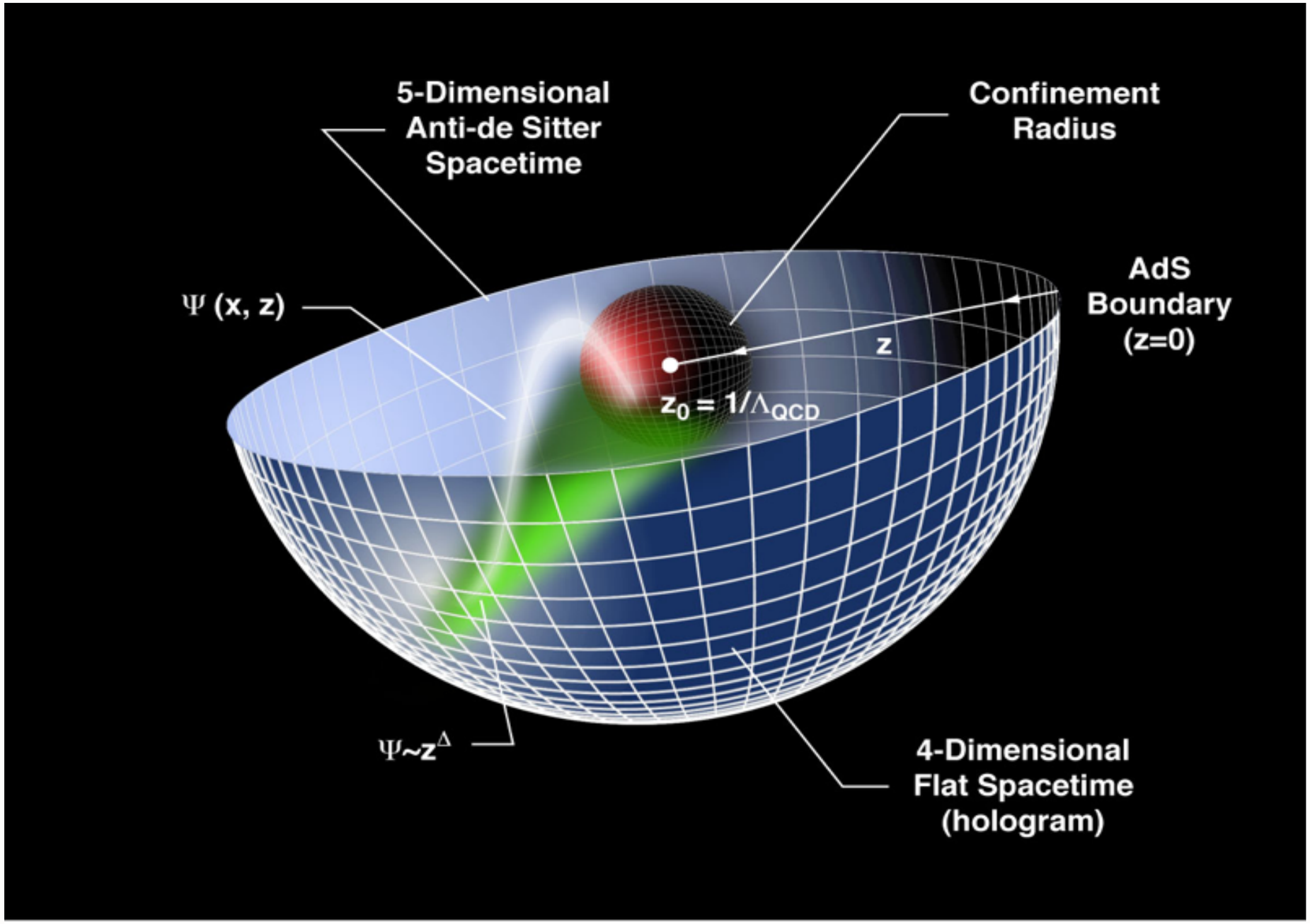
$$\Psi_n(x_i, \vec{k}_{\perp i}, \lambda_i)$$



- *Light Front Wavefunctions:*

  Schrödinger Wavefunctions of Hadron Physics

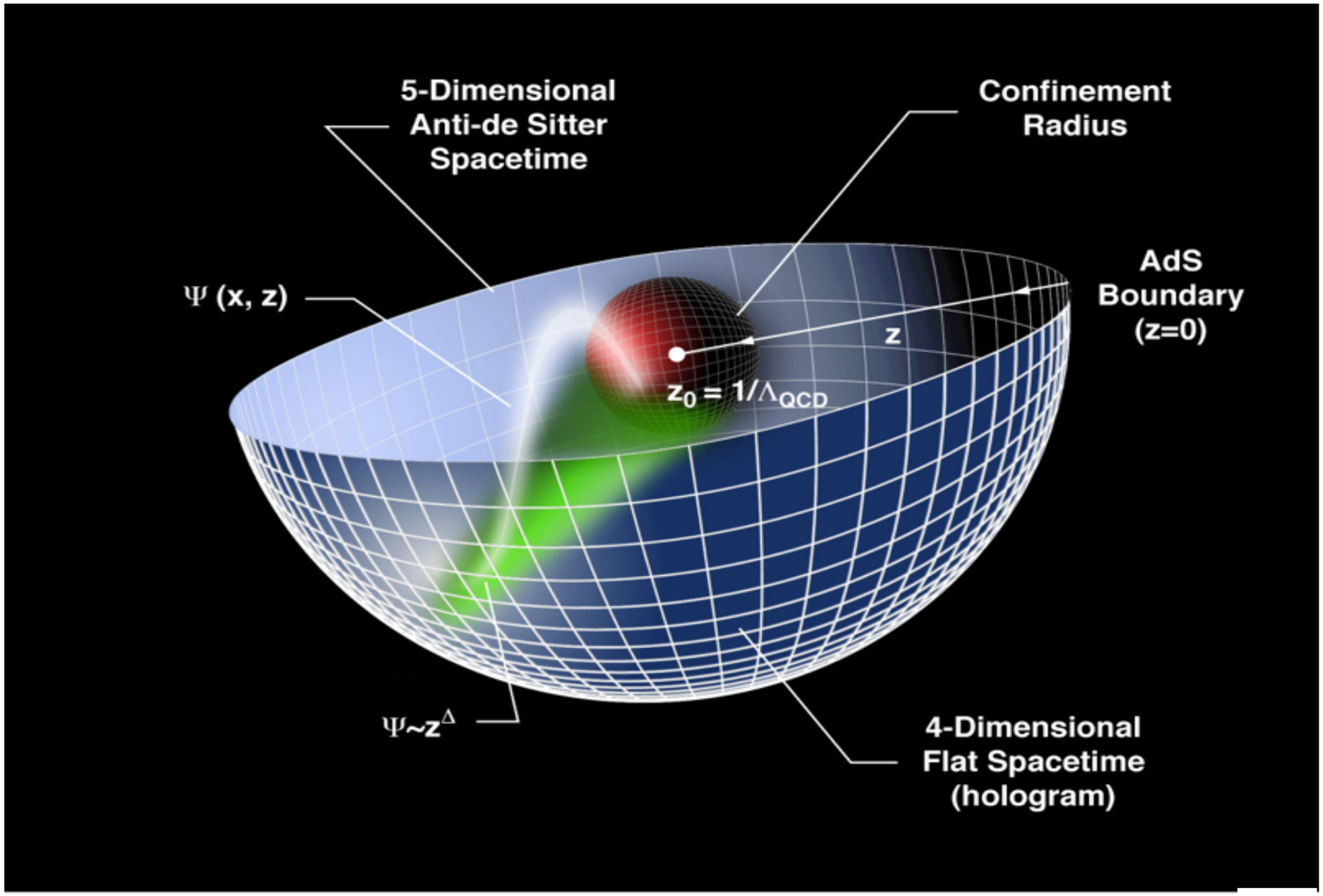
Applications of AdS/CFT to QCD



Changes in physical length scale mapped to evolution in the 5th dimension z

in collaboration with Guy de Teramond

Applications of AdS/CFT to QCD



Changes in physical length scale mapped to evolution in the 5th dimension z

de Teramond, sjb

Bottom-Up

String Theory

Top-Down

Goal:

- **Use AdS/CFT to provide an approximate, covariant, and analytic model of hadron structure with confinement at large distances, conformal behavior at short distances**
- **Analogous to the Schrodinger Theory for Atomic Physics**
- *AdS/QCD Light-Front Holography*
- *Hadronic Spectra and Light-Front Wavefunctions*

Conformal Theories are invariant under the Poincare and conformal transformations with

$$\mathbf{M}^{\mu\nu}, \mathbf{P}^{\mu}, \mathbf{D}, \mathbf{K}^{\mu},$$

the generators of SO(4,2)

SO(4,2) has a mathematical representation on AdS5

Scale Transformations

- Isomorphism of $SO(4,2)$ of conformal QCD with the group of isometries of AdS space

$$ds^2 = \frac{R^2}{z^2}(\eta_{\mu\nu}dx^\mu dx^\nu - dz^2),$$

invariant measure

$x^\mu \to \lambda x^\mu,\ z \to \lambda z$, maps scale transformations into the holographic coordinate z.

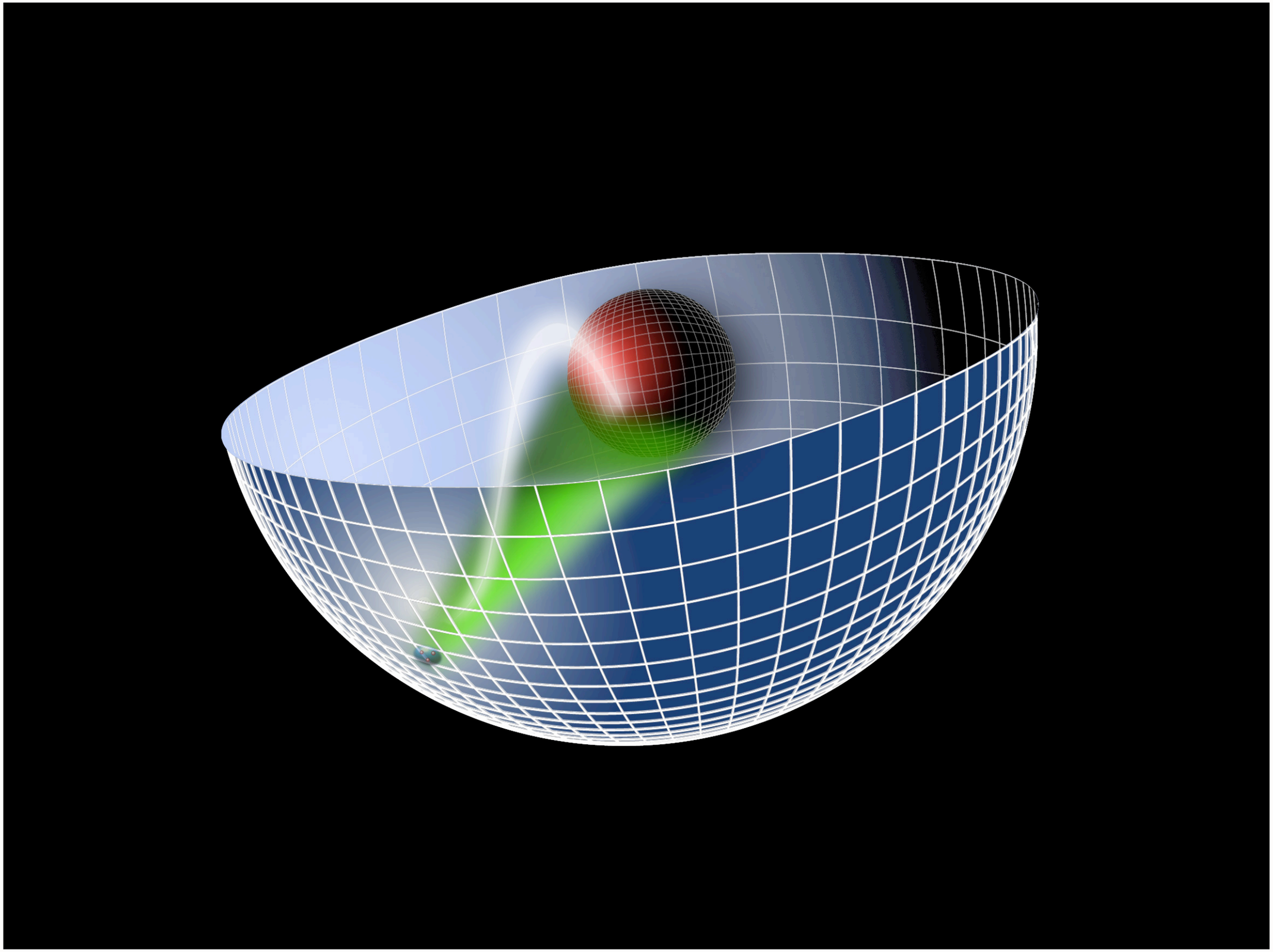
- AdS mode in z is the extension of the hadron wf into the fifth dimension.

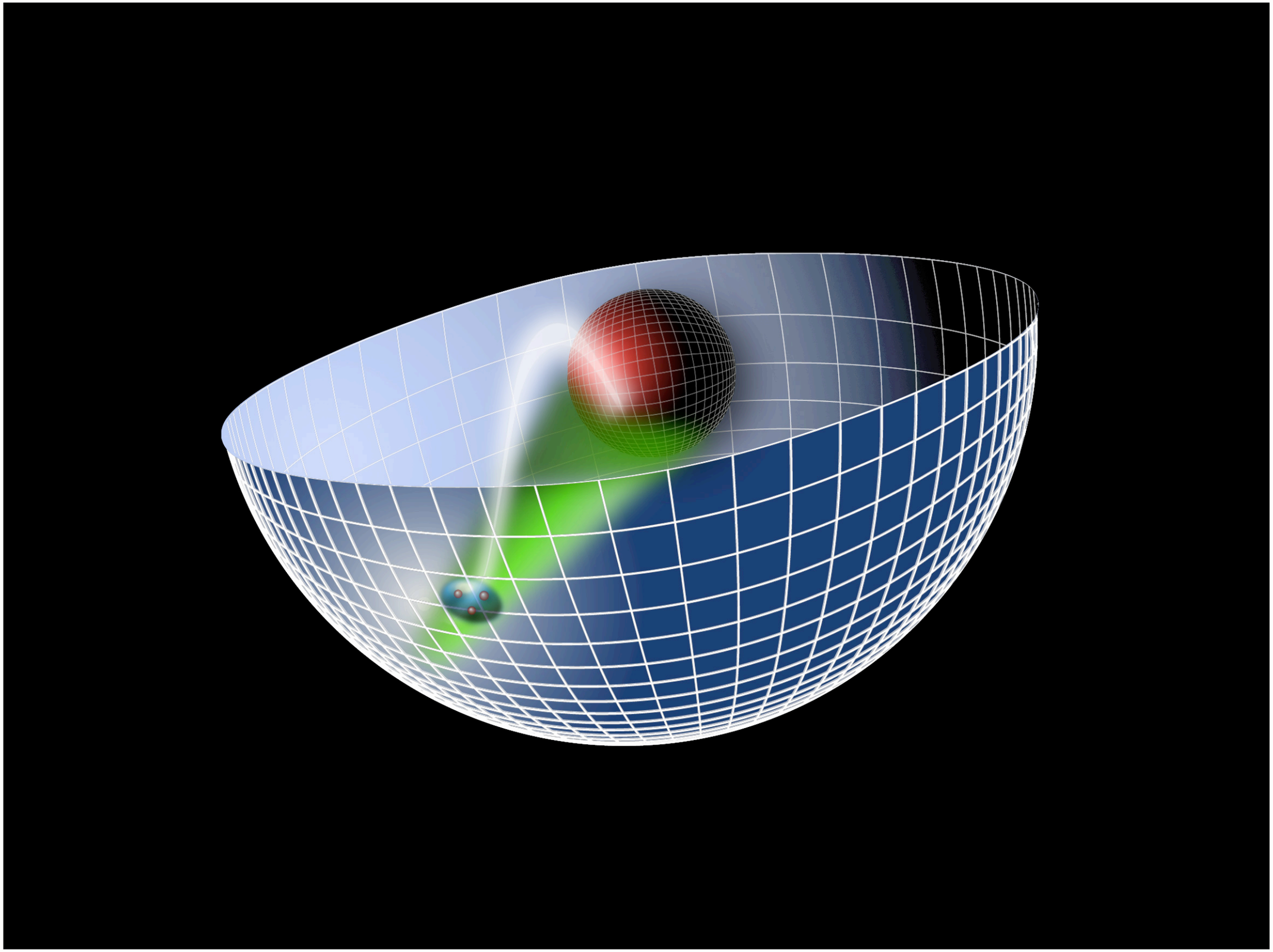
- Different values of z correspond to different scales at which the hadron is examined.

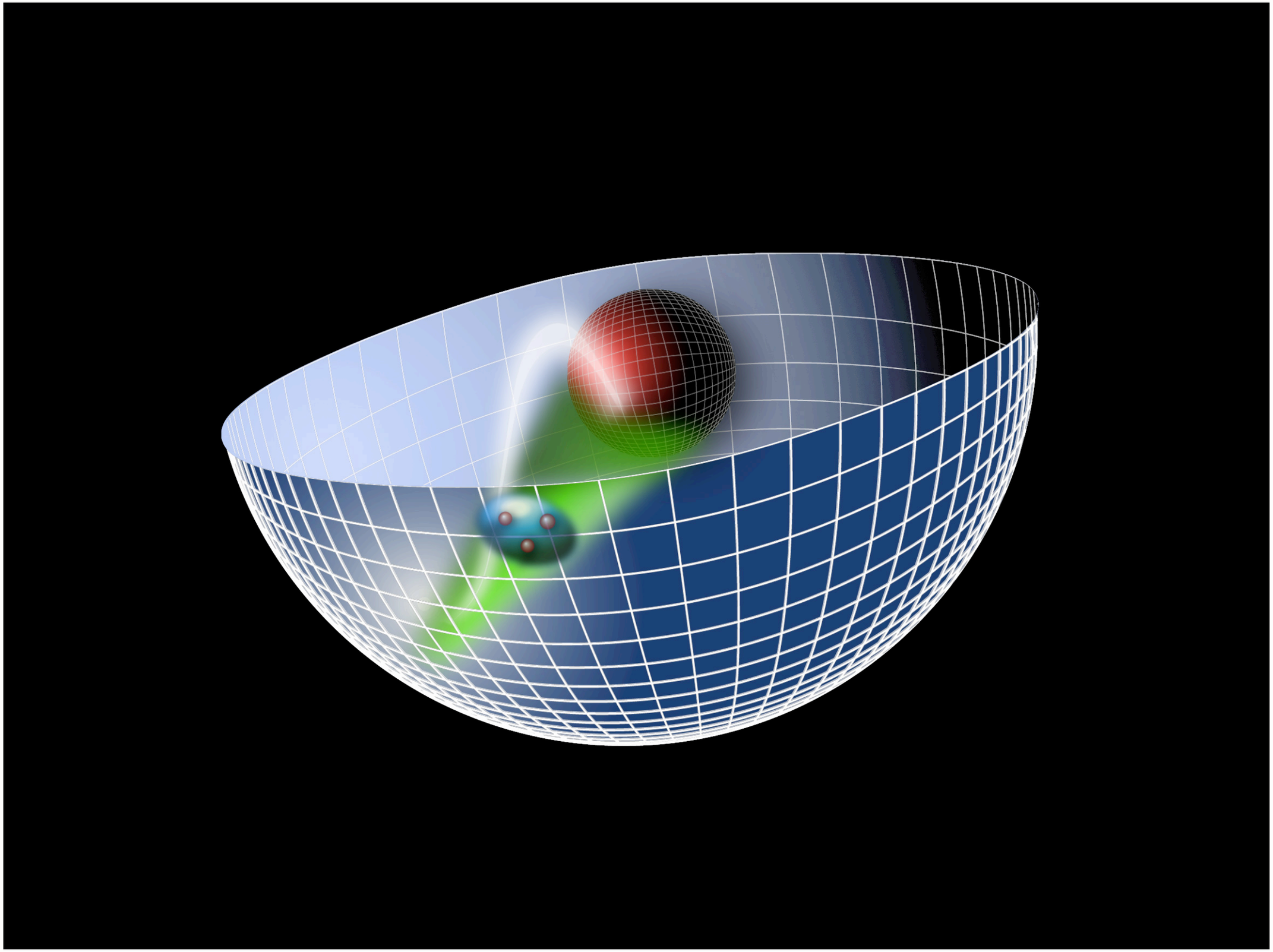
$$x^2 \to \lambda^2 x^2, \quad z \to \lambda z.$$

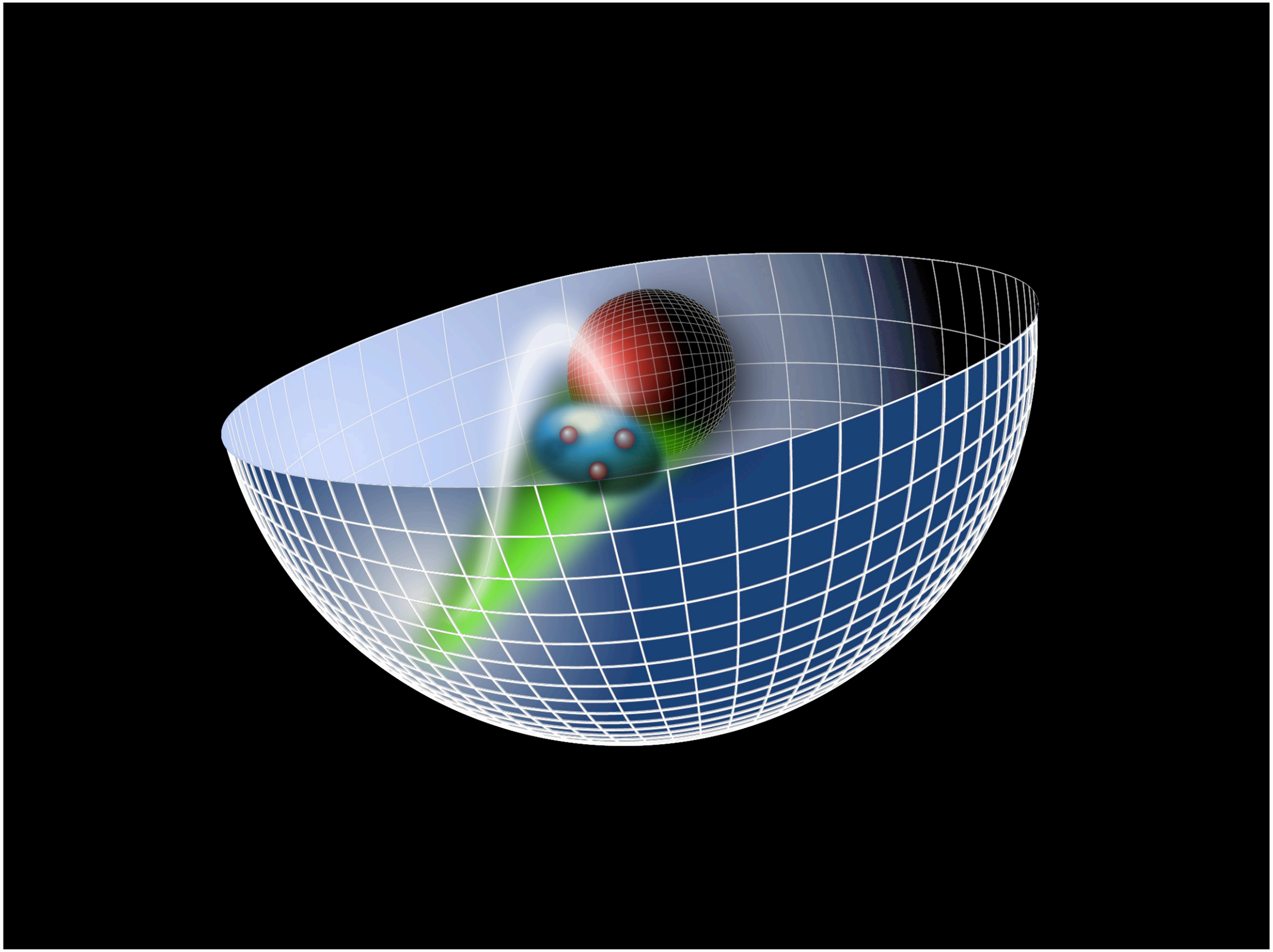
$x^2 = x_\mu x^\mu$: invariant separation between quarks

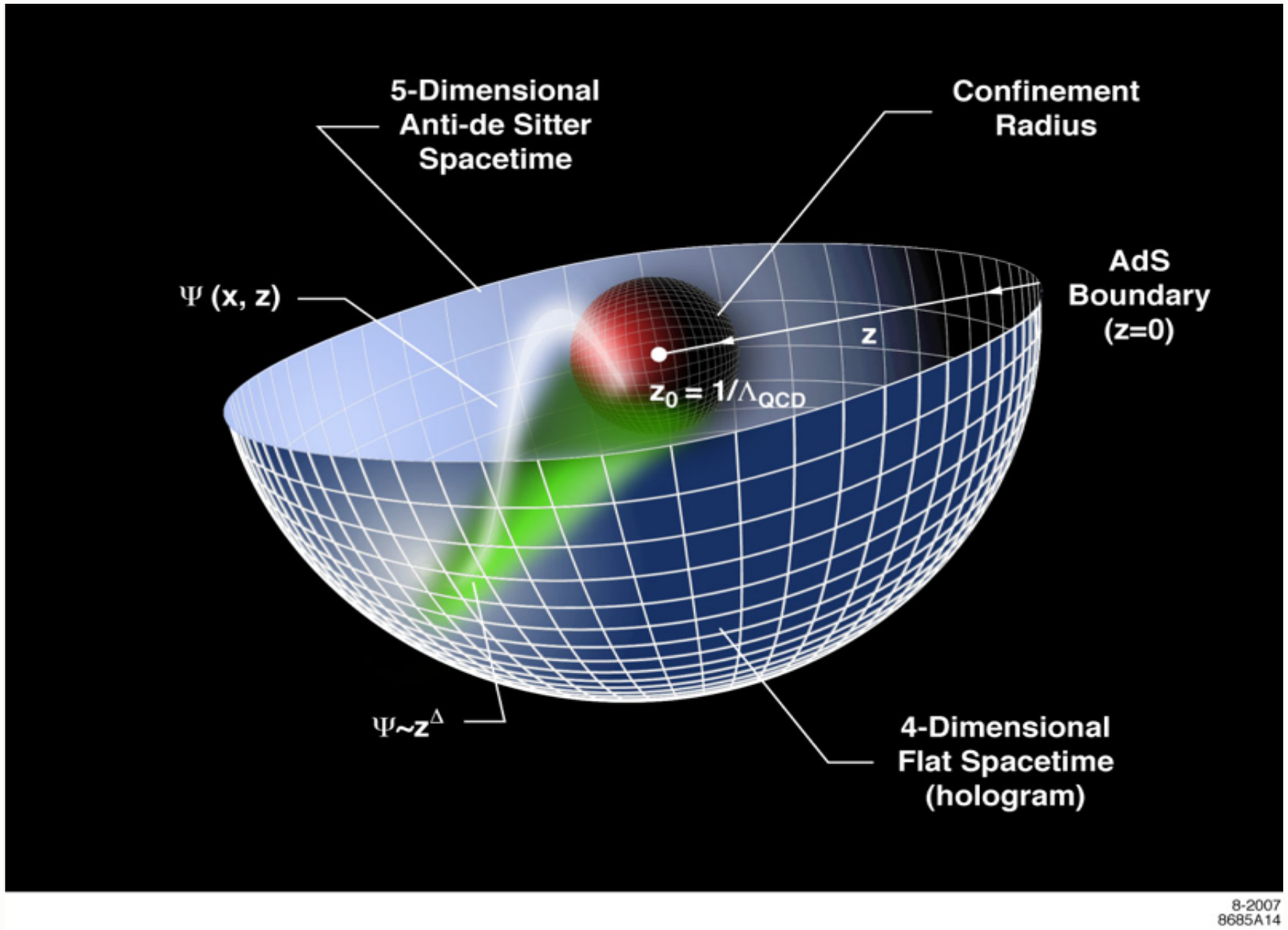
- The AdS boundary at $z \to 0$ correspond to the $Q \to \infty$, UV zero separation limit.











Related AdS/QCD model: Schmidt and Vega

- Truncated AdS/CFT (Hard-Wall) model: cut-off at $z_0 = 1/\Lambda_{\rm QCD}$ breaks conformal invariance and allows the introduction of the QCD scale (Hard-Wall Model) Polchinski and Strassler (2001).
- Smooth cutoff: introduction of a background dilaton field $\varphi(z)$ – usual linear Regge dependence can be obtained (Soft-Wall Model) Karch, Katz, Son and Stephanov (2006).

We will consider both holographic models

AdS/CFT: Anti-de Sitter Space / Conformal Field Theory

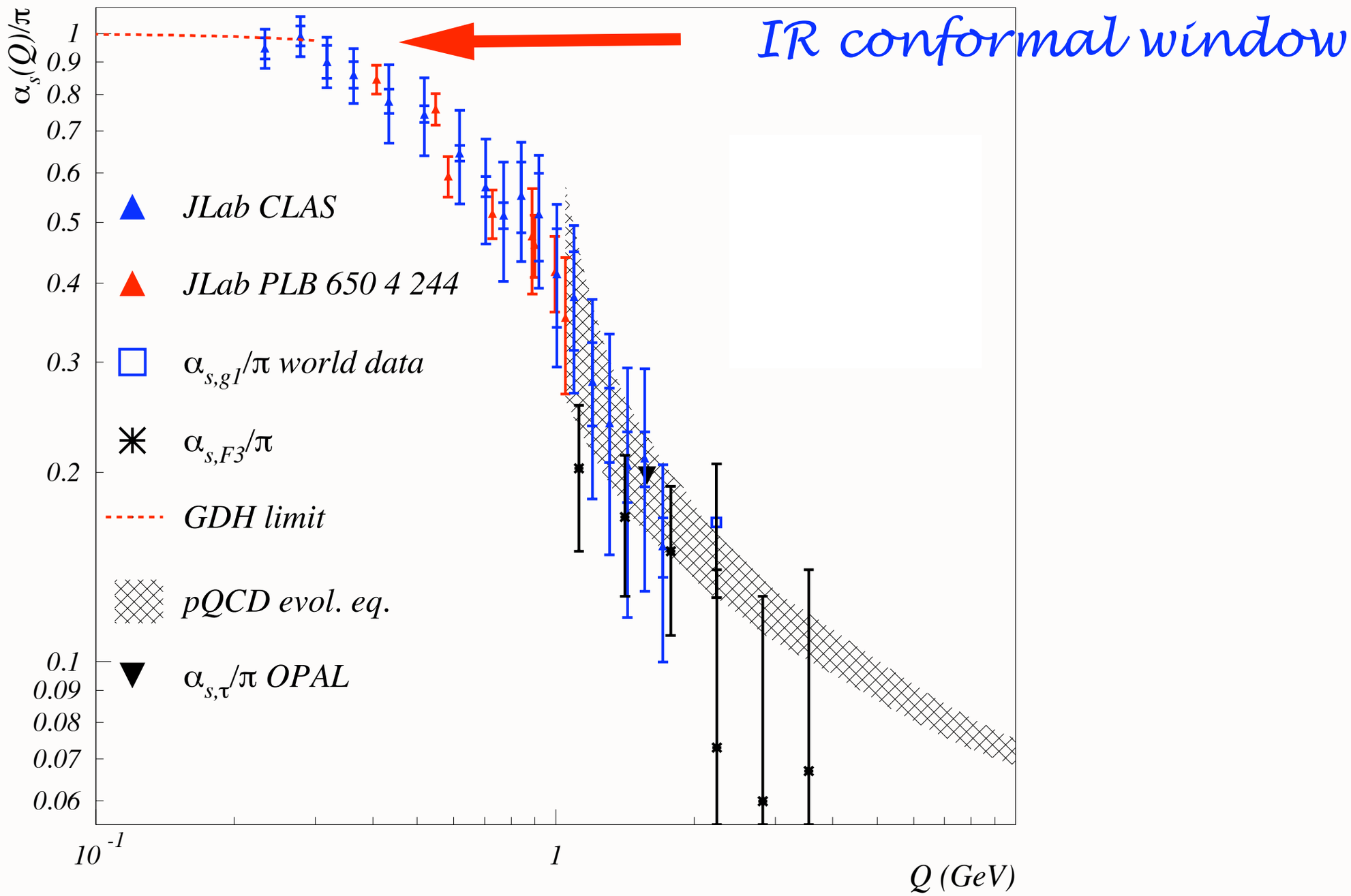
Maldacena:

Map $AdS_5 \times S_5$ to conformal N=4 SUSY

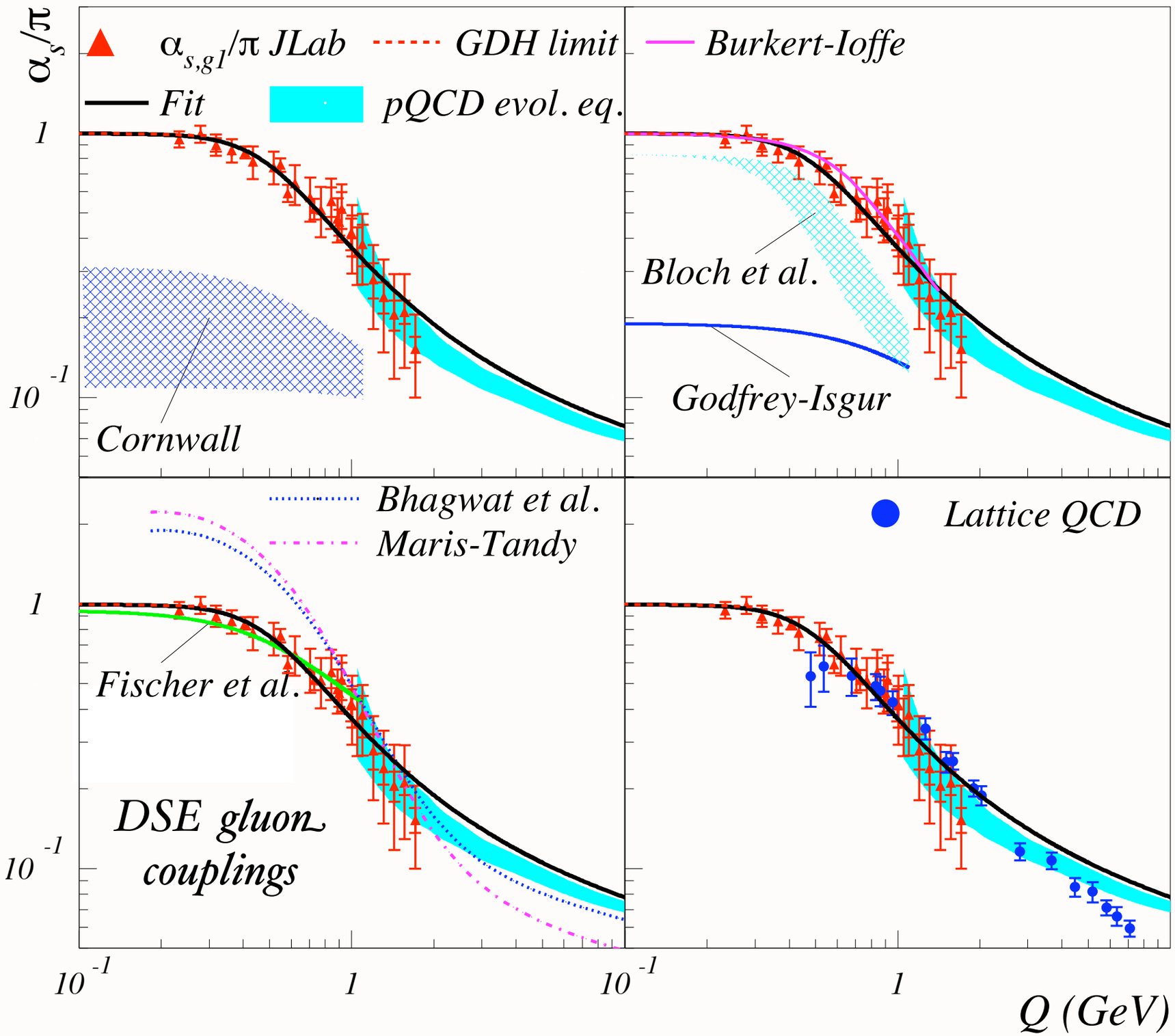
- **QCD is not conformal**; however, it has manifestations of a scale-invariant theory: Bjorken scaling, dimensional counting for hard exclusive processes
- **Conformal window:** $\alpha_s(Q^2) \simeq$ const at small Q^2
- **Use mathematical mapping of the conformal group SO(4,2) to AdS5 space**

Deur, Korsch, et al: Effective Charge from Bjorken Sum Rule

$$\Gamma_{bj}^{p-n}(Q^2) \equiv \frac{g_A}{6}[1 - \frac{\alpha_s^{g_1}(Q^2)}{\pi}]$$



Deur, Korsch, et al.



IR Conformal Window for QCD?

- *Dyson-Schwinger Analysis:* **QCD gluon coupling has IR Fixed Point**

- *Evidence from Lattice Gauge Theory*

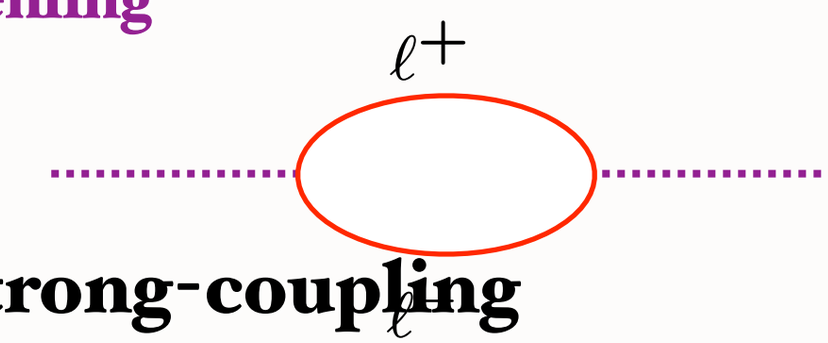
- Define coupling from observable: **indications of IR fixed point for QCD effective charges**

Shrock, de Teramond, sjb

- Confined gluons and quarks have maximum wavelength: **Decoupling of QCD vacuum polarization at small Q^2**

Serber-Uehling

$$\Pi(Q^2) \rightarrow \frac{\alpha}{15\pi}\frac{Q^2}{m^2} \qquad Q^2 << 4m^2$$



- **Justifies application of AdS/CFT in strong-coupling conformal window**

AdS/CFT

- Use mapping of conformal group SO(4,2) to AdS5
- Scale Transformations represented by wavefunction in $\psi(z)$ 5th dimension $x_\mu^2 \to \lambda^2 x_\mu^2$ $z \to \lambda z$
- Match solutions at small z to conformal dimension of hadron wavefunction at short distances $\psi(z) \sim z^\Delta$ at $z \to 0$
- Hard wall model: Confinement at large distances and conformal symmetry in interior
- Truncated space simulates "bag" boundary conditions

$$0 < z < z_0 \qquad \psi(z_0) = 0 \qquad z_0 = \frac{1}{\Lambda_{QCD}}$$

Bosonic Solutions: Hard Wall Model

- Conformal metric: $ds^2 = g_{\ell m} dx^\ell dx^m$. $x^\ell = (x^\mu, z)$, $g_{\ell m} \to \left(R^2/z^2\right) \eta_{\ell m}$.
- Action for massive scalar modes on AdS$_{d+1}$:

$$S[\Phi] = \frac{1}{2} \int d^{d+1}x \sqrt{g}\, \tfrac{1}{2} \left[g^{\ell m} \partial_\ell \Phi \partial_m \Phi - \mu^2 \Phi^2 \right], \quad \sqrt{g} \to (R/z)^{d+1}.$$

- Equation of motion

$$\frac{1}{\sqrt{g}} \frac{\partial}{\partial x^\ell} \big(\sqrt{g}\, g^{\ell m} \frac{\partial}{\partial x^m} \Phi \big) + \mu^2 \Phi = 0.$$

- Factor out dependence along x^μ-coordinates , $\Phi_P(x,z) = e^{-iP\cdot x}\, \Phi(z)$, $P_\mu P^\mu = \mathcal{M}^2$:

$$\left[z^2 \partial_z^2 - (d-1) z\, \partial_z + z^2 \mathcal{M}^2 - (\mu R)^2 \right] \Phi(z) = 0.$$

- Solution: $\Phi(z) \to z^\Delta$ as $z \to 0$,

$$\Phi(z) = C z^{d/2} J_{\Delta - d/2}(z\mathcal{M}) \qquad \Delta = \tfrac{1}{2}\Big(d + \sqrt{d^2 + 4\mu^2 R^2} \Big).$$

$$\Delta = 2 + L \qquad d = 4 \qquad (\mu R)^2 = L^2 - 4$$

Let $\Phi(z) = z^{3/2}\phi(z)$

AdS Schrodinger Equation for bound state of two scalar constituents:

$$\left[-\frac{d^2}{dz^2} - \frac{1-4L^2}{4z^2} \right] \phi(z) = \mathcal{M}^2 \phi(z)$$

L: orbital angular momentum

Derived from variation of Action in AdS_5

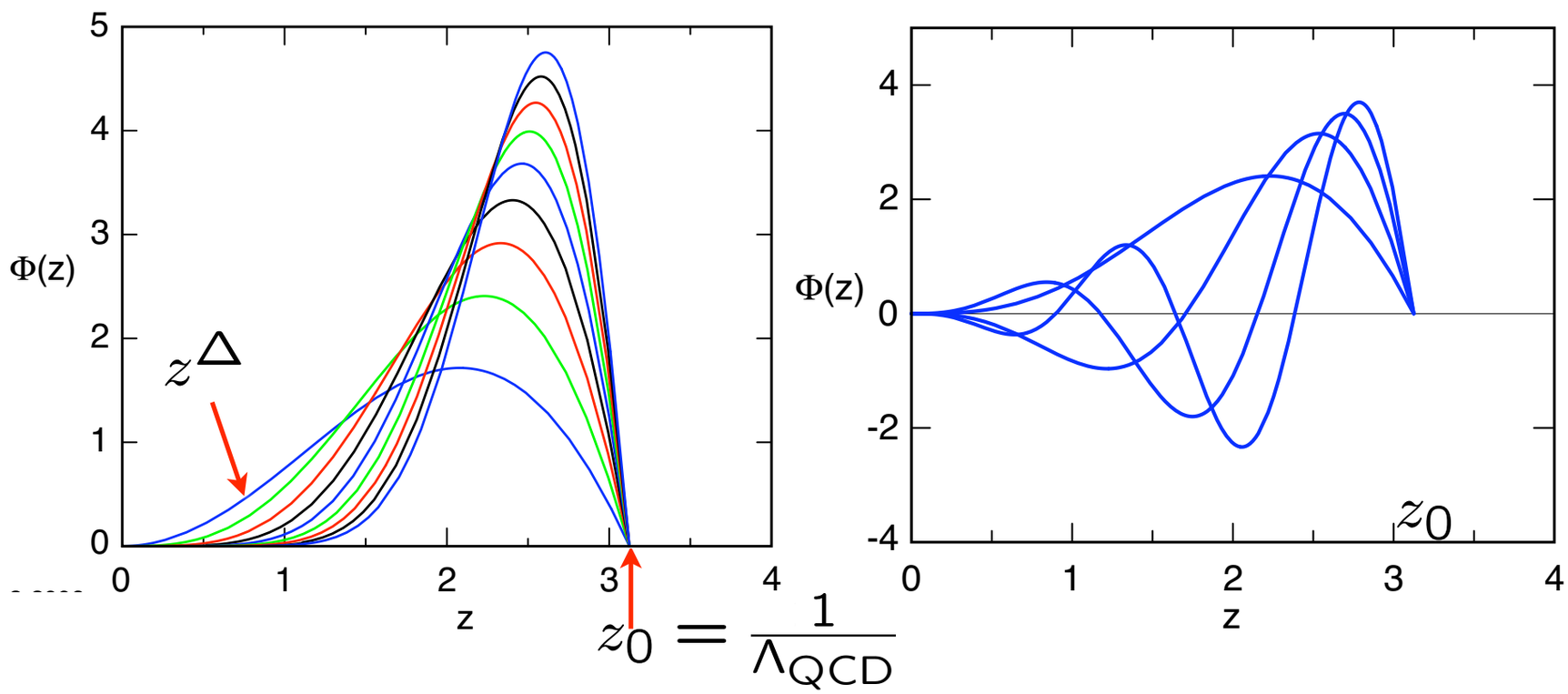
Hard wall model: truncated space

$$\phi(z = z_0 = \frac{1}{\Lambda_c}) = 0.$$

Match fall-off at small z to conformal twist-dimension at short distances

twist

- Pseudoscalar mesons: $\mathcal{O}_{2+L} = \overline{\psi}\gamma_5 D_{\{\ell_1} \ldots D_{\ell_m\}}\psi$ ($\Phi_\mu = 0$ gauge). $\Delta = 2 + L$
- 4-d mass spectrum from boundary conditions on the normalizable string modes at $z = z_0$, $\Phi(x, z_o) = 0$, given by the zeros of Bessel functions $\beta_{\alpha,k}$: $\mathcal{M}_{\alpha,k} = \beta_{\alpha,k}\Lambda_{QCD}$
- Normalizable AdS modes $\Phi(z)$



$S = 0$ Meson orbital and radial AdS modes for $\Lambda_{QCD} = 0.32$ GeV.

