Dyson-Schwinger Equations and Observables

Craig D. Roberts

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Fachbereich Physik, Universität Rostock

- *Dyson-Schwinger Equations: Density, Temperature and Continuum Strong QCD*
  C.D. Roberts and S.M. Schmidt, nu-th/0005064, Prog. Part. Nucl. Phys. **45** (2000) S1
- *The IR behavior of QCD Green's functions: Confinement, DCSB, and hadrons . . .*
  R. Alkofer and L. von Smekal, he-ph/0007355, Phys. Rept. **353**, 281 (2001)
- *Dyson-Schwinger equations: A Tool for Hadron Physics*
  P. Maris and C.D. Roberts, nu-th/0301049

ANL Physics Division

Dyson-Schwinger Equations

Dyson-Schwinger Equations

- NonPerturbative, Continuum approach to QCD

Dyson-Schwinger Equations

- NonPerturbative, Continuum approach to QCD
- Simplest level: Generating Tool for Perturbation Theory

  ..... Materially Reduces Model Dependence

Dyson-Schwinger Equations

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  - Hadrons as Composites of Quarks and Gluons

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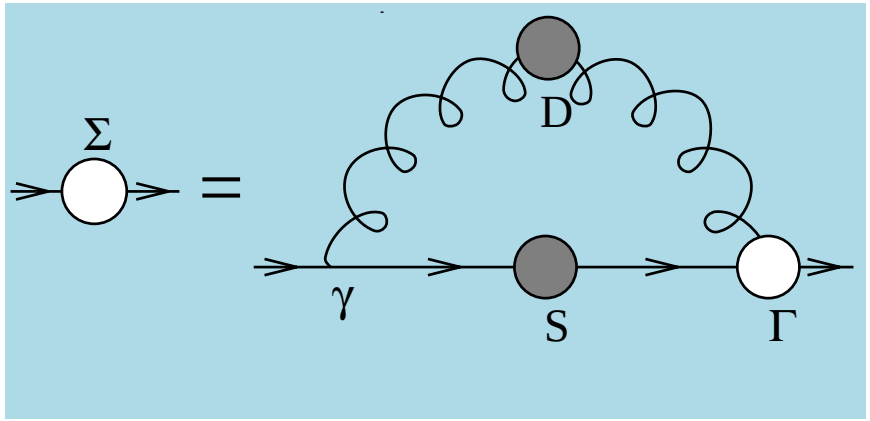
Cross-Sections built from Schwinger Functions

Persistent Challenge

Persistent Challenge

- Infinitely Many Coupled Equations



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    Not useful for the nonperturbative problems in which we're interested

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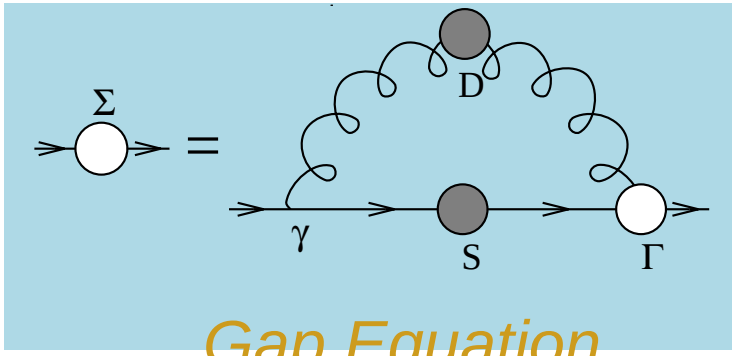
Describe this

ANL Physics Division

Dressed-quark Propagator

Dressed-quark Propagator

$$S(p) = \frac{Z(p^2)}{i\gamma \cdot p + M(p^2)}$$



Gap Equation

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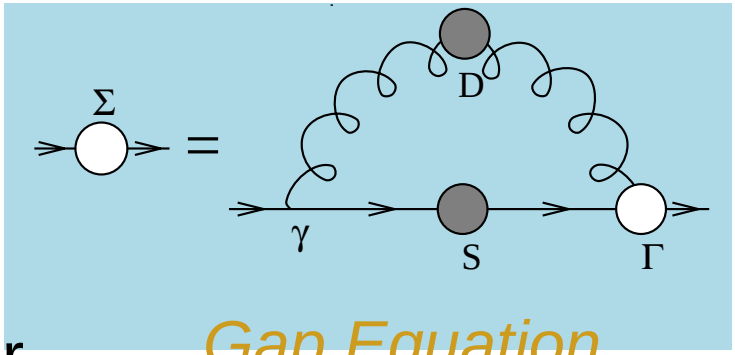

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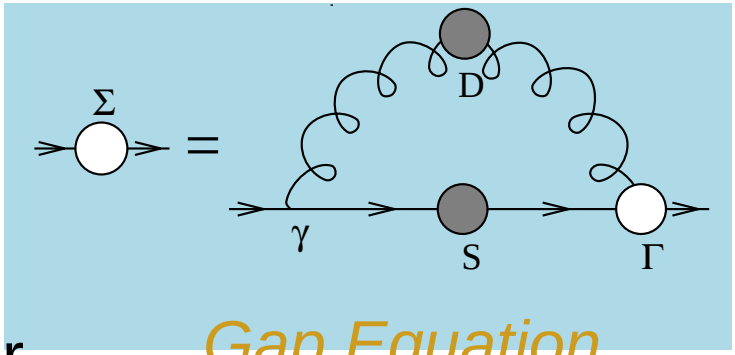

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- Weak Coupling Expansion

  Reproduces Every Diagram in Perturbation Theory

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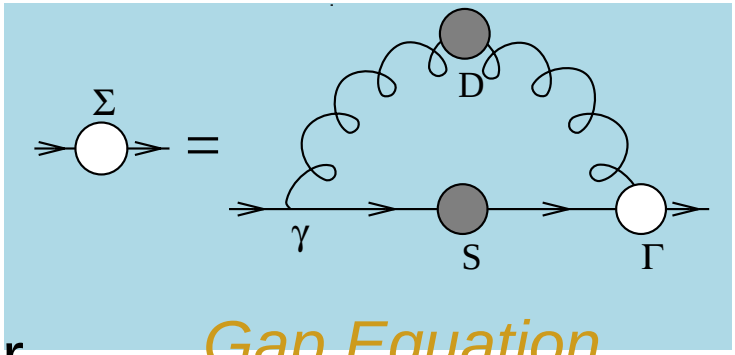

Gap Equation

- dressed-quark propagator

$$S(p) = \frac{1}{i\gamma \cdot p\, A(p^2) + B(p^2)}$$

- But in Perturbation Theory

$$B(p^2) = m\left(1 - \frac{\alpha}{\pi}\ln\left[\frac{p^2}{m^2}\right] + \ldots\right) \stackrel{m\to 0}{\longrightarrow} 0$$

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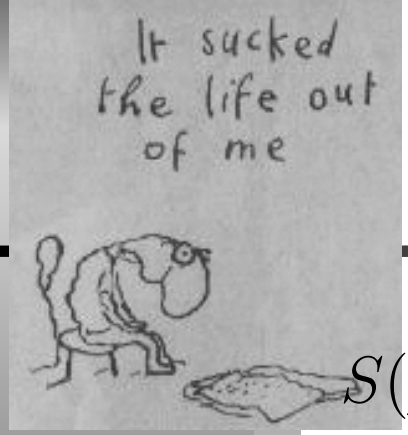

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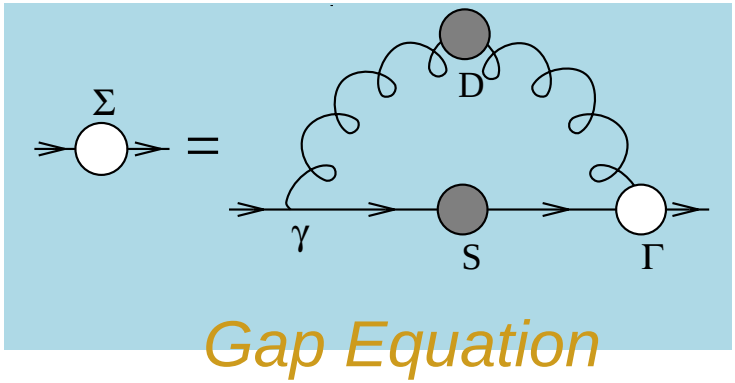
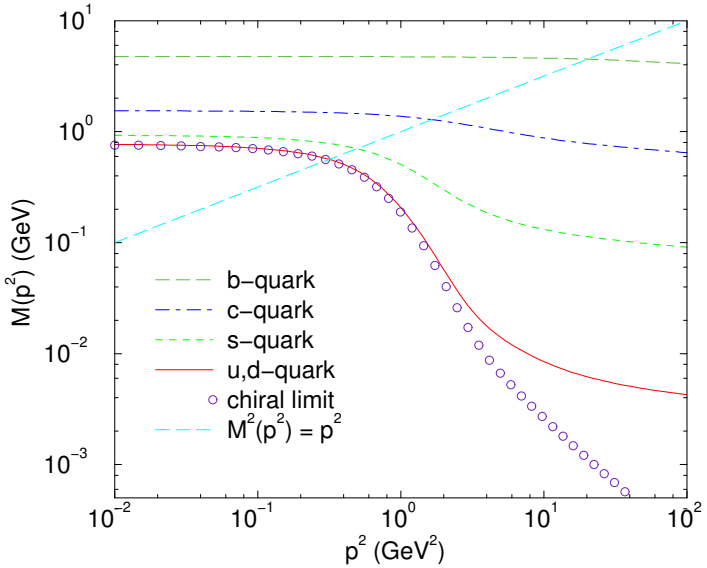
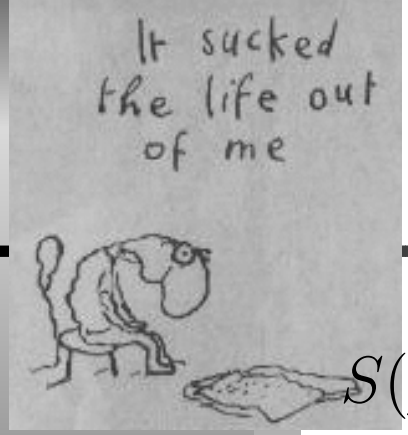


- Enhancement of Gap Equation's Kernel on IR domain

  $\Rightarrow$ IR Enhancement of $M(p^2)$





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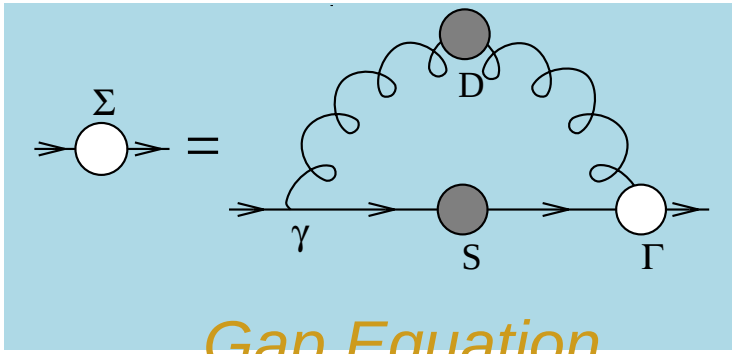
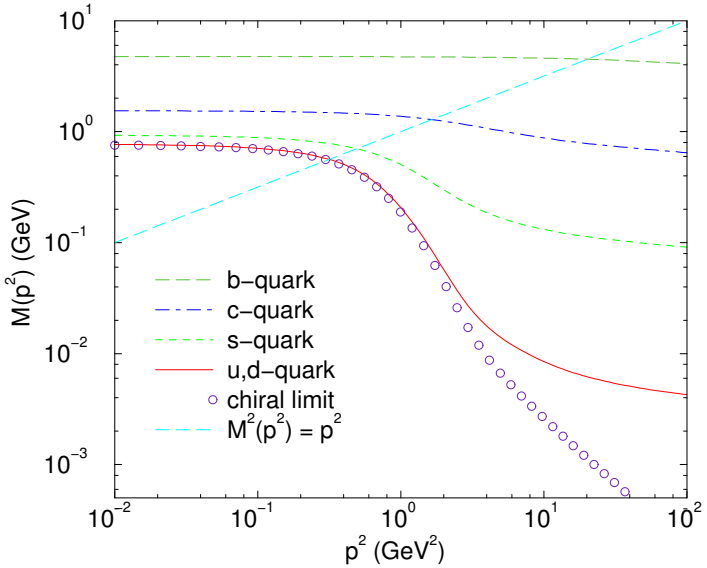
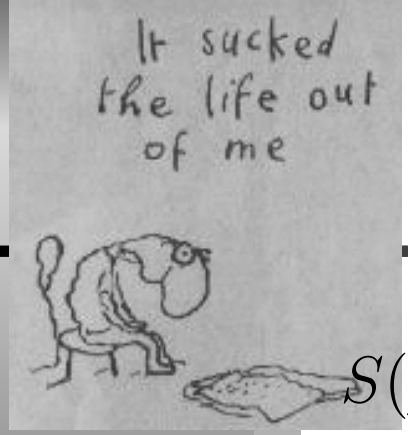


Gap Equation

- Enhancement of Gap Equation's Kernel on IR domain
  $\Rightarrow$ IR Enhancement of $M(p^2)$
- Euclidean Constituent–Quark Mass: M_f^E: $p^2 = M(p^2)^2$





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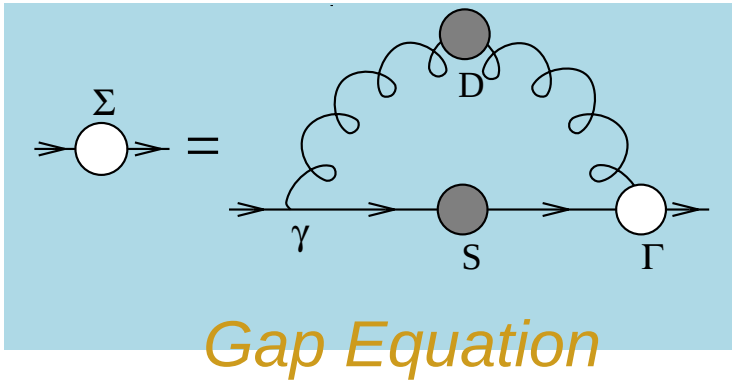

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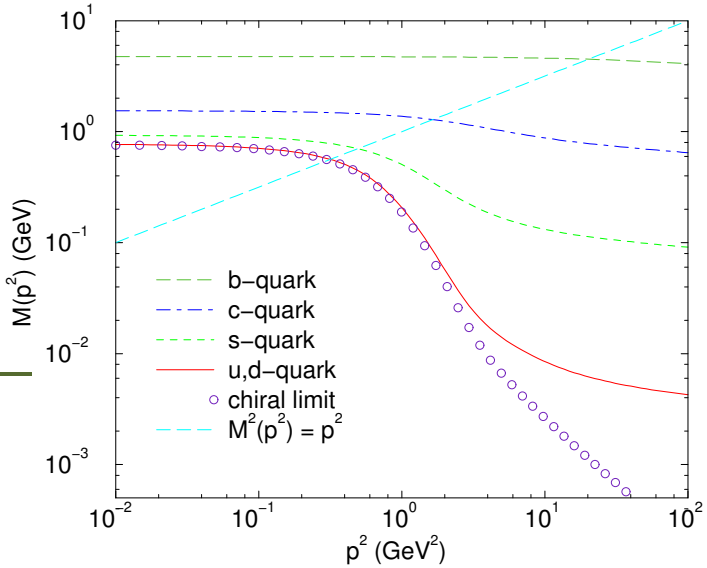
flavour	u/d	s	c	b	t
$\frac{M^E}{m_{\mu\sim 20\,\mathrm{GeV}}}$	$\sim 10^2$	~ 10	1.4	1.1	$\to 1$
			-2.2	-1.2	



Vacuum quark condensate

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Trace of the Chiral-limit dressed-quark propagator

Vacuum quark condensate

$$-\langle \bar{q}q \rangle^0_\zeta \Big|_{\zeta=19\,\mathrm{GeV}} =$$

$$Z_4(\zeta,\Lambda)\;\; N_c \int_q^\Lambda \mathrm{tr}_{\mathrm{Dirac}}\left[S_{\hat{m}=0}(q)\right] = \;\; (0.275\,\mathrm{GeV})^3$$

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- Mass renormalisation constant

  Ensures finiteness and correct renormalisation group flow

  $m(\zeta)\,\langle \bar{q}q \rangle^0_\zeta =$ const.

