# Holographic Monopole Catalysis of Baryon Decay

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# Monopole catalysis

Callan and Rubakov Effect

In the presence of magnetic monopole, baryon number can be violated by anomaly

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However, at low energy QCD, baryons are described by topological solitons called Skyrmions

Q : How can the topological solitons can decay in the presence of magnetic monopole ???

# Monopole catalysis of Skyrmion decay

Callan and Witten showed that

it is possible for the Skyrmions to decay in the presence of magnetic monopole

- Skyrmions are topological solitons from $\pi_3(SU(2)) = Z$ with the Skyrme action

$$L = \frac{f_\pi^2}{4} \mathrm{tr} \left( U^{-1} \partial_\mu U \right)^2 + \frac{1}{32e^2} \mathrm{tr} \left[ U^{-1} \partial_\mu U, U^{-1} \partial_\nu U \right]^2$$

- There is a topologically conserved current

$$B^\mu = \frac{1}{24\pi^2} \epsilon^{\mu\nu\alpha\beta} \mathrm{tr} \left( U^{-1} \partial_\nu U U^{-1} \partial_\alpha U U^{-1} \partial_\beta U \right)$$

Naturally, the topological baryon number $B = \int d^3 x B^0$ is always conserved in any finite time processes.

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# How about the magnetic monopole ?

- Recall that the $SU(2)$ valued group field $U$ transforms under chiral symmetry $U(2)_L \times U(2)_R$ as

$$U \rightarrow U_L U U_R^\dagger$$

- Electromagnetism is

$$Q = \frac{B}{2} + I_3$$

where $B$ is from $U(1)_V$ and $I_3$ is the third component of $SU(2)_V$. We weakly gauge these symmetries to introduce electromagnetism

- Therefore, our topological baryon current $B^\mu$ should also respect this weakly gauged symmetry to be gauge invariant. For example,

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# Conserved gauge invariant baryon current

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Witten found that

$$\begin{aligned} B^\mu &= \frac{1}{24\pi^2}\epsilon^{\mu\nu\alpha\beta}\mathrm{tr}\left(U^{-1}\partial_\nu U U^{-1}\partial_\alpha U U^{-1}\partial_\beta U\right) \\ &- \frac{1}{24\pi^2}\epsilon^{\mu\nu\alpha\beta}\partial_\nu\left[3A_\alpha^{EM}\mathrm{tr}\left(Q\left(U^{-1}\partial_\beta U + \partial_\beta U U^{-1}\right)\right)\right] \end{aligned}$$

# Monopole background is singular

- The $B^\mu$ is now both gauge invariant and conserved only for smooth EM potential $A^{EM}$
- The tricky point is that the magnetic monopole potential is singular : Dirac string

$$A^{EM} = \frac{-i}{2}(1-\cos\theta)d\phi$$

- Interestingly, the baryon number conservation can be violated precisely at the center of the monopole

**This is the low energy manifestation of Callan-Rubakov effect**

- Charged pions are suppressed at the monopole core, but neutral pions can be non-zero $U = \exp(\frac{2i}{F_\pi}\pi^0\sigma^3)$. Then there is a radial flux

$$B^r = \frac{(\partial_t \pi^0)}{4\pi^2 F_\pi r^2}$$

which results in the baryon number violation

$$\frac{dB}{dt} = \frac{1}{\pi F_\pi}\left(\partial_t\pi^0\right)\Big|_{r=0}$$

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Solution :

- In the presence of magnetic monopole, one is allowed to perform singular gauge transformation moving Dirac string from one place to another
- Under this singular gauge transformations, topological baryon number $\pi_3(SU(2)) = Z$ is not conserved. Different $\pi_3(SU(2)) = Z$ can be connected by our allowed gauge transformations, and thus **it is not physical**
- Gauge invariant physical baryon number is given by above, and it is not wholly given by topological numbers, and can disappear fractionally at the monopole core

# Holographic QCD

- Holographic QCD is an alternative approach to low energy strongly coupled QCD based on large $N$ limit
- In large N limit, the theory becomes classical with new variables called master fields
- The renormalization group survives in the large N limit. Asymptotic freedom and dimensional transmutation persist. One needs to renormalize the theory after fixing the renormalization scale.
- One ends up with a classical theory of master fields with the features of renormaliztion group, especially the formalism is invariant under shifts in the renormalization point, kind of general covariance in energy direction
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- 5 dimensional theory is a gravity theory, realizing general covariance in energy direction as a renormalization group
- When the field theory is strongly coupled, the 5 dimensional theory becomes a local theory

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$$A(x^\mu, Z) = A^{(0)} + \sum_{n\geq 1} f_n(Z) A^{(n)}$$

The point is that the zero mode $A^{(0)}$ is non-normalizable due to the specific metric property of 5 dimensions, and therefore it is non-dynamical, something like external gauge potential coupled to the currents

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Important : By turning on $A^{(0)}$, we can introduce weakly gauged external gauge potential coupled to the global currents, and study how the system responds to this external potential

# More concretely,

- Corresponding to $U(2)_L \times U(2)_R$ chiral symmetry in QCD, we have $U(2)_L \times U(2)_R$ gauge symmetry in 5 dimensional holographic QCD
- Electromagnetism is an external potential coupled to $Q = \frac{B}{2} + I_3$ symmetry current, so $A^{EM}$ is encode as the zero mode $A^{(0)}$ of the corresponding 5 dimensional gauge field to $Q = \frac{B}{2} + I_3$
- Chiral symmetry breaking is realized as a gauge symmetry breaking in 5 dimensions. Goldstone pions are realized as Wilson lines along the 5'th direction
- The upshot is that

