

Quarkonium at Finite Temperature

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Outline

Static Quark Correlators At Finite Temperature

- Introduction: Static quark-antiquark correlators
- Problems with extracting triplet
- Numerical results
- Screening function

Meson Correlators and Spectral Functions

- Reconstruction of the Spectral Function

- Test of MEM

- Charmonium

- Zero Temperature

- Finite Temperature

- Bottomonium

- Zero modes contribution

- Potential Models

Introduction

- ▶ Heavy quarkonium in the medium - Matsui-Satz conjecture
- ▶ Temperature dependent potential - Potential models
- ▶ Non-perturbative aspects: Spatial correlation functions of static quarks
 - ▶ melting of the hadronic states in the deconfined phase
 - ▶ color screening in the deconfined phase
 - ▶ quarkonium dissociation
- ▶ Spectral functions
 - ▶ Euclidean meson correlators (time-like)
 - ▶ Reconstruction of SF: MEM

Review: A. Bazavov, P. Petreczky, A.V.; 0904.1748

Basics of lattice gauge theory

The theory is defined by the partition function

$$Z = \int DUD\bar{\psi}D\psi \exp(-S) \quad (1)$$

$$S = S_g + S_f \quad (2)$$

$$S_f = \bar{\psi}M\psi \quad (3)$$

$$S_g = \beta \sum_P \left(1 - \frac{1}{N} \text{Tr} U_P \right), \quad (4)$$

here $\beta = 2N/g^2$.

The expectation value of an operator \hat{O} is given then by

$$\langle \hat{O} \rangle = \frac{1}{Z} \int DUD\bar{\psi}D\psi \hat{O} \exp(-S). \quad (5)$$

Integration over the fermion fields

$$Z = \int DU \det M[U] \exp(-S_g) \equiv \int DU \exp(-S_{\text{eff}}), \quad (6)$$

where $S_{\text{eff}} = S_g - \ln \det M[U]$ is the effective action.

Static quark correlators

A. Bazavov, P. Petreczky, A.V.; Phys.Rev.D78:114026,2008; arXiv:0809.2062
[hep-lat]

Static meson operators:

$$O(x, y; t) = \bar{\psi}(x, t) U(x, y; t) \psi(y, t), \quad (7)$$

$$O^a(x, y; t) = \bar{\psi}(x, t) U(x, x_0; t) T^a U(x_0, y; t) \psi(y, t). \quad (8)$$

Correlators of these operators at $t = 1/T$ (integrating out the static fields):

$$G_1(r, T) = \frac{1}{2} \langle \text{Tr} L^\dagger(x) U(x, y; 0) L(y) U^\dagger(x, y, 1/T) \rangle, \quad (9)$$

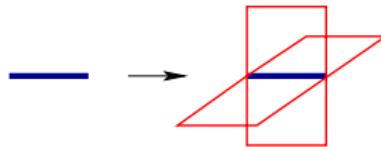
$$\begin{aligned} G_3(r, T) &= \frac{1}{3} \langle \text{Tr} L^\dagger(x) \text{Tr} L(y) \rangle \\ &- \frac{1}{6} \langle \text{Tr} L^\dagger(x) U(x, y; 0) L(y) U^\dagger(x, y, 1/T) \rangle. \end{aligned} \quad (10)$$

Wilson line $L(\vec{x}) = \prod_{t=0}^{N_\tau-1} U_{(\vec{x}, t), 0}$.

Static quark correlators

These correlators:

- depend on the choice of the spatial transporters $U(x, y; t)$,
APE smearing



$$U_{x,\mu} \rightarrow U'_{x,\mu} = (1 - 6c)U_{x,\mu} + c \sum_{\nu \neq \mu} U_{x,\mu} U_{x+\hat{\nu},\mu} U_{x+\hat{\mu},\nu}^\dagger. \quad (11)$$

- in the special gauge, where $U(x, y, z; t) = 1$ give standard definition of the singlet and triplet free energies

$$\exp(-F_1(r, T)/T + C) = \frac{1}{2} \langle \text{Tr} L^\dagger(x) L(y) \rangle, \quad (12)$$

$$\begin{aligned} \exp(-F_3(r, T)/T + C) &= \frac{1}{3} \langle \text{Tr} L^\dagger(x) \text{Tr} L(y) \rangle - \frac{1}{6} \langle \text{Tr} L^\dagger(x) L(y) \rangle, \\ r &= |x - y|. \end{aligned}$$

Static quark-antiquark free energies

The physical free energy of a static quark anti-quark pair is given by the thermal average of the singlet and triplet free energy

$$\begin{aligned} \exp(-F_a(r, T)/T) &= \\ \frac{1}{4} \exp(-F_1(r, T)/T) + \frac{3}{4} \exp(-F_3(r, T)/T) \\ &= \frac{1}{4} \langle \text{Tr}L(r) \text{Tr}L(0) \rangle. \end{aligned} \quad (13)$$

At high temperature in the leading order HTL approximation the singlet and triplet free energies are

$$F_1(r, T) = -\frac{3}{4} \frac{\alpha_s}{r} \exp(-m_D r) - \frac{3}{4} \alpha_s m_D \quad (14)$$

$$F_3(r, T) = +\frac{1}{4} \frac{\alpha_s}{r} \exp(-m_D r) - \frac{3}{4} \alpha_s m_D \quad (15)$$

Static quark-antiquark free energies

Using the transfer matrix one can show that

$$G_1(r, T) = \sum_{n=1}^{\infty} c_n(r) e^{-E_n(r, T)/T}, \quad (16)$$

$$G(r, T) = \langle \text{Tr} L(r) \text{Tr} L(0) \rangle = \sum_{n=1}^{\infty} e^{-E_n(r, T)/T}, \quad (17)$$

where E_n are the energy levels of static quark and anti-quark pair. The coefficients $c_n(r)$ depend on the choice of the transporters U .

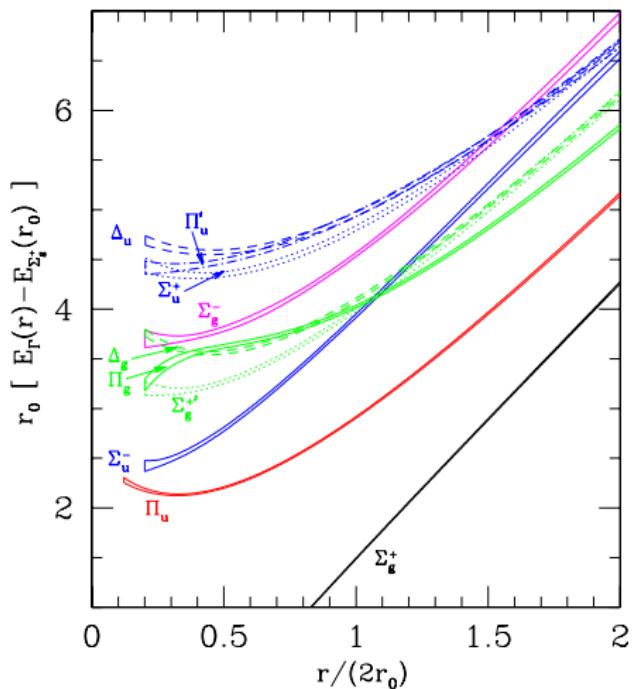
If $c_1 = 1$ the dominant contribution to G_3 would be the 1st excited state E_2 , thus justifying the name singlet and triplet free energy.

In perturbation theory $c_1 = 1$ up to $\mathcal{O}(g^6)$ corrections ¹ and therefore at short distances, $r \ll 1/T$ the color singlet and color averaged free energy are related $F_a(r, T) = F_1(r, T) + T \ln 4$.

Also at small distances, $c_2(r) \sim (r \Lambda_{QCD})^4$.

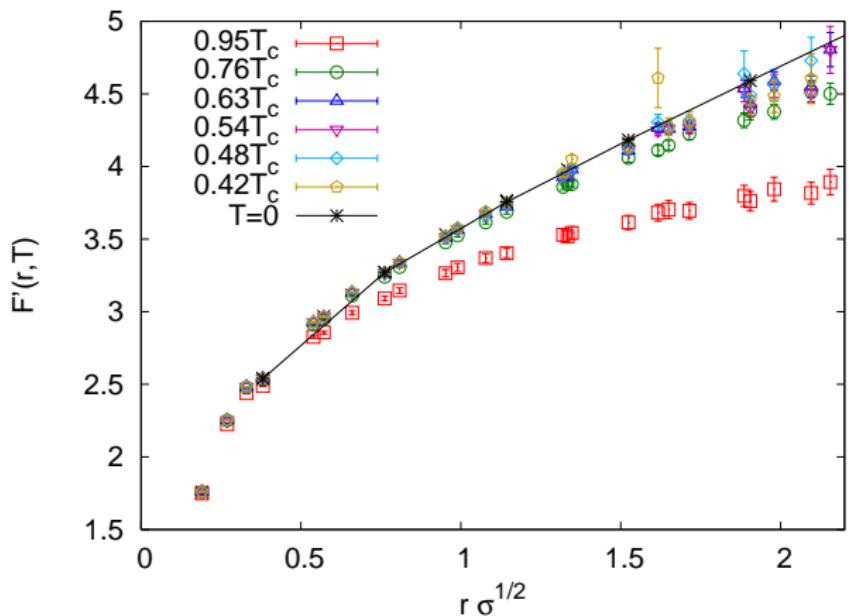
¹N. Brambilla, A. Pineda, J. Soto, A. Vairo, NPB 566, 275 (2000)