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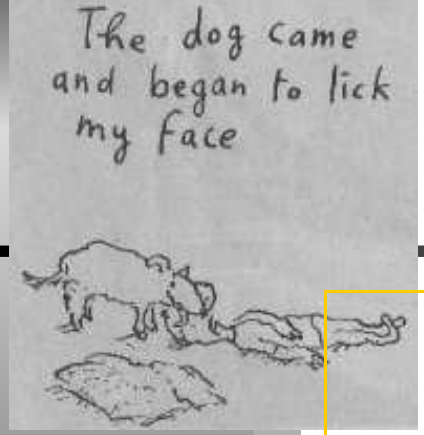
  AND ensures Gauge Invariance



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Numerical Value

π and K observables

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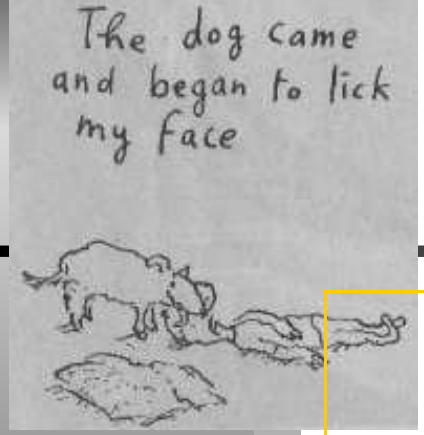

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Corresponds to

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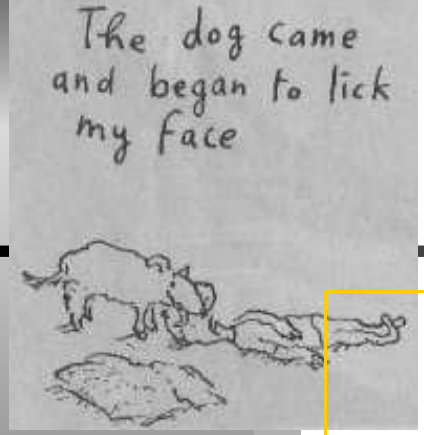

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Close packed spheres: $1.8\,\text{fm}^{-3}$

$$\Rightarrow V_{\langle\bar{q}q\rangle} = 0.55\,\mathrm{fm}^3 \Rightarrow r_{\langle\bar{q}q\rangle} = 0.51\,\text{fm} = 0.77\,r_\pi = 0.58\,r_p$$

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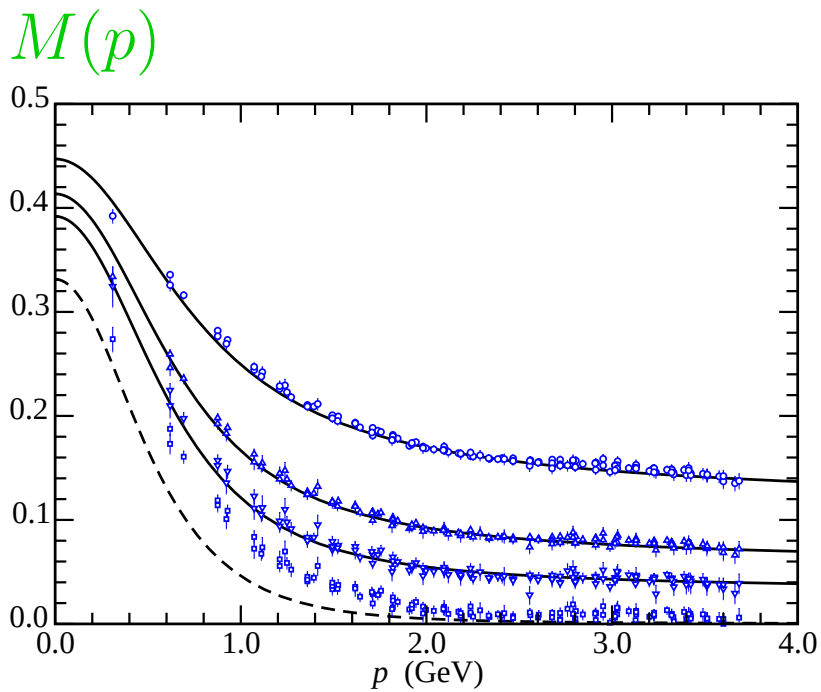
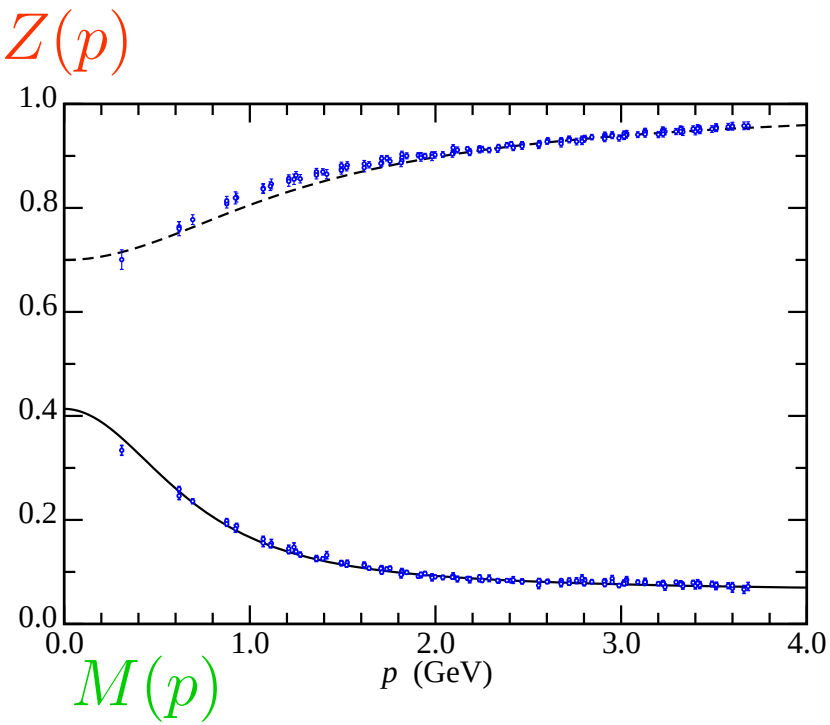
cf. Quenched lattice simulations





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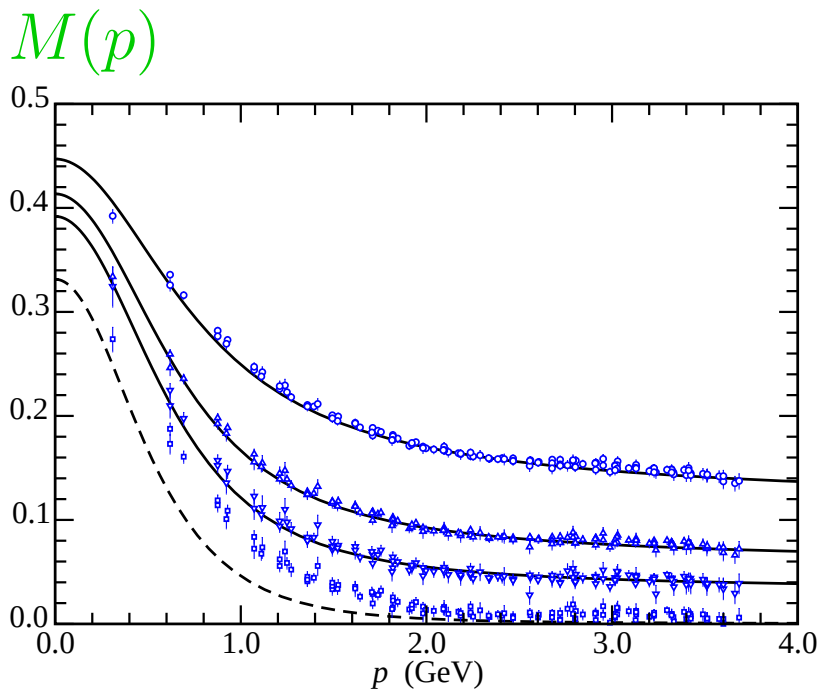

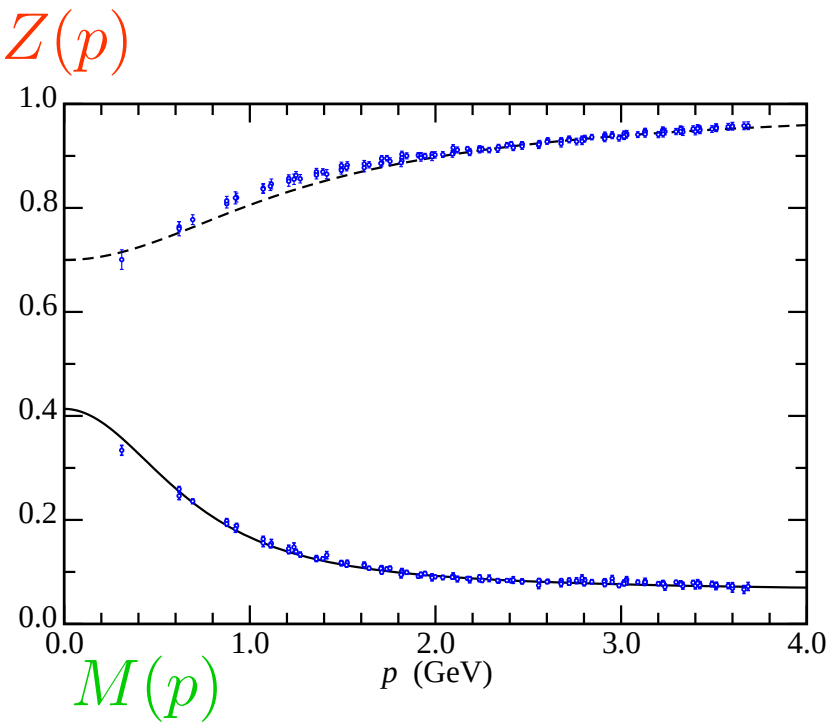

- Lattice Meas.
  - Bowman, Heller, Leinweber, Williams: he-lat/0209129

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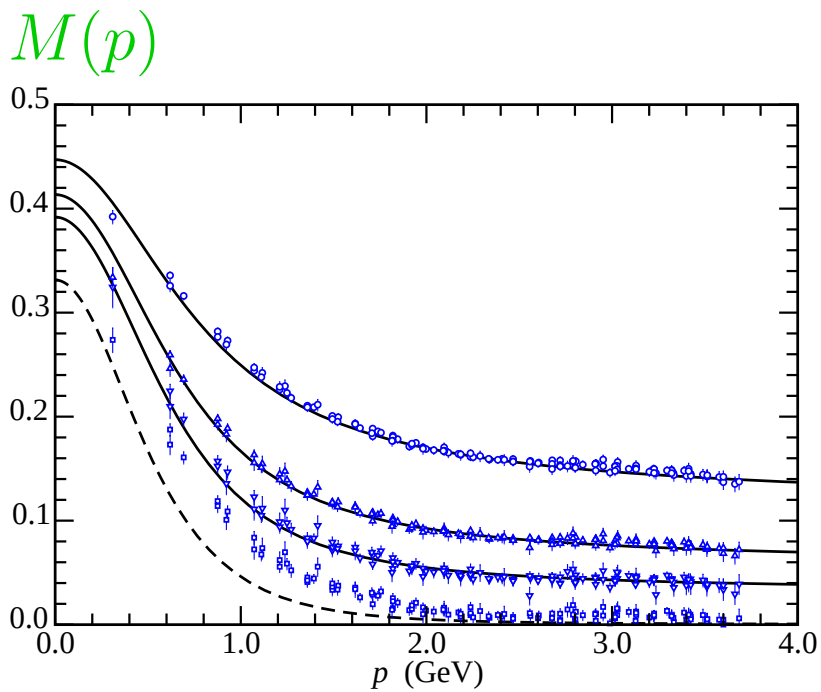

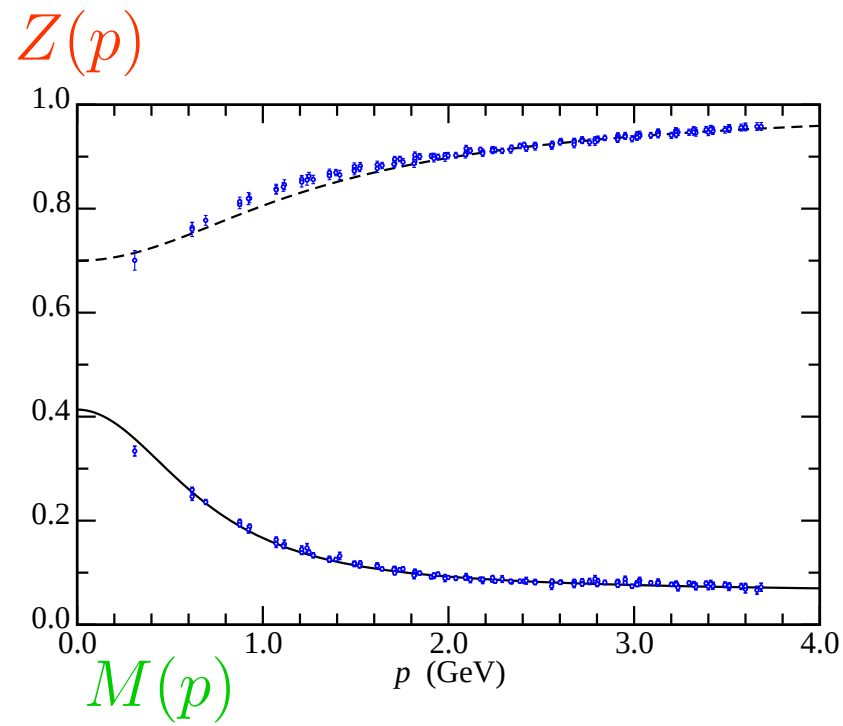

- DSE Cal.– Bhagwat, Pichowsky, CDR, Tandy nu-th/0304003

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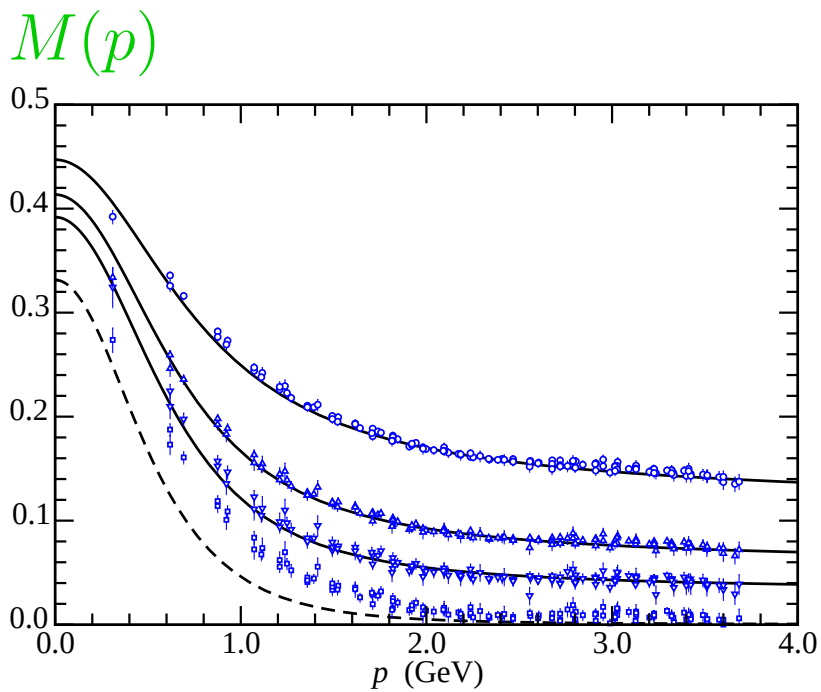

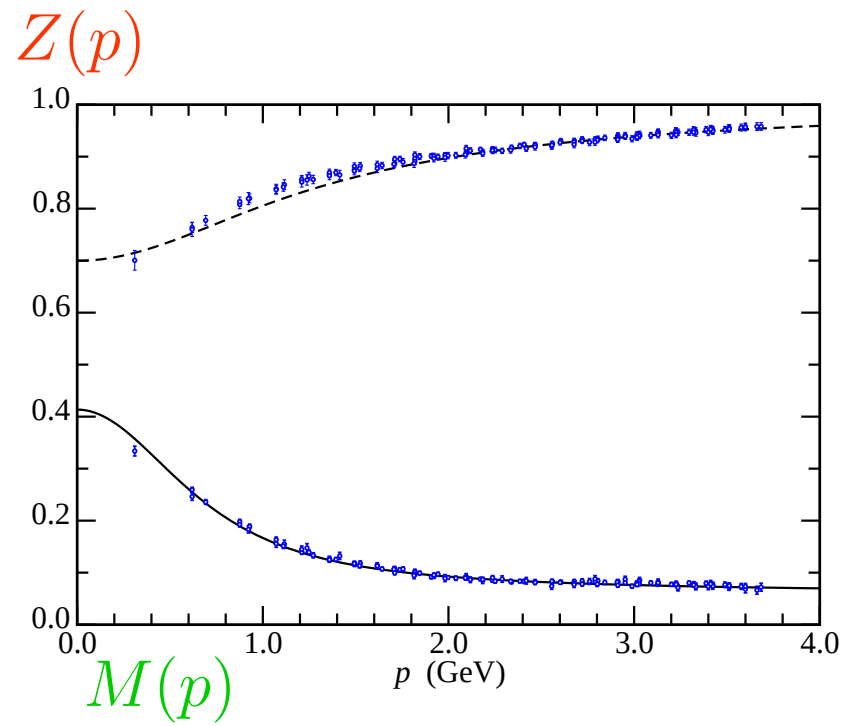

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$$f_\pi^0 = 0.068\,\mathrm{GeV} \quad \langle \bar{q}q \rangle^0_{1\,\mathrm{GeV}} = (-0.19\,\mathrm{GeV})^3\,.$$

ANL Physics Division

Dichotomy of the Pion

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- How does one make an almost massless particle
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The correct understanding of pion observables;
e.g. mass, decay constant and form factors,
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a well-defined and valid chiral limit.

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- Requires detailed understanding of Connection between Current-quark and Constituent-quark masses

Hadrons

- Established understanding
  of two- and three-point functions

Hadrons

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- What about bound states?

Hadrons

- Without bound states, Comparison with experiment is impossible

Hadrons

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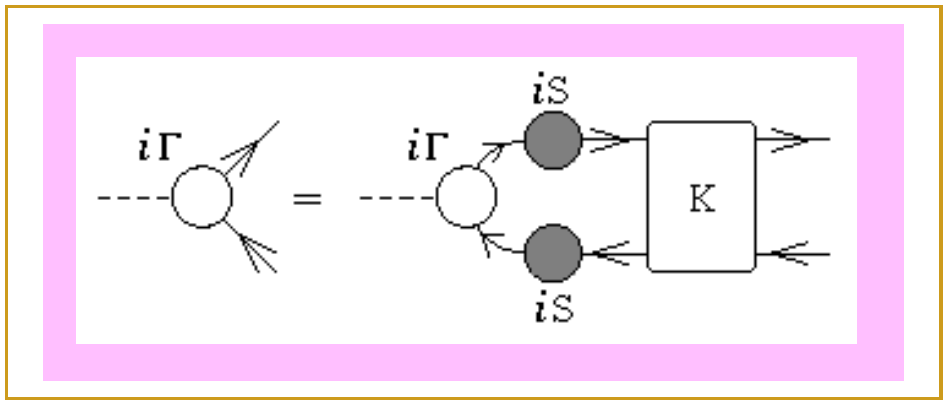
- They're pole contributions to $n \geq 3$-point colour-singlet functions

Hadrons

- Without bound states, Comparison with experiment is impossible
- Bethe-Salpeter Equation



QFT Generalisation of Lippman-Schwinger Equation.