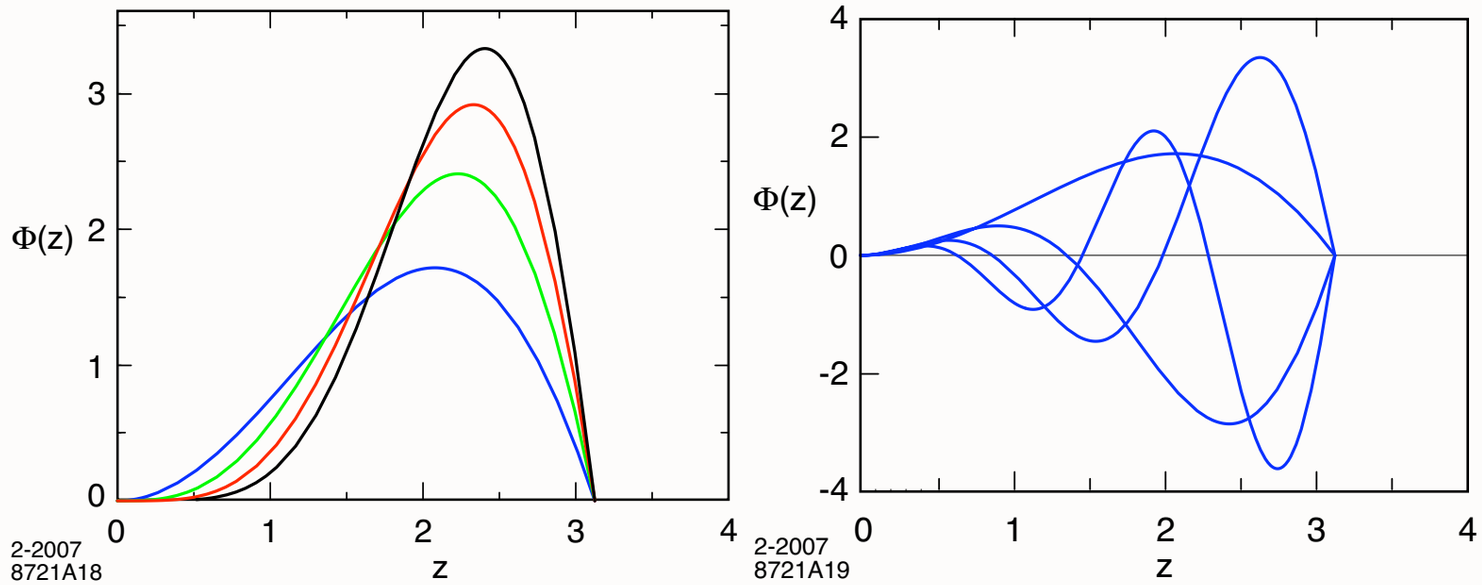


Fig: Orbital and radial AdS modes in the hard wall model for $\Lambda_{\rm QCD}$ = 0.32 GeV .

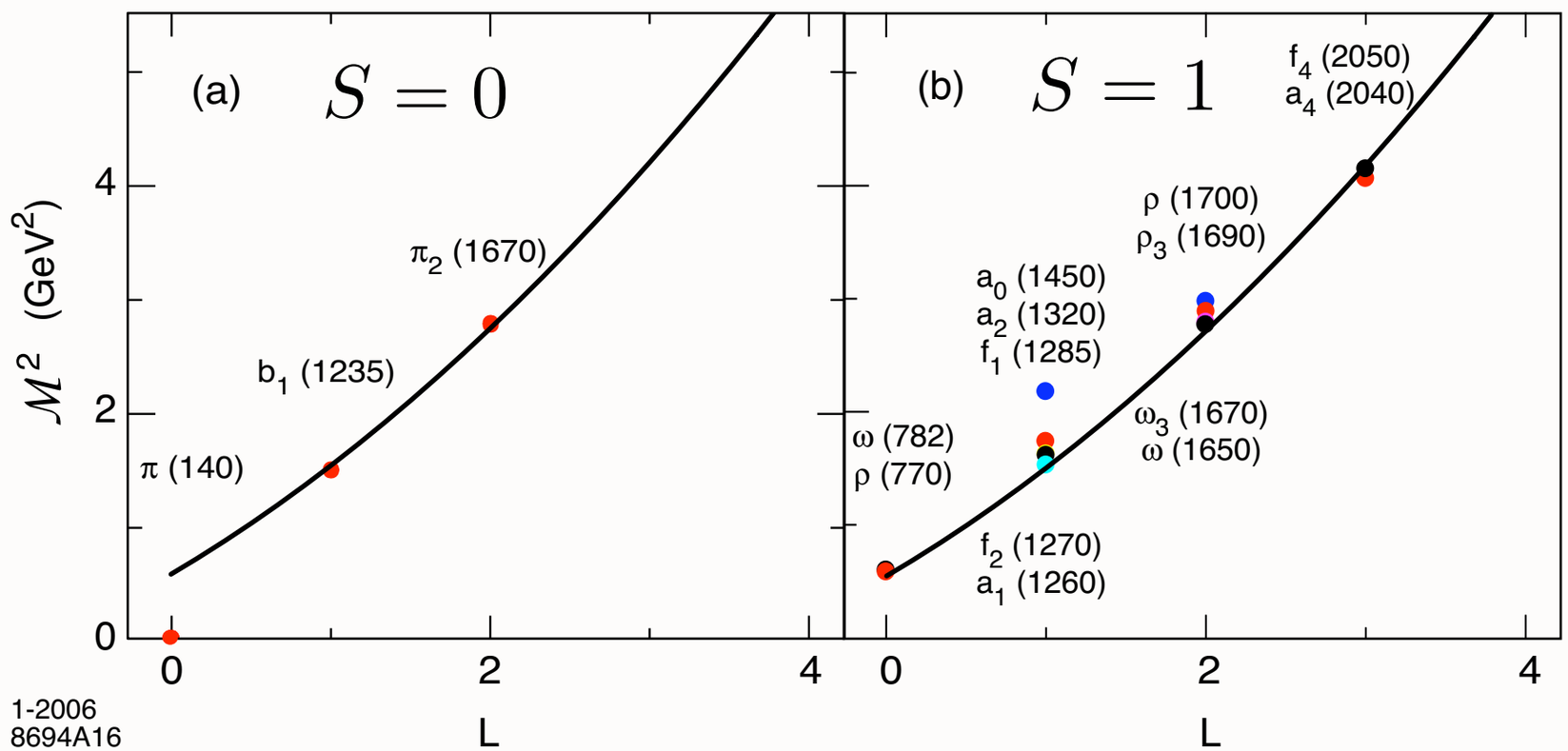


Fig: Light meson and vector meson orbital spectrum $\Lambda_{QCD} = 0.32$ GeV

Higher Spin Bosonic Modes HW

- Each hadronic state of integer spin $S \leq 2$ is dual to a normalizable string mode

$$\Phi(x,z)_{\mu_1\mu_2\cdots\mu_S} = \epsilon_{\mu_1\mu_2\cdots\mu_S}\, e^{-iP\cdot x}\, \Phi_S(z).$$

with four-momentum P_μ and spin polarization indices along the 3+1 physical coordinates.

- Wave equation for spin S-mode W. S. I'Yi, Phys. Lett. B **448**, 218 (1999)

$$\left[z^2\partial_z^2 - (d+1-2S)z\,\partial_z + z^2\mathcal{M}^2 - (\mu R)^2\right]\Phi_S(z) = 0,$$

- Solution

$$\widetilde{\Phi}(z)_S = \left(\frac{z}{R}\right)^S \Phi(z)_S = C e^{-iP\cdot x} z^{\frac{d}{2}} J_{\Delta-\frac{d}{2}}(z\mathcal{M})\, \epsilon(P)_{\mu_1\mu_2\cdots\mu_S},$$

- We can identify the conformal dimension:

$$\Delta = \frac{1}{2}\left(d + \sqrt{(d-2S)^2 + 4\mu^2R^2}\right).$$

- Normalization:

$$R^{d-2S-1}\int_0^{\Lambda_{\rm QCD}^{-1}} \frac{dz}{z^{d-2S-1}}\,\Phi_S^2(z) = 1.$$

- Erlich, Karch, Katz, Son, Stephanov
- de Teramond, sjb

AdS Soft-Wall Schrodinger Equation for bound state of two scalar constituents:

$$\Big[-\frac{d^2}{dz^2} - \frac{1-4L^2}{4z^2} + U(z)\Big]\phi(z) = \mathcal{M}^2\phi(z)$$

$$U(z) = \kappa^4 z^2 + 2\kappa^2(L+S-1)$$

Derived from variation of Action Dilaton-Modified AdS_5

$$e^{\Phi(z)} = e^{+\kappa^2 z^2}$$

Quark separation increases with L

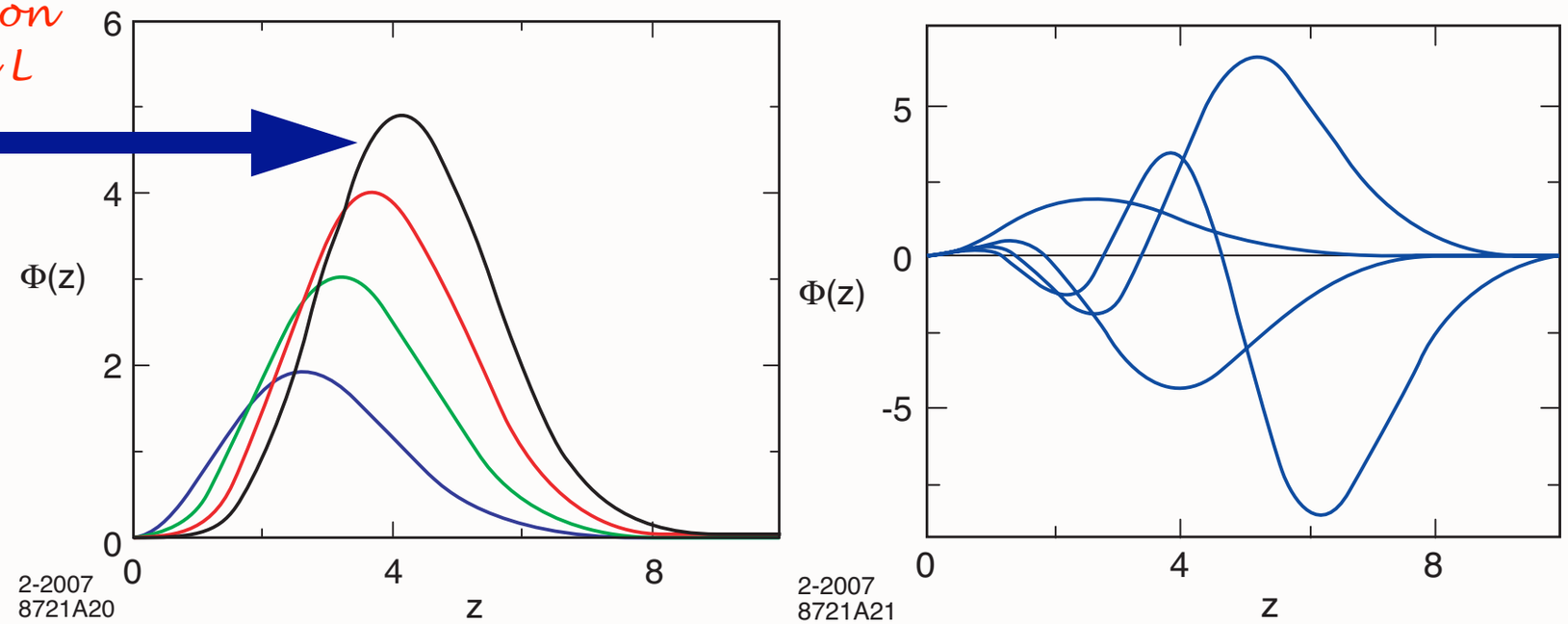
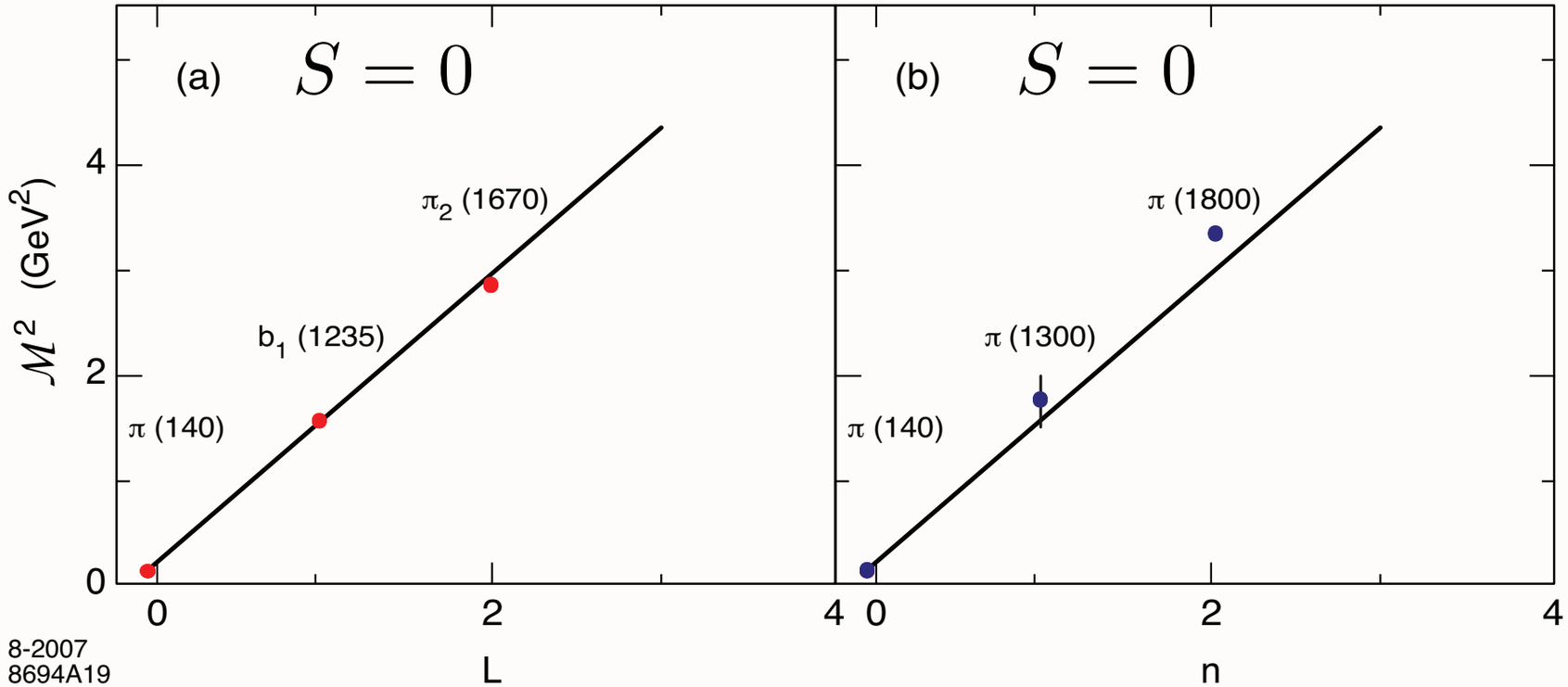


Fig: Orbital and radial AdS modes in the soft wall model for κ = 0.6 GeV .

Soft Wall Model

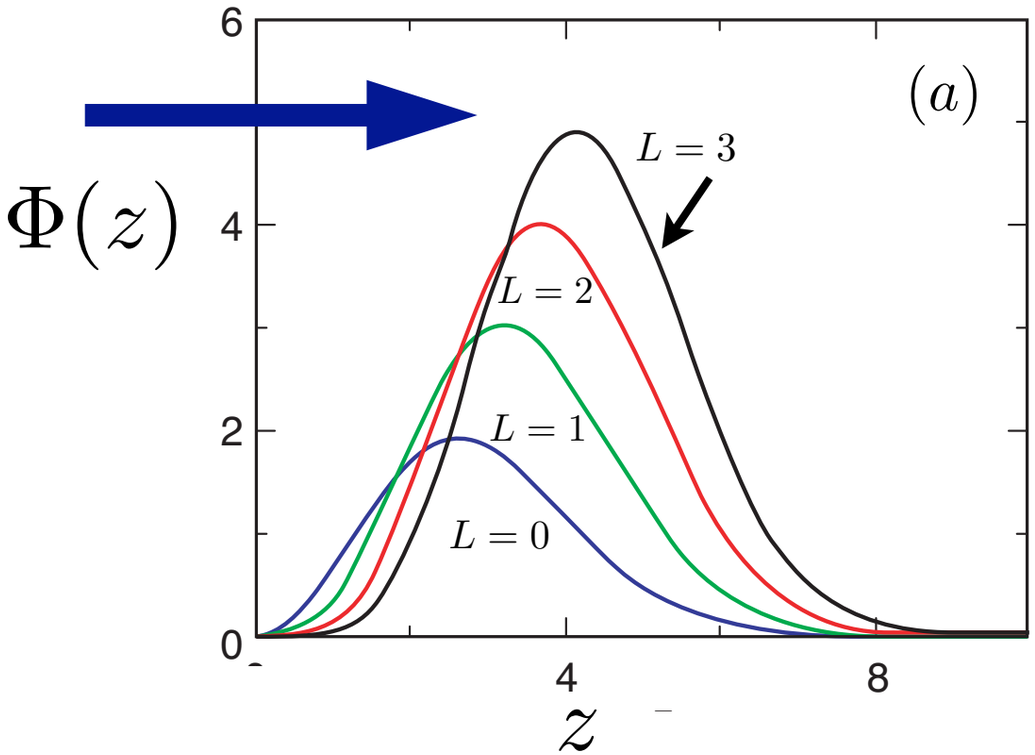


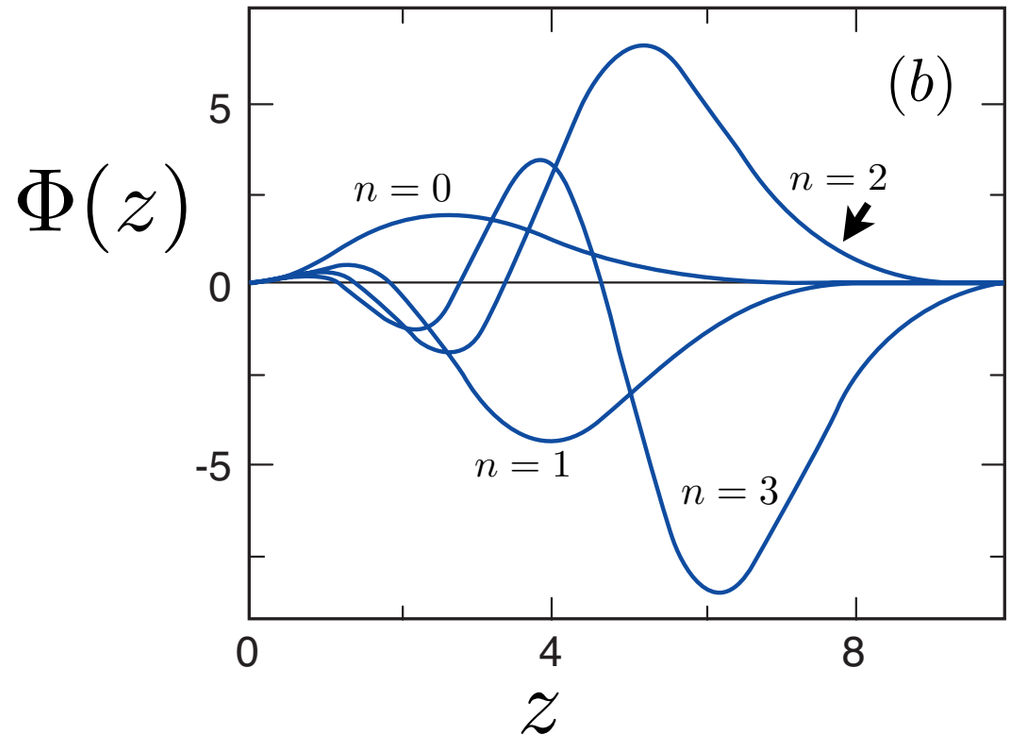
Light meson orbital (a) and radial (b) spectrum for $\kappa = 0.6$ GeV.

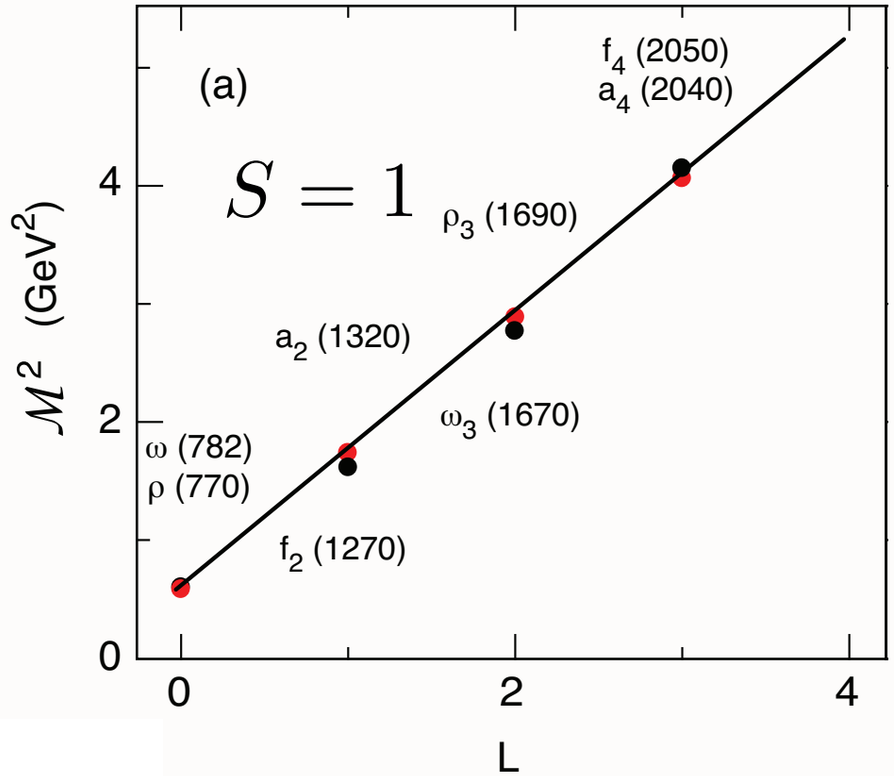
Pion mass automatically zero!

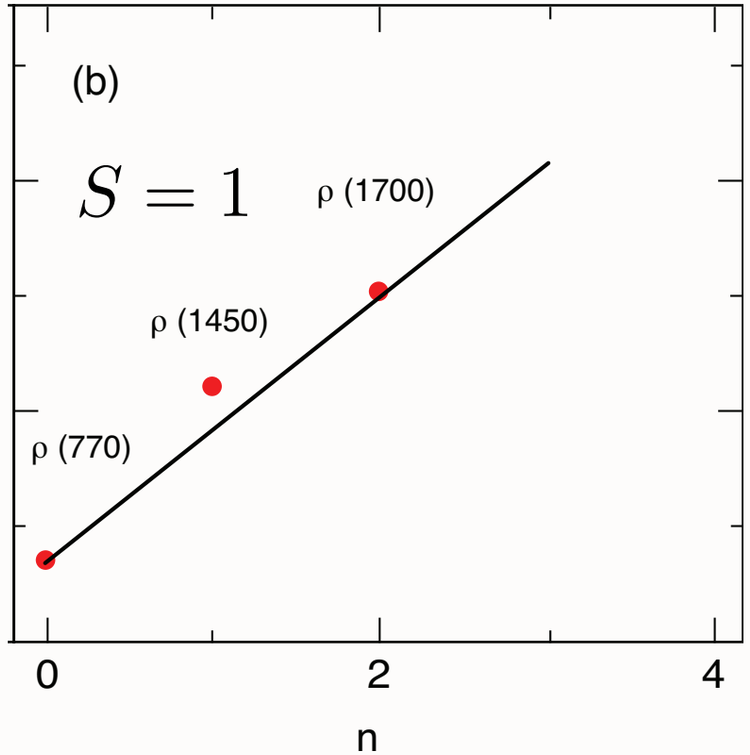
$$m_q = 0$$

Quark separation increases with L









Higher Spin Bosonic Modes SW

Soft-wall model

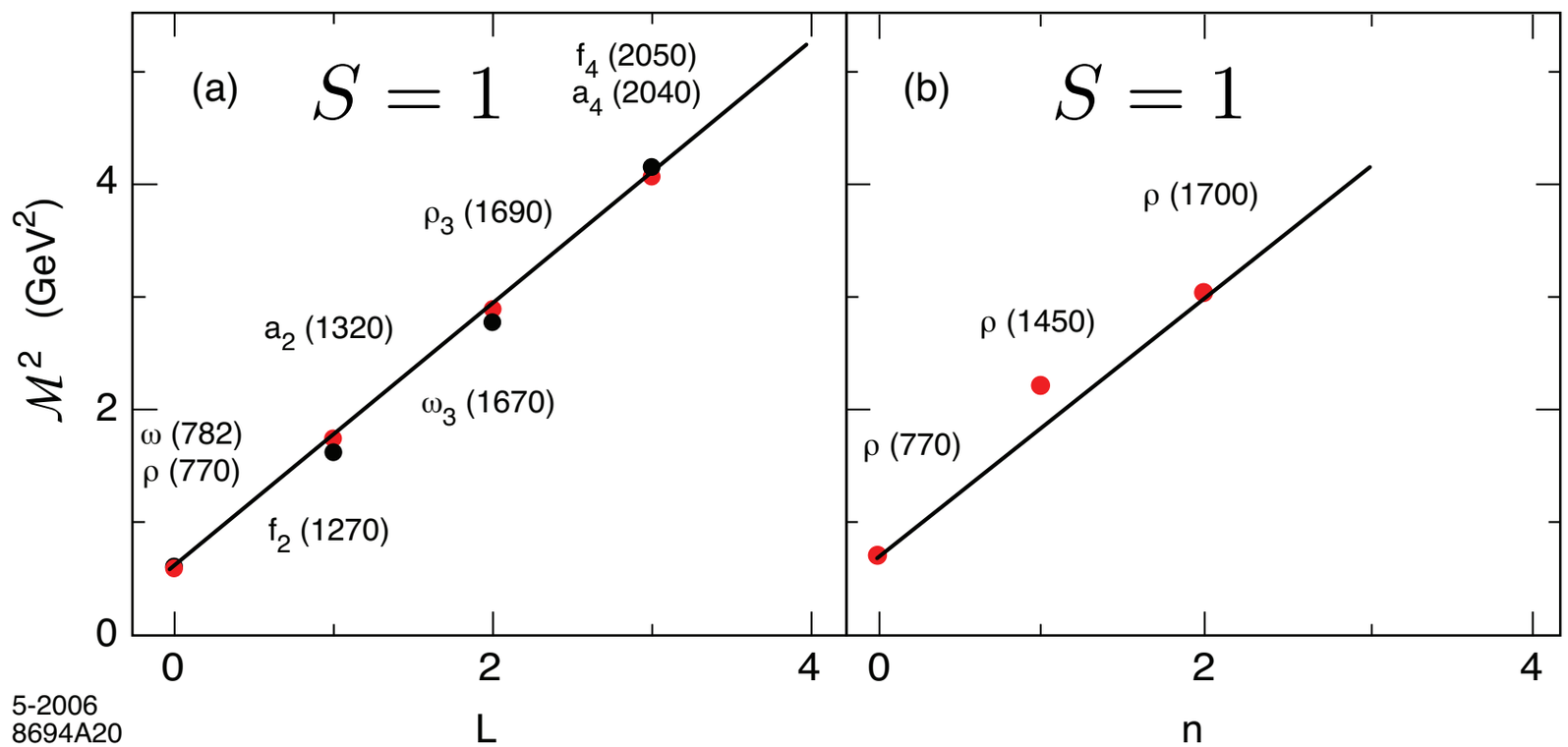
- Effective LF Schrödinger wave equation

$$\left[-\frac{d^2}{dz^2} - \frac{1-4L^2}{4z^2} + \kappa^4 z^2 + 2\kappa^2(L+S-1)\right]\phi_S(z) = \mathcal{M}^2\phi_S(z)$$

with eigenvalues $\mathcal{M}^2 = 2\kappa^2(2n+2L+S)$.

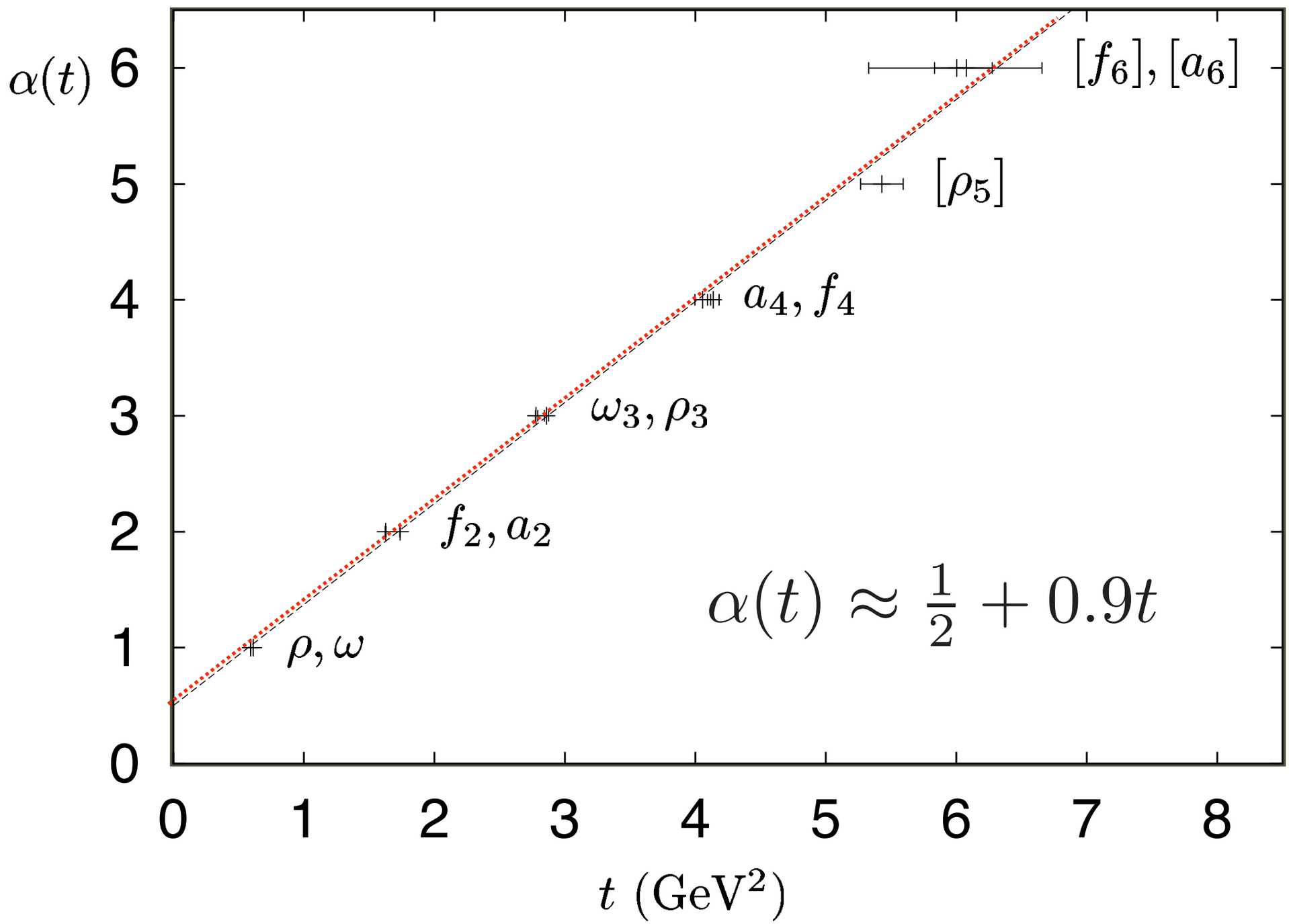
Same slope in n and L

- Compare with Nambu string result (rotating flux tube): $M_n^2(L) = 2\pi\sigma\,(n+L+1/2)\,.$



Vector mesons orbital (a) and radial (b) spectrum for $\kappa = 0.54$ GeV.

- Glueballs in the bottom-up approach: (HW) Boschi-Filho, Braga and Carrion (2005); (SW) Colangelo, De Facio, Jugeau and Nicotri(2007).



AdS/QCD Soft Wall Model -- Reproduces Linear Regge Trajectories

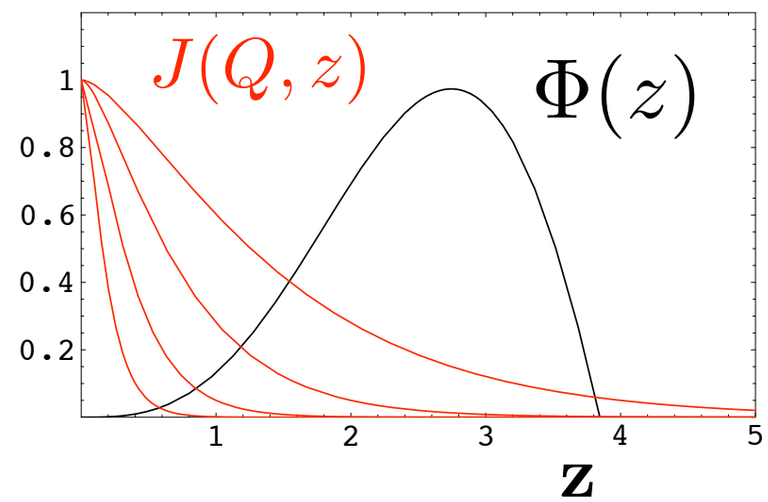
Hadron Form Factors from AdS/CFT

Propagation of external perturbation suppressed inside AdS.

$$J(Q,z) = zQK_1(zQ)$$

$$F(Q^2)_{I\to F} = \int \frac{dz}{z^3}\Phi_F(z)J(Q,z)\Phi_I(z)$$

High Q^2
from
small z ~ 1/Q



Polchinski, Strassler
de Teramond, sjb

Consider a specific AdS mode $\Phi^{(n)}$ dual to an n partonic Fock state $|n\rangle$. At small z, Φ scales as $\Phi^{(n)} \sim z^{\Delta_n}$. Thus:

$$F(Q^2) \to \left[\frac{1}{Q^2}\right]^{\tau-1},$$

Dimensional Quark Counting Rules
General result from
AdS/CFT

where $\tau = \Delta_n - \sigma_n$, $\sigma_n = \sum_{i=1}^{n} \sigma_i$. The twist is equal to the number of partons, $\tau = n$.

Current Matrix Elements in AdS Space (SW)

sjb and GdT
Grigoryan and Radyushkin

- Propagation of external current inside AdS space described by the AdS wave equation

$$\left[z^2\partial_z^2 - z\left(1+2\kappa^2 z^2\right)\partial_z - Q^2 z^2\right] J_\kappa(Q,z) = 0.$$

- Solution bulk-to-boundary propagator

$$J_\kappa(Q,z) = \Gamma\left(1+\frac{Q^2}{4\kappa^2}\right) U\left(\frac{Q^2}{4\kappa^2}, 0, \kappa^2 z^2\right),$$

where $U(a,b,c)$ is the confluent hypergeometric function

$$\Gamma(a)U(a,b,z) = \int_0^\infty e^{-zt} t^{a-1}(1+t)^{b-a-1} dt.$$

Soft Wall Model

- Form factor in presence of the dilaton background $\varphi = \kappa^2 z^2$

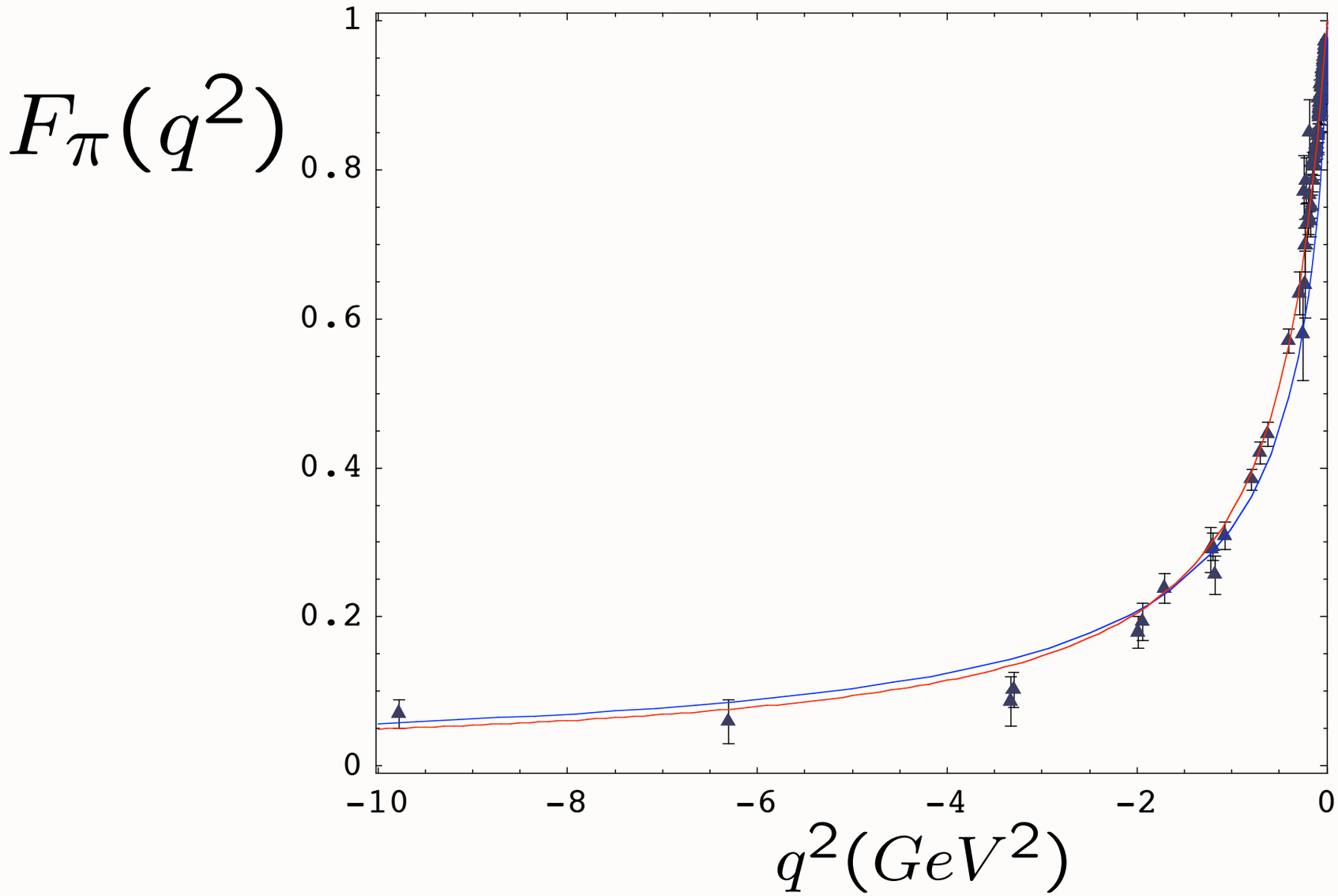
$$F(Q^2) = R^3 \int \frac{dz}{z^3} e^{-\kappa^2 z^2} \Phi(z) J_\kappa(Q,z) \Phi(z).$$

- For large $Q^2 \gg 4\kappa^2$

$$J_\kappa(Q,z) \to zQK_1(zQ) = J(Q,z),$$

the external current decouples from the dilaton field.

Spacelike pion form factor from AdS/CFT



Data Compilation
Baldini, Kloe and Volmer

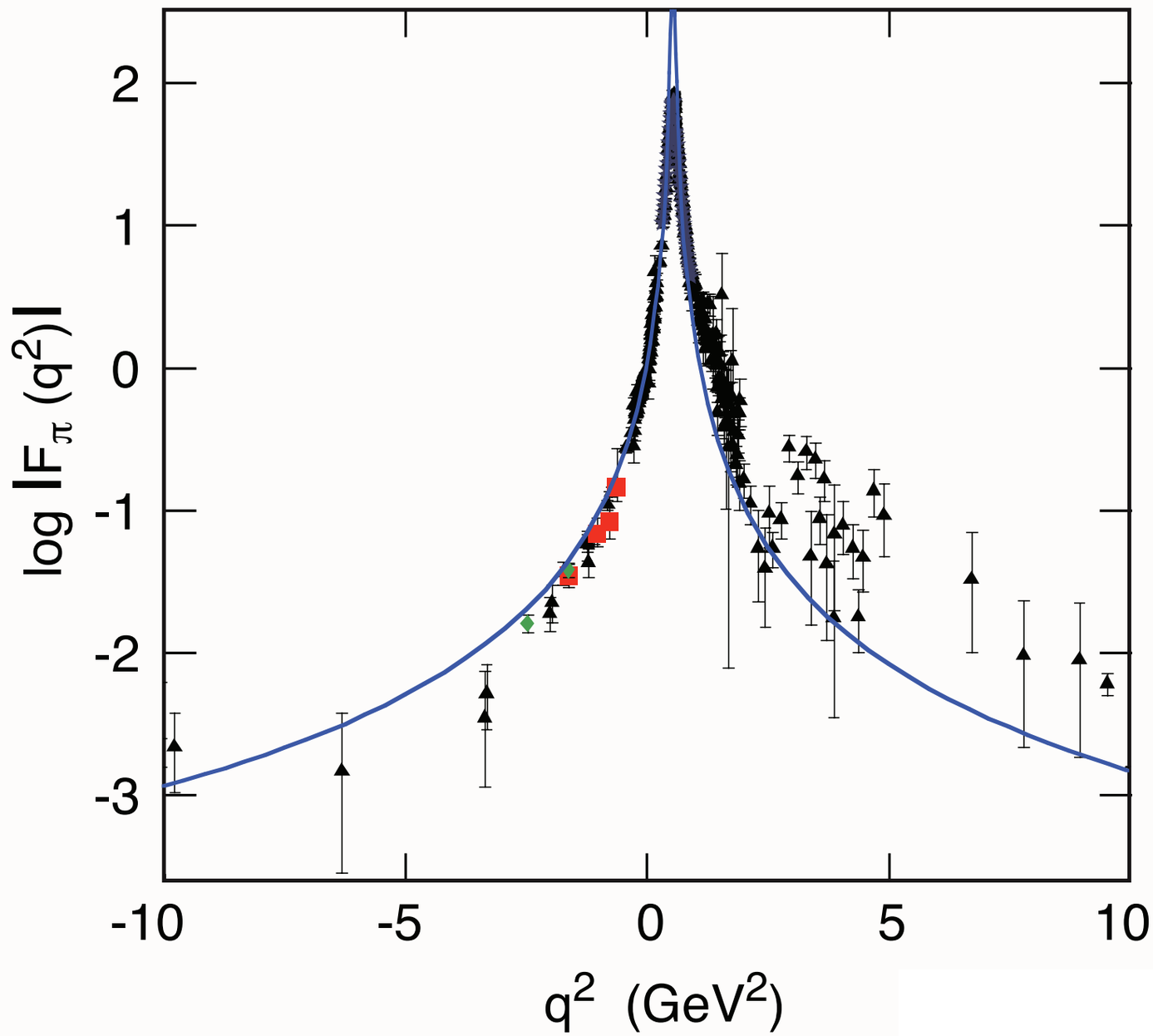
Soft Wall: Harmonic Oscillator Confinement

Hard Wall: Truncated Space Confinement

One parameter - set by pion decay constant

de Teramond, sjb
See also: Radyushkin

- Analytical continuation to time-like region $q^2 \to -q^2$ $\qquad M_\rho = 2\kappa = 750 \text{ MeV}$
- Strongly coupled semiclassical gauge/gravity limit hadrons have zero widths (stable).



Space and time-like pion form factor for $\kappa = 0.375$ GeV in the SW model.

- Vector Mesons: Hong, Yoon and Strassler (2004); Grigoryan and Radyushkin (2007).