Hybrid potentials



C. Michael, hep-lat 9809211

Color averaged free energy at $\beta = 2.5$



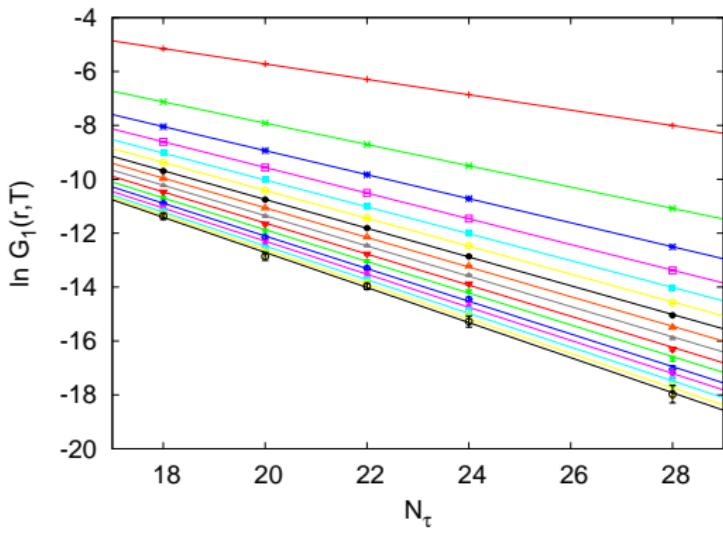
$F'(r, T) = F(r, T) - T \ln 4$. Almost no T -dependence up to $T/T_c = 0.76$.

Extracting c_1

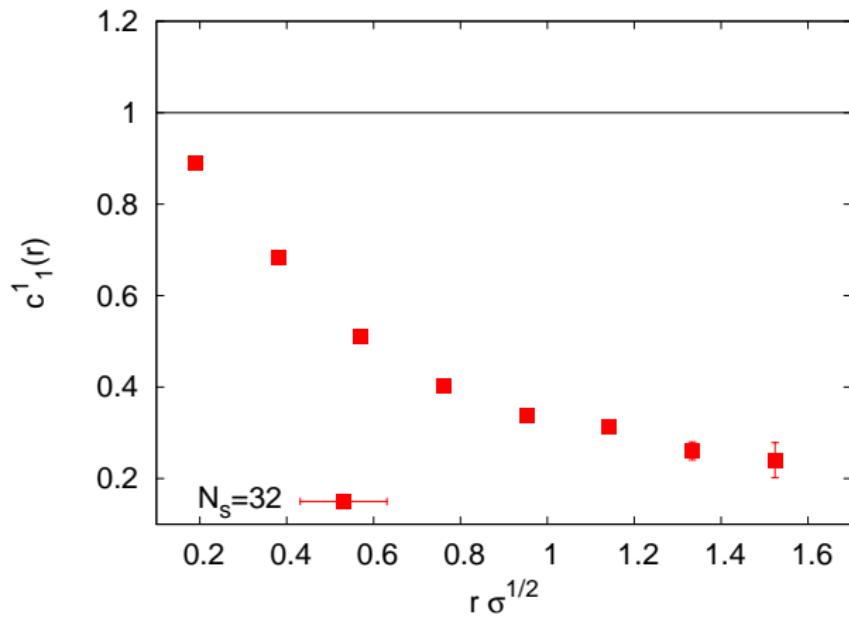
Truncate and fit at fixed r :

$$G_1(r, T) = \sum_{n=1}^{\infty} c_n^1(r) e^{-E_n(r, T)/T}$$

$$G_1(r, T) = c_1^1(r) e^{-a(r) N_\tau}$$



Coefficient $c_1^1(r)$ at $\beta = 2.5$



Problems with extracting the triplet

$$e^{-\tilde{F}_a(r)/T} = G(r, t) = e^{-E_1(r)/T} + e^{-E_2(r)/T},$$

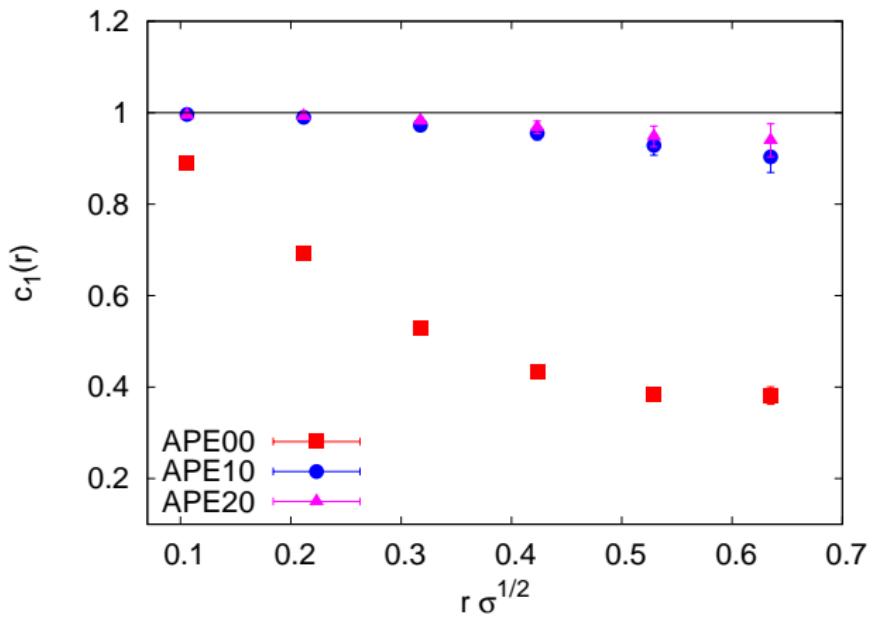
$$e^{-\tilde{F}_1(r)/T} = G_1(r, T) = c_1(r) e^{-E_1(r)/T} + c_2(r) e^{-E_2(r)/T},$$

$$e^{-\tilde{F}_3(r)/T} = (1 - c_1(r)) e^{-E_1(r)/T} + \dots$$

$$F_3(r, T) = E_2(r) - T \ln \left(1 - c_2(r) + \frac{1}{3}(1 - c_1(r)) e^{\Delta E_{12}(r)/T} \right),$$

If $c_1^1 \neq 1$ then $\tilde{F}_3(r, T)$ has a contribution from the singlet!

Coefficient c_1 at $\beta = 2.7$



Singlet free energy

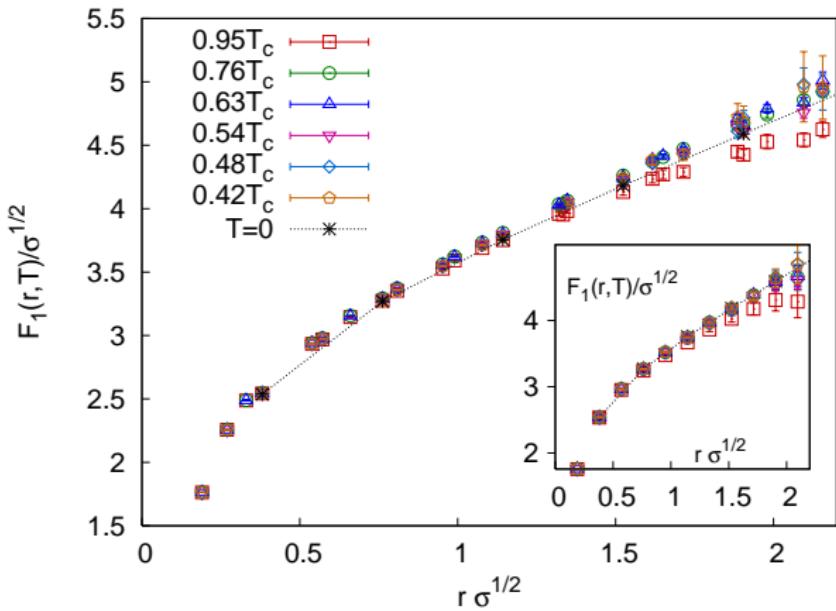


Figure: The color singlet free energy in $SU(2)$ gauge theory below the deconfinement temperature at $\beta = 2.5$ calculated on $32^3 \times N_\tau$ lattices. Also shown is the $T = 0$ potential. The inset shows the color singlet free energy from which the contribution from the matrix element $T \ln c_1$ has been subtracted.

Adjoint free energy

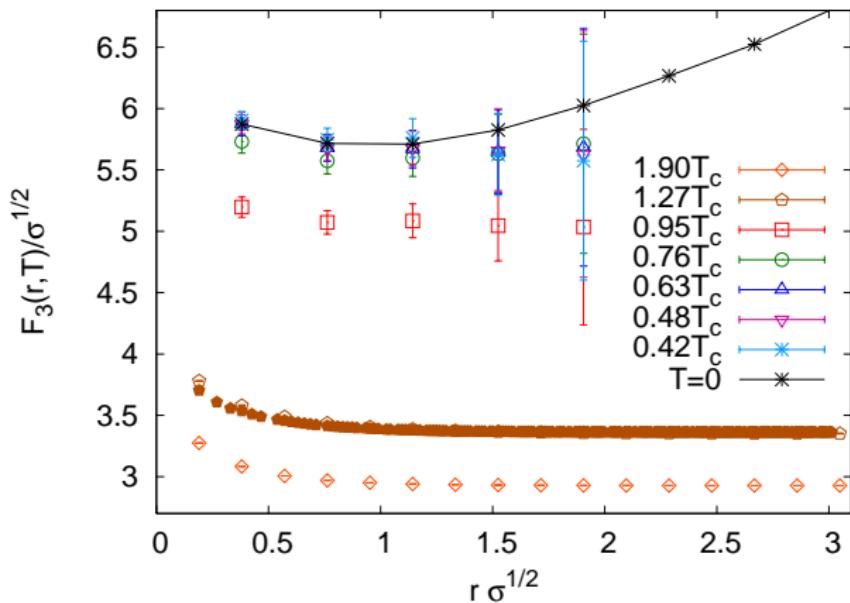


Figure: The triplet free energy at different temperatures calculated at $\beta = 2.5$. The filled symbols correspond to calculations in Coulomb gauge.

