Dyson-Schwinger Equations: A Tool for Hadron Physics

Craig D. Roberts

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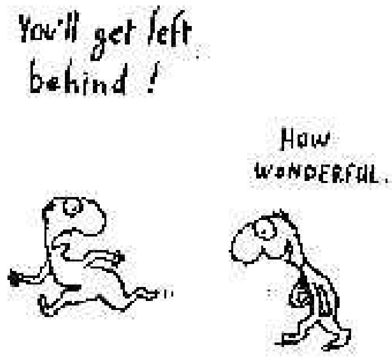
Physics Division, Argonne National Laboratory

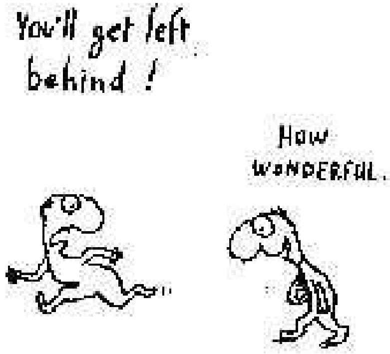
- *Dyson-Schwinger Equations: Density, Temperature and Continuum Strong QCD*
  C.D. Roberts and S.M. Schmidt, nu-th/0005064, Prog. Part. Nucl. Phys. **45** (2000) S1
- *The IR behavior of QCD Green's functions: Confinement, DCSB, and hadrons . . .*
  R. Alkofer and L. von Smekal, he-ph/0007355, Phys. Rept. **353**, 281 (2001)
- *Dyson-Schwinger equations: A Tool for Hadron Physics*
  P. Maris and C.D. Roberts, nu-th/0301049

Dyson-Schwinger Equations



Dyson-Schwinger Equations

- NonPerturbative, Continuum approach to QCD



Dyson-Schwinger Equations

- NonPerturbative, Continuum approach to QCD
- Simplest level: Generating Tool for Perturbation Theory
  .... Materially Reduces Model Dependence

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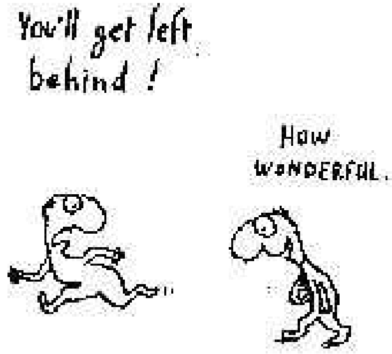

Dyson-Schwinger Equations

- NonPerturbative, Continuum approach to QCD
  - Hadrons as Composites of Quarks and Gluons

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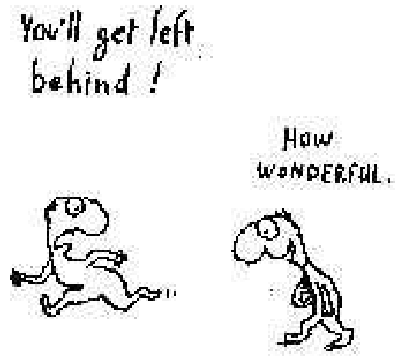

Dyson-Schwinger Equations

- NonPerturbative, Continuum approach to QCD
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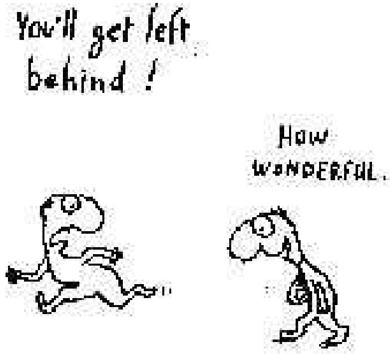
    - Qualitative and Quantitative Importance of:
      - Dynamical Chiral Symmetry Breaking



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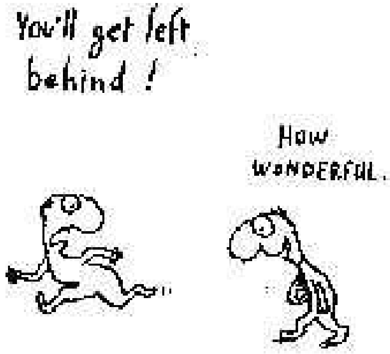

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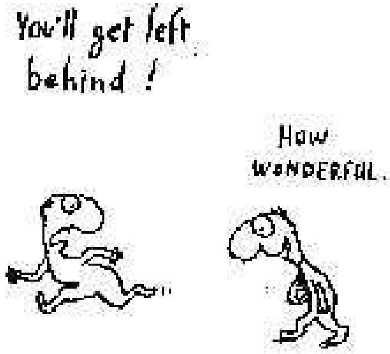
    - $\Rightarrow$ Understanding InfraRed (long-range)

   . . . . behaviour of $\alpha_s(Q^2)$



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Cross-Sections built from Schwinger Functions

Persistent Challenge

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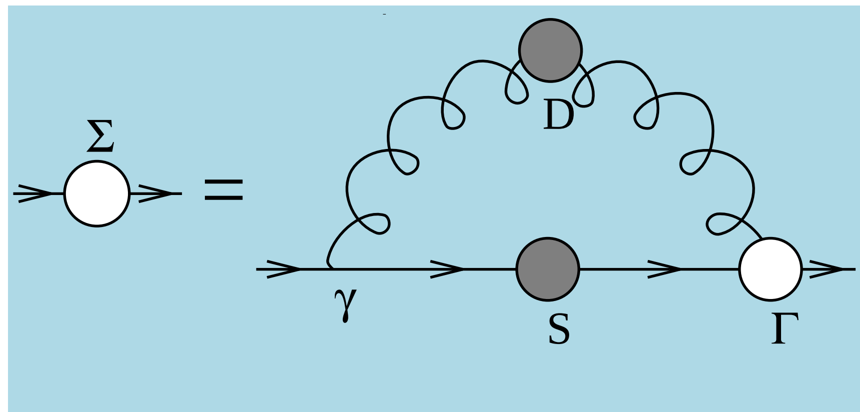
Persistent Challenge

- Infinitely Many Coupled Equations



Persistent Challenge

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  - Weak coupling expansion $\Rightarrow$ Perturbation Theory

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    Not useful for the nonperturbative problems in which we're interested

Persistent Challenge

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- “Many-body physics Approach”

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  - Rainbow-Ladder ($\sim$ Hartree-Fock) truncation Widespread exploration of consequences

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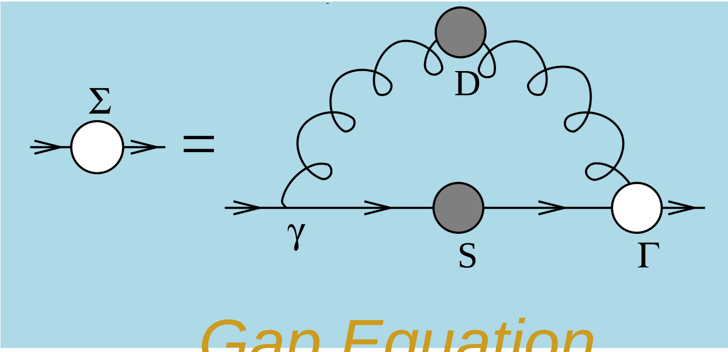
Describe this in some detail

Dressed-quark Propagator

Dressed-quark Propagator

$$S(p) = \frac{Z(p^2)}{i\gamma \cdot p + M(p^2)}$$



Gap Equation

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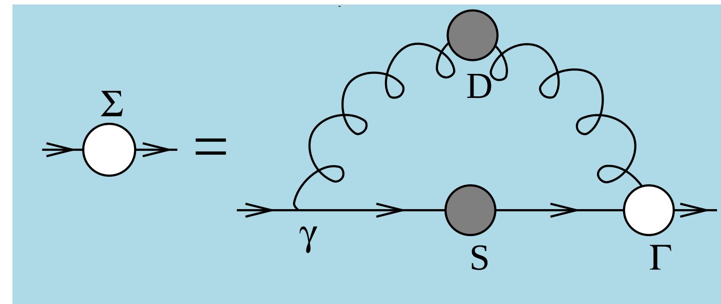

Gap Equation

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$$S(p) = \frac{1}{i\gamma \cdot p\, A(p^2) + B(p^2)}$$

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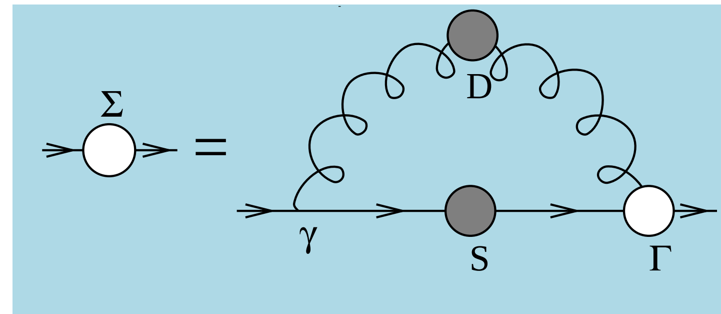

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$$S(p) = \frac{1}{i\gamma \cdot p\, A(p^2) + B(p^2)}$$

- Weak Coupling Expansion

  Reproduces Every Diagram in Perturbation Theory

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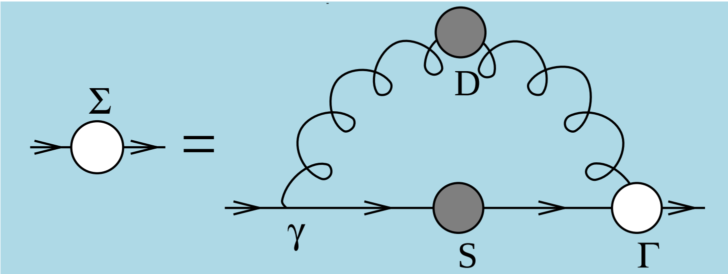