Light-Front Representation of Two-Body Meson Form Factor

- Drell-Yan-West form factor

$$\vec{q}_\perp^{\,2} = Q^2 = -q^2$$

$$F(q^2) = \sum_q e_q \int_0^1 dx \int \frac{d^2\vec{k}_\perp}{16\pi^3}\, \psi^*_{P'}(x, \vec{k}_\perp - x\vec{q}_\perp)\, \psi_P(x, \vec{k}_\perp).$$

- Fourrier transform to impact parameter space $\vec{b}_\perp$

$$\psi(x, \vec{k}_\perp) = \sqrt{4\pi} \int d^2\vec{b}_\perp\, e^{i\vec{b}_\perp\cdot\vec{k}_\perp} \widetilde{\psi}(x, \vec{b}_\perp)$$

- Find ($b = |\vec{b}_\perp|$) :

$$\begin{aligned} F(q^2) &= \int_0^1 dx \int d^2\vec{b}_\perp\, e^{ix\vec{b}_\perp\cdot\vec{q}_\perp} \left|\widetilde{\psi}(x, b)\right|^2 \\ &= 2\pi \int_0^1 dx \int_0^\infty b\, db\, J_0\left(bqx\right)\, \left|\widetilde{\psi}(x, b)\right|^2, \end{aligned}$$

Soper

Holographic Mapping of AdS Modes to QCD LFWFs

- Integrate Soper formula over angles:

$$F(q^2) = 2\pi \int_0^1 dx \, \frac{(1-x)}{x} \int \zeta d\zeta J_0\left(\zeta q \sqrt{\frac{1-x}{x}}\right) \tilde{\rho}(x,\zeta),$$

with $\widetilde{\rho}(x,\zeta)$ QCD effective transverse charge density.

- Transversality variable

$$\boxed{\zeta = \sqrt{x(1-x)\vec{b}_\perp^2}}$$

- Compare AdS and QCD expressions of FFs for arbitrary Q using identity:

$$\int_0^1 dx J_0\left(\zeta Q \sqrt{\frac{1-x}{x}}\right) = \zeta Q K_1(\zeta Q),$$

the solution for $J(Q,\zeta) = \zeta Q K_1(\zeta Q)$!

- Electromagnetic form-factor in AdS space:

$$F_{\pi^+}(Q^2) = R^3 \int \frac{dz}{z^3}\, J(Q^2, z)\, |\Phi_{\pi^+}(z)|^2 \,,$$

where $J(Q^2, z) = zQK_1(zQ)$.

- Use integral representation for $J(Q^2, z)$

$$J(Q^2, z) = \int_0^1 dx\, J_0\left(\zeta Q \sqrt{\frac{1-x}{x}}\right)$$

- Write the AdS electromagnetic form-factor as

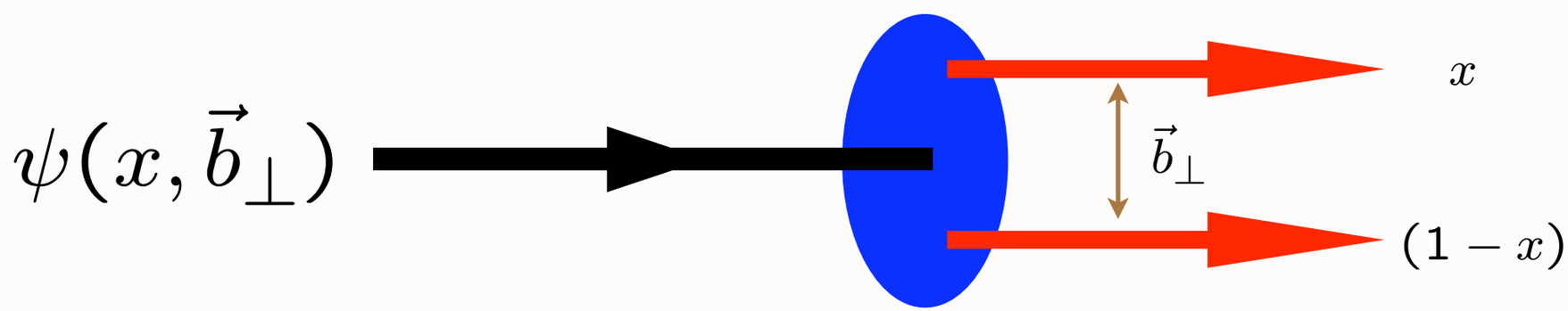
$$F_{\pi^+}(Q^2) = R^3 \int_0^1 dx \int \frac{dz}{z^3}\, J_0\left(zQ\sqrt{\frac{1-x}{x}}\right) |\Phi_{\pi^+}(z)|^2$$

- Compare with electromagnetic form-factor in light-front QCD for arbitrary Q

$$\boxed{\left|\tilde{\psi}_{q\bar{q}/\pi}(x,\zeta)\right|^2 = \frac{R^3}{2\pi}\, x(1-x) \frac{|\Phi_\pi(\zeta)|^2}{\zeta^4}}$$

with $\zeta = z,\ 0 \leq \zeta \leq \Lambda_{\rm QCD}$

LF(3+1)		AdS_5
$\psi(x, \vec{b}_\perp)$	$\longleftrightarrow$	$\phi(z)$
$\zeta = \sqrt{x(1-x)\vec{b}_\perp^2}$	$\longleftrightarrow$	z



$$\psi(x, \vec{b}_\perp) = \sqrt{\frac{x(1-x)}{2\pi\zeta}}\phi(\zeta)$$

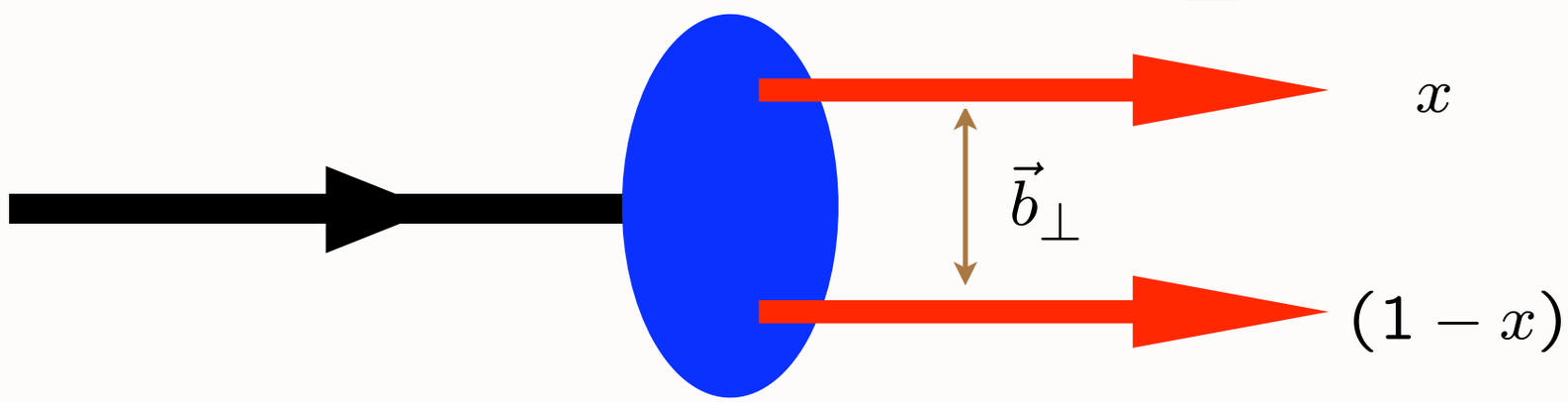
Light-Front Holography: Unique mapping derived from equality of LF and AdS formula for current matrix elements

Light-Front Holography: Map AdS/CFT to 3+1 LF Theory

Relativistic LF radial equation! Frame Independent

$$\Big[-\frac{d^2}{d\zeta^2}+\frac{1-4L^2}{4\zeta^2}+U(\zeta)\Big]\phi(\zeta)=\mathcal{M}^2\phi(\zeta)$$

$$\zeta^2=x(1-x)\mathbf{b}_\perp^2.$$



$$U(\zeta)=\kappa^4\zeta^2+2\kappa^2(L+S-1)$$

G. de Teramond, sjb

soft wall confining potential:

Derivation of the Light-Front Radial Schrodinger Equation directly from LF QCD

$$\mathcal{M}^2 = \int_0^1 dx \int \frac{d^2\vec{k}_\perp}{16\pi^3} \frac{\vec{k}_\perp^2}{x(1-x)} \left|\psi(x,\vec{k}_\perp)\right|^2 + \text{interactions}$$

$$= \int_0^1 \frac{dx}{x(1-x)} \int d^2\vec{b}_\perp \, \psi^*(x,\vec{b}_\perp) \left(-\vec{\nabla}^2_{\vec{b}_{\perp\ell}}\right) \psi(x,\vec{b}_\perp) + \text{interactions.}$$

Change variables $(\vec{\zeta},\varphi)$, $\vec{\zeta} = \sqrt{x(1-x)}\vec{b}_\perp$: $\quad \nabla^2 = \frac{1}{\zeta}\frac{d}{d\zeta}\left(\zeta\frac{d}{d\zeta}\right) + \frac{1}{\zeta^2}\frac{\partial^2}{\partial\varphi^2}$

$$\mathcal{M}^2 = \int d\zeta \, \phi^*(\zeta)\sqrt{\zeta}\left(-\frac{d^2}{d\zeta^2} - \frac{1}{\zeta}\frac{d}{d\zeta} + \frac{L^2}{\zeta^2}\right)\frac{\phi(\zeta)}{\sqrt{\zeta}}$$

$$+ \int d\zeta \, \phi^*(\zeta) U(\zeta)\phi(\zeta)$$

$$= \int d\zeta \, \phi^*(\zeta)\left(-\frac{d^2}{d\zeta^2} - \frac{1-4L^2}{4\zeta^2} + U(\zeta)\right)\phi(\zeta)$$

Consider the AdS_5 metric:

$ds^2 = \frac{R^2}{z^2}(\eta_{\mu\nu}dx^\mu dx^\nu - dz^2)$.

ds^2 invariant if $x^\mu \to \lambda x^\mu,\ z \to \lambda z$,

Maps scale transformations to scale changes of the the holographic coordinate z.

We define light-front coordinates $x^\pm = x^0 \pm x^3$.

Then $\eta^{\mu\nu}dx_\mu dx_\nu = dx_0{}^2 - dx_3{}^2 - dx_\perp{}^2 = dx^+dx^- - dx_\perp{}^2$

and

$$\boxed{ds^2 = -\frac{R^2}{z^2}(dx_\perp{}^2 + dz^2) \text{ for } x^+ = 0.}$$

Light-Front/AdS$_5$ Duality

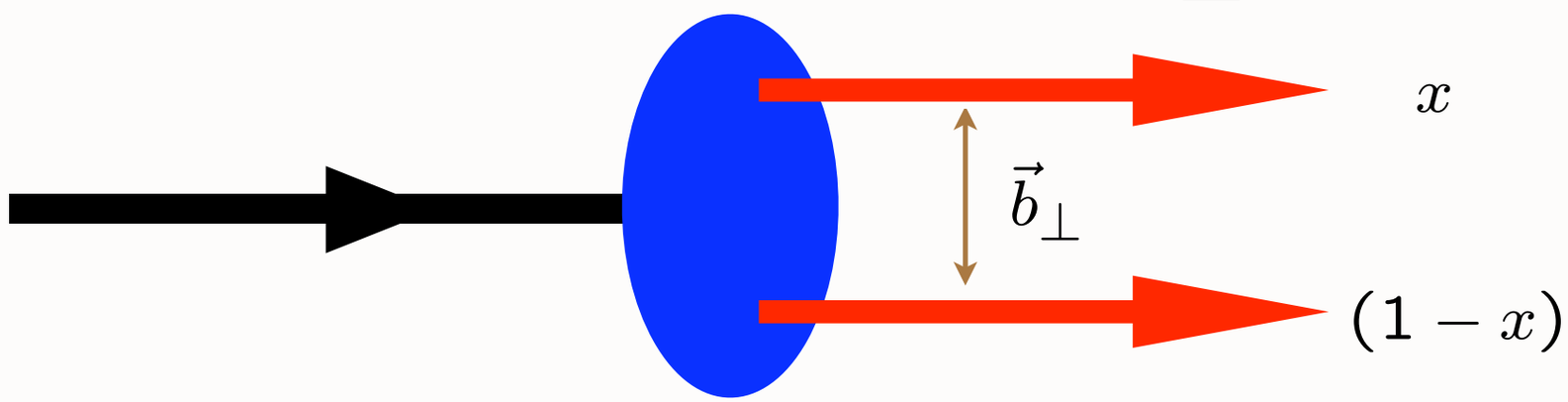
- ds^2 is invariant if $dx_\perp{}^2 \to \lambda^2 dx_\perp{}^2$, and $z \to \lambda z$, at equal LF time.
- Maps scale transformations in transverse LF space to scale changes of the holographic coordinate z.
- Holographic connection of AdS_5 to the light-front.
- The effective wave equation in the two-dim transverse LF plane has the Casimir representation L^2 corresponding to the $SO(2)$ rotation group [The Casimir for $SO(N) \sim S^{N-1}$ is $L(L+N-2)$].

Light-Front Holography: Map AdS/CFT to 3+1 LF Theory

Relativistic LF radial equation! Frame Independent

$$\Big[-\frac{d^2}{d\zeta^2}+\frac{1-4L^2}{4\zeta^2}+U(\zeta)\Big]\phi(\zeta)=\mathcal{M}^2\phi(\zeta)$$

$$\zeta^2=x(1-x)\mathbf{b}_\perp^2.$$



$$U(\zeta)=\kappa^4\zeta^2+2\kappa^2(L+S-1)$$

G. de Teramond, sjb

soft wall confining potential:

Example: Two-parton Pion LFWF

- Hard-Wall Model (P-S)

$$\tilde{\psi}^{HW}_{\bar{q}q/\pi}(x,\mathbf{b}_\perp) = \frac{\Lambda_{\rm QCD}\sqrt{x(1-x)}}{\sqrt{\pi}J_{1+L}(\beta_{L,k})} J_L\left(\sqrt{x(1-x)}\,|\mathbf{b}_\perp|\beta_{L,k}\Lambda_{\rm QCD}\right)\theta\left(\mathbf{b}_\perp^2 \leq \frac{\Lambda^{-2}_{\rm QCD}}{x(1-x)}\right)$$

- Soft-Wall Model (K-K-S-S)

$$\tilde{\psi}^{SW}_{\bar{q}q/\pi}(x,\mathbf{b}_\perp) = \kappa^{L+1}\sqrt{\frac{2n!}{(n+L)!}}\,[x(1-x)]^{\frac{1}{2}+L}|\mathbf{b}_\perp|^L e^{-\frac{1}{2}\kappa^2 x(1-x)\mathbf{b}_\perp^2} L_n^L\left(\kappa^2 x(1-x)\mathbf{b}_\perp^2\right)$$

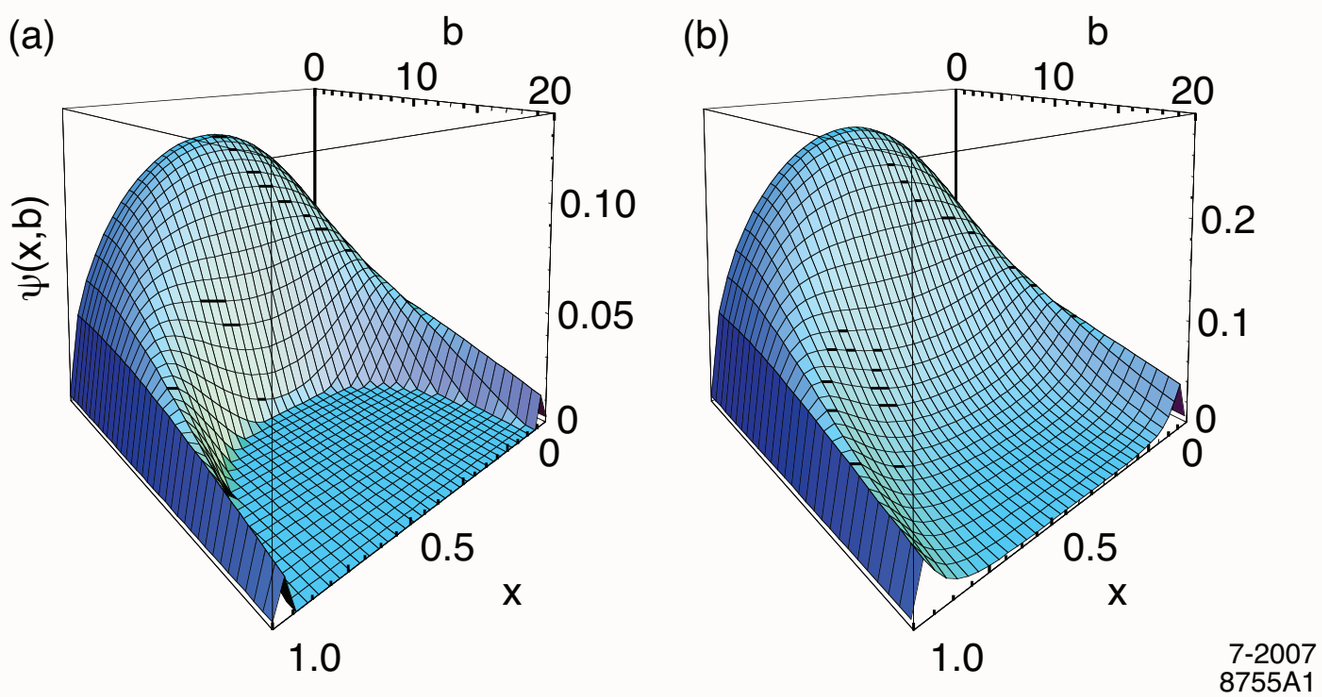


Fig: Ground state pion LFWF in impact space: (a) HW model $\Lambda_{\rm QCD} = 0.32$ GeV, (b) SW model $\kappa = 0.375$ GeV

Example: Evaluation of QCD Matrix Elements

- Pion decay constant f_π defined by the matrix element of EW current J^+_W:

$$\left\langle 0 \left| \overline{\psi}_u \gamma^+ \tfrac{1}{2}(1-\gamma_5)\psi_d \right| \pi^- \right\rangle = i \frac{P^+ f_\pi}{\sqrt{2}}$$

with

$$\left|\pi^-\right\rangle = |d\overline{u}\rangle = \frac{1}{\sqrt{N_C}} \frac{1}{\sqrt{2}} \sum_{c=1}^{N_C} \left(b^\dagger_{c\,d\downarrow} d^\dagger_{c\,u\uparrow} - b^\dagger_{c\,d\uparrow} d^\dagger_{c\,u\downarrow} \right) |0\rangle.$$

- Find light-front expression (Lepage and Brodsky '80):

$$f_\pi = 2\sqrt{N_C} \int_0^1 dx \int \frac{d^2\vec{k}_\perp}{16\pi^3}\, \psi_{\overline{q}q/\pi}(x, k_\perp).$$

- Using relation between AdS modes and QCD LFWF in the $\zeta \to 0$ limit

$$f_\pi = \frac{1}{8}\sqrt{\frac{3}{2}}\, R^{3/2} \lim_{\zeta \to 0} \frac{\Phi(\zeta)}{\zeta^2}.$$

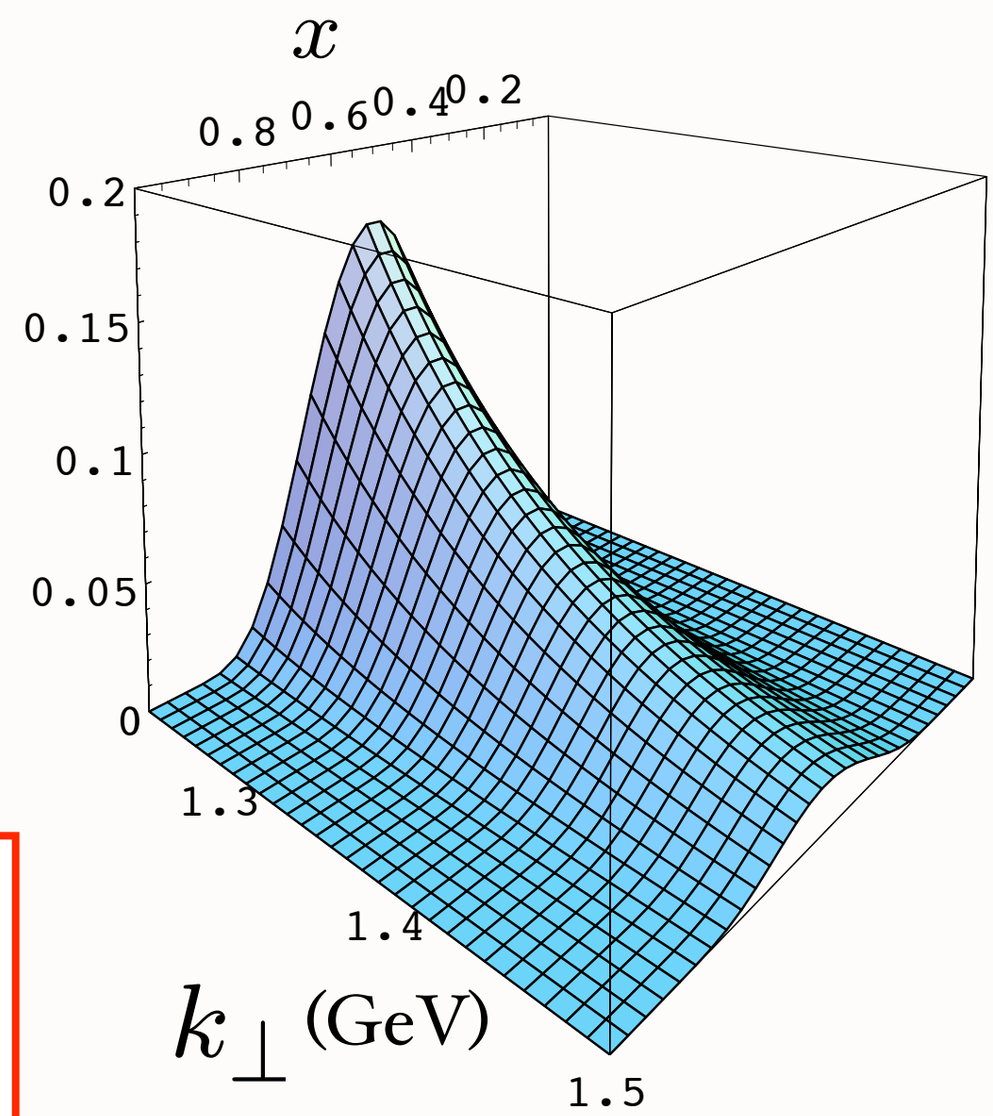
- Holographic result ($\Lambda_{\rm QCD} = 0.22$ GeV and $\kappa = 0.375$ GeV from pion FF data): Exp: $f_\pi = 92.4$ MeV

$$f_\pi^{HW} = \frac{\sqrt{3}}{8 J_1(\beta_{0,k})}\, \Lambda_{\rm QCD} = 91.7\ \mathrm{MeV},\ f_\pi^{SW} = \frac{\sqrt{3}}{8}\, \kappa = 81.2\ \mathrm{MeV},$$

Prediction from AdS/CFT: Meson LFWF

de Teramond, sjb

$\psi_M(x, k_\perp^2)$



"Soft Wall" model

$\kappa = 0.375$ GeV

massless quarks

Note coupling

$k_\perp^2, x$

$$\psi_M(x, k_\perp) = \frac{4\pi}{\kappa\sqrt{x(1-x}} e^{-\frac{k_\perp^2}{2\kappa^2 x(1-x)}}$$

$$\phi_M(x, Q_0) \propto \sqrt{x(1-x)}$$

Connection of Confinement to TMDs

Gravitational Form Factor of Composite Hadrons

- Gravitational FF defined by matrix elements of the energy momentum tensor $\Theta^{++}(x)$

$$\left\langle P' \left| \Theta^{++}(0) \right| P \right\rangle = 2 \left(P^+\right)^2 A(Q^2)$$

- $\Theta^{\mu\nu}$ is computed for each constituent in the hadron from the QCD Lagrangian

$$\mathcal{L}_{\rm QCD} = \overline{\psi} \left(i\gamma^\mu D_\mu - m \right) \psi - \tfrac{1}{4} G^a_{\mu\nu} G^{a\,\mu\nu}$$

- Symmetric and gauge invariant $\Theta^{\mu\nu}$ from variation of $S_{\rm QCD} = \int d^4x \sqrt{g}\, \mathcal{L}_{\rm QCD}$ with respect to four-dim Minkowski metric $g_{\mu\nu}$, $\Theta^{\mu\nu}(x) = -\frac{2}{\sqrt{g}} \frac{\delta S_{\rm QCD}}{\delta g_{\mu\nu}(x)}$:

$$\Theta^{\mu\nu} = \tfrac{1}{2} \overline{\psi} i (\gamma^\mu D^\nu + \gamma^\nu D^\mu) \psi - g^{\mu\nu} \overline{\psi} \left(i \not{D} - m \right) \psi - G^{a\,\mu\lambda} G^{a\,\nu}{}_\lambda + \tfrac{1}{4} g^{\mu\nu} G^a_{\mu\nu} G^{a\,\mu\nu}$$

- Quark contribution in light front gauge ($A^+ = 0,\ g^{++} = 0$)

$$\Theta^{++}(x) = \frac{i}{2} \sum_f \overline{\psi}^f(x) \gamma^+ \overleftrightarrow{\partial}^+ \psi^f(x)$$

- Particle number representation

$$\Theta^{++} = \frac{1}{2}\sum_f \int \frac{dq^+ d^2\mathbf{q}_\perp}{(2\pi)^3} \int \frac{dq'^+ d^2\mathbf{q}'_\perp}{(2\pi)^3} \left(q^+ + q'^+\right) \left\{ b^{f\dagger}(q) b^f(q') + d^{f\dagger}(q) d^f(q') \right\}$$

- Gravitational form-factor in momentum space

$$A(q^2) = \sum_n \int \left[dx_i\right] \left[d^2\mathbf{k}_{\perp i}\right] \sum_f x_f \, \psi^*_{n/P'}(x_i, \mathbf{k}'_{\perp i}) \psi_{n/P}(x_i, \mathbf{k}_{\perp i}),$$

where $\mathbf{k}'_{\perp i} = \mathbf{k}_{\perp i} + (1 - x_i)\, \mathbf{q}_\perp$ for a struck quark and $\mathbf{k}'_{\perp i} = \mathbf{k}_{\perp i} - x_i\, \mathbf{q}_\perp$ for each spectator

- Gravitational form-factor in impact space

$$A(q^2) = \sum_n \prod_{j=1}^{n-1} \int dx_j d^2\mathbf{b}_{\perp j} \sum_f x_f \exp\Big(i\mathbf{q}_\perp \cdot \sum_{j=1}^{n-1} x_j \mathbf{b}_{\perp j} \Big) \left| \tilde{\psi}_n(x_j, \mathbf{b}_{\perp j}) \right|^2$$

Gravitational Form Factor on the LF

$$A_f(q^2) = \int_0^1 x\, dx \int d^2\vec{\eta}_\perp e^{i\vec{\eta}_\perp\cdot\vec{q}_\perp}\, \tilde{\rho}_f(x,\vec{\eta}_\perp),$$

where

$$\tilde{\rho}_f(x,\vec{\eta}_\perp) = \int \frac{d^2\vec{q}_\perp}{(2\pi)^2}\, e^{-i\vec{\eta}_\perp\cdot\vec{q}_\perp}\, \rho_f(x,\vec{q}_\perp)$$

$$= \sum_n \prod_{j=1}^{n-1} \int dx_j\, d^2\vec{b}_{\perp j}\, \delta\Big(1 - x - \sum_{j=1}^{n-1} x_j\Big)$$

$$\times \delta^{(2)}\Big(\sum_{j=1}^{n-1} x_j \vec{b}_{\perp j} - \vec{\eta}_\perp\Big) \Big|\tilde{\psi}_n(x_j, \vec{b}_{\perp j})\Big|^2 .$$

Extra factor of x relativ
to charge
form factor

For each quark and
gluon field $x=x_f$

Integrate over angle

$$A_f(q^2) = 2\pi \int_0^1 dx\,(1-x) \int \zeta d\zeta\, J_0\Big(\zeta q \sqrt{\frac{1-x}{x}}\Big)\, \tilde{\rho}_f(x,\zeta)$$

$$\zeta = \sqrt{\frac{x}{1-x}}\, \Big|\sum_{j=1}^{n-1} x_j \mathbf{b}_{\perp j}\Big|$$

Gravitational Form Factor in AdS space

- Hadronic gravitational form-factor in AdS space

$$A_\pi(Q^2) = R^3 \int \frac{dz}{z^3}\, H(Q^2, z)\, |\Phi_\pi(z)|^2 \,,$$

Abidin & Carlson

where $H(Q^2, z) = \frac{1}{2} Q^2 z^2 K_2(zQ)$

- Use integral representation for $H(Q^2, z)$

$$H(Q^2, z) = 2 \int_0^1 x\, dx\, J_0\left(zQ\sqrt{\frac{1-x}{x}}\right)$$

- Write the AdS gravitational form-factor as

$$A_\pi(Q^2) = 2R^3 \int_0^1 x\, dx \int \frac{dz}{z^3}\, J_0\left(zQ\sqrt{\frac{1-x}{x}}\right) |\Phi_\pi(z)|^2$$

- Compare with gravitational form-factor in light-front QCD for arbitrary Q

$$\boxed{\left|\tilde{\psi}_{q\bar{q}/\pi}(x, \zeta)\right|^2 = \frac{R^3}{2\pi}\, x(1-x) \frac{|\Phi_\pi(\zeta)|^2}{\zeta^4},}$$

Identical to LF Holography obtained from electromagnetic current

The AdS Gravitational Form Factor

Abidin & Carlson

$$ds^2 = \frac{R^2}{z^2}\left((\eta_{\mu\nu} + h_{\mu\nu})dx^\mu dx^\nu - dz^2\right).$$ **linearized metric**

$$h_{zz} = h_{z\mu} = 0$$ **gauge choice**

$$\int d^4x\, dz\, \sqrt{g}\, h^{\ell m}(x,z)\partial_\ell \Phi^*_{P'}(x,z)\partial_m \Phi_P(x,z)$$ **gravitational coupling**

$$z^3\partial_z\left(\frac{1}{z^3}\partial_z h^\nu_\mu\right) - \partial_\rho\partial^\rho h^\nu_\mu = 0.$$ **eqn. of motion from action**

propagation of graviton into AdS from external source

$$h^\nu_\mu(x,z) = \eta^\nu_\mu\, e^{-iq\cdot x} H(q^2,z) \qquad H(q^2=0,z) = H(q^2, z=0) = 1.$$

$$H(Q^2,z) = \frac{1}{2}Q^2 z^2 K_2(zQ).$$ **solution!**

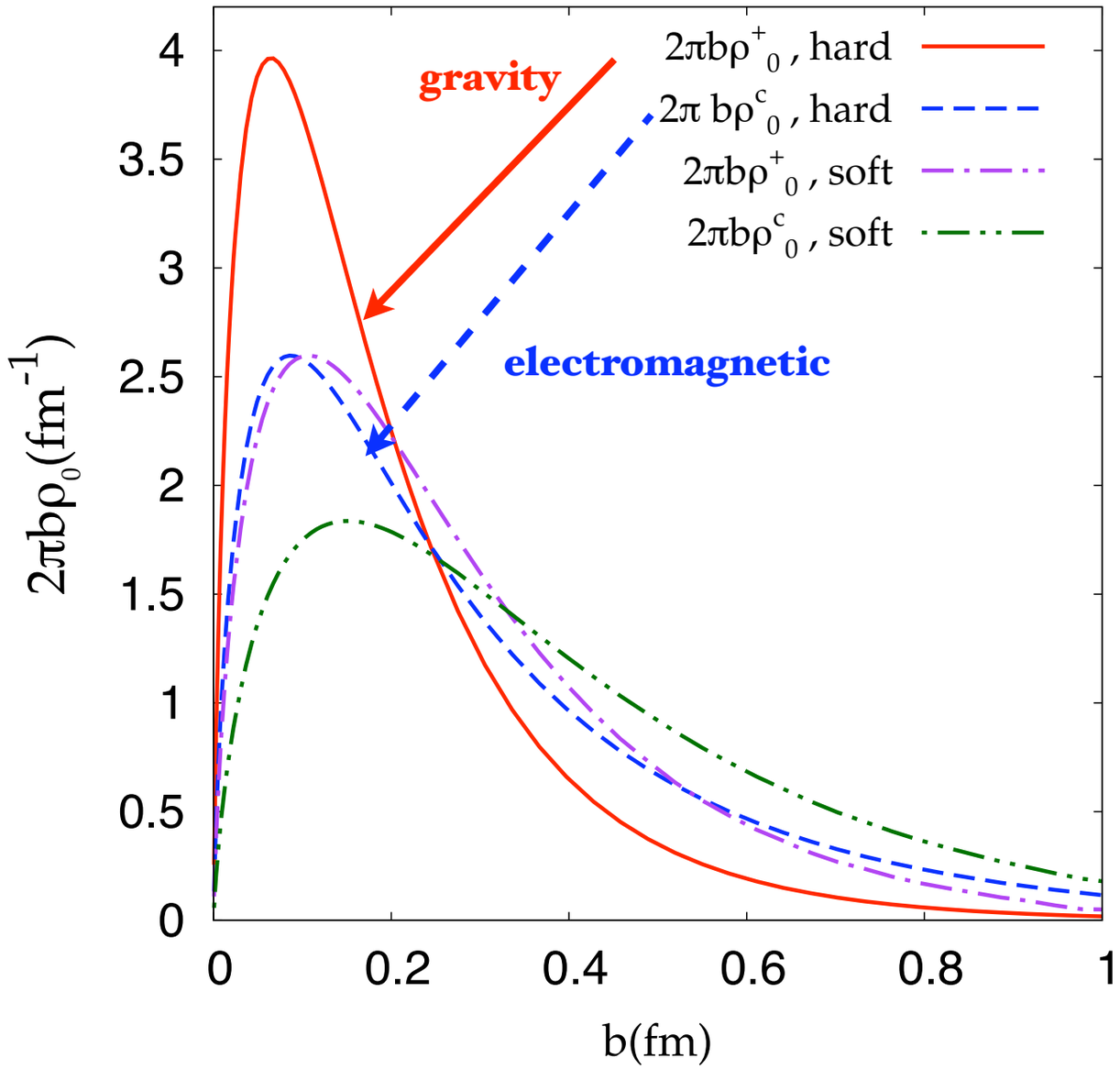
$$A(Q^2) = R^3\int \frac{dz}{z^3}\,\Phi(z)H(Q^2,z)\Phi(z).$$ **Gravitational Form Factor**

Holographic result for LFWF identical for electroweak and gravity couplings! Highly nontrivial consistency test

AdS/QCD can predict

- Momentum fractions for each quark flavor and the gluons $A_f(0) =< x_f >, \sum_f A_f(0) = A(0) = 1$
- Orbital Angular Momentum for each quark flavor and the gluons $B_f(0) =< L^3_f >, \sum_f B_f(0) = B(0) = 0$
- Vanishing Anomalous Gravitomagnetic Moment
- Shape and Asymptotic Behavior of $A_f(Q^2), B_f(Q^2)$

Momentum Density more Compact than Charge Density



Z. Abidin and C. E. Carlson,
"Hadronic Momentum Densities in the Transverse
arXiv:0808.3097 [hep-ph].

Immediate property of LF Holography

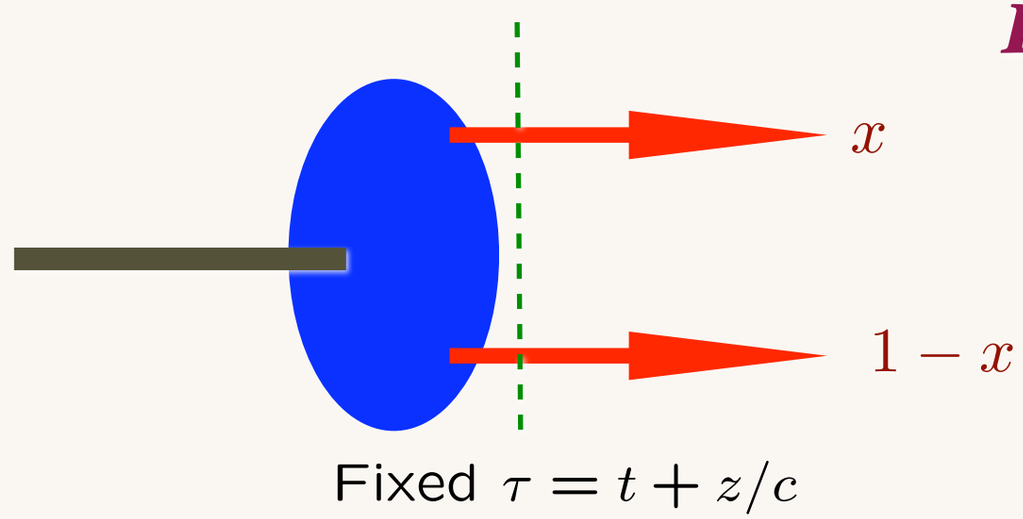
$$\psi_M(x, k_\perp) = \frac{4\pi}{\kappa\sqrt{x(1-x}} F\left(\frac{k_\perp^2}{2\kappa^2 x(1-x)}\right)$$

Hadron Distribution Amplitudes

Lepage, sjb

$$\phi_H(x_i, Q)$$

$$\sum_i x_i = 1$$



$k_\perp^2 < Q^2$

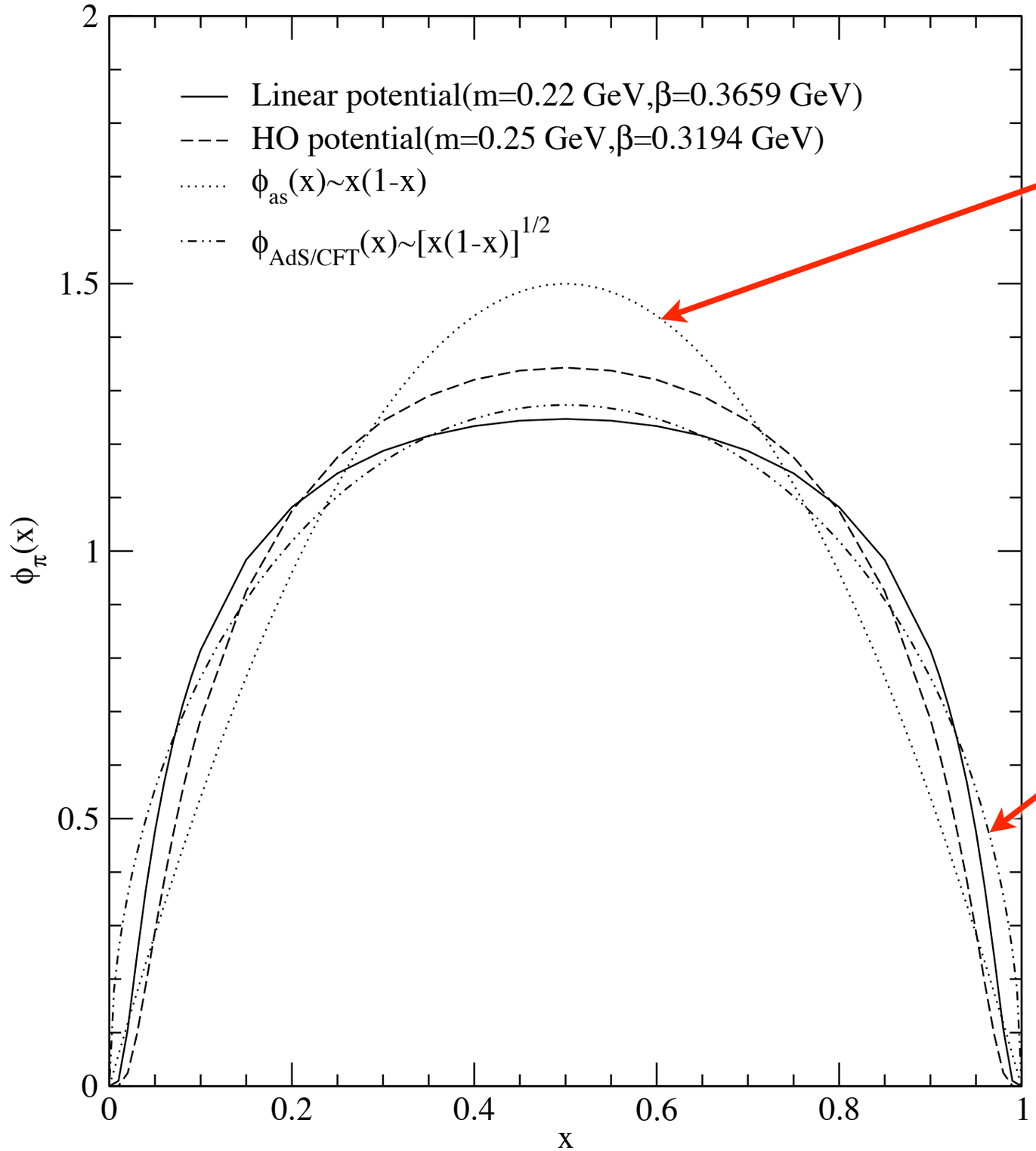
- Fundamental gauge invariant non-perturbative input to hard exclusive processes, heavy hadron decays. Defined for Mesons, Baryons

- Evolution Equations from PQCD, OPE, Conformal Invariance

Lepage, sjb
Efremov, Radyushkin
Sachrajda, Frishman Lepage, sjb
Braun, Gardi

- Compute from valence light-front wavefunction in light-cone gauge

$$\phi_M(x, Q) = \int^Q d^2\vec{k}\ \psi_{q\bar{q}}(x, \vec{k}_\perp)$$



$$\phi_{asympt} \sim x(1-x)$$

AdS/CFT:

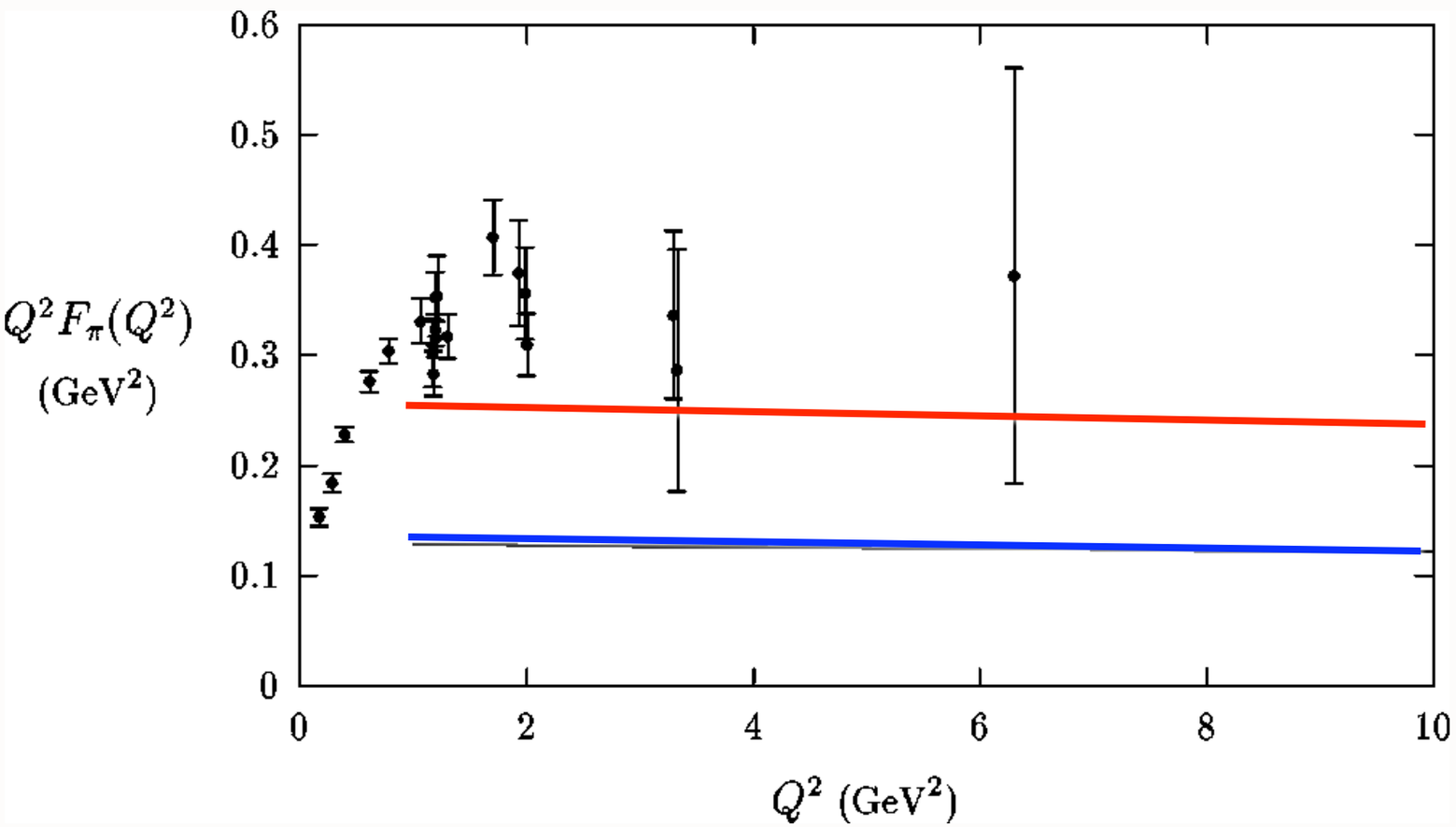
$$\phi(x, Q_0) \propto \sqrt{x(1-x)}$$

Increases PQCD leading twist predictio
$F_\pi(Q^2)$ by factor 16/9

Lepage, sjb

C. Ji, A. Pang, D. Robertson, sjb
Choi, Ji

$$F_\pi(Q^2) = \int_0^1 dx\, \phi_\pi(x) \int_0^1 dy\, \phi_\pi(y) \frac{16\pi C_F \alpha_V(Q_V)}{(1-x)(1-y)Q^2}$$



$\phi(x, Q_0) \propto \sqrt{x(1-x)}$

$\phi_{asymptotic} \propto x(1-x)$

Normalized to f_π

AdS/CFT: Increases PQCD leading twist prediction for $F_\pi(Q^2)$ by factor 16/9

Second Moment of Pion Distribution Amplitude

$$< \xi^2 >= \int_{-1}^{1} d\xi \; \xi^2 \phi(\xi)$$

$$\xi = 1 - 2x$$

$$< \xi^2 >_\pi = 1/5 = 0.20 \qquad \phi_{asympt} \propto x(1-x)$$

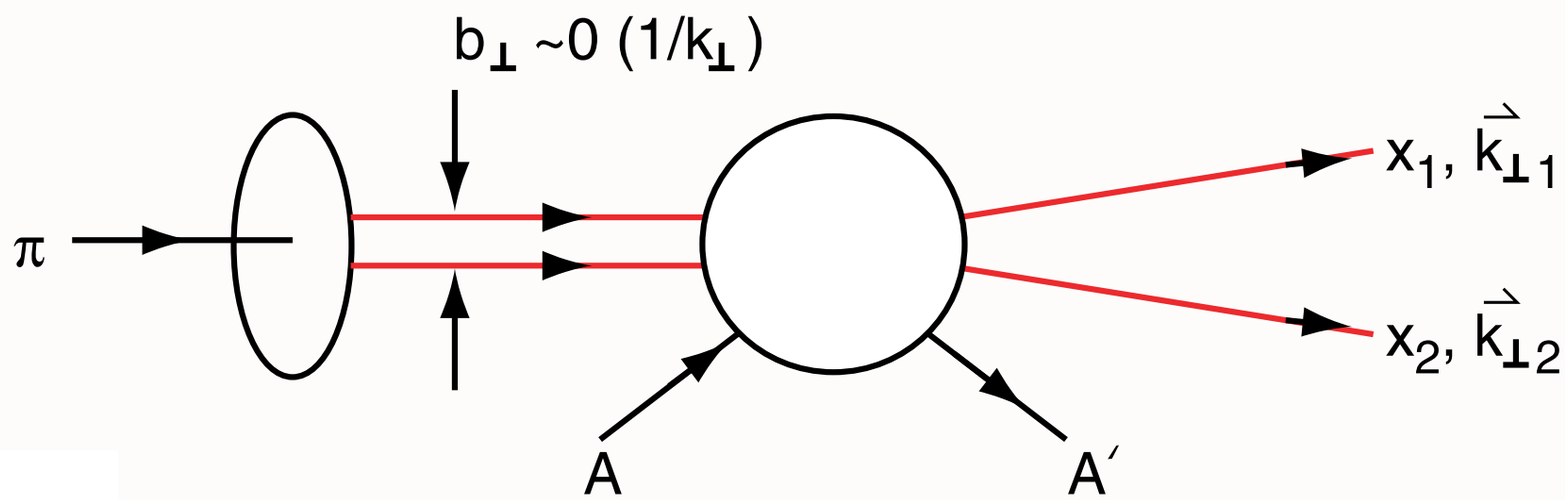
$$< \xi^2 >_\pi = 1/4 = 0.25 \qquad \phi_{AdS/QCD} \propto \sqrt{x(1-x)}$$

Lattice (I) $< \xi^2 >_\pi = 0.28 \pm 0.03$ Donnellan et al.