Comparison to C. Michael, S.J. Perantonis, J. Phys. G 18, 1725 (1992)

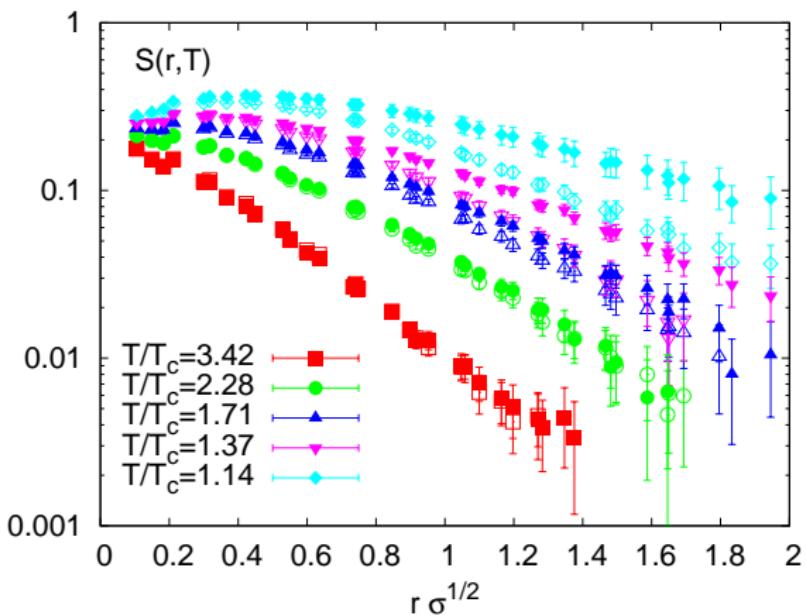
Screening function

$$S(r, T) = (F_\infty(T) - F_1(r, T)) r = -(r T) \ln \frac{\langle \text{Tr} L^\dagger(x) L(y) \rangle}{| \langle L \rangle |^2}$$

Its exponential fall-off is governed by the Debye screening mass:

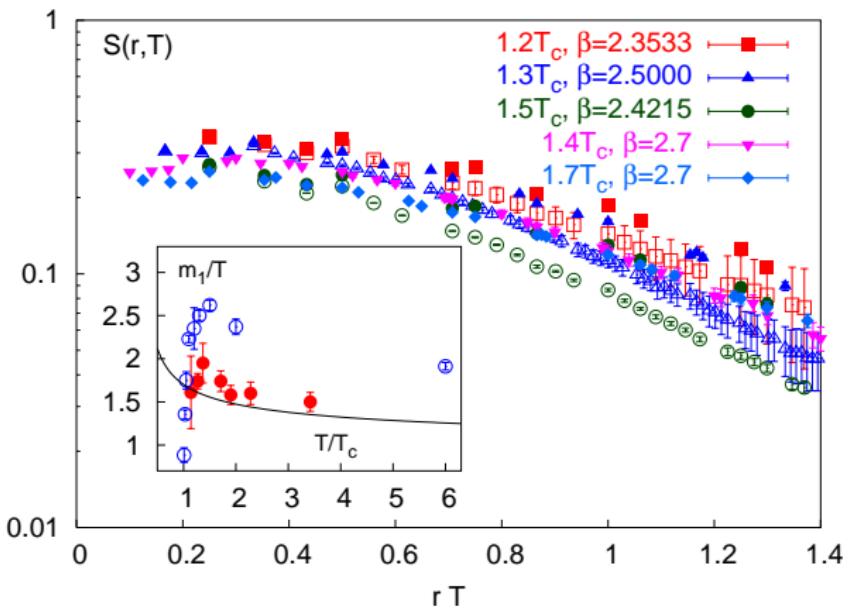
$$\ln S(r, T) \sim -m_D r$$

Screening function



APE10 vs. APE20 at different temperatures

Screening function



Conclusion 1

- ▶ The singlet, triplet and color averaged static meson correlators calculated using different levels of APE smearing
- ▶ APE smearing procedure allows to remove distance dependence from the matrix elements c_1 in the singlet channel thus providing the correct interpretation of the triplet correlator
- ▶ Compared to fixing Coulomb gauge APE smearing procedure offers improvement in assessing triplet free energy contribution of static quark anti-quark pair
- ▶ For higher temperatures dependence on levels of APE smearing vanishes
- ▶ The screening function shows correct exponential fall-off behaviour

Quakronium correlators and spectral functions

P.Petreczky, K. Petrov, A.V.; Phys.Rev.D75:014506,2007; hep-lat/0611017

Point meson operator

$$J_H(t, x) = \bar{q}(t, x)\Gamma_H q(t, x),$$

where $\Gamma_H = 1, \gamma_5, \gamma_\mu, \gamma_5\gamma_\mu, \gamma_\mu\gamma_\nu$.

Meson states in different channels:

Γ	$^{2S+1}L_J$	J^{PC}	$c\bar{c}$ (n=1)	$c\bar{c}$ (n=2)	$b\bar{b}(n=1)$ (n=1)	$b\bar{b}(n=2)$ (n=2)
γ_5	1S_0	0^{-+}	η_c	η'_c	η_b	η'_b
γ_s	3S_1	1^{--}	J/ψ	ψ'	$\Upsilon(1S)$	$\Upsilon(2S)$
$\gamma_s\gamma_{s'}$	1P_1	1^{+-}	h_c		h_b	
1	3P_0	0^{++}	χ_{c0}		$\chi_{b0}(1P)$	$\chi_{b0}(2S)$
	3P_1	1^{++}	χ_{c1}		$\chi_{b1}(1P)$	$\chi_{b1}(2P)$
		2^{++}	χ_{c2}		$\chi_{b2}(1P)$	$\chi_{b2}(2P)$

The spectral function

$$\sigma_H(p_0, \vec{p}) = \frac{1}{2\pi} (D_H^>(p_0, \vec{p}) - D_H^<(p_0, \vec{p})) = \frac{1}{\pi} \text{Im} D_H^R(p_0, \vec{p})$$

$$D_H^{>(<)}(p_0, \vec{p}) = \int \frac{d^4 p}{(2\pi)^4} e^{ip.x} D_H^{>(<)}(x_0, \vec{x}) \quad (18)$$

$$\begin{aligned} D_H^>(x_0, \vec{x}) &= \langle J_H(x_0, \vec{x}), J_H(0, \vec{0}) \rangle \\ D_H^<(x_0, \vec{x}) &= \langle J_H(0, \vec{0}), J_H(x_0, \vec{x}) \rangle, x_0 > 0 \end{aligned} \quad (19)$$

The Euclidean propagator

$$G_H(\tau, \vec{p}) = \int d^3 x e^{i\vec{p}.\vec{x}} \langle T_\tau J_H(\tau, \vec{x}) J_H(0, \vec{0}) \rangle$$

is related to the spectral function through the integral representation

$$G(\tau, \vec{p}) = \int_0^\infty d\omega \sigma(\omega, \vec{p}) K(\omega, \tau), \quad K(\omega, \tau) = \frac{\cosh(\omega(\tau - 1/2T))}{\sinh(\omega/2T)}.$$

Reconstruction of the Spectral Function

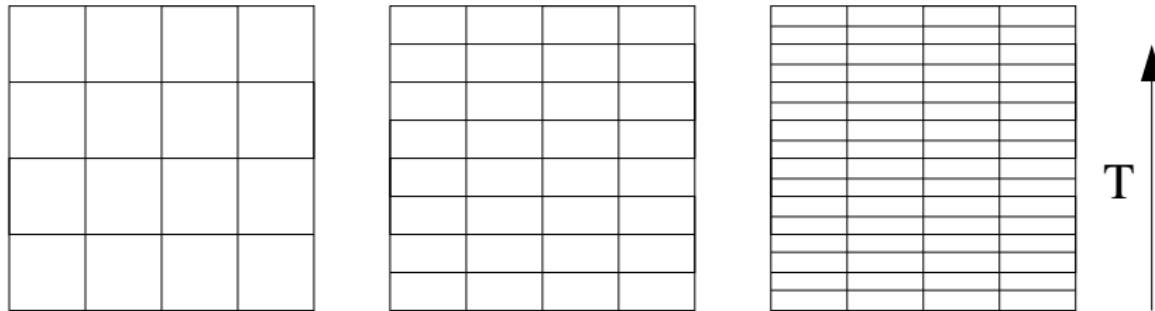
- ▶ $G(\tau, \vec{p}) = \int_0^\infty d\omega \sigma(\omega, \vec{p}, T) K(\omega, \tau, T)$
- ▶ $O(10)$ data and $O(100)$ degrees of freedom to reconstruct.
- ▶ Bayesian technique: find $\sigma(\omega, T)$ that maximizes $P[\sigma | DH]$.
 - ▶ D data
 - ▶ H prior knowledge: $\sigma(\omega, T) > 0$

Maximum Entropy Method: Asakawa, Hatsuda, Nakahara, PRD 60 (99) 091503, Prog. Part. Nucl. Phys. 46 (01) 459

$$P[\sigma | DH] = \exp\left(-\frac{1}{2}\chi^2 + \alpha S\right) \quad (20)$$

Shannon-Janes entropy: $S = \int d\omega \left[\sigma(\omega) - m(\omega) - \sigma(\omega) \ln\left(\frac{\sigma(\omega)}{m(\omega)}\right) \right]$, $m(\omega)$ - the default model, $m(\omega \gg \Lambda_{QCD}) = m_0 \omega^2$ - perturbation theory.