Gap Equation

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$$S(p) = \frac{1}{i\gamma\cdot p\, A(p^2) + B(p^2)}$$

- But in Perturbation Theory

$$B(p^2) = m\left(1 - \frac{\alpha}{\pi}\ln\left[\frac{p^2}{m^2}\right] + \ldots\right) \stackrel{m\to 0}{\to} 0$$

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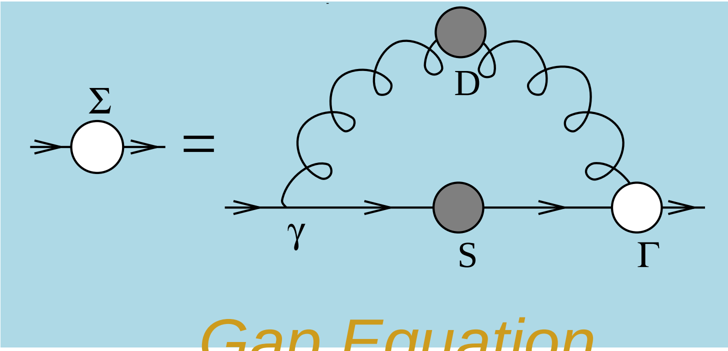
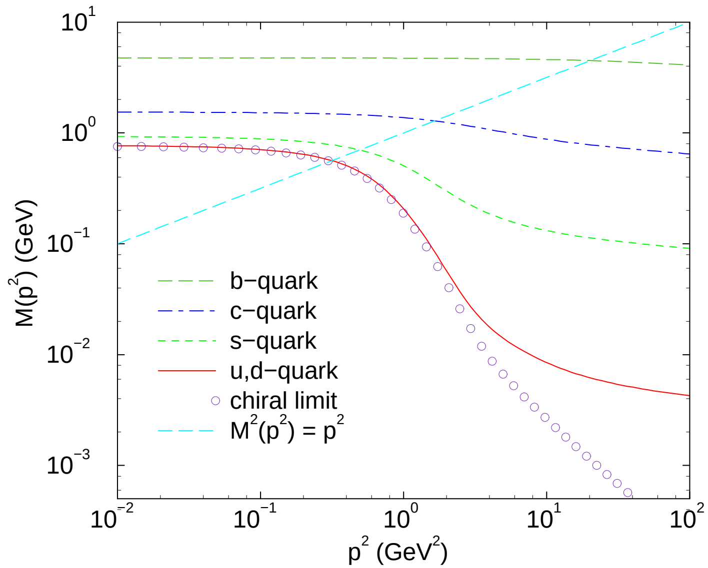


Gap Equation

- Enhancement of Gap Equation's Kernel on IR domain
  $\Rightarrow$ IR Enhancement of $M(p^2)$



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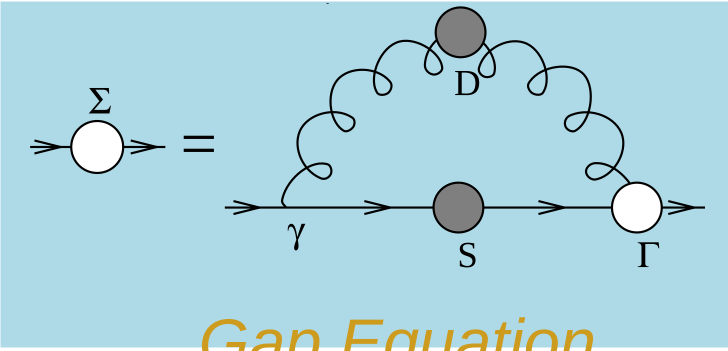

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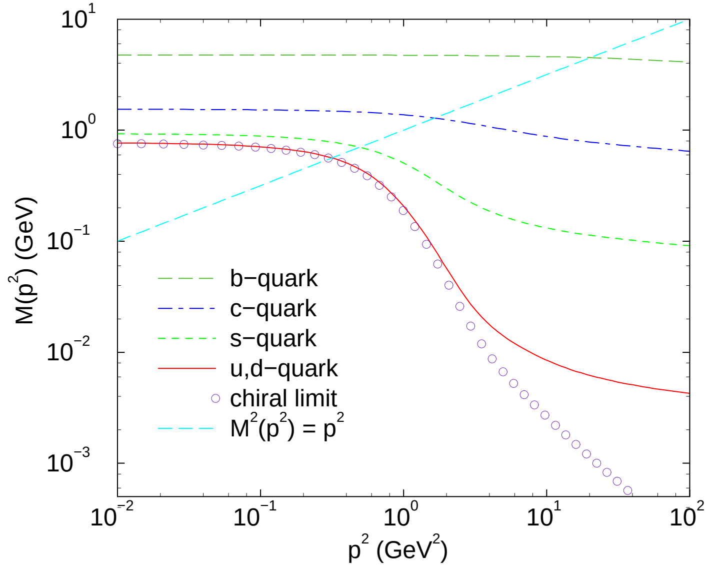
- Euclidean Constituent–Quark Mass: M_f^E: $p^2 = M(p^2)^2$



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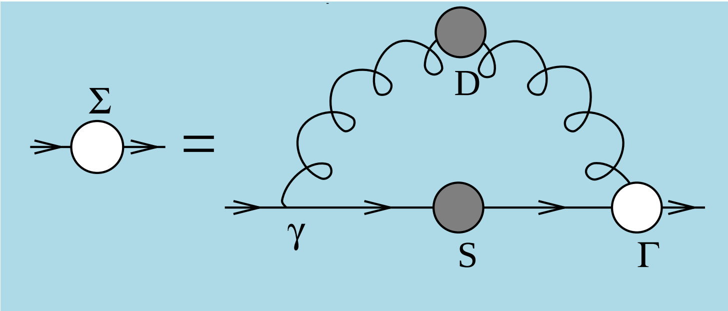

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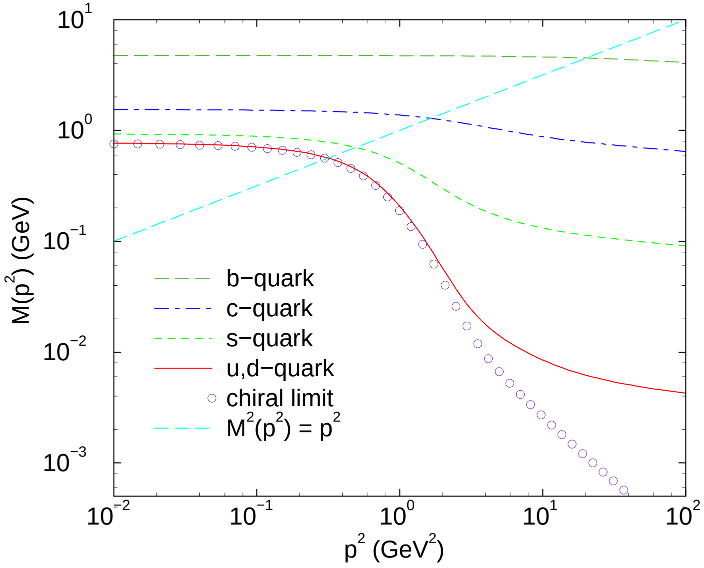
flavour	u/d	s	c	b	t
$\frac{M^E}{m_{\mu\sim 20\,\mathrm{GeV}}}$	150	10	2.2	1.2	$\rightarrow 1$



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Vacuum quark condensate

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Vacuum quark condensate

Trace of the Chiral-limit dressed-quark propagator

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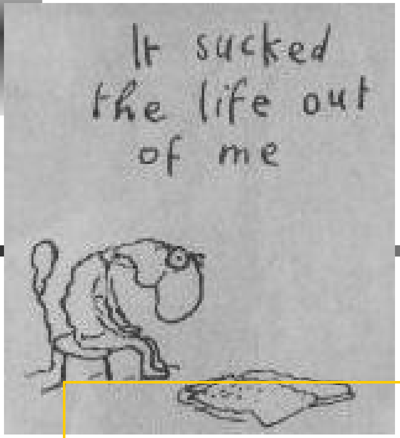
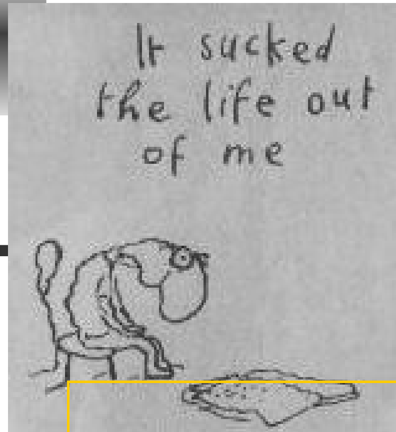


Vacuum quark condensate

$$- \langle \bar{q}q \rangle_\zeta^0 \Big|_{\zeta=19\,\mathrm{GeV}} =$$

$$Z_4(\zeta,\Lambda)\;N_c \int_q^\Lambda \mathrm{tr}_{\mathrm{Dirac}}\left[S_{\hat{m}=0}(q)\right] = \;(0.275\,\mathrm{GeV})^3$$



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- Mass renormalisation constant

  Ensures finiteness and correct renormalisation group flow

  $m(\zeta)\,\langle \bar{q}q\rangle_\zeta^0 =$ const.