Lattice (II) $< \xi^2 >_\pi = 0.269 \pm 0.039$ Braun et al.

Diffractive Dissociation of Pion into Quark Jets

E791 Ashery et al.



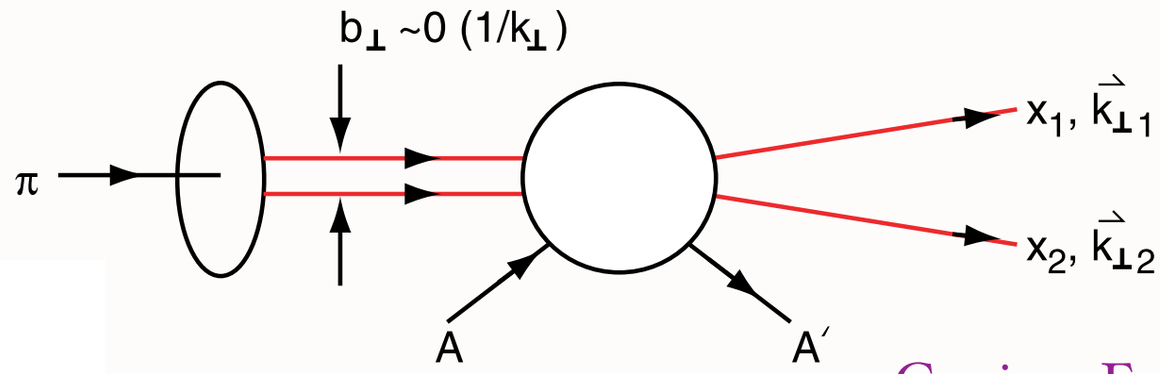
$$M \propto \frac{\partial^2}{\partial^2 k_\perp} \psi_\pi(x, k_\perp)$$

Measure Light-Front Wavefunction of Pion

Minimal momentum transfer to nucleus

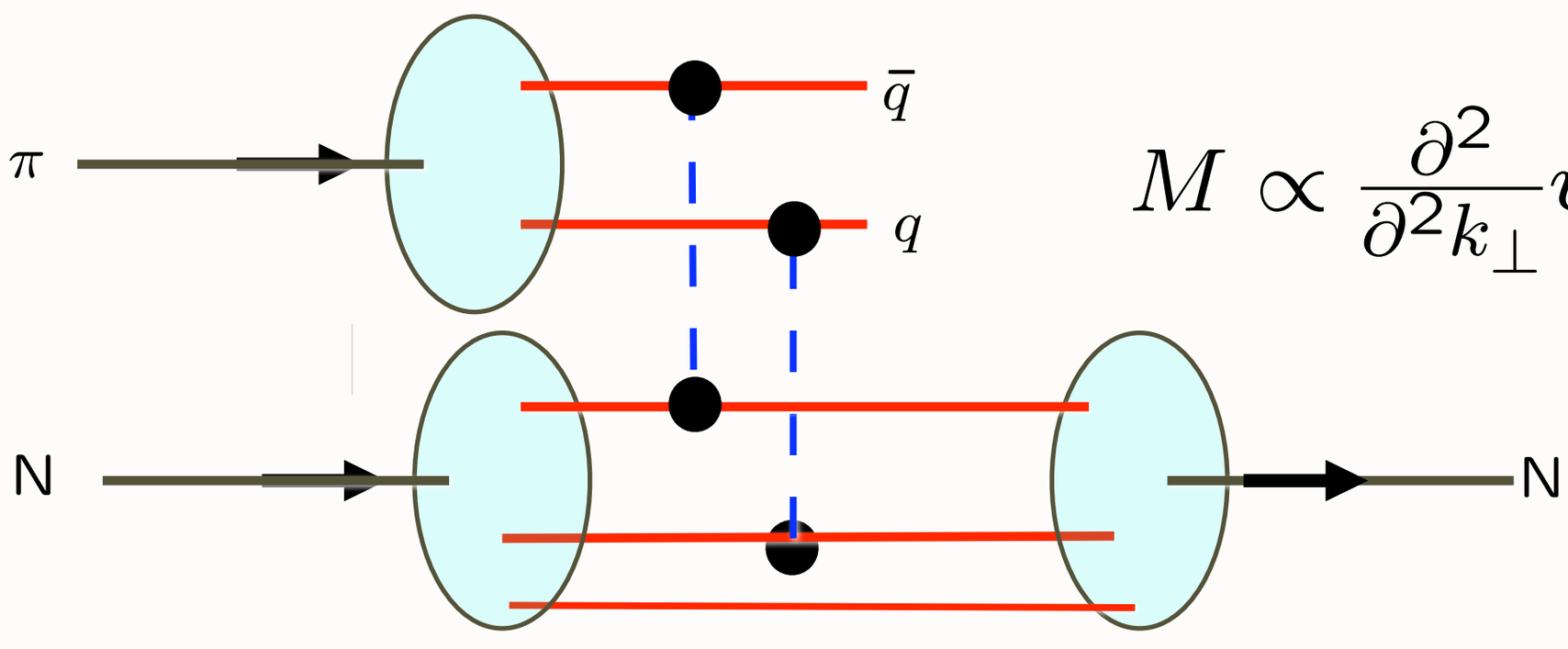
Nucleus left Intact!

E791 FNAL Diffractive DiJet



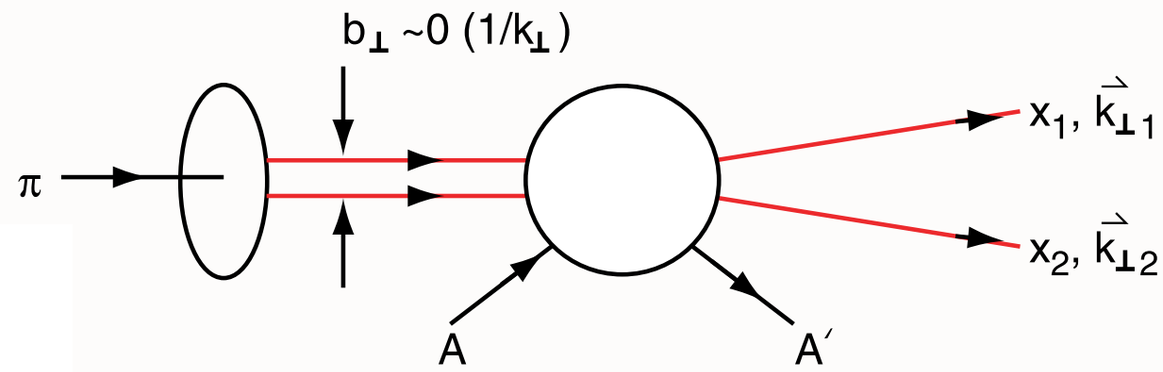
Gunion, Frankfurt, Mueller, Strikman, sjb
Frankfurt, Miller, Strikman

Two-gluon exchange measures the second derivative of the pion light-front wavefunction



$$M \propto \frac{\partial^2}{\partial^2 k_\perp} \psi_\pi(x, k_\perp)$$

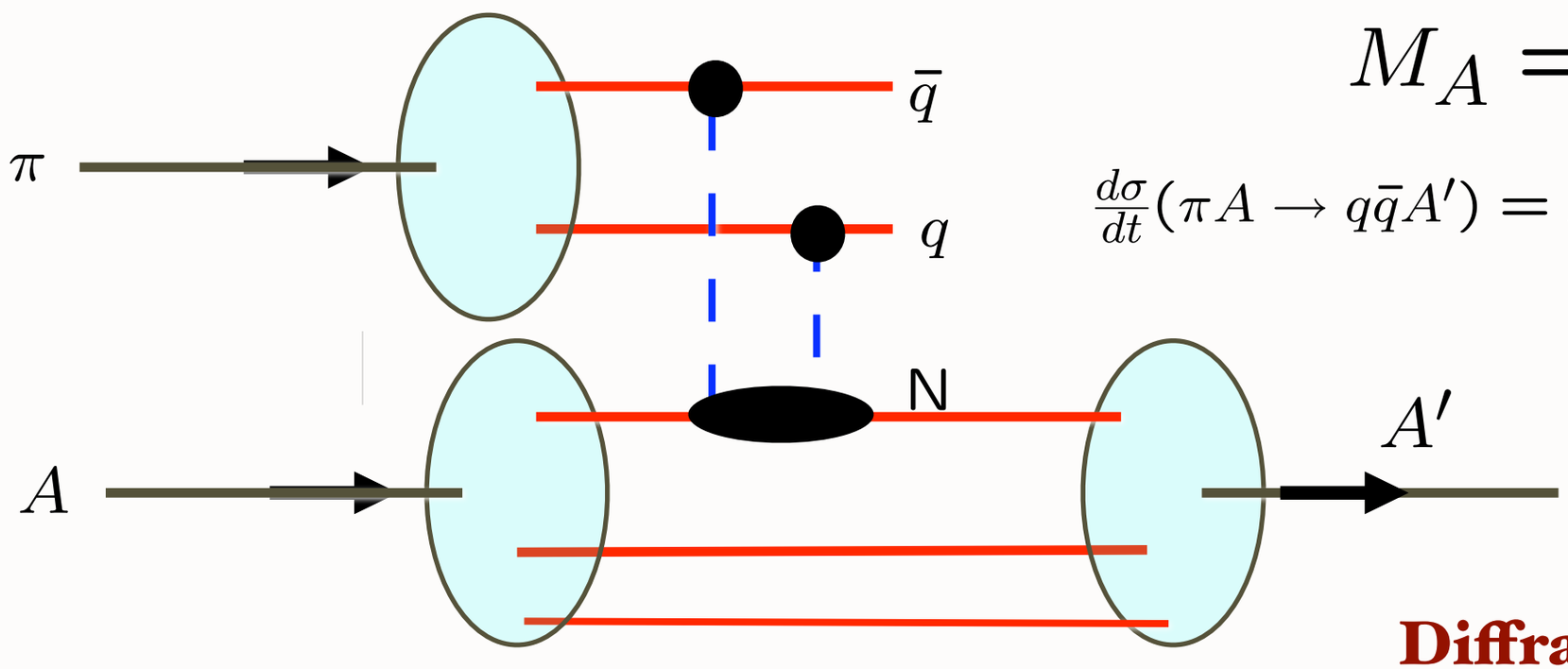
Key Ingredients in E791 Experiment



Brodsky Mueller
Frankfurt Miller Strikman

Small color-dipole moment pion not absorbed; interacts with each nucleon coherently

QCD COLOR Transparency



$$M_A = A\ M_N$$

$$\frac{d\sigma}{dt}(\pi A \to q\bar{q}A') = A^2\ \frac{d\sigma}{dt}(\pi N \to q\bar{q}N')\ F_A^2(t)$$

Target left intact

Diffraction, Rapidity gap

Color Transparency

Bertsch, Gunion, Goldhaber, sjb

A. H. Mueller, sjb

- Fundamental test of gauge theory in hadron physics
- Small color dipole moments interact weakly in nuclei
- Complete coherence at high energies
- Clear Demonstration of CT from Diffractive Di-Jets

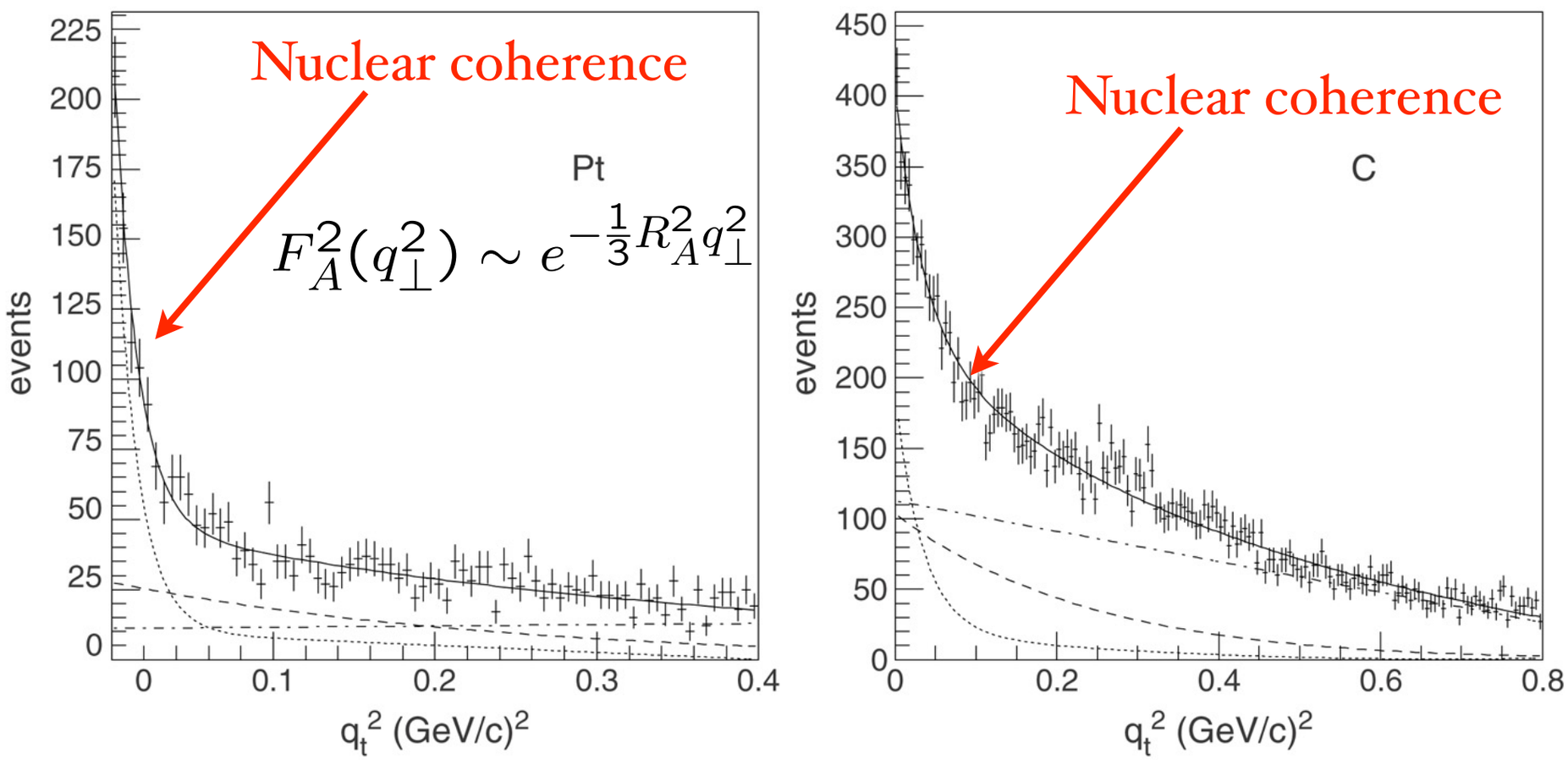
- **Fully coherent interactions between pion and nucleons.**

- **Emerging Di-Jets do not interact with nucleus.**

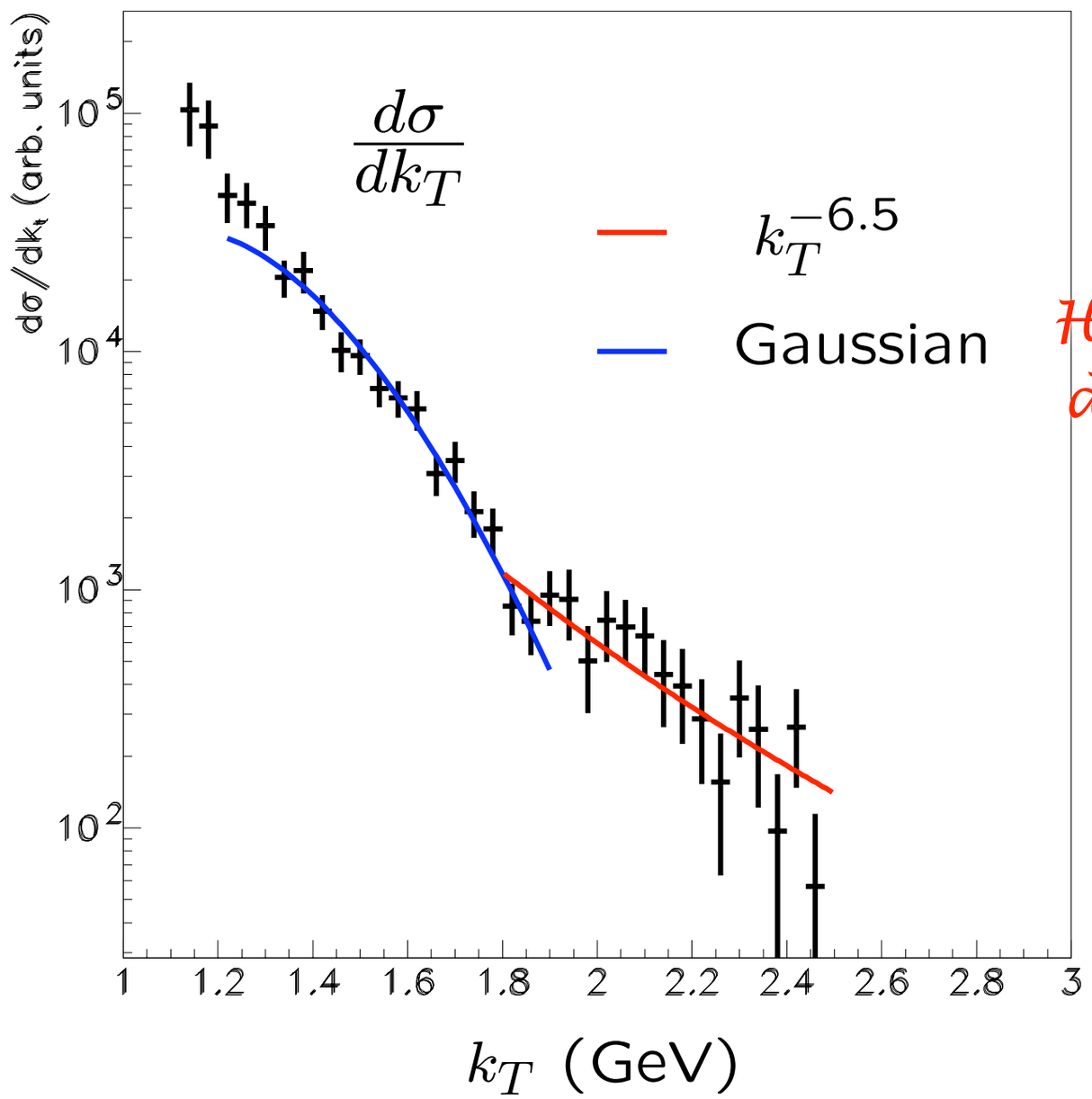
$$\mathcal{M}(\mathcal{A}) = \mathcal{A} \cdot \mathcal{M}(\mathcal{N})$$

$$\frac{d\sigma}{dq_t^2} \propto A^2 \quad q_t^2 \sim 0$$

$$\sigma \propto A^{4/3}$$



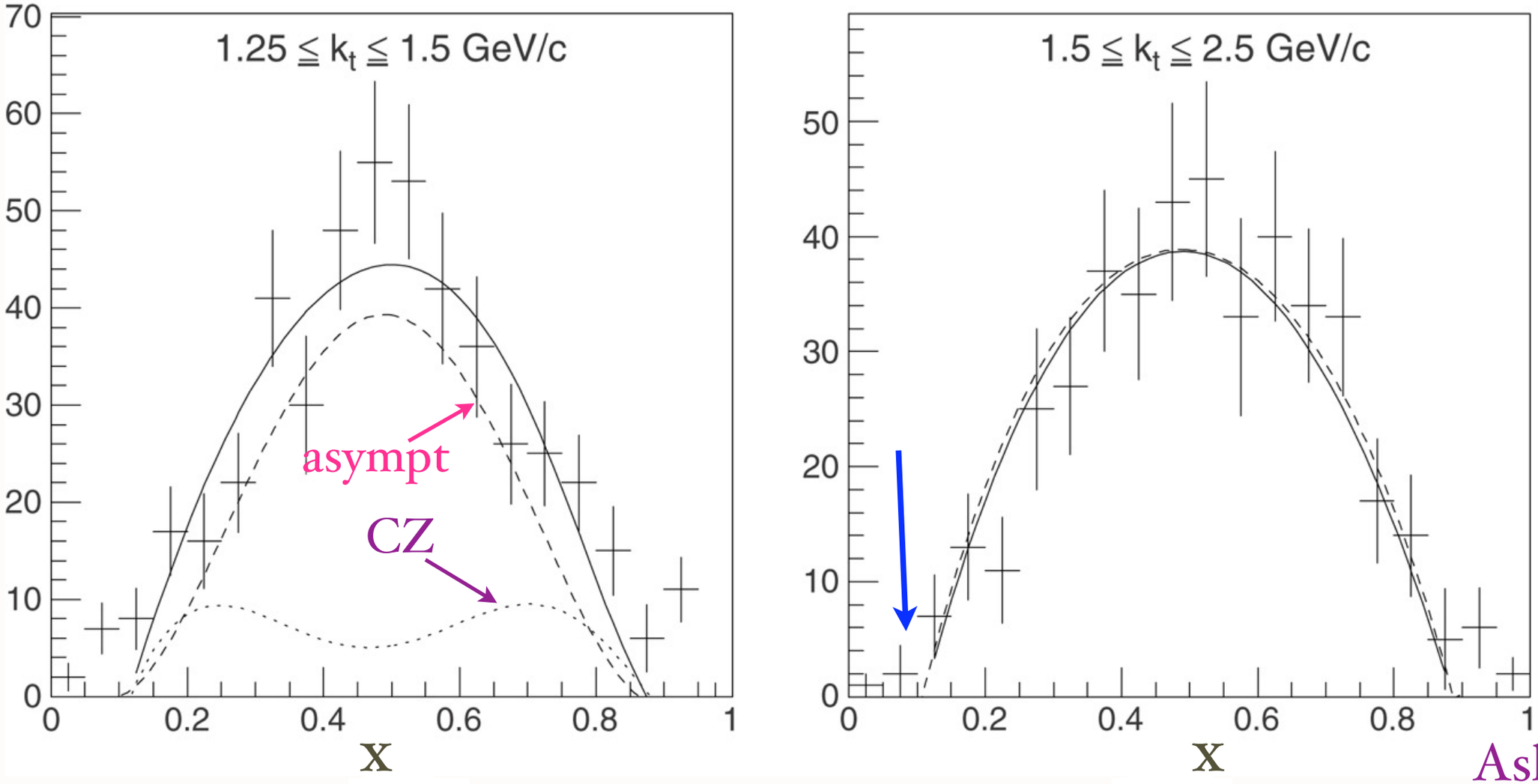
E791 Diffractive Di-Jet transverse momentum distribution



Two Components

High Transverse momentum dependence consistent with $k_T^{-6.5}$ *PQCD, ERBL Evolution*

Gaussian component similar to AdS/CFT HO LFWF



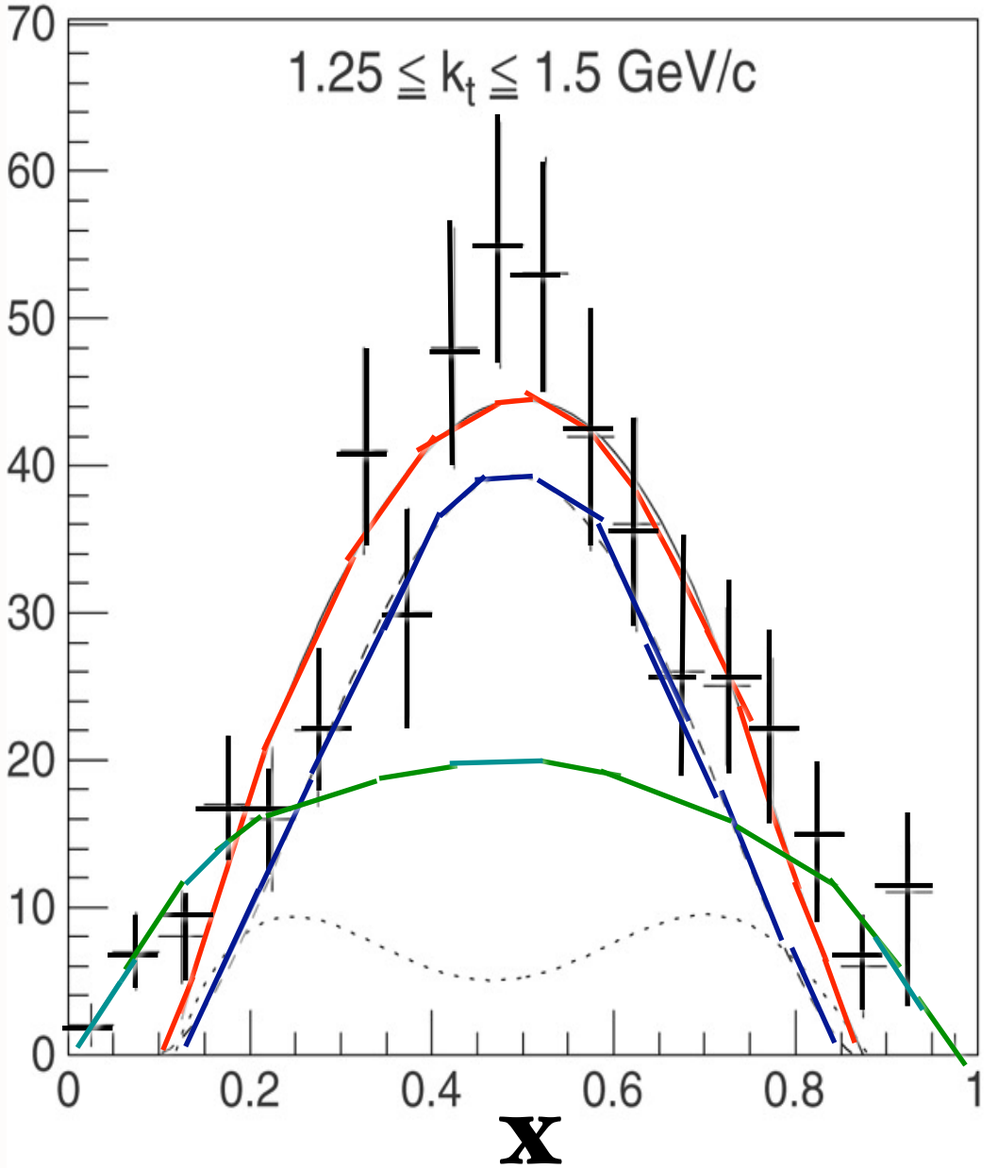
Ashery E791

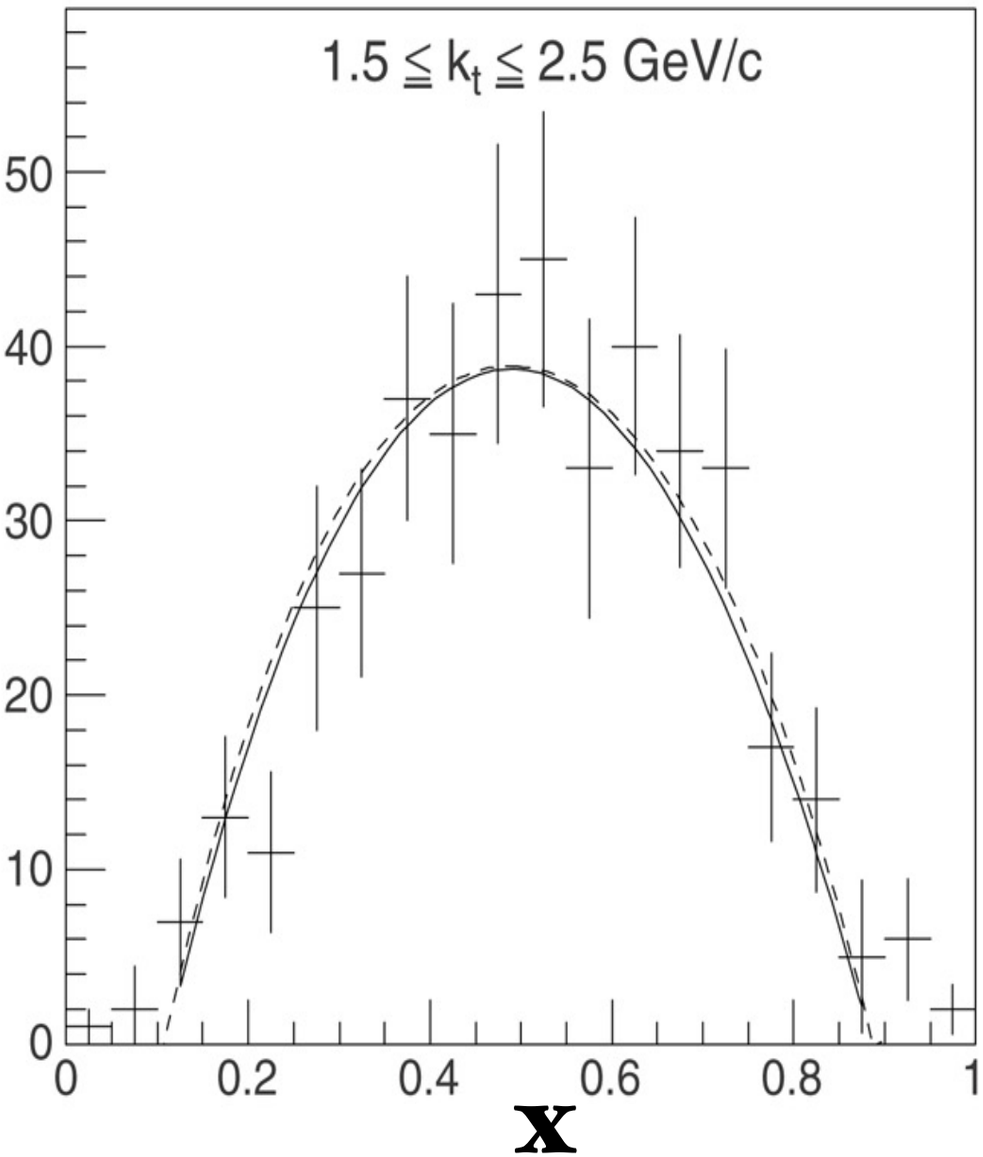
Narrowing of x distribution at higher jet transverse momentum

x distribution of diffractive dijets from the platinum target for $1.25 \leq k_t \leq 1.5$ GeV/c (left) and for $1.5 \leq k_t \leq 2.5$ GeV/c (right). The solid line is a fit to a combination of the asymptotic and CZ distribution amplitudes. The dashed line shows the contribution from the asymptotic function and the dotted line that of the CZ function.

Possibly two components: Nonperturbative (AdS/CFT) and Perturbative (ERBL) $\phi(x) \propto \sqrt{x(1-x)}$

Evolution to asymptotic distribution





Ashery
E791

Possibly two components:
Perturbative (ERBL) + Nonperturbative (AdS/CFT)

$$\phi(x) = A_{\rm pert}(k_\perp^2)x(1-x) + B_{\rm nonpert}(k_\perp^2)\sqrt{x(1-x)}$$

Narrowing of x distribution at high jet transverse momentum

Note: Contributions to Mesons Form Factors at Large Q in AdS/QCD

- Write form factor in terms of an effective partonic transverse density in impact space $\mathbf{b}_\perp$

$$F_\pi(q^2) = \int_0^1 dx \int db^2\, \widetilde{\rho}(x, b, Q),$$

with $\widetilde{\rho}(x, b, Q) = \pi J_0\left[b\, Q(1-x)\right] |\widetilde{\psi}(x, b)|^2$ and $b = |\mathbf{b}_\perp|$.

- Contribution from $\rho(x, b, Q)$ is shifted towards small $|\mathbf{b}_\perp|$ and large $x \to 1$ as Q increases.

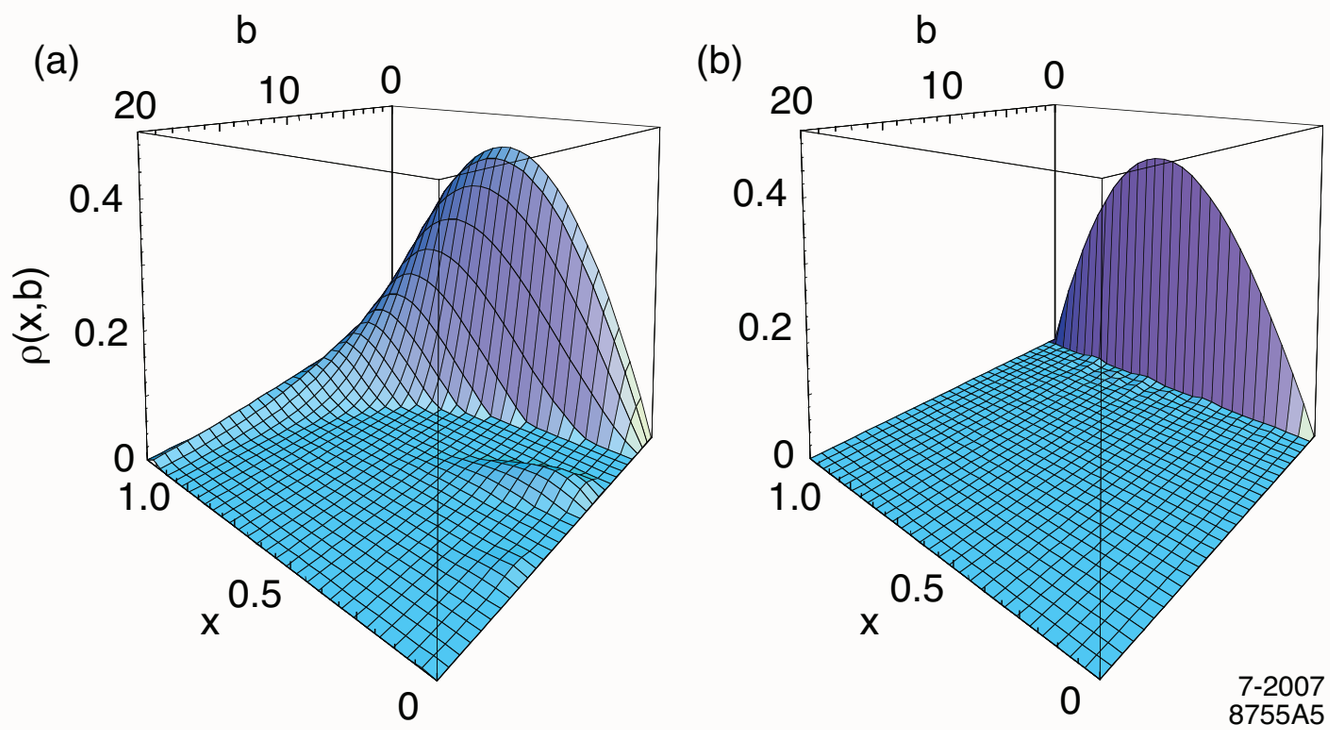


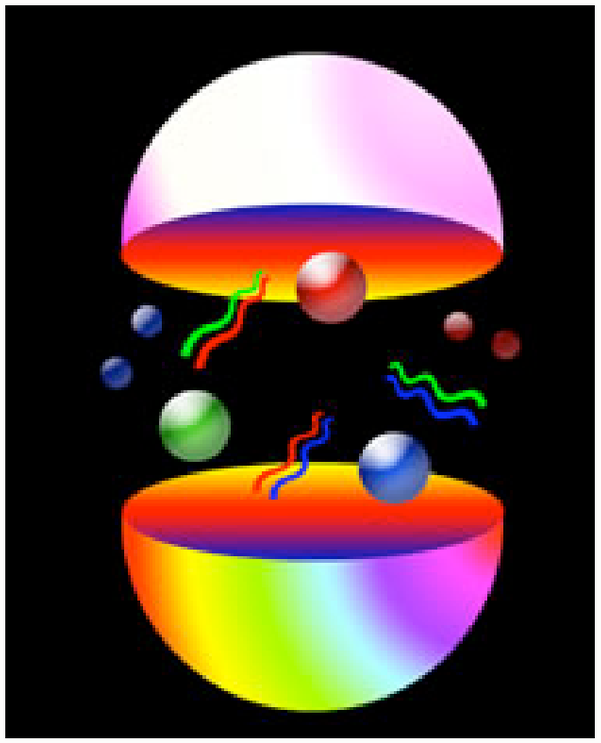
Fig: LF partonic density $\rho(x, b, Q)$: (a) Q = 1 GeV/c, (b) very large Q.

- Baryons Spectrum in "bottom-up" holographic QCD

GdT and Sjb hep-th/0409074, hep-th/0501022.

See also T. Sakai and S. Sugimoto

Baryons in Ads/CFT



- Action for massive fermionic modes on AdS$_{d+1}$:

$$S[\overline{\Psi},\Psi] = \int d^{d+1}x\,\sqrt{g}\,\overline{\Psi}(x,z)\left(i\Gamma^{\ell}D_{\ell}-\mu\right)\Psi(x,z).$$

- Equation of motion: $\left(i\Gamma^{\ell}D_{\ell}-\mu\right)\Psi(x,z)=0$

$$\left[i\left(z\eta^{\ell m}\Gamma_{\ell}\partial_m+\frac{d}{2}\Gamma_z\right)+\mu R\right]\Psi(x^{\ell})=0.$$

Baryons

Holographic Light-Front Integrable Form and Spectrum

- In the conformal limit fermionic spin-$\frac{1}{2}$ modes $\psi(\zeta)$ and spin-$\frac{3}{2}$ modes $\psi_\mu(\zeta)$ are two-component spinor solutions of the Dirac light-front equation

$$\alpha\Pi(\zeta)\psi(\zeta) = \mathcal{M}\psi(\zeta),$$

where $H_{LF} = \alpha\Pi$ and the operator

$$\Pi_L(\zeta) = -i\left(\frac{d}{d\zeta} - \frac{L+\frac{1}{2}}{\zeta}\gamma_5\right),$$

and its adjoint $\Pi_L^\dagger(\zeta)$ satisfy the commutation relations

$$\left[\Pi_L(\zeta), \Pi_L^\dagger(\zeta)\right] = \frac{2L+1}{\zeta^2}\gamma_5.$$

- Note: in the Weyl representation ($i\alpha = \gamma_5\beta$)

$$i\alpha = \begin{pmatrix} 0 & I \\ -I & 0 \end{pmatrix}, \qquad \beta = \begin{pmatrix} 0 & I \\ I & 0 \end{pmatrix}, \qquad \gamma_5 = \begin{pmatrix} I & 0 \\ 0 & -I \end{pmatrix}.$$

- Baryon: twist-dimension $3 + L \quad (\nu = L + 1)$

$$\mathcal{O}_{3+L} = \psi D_{\{\ell_1} \dots D_{\ell_q} \psi D_{\ell_{q+1}} \dots D_{\ell_m\}} \psi, \qquad L = \sum_{i=1}^{m} \ell_i.$$

- Solution to Dirac eigenvalue equation with UV matching boundary conditions

$$\psi(\zeta) = C\sqrt{\zeta}\left[J_{L+1}(\zeta\mathcal{M})u_+ + J_{L+2}(\zeta\mathcal{M})u_-\right].$$

Baryonic modes propagating in AdS space have two components: orbital L and $L + 1$.

- Hadronic mass spectrum determined from IR boundary conditions

$$\psi_\pm\left(\zeta = 1/\Lambda_{\rm QCD}\right) = 0,$$

given by

$$\mathcal{M}^+_{\nu,k} = \beta_{\nu,k}\Lambda_{\rm QCD}, \quad \mathcal{M}^-_{\nu,k} = \beta_{\nu+1,k}\Lambda_{\rm QCD},$$

with a scale independent mass ratio.