Anisotropic lattice $\xi = a_s/a_\tau = 2$ and 4.



Standard Wilson action in the gauge sector and the anisotropic clover improved action for heavy fermions. Quenched approximation. Sommer scale to fix the physical units.

Model spectral functions

- ▶ Dirac's delta function

$$\hat{\sigma}(\omega) = \frac{3}{2\pi} R_1 \delta(\omega - m)$$

- ▶ Breit-Wigner

$$\hat{\sigma}(\omega) = \frac{3}{2\pi} R_1 \frac{1}{\pi} \frac{\gamma}{(\omega - m)^2 + \gamma^2} \frac{\omega^3}{m^3},$$

- ▶ + continuum

$$\sigma(\omega) = \hat{\sigma}(\omega) + \Theta(\omega - s_0) m_0 \omega^2$$

Model spectral function is integrated with the kernel to produce the propagator. The covariance matrix is taken from the lattice simulation ($\beta = 6.5$, $\xi = 4$, $N_t = 160$)

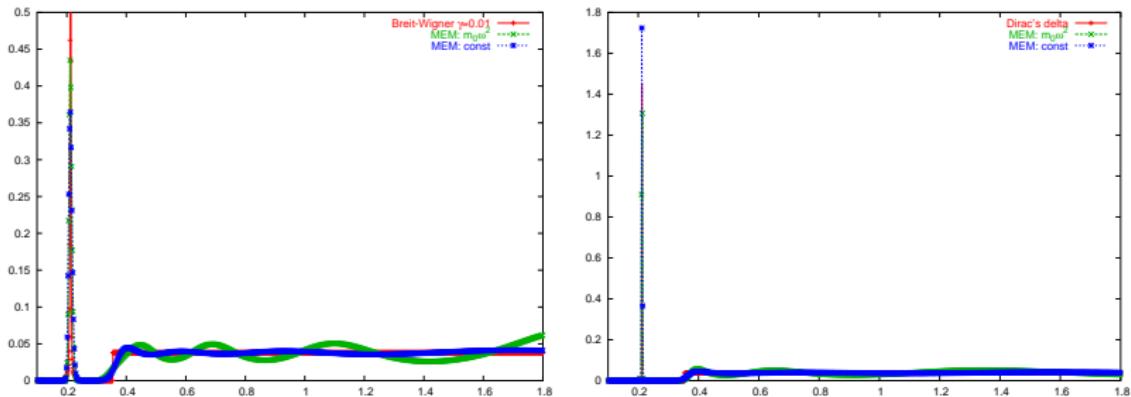


Figure: The $\gamma = 0.01$ Breit-Wigner (left) and Dirac's delta (right) spectral function reconstruction.

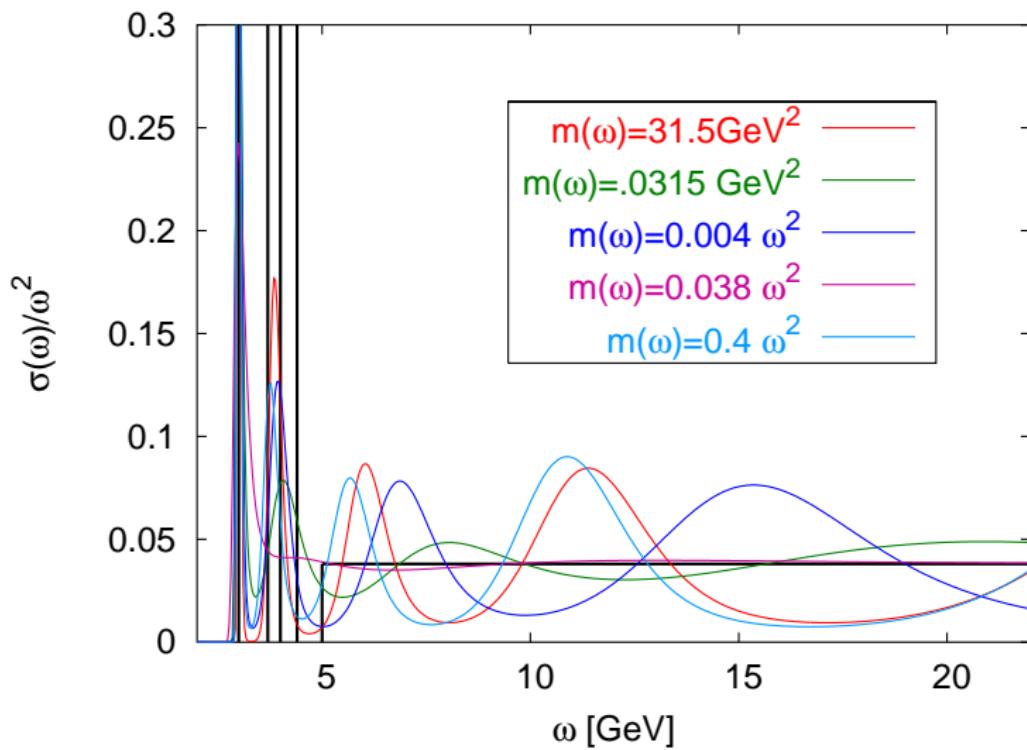


Figure: The Dirac's delta (4 state) spectral function reconstruction.

Charmonium: $T = 0$

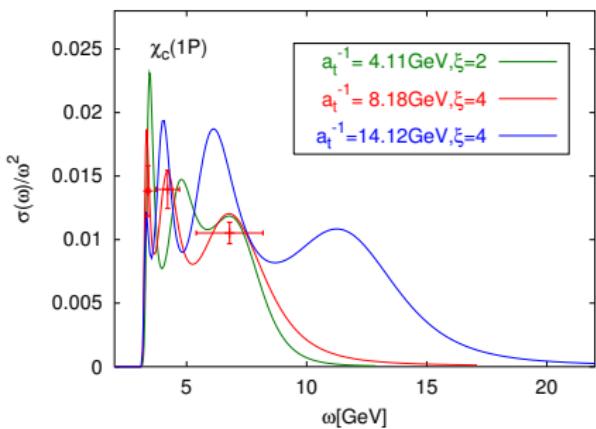
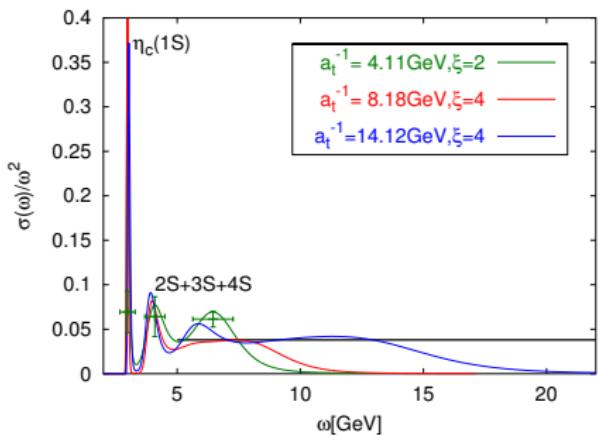


Figure: Charmonium spectral function in the pseudo-scalar channel (left) and the scalar channel (right) at different lattice spacings and zero temperature.

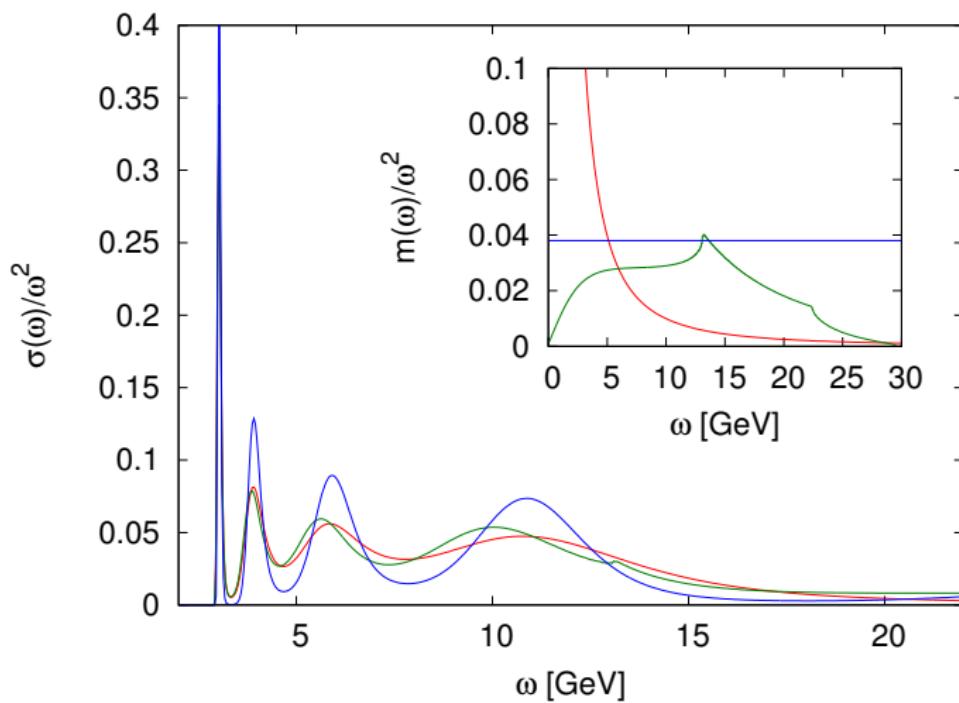


Figure: Charmonium spectral function dependence on the default model. Pseudo-scalar channel at $a_t^{-1} = 14.12 \text{ GeV}$ and zero temperature.