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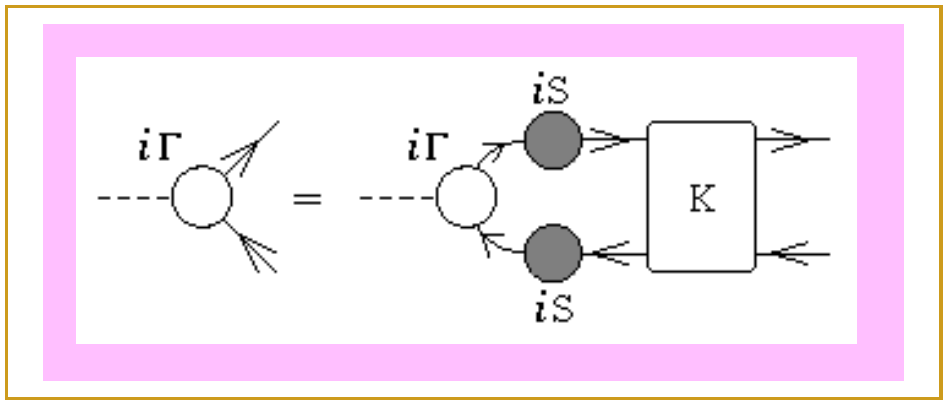

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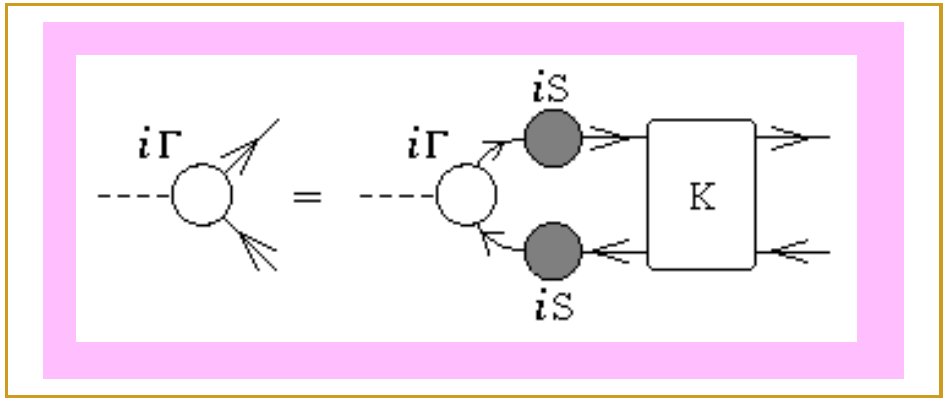

QFT Generalisation of Lippman-Schwinger Equation.

- What is the kernel, K?

or What is the long-range potential in QCD?

Bethe-Salpeter Kernel

Bethe-Salpeter Kernel

- Axial-vector Ward-Takahashi identity

$$P_\mu \; \Gamma^l_{5\mu}(k;P) \quad = \quad \mathcal{S}^{-1}(k_+)\,\frac{1}{2}\lambda^l_f i\gamma_5 + \frac{1}{2}\lambda^l_f i\gamma_5 \;\; \mathcal{S}^{-1}(k_-)$$

$$-M_\zeta\, i\Gamma^l_5(k;P) - i\Gamma^l_5(k;P)\, M_\zeta$$

QFT Statement of Chiral Symmetry

Bethe-Salpeter Kernel

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Satisfies BSE

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Kernels must be intimately related

Bethe-Salpeter Kernel

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Kernels must be intimately related

- Relation must be preserved by truncation

Bethe-Salpeter Kernel

- Axial-vector Ward-Takahashi identity

$$P_\mu \Gamma^l_{5\mu}(k;P) = \mathcal{S}^{-1}(k_+)\,\frac{1}{2}\lambda^l_f i\gamma_5 + \frac{1}{2}\lambda^l_f i\gamma_5\,\mathcal{S}^{-1}(k_-)$$

$$-M_\zeta\, i\Gamma^l_5(k;P) - i\Gamma^l_5(k;P)\, M_\zeta$$

Satisfies BSE Satisfies DSE

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Satisfies BSE — Satisfies DSE

Kernels must be intimately related

- Relation must be preserved by truncation
- Failure $\Rightarrow$ Violation of Goldstone's Theorem

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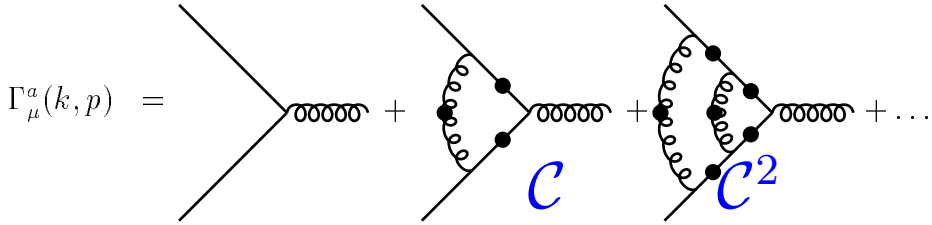
Colour-singlet Bethe-Salpeter equation

Colour-singlet Bethe-Salpeter equation

- Coupling-modified dressed-ladder vertex



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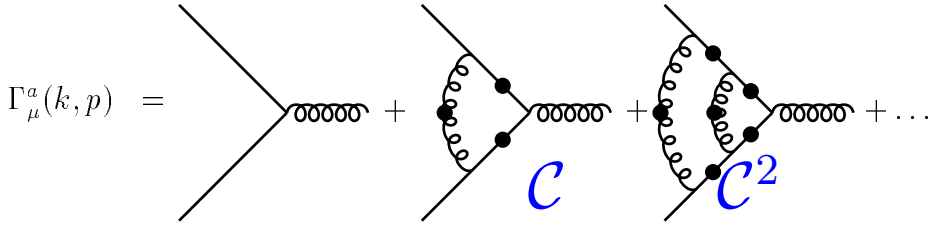
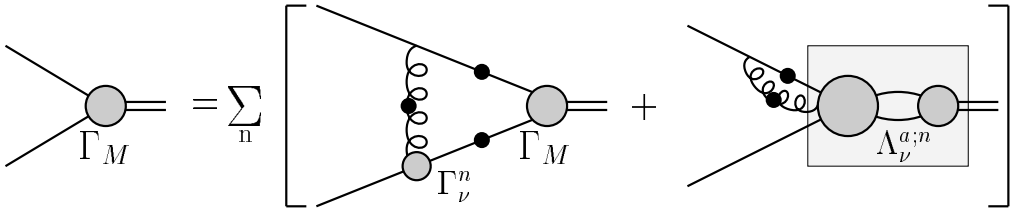


- BSE consistent with vertex



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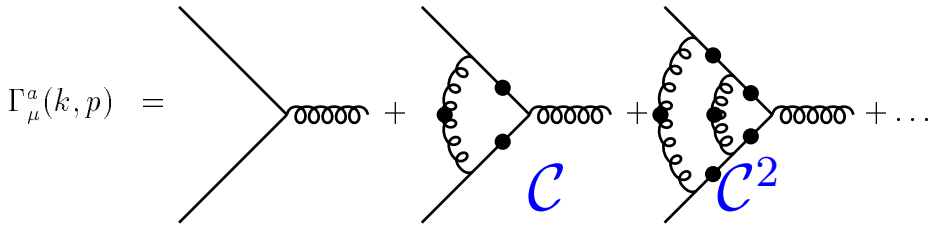

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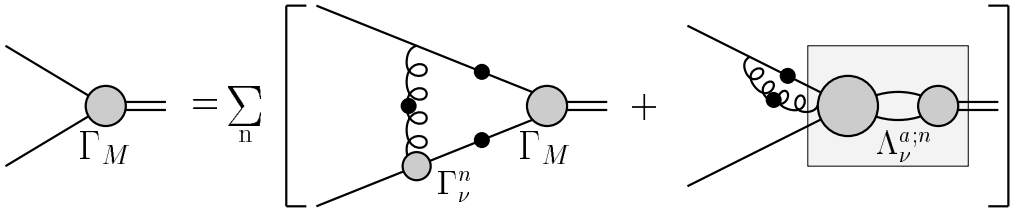
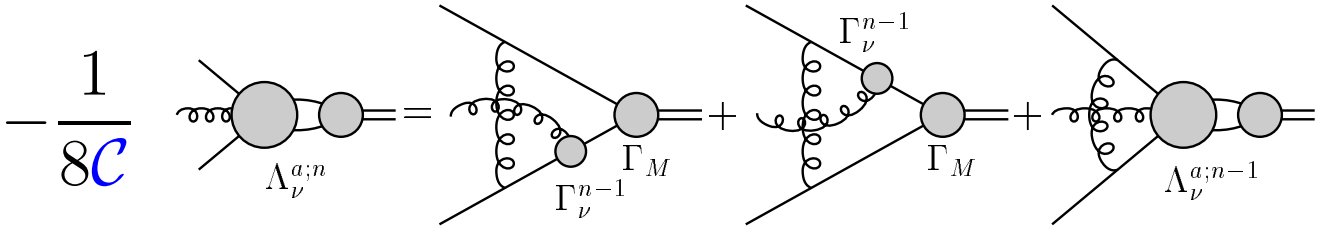


- Bethe-Salpeter kernel . . . recursion relation



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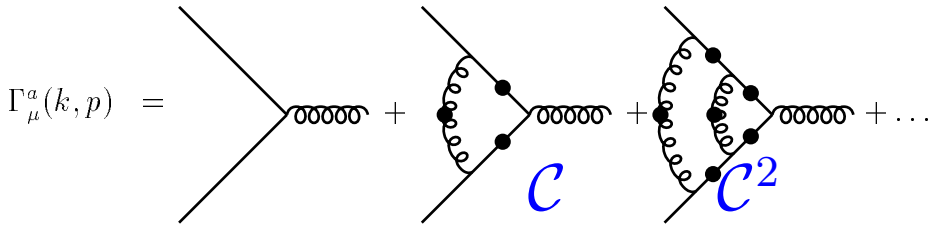

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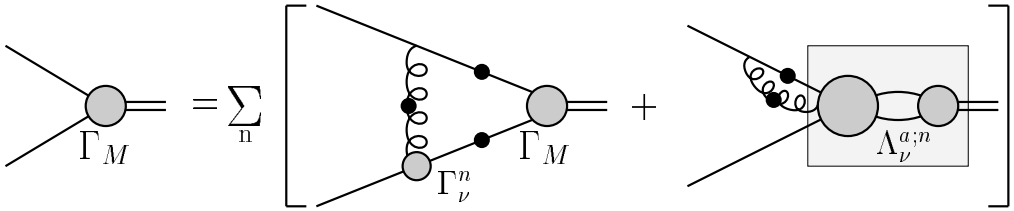

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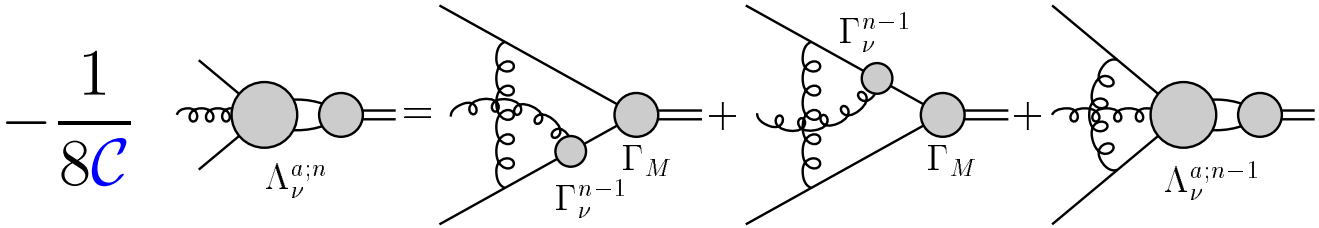

- Kernel necessarily non-planar,

  even with planar vertex

Goldberger-Treiman for pion

Goldberger-Treiman for pion

- Pseudoscalar Bethe-Salpeter amplitude

$$\Gamma_{\pi^j}(k;P) = \tau^{\pi^j}\gamma_5\Big[iE_\pi(k;P) + \gamma\cdot P F_\pi(k;P) + \gamma\cdot k\, k\cdot P\, G_\pi(k;P) + \sigma_{\mu\nu}\, k_\mu P_\nu\, H_\pi(k;P)\Big]$$

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- Dressed-quark Propagator: $S(p) = \dfrac{1}{i\gamma\cdot p\, A(p^2) + B(p^2)}$

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$$\Rightarrow \quad \begin{aligned} f_\pi E_\pi(k;P=0) &= B(p^2) \\ F_R(k;0) + 2\, f_\pi F_\pi(k;0) &= A(k^2) \\ G_R(k;0) + 2\, f_\pi G_\pi(k;0) &= 2A'(k^2) \\ H_R(k;0) + 2\, f_\pi H_\pi(k;0) &= 0 \end{aligned}$$

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Goldberger-Treiman for pion

- Pseudoscalar Bethe-Salpeter amplitude

Pseudovector components necessarily nonzero

$$\Gamma_{\pi^j}(k;P) = \tau^{\pi^j}\gamma_5\left[iE_\pi(k;P) + \gamma\cdot P F_\pi(k;P) + \gamma\cdot k\, k\cdot P\, G_\pi(k;P) + \sigma_{\mu\nu}\, k_\mu P_\nu\, H_\pi(k;P)\right]$$

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$\Rightarrow$ Exact in Chiral QCD

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QCD mass formula

(Maris, Roberts, Tandy, nucl-th/9707003, Krassnigg & Roberts)

$$f_H \quad m_H^2 = - \quad \rho_\zeta^H \quad \mathcal{M}_H$$

- Light-quarks; i.e., $m_q \sim 0$,

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Hence $m_H^2 = \dfrac{-\langle \bar{q}q \rangle_\zeta^0}{(f_\pi^0)^2} \, m_q$

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- Applicable to ALL pseudoscalar mesons, irrespective of their mass (and that of their constituents).

Andreas Krassnigg

Future President
. . . almost Blood
Relative of Arnold

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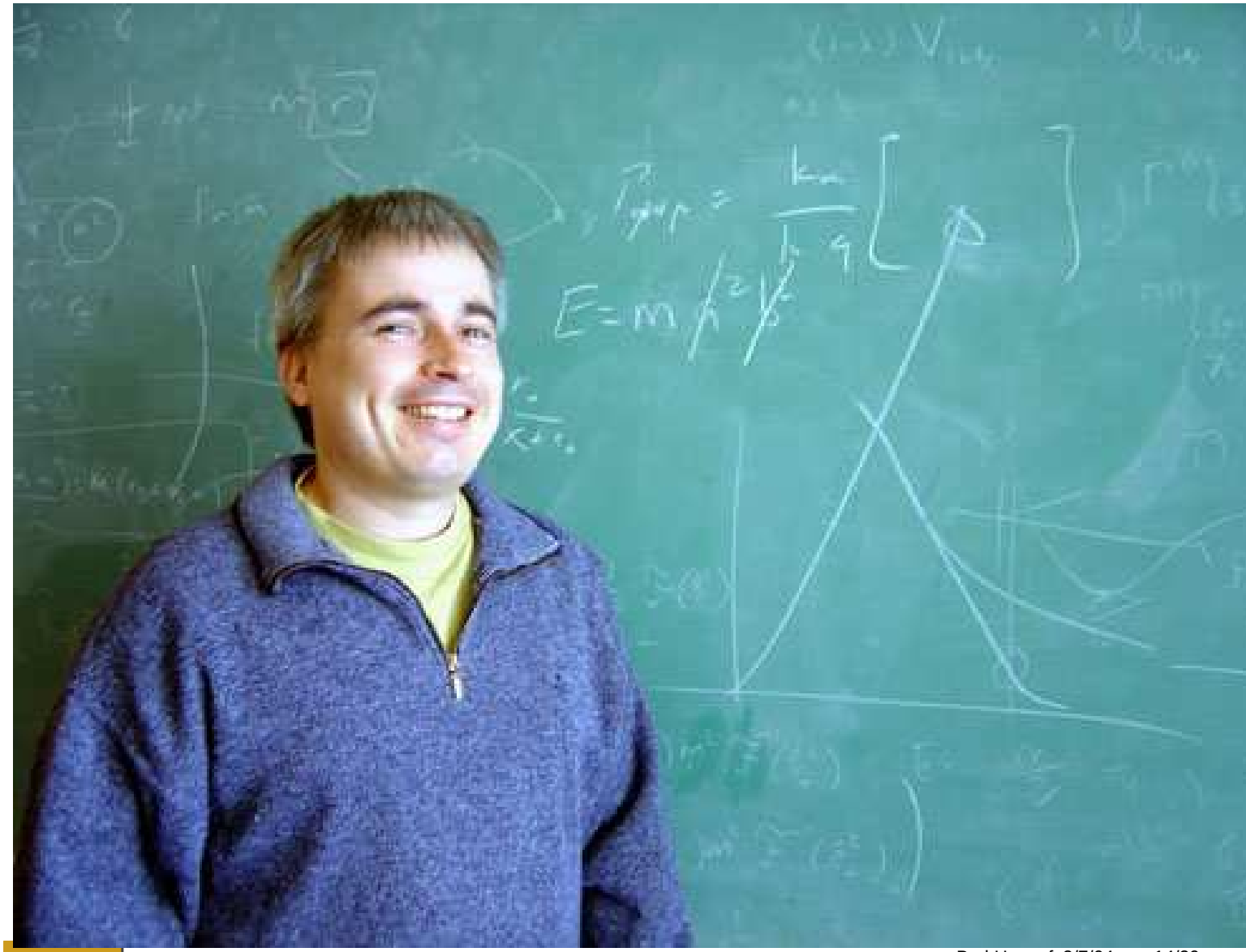
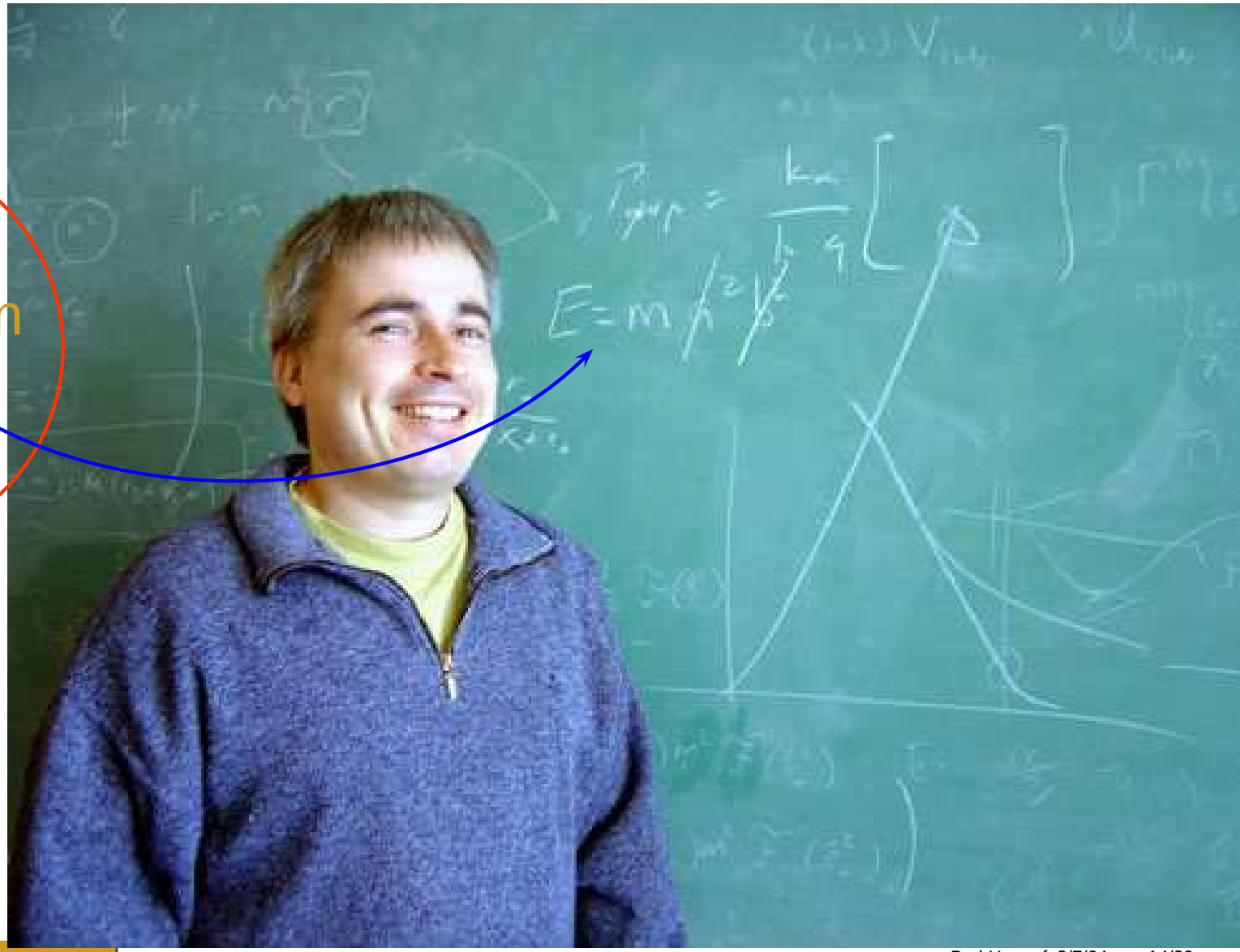
Arne Höll