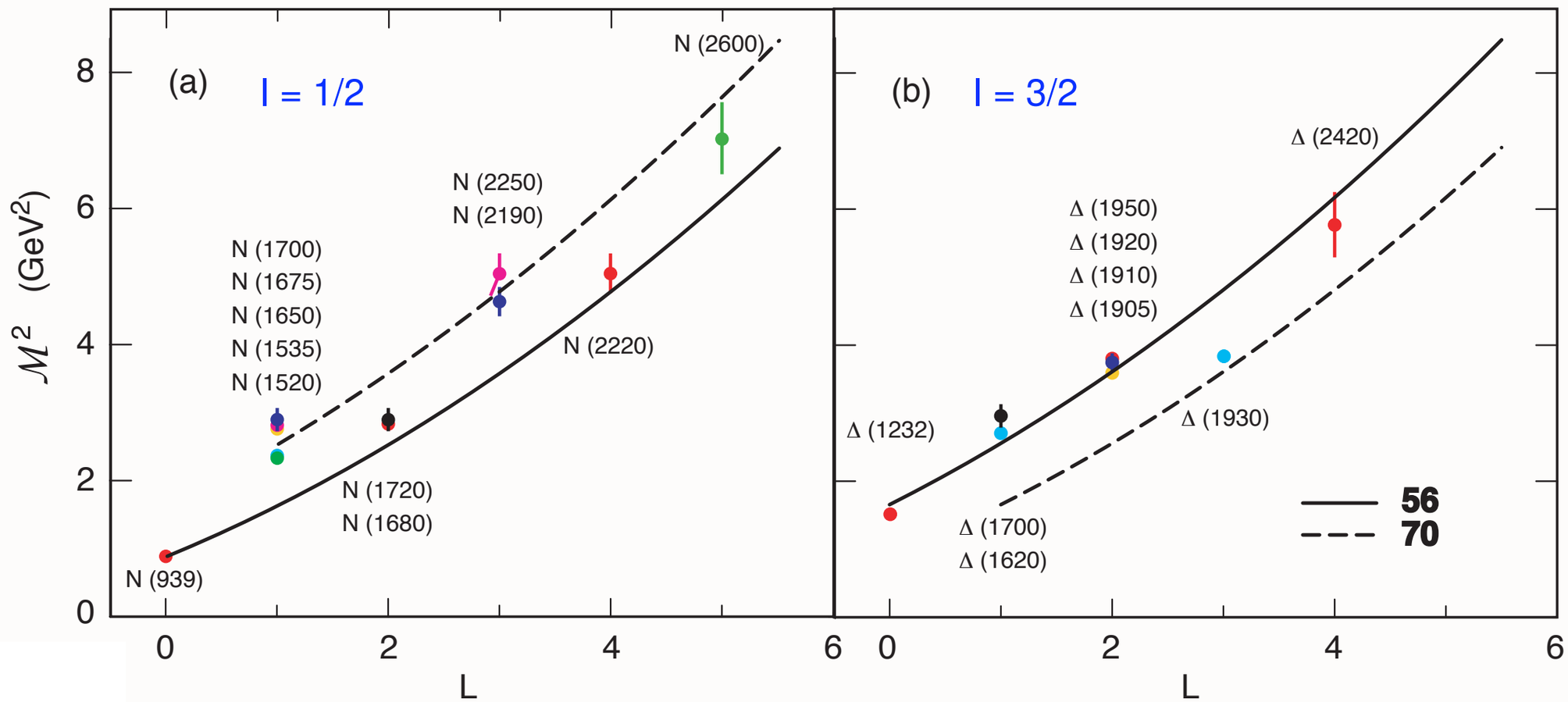


Fig: Light baryon orbital spectrum for Λ_{QCD} = 0.25 GeV in the HW model. The $\mathbf{56}$ trajectory corresponds to L even $P = +$ states, and the $\mathbf{70}$ to L odd $P = -$ states.

$SU(6)$	S	L	Baryon State
56	$\frac{1}{2}$	0	$N\frac{1}{2}^+(939)$
	$\frac{3}{2}$	0	$\Delta\frac{3}{2}^+(1232)$
70	$\frac{1}{2}$	1	$N\frac{1}{2}^-(1535)$ $N\frac{3}{2}^-(1520)$
	$\frac{3}{2}$	1	$N\frac{1}{2}^-(1650)$ $N\frac{3}{2}^-(1700)$ $N\frac{5}{2}^-(1675)$
	$\frac{1}{2}$	1	$\Delta\frac{1}{2}^-(1620)$ $\Delta\frac{3}{2}^-(1700)$
56	$\frac{1}{2}$	2	$N\frac{3}{2}^+(1720)$ $N\frac{5}{2}^+(1680)$
	$\frac{3}{2}$	2	$\Delta\frac{1}{2}^+(1910)$ $\Delta\frac{3}{2}^+(1920)$ $\Delta\frac{5}{2}^+(1905)$ $\Delta\frac{7}{2}^+(1950)$
70	$\frac{1}{2}$	3	$N\frac{5}{2}^-$ $N\frac{7}{2}^-$
	$\frac{3}{2}$	3	$N\frac{3}{2}^-$ $N\frac{5}{2}^-$ $N\frac{7}{2}^-(2190)$ $N\frac{9}{2}^-(2250)$
	$\frac{1}{2}$	3	$\Delta\frac{5}{2}^-(1930)$ $\Delta\frac{7}{2}^-$
56	$\frac{1}{2}$	4	$N\frac{7}{2}^+$ $N\frac{9}{2}^+(2220)$
	$\frac{3}{2}$	4	$\Delta\frac{5}{2}^+$ $\Delta\frac{7}{2}^+$ $\Delta\frac{9}{2}^+$ $\Delta\frac{11}{2}^+(2420)$
70	$\frac{1}{2}$	5	$N\frac{9}{2}^-$ $N\frac{11}{2}^-(2600)$
	$\frac{3}{2}$	5	$N\frac{7}{2}^-$ $N\frac{9}{2}^-$ $N\frac{11}{2}^-$ $N\frac{13}{2}^-$

Non-Conformal Extension of Algebraic Structure (Soft Wall Model)

- We write the Dirac equation

$$(\alpha\Pi(\zeta) - \mathcal{M})\,\psi(\zeta) = 0,$$

in terms of the matrix-valued operator Π

$$\Pi_\nu(\zeta) = -i\left(\frac{d}{d\zeta} - \frac{\nu+\frac{1}{2}}{\zeta}\gamma_5 - \kappa^2\zeta\gamma_5\right),$$

and its adjoint $\Pi^\dagger$, with commutation relations

$$\left[\Pi_\nu(\zeta), \Pi_\nu^\dagger(\zeta)\right] = \left(\frac{2\nu+1}{\zeta^2} - 2\kappa^2\right)\gamma_5.$$

- Solutions to the Dirac equation

$$\begin{aligned}\psi_+(\zeta) &\sim z^{\frac{1}{2}+\nu}e^{-\kappa^2\zeta^2/2}L_n^\nu(\kappa^2\zeta^2),\\ \psi_-(\zeta) &\sim z^{\frac{3}{2}+\nu}e^{-\kappa^2\zeta^2/2}L_n^{\nu+1}(\kappa^2\zeta^2).\end{aligned}$$

- Eigenvalues

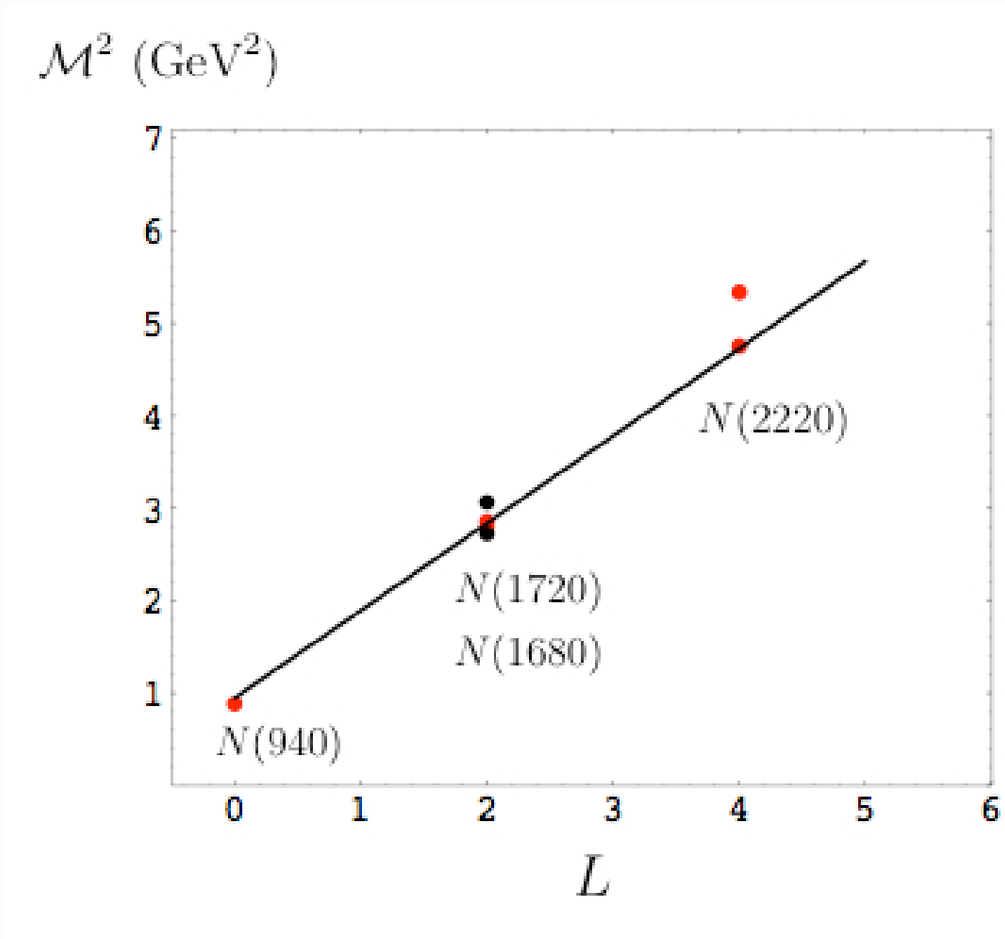
$$\mathcal{M}^2 = 4\kappa^2(n+\nu+1).$$

- Baryon: twist-dimension $3+L \quad (\nu = L+1)$

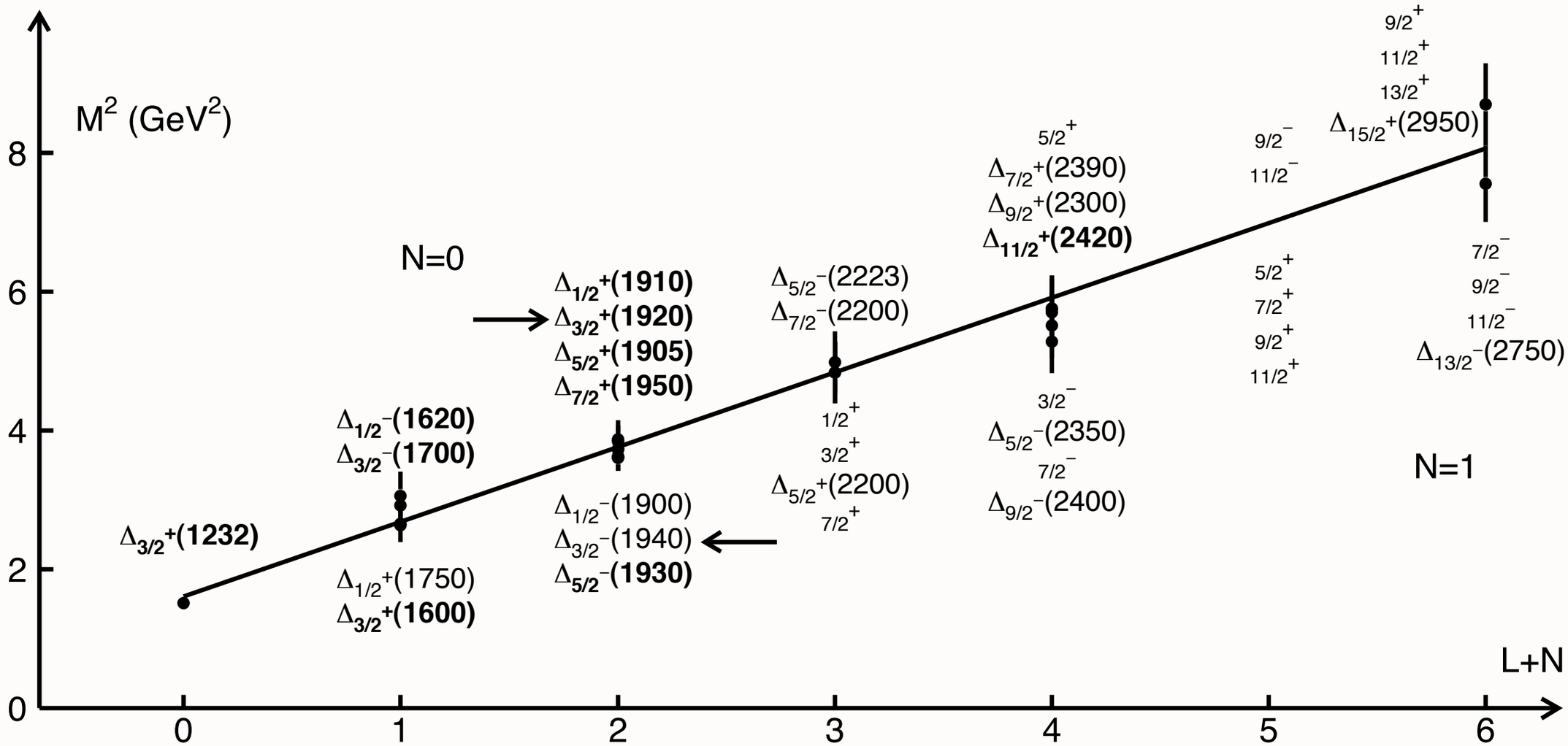
$$\mathcal{O}_{3+L} = \psi D_{\{\ell_1} \ldots D_{\ell_q} \psi D_{\ell_{q+1}} \ldots D_{\ell_m\}} \psi, \qquad L = \sum_{i=1}^{m} \ell_i.$$

- Define the zero point energy (identical as in the meson case) $\mathcal{M}^2 \to \mathcal{M}^2 - 4\kappa^2$:

$$\mathcal{M}^2 = 4\kappa^2(n+L+1).$$



Proton Regge Trajectory $\quad \kappa = 0.49$GeV



E. Klempt *et al.*: Δ^* resonances, quark models, chiral symmetry and AdS/QCD

H. Forkel, M. Beyer and T. Frederico, JHEP **0707** (2007) 077.
H. Forkel, M. Beyer and T. Frederico, Int. J. Mod. Phys. E **16** (2007) 2794.

Space-Like Dirac Proton Form Factor

- Consider the spin non-flip form factors

$$F_+(Q^2) = g_+ \int d\zeta\, J(Q,\zeta)|\psi_+(\zeta)|^2,$$
$$F_-(Q^2) = g_- \int d\zeta\, J(Q,\zeta)|\psi_-(\zeta)|^2,$$

where the effective charges g_+ and g_- are determined from the spin-flavor structure of the theory.

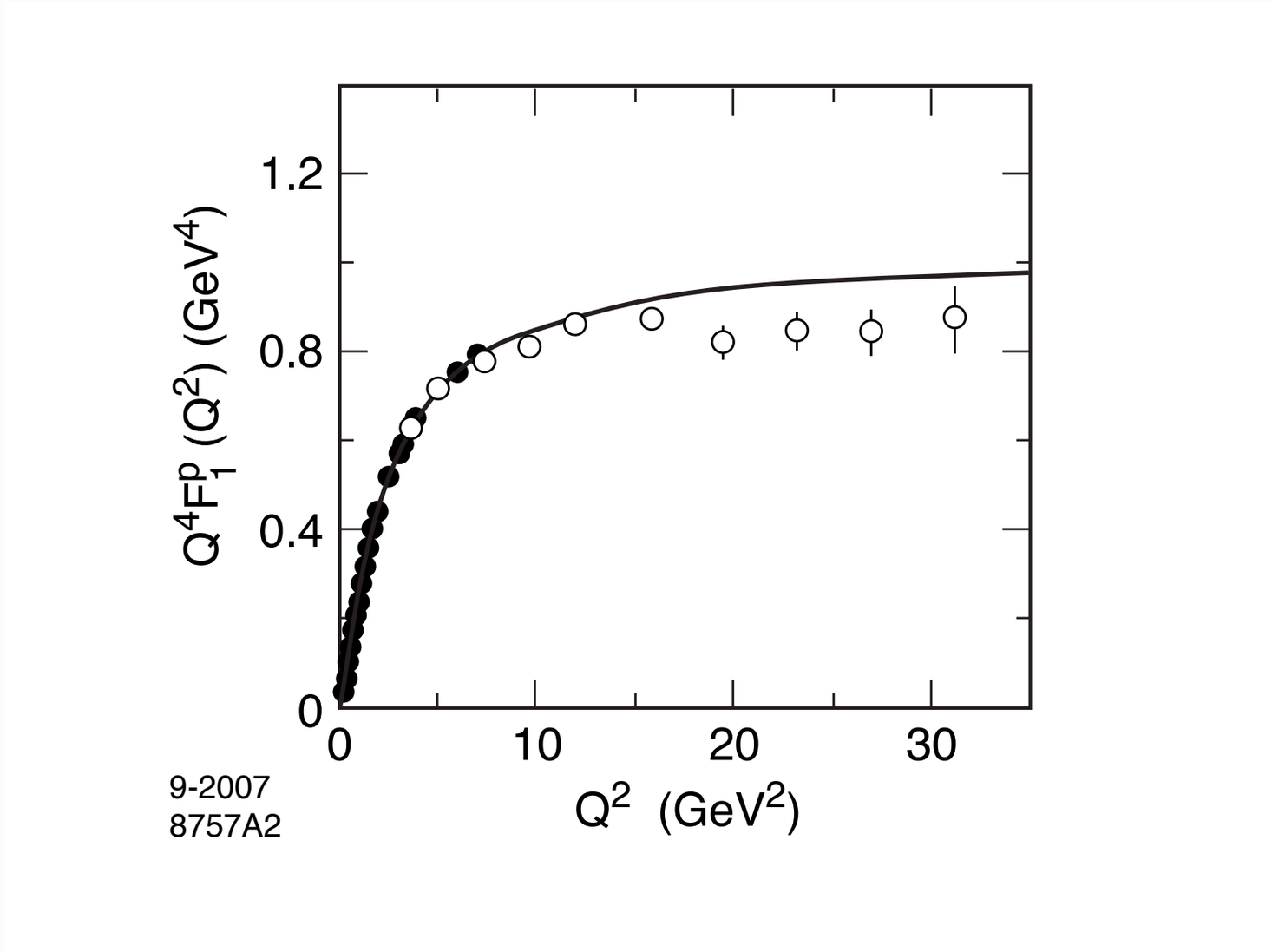
- Choose the struck quark to have $S^z = +1/2$. The two AdS solutions $\psi_+(\zeta)$ and $\psi_-(\zeta)$ correspond to nucleons with $J^z = +1/2$ and $-1/2$.

- For $SU(6)$ spin-flavor symmetry

$$F_1^p(Q^2) = \int d\zeta\, J(Q,\zeta)|\psi_+(\zeta)|^2,$$
$$F_1^n(Q^2) = -\frac{1}{3}\int d\zeta\, J(Q,\zeta)\left[|\psi_+(\zeta)|^2 - |\psi_-(\zeta)|^2\right],$$

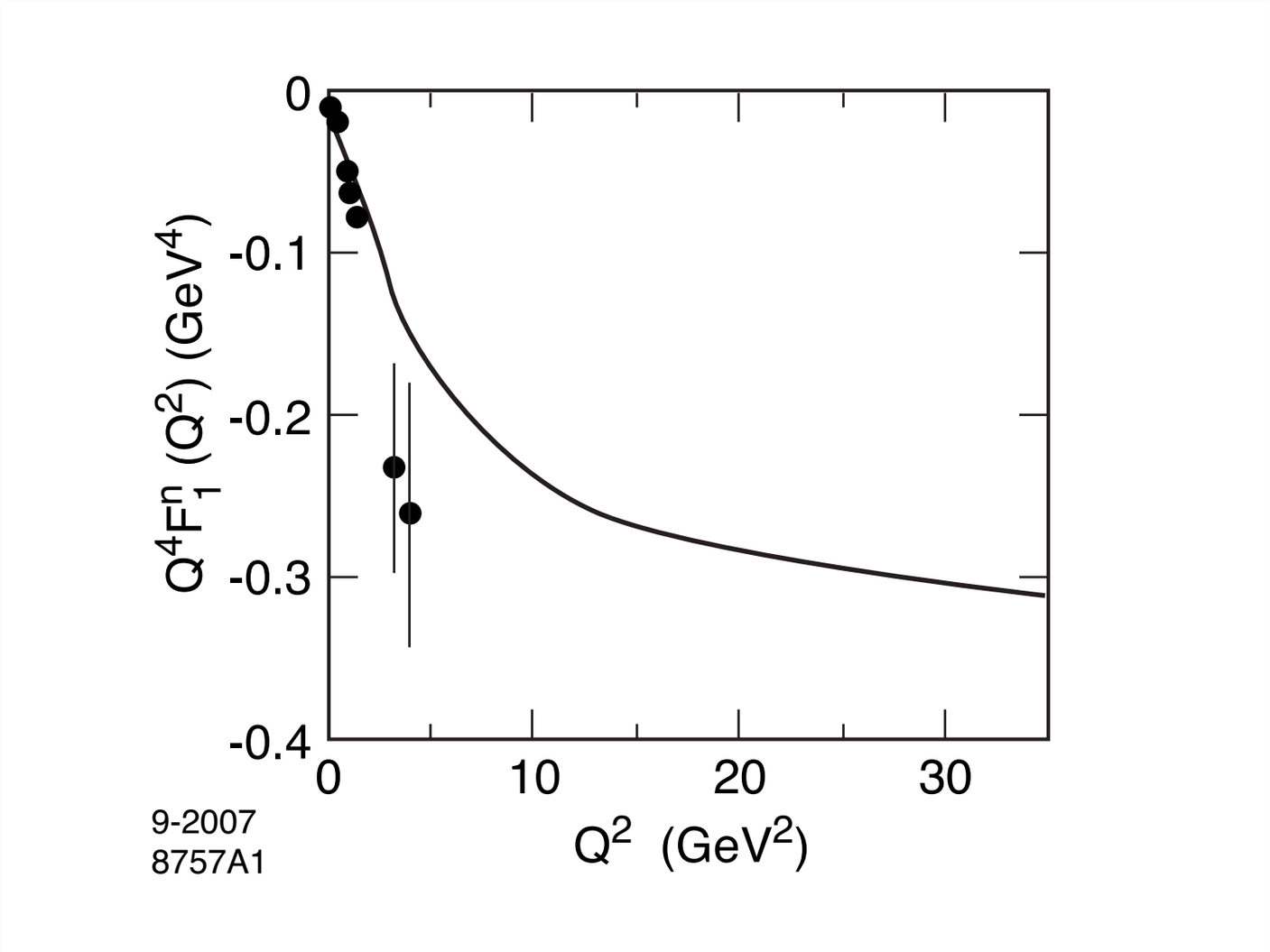
where $F_1^p(0) = 1,\ F_1^n(0) = 0.$

- Scaling behavior for large Q^2: $Q^4 F_1^p(Q^2) \to \text{constant}$ $\boxed{\text{Proton } \tau = 3}$



SW model predictions for $\kappa = 0.424$ GeV. Data analysis from: M. Diehl *et al.* Eur. Phys. J. C **39**, 1 (2005).

- Scaling behavior for large Q^2: $Q^4 F_1^n(Q^2) \to \text{constant}$ $\boxed{\text{Neutron } \tau = 3}$

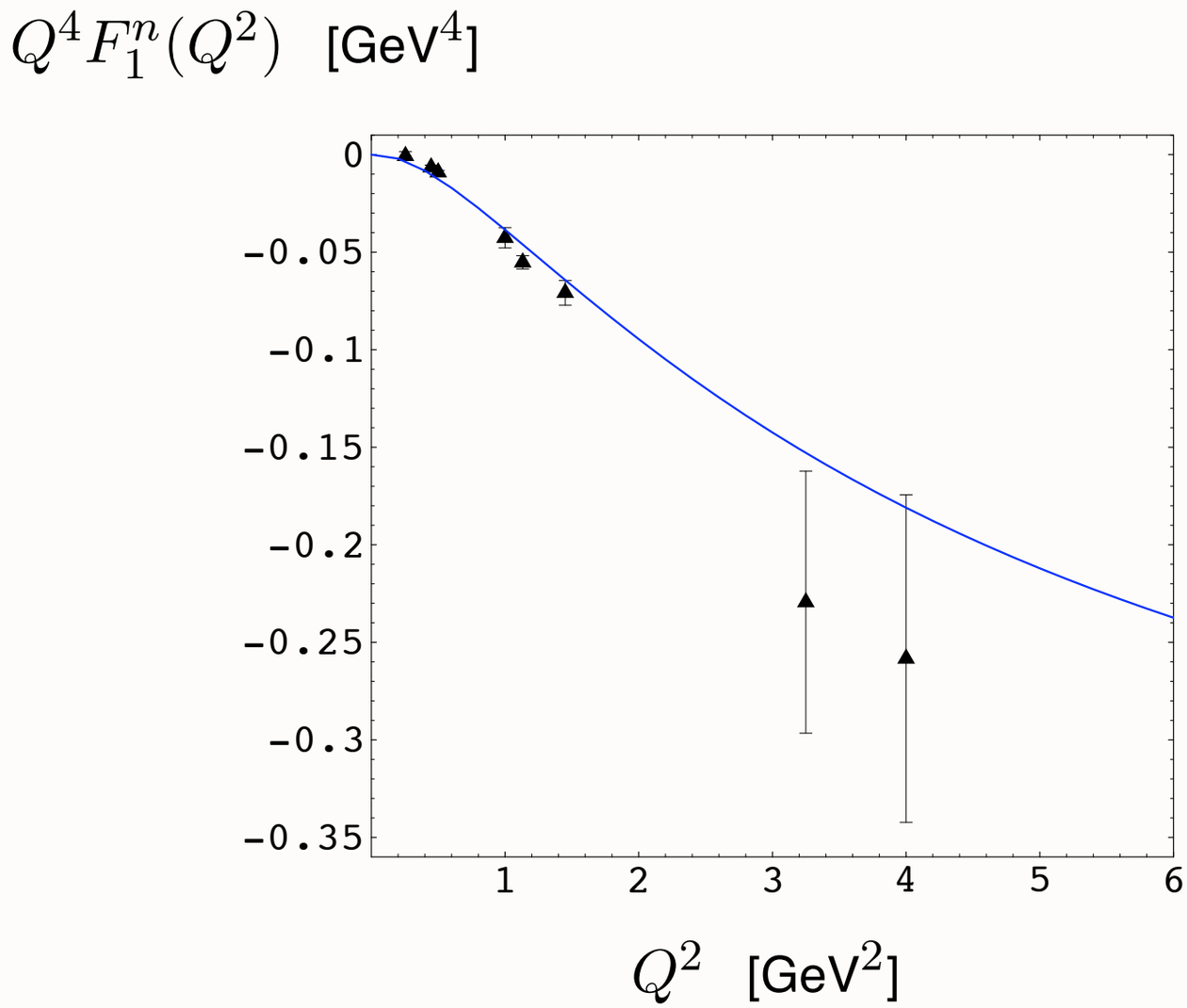


SW model predictions for $\kappa = 0.424$ GeV. Data analysis from M. Diehl *et al.* Eur. Phys. J. C **39**, 1 (2005).

Dirac Neutron Form Factor

(Valence Approximation)

Truncated Space Confinement

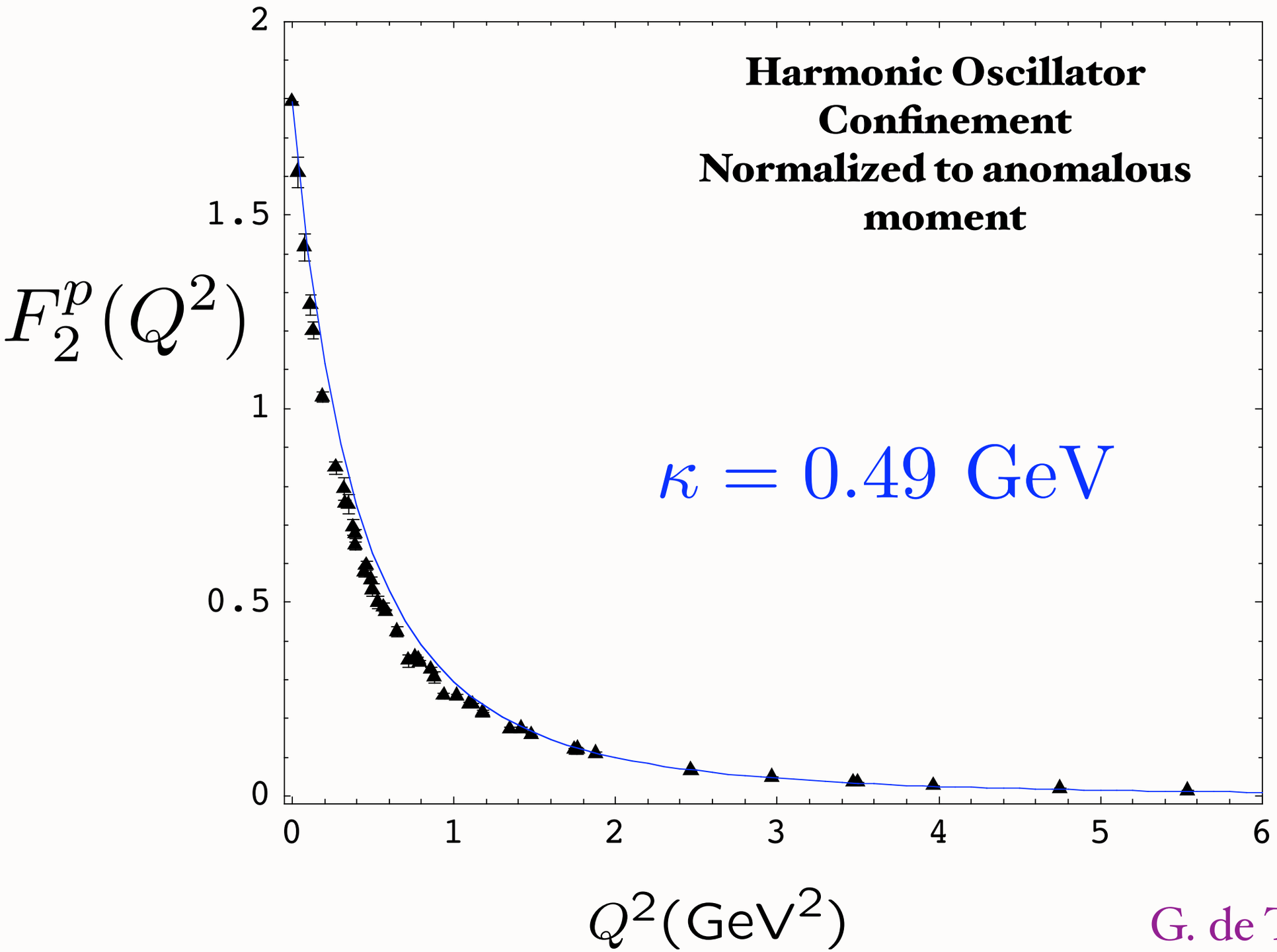


Prediction for $Q^4 F_1^n(Q^2)$ for $\Lambda_{\rm QCD} = 0.21$ GeV in the hard wall approximation. Data analysis from Diehl (2005).

Spacelike Pauli Form Factor

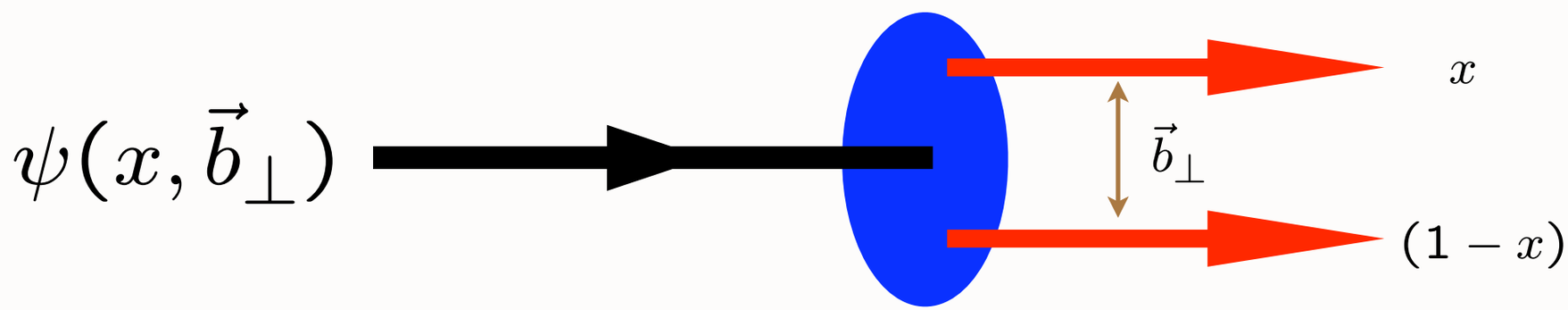
Preliminary

From overlap of $L = 1$ and $L = 0$ LFWFs



G. de Teramond, sjb

LF(3+1)		AdS_5
$\psi(x, \vec{b}_\perp)$	$\longleftrightarrow$	$\phi(z)$
$\zeta = \sqrt{x(1-x)\vec{b}_\perp^2}$	$\longleftrightarrow$	z

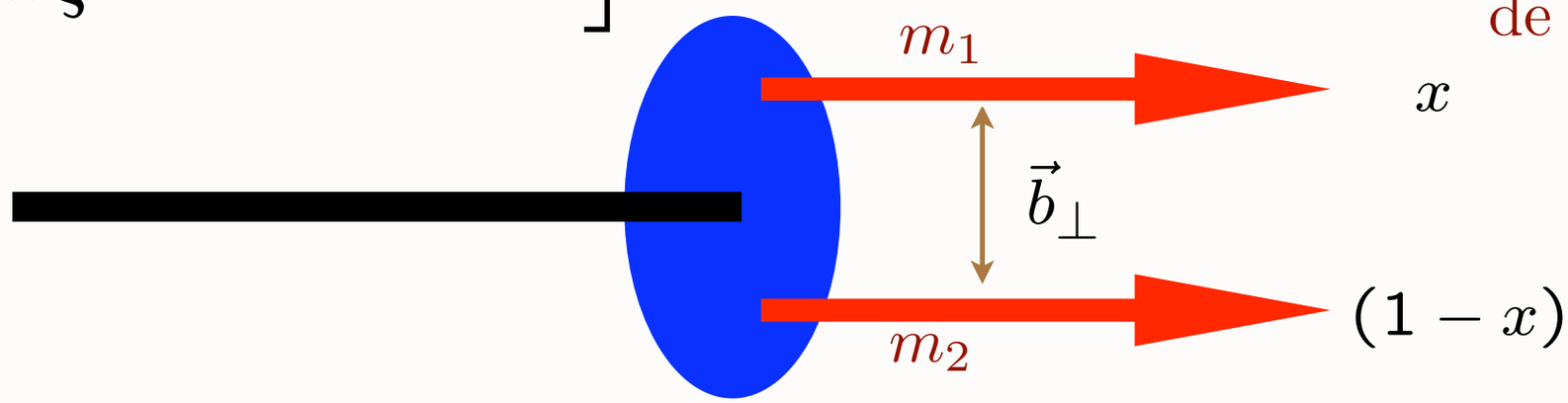


$$\psi(x, \vec{b}_\perp) = \sqrt{\frac{x(1-x)}{2\pi\zeta}}\phi(\zeta)$$

Light-Front Holography: Unique mapping derived from equality of LF and AdS formula for current matrix elements

$$\left[-\frac{d^2}{d\zeta^2} + V(\zeta)\right]\phi(\zeta) = \mathcal{M}^2\phi(\zeta)$$

de Teramond, sjb



$$\zeta = \sqrt{x(1-x)\vec{b}_\perp^2}$$

Holographic Variable

$$-\frac{d}{d\zeta^2} \equiv \frac{k_\perp^2}{x(1-x)}$$

LF Kinetic Energy in momentum space

Assume LFWF is a dynamical function of the quark-antiquark invariant mass squared

$$-\frac{d}{d\zeta^2} \to -\frac{d}{d\zeta^2} + \frac{m_1^2}{x} + \frac{m_2^2}{1-x} \equiv \frac{k_\perp^2 + m_1^2}{x} + \frac{k_\perp^2 + m_2^2}{1-x}$$

Result: Soft-Wall LFWF for massive constituents

$$\psi(x, \mathbf{k}_\perp) = \frac{4\pi c}{\kappa\sqrt{x(1-x)}}\, e^{-\frac{1}{2\kappa^2}\left(\frac{\mathbf{k}_\perp^2}{x(1-x)} + \frac{m_1^2}{x} + \frac{m_2^2}{1-x}\right)}$$

LF WF in impact space: soft-wall model with massive quarks

$$\psi(x, \mathbf{b}_\perp) = \frac{c\,\kappa}{\sqrt{\pi}}\sqrt{x(1-x)}\, e^{-\frac{1}{2}\kappa^2 x(1-x)\mathbf{b}_\perp^2 - \frac{1}{2\kappa^2}\left[\frac{m_1^2}{x} + \frac{m_2^2}{1-x}\right]}$$

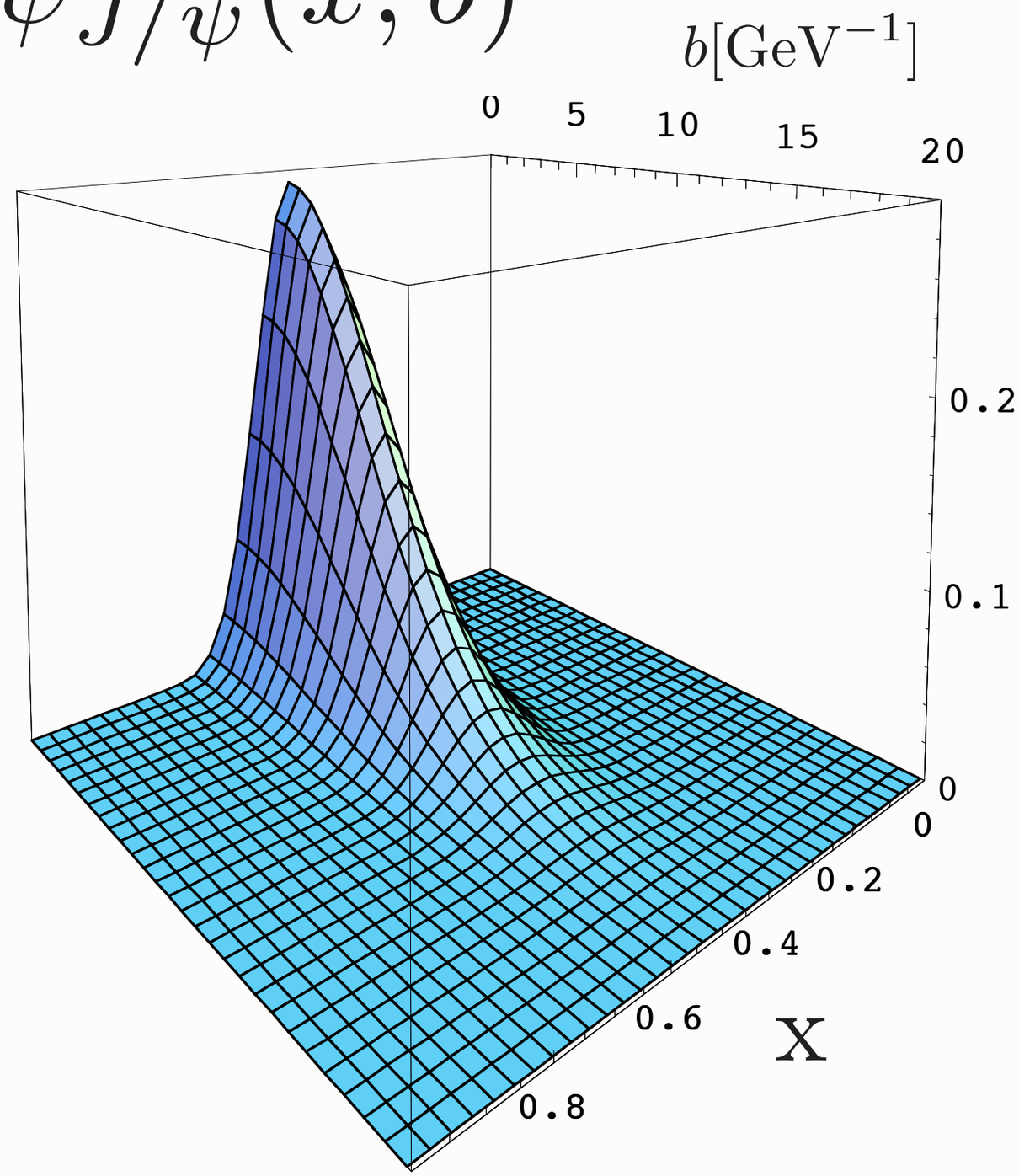
ground state LFWF

$$z \to \zeta \to \chi$$

$$\chi^2 = b^2 x(1-x) + \frac{1}{\kappa^4}\left[\frac{m_1^2}{x} + \frac{m_2^2}{1-x}\right]$$

J/ψ

$\psi_{J/\psi}(x, b)$



LFWF peaks at

$$x_i = \frac{m_{\perp i}}{\sum_j^n m_{\perp j}}$$

where

$$m_{\perp i} = \sqrt{m^2 + k_\perp^2}$$

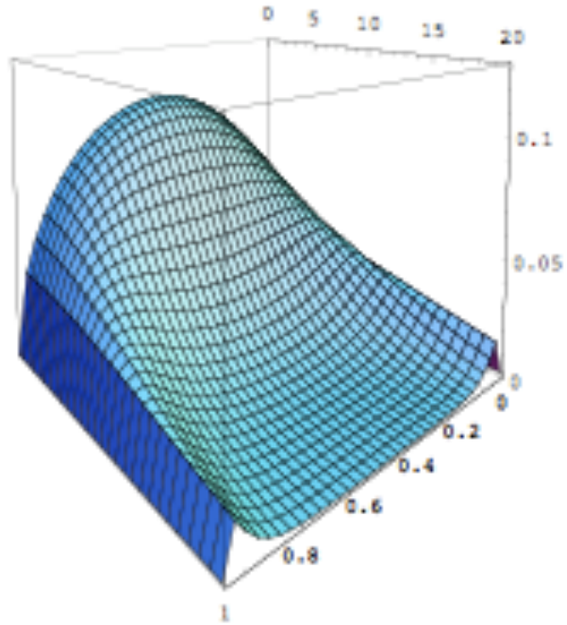
minimum of LF energy denominator

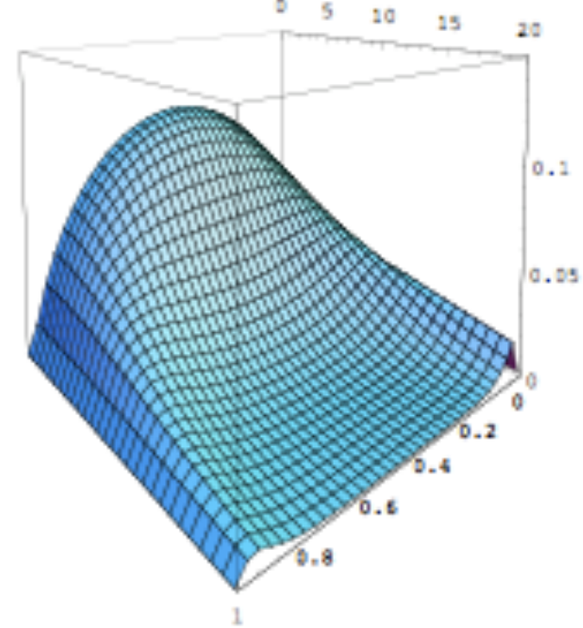
$$\kappa = 0.375 \text{ GeV}$$

$$m_a = m_b = 1.25 \text{ GeV}$$

$|\pi^+ >= |u\bar{d} >$

$m_u = 2\ MeV$
$m_d = 5\ MeV$



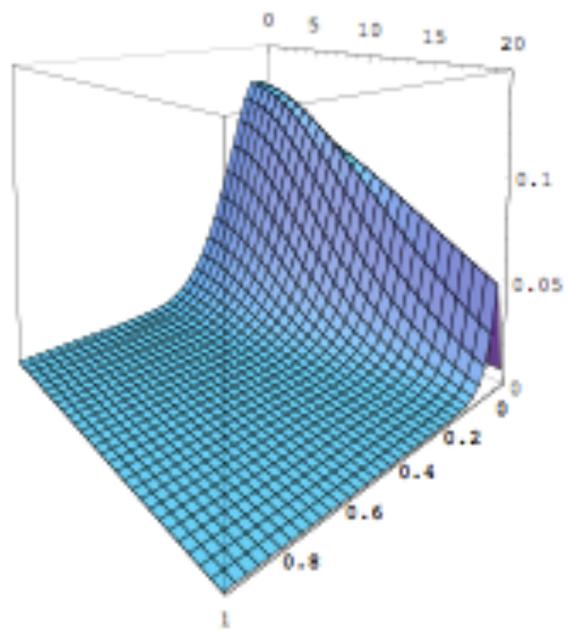


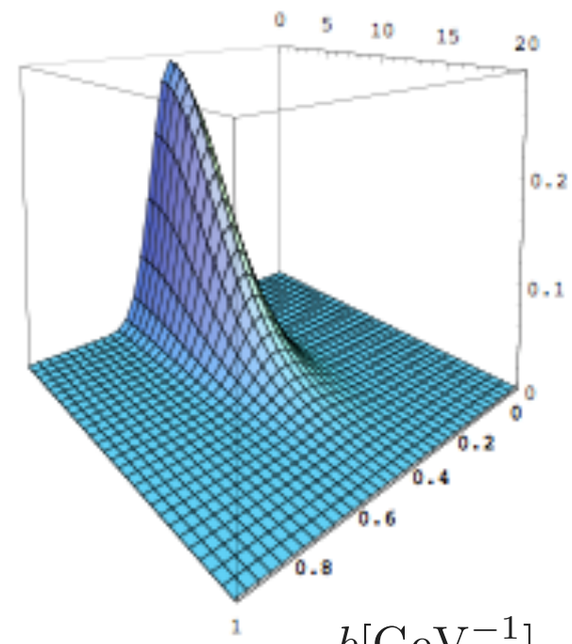
$|K^+ >= |u\bar{s} >$

$m_s = 95\ MeV$

$|D^+ >= |c\bar{d} >$

$m_c = 1.25\ GeV$

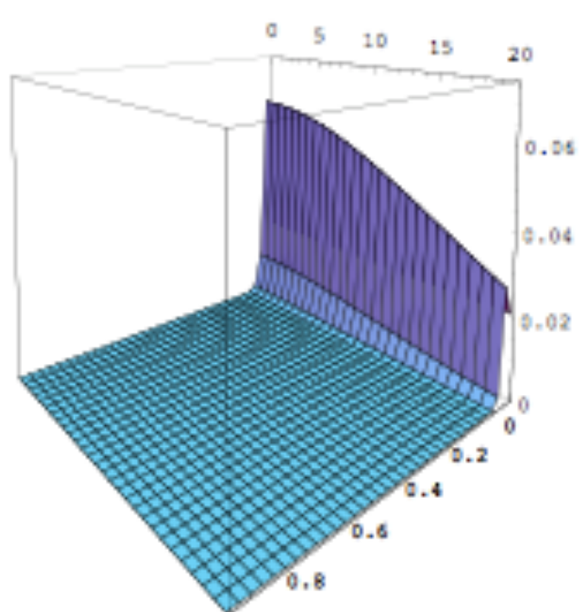


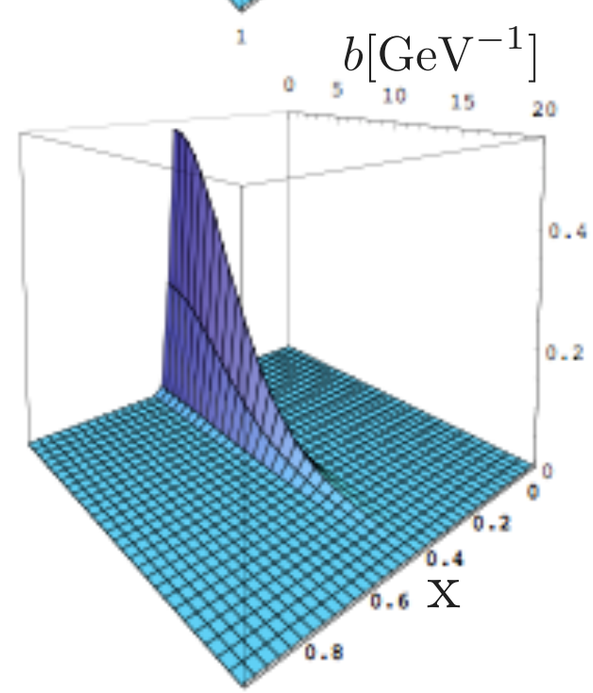


$|\eta_c >= |c\bar{c} >$

$|B^+ >= |u\bar{b} >$

$m_b = 4.2\ GeV$





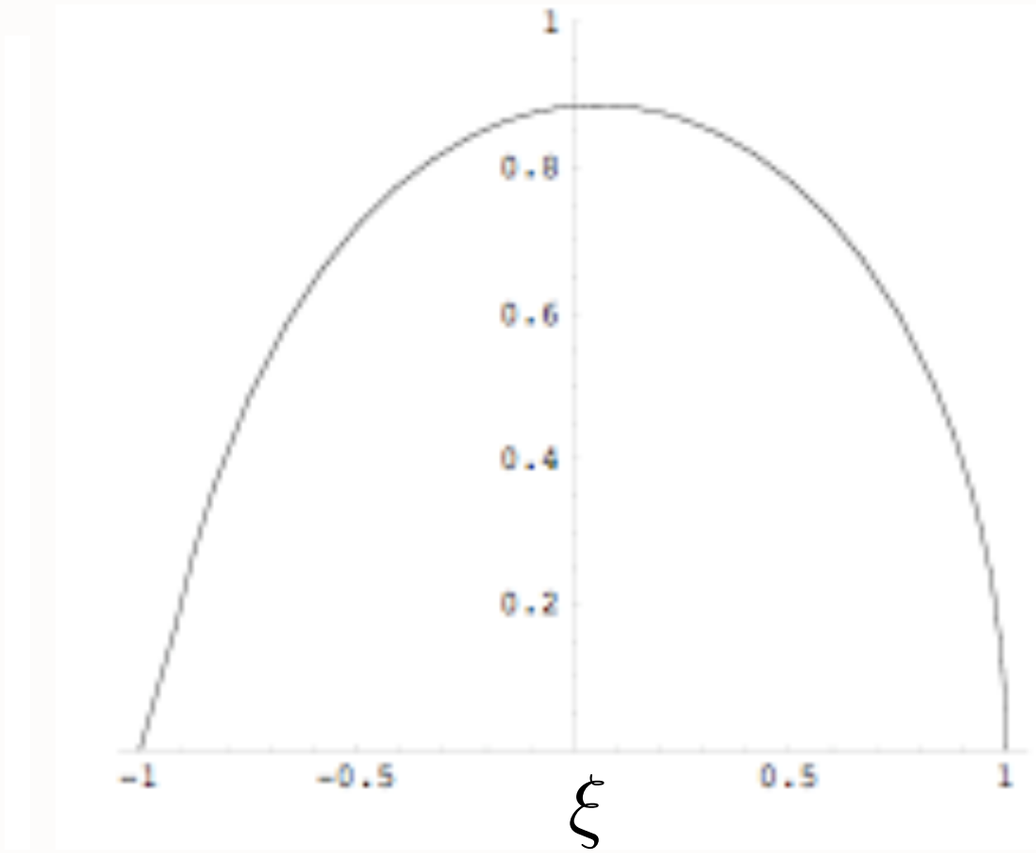
$|\eta_b >= |b\bar{b} >$

$\kappa = 375\ MeV$

First Moment of Kaon Distribution Amplitude

$$< \xi >= \int_{-1}^{1} d\xi \; \xi \; \phi(\xi)$$

$$\xi = 1 - 2x$$



$$< \xi >_K = 0.04 \pm 0.02 \qquad \kappa = 375 \; MeV$$

Range from $m_s = 65 \pm 25$ MeV (PDG)

$< \xi >_K = 0.029 \pm 0.002$ Donnellan et al.