Charmonium: $T > 0$

$$G_{recon}(\tau, T) = \int_0^{\infty} d\omega \sigma(\omega, T=0) K(\tau, \omega, T)$$

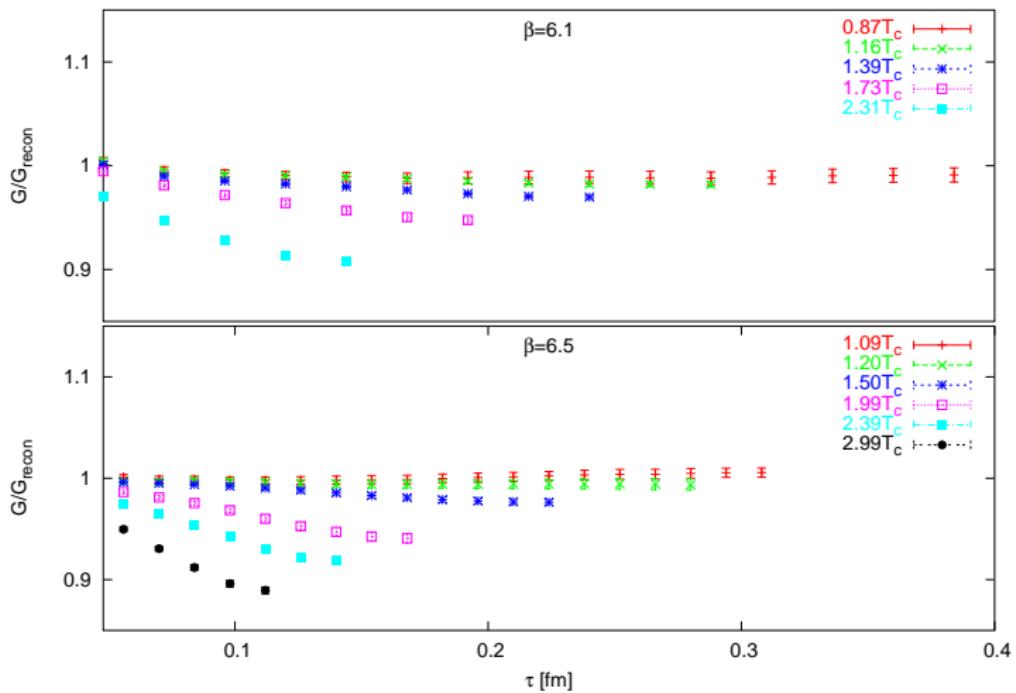


Figure: The ratio $G(\tau, T)/G_{\text{recon}}(\tau, T)$ of charmonium for the pseudo-scalar channel at $a_t^{-2} = 8.18$ and 14.11 GeV at different temperatures.

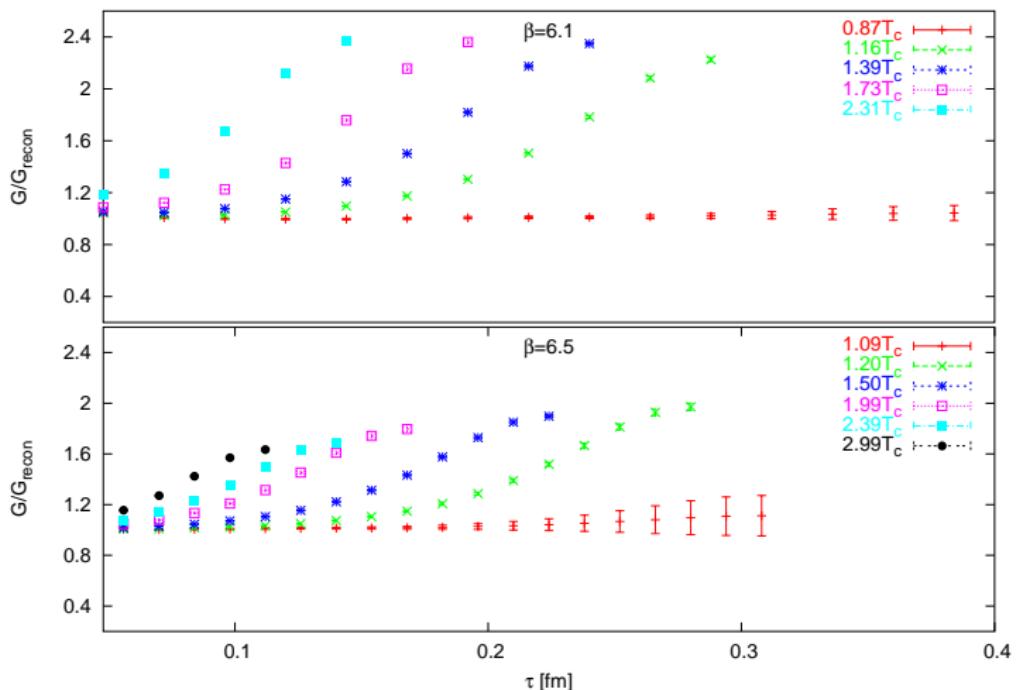


Figure: The ratio $G(\tau, T)/G_{\text{recon}}(\tau, T)$ of charmonium for the scalar channel at $a_t^{-2} = 8.18$ and 14.11 GeV at different temperatures.

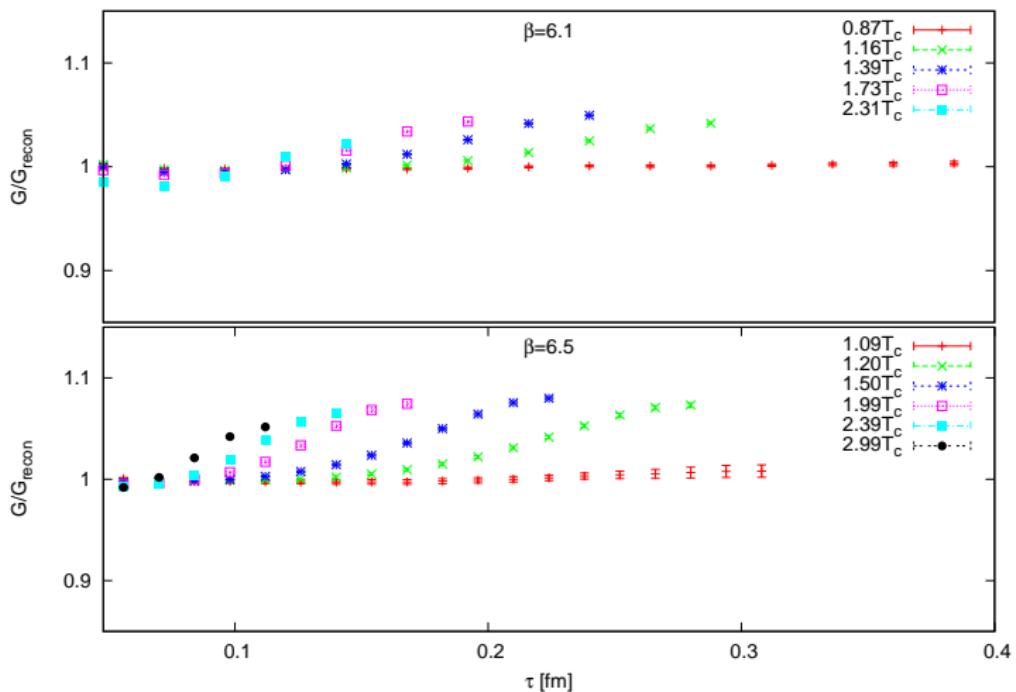


Figure: The ratio $G(\tau, T)/G_{\text{recon}}(\tau, T)$ of charmonium for the vector channel at $a_t^{-2} = 8.18$ and 14.11 GeV at different temperatures.