Arne Höll

Closing in on something



Radial Excitations

Radial Excitations

- Spectrum contains 3 pseudoscalars [$I^G(J^P)L = 1^-(0^-)S$]

  masses below 2 GeV: $\pi(140)$; $\pi(1300)$; and $\pi(1800)$

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- The Pion

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- Consituent-Q Model: 1st three members of $\boldsymbol{n\,^1S_0}$ trajectory; i.e., ground state plus radial excitations

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- Radial excitations $\Rightarrow$ Long-range radial wave functions

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- Radial excitations $\Rightarrow$ Long-range radial wave functions $\Rightarrow$ sensitive to confinement
- NSAC Long-Range Plan, 2002:

  . . . an understanding of confinement "remains one of the greatest intellectual challenges in physics"

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Radial Excitations & Chiral Symmetry

Radial Excitations & Chiral Symmetry

Höll, Krassnigg, Roberts
nu-th/0406030

- Fundamental properties of QCD

Radial Excitations & Chiral Symmetry

Höll, Krassnigg, Roberts
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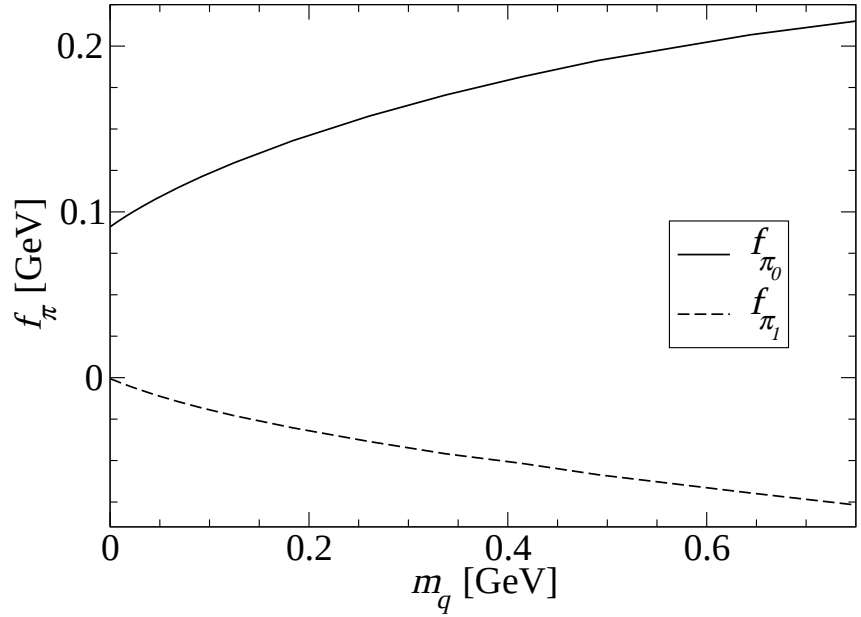
- Fundamental properties of QCD
  - If chiral symmetry is dynamically broken, then in the chiral limit every pseudoscalar meson is blind to the weak interaction *except* $\pi(140)$.



Höll, Krassnigg, Roberts
nu-th/0406030

Radial Excitations & Chiral Symmetry

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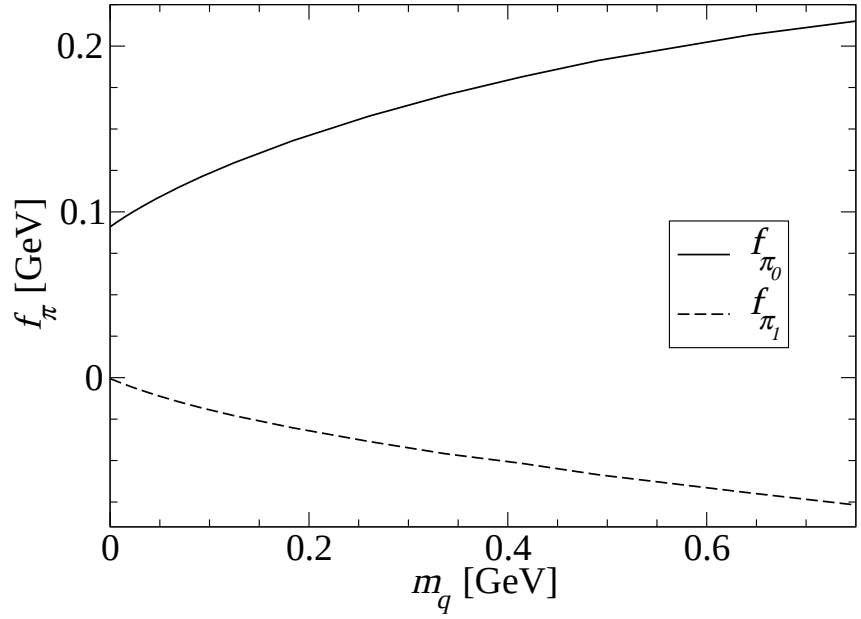

Radial Excitations & Chiral Symmetry

Höll, Krassnigg, Roberts
nu-th/0406030

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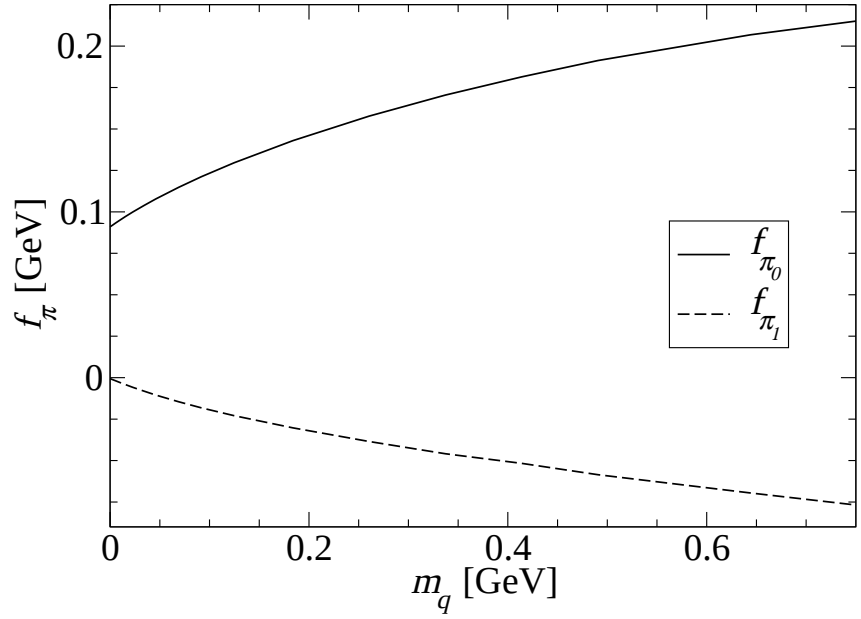
- All theories with confinement exhibit Dynamical Chiral Symmetry Breaking



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Ab-Initio **Calculations**

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Pieter Maris

Peter Tandy

Ab-Initio **Calculations**

Maris & Tandy, Series of Five Articles: 1999 – Present

Perfected a Renormalisation-Group Improved Rainbow-Ladder Model of Quark-Quark Interaction

Ab-Initio **Calculations**

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Perfected a Renormalisation-Group Improved Rainbow-Ladder Model of Quark-Quark Interaction

- Rainbow-Ladder $=$ First Order in Truncation Described Above

- Anticipate Accurate for 0^- & 1^- Mesons

Ab-Initio **Calculations**

Maris & Tandy, Series of Five Articles: 1999 – Present

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- One Parameter $=$ Interaction Energy:

$$\boxed{\mathcal{E} \approx 700\,\text{MeV}}$$

- Dressed-Glue Mass scale:

  Characterises DCSB and light-quark Confinement

- Both Phenomena Disappear for $\mathcal{E} \lesssim 200\,\text{MeV}$

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- *Dyson-Schwinger equations:*
  A Tool for Hadron Physics
  P. Maris and C.D. Roberts, nu-th/0301049

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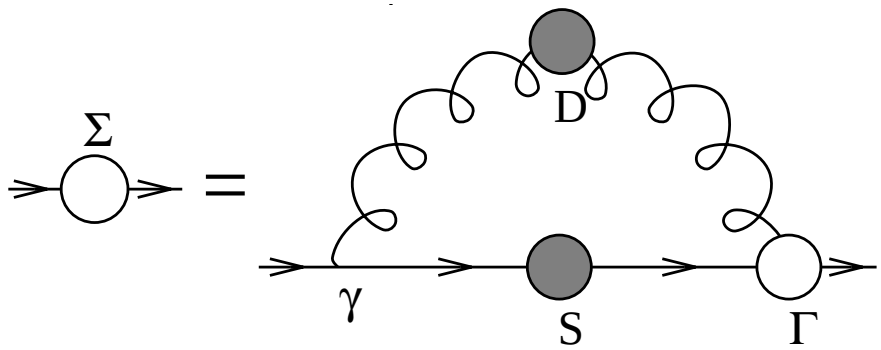
Pion Form Factor

Procedure Now Straightforward

Pion Form Factor

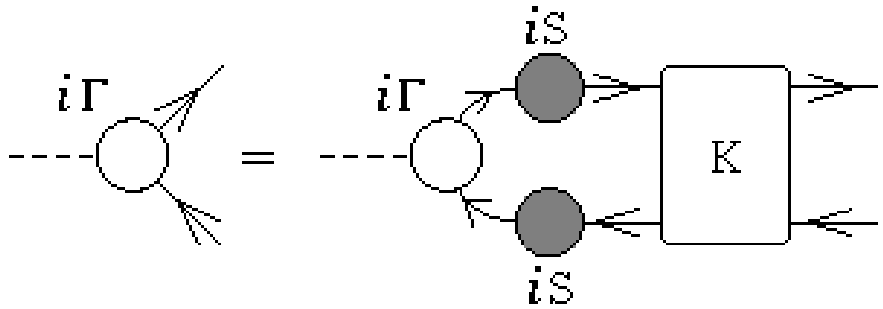
- Solve Gap Equation

  $\Rightarrow$ Dressed-Quark Propagator, $S(p)$



Pion Form Factor

- Use that to Complete Bethe Salpeter Kernel, K
- Solve Homogeneous Bethe-Salpeter Equation for Pion Bethe-Salpeter Amplitude, Γ_π



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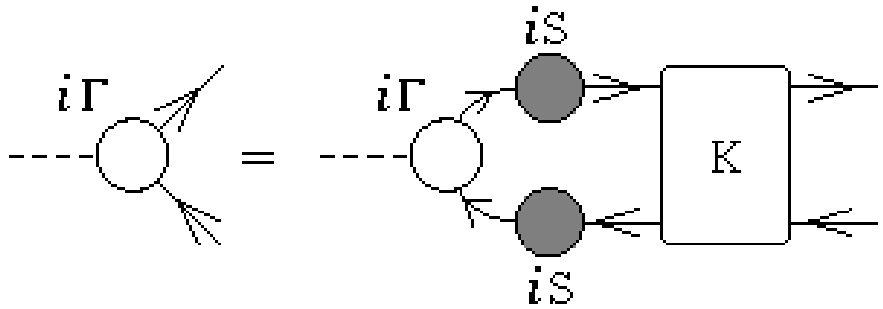

- Solve Inhomogeneous Bethe-Salpeter Equation for Dressed-Quark-Gluon Vertex, Γ_μ

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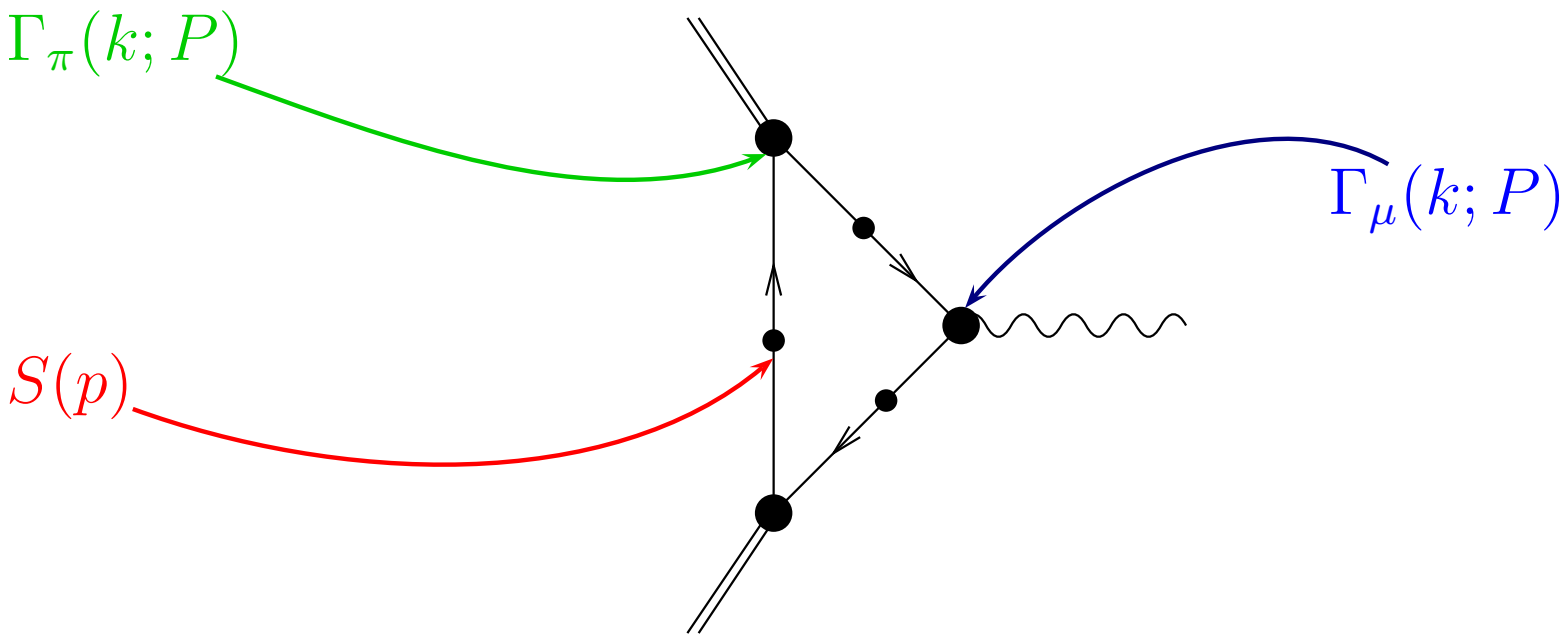
Pion Form Factor

- Now have all elements for Impulse Approximation to Electromagnetic Pion Form factor