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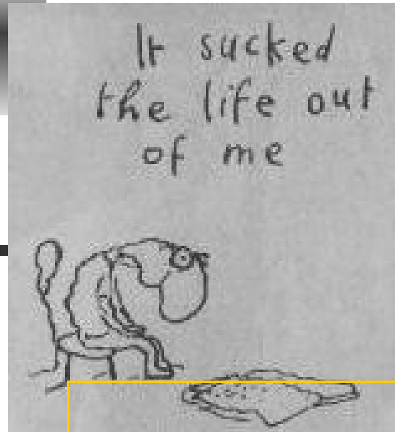

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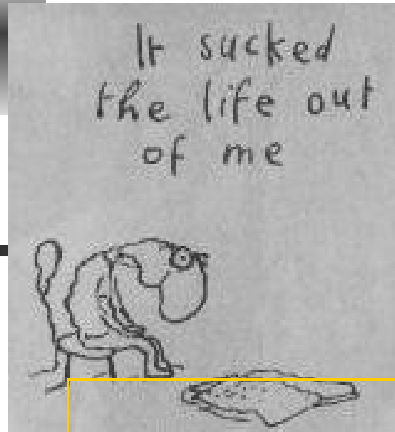

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Numerical Value

π and K observables

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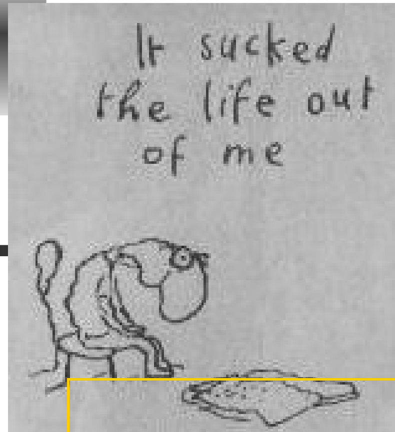

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Corresponds to

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Close packed spheres:

$$1.8\,\mathrm{fm}^{-3} \Rightarrow V_{\langle \bar{q}q\rangle} = 0.55\,\mathrm{fm}^3 \Rightarrow r_{\langle \bar{q}q\rangle} = 0.51\,\mathrm{fm} = 0.77\,r_\pi = 0.58\,r_p$$

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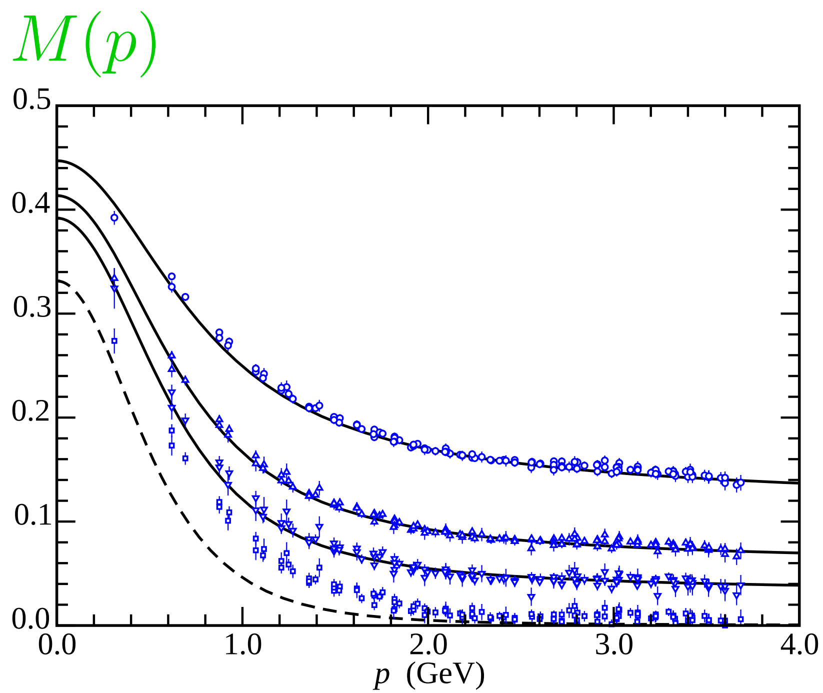
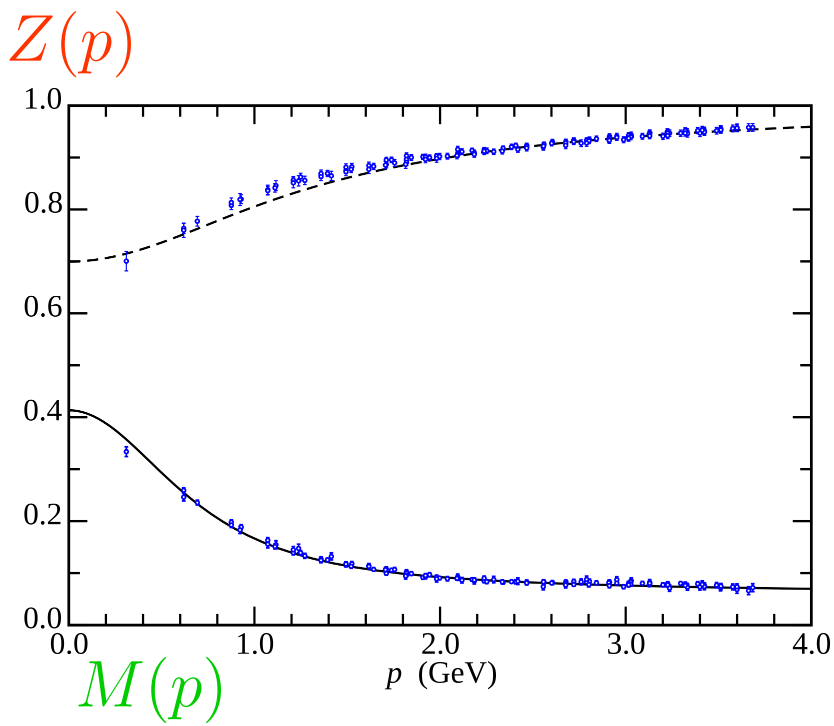
Dressing is a Fact

Fundamentally important in QCD, Observable consequences

Long-standing prediction of DSEs

cf. Quenched lattice simulations





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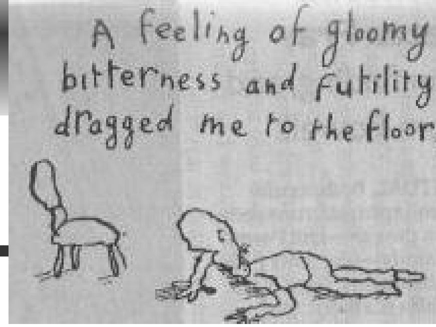

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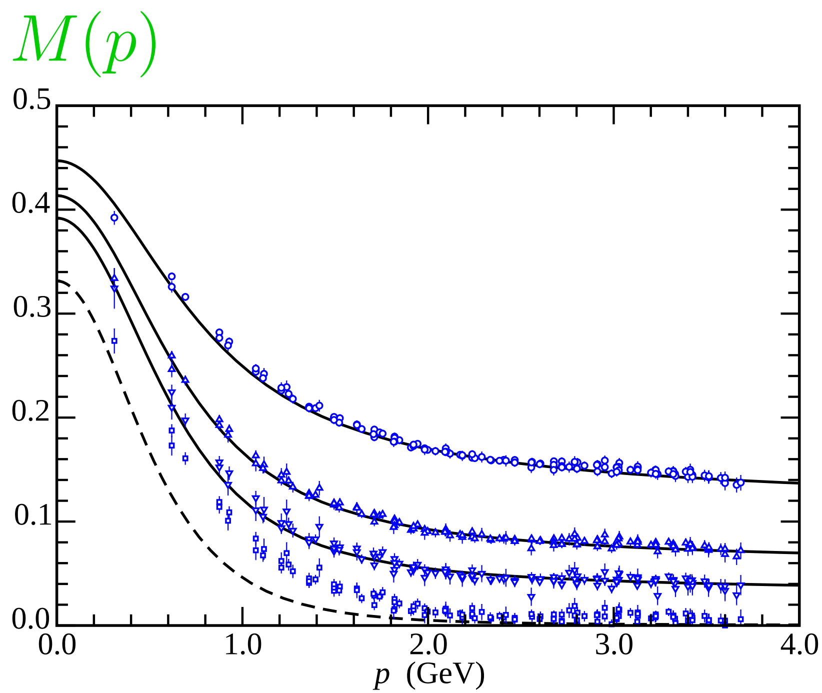

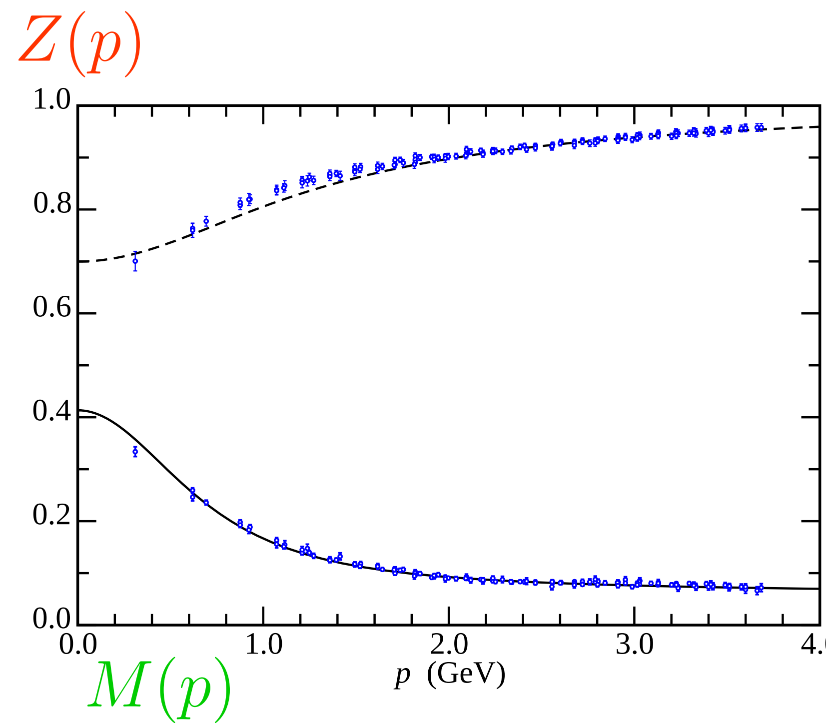

- Lattice Meas. – Bowman, Heller, Williams: he-lat/0203001

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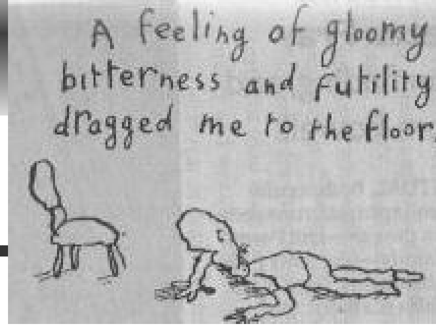