  **5 dimensional gauge theory encapsulates both the dynamical mesons such as pions and the external potential coupled to the symmetry currents into a single framework**

$$A_\mu(x,Z) = A_\mu^{EM} + f_0(Z)(\partial_\mu \pi(x)) + \text{higher vector mesons}$$

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$\longrightarrow$ we have 4 space directions $x^i$ and $Z$, and one can have a topological soliton with

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In fact, $B = \int_{R^3 \times Z} B^0$ can be shown to reduce to the Skyrmion number upon reduction to 4 dimensions

# Holographic baryon number current

By integrating over 5'th dimension $Z$, one in fact gets 4 dimensional baryon current

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Important : This 4 dimensional current is both gauge invariant and conserved as the one Witten found before

Recall that the 5 dimensional gauge field $A(x,Z)$ contains both pion fields as well as external EM potential in a single framework

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**The holographic baryon current for the above expansion of pions and the $A^{EM}$ reproduces the Witten's result**

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**Any phenomenon that involves these two, such as monopole catalysis, should finds its manifestation within our set-up**

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**Monopole catalysis = Violation of Bianchi identity in 5D by an EM monopole field**

# Details...

- Recall that the external EM potential is encoded in the 5D gauge field as

$$A = QA^{EM} + \psi_+(Z)U^{-1}\partial_\mu U + \cdots$$

Note that $A^{EM}$ is a constant mode along $Z$

- We then put the monopole background to $A^{EM}$ with the violation of Bianchi identity

$$-2\pi i = \int_{S^2} F^{EM} = \int_{S^2} dA^{EM} = \int_{B^3} d^2 A^{EM} \longrightarrow d^2 A^{EM} = -2\pi i \delta_3(\vec{0})$$

- The result is having a string of monopole along $Z$
- In 5D, Bianchi identity is violated by $A^{EM}$

$$DF \equiv dF + A \wedge F - F \wedge A = Qd^2 A^{EM} = -2\pi i \delta_3(\vec{0})$$
$$d\,\mathrm{tr}\,(F \wedge F) = 2\mathrm{tr}(DF \wedge F) = -4\pi i \mathrm{tr}(QF) \wedge \delta_3(\vec{0})$$

In components, this is

$$\partial_\mu \left(\epsilon^{\mu\nu\alpha\beta}\mathrm{tr}(F_{\nu\alpha}F_{\beta Z})\right) = -\frac{1}{4}\partial_Z \left(\epsilon^{\mu\nu\alpha\beta}\mathrm{tr}(F_{\mu\nu}F_{\alpha\beta})\right) + 4\pi i \delta^{(3)}(\vec{x})\mathrm{tr}(QF_{tZ})$$

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$$d\,\mathrm{tr}\,(F \wedge F) = 2\mathrm{tr}(DF \wedge F) = -4\pi i\,\mathrm{tr}(QF) \wedge \delta_3(\vec{0})$$

In components, this is

$$\partial_\mu \left(\epsilon^{\mu\nu\alpha\beta}\mathrm{tr}(F_{\nu\alpha}F_{\beta Z})\right) = -\frac{1}{4}\partial_Z \left(\epsilon^{\mu\nu\alpha\beta}\mathrm{tr}(F_{\mu\nu}F_{\alpha\beta})\right) + 4\pi i \delta^{(3)}(\vec{x})\mathrm{tr}(QF_{tZ})$$

# Details...

- Recall that the external EM potential is encoded in the 5D gauge field as

$$A = QA^{EM} + \psi_+(Z)U^{-1}\partial_\mu U + \cdots$$

Note that $A^{EM}$ is a constant mode along $Z$

- We then put the monopole background to $A^{EM}$ with the violation of Bianchi identity

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By integrating over $Z$, we precisely get the violation of baryon number

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Indeed, by plugging in the mode expansion of 5D gauge field

$$A_\mu = [(U^{-1}(QA^{EM}_\mu + A_{L\mu})U)\psi_+(Z) + (QA^{EM}_\mu + A_{R\mu})\psi_-] + \psi_+ U^{-1}\partial_\mu U + \cdots$$

one finally gets

$$\partial_\mu B^\mu = \mathrm{tr}(Q\sigma^3)\frac{(\partial_t\pi)}{\pi F_\pi}\delta^{(3)}(\vec{x}) - \frac{i\delta^{(3)}(\vec{x})}{2\pi}\mathrm{tr}[QU^{-1}A_{Lt}U - QA_{Rt}]$$

**The first piece is precisely monopole catalysis formula !!!**

# Speculations

The second piece

$$\partial_\mu B^\mu \sim -\frac{i\delta^{(3)}(\vec{x})}{2\pi}\mathrm{tr}[QU^{-1}A_{Lt}U - QA_{Rt}]$$

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Recall that the non-normalizable external potentials $A_{Lt}$ and $A_{Rt}$ couple to the chiral current density

$$J_L^0 A_{Lt} + J_R^0 A_{Rt}$$

so that they represent chemical potential for chiral densities

# Thank you very much

- In summary, monopole catalysis in holographic QCD is simply a violation of Bianchi identity
- The reason for this simplification is that holographic QCD unifies mesons and the external weakly gauged chiral symmetry potential in a single framework of 5D gauge theory
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