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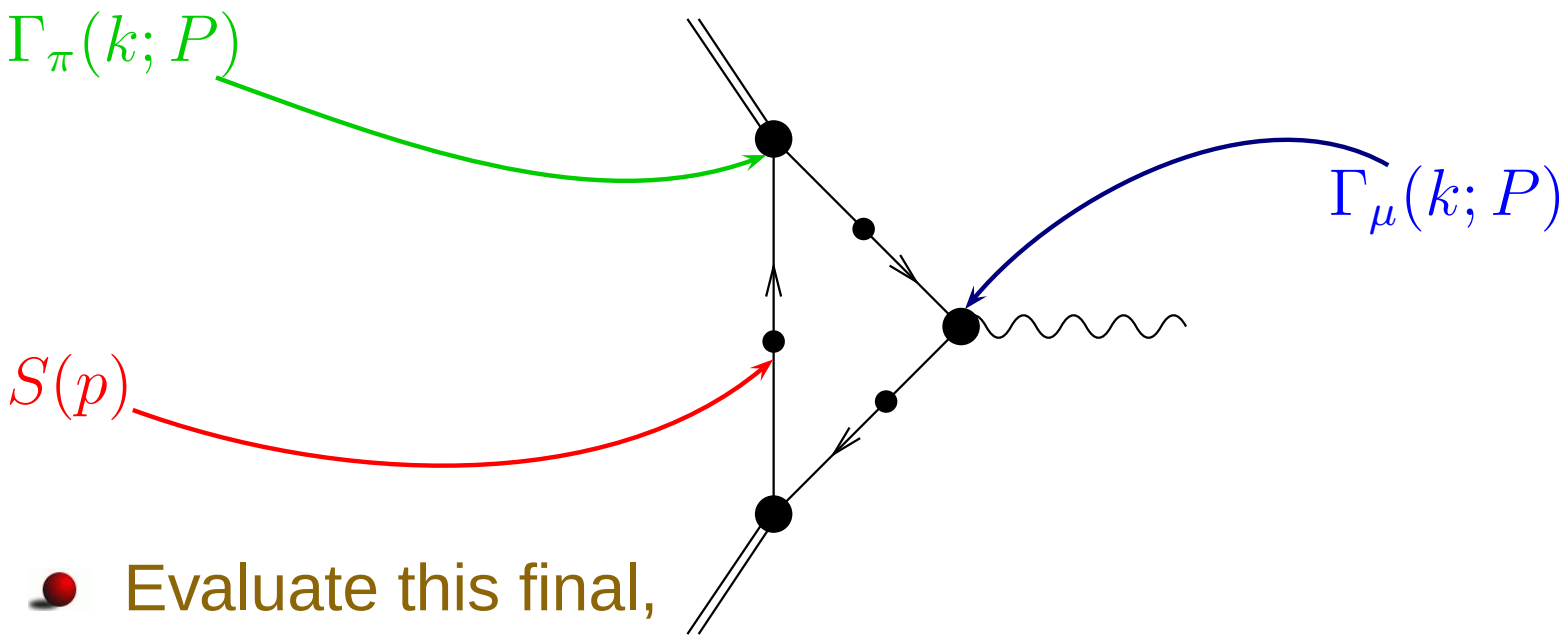
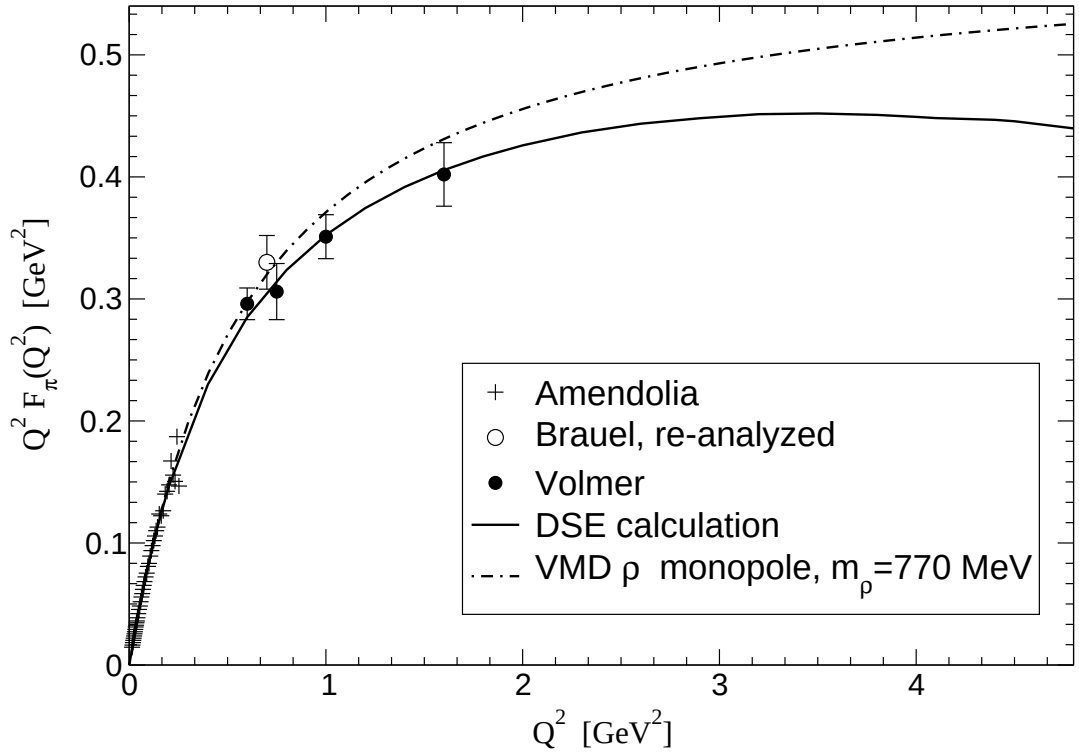


- Evaluate this final, three-dimensional integral

Calculated Pion Form Factor

Calculation published in 1999; No Parameters Varied



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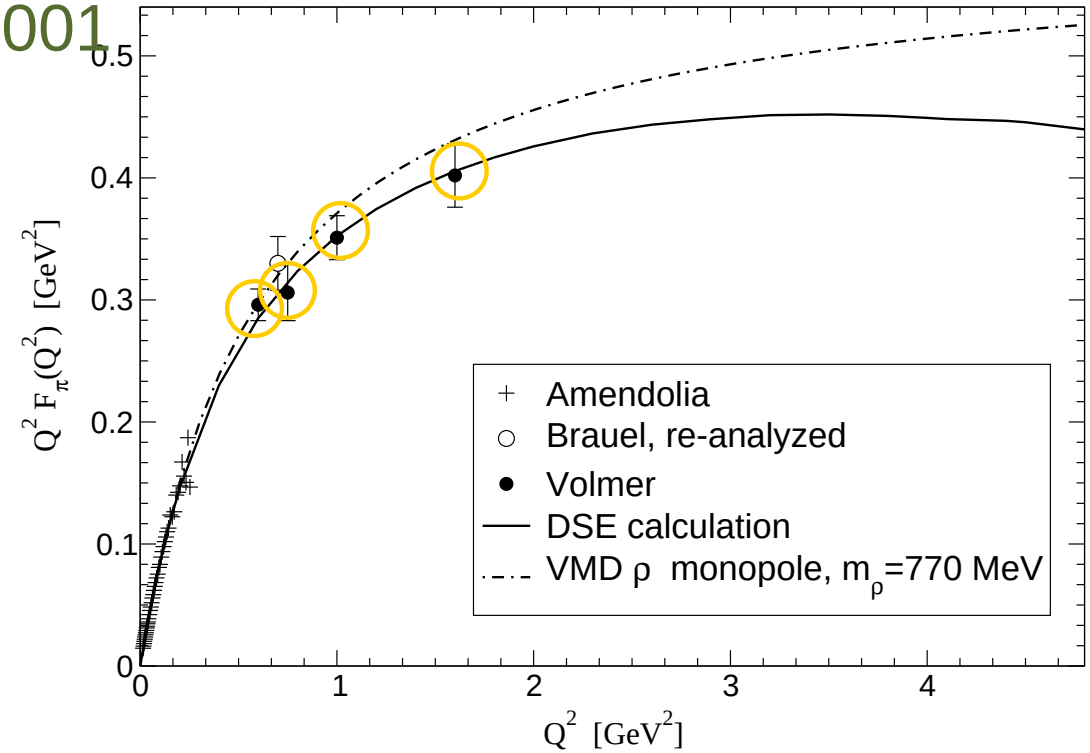
Calculated Pion Form Factor

Calculation published in 1999; No Parameters Varied

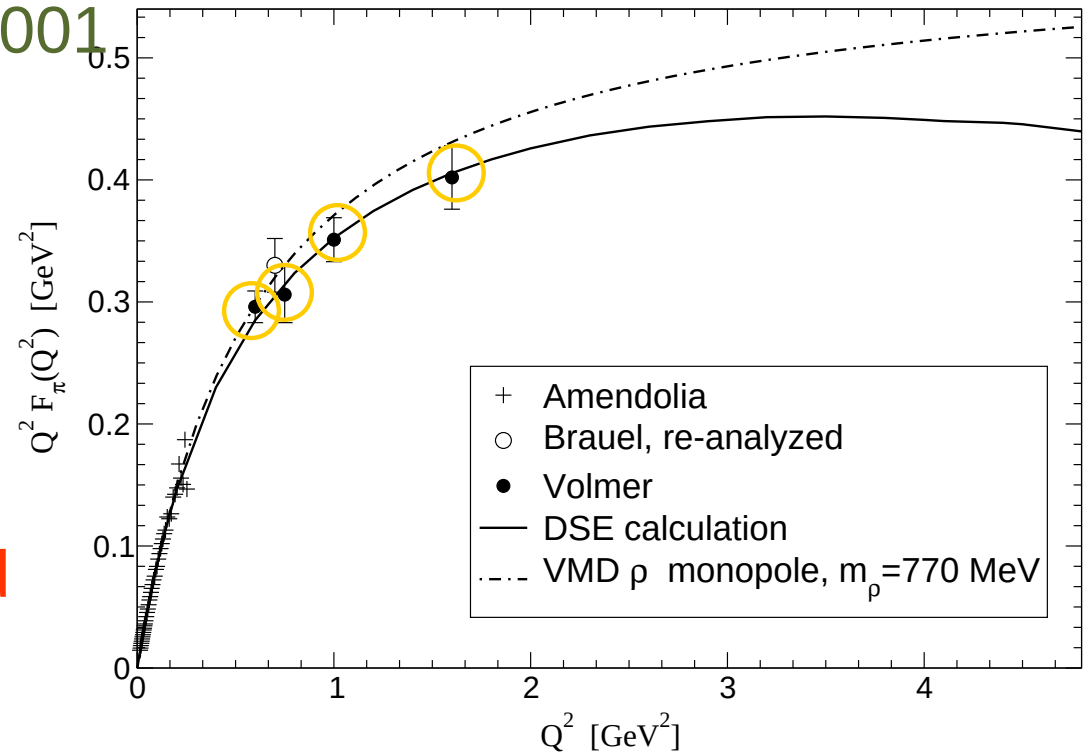
Data published in 2001



Calculated Pion Form Factor

Calculation published in 1999; No Parameters Varied

Data published in 2001



Many subsequent successful applications

. . Again, parameters Fixed

Notably $\pi\ \pi$ Scattering

Maris, et al., Phys. Rev. D 65, 076008

Bicudo, Phys. Rev. C 67, 035201

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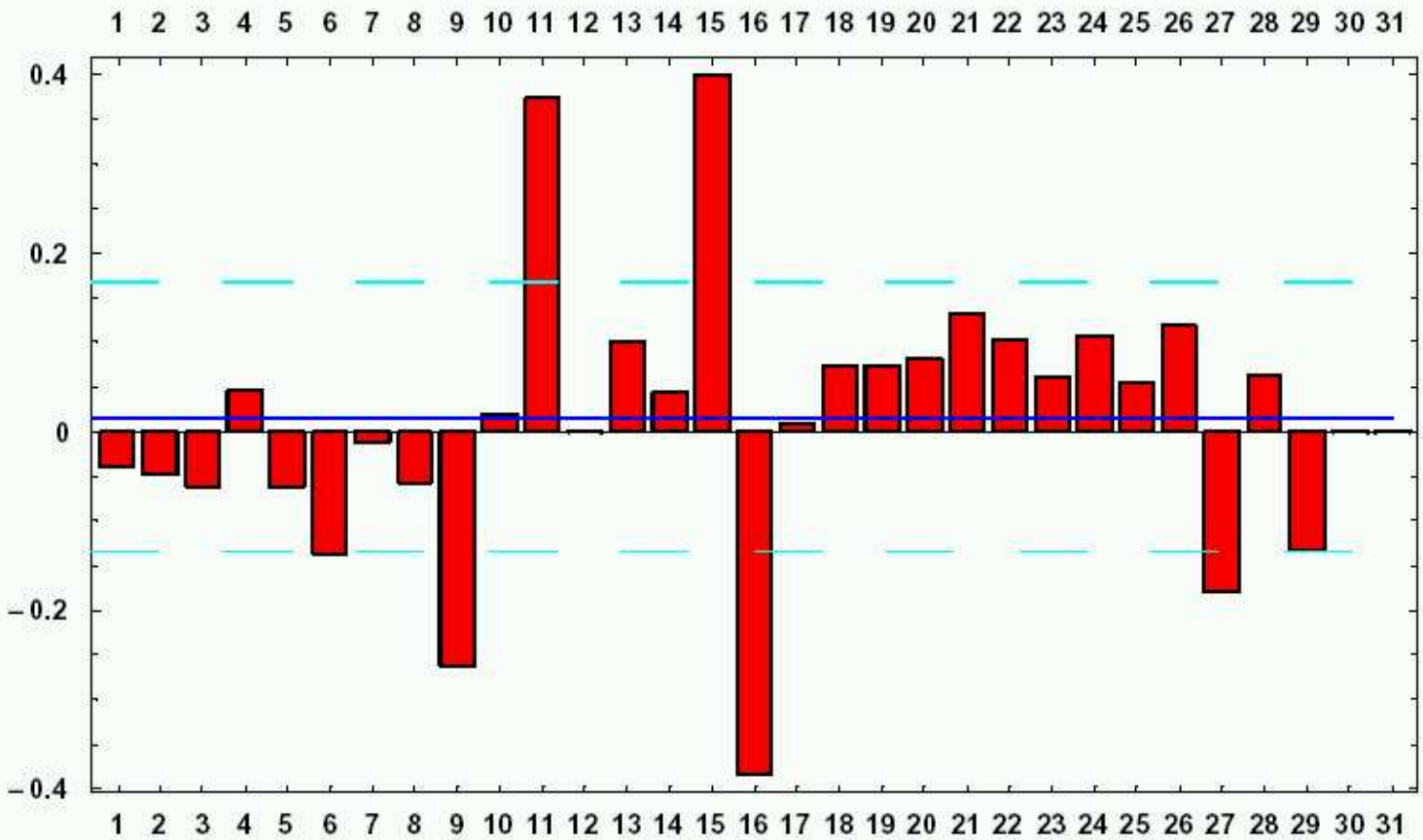
Maris & Tandy
Relative Error



Relative Error, Predictions of Maris and Tandy Model

All tabulated quantities in nu-th/0301049

<error> = 1.6%, Sqrt[<error^2>] = 15%

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π and ρ mesons

π and ρ mesons

	$M_H^{n=0}$	$M_H^{n=1}$	$M_H^{n=2}$	$M_H^{n=\infty}$
π, $m = 0$	0	0	0	0
π, $m = 0.011$	0.147	0.135	0.139	0.138
ρ, $m = 0$	0.920	0.648	0.782	0.754
ρ, $m = 0.011$	0.936	0.667	0.798	0.770

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  Not constituent-quark-model-like hyperfine splitting

π and ρ mesons

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π, $m = 0$	0	0	0	0
π, $m = 0.011$	0.147	0.135	0.139	0.138
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ANL Physics Division

Epilogue

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Dyson-Schwinger Equations

- Provide Understanding of

  Dynamical Chiral Symmetry Breaking:

  $\Rightarrow$ π is quark-antiquark Bound State

  AND QCD's Goldstone Mode

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  e.g., Quark Goldberger-Treiman

  Properties of Pseudoscalar Mesons

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- Renormalisation-Group-Improved Rainbow-Ladder

  $\Rightarrow$ Practical Phenomenological Tool

  Corrections Quantifiable

ANL Physics Division

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cf. Lattice
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ANL Physics Division

Current-quark mass-dependence of meson masses

Current-quark mass-dependence of meson masses

$m_{u,d} = 0.01$	$m_s = 0.166$	$m_c = 1.33$	$m_b = 4.62$
$m_\rho = 0.77$	$m_\phi = 1.02$	$m_{J/\psi} = 3.10$	$m_{\Upsilon(1S)} = 9.46$
$m_\pi = 0.14$	$m_{0^-_{s\bar{s}}} = 0.63$	$m_{\eta_c} = 2.97$	$m_{\eta_b} = 9.42$

- Fit current-quark masses to vector meson spectrum

ANL Physics Division

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- Fit current-quark masses to vector meson spectrum
- Predictions $m_{\eta_c} = 2.97$ and $m_{\eta_b} = 9.42$

  cf. $m_{\eta_c}^{\text{expt.}} = 2.98$ and $m_{\eta_b}^{\text{expt.}} = 9.30 \pm 0.03$

  NB. $m_{\Upsilon(1S)} - m_{\eta_b}^{\text{expt.}}$
  - Calc.: 40 MeV
  - Expt.: 160 MeV, which is $>$ Expt. $m_{J/\psi} - m_{\eta_c}^{\text{expt.}}$
  - Unlikely

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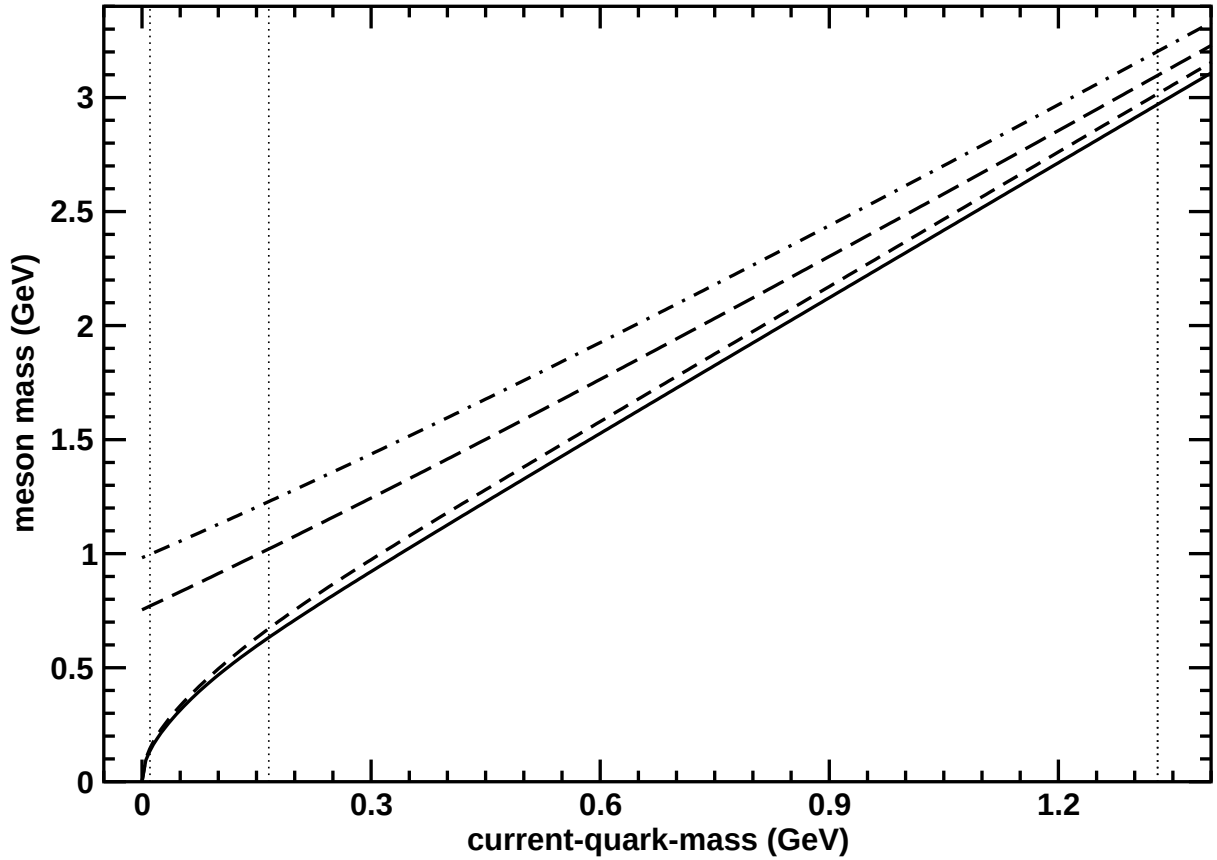
Current-quark mass-dependence of meson masses