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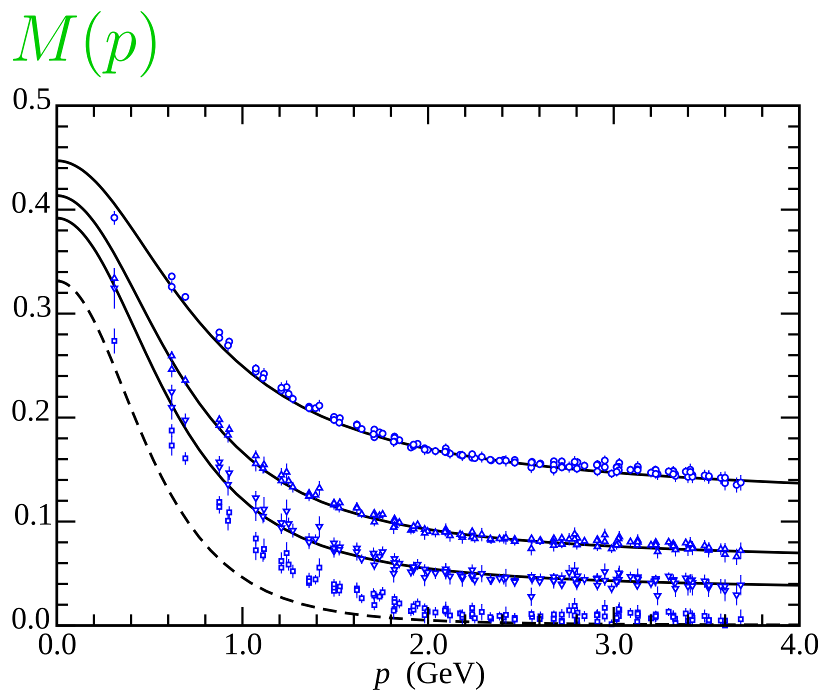

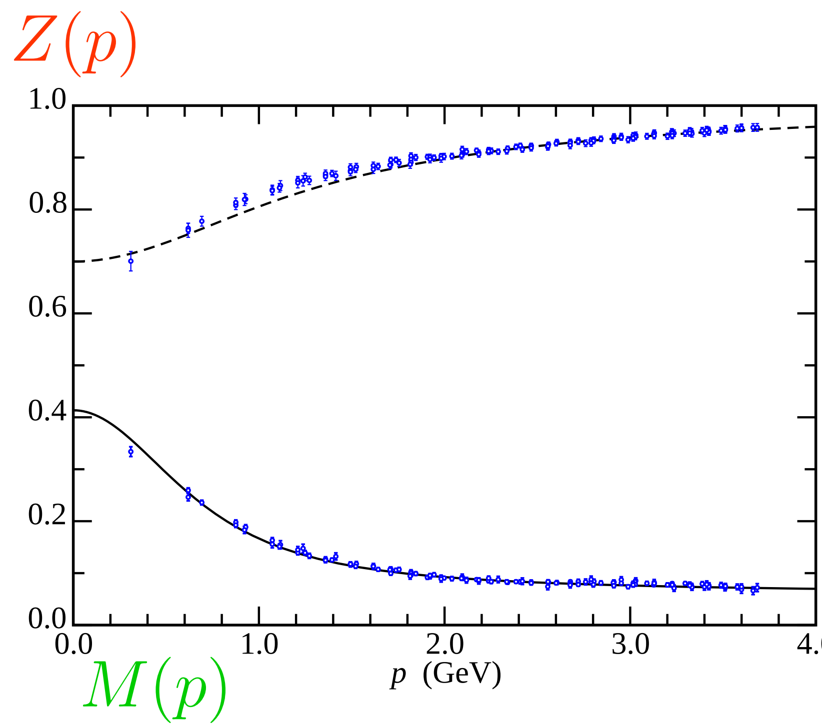

- Lattice Meas. – Bowman, Heller, Williams: he-lat/0203001
- DSE Cal.– Bhagwat, Pichowsky, CDR, Tandy nu-th/0304003

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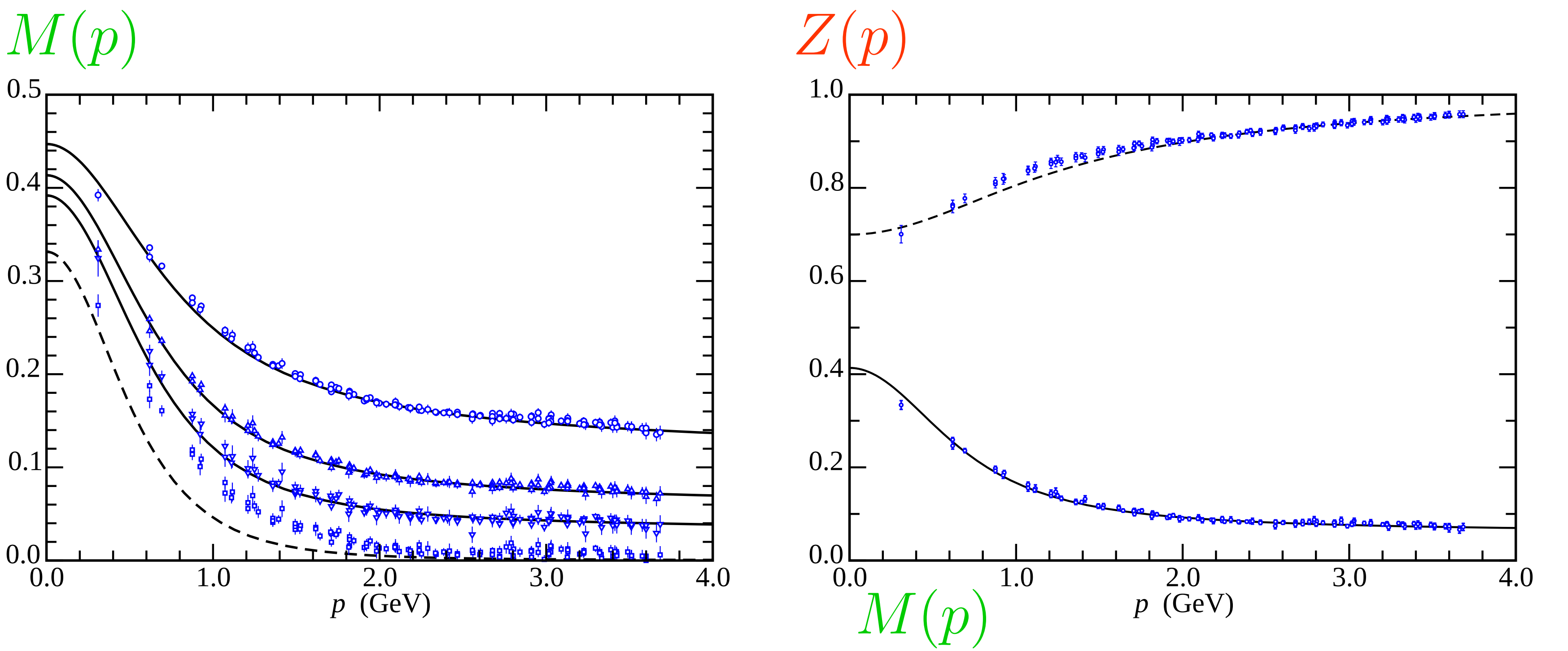

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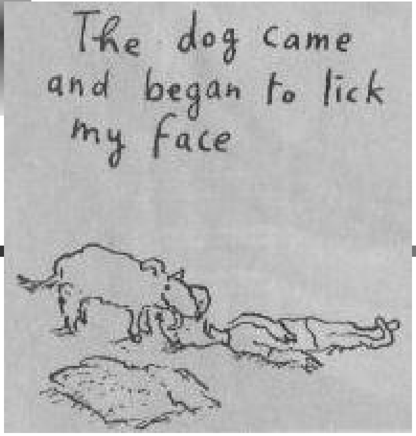
$$f_\pi^0 = 0.068\,\mathrm{GeV} \quad \langle \bar{q}q \rangle^0_{1\,\mathrm{GeV}} = (-0.19\,\mathrm{GeV})^3\,.$$

Hadrons

Hadrons

- Established understanding of two point functions
- What about bound states?