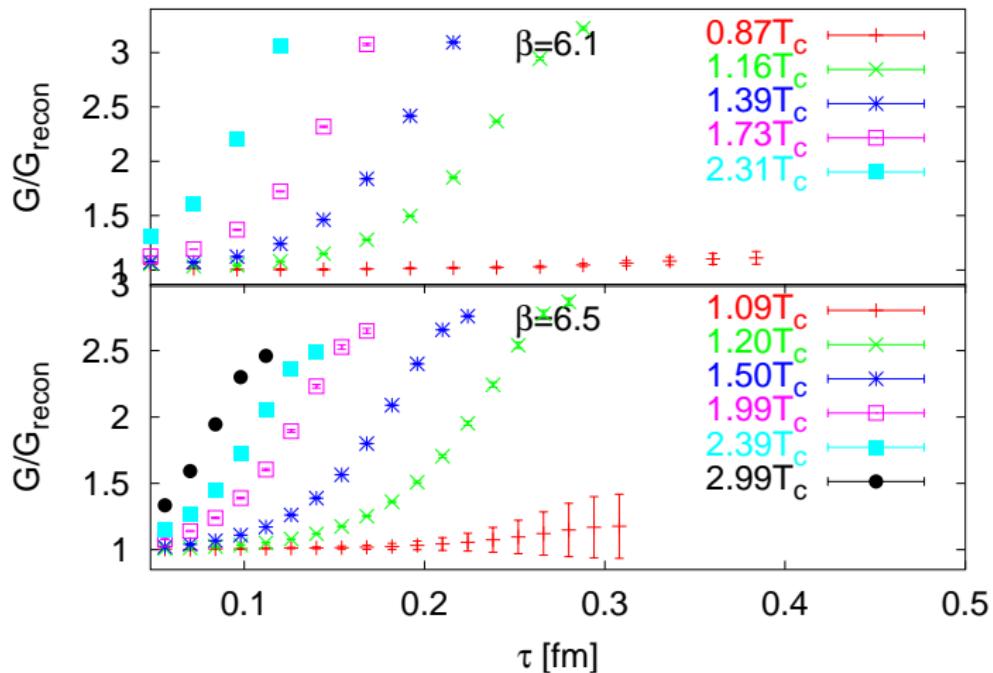


Figure: The ratio $G(\tau, T)/G_{\text{recon}}(\tau, T)$ of charmonium for the axial vector channel at $a_t^{-2} = 8.18$ and 14.11 GeV at different temperatures.

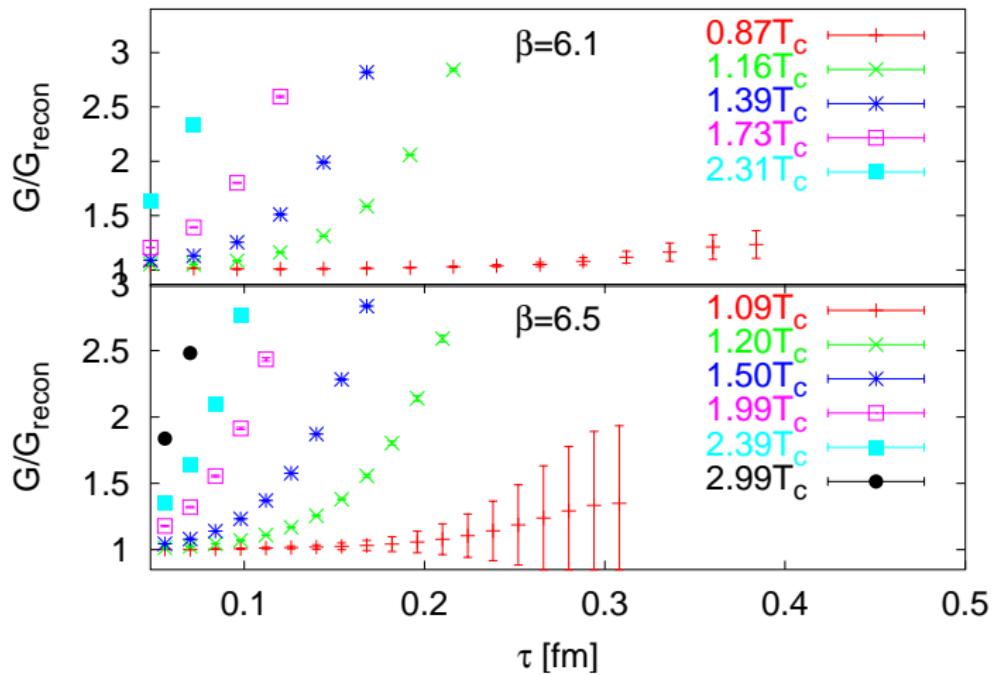


Figure: The ratio $G(\tau, T)/G_{\text{recon}}(\tau, T)$ of charmonium for the tensor channel at $a_t^{-2} = 8.18$ and 14.11 GeV at different temperatures.

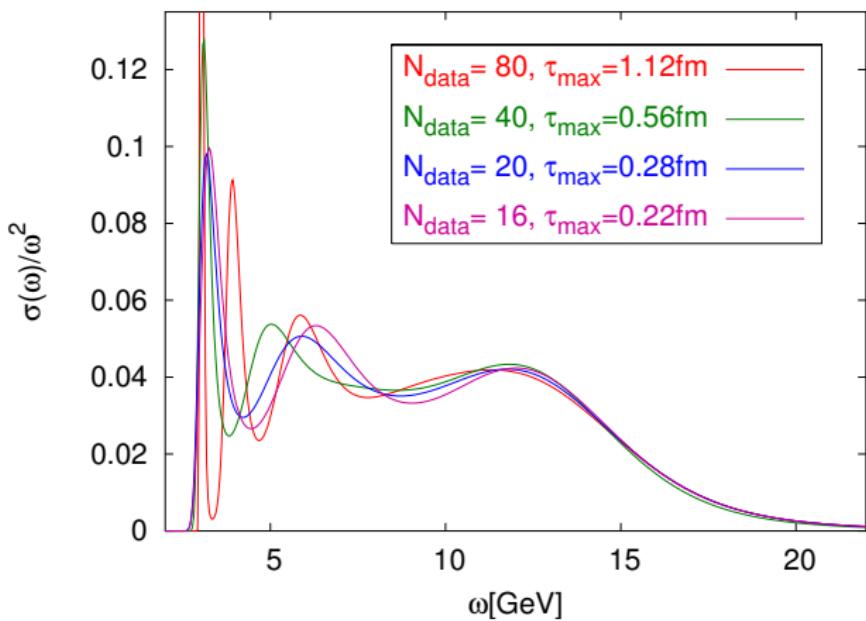


Figure: The dependence of the reconstructed pseudo-scalar spectral function on the maximal temporal extent for $\beta = 6.5$. In the analysis the default model $m(\omega) = 1$ has been used.

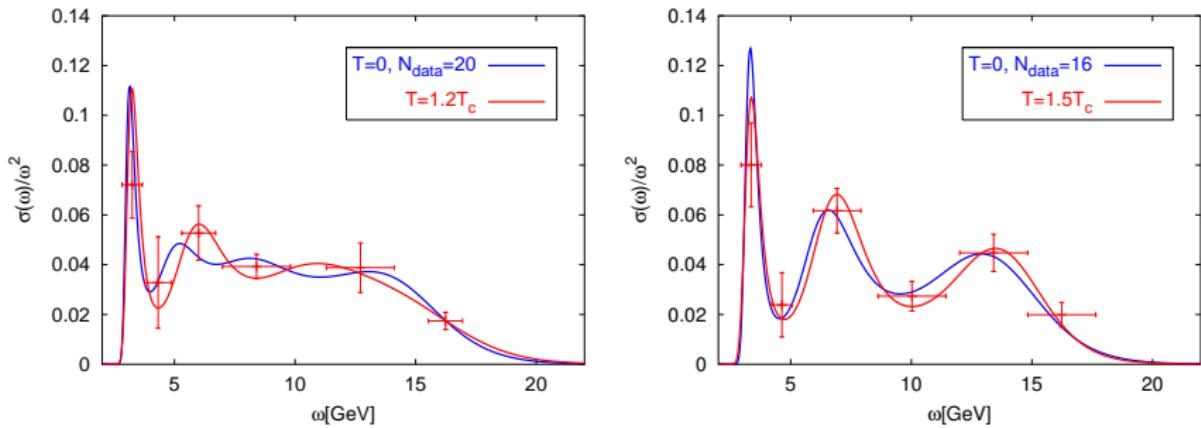


Figure: Charmonium spectral functions in the pseudo-scalar channel at $a_t^{-2} = 14.11 \text{ GeV}$ at zero and above deconfinement temperatures.

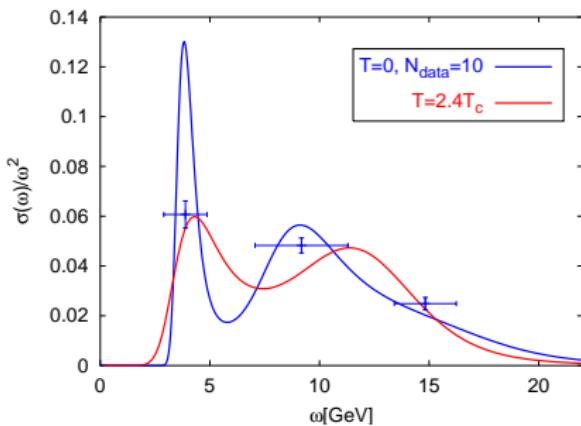


Figure: Charmonium spectral functions in the pseudoscalar channel at finite temperature.