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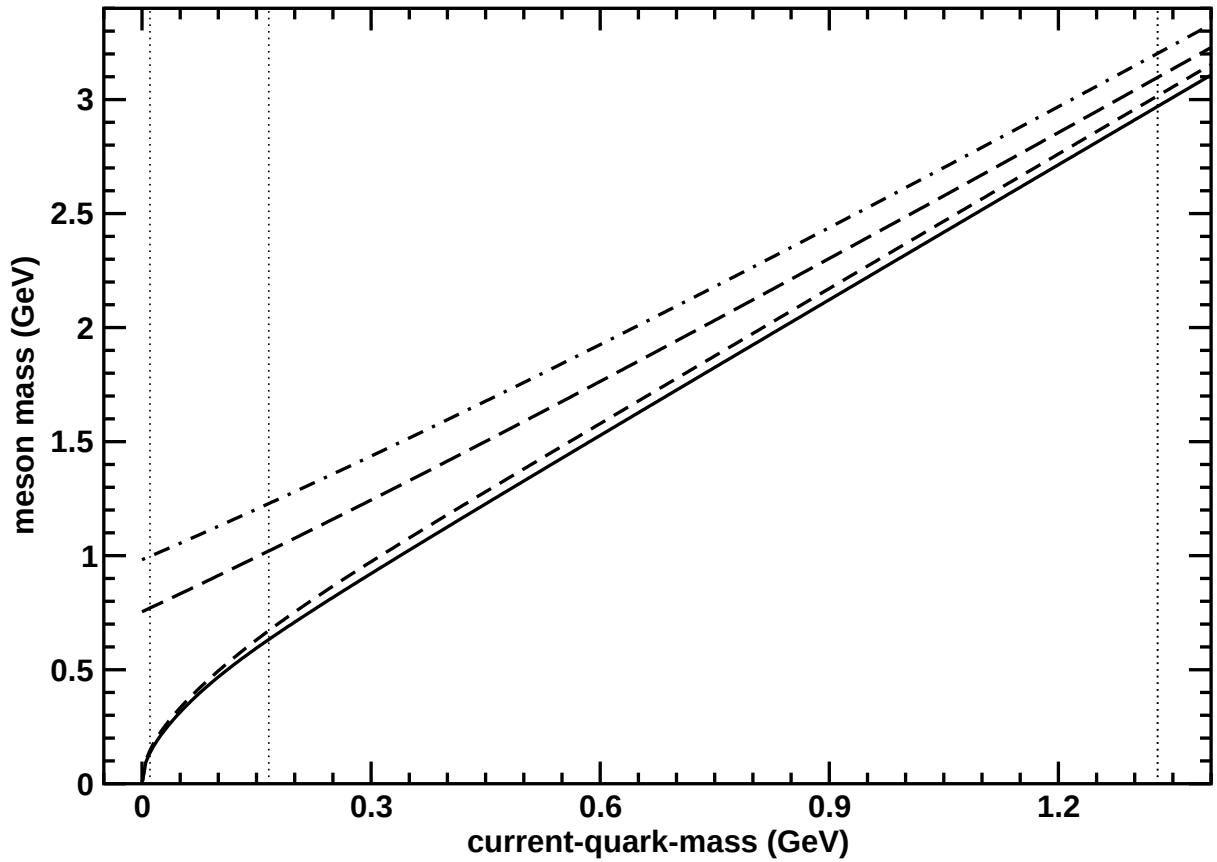
Current-quark mass-dependence of meson masses

- For $m \ll \mathcal{G}$
  $m_{0^-}^2 = 1.33\, m$



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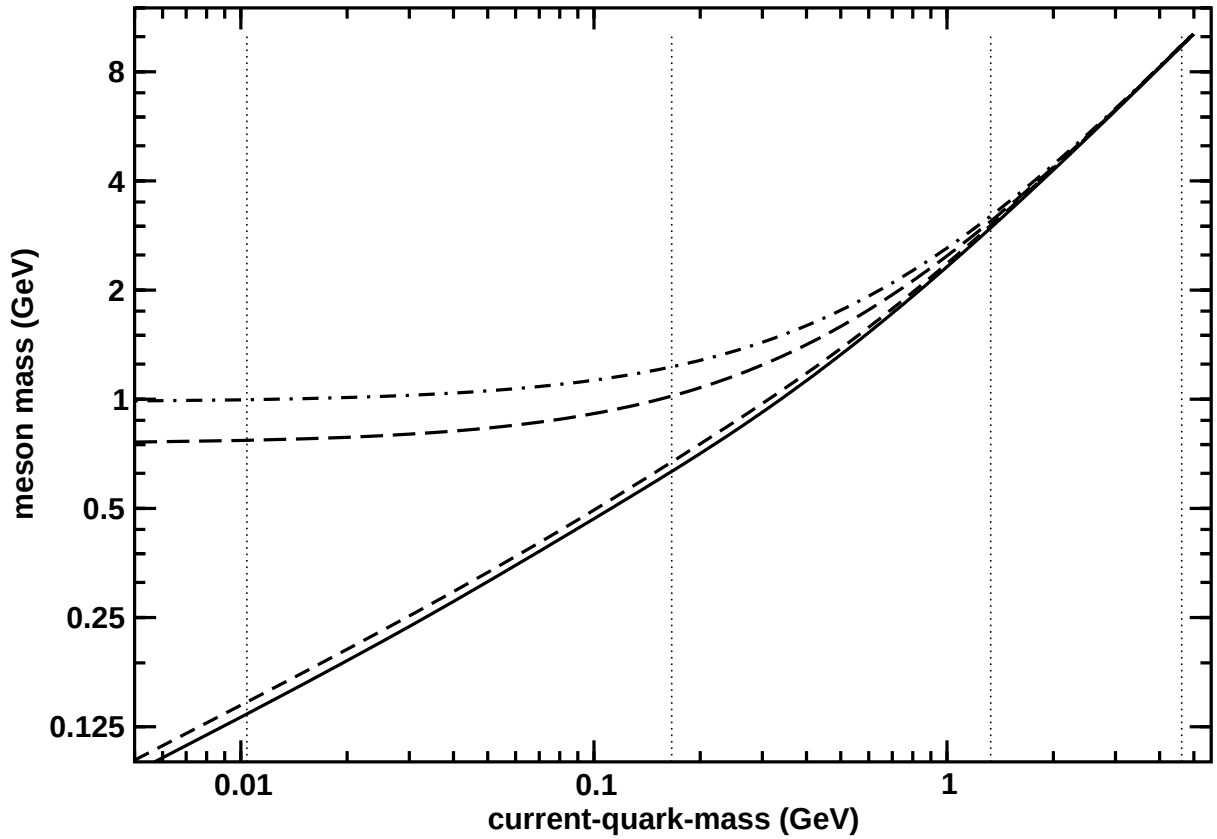

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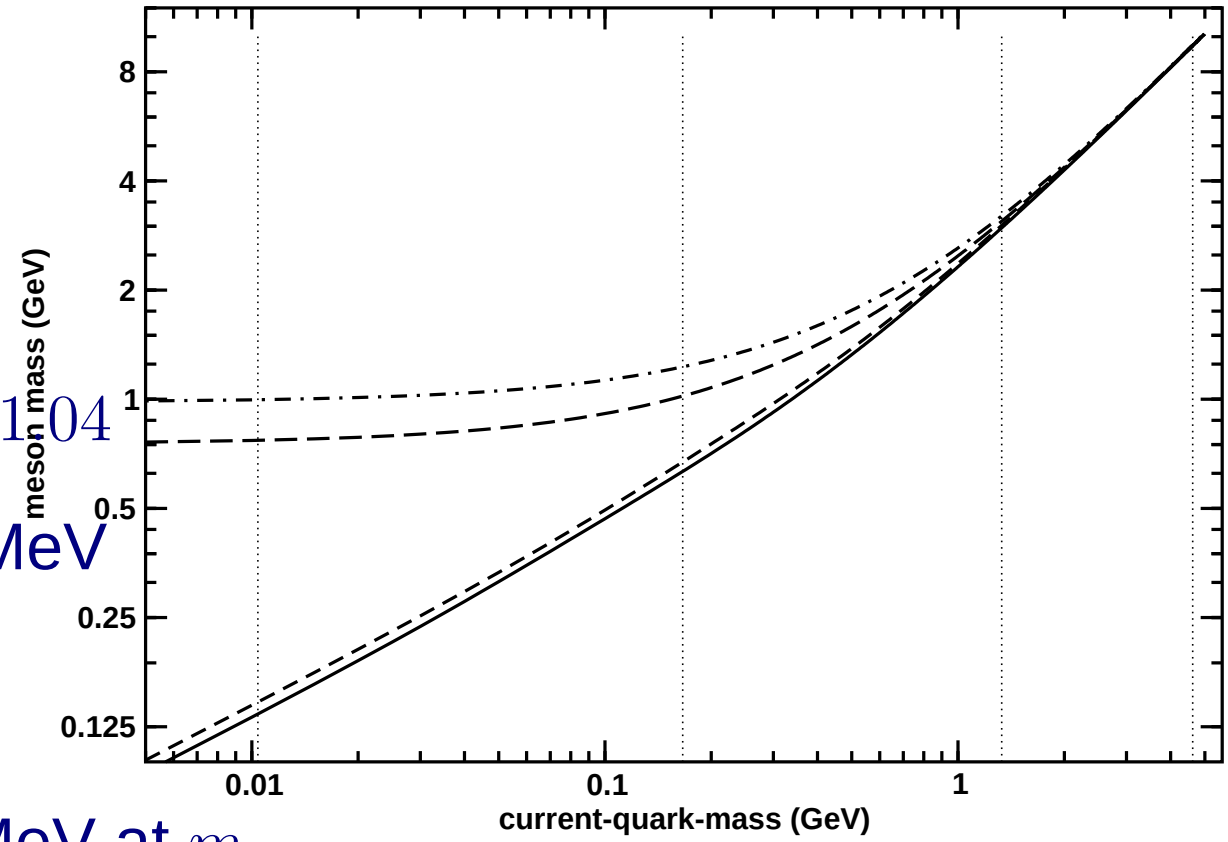
Current-quark mass-dependence of meson masses

- For $m \gg \mathcal{G}$

  $m_{0^-} = m_{1^+}$

- $\left.\dfrac{m_{1^-}}{m_{0^-}}\right|_{m=m_c} = 1.04$

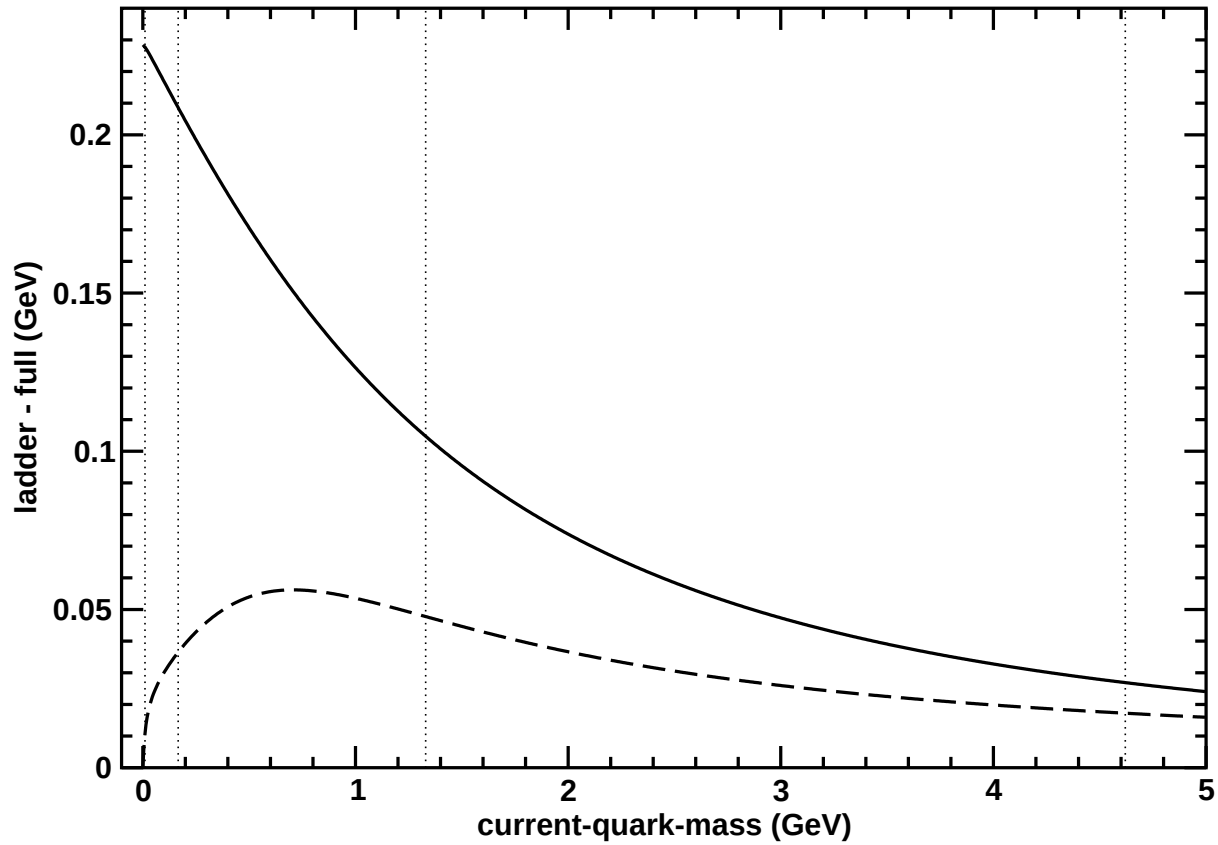
  splitting 130 MeV



- Drops to 40 MeV at m_b

  just 5% of its chiral limit value

Current-quark mass-dependence of meson masses



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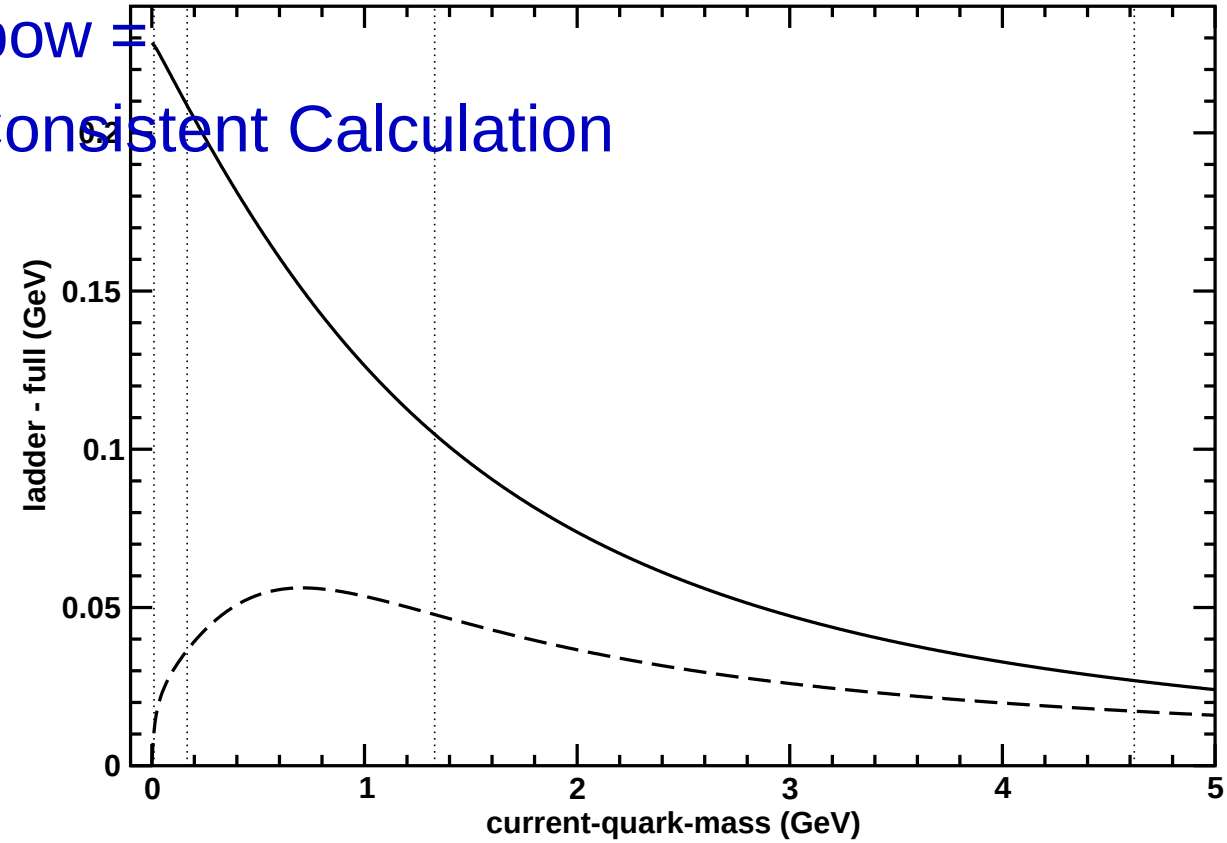
Current-quark mass-dependence of meson masses