Hadrons

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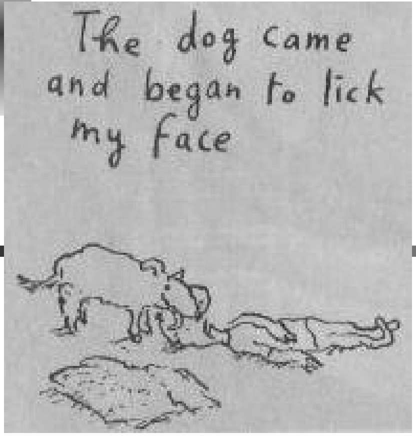
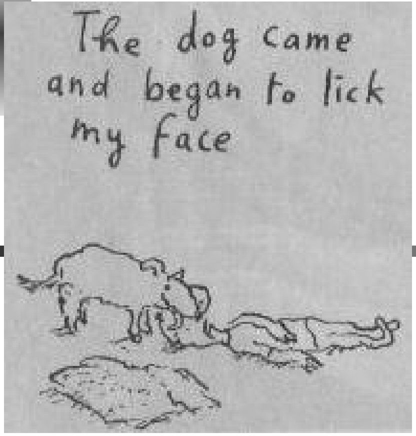


Hadrons

- Without bound states, Comparison with experiment is impossible
- They're pole contributions to $n \geq 3$-point functions



Hadrons

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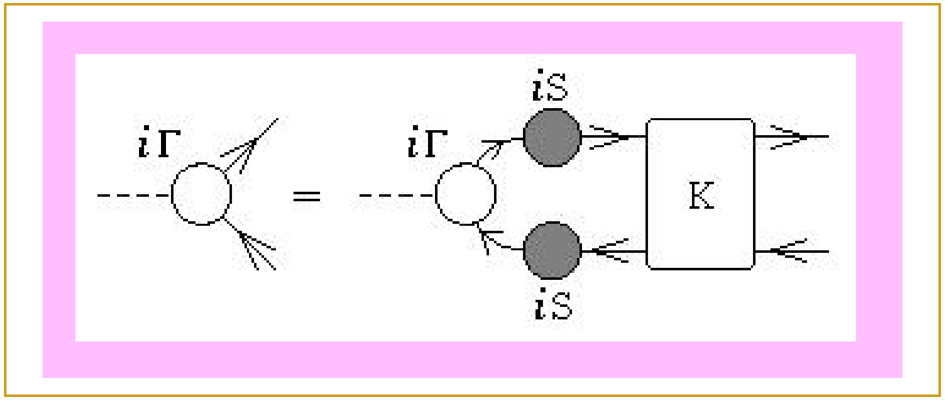
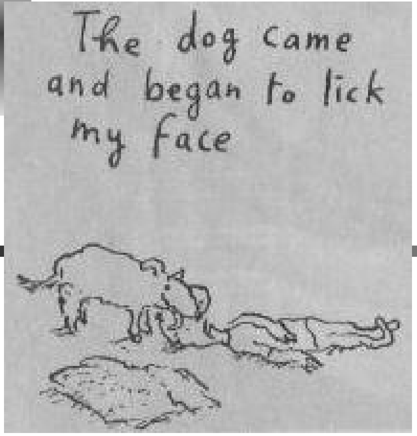


QFT Generalisation of Lippman-Schwinger Equation.



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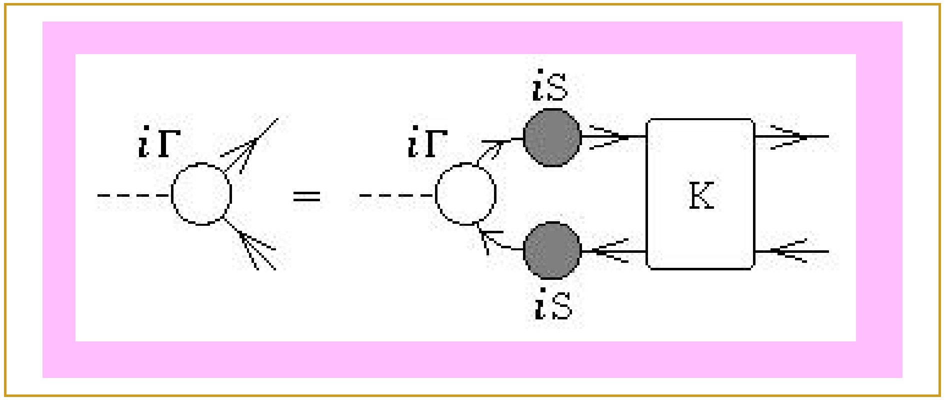

QFT Generalisation of Lippman-Schwinger Equation.

- What is the kernel, K?

- Without bound states, Comparison with experiment is impossible
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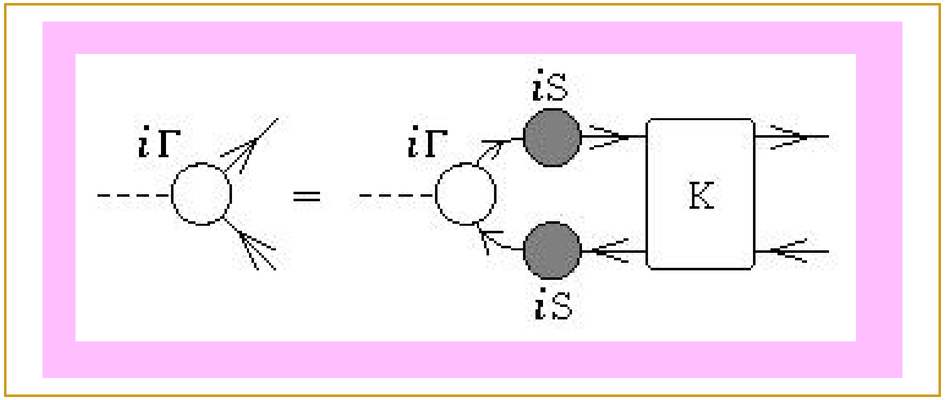

QFT Generalisation of Lippman-Schwinger Equation.

- What is the kernel, K?

or What is the long-range potential in QCD?

Dichotomy of the Pion

- How does one make an almost massless particle
  . . . . from two heavy constituents?

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a well-defined and valid chiral limit.

- Requires detailed understanding of Connection between Current-quark and Constituent-quark masses

Bethe-Salpeter Kernel

Bethe-Salpeter Kernel

- Axial-vector Ward-Takahashi identity

$$P_\mu \; \Gamma^l_{5\mu}(k;P) \quad = \quad \mathcal{S}^{-1}(k_+)\,\frac{1}{2}\lambda^l_f i\gamma_5 + \frac{1}{2}\lambda^l_f i\gamma_5 \; \mathcal{S}^{-1}(k_-)$$

$$-M_\zeta \, i\Gamma^l_5(k;P) - i\Gamma^l_5(k;P)\, M_\zeta$$

QFT Statement of Chiral Symmetry

Bethe-Salpeter Kernel

- Axial-vector Ward-Takahashi identity

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Satisfies BSE

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Kernels must be intimately related

Bethe-Salpeter Kernel

- Axial-vector Ward-Takahashi identity

$$
\begin{aligned}
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& - M_\zeta \, i\Gamma^l_5(k;P) - i\Gamma^l_5(k;P)\, M_\zeta
\end{aligned}
$$

Satisfies BSE　　　　Satisfies DSE

Kernels must be intimately related

- Relation must be preserved by truncation

Bethe-Salpeter Kernel

- Axial-vector Ward-Takahashi identity

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Satisfies BSE | Satisfies DSE

Kernels must be intimately related

- Relation must be preserved by truncation
- Nontrivial constraint

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Satisfies BSE | Satisfies DSE

Kernels must be intimately related

- Relation must be preserved by truncation
- Failure $\Rightarrow$ Violation of Goldstone's Theorem

Bad Idea

Laying a snare for oneself

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Goldberger-Treiman for pion

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Goldberger-Treiman for pion

- Pseudoscalar Bethe-Salpeter amplitude

$$\begin{aligned}\Gamma_{\pi^j}(k;P) &= \tau^{\pi^j}\gamma_5\Big[iE_\pi(k;P) + \gamma\cdot P F_\pi(k;P) \\ &\quad + \gamma\cdot k\, k\cdot P\, G_\pi(k;P) + \sigma_{\mu\nu}\, k_\mu P_\nu\, H_\pi(k;P)\Big]\end{aligned}$$

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$\Rightarrow$ $$f_\pi E_\pi(k;P=0) = B(k^2)$$

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Goldberger-Treiman for pion

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$\Rightarrow$

$$\begin{aligned} f_\pi E_\pi(k;P=0) &= B(k^2) \\ F_R(k;0) + 2\, f_\pi F_\pi(k;0) &= A(k^2) \\ G_R(k;0) + 2\, f_\pi G_\pi(k;0) &= 2A'(k^2) \\ H_R(k;0) + 2\, f_\pi H_\pi(k;0) &= 0 \end{aligned}$$

Goldberger-Treiman for pion

- Pseudoscalar Bethe-Salpeter amplitude

Pseudovector components necessarily nonzero

$$\Gamma_{\pi^j}(k;P) = \tau^{\pi^j}\gamma_5\left[iE_\pi(k;P) + \gamma\cdot P F_\pi(k;P) + \gamma\cdot k\, k\cdot P\, G_\pi(k;P) + \sigma_{\mu\nu}\, k_\mu P_\nu\, H_\pi(k;P)\right]$$

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Exact in Chiral QCD

Bethe-Salpeter equation

Bethe-Salpeter equation

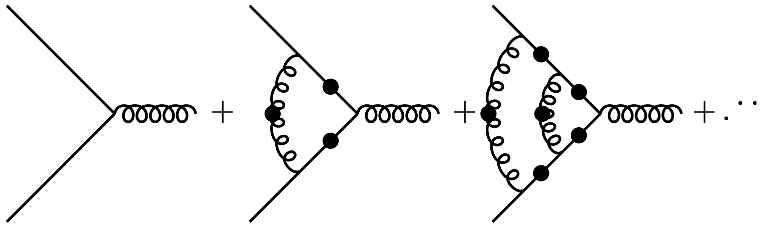
- Electromagnetic & chiral current conservation

  Expressed via Ward-Takahashi identities

Bethe-Salpeter equation

- Electromagnetic & chiral current conservation
  Expressed via Ward-Takahashi identities
- NB. Pole contributions in colour-singlet vertices
  completely describe colour singlet bound states