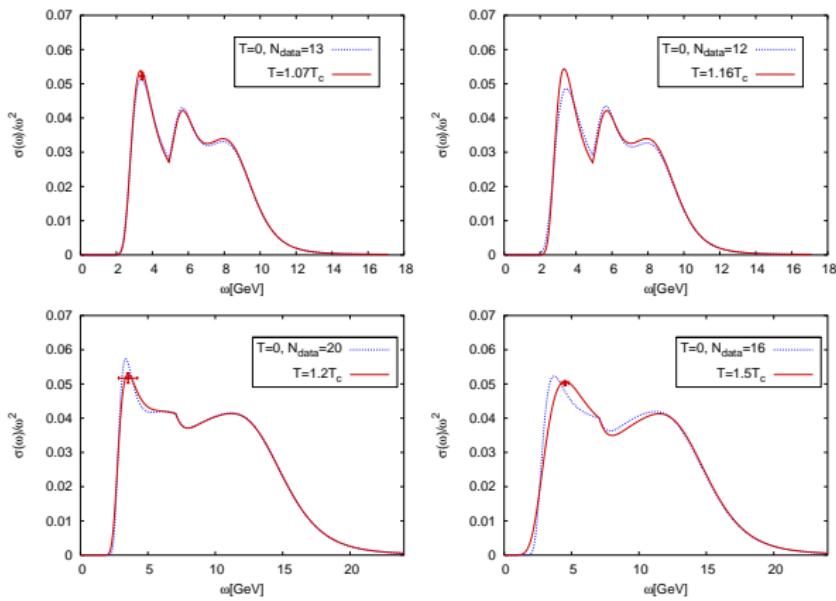


Figure: The pseudo-scalar spectral function at different temperatures together with the zero temperature spectral functions reconstructed using default model coming from the high energy part of the zero temperature spectral function.

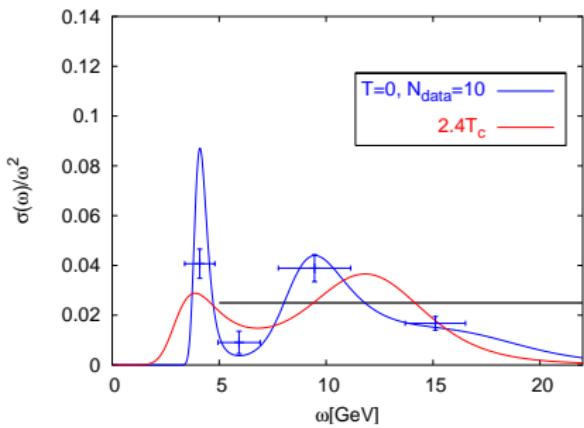
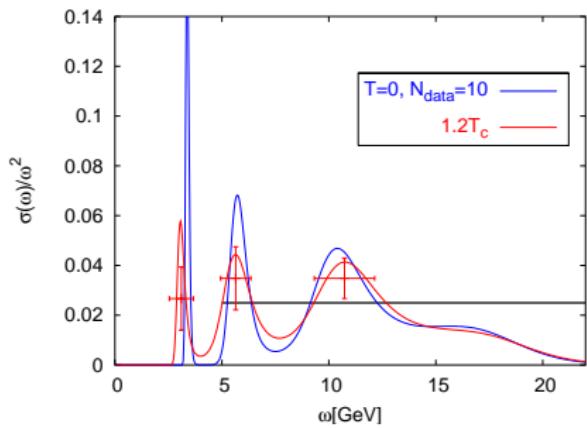


Figure: Charmonium spectral functions in the vector channel at finite temperature.

Bottomonium

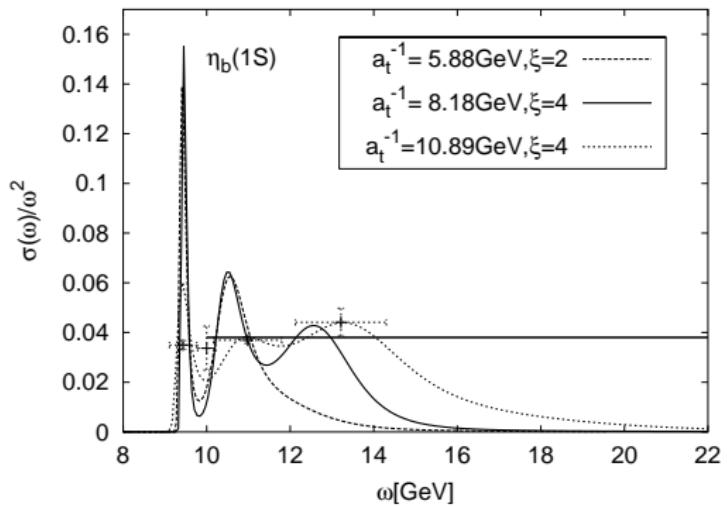


Figure: The pseudo-scalar bottomonium spectral function at zero temperature for different lattice spacings.

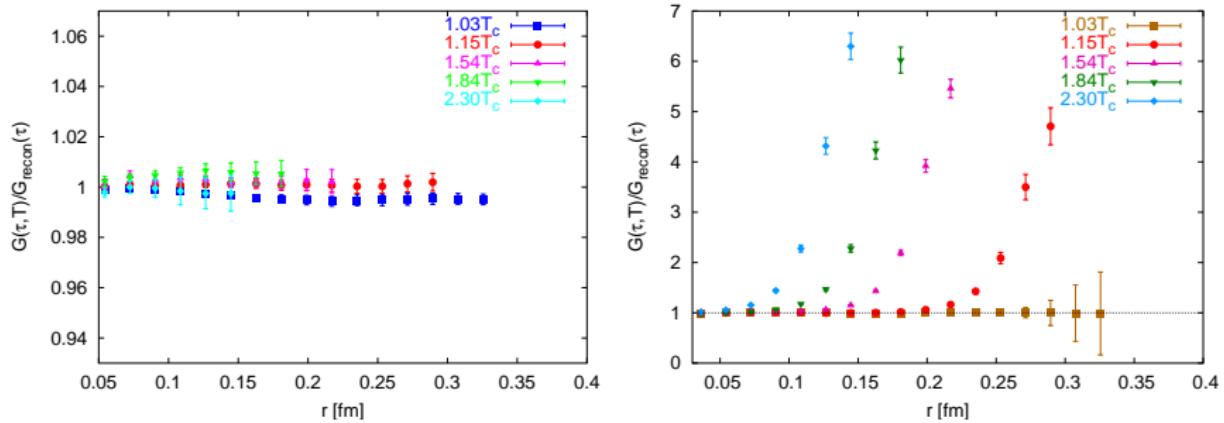


Figure: Bottomonia correlators in pseudo-scalar (left) and scalar (right) channels for different temperatures.

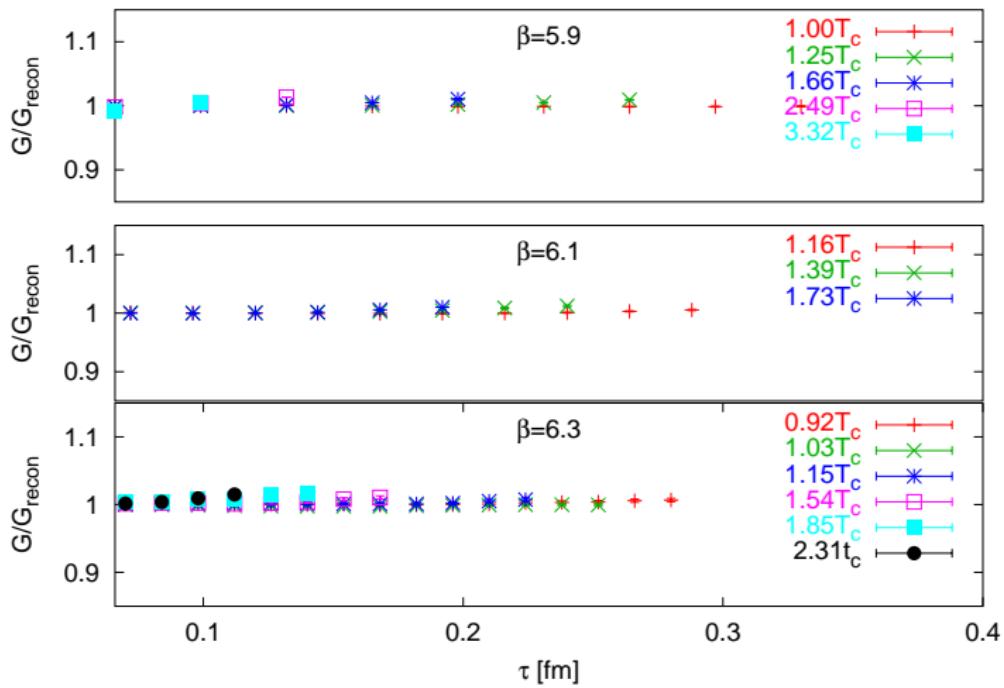


Figure: Bottomonia correlators for vector channel for different temperatures and lattices.