$< \xi >_K = 0.0272 \pm 0.0005$ Braun et al.

Use AdS/CFT orthonormal LFWFs as a basis for diagonalizing the QCD LF Hamiltonian

- Good initial approximant
- Better than plane wave basis

Pauli, Hornbostel, Hiller, McCartor, sjb

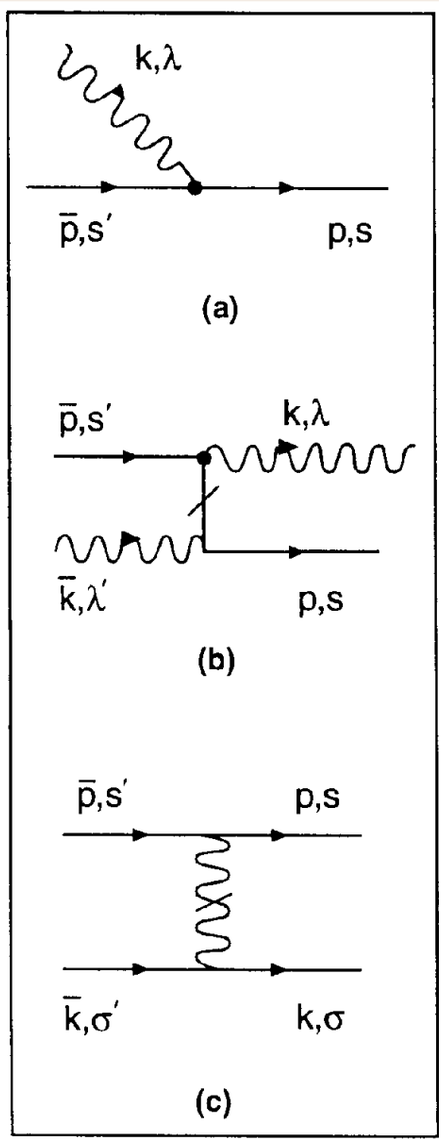
- DLCQ discretization -- highly successful 1+1
- Use independent HO LFWFs, remove CM motion

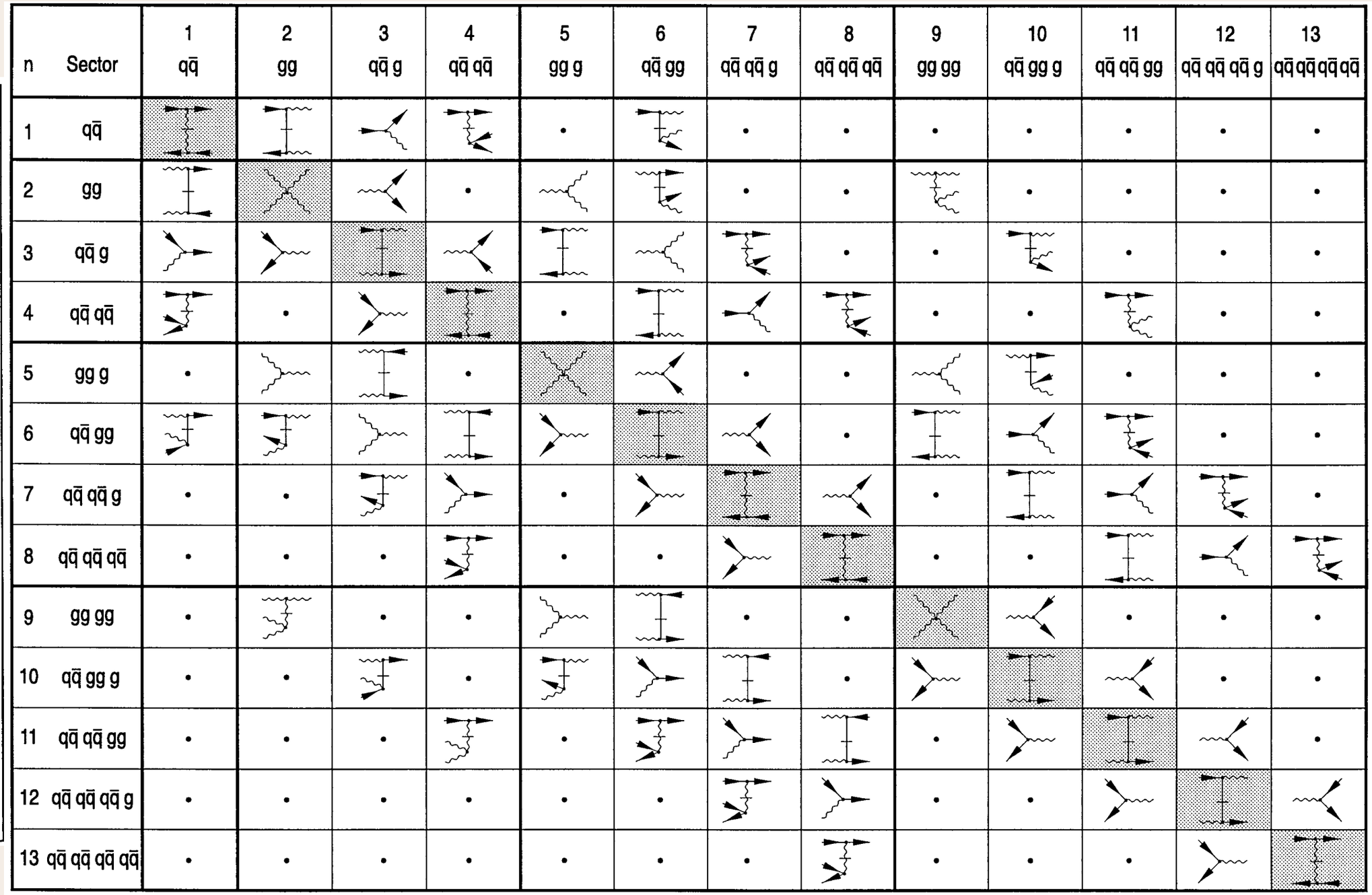
Vary, Harinandrath, Maris, sjb

- Similar to Shell Model calculations

Light-Front QCD Heisenberg Equation

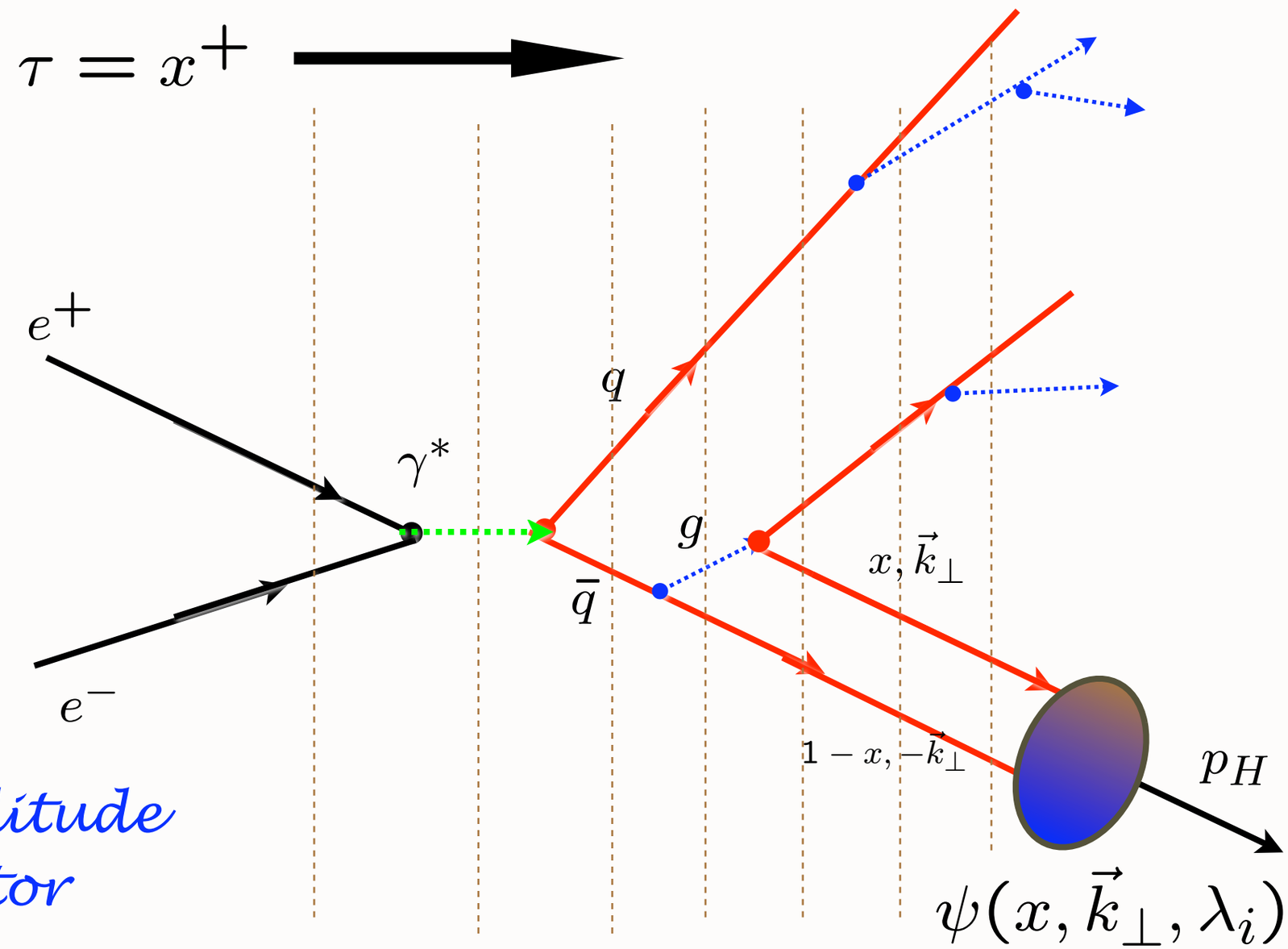
$$H_{LC}^{QCD}|\Psi_h\rangle = \mathcal{M}_h^2\ |\Psi_h\rangle$$





Use AdS/QCD basis functions

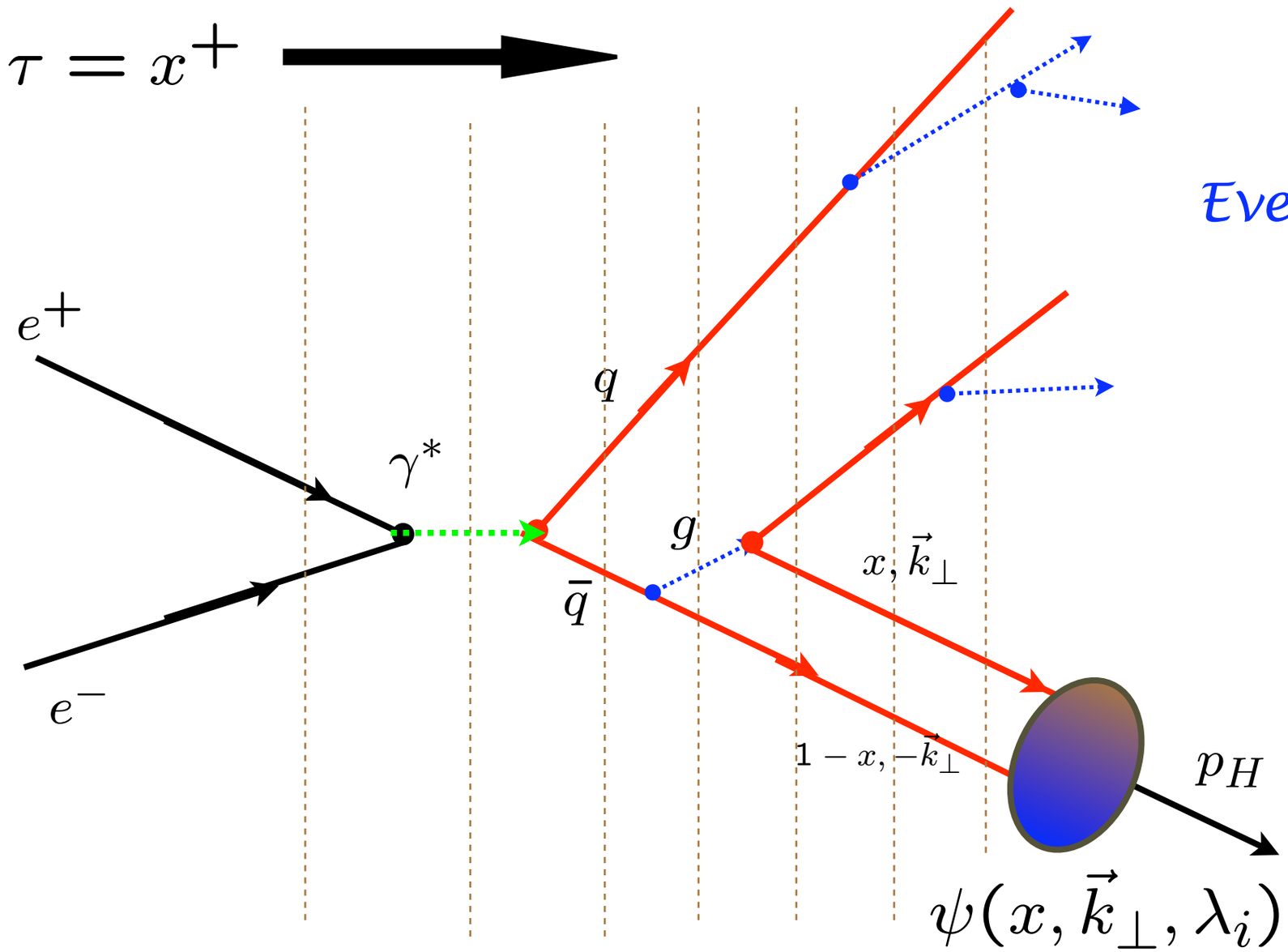
Hadronization at the Amplitude Level



Event amplitude generator

Construct helicity amplitude using Light-Front Perturbation theory; coalesce quarks via LFWFs

Hadronization at the Amplitude Level



Event amplitude generator

AdS/QCD Hard Wall Confinement:

Capture if $\zeta^2 = x(1-x)b_\perp^2 > \frac{1}{\Lambda_{QCD}^2}$

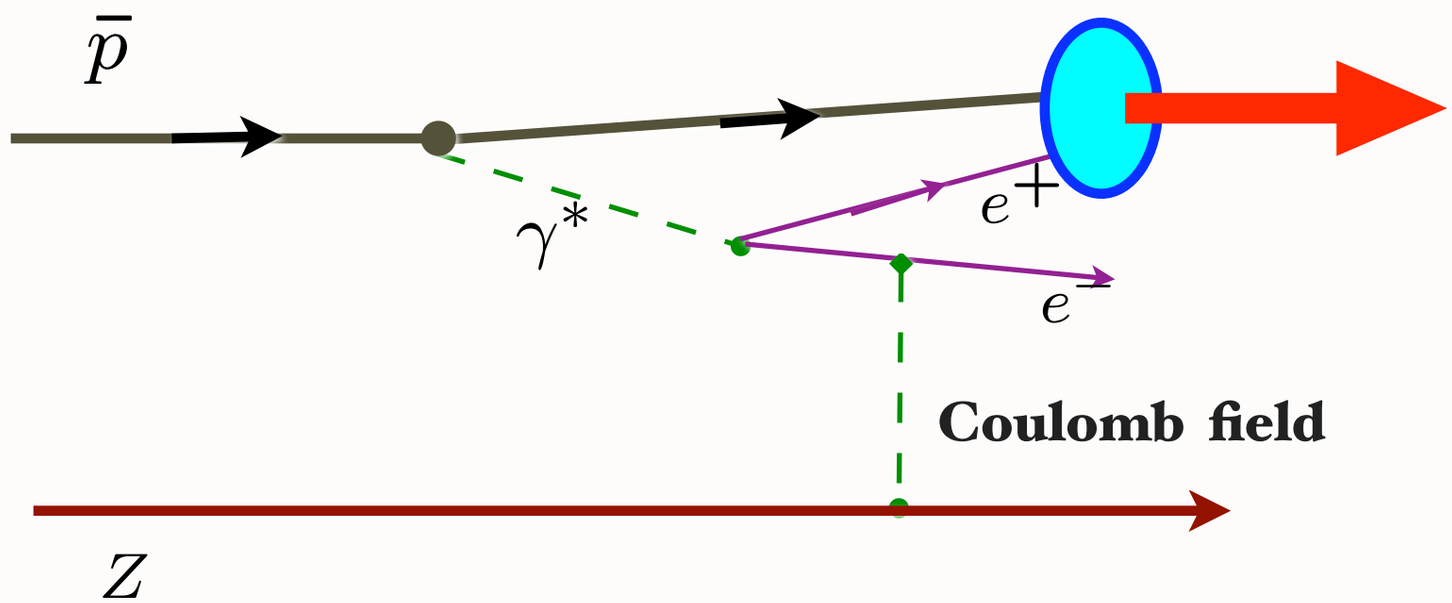
i.e.,

$$\mathcal{M}^2 = \frac{k_\perp^2}{x(1-x)} < \Lambda_{QCD}^2$$

Formation of Relativistic Anti-Hydrogen

Measured at CERN-LEAR and FermiLab

Munger, Schmidt, sjb



$\bar{H}(\bar{p}e^+)$

$b_\perp \leq \frac{1}{m_{red}\alpha}$

$y_{\bar{p}} \simeq y_{e^+}$

Coalescence of off-shell co-moving positron and antiproton

Wavefunction maximal at small impact separation and equal rapidity

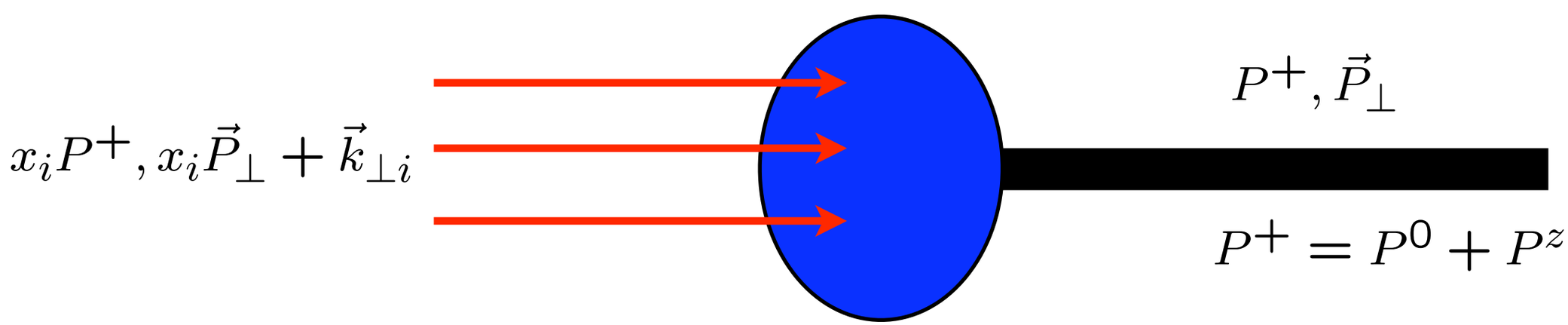
"Hadronization" at the Amplitude Level

Features of LF T-Matrix Formalism
"Event Amplitude Generator"

- Coalesce color-singlet cluster to hadronic state if

$$\mathcal{M}_n^2 = \sum_{i=1}^{n} \frac{k_{\perp i}^2 + m_i^2}{x_i} < \Lambda_{QCD}^2$$

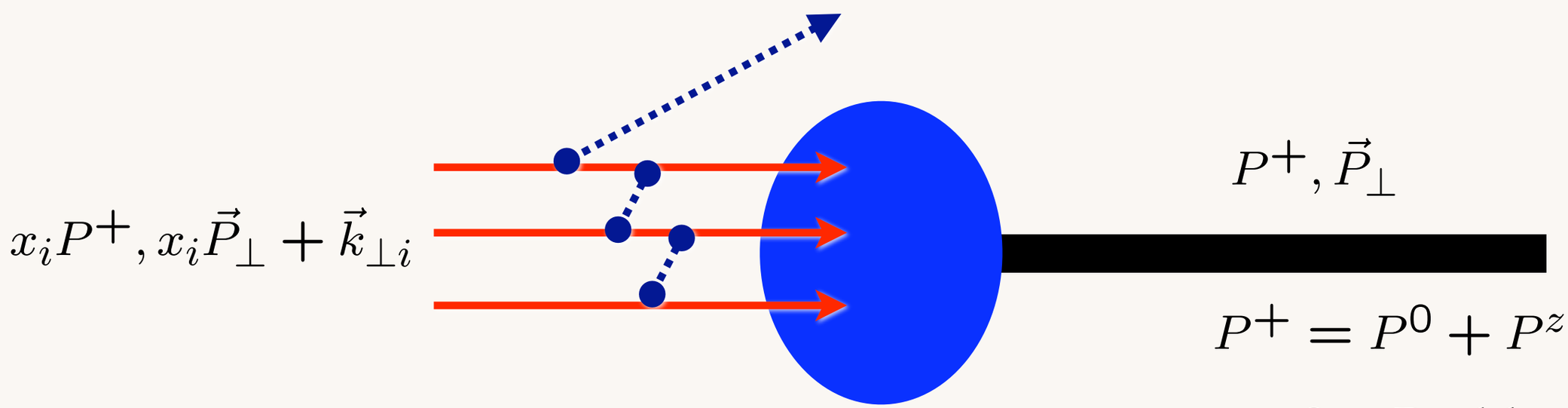
- The coalescence probability amplitude is the LF wavefunction $\Psi_n(x_i, \vec{k}_{\perp i}, \lambda_i)$
- No IR divergences: Maximal gluon and quark wavelength from confinement



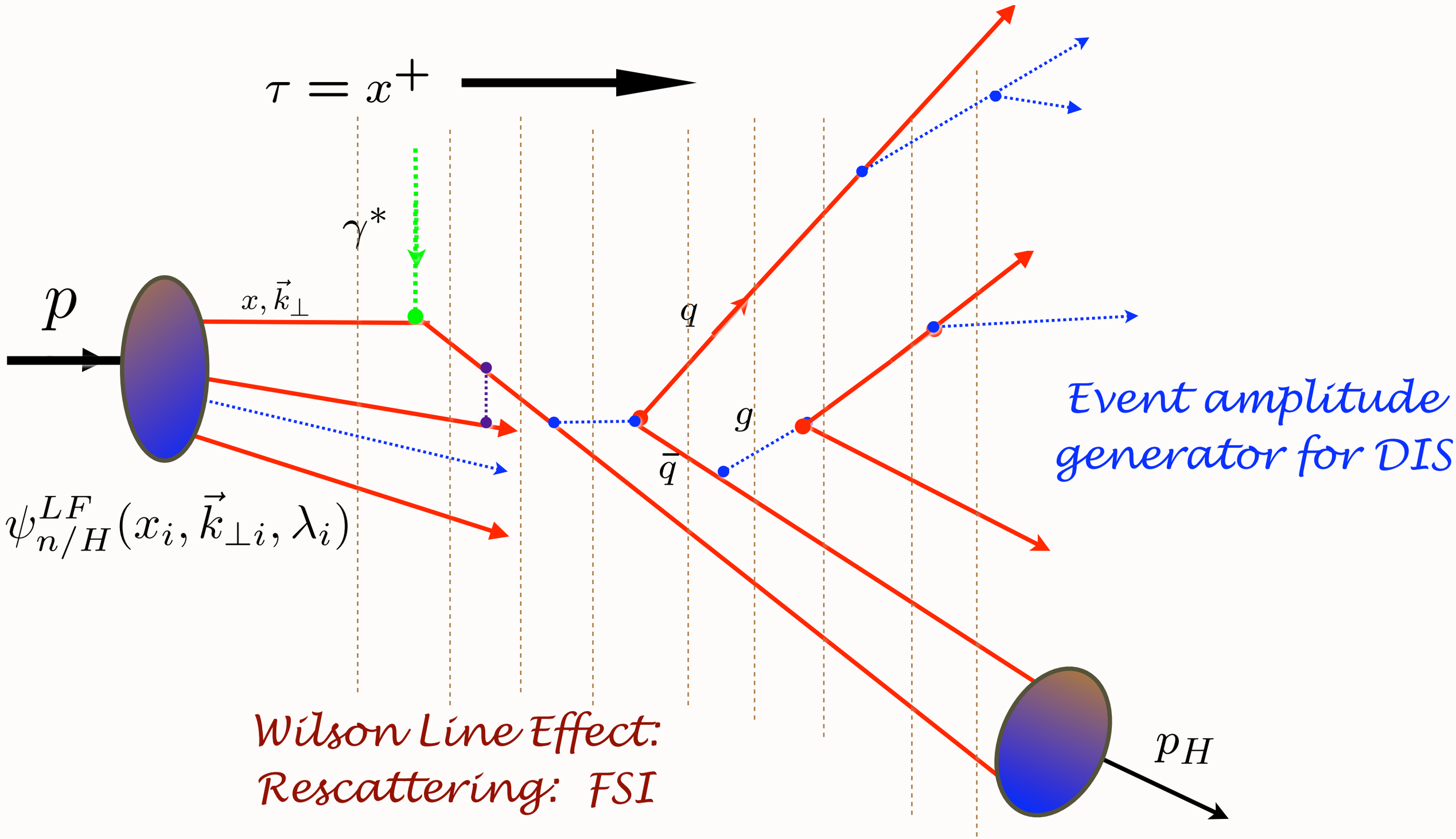
Features of LF T-Matrix Formalism
"Event Amplitude Generator"

If $\mathcal{M}_n^2 \geq \Lambda_{QCD}^2$ use PQCD hard gluon exchange

- DGLAP and ERBL Evolution from gluon emission and exchange
- Factorization Scale for structure functions and fragmentation functions set: $\mu_{fact} = \Lambda_{QCD}$



Hadronization at the Amplitude Level

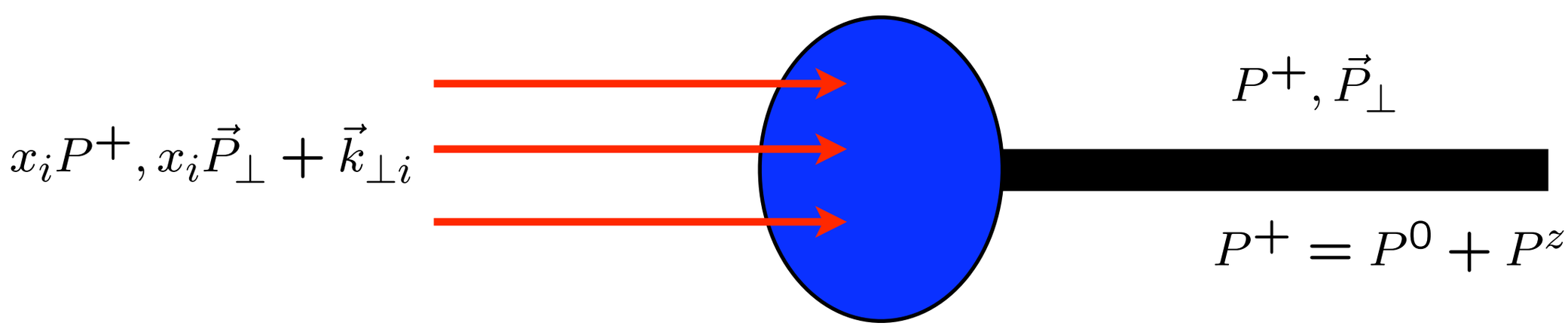


Features of LF T-Matrix Formalism

- Only positive + momenta; no backward time-ordered diagrams
- Frame-independent! Independent of P^+ and P^z
- LC gauge: No ghosts; physical helicity
- $J^z = L^z + S^z$ conservation at every vertex
- Sum all amplitudes with same initial-and final-state helicity, then square to get rate
- Renormalize each UV-divergent amplitude using "alternating denominator" method
- Multiple renormalization scales (BLM)

Features of LF T-Matrix Formalism
"Event Amplitude Generator"

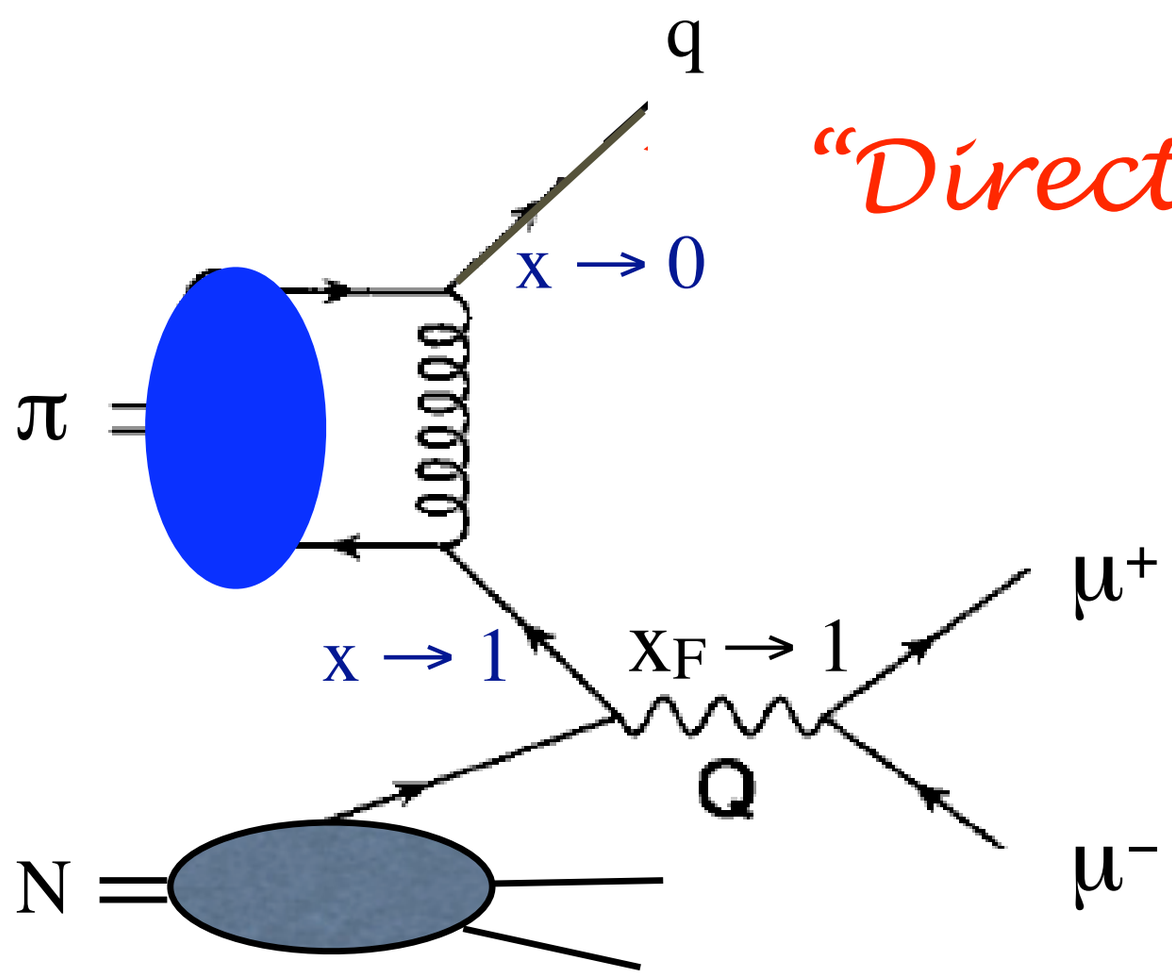
- Same principle as antihydrogen production: off-shell coalescence
- coalescence to hadron favored at equal rapidity, small transverse momenta
- leading heavy hadron production: D and B mesons produced at large z
- hadron helicity conservation if hadron LFWF has $L^z = 0$
- Baryon AdS/QCD LFWF has aligned and anti-aligned quark spin



$\pi N \to \mu^+ \mu^- X$ at high x_F

In the limit where $(1-x_F)Q^2$ is fixed as $Q^2 \to \infty$

Entire pion wf contributes to hard process



"Direct" Subprocess

Virtual photon is longitudinally polarized

Berger, sjb
Khoze, Brandenburg, Muller, sjb
Hoyer Vanttinen

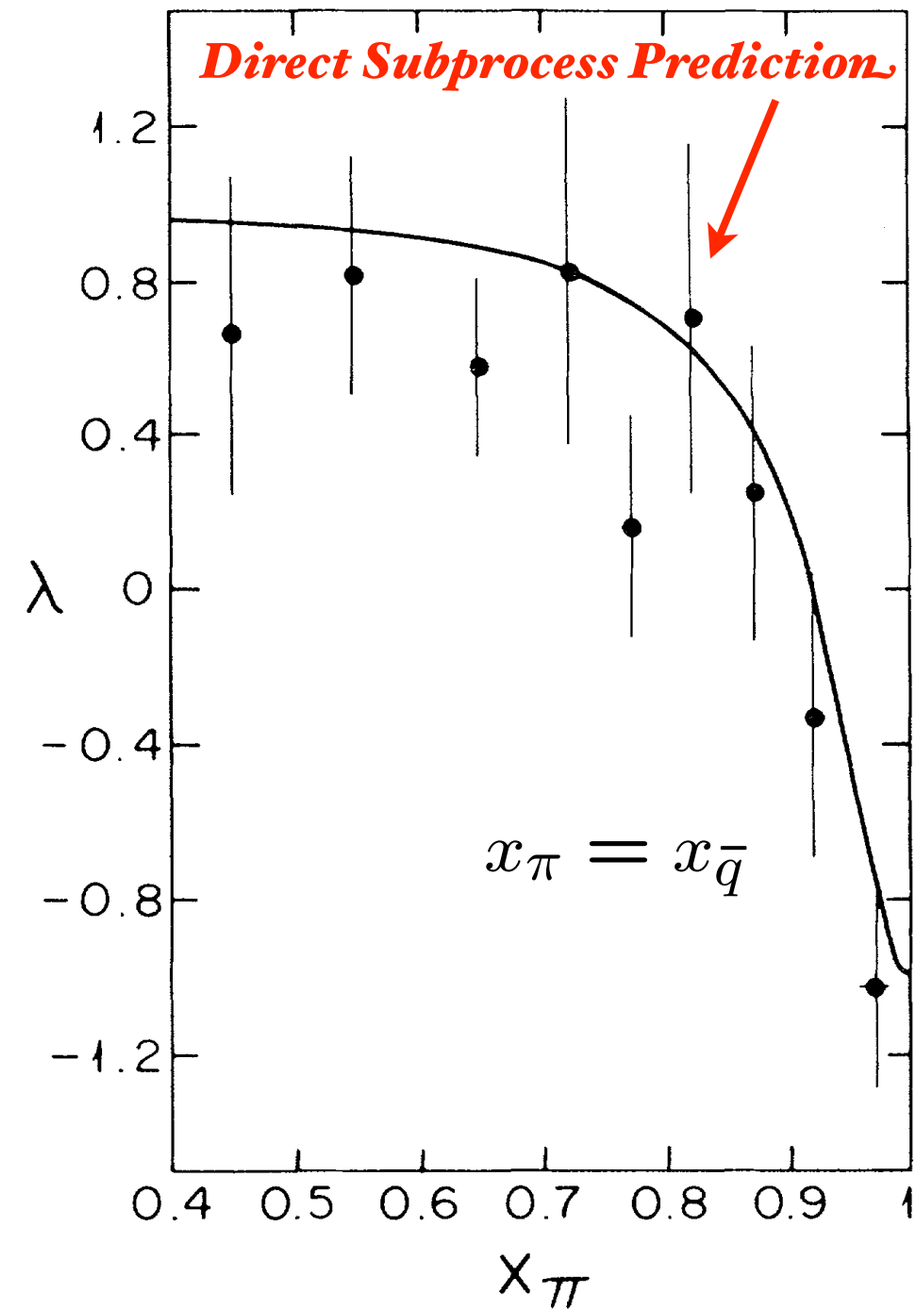
$\pi^- N \to \mu^+ \mu^- X$ at 80 GeV/c

$$\frac{d\sigma}{d\Omega} \propto 1 + \lambda \cos^2\theta + \rho \sin 2\theta \cos\phi + \omega \sin^2\theta \cos 2\phi.$$

$$\frac{d^2\sigma}{dx_\pi d\cos\theta} \propto x_\pi \left[(1-x_\pi)^2 (1+\cos^2\theta) + \frac{4}{9} \frac{\langle k_T^2 \rangle}{M^2} \sin^2\theta \right]$$

$$\langle k_T^2 \rangle = 0.62 \pm 0.16 \ \ \mathrm{GeV}^2/c^2$$

Dramatic change in angular distribution at large x_F



Chicago-Princeton Collaboration

Phys.Rev.Lett.55:2649,1985

Example of a higher-twist direct subprocess

Crucial Test of Leading -Twist QCD: Scaling at fixed x_T

$$x_T = \frac{2p_T}{\sqrt{s}}$$

$$E\frac{d\sigma}{d^3p}(pN \to \pi X) = \frac{F(x_T,\theta_{CM})}{p_T^{n_{eff}}}$$