Bethe-Salpeter equation

- Dressed-ladder vertex $\Gamma^a_\mu(k,p) \; =$ 

Bethe-Salpeter equation

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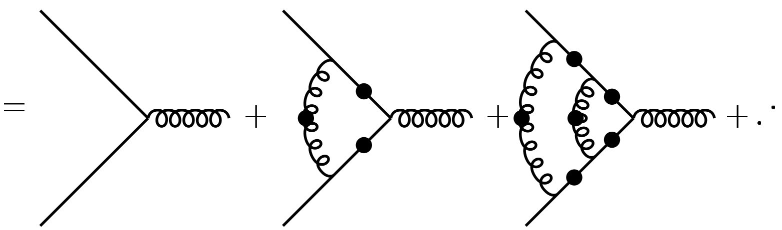
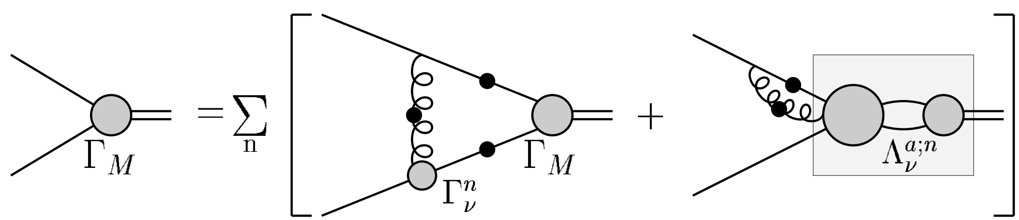


- BSE consistent with dressed-ladder vertex



Bethe-Salpeter equation

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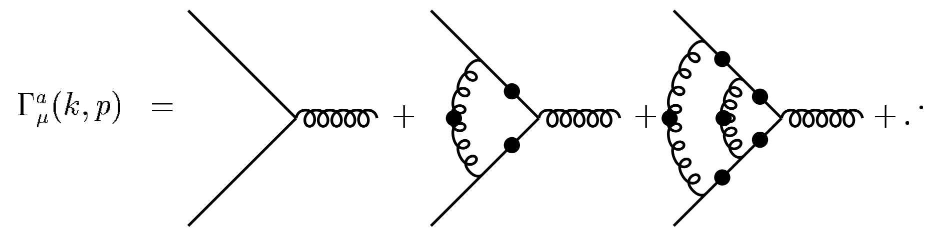

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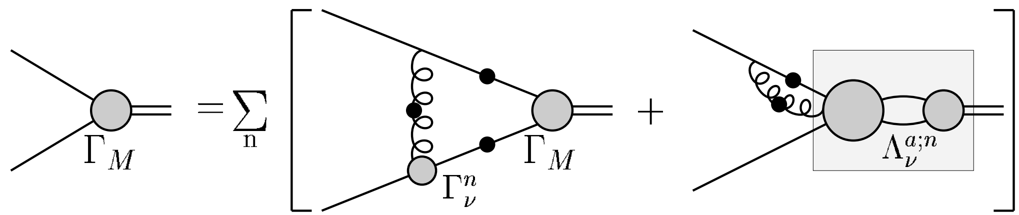
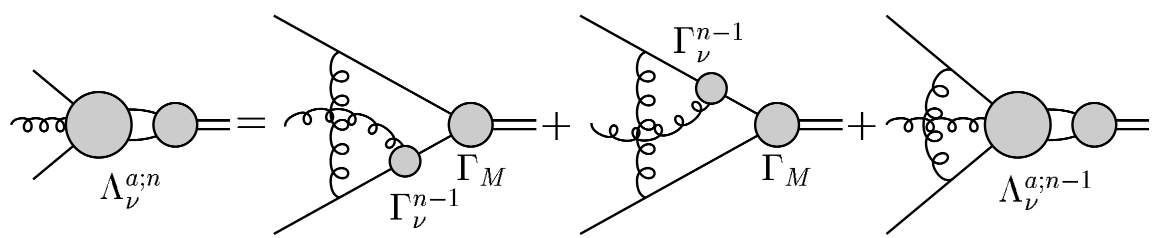


- Bethe-Salpeter kernel . . . recursion relation



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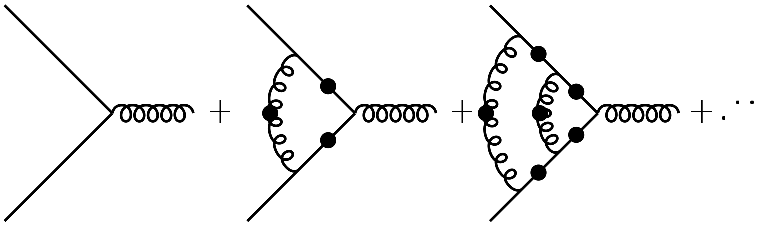
- Dressed-ladder vertex $\Gamma^a_\mu(k,p) =$ 

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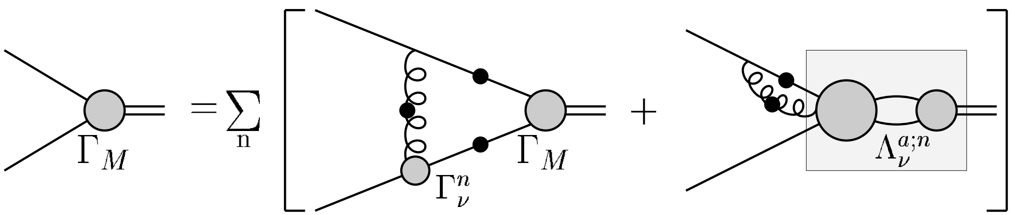

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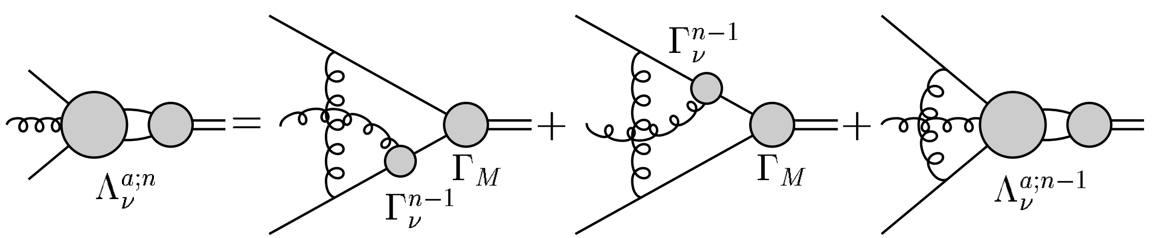

- Kernel necessarily non-planar, even with planar vertex

π and ρ mesons

π and ρ mesons

	$M_H^{n=0}$	$M_H^{n=1}$	$M_H^{n=2}$	$M_H^{n=\infty}$
π, $m = 0$	0	0	0	0
π, $m = 0.011$	0.152	0.152	0.152	0.152
ρ, $m = 0$	0.678	0.745	0.754	0.754
ρ, $m = 0.011$	0.695	0.762	0.770	0.770

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  For $\boldsymbol{m_\rho}$ – zeroth order, accurate to 10%

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  For m_ρ – zeroth order, accurate to 10%

  – one loop, accurate to 1%

Ab-Initio **Calculations**

Ab-Initio **Calculations**

Pieter Maris

Peter Tandy

Ab-Initio **Calculations**

Maris & Tandy, Series of Five Articles: 1999 – Present

Perfected a Renormalisation-Group Improved Rainbow-Ladder Model of Quark-Quark Interaction

Ab-Initio **Calculations**

Maris & Tandy, Series of Five Articles: 1999 – Present

Perfected a Renormalisation-Group Improved Rainbow-Ladder Model of Quark-Quark Interaction

- Rainbow-Ladder $=$ First Order in Truncation Described Above
- Anticipate Accurate for 0^- & 1^- Mesons

Ab-Initio **Calculations**

Maris & Tandy, Series of Five Articles: 1999 – Present

Perfected a Renormalisation-Group Improved Rainbow-Ladder Model of Quark-Quark Interaction

- One Parameter $=$ Interaction Energy:

  $$\boxed{\mathcal{E} \approx 700\,\text{MeV}}$$

- Dressed-Glue Mass scale:

  Characterises DCSB and light-quark Confinement

- Both Phenomena Disappear for $\mathcal{E} \lesssim 200\,\text{MeV}$

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- *Dyson-Schwinger equations:*
  A Tool for Hadron Physics
  P. Maris and C.D. Roberts, nu-th/0301049

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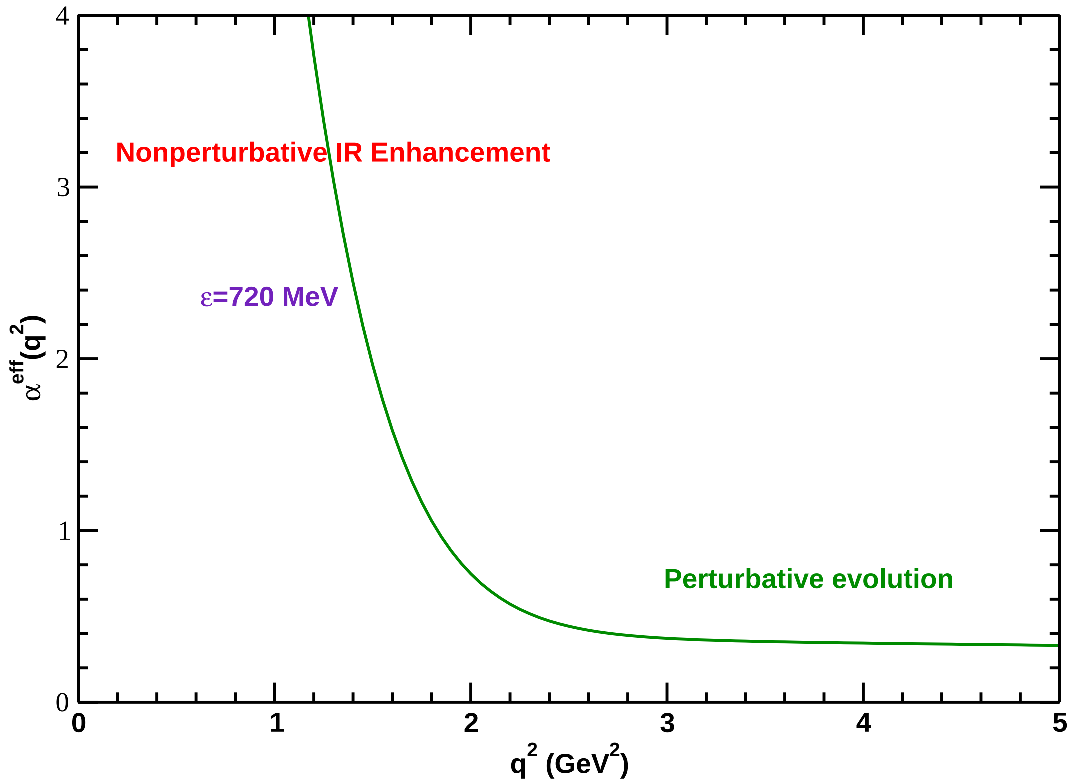
Interaction