Zero modes

Finite temperature quarkonium spectral functions:

- ▶ quark anti-quark pair: ($\omega > 2m$)
- ▶ scattering of the external probe off a heavy quark from the medium: ($\omega < k$), as $k \rightarrow 0 \rightarrow \chi^i(T)\omega\delta(\omega)$ in the free theory.

$$\sigma^i(\omega, T) = \sigma_{\text{high}}^i(\omega, T) + \sigma_{\text{low}}^i(\omega, T). \quad (21)$$

$$G^i(\tau, T) = G_{\text{high}}^i(\tau, T) + G_{\text{low}}^i(\tau, T). \quad (22)$$

To a very good approximation $G_{\text{low}}^i(\tau, T) = \chi^i(T)T$, i.e. constant.

$$G'(\tau, T)/G'_{\text{recon}}(\tau, T)$$

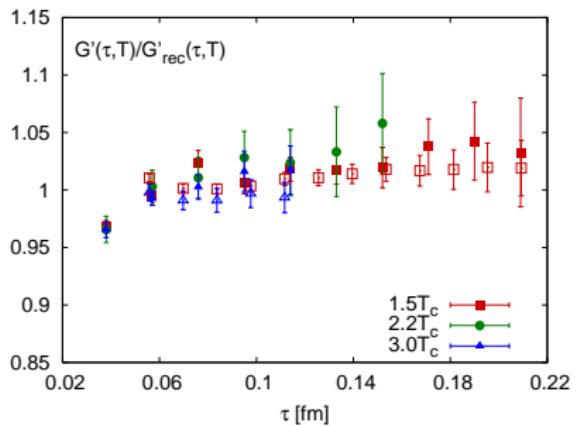
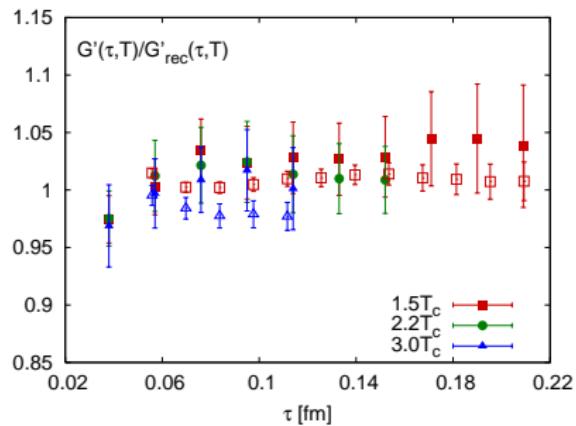


Figure: The ratio of the derivatives $G'(\tau, T) / G'_{rec}(\tau, T)$ in the scalar channel (left) and axial-vector channel (right) calculated at $\beta = 7.192$.

Potential Models

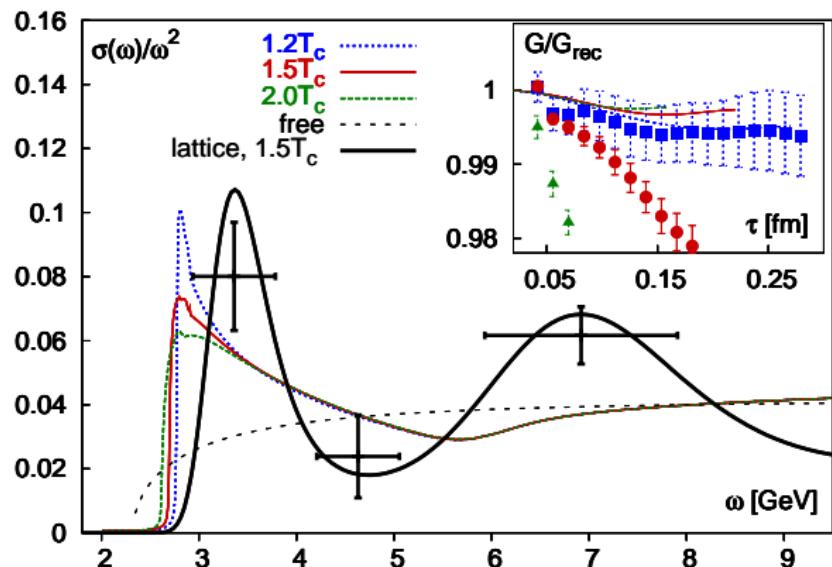
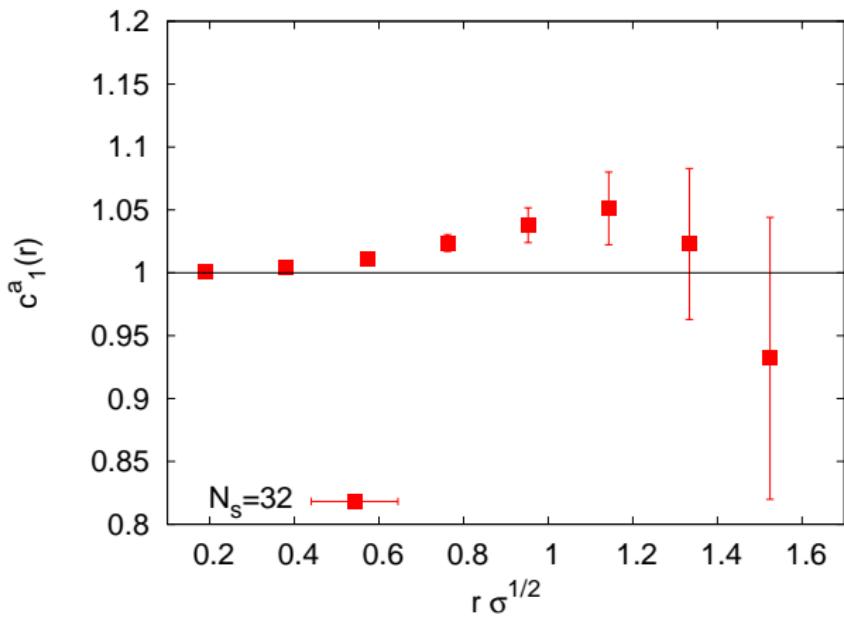


Figure: The charmonium spectral functions at different temperatures (S channel).

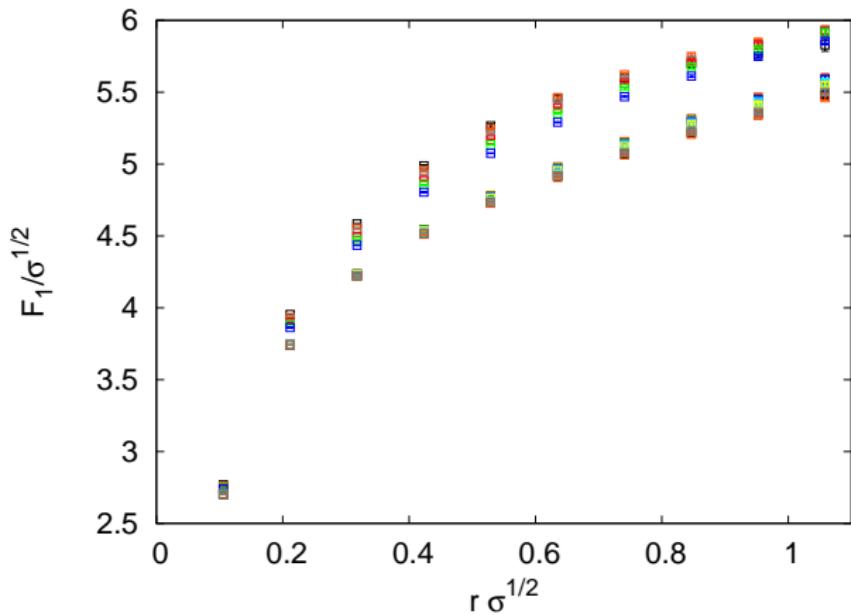
Conclusion 2

- ▶ MEM can resolve finite width bound states.
- ▶ The $1S$ (η_c , J/ψ) charmonium correlators do not change in the deconfined phase up to $T \simeq 1.5 T_c$.
- ▶ The $1S$ spectral function at this temperature within precision of MEM corresponds to zero temperature spectral function.
- ▶ $1P$ (χ_{c0} , χ_{c1}) charmonium correlators and spectral function show significant change at $1.1 T_c$.
- ▶ Bottomonium states show similar behavior.
- ▶ Zero modes account on the change of the $1P$ correlators.
- ▶ Potential models indicate melting of quarkonia (while preserving the correlators).

Coefficient c_1^a at $\beta = 2.5$

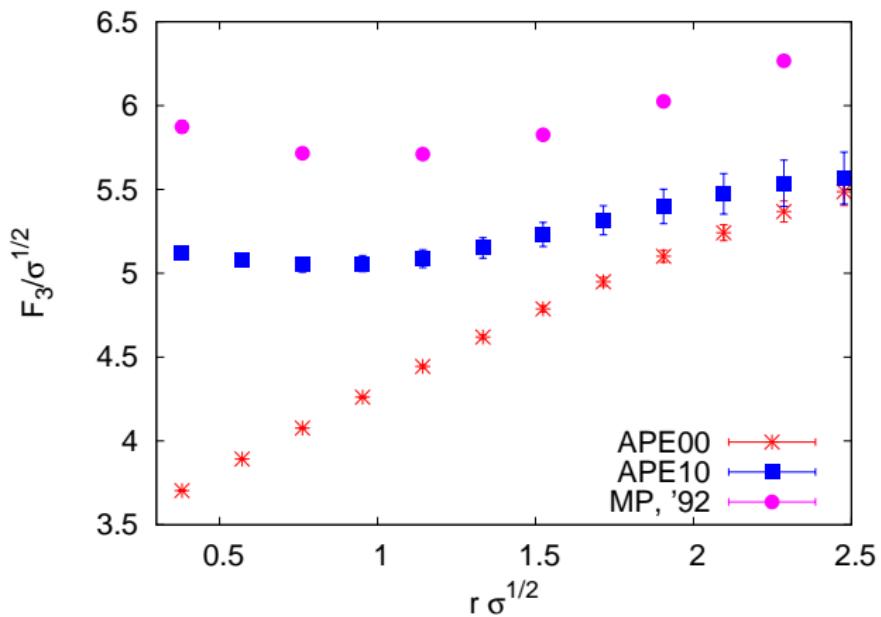


Singlet free energy



T -dependence is greatly reduced with APE10, 20 vs. APE0 (upper curve),
 $T/T_c = 0.49, 0.57, 0.62, 0.68, 0.76, 0.86$.

Triplet free energy



Great improvement with APE10 vs. APE0, $T/T_c = 0.76$.