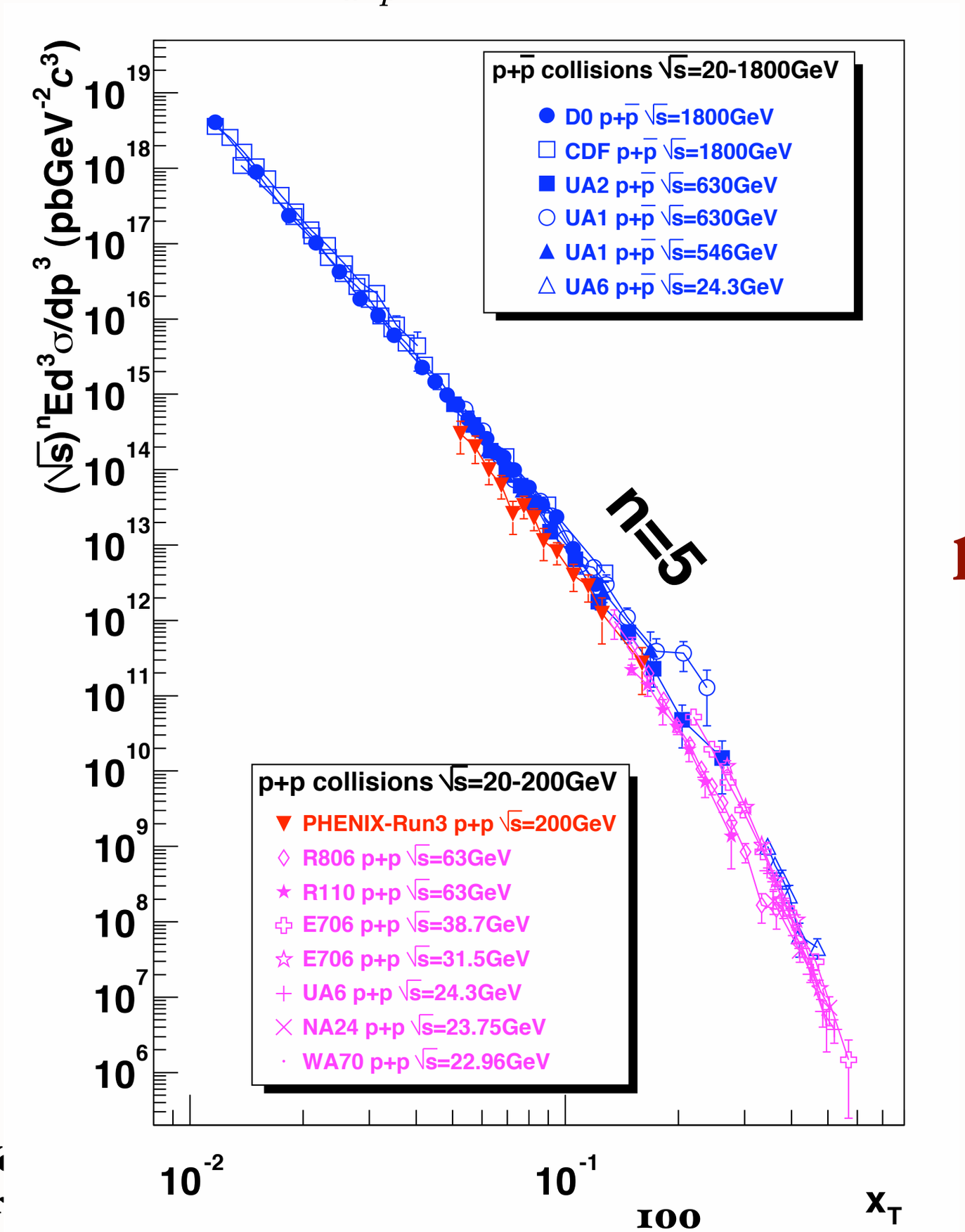
Parton model: $n_{eff} = 4$

As fundamental as Bjorken scaling in DIS

Conformal scaling: $n_{eff} = 2\, n_{active} - 4$

$\sqrt{s}^n E \frac{d\sigma}{d^3p}(pp \to \gamma X)$ at fixed x_T

Tannenbaum



Scaling of direct photon production consistent with PQCD

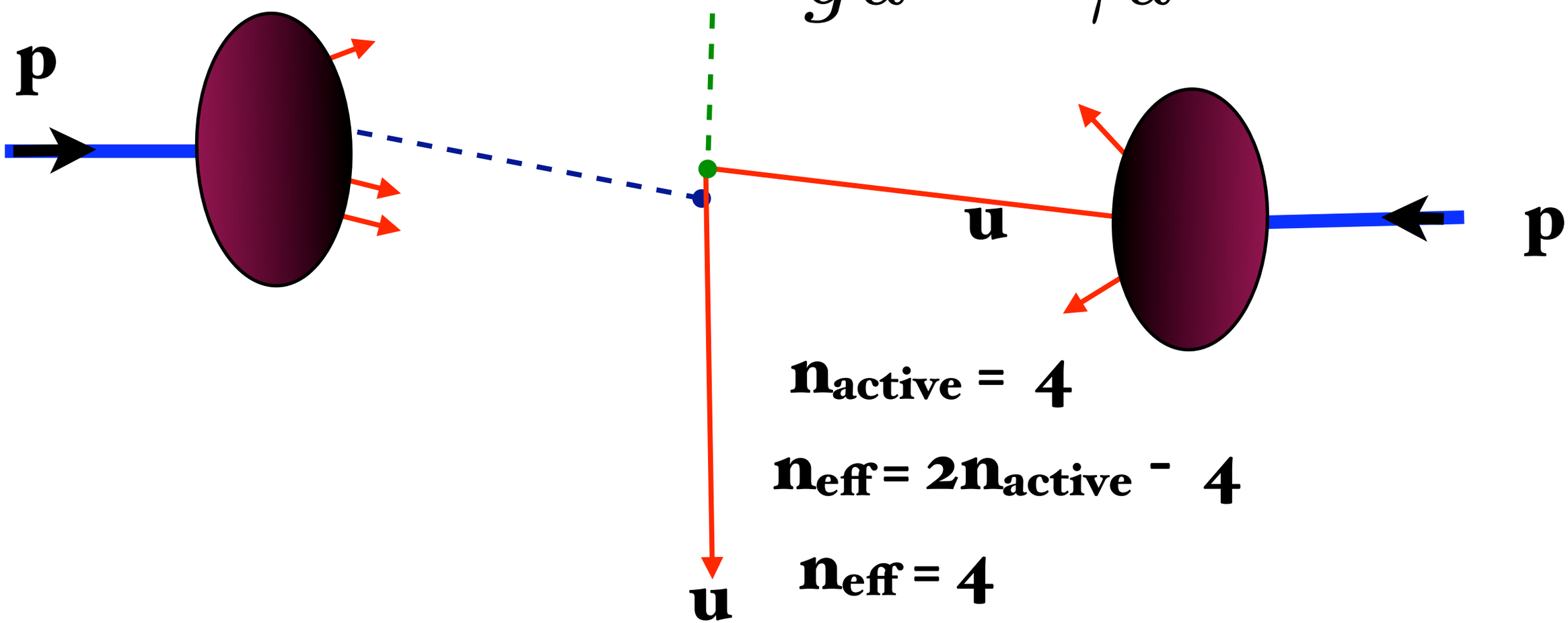
JTI W
Apr

Stan Brodsky
SLAC

$$pp \longrightarrow \gamma X$$

$$E\frac{d\sigma}{d^3p}(pp \to \gamma X) = \frac{F(\theta_{cm}, x_T)}{p_T^4}$$

$$gu \longrightarrow \gamma u$$

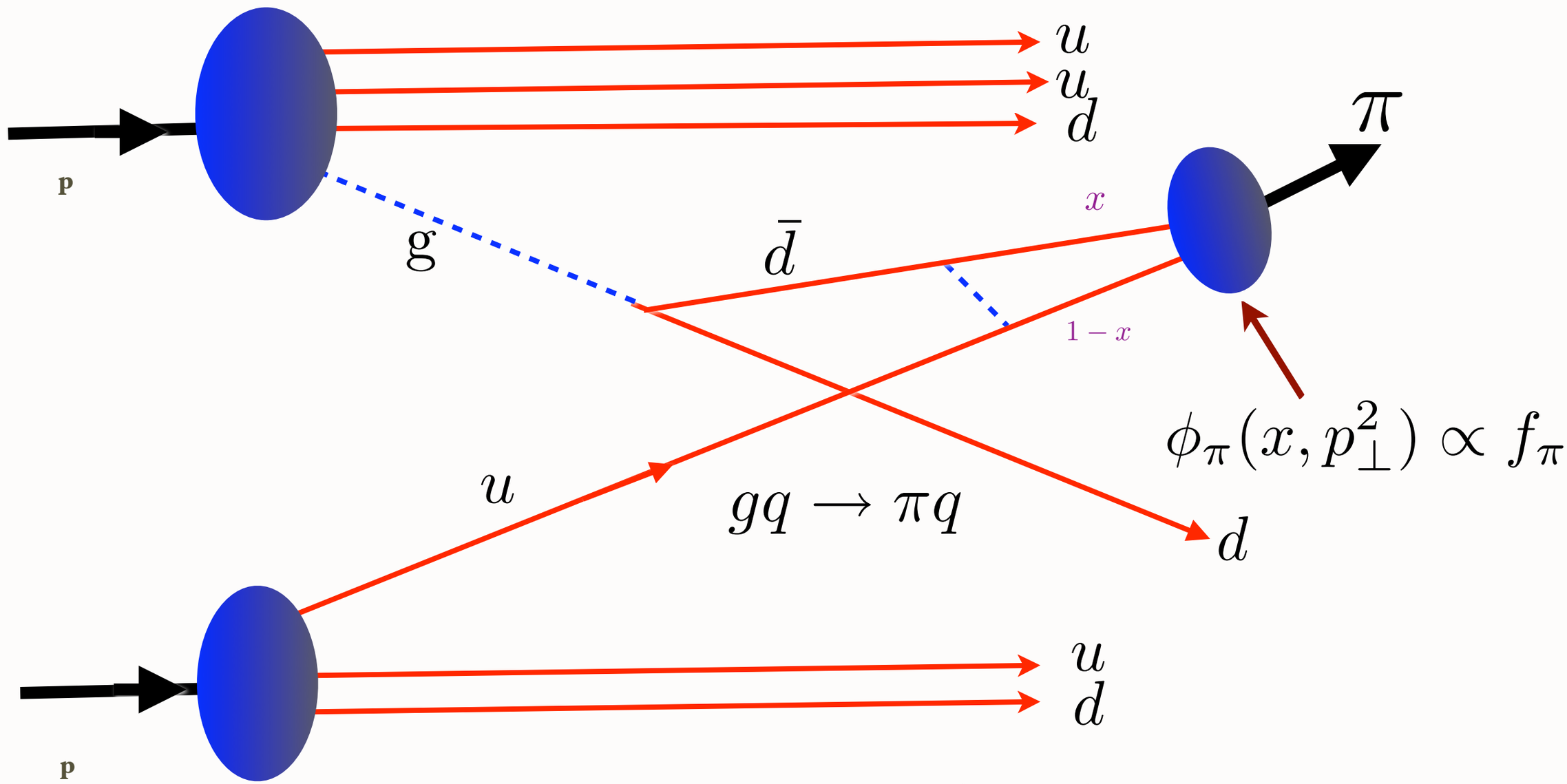


$n_{active} = 4$

$n_{eff} = 2n_{active} - 4$

$n_{eff} = 4$

Higher-Twist Contribution to Hadron Production

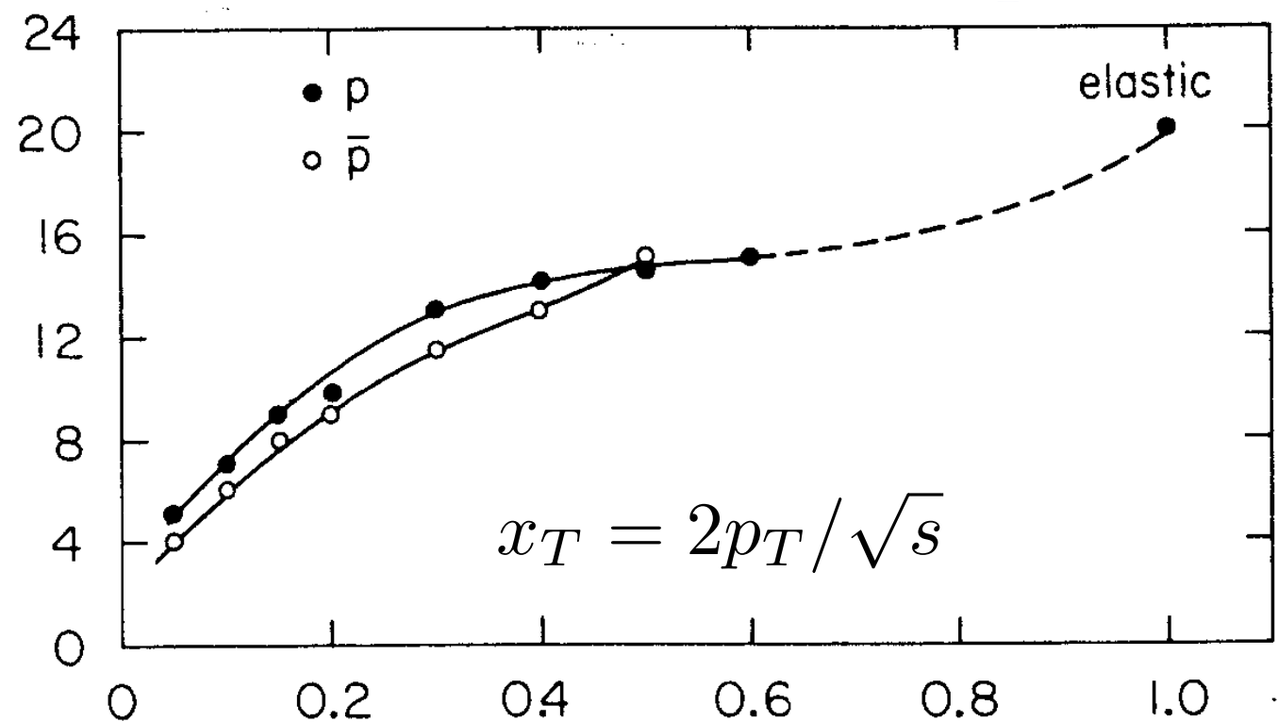


$$\frac{d\sigma}{d^3p/E} = \alpha_s^3 f_\pi^2 \frac{F(x_\perp, y)}{p_\perp^6}$$

No Fragmentation Function

$$E\frac{d\sigma}{d^3p}(pp \to HX) = \frac{F(x_T, \theta_{cm} = \pi/2)}{p_T^{n_{eff}}}$$

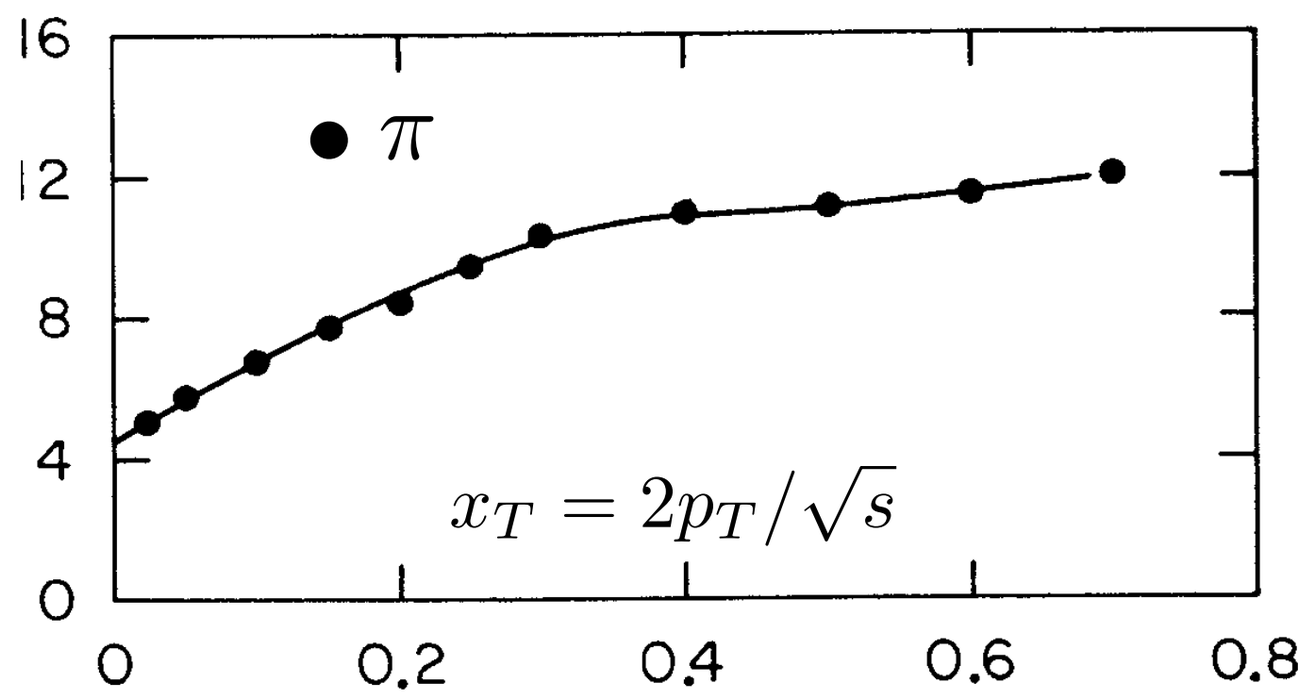
$n_{\rm eff}$



Clear evidence for higher-twist contributions

Fermilab, ISR data

$n_{\rm eff}$

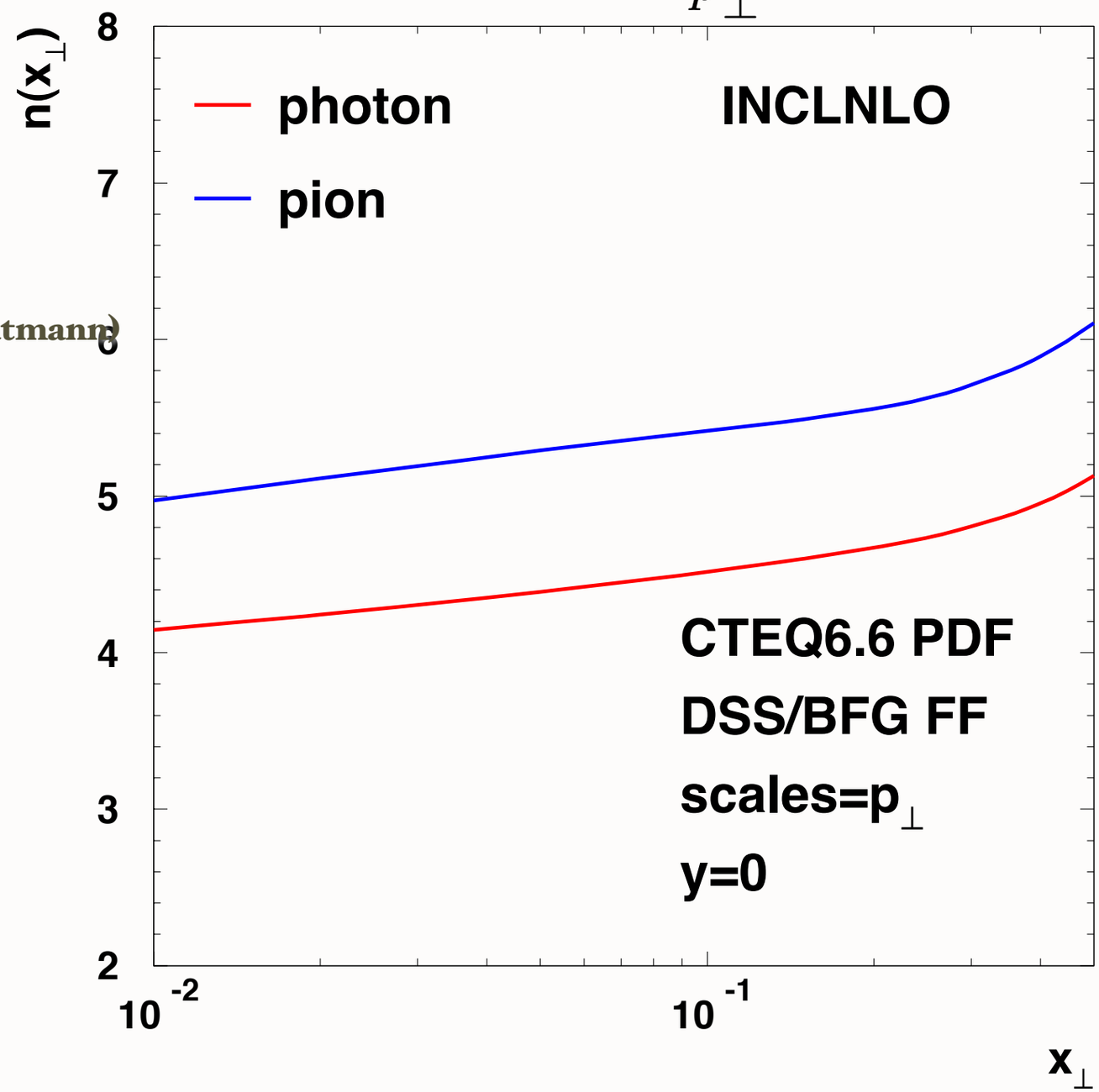


Continuous Rise of n_{eff}

QCD prediction: Modification of power fall-off due to DGLAP evolution and the Running Coupling

$$\frac{d\sigma}{d^3p/E} = \frac{F(x_\perp, y)}{p_\perp^{n(x_\perp)}}$$

DSS
(De Florian-Sassot-Stratmann)



$$pp \to \pi X$$

$$pp \to \gamma X$$

$$5 < p_\perp < 20\ GeV$$

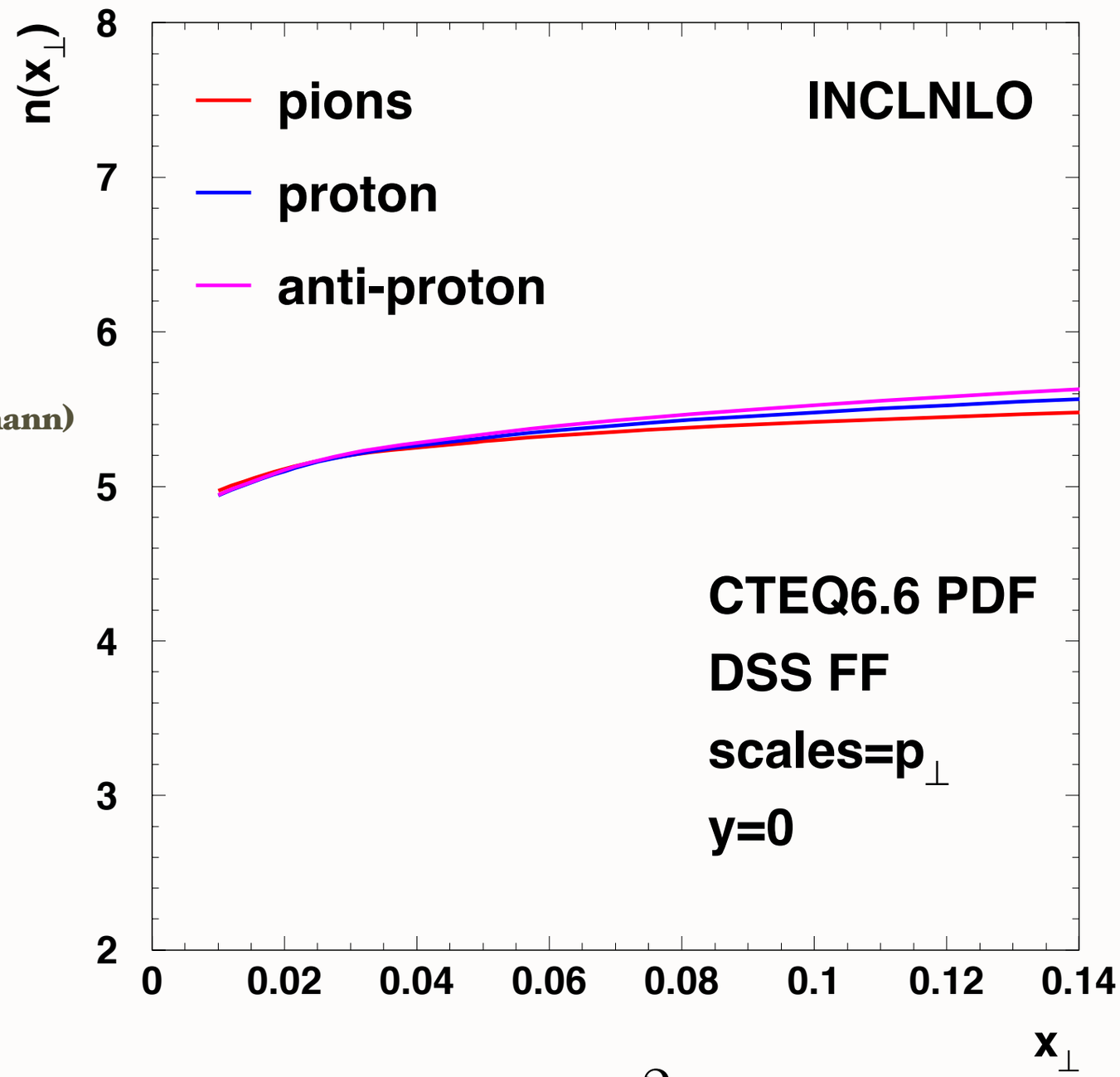
$$70\ GeV < \sqrt{s} < 4\ TeV$$

$$5 < p_\perp < 20\ GeV \qquad 70\ GeV < \sqrt{s} < 4\ TeV$$

$$pp \to \pi X$$

$$pp \to pX$$

$$pp \to \bar{p}X$$



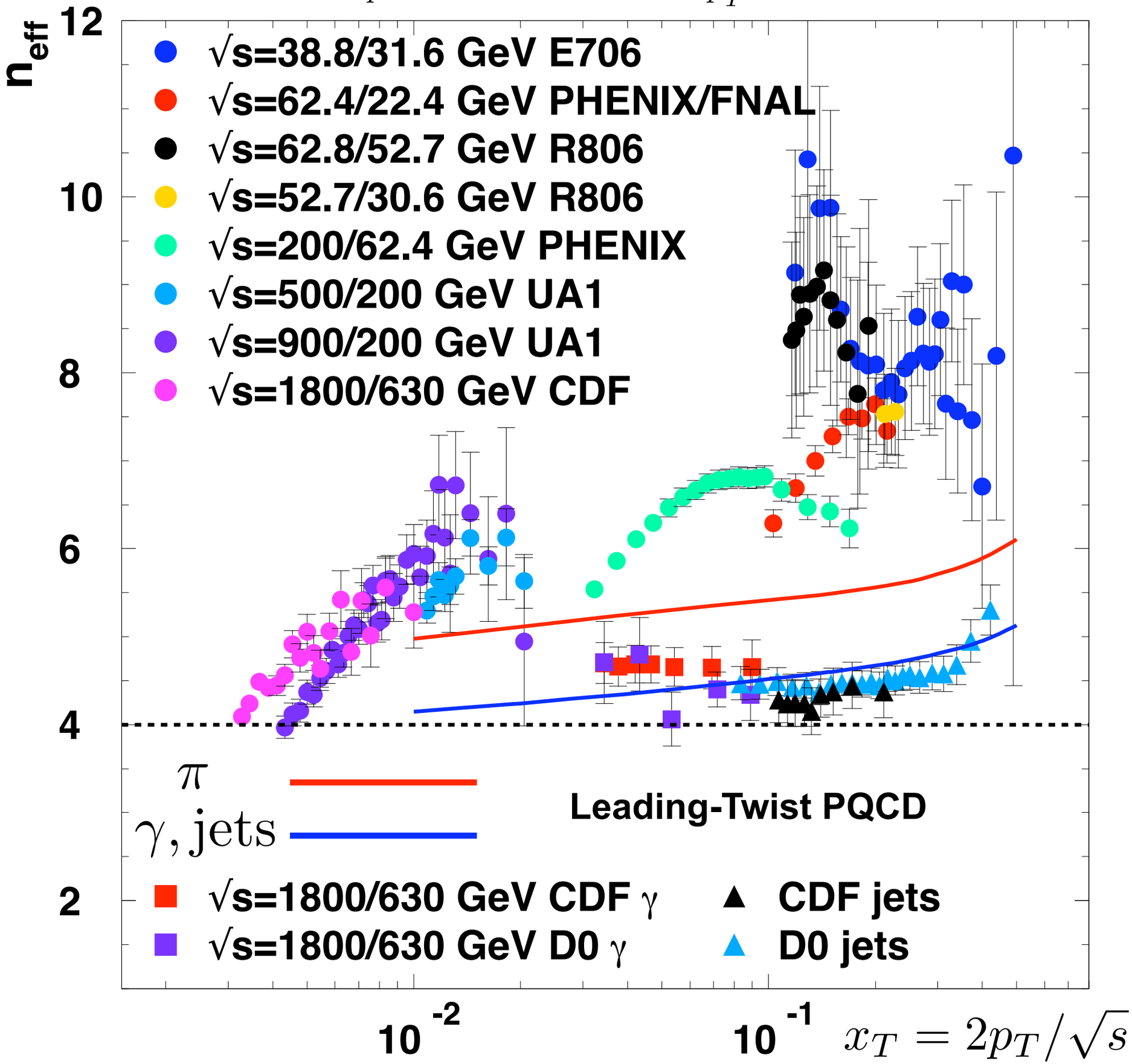
DSS
(De Florian-Sassot-Stratmann)

$$\frac{d\sigma}{d^3p/E} = \frac{F(x_\perp, y)}{p_\perp^{n(x_\perp)}}$$

Arleo, Aurenche

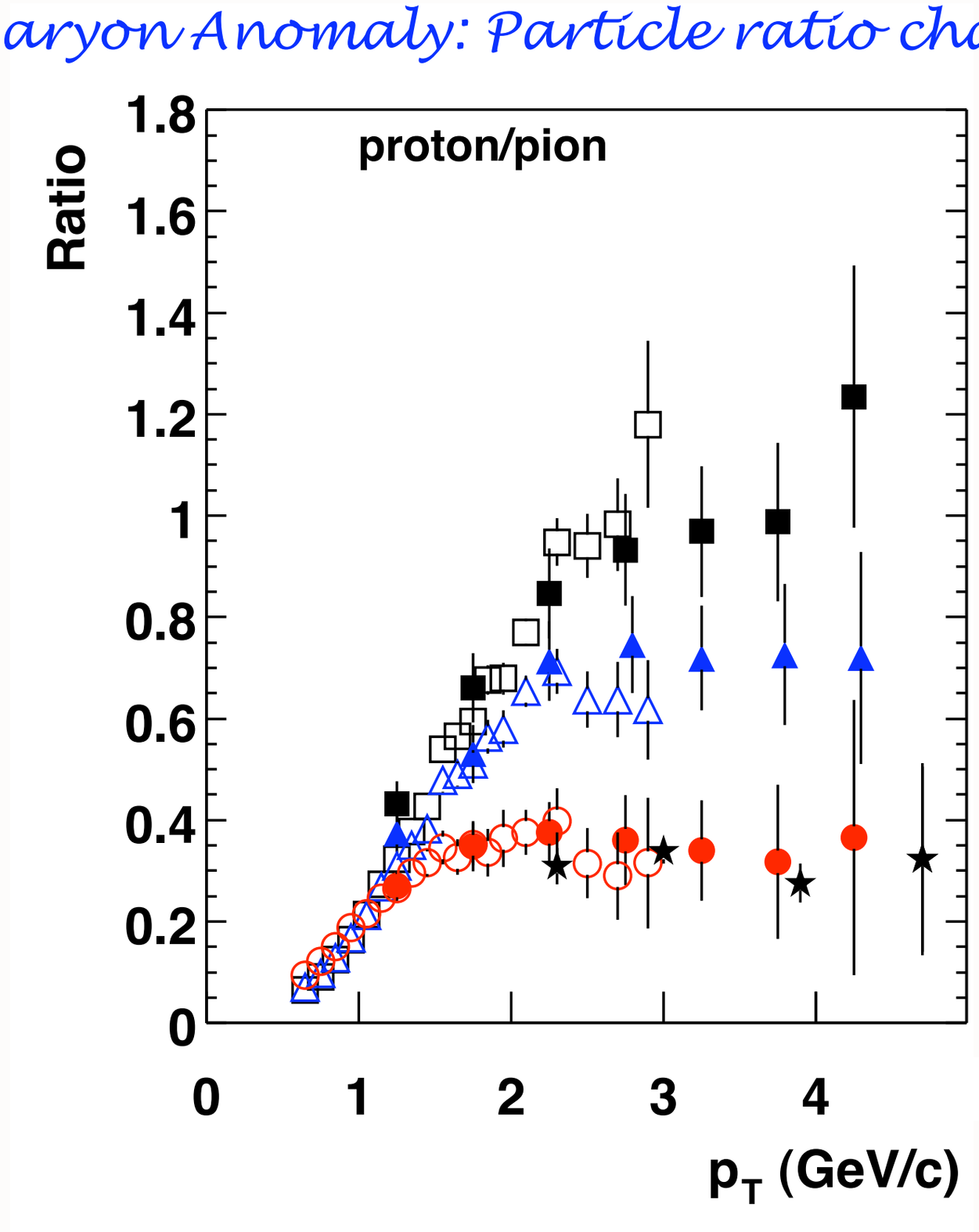
$$x_\perp = \frac{2p_\perp}{\sqrt{s}}$$

$$E\frac{d\sigma}{d^3p}(pp \to HX) = \frac{F(x_T, \theta_{CM} = \pi/2)}{p_T^{n_{\rm eff}}}$$



S. S. Adler *et al*. PHENIX Collaboration *Phys. Rev. Lett.* **91**, 172301 (2003).

Baryon Anomaly: Particle ratio changes with centrality!



Protons less absorbed in nuclear collisions than pions

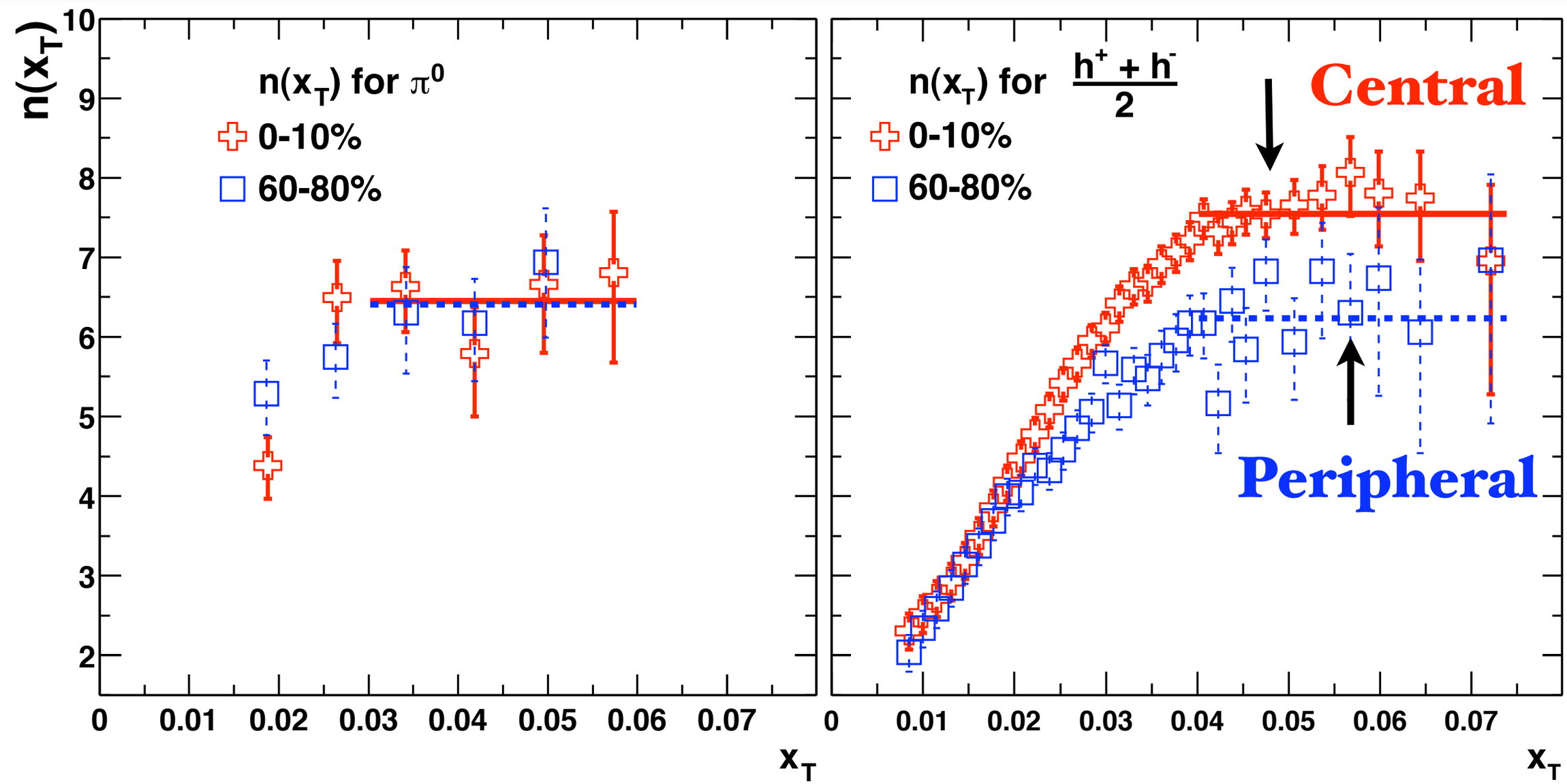
← **Central**

Au+Au 0-10%
Au+Au 20-30%
Au+Au 60-92%
p+p, $\sqrt{s}$ = 53 GeV, ISR
e^+e^-, gluon jets, DELPHI
e^+e^-, quark jets, DELPHI

← **Peripheral**

Sickles, sjb

$$\sqrt{s_{NN}} = 130 \text{ and } 200 \text{ GeV}$$

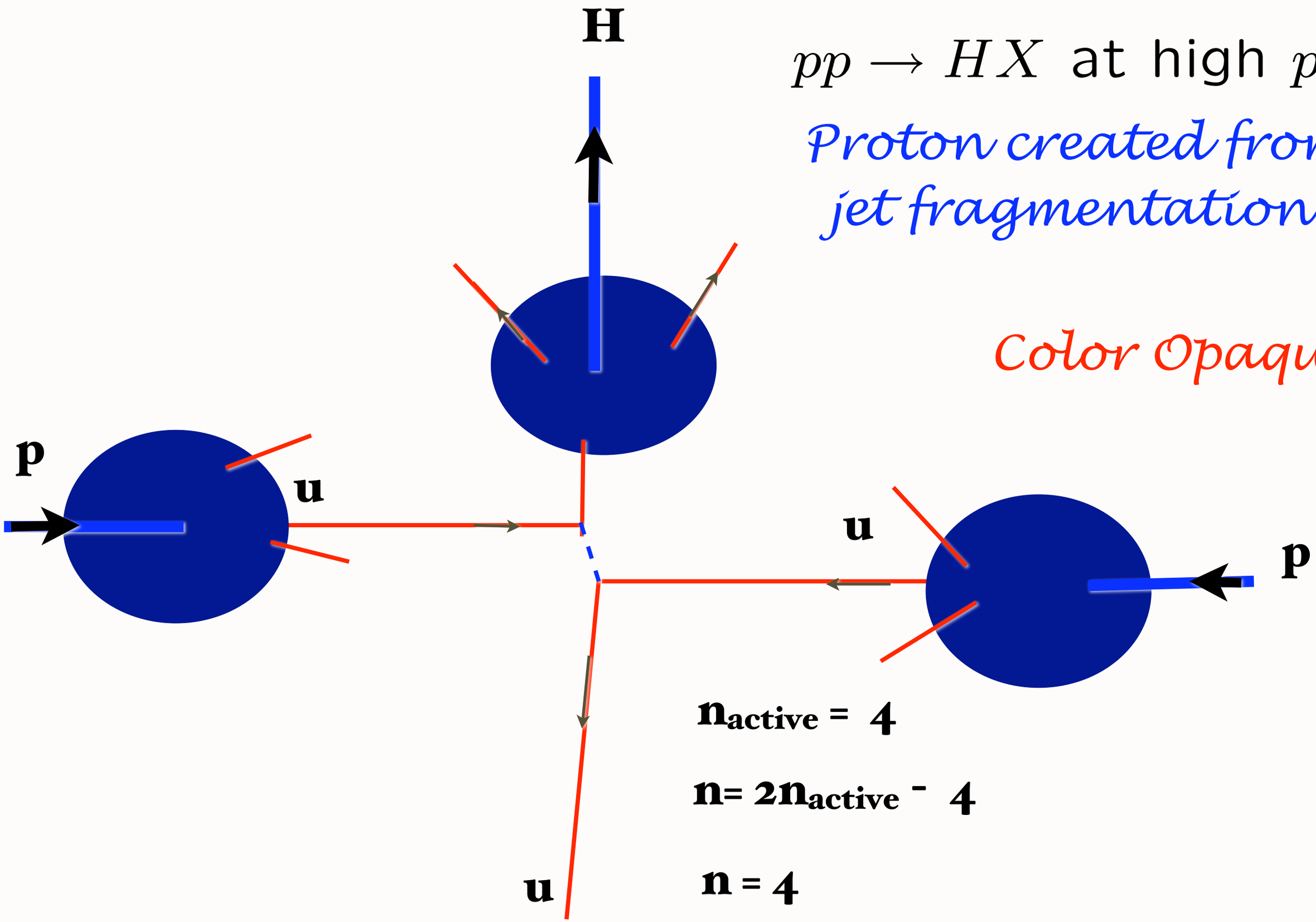


Proton power changes with centrality !