Kernel of Bethe-Salpeter Equation

$$K(p,k;P) \approx \frac{\alpha^{\rm eff}((p-k)^2)}{(p-k)^2}$$



Interaction

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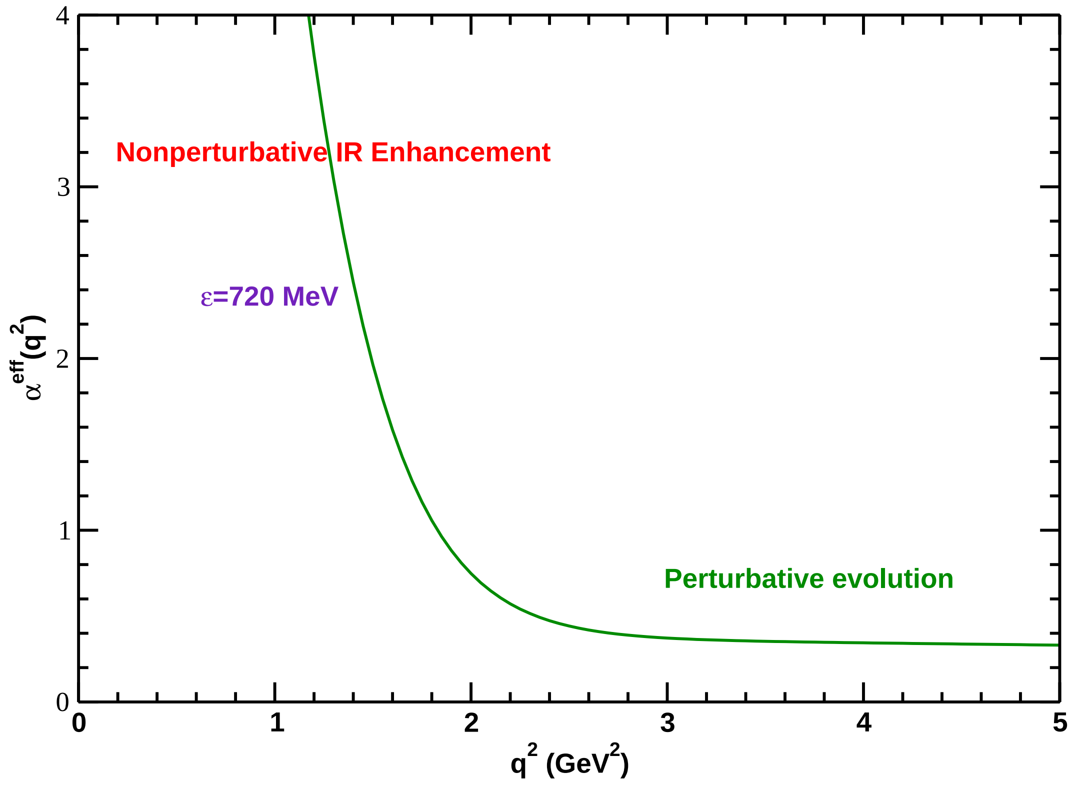
Prescribes Gap Equation's Kernel



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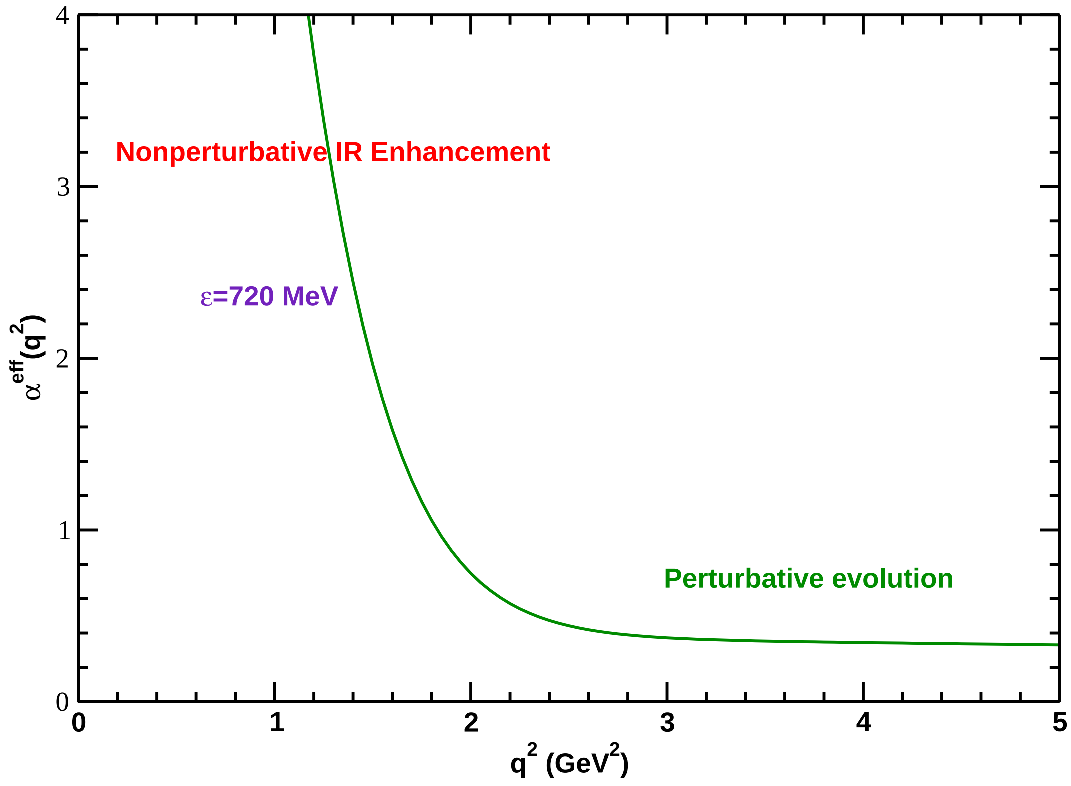


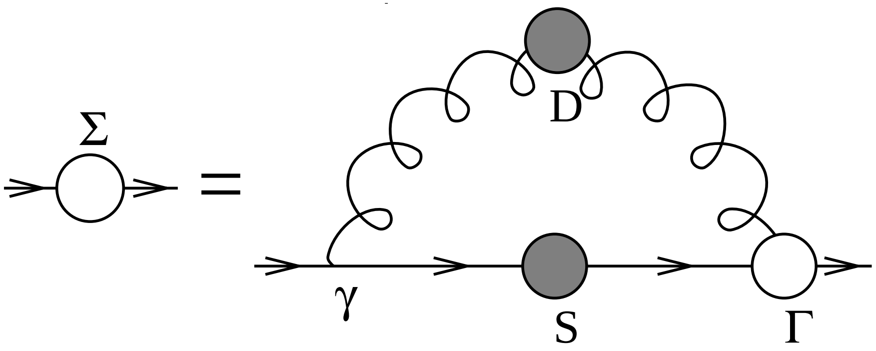
Connects *Ansatz* for long-range part of QCD's interaction with Observables.

Pion Form Factor

Procedure Now Straightforward

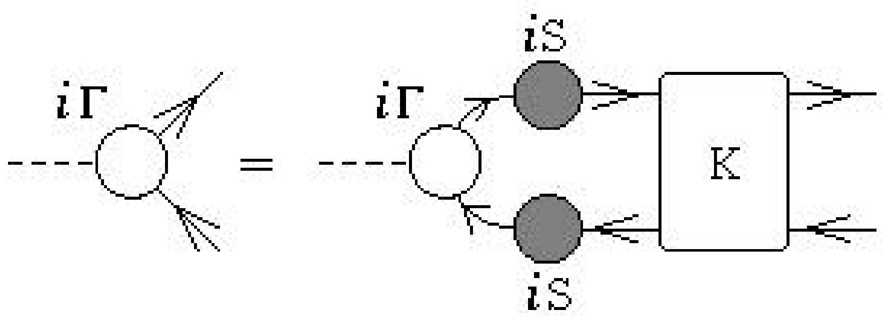
Pion Form Factor

- Solve Gap Equation
  $\Rightarrow$ Dressed-Quark Propagator, $S(p)$



Pion Form Factor

- Use that to Complete Bethe Salpeter Kernel, K
- Solve Homogeneous Bethe-Salpeter Equation for Pion Bethe-Salpeter Amplitude, Γ_π