- For $m \gg \mathcal{G}$
  Ladder-Rainbow =
  Completely Consistent Calculation



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- Vector Mesons
  - 20% at m_s
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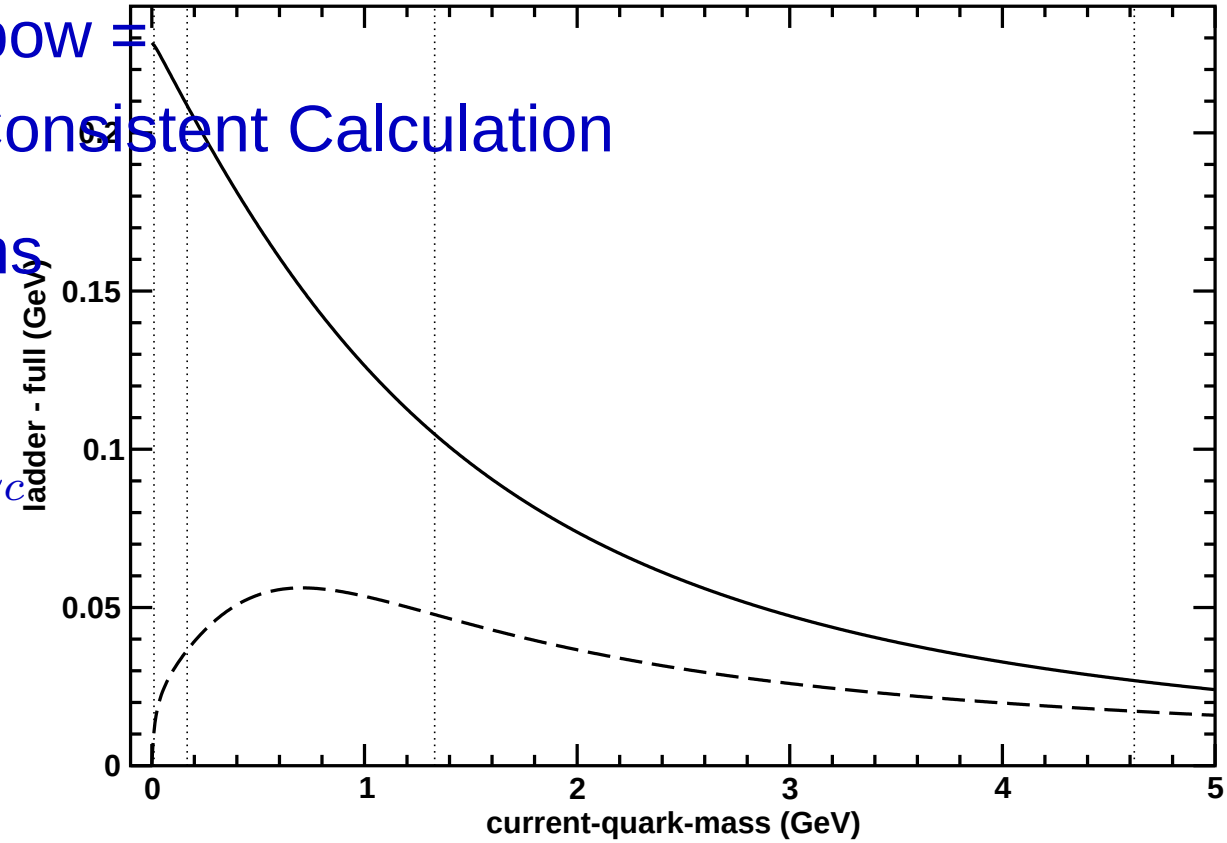

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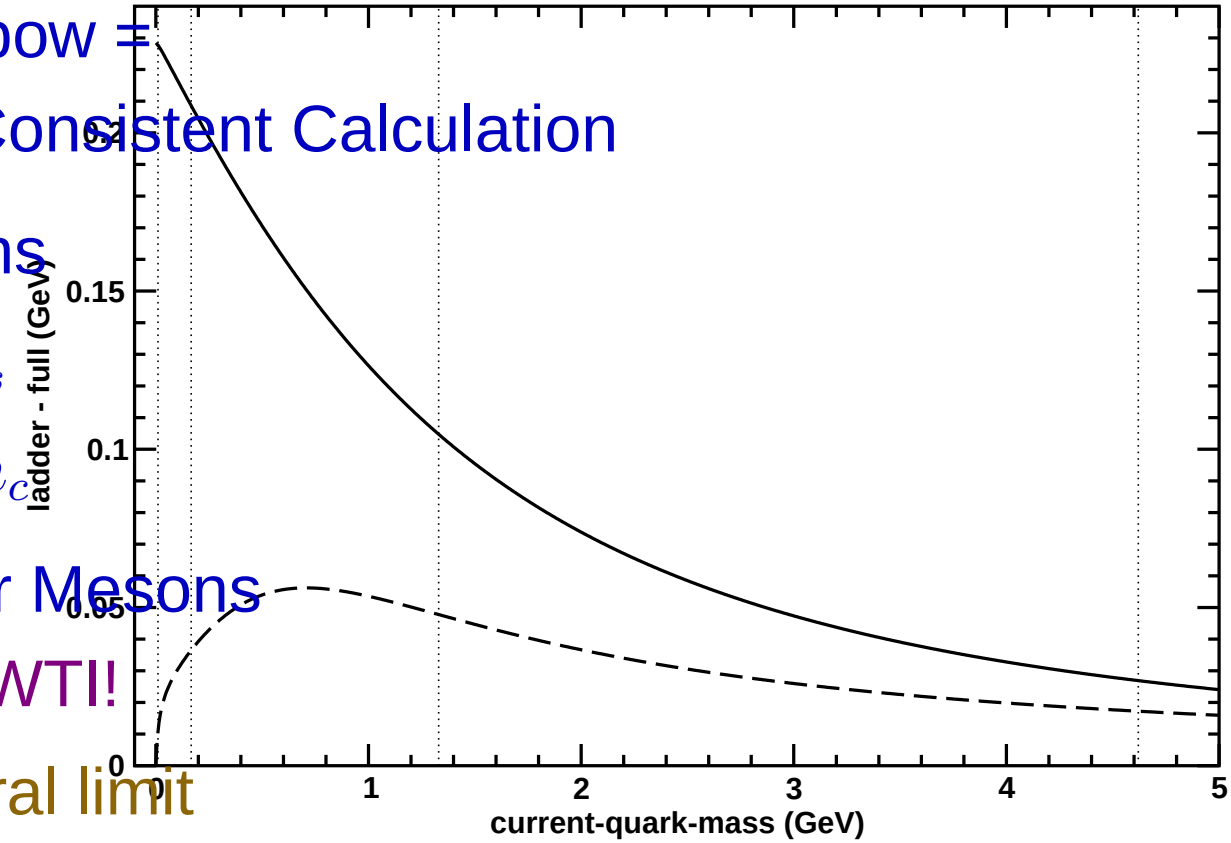

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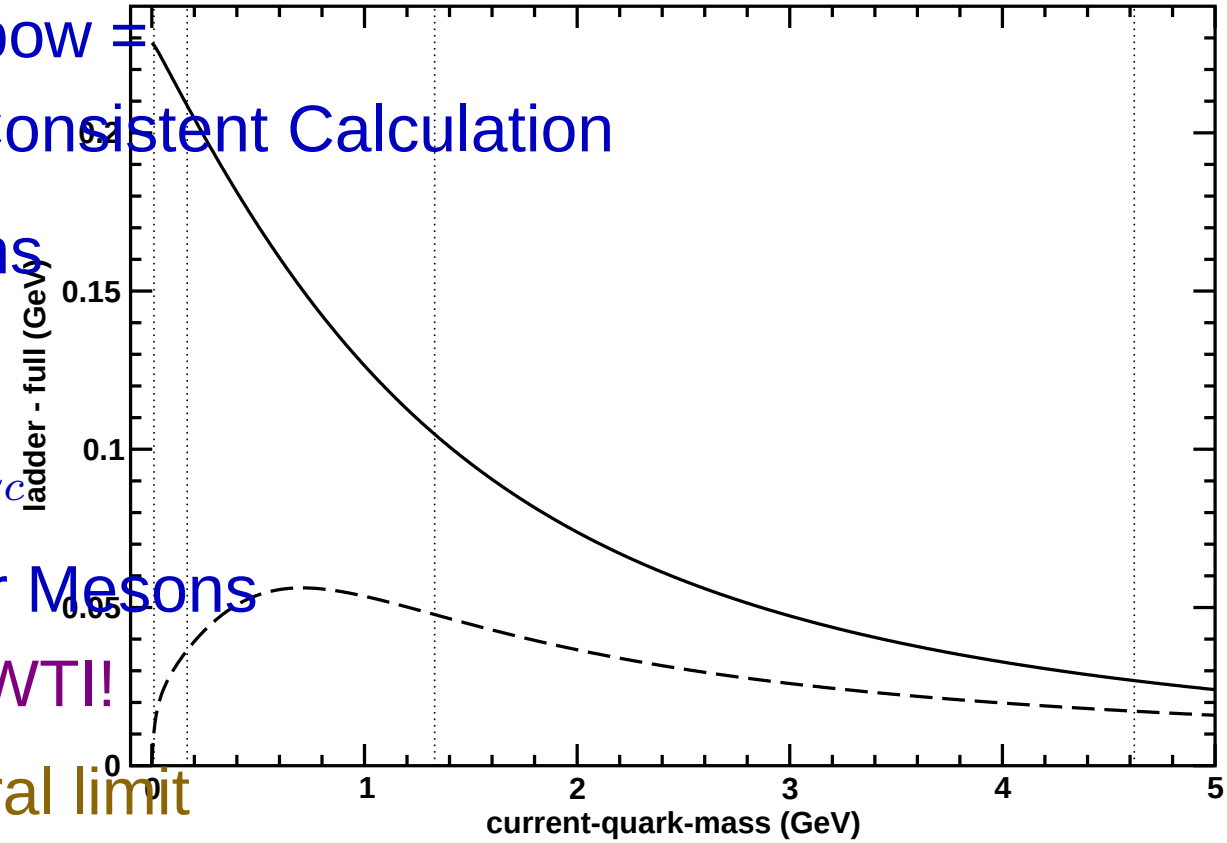
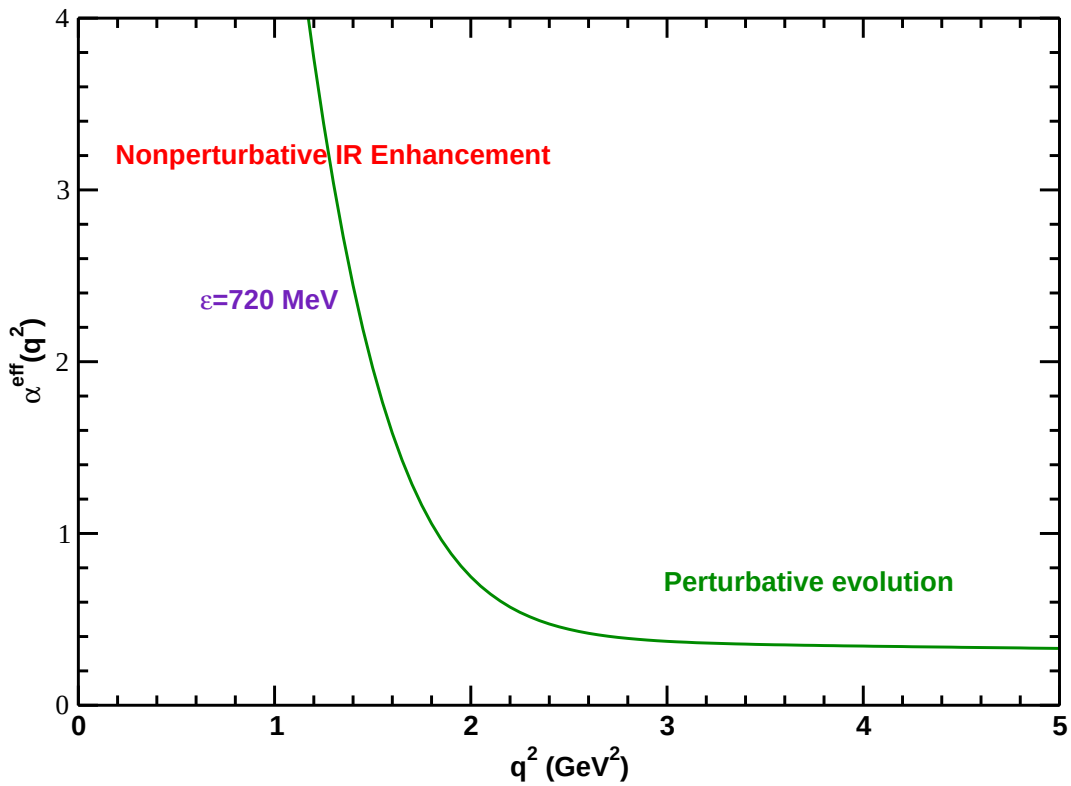


Explains Pattern of Maris-Tandy Success

Interaction

Kernel of Bethe-Salpeter Equation

$$K(p,k;P) \approx \frac{\alpha^{\rm eff}((p-k)^2)}{(p-k)^2}$$



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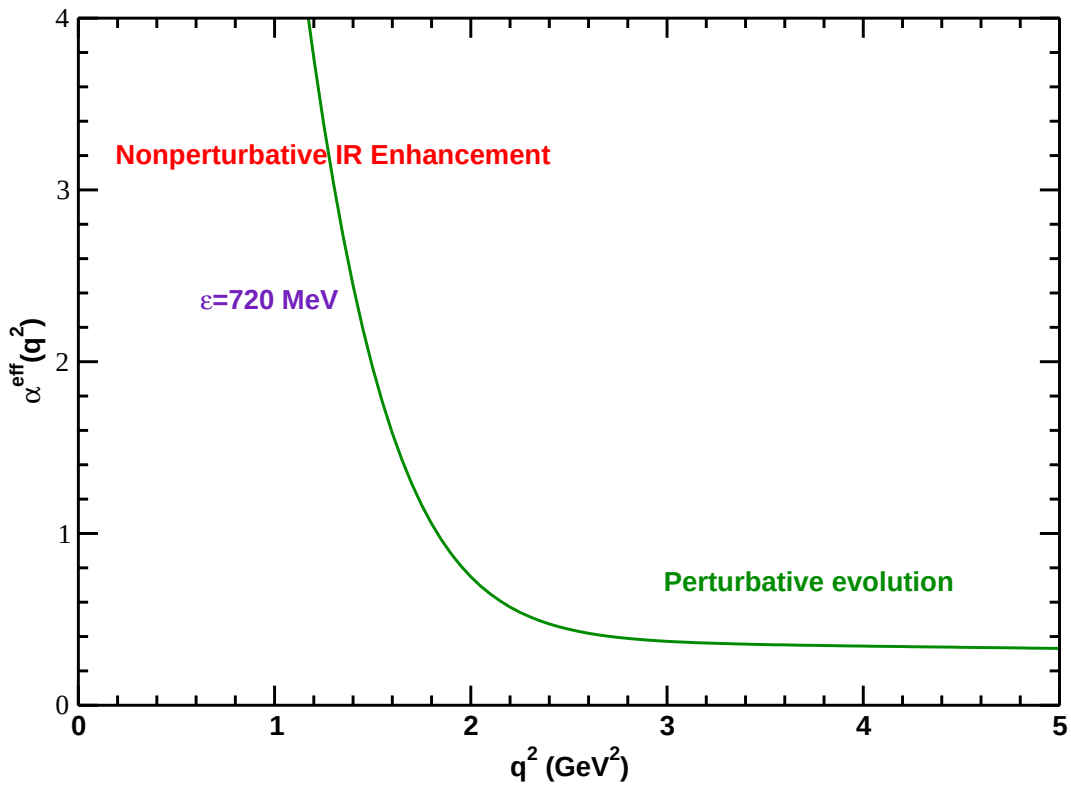

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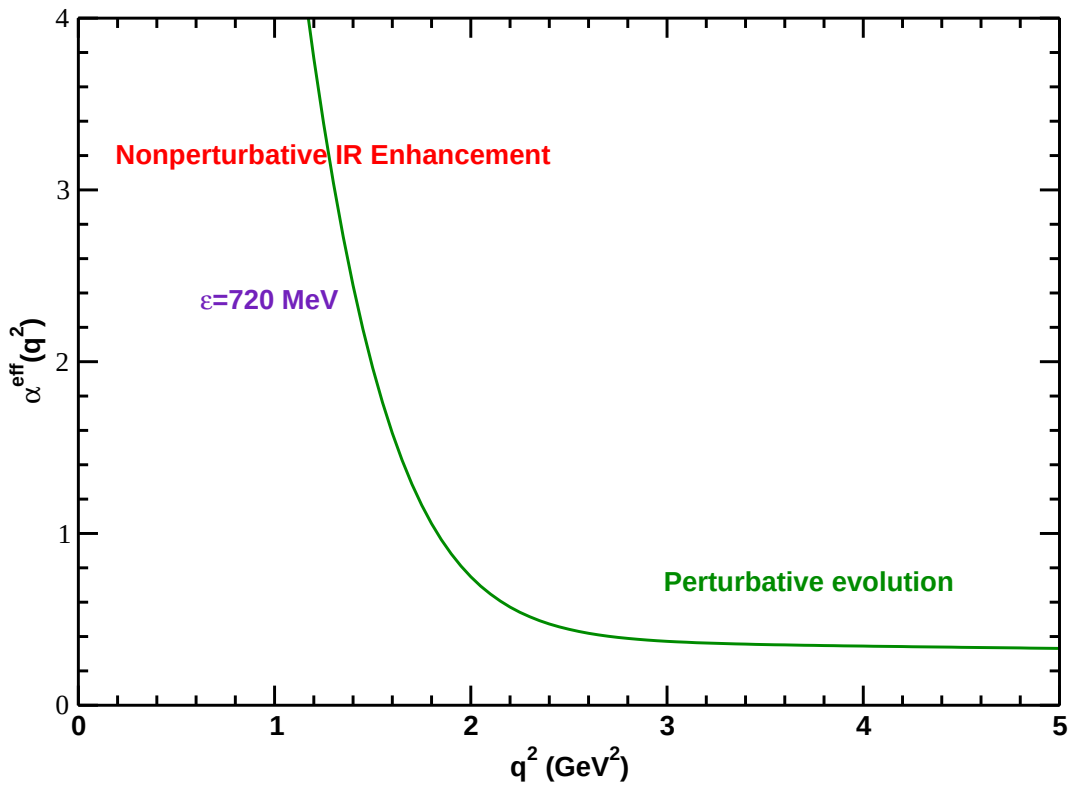


Prescribes Gap Equation's Kernel

Connects *Ansatz* for long-range part of QCD's interaction with Observables.