$pp \to HX$ at high p_T

Proton created from jet fragmentation

Color Opaque



$n_{active} = 4$

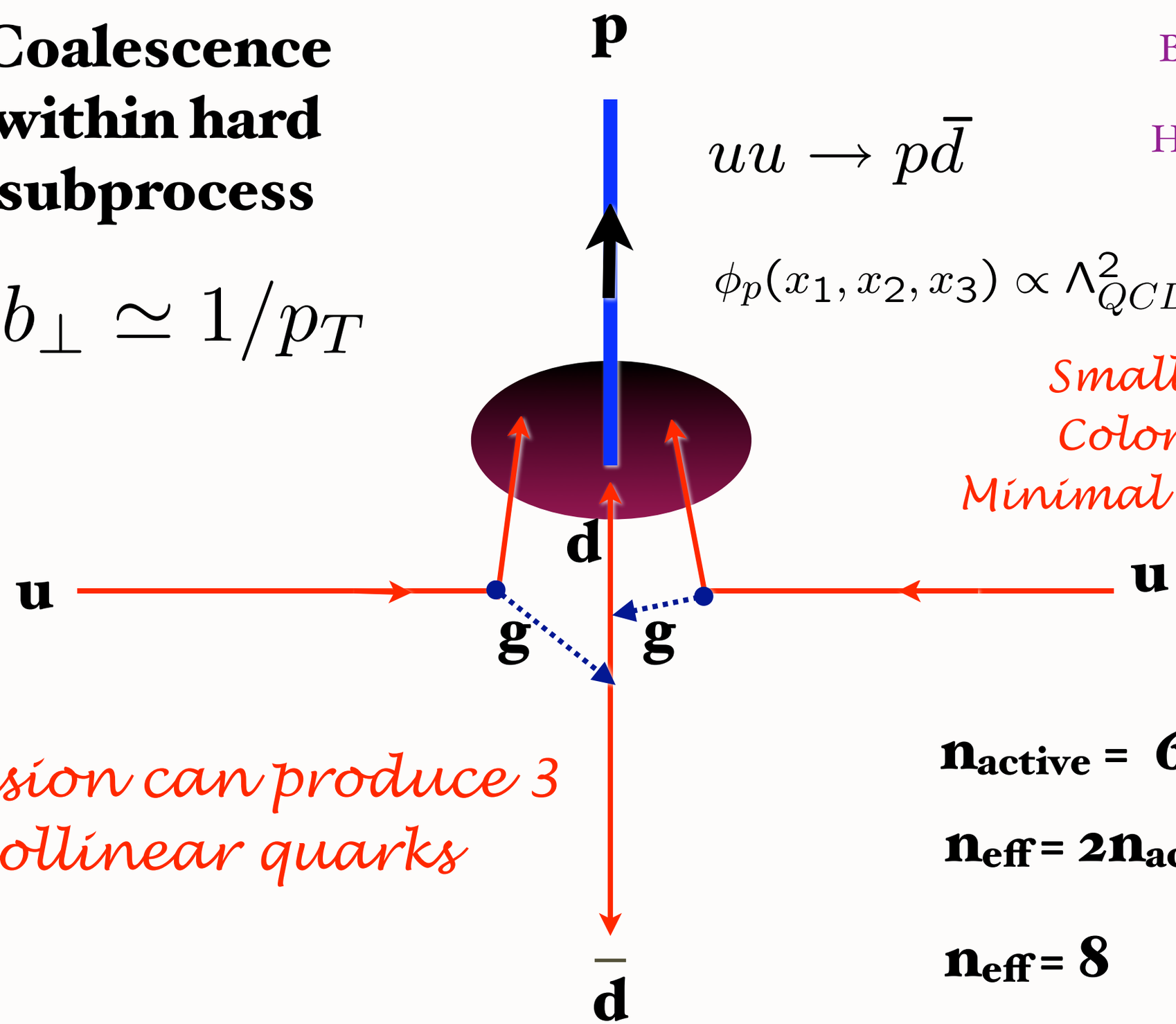
$n = 2n_{active} - 4$

$n = 4$

Baryon can be made directly within hard subprocess

Coalescence within hard subprocess

$$b_\perp \simeq 1/p_T$$



$$uu \to p\bar{d}$$

$$\phi_p(x_1, x_2, x_3) \propto \Lambda^2_{QCD}$$

Bjorken
Blankenbecler, Gunion, sjb
Berger, sjb
Hoyer, et al: Semi-Exclusive

Sickles, sjb

Small color-singlet
Color Transparent
Minimal same-side energy

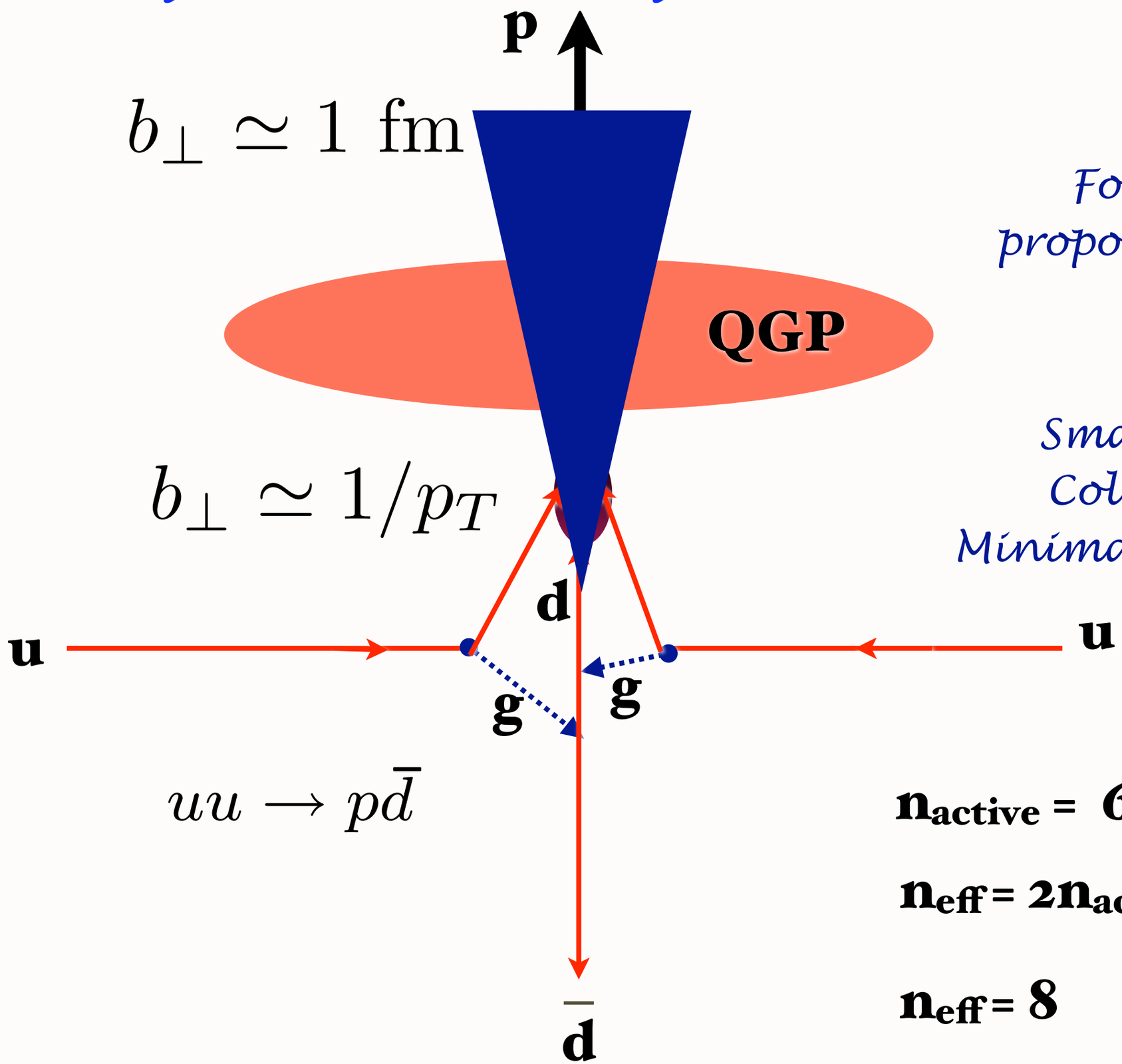
Collision can produce 3 collinear quarks

$n_{active} = 6$

$n_{eff} = 2n_{active} - 4$

$n_{eff} = 8$

Baryon made directly within hard subprocess



Formation Time
proportional to Energy

Small color-singlet
Color Transparent
Minimal same-side energy

$uu \longrightarrow p\bar{d}$

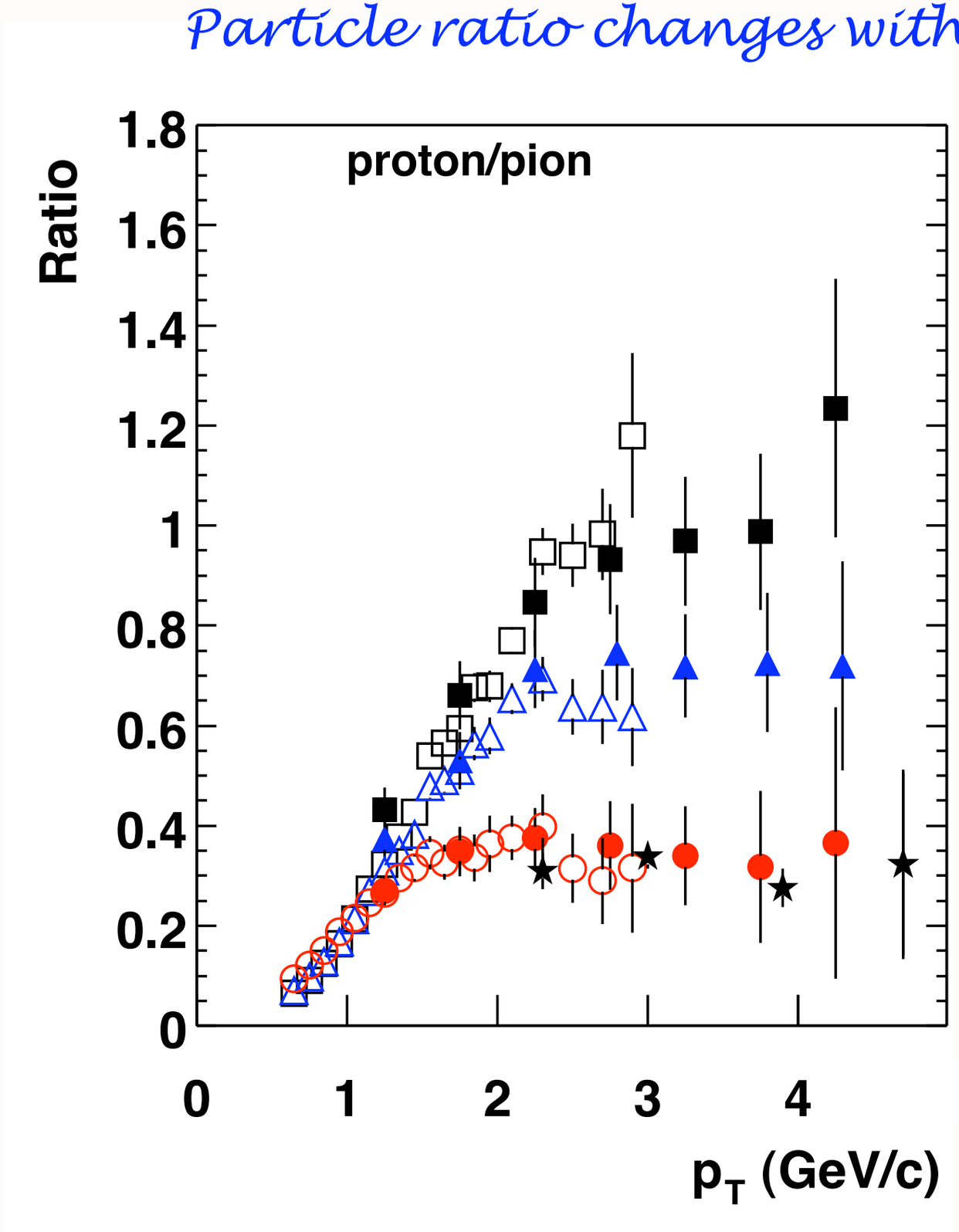
$n_{active} = 6$

$n_{eff} = 2n_{active} - 4$

$n_{eff} = 8$

S. S. Adler *et al*. PHENIX Collaboration *Phys. Rev. Lett.* **91**, 172301 (2003).

Particle ratio changes with centrality!



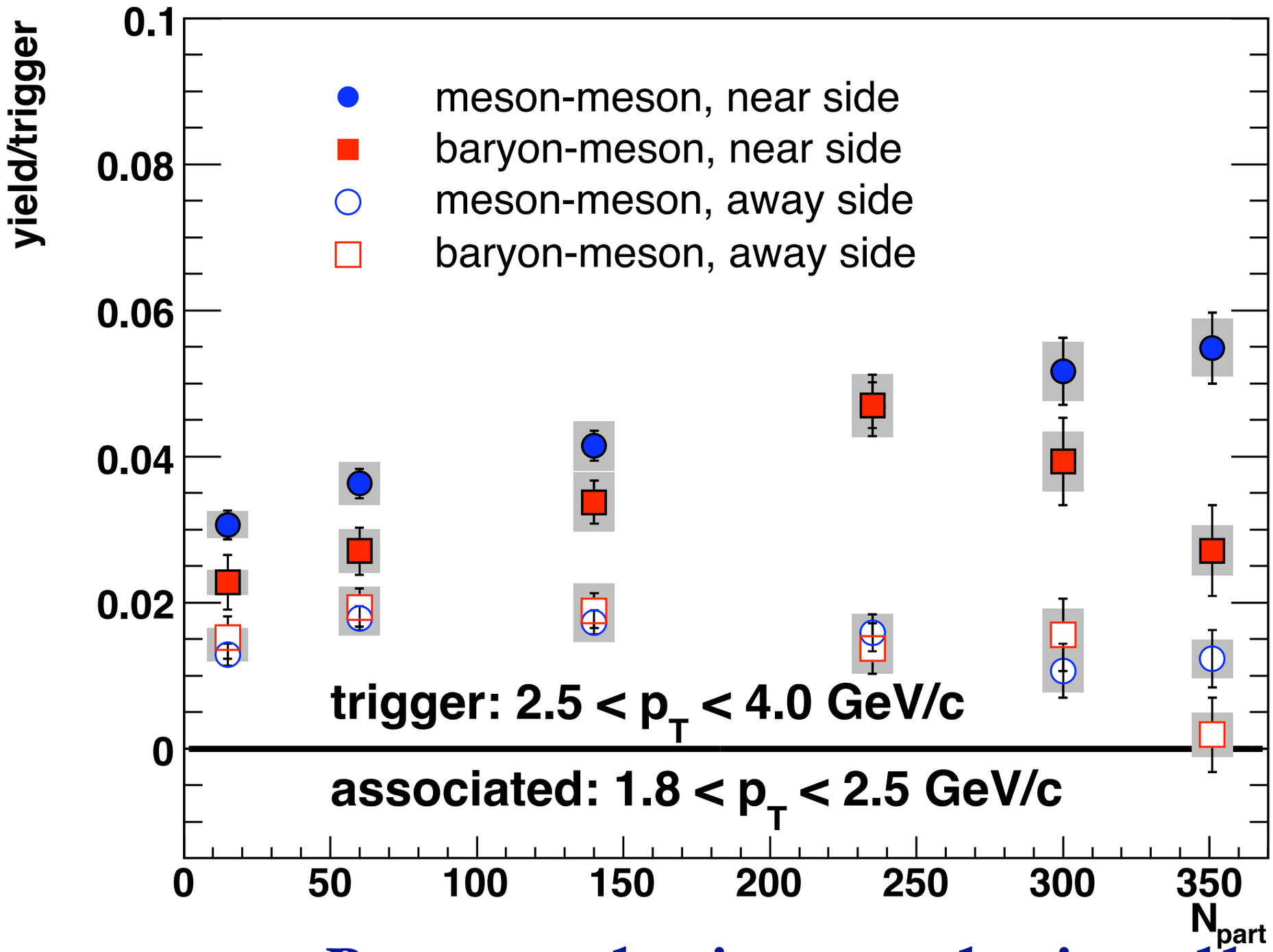
Protons less absorbed in nuclear collisions than pions because of dominant color transparent higher twist process

← **Central**

- □ ■ Au+Au 0-10%
- △ ▲ Au+Au 20-30%
- ○ ● Au+Au 60-92%
- ★ p+p, $\sqrt{s}$ = 53 GeV, ISR
- - - - e^+e^-, gluon jets, DELPHI
- ..... e^+e^-, quark jets, DELPHI

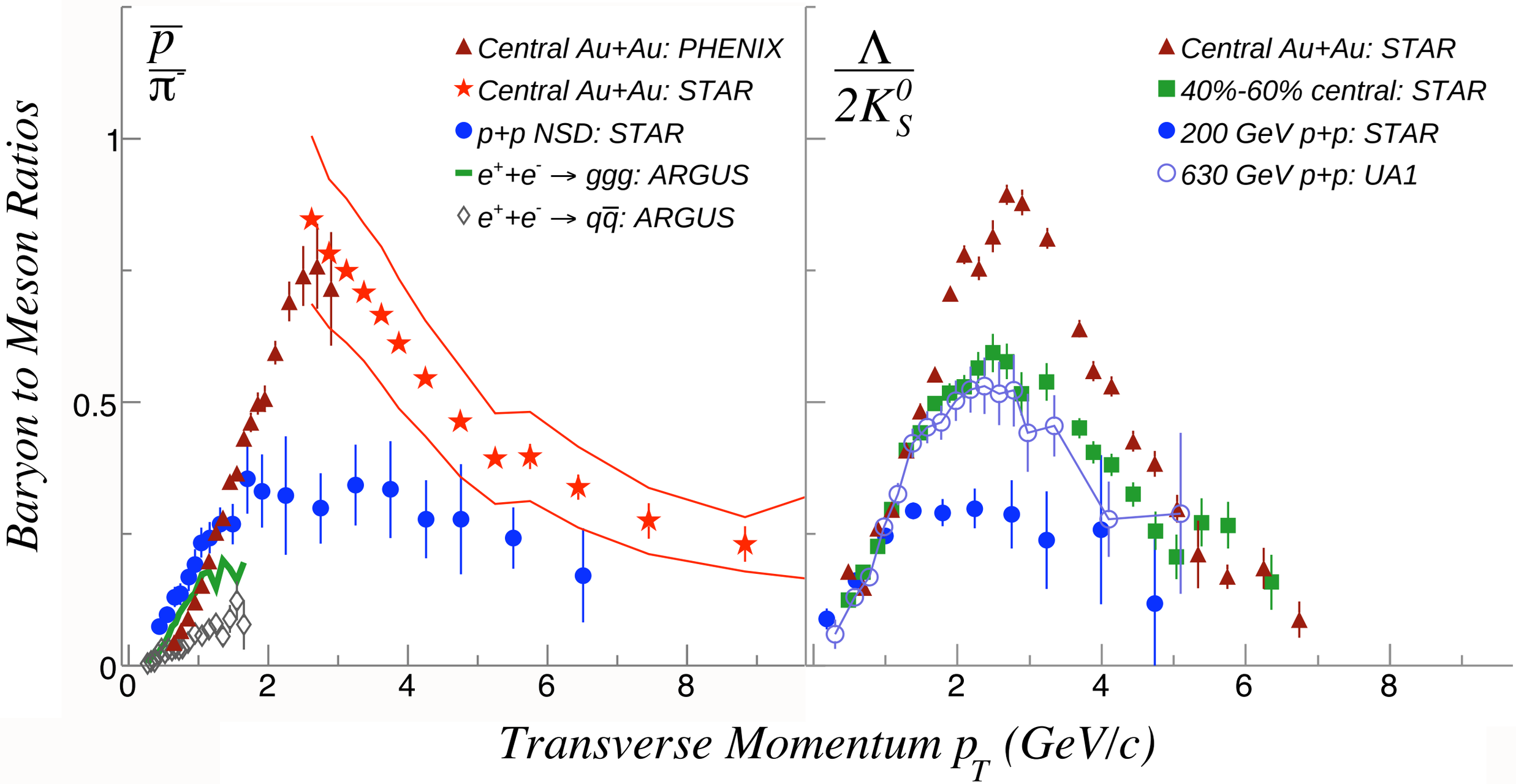
← **Peripheral**

Anne Sickles



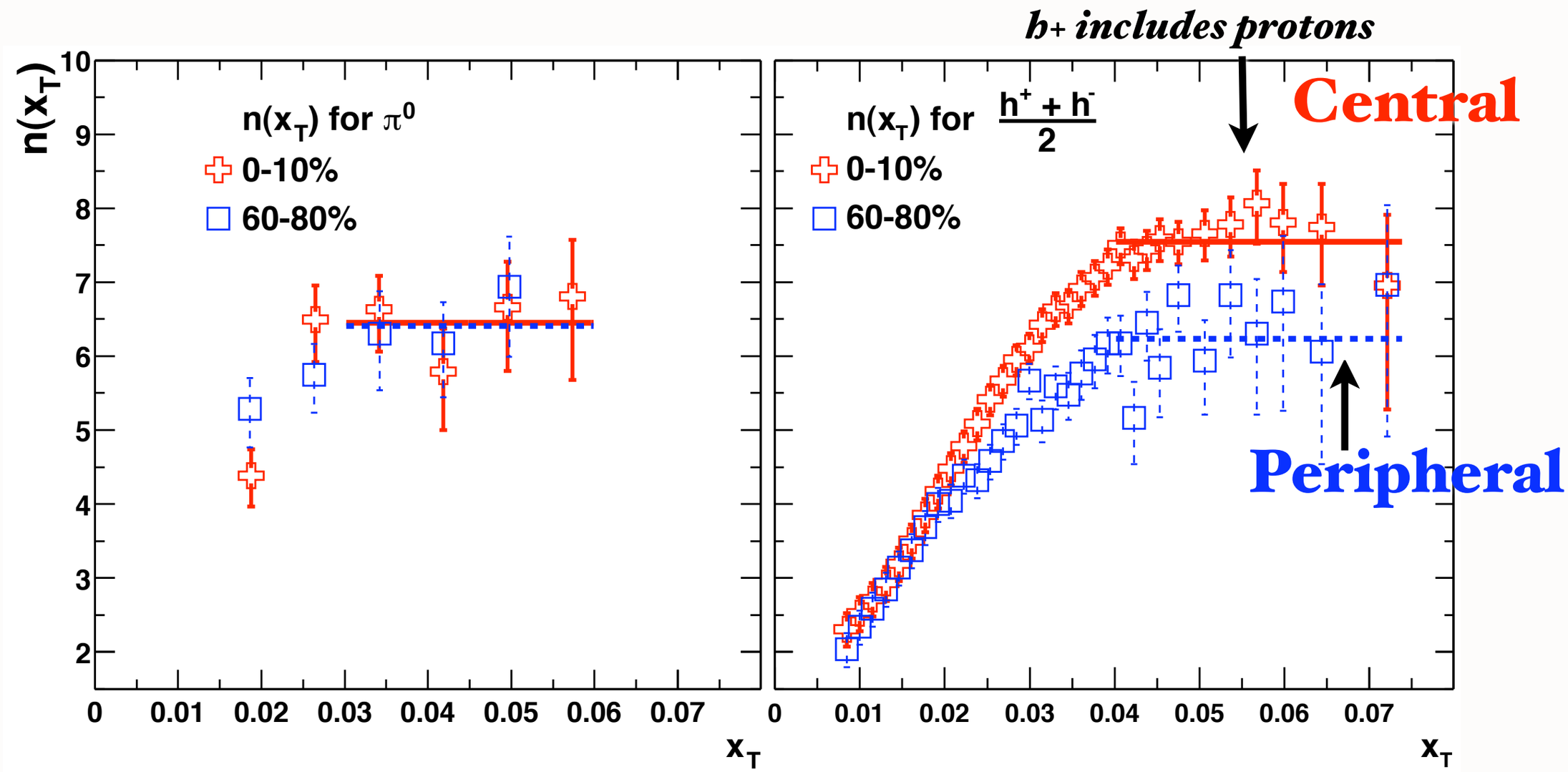
proton trigger: # same-side particles decreases with centrality

Proton production more dominated by color-transparent direct high-n_{eff} subprocesses


Baryon to Meson Ratios
$\frac{\bar{p}}{\pi^-}$
Central Au+Au: PHENIX
Central Au+Au: STAR
p+p NSD: STAR
$e^+ + e^- \rightarrow ggg$: ARGUS
$e^+ + e^- \rightarrow q\bar{q}$: ARGUS
$\frac{\Lambda}{2K_S^0}$
Central Au+Au: STAR
40%-60% central: STAR
200 GeV p+p: STAR
630 GeV p+p: UA1
0
0.5
1
0
2
4
6
8
Transverse Momentum p_T (GeV/c)

Power-law exponent $n(x_T)$ for π^0 and h spectra in central and peripheral Au+Au collisions at $\sqrt{s_{NN}} = 130$ and 200 GeV

S. S. Adler, *et al.*, PHENIX Collaboration, *Phys. Rev.* C **69**, 034910 (2004) [nucl-ex/0308006].

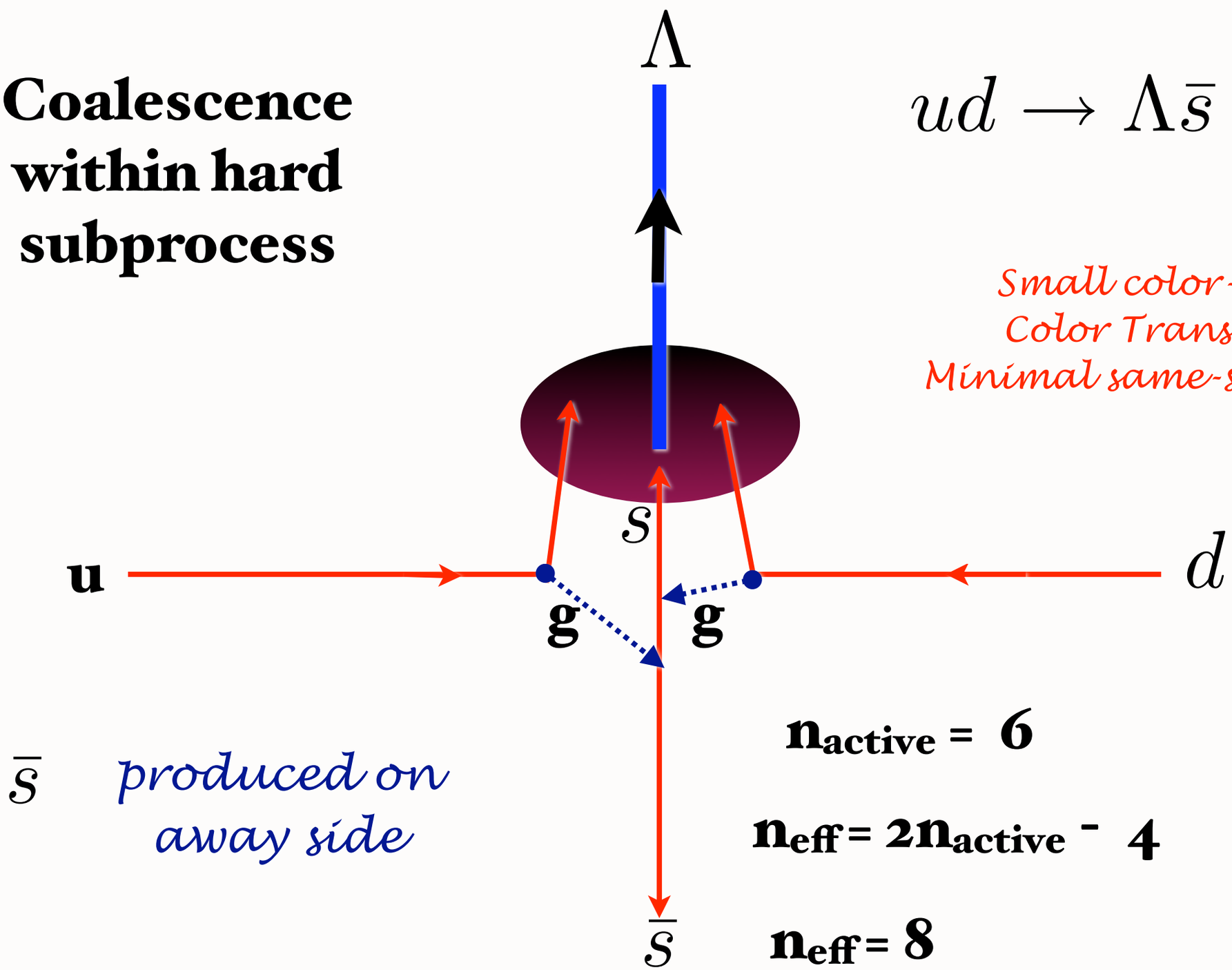


Proton production dominated by color-transparent direct high n_{eff} subprocesses

Lambda can be made directly within hard subprocess

Coalescence within hard subprocess

$$ud \to \Lambda \bar{s}$$



Small color-singlet
Color Transparent
Minimal same-side energy

produced on away side

$n_{active} = 6$

$n_{eff} = 2n_{active} - 4$

$n_{eff} = 8$

Sickles, sjb

Baryon Anomaly: Evidence for Direct, Higher-Twist Subprocesses

- Explains anomalous power behavior at fixed x_T
- Protons more likely to come from direct higher-twist subprocess than pions
- Protons less absorbed than pions in central nuclear collisions because of color transparency
- Predicts increasing proton to pion ratio in central collisions
- Proton power n_{eff} increases with centrality since leading twist contribution absorbed
- Fewer same-side hadrons for proton trigger at high centrality
- Exclusive-inclusive connection at $x_T = 1$

Chiral Symmetry Breaking in AdS/QCD

We consider the action of the X field which encodes the effects of CSB in AdS/QCD:

$$S_X = \int d^4x dz \sqrt{g} \left(g^{\ell m} \partial_\ell X \partial_m X - \mu_X^2 X^2 \right), \tag{1}$$

with equations of motion

$$z^3 \partial_z \left(\frac{1}{z^3} \partial_z X \right) - \partial_\rho \partial^\rho X - \left(\frac{\mu_X R}{z} \right)^2 X = 0. \tag{2}$$

Ehrlich, Katz, Son, Stephanov

Babington, Erdmenger, Evans, Kirsch, Guralnik, Thelfall

The zero mode has no variation along Minkowski coordinates

$$\partial_\mu X(x, z) = 0,$$

thus the equation of motion reduces to

$$\left[z^2 \partial_z^2 - 3z\, \partial_z + 3 \right] X(z) = 0. \tag{3}$$

for $(\mu_X R)^2 = -3$, which corresponds to scaling dimension $\Delta_X = 3$. The solution is

$$X(z) = \langle X \rangle = Az + Bz^3, \tag{4}$$

where A and B are determined by the boundary conditions.

de Teramond, Shrock, sjb (preliminary)

$$A \propto m_q \qquad\qquad B \propto < \bar{\psi}\psi >$$

Expectation value taken inside hadron

In presence of quark masses the Holographic LF wave equation is ($\zeta = z$)

$$\left[-\frac{d^2}{d\zeta^2} + V(\zeta) + \frac{X^2(\zeta)}{\zeta^2}\right]\phi(\zeta) = \mathcal{M}^2\phi(\zeta), \tag{1}$$

and thus

$$\boxed{\delta M^2 = \left\langle \frac{X^2}{\zeta^2} \right\rangle .} \tag{2}$$

The parameter a is determined by the Weisberger term

$$a = \frac{2}{\sqrt{x}}.$$

Thus

$$\boxed{X(z) = \frac{m}{\sqrt{x}} z - \sqrt{x}\langle\bar{\psi}\psi\rangle z^3,} \tag{3}$$

and

$$\delta M^2 = \sum_i \left\langle \frac{m_i^2}{x_i} \right\rangle - 2\sum_i m_i \langle\bar{\psi}\psi\rangle\langle z^2\rangle + \langle\bar{\psi}\psi\rangle^2\langle z^4\rangle, \tag{4}$$

where we have used the sum over fractional longitudinal momentum $\sum_i x_i = 1$.

Mass shift from dynamics inside hadronic boundary

Chiral Symmetry Breaking in AdS/QCD

- **Chiral symmetry breaking effect in AdS/QCD depends on weighted z^2 distribution, not constant condensate**

$$\delta M^2 = -2m_q < \bar{\psi}\psi > \times \int dz\ \phi^2(z) z^2$$

- **z^2 weighting consistent with higher Fock states at periphery of hadron wavefunction**

- **AdS/QCD supports confined condensate picture**

de Teramond, Shrock, sjb

Quark and Gluon condensates reside within hadrons, not vacuum

Casher and Susskind Roberts et al. Shrock and sjb

- **Bound-State Dyson-Schwinger Equations**

  Roberts et al.

- **LF vacuum trivial up to $k^+ = 0$ zero modes**
- **Analogous to finite size superconductor**
- **Implications for cosmological constant -- Eliminates 45 orders of magnitude conflict**

  Shrock and sjb

Pion mass **and decay constant.**
Pieter Maris, Craig D. Roberts (Argonne, PHY) , Peter C. Tandy (Kent State U.) . ANL-PHY-8753-TH-97, KSUCNR-103-97, Jul 1997. 12pp.
Published in **Phys.Lett.B420:267-273,1998**.
e-Print: **nucl-th/9707003**

Pi- and K meson Bethe-Salpeter amplitudes.
Pieter Maris, Craig D. Roberts (Argonne, PHY) . ANL-PHY-8788-TH-97, Aug 1997. 34pp.
Published in **Phys.Rev.C56:3369-3383,1997**.
e-Print: **nucl-th/9708029**

Concerning the quark condensate.
K. Langfeld (Tubingen U.) , H. Markum (Vienna, Tech. U.) , R. Pullirsch (Regensburg U.) , C.D. Roberts (Argonne, PHY & Rostock U.) , S.M. Schmidt (Tubingen U. & HGF, Bonn) . ANL-PHY-10460-TH-2002, MPG-VT-UR-239-02, Jan 2003. 7pp.
Published in **Phys.Rev.C67:065206,2003**.
e-Print: **nucl-th/0301024**

"In-Meson Condensate"

$$-\langle \bar{q}q\rangle_\zeta^\pi = f_\pi \langle 0|\bar{q}\gamma_5 q|\pi\rangle \,.$$

Valid even for $m_q \to 0$

f_π nonzero

QCD Symmetries

- Color Confinement: Maximum Wavelength of Quark and Gluons
- Conformal symmetry of QCD coupling in IR
- Provides Conformal Template
- Motivation for AdS/QCD
- QCD Condensates inside of hadronic LFWFs
- Technicolor: confined condensates inside of technihadrons -- alternative to Higgs
- Simple physical solution to cosmological constant conflict

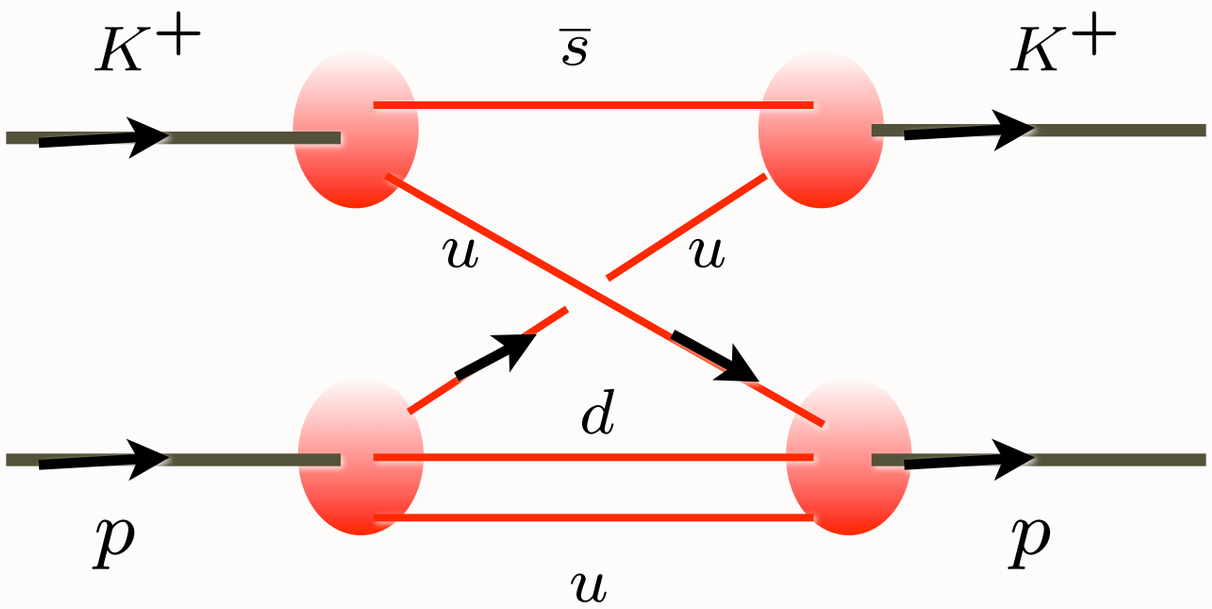
Hadron Dynamics at the Amplitude Level

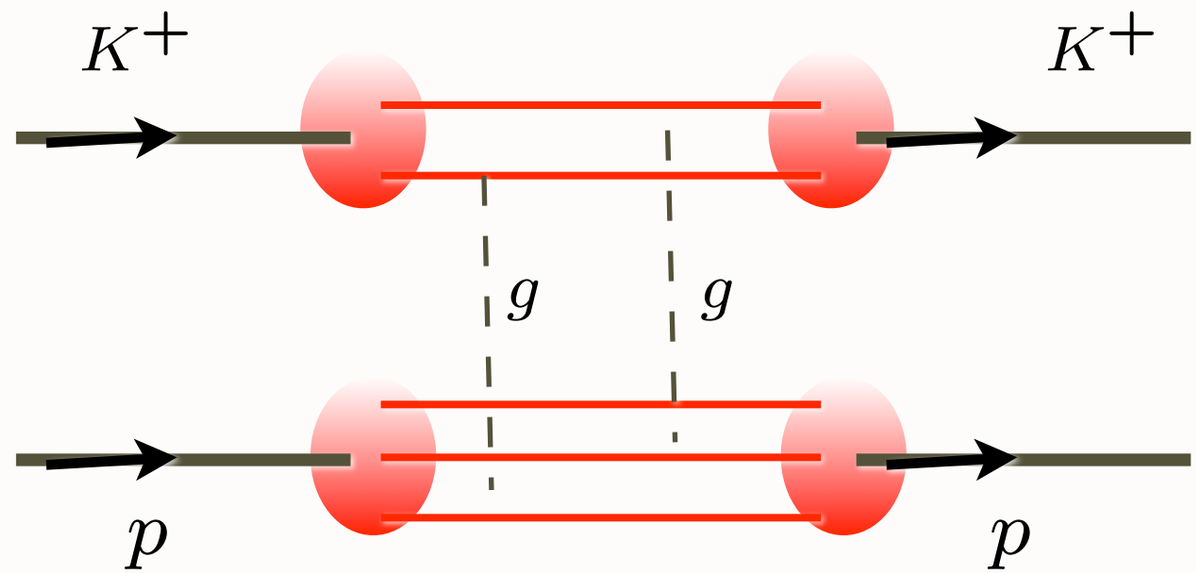
- LFWFS are the universal hadronic amplitudes which underlie structure functions, GPDs, exclusive processes, distribution amplitudes, direct subprocesses, hadronization.
- Relation of spin, momentum, and other distributions to physics of the hadron itself.
- Connections between observables, orbital angular momentum
- Role of FSI and ISIs--Sivers effect
- Higher Fock States give GMOR Relations, Chiral Symmetry Breaking

New Perspectives for QCD from AdS/CFT

- LFWFs: Fundamental frame-independent description of hadrons at amplitude level
- Holographic Model from AdS/CFT : Confinement at large distances and conformal behavior at short distances
- Model for LFWFs, meson and baryon spectra: many applications!
- New basis for diagonalizing Light-Front Hamiltonian
- Physics similar to MIT bag model, but covariant. No problem with support $0 < x < 1$.
- Quark Interchange dominant force at short distances

CIM: Blankenbecler, Gunion, sjb





Quark Interchange

(Spin exchange in atom-atom scattering)

Gluon Exchange

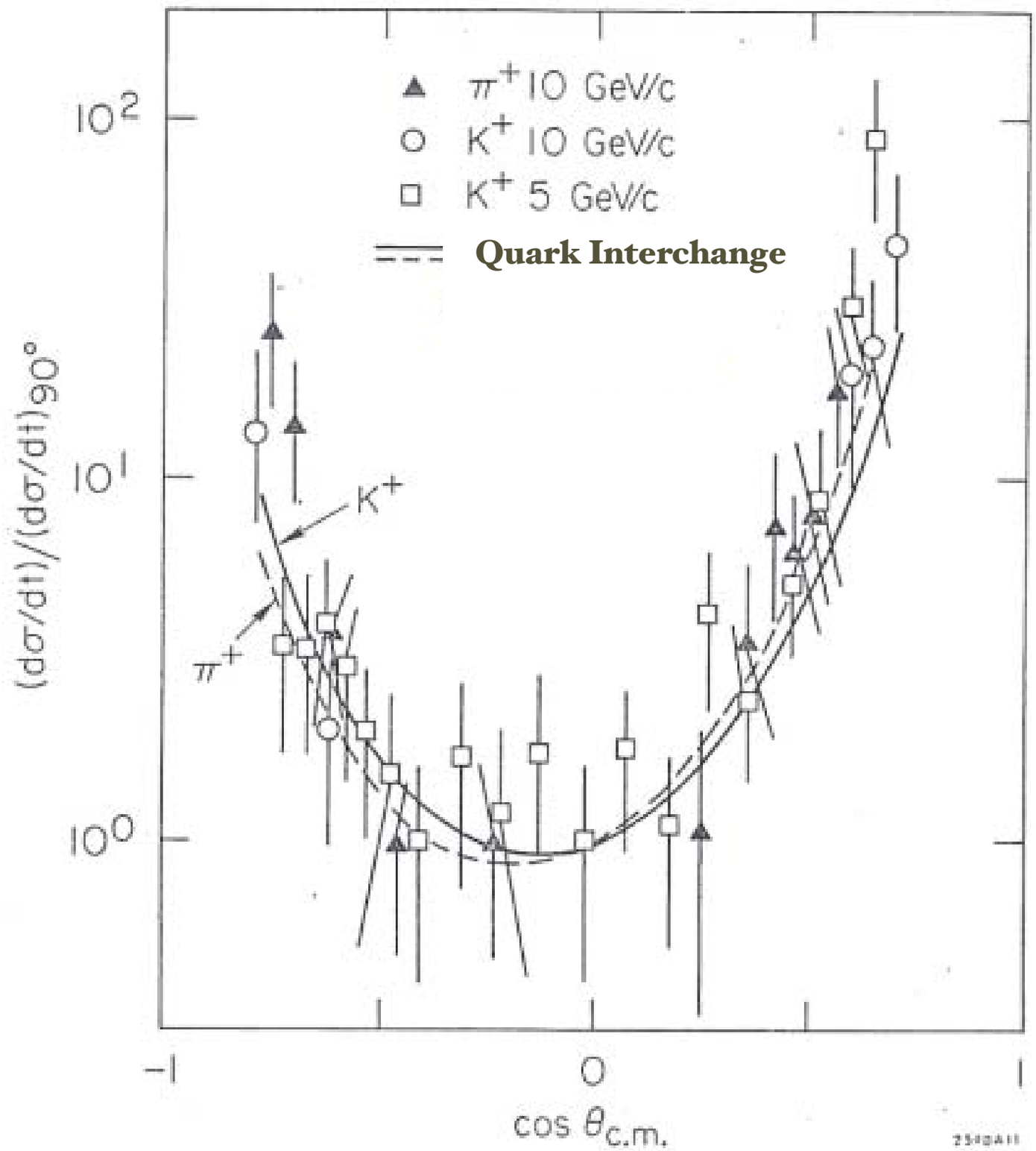
(Van der Waal -- Landshoff)

$$\frac{d\sigma}{dt} = \frac{|M(s,t)|^2}{s^2}$$

$$M(t,u)_{\text{interchange}} \propto \frac{1}{ut^2}$$

$$M(s,t)_{\text{gluonexchange}} \propto sF(t)$$

MIT Bag Model (de Tar), large N_C, ('t Hooft), AdS/CFT all predict dominance of quark interchange:



AdS/CFT explains why quark interchange is dominant interaction at high momentum transfer in exclusive reactions

$$M(t,u)_{\text{interchange}} \propto \frac{1}{ut^2}$$

Non-linear Regge behavior:

$$\alpha_R(t) \to -1$$

Why is quark-interchange dominant over gluon exchange?

Example: $M(K^+p \to K^+p) \propto \frac{1}{ut^2}$

Exchange of common u quark

$M_{QIM} = \int d^2k_\perp dx \; \psi_C^\dagger \psi_D^\dagger \Delta \psi_A \psi_B$

Holographic model (Classical level):

Hadrons enter 5th dimension of AdS_5

Quarks travel freely within cavity as long as separation $z < z_0 = \frac{1}{\Lambda_{QCD}}$

LFWFs obey conformal symmetry producing quark counting rules.

Comparison of Exclusive Reactions at Large t

B. R. Baller,[a] G. C. Blazey,[b] H. Courant, K. J. Heller, S. Heppelmann,[c] M. L. Marshak, E. A. Peterson, M. A. Shupe, and D. S. Wahl[d]
University of Minnesota, Minneapolis, Minnesota 55455

D. S. Barton, G. Bunce, A. S. Carroll, and Y. I. Makdisi
Brookhaven National Laboratory, Upton, New York 11973

and

S. Gushue[e] and J. J. Russell
Southeastern Massachusetts University, North Dartmouth, Massachusetts 02747
(Received 28 October 1987; revised manuscript received 3 February 1988)

Cross sections or upper limits are reported for twelve meson-baryon and two baryon-baryon reactions for an incident momentum of 9.9 GeV/c, near 90° c.m.: $\pi^{\pm}p \to p\pi^{\pm}, p\rho^{\pm}, \pi^{+}\Delta^{\pm}, K^{+}\Sigma^{\pm}, (\Lambda^{0}/\Sigma^{0})K^{0}$; $K^{\pm}p \to pK^{\pm}$; $p^{\pm}p \to pp^{\pm}$. By studying the flavor dependence of the different reactions, we have been able to isolate the quark-interchange mechanism as dominant over gluon exchange and quark-antiquark annihilation.

$\pi^{\pm}p \to p\pi^{\pm}$,

$K^{\pm}p \to pK^{\pm}$,

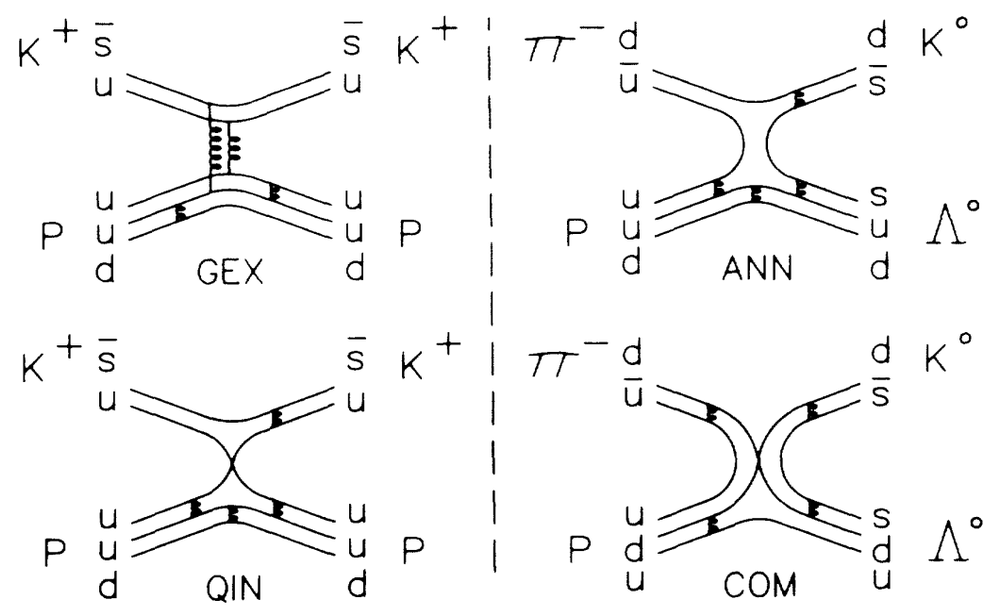
$\pi^{\pm}p \to p\rho^{\pm}$,

$\pi^{\pm}p \to \pi^{+}\Delta^{\pm}$,

$\pi^{\pm}p \to K^{+}\Sigma^{\pm}$,

$\pi^{-}p \to \Lambda^{0}K^{0}, \Sigma^{0}K^{0}$,

$p^{\pm}p \to pp^{\pm}$.



Features of Soft-Wall AdS/QCD

- Single-variable frame-independent radial Schrodinger equation
- Massless pion (m_q =0)
- Regge Trajectories: universal slope in n and L
- Valid for all integer J & S. Spectrum is independent of S
- Dimensional Counting Rules for Hard Exclusive Processes
- Phenomenology: Space-like and Time-like Form Factors
- LF Holography: LFWFs; broad distribution amplitude
- No large Nc limit
- Add quark masses to LF kinetic energy
- Systematically improvable -- diagonalize H_{LF} on AdS basis

String Theory

AdS/CFT

Mapping of Poincare' and Conformal SO(4,2) symmetries of 3+1 space to AdS5 space

Goal: First Approximant to QCD

Counting rules for Hard Exclusive Scattering

Regge Trajectories

QCD at the Amplitude Level

AdS/QCD

Conformal behavior at short distances + Confinement at large distance

Semi-Classical QCD / Wave Equations

Holography

Boost Invariant 3+1 Light-Front Wave Equations

J =0,1,1/2,3/2 plus L

Integrable!

Hadron Spectra, Wavefunctions, Dynamics