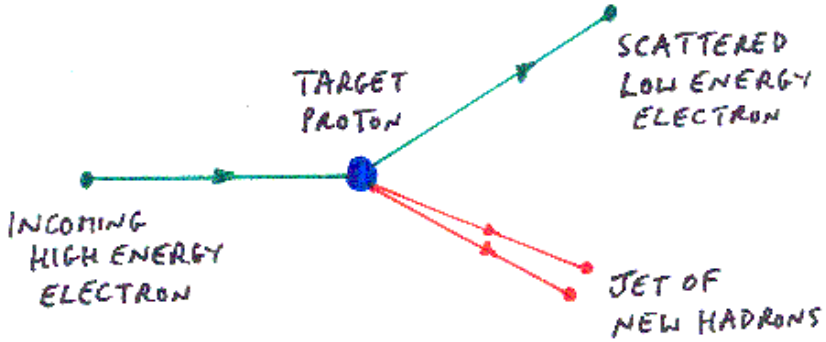
ANL Physics Division

Deep-inelastic scattering

Deep-inelastic scattering

- Looking for Quarks

Deep-inelastic scattering



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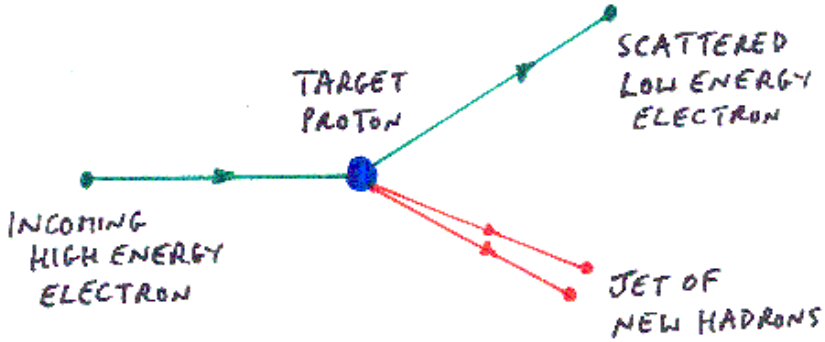

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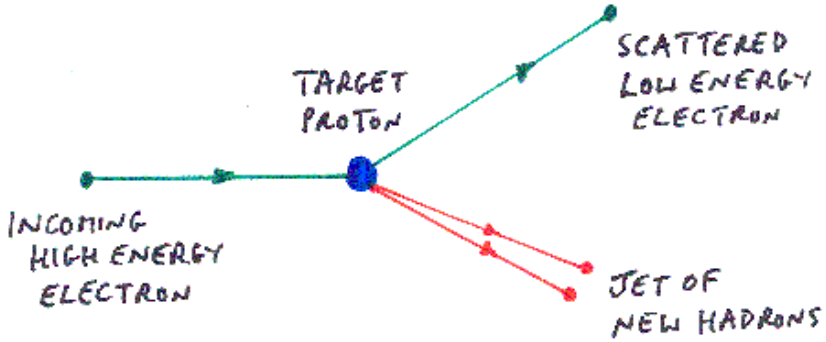
- Signature Experiment for QCD:

  Discovery of Quarks at SLAC

Deep-inelastic scattering



- Looking for Quarks

- Signature Experiment for QCD:

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- Cross-section: Interpreted as Measurement of Momentum-Fraction Prob. Distribution: $q(x)$, $g(x)$

ANL Physics Division

Pion's valence quark distn

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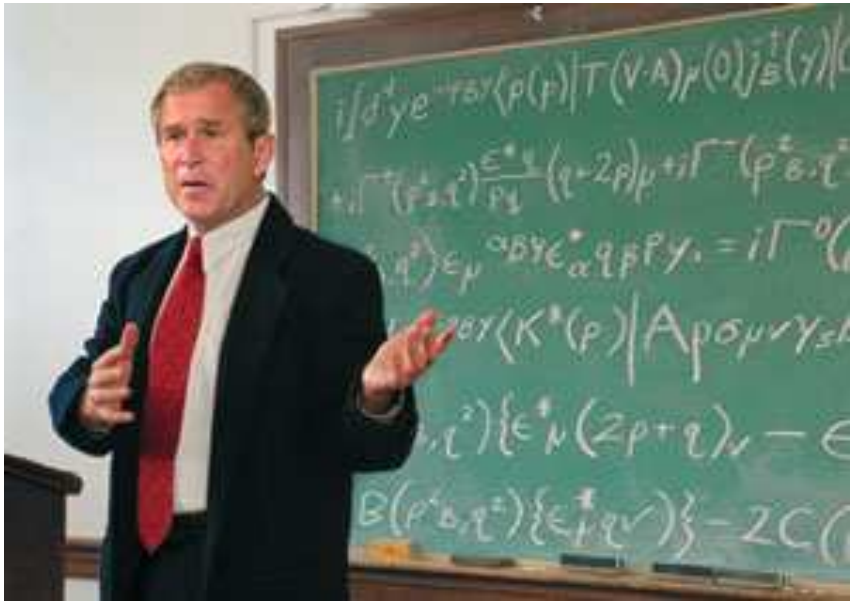
- Proved recently (22/July, ANL)



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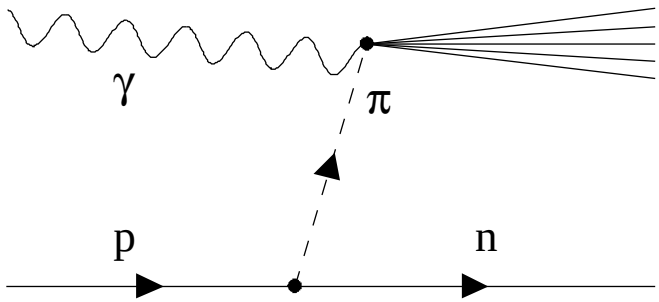
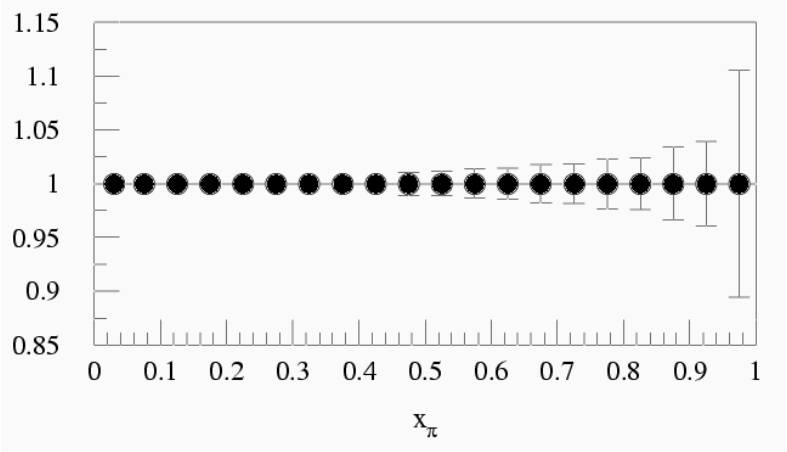
- Proposal `(Holt & Reimer, ANL, nu-ex/0010004)`

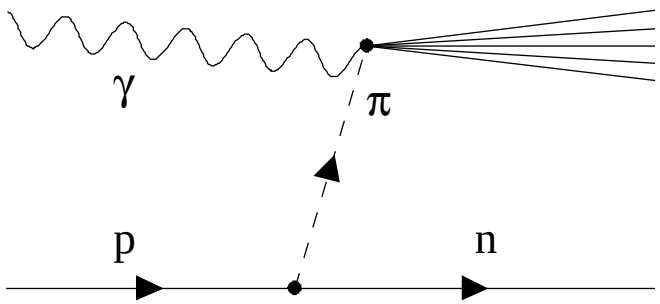
$e^-_{5\mathrm{GeV}} - p_{25\,\mathrm{GeV}}$ Collider $\to$ Accurate "Measurement"

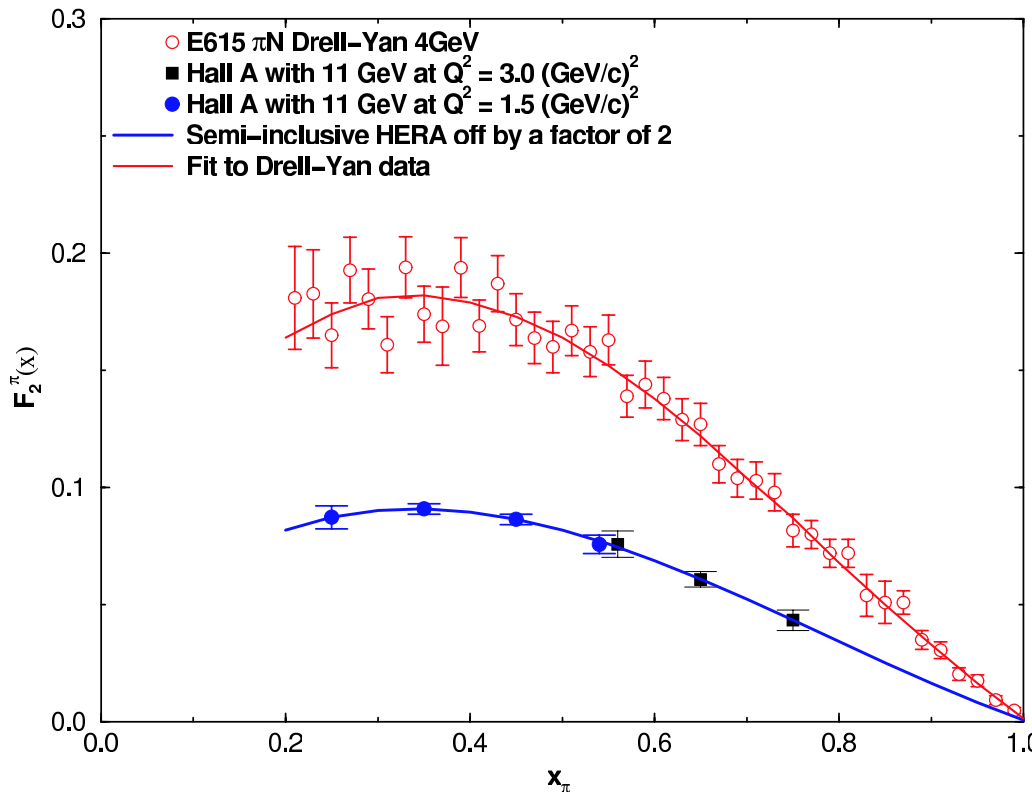




Pion's valence quark distn

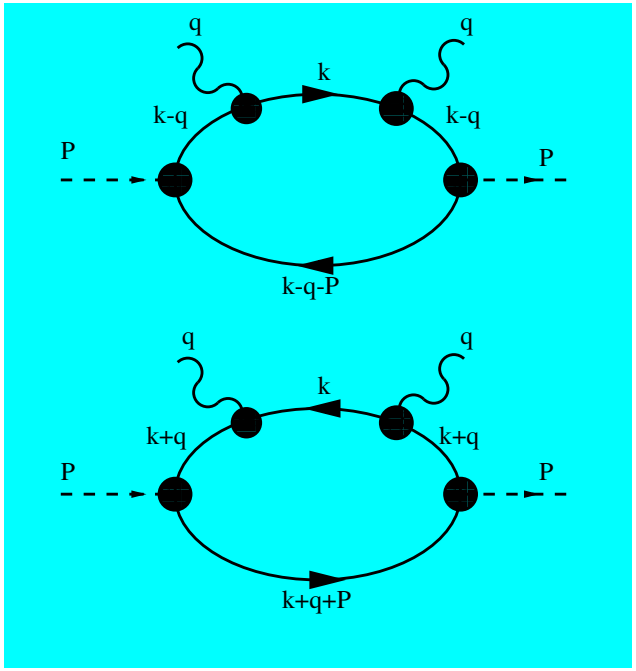
- Proposal at JLab
  (Holt, Reimer, Wijesooriya, et al., JLab at 12 GeV)





ANL Physics Division

Handbag diagrams



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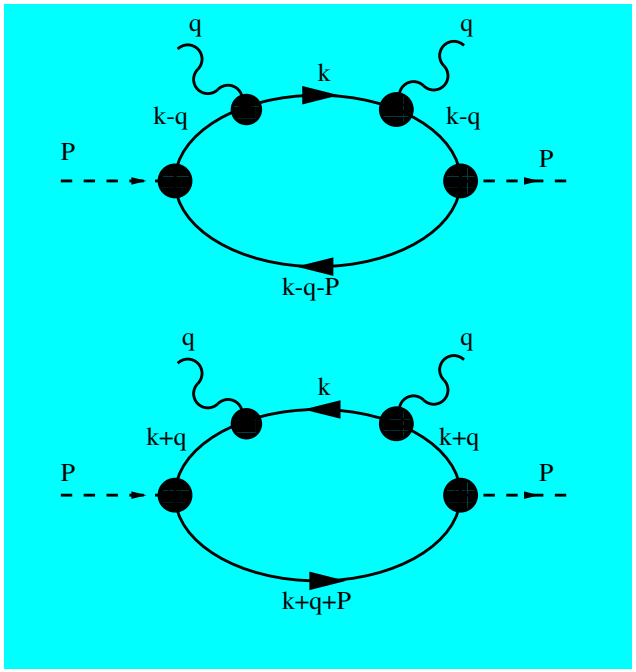


$$W_{\mu\nu}(q;P) = \frac{1}{2\pi}\mathrm{Im}\left[T^{+}_{\mu\nu}(q;P) + T^{-}_{\mu\nu}(q;P)\right]$$

$$T^{+}_{\mu\nu}(q,P) = \mathrm{tr}\int \frac{d^4k}{(2\pi)^4}\,\tau_{-}\bar{\Gamma}_{\pi}(k_{-\frac{1}{2}};-P)\,S(k_{-0})\,ieQ\Gamma_{\nu}(k_{-0},k)$$
$$\times\, S(k)\,ieQ\Gamma_{\mu}(k,k_{-0})\,S(k_{-0})\tau_{+}\Gamma_{\pi}(k_{-\frac{1}{2}};P)\,S(k_{--})$$

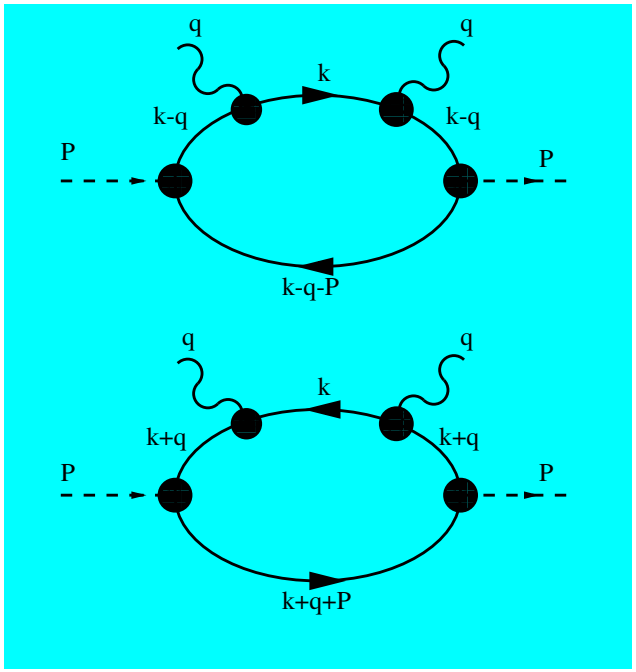
ANL Physics Division

Handbag diagrams

Bjorken Limit: $q^2 \to \infty\,, \quad P \cdot q \to -\infty$

but $x := -\dfrac{q^2}{2P \cdot q}$ fixed.

Numerous algebraic simplifications



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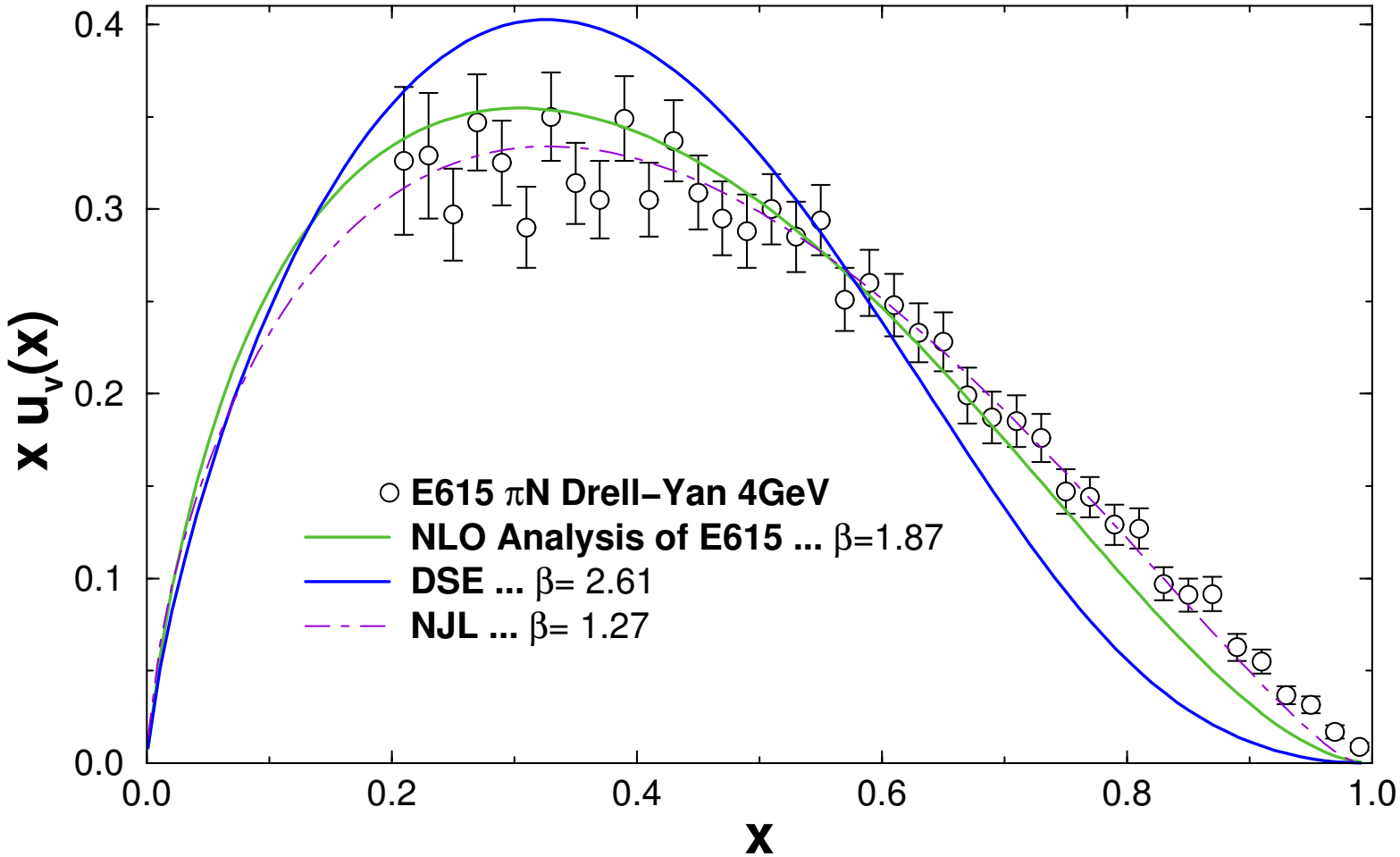
Extant theory vs. experiment

Krishni Wijersooriya, Paul Reimer and Roy Holt, soon to be submitted



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