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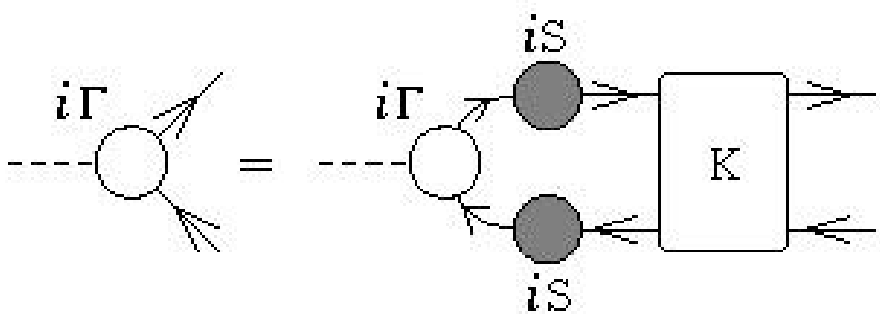
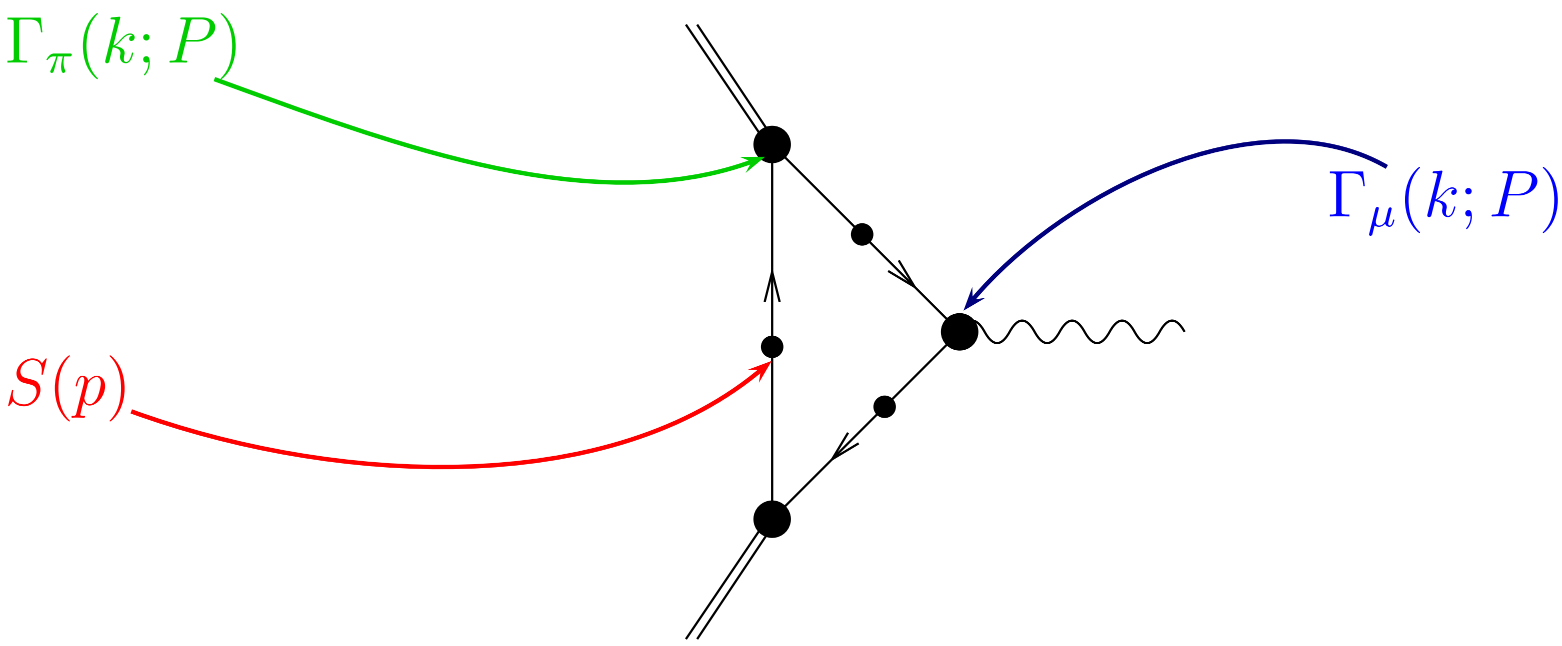


- Solve Inhomogeneous Bethe-Salpeter Equation for Dressed-Quark-Gluon Vertex, Γ_μ

Pion Form Factor

- Now have all elements for Impulse Approximation to Electromagnetic Pion Form factor



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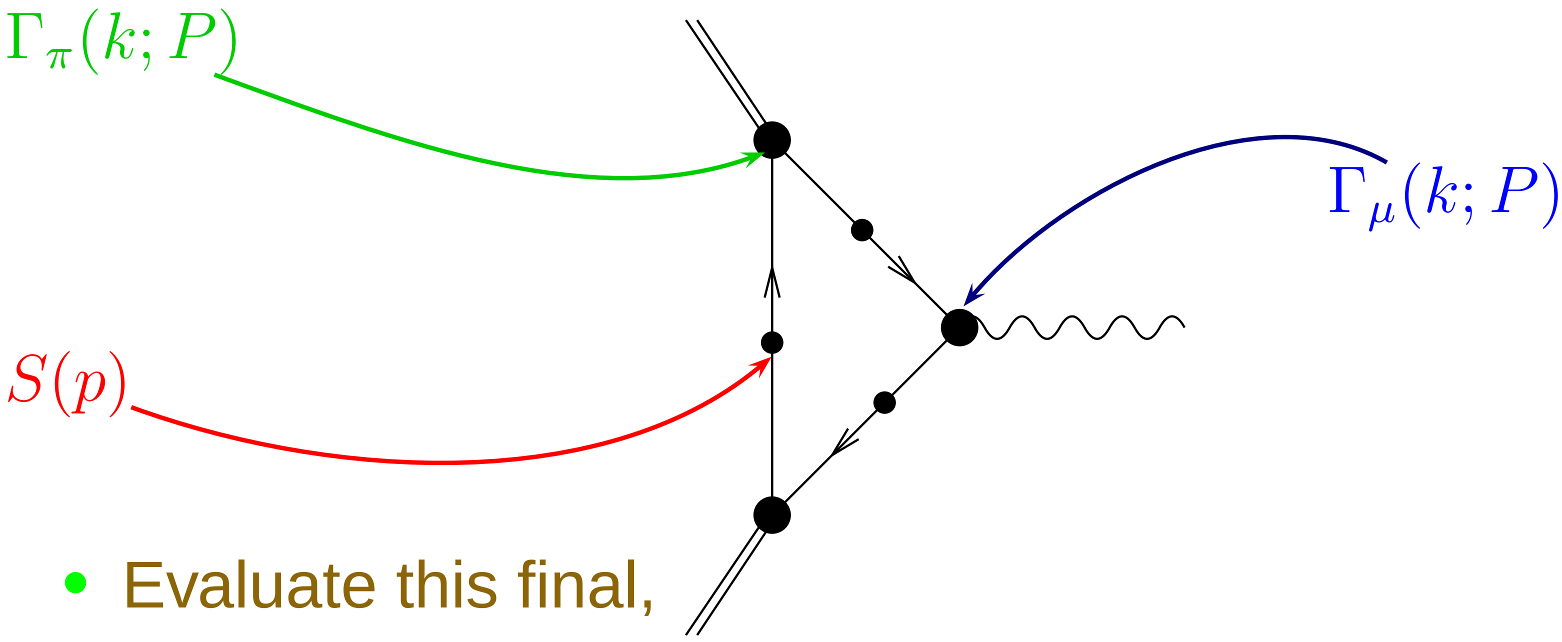
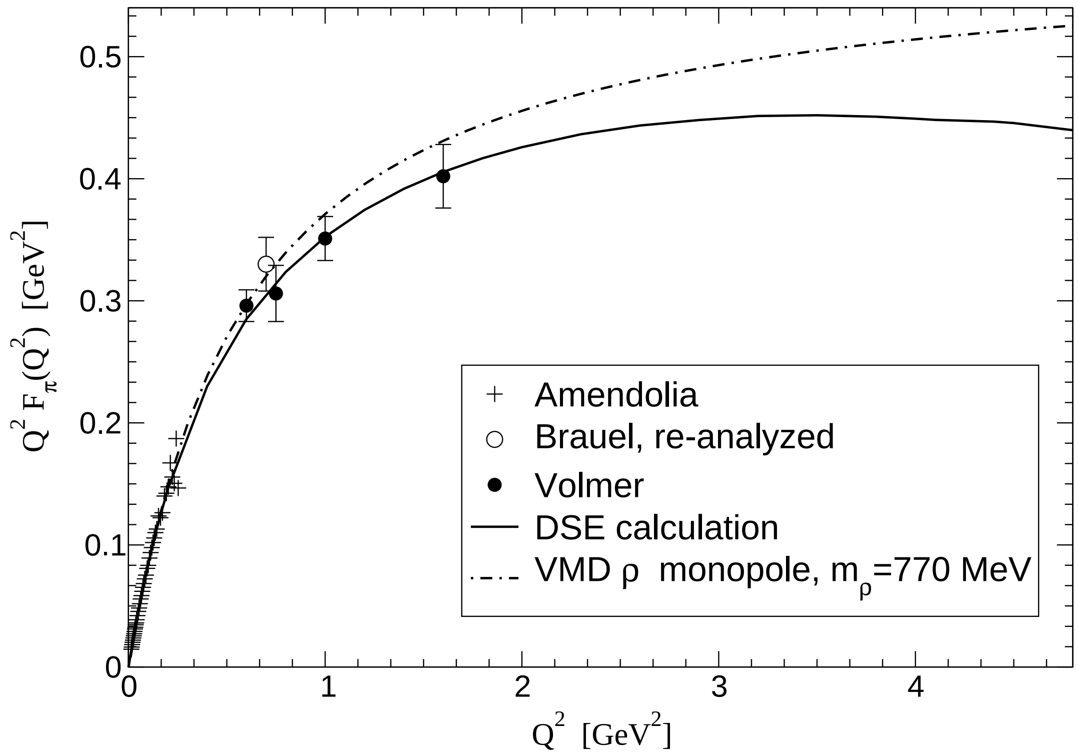


- Evaluate this final, three-dimensional integral

Calculated Pion Form Factor

Calculation published in 1999; No Parameters Varied

Data published in 2001



Calculated Pion Form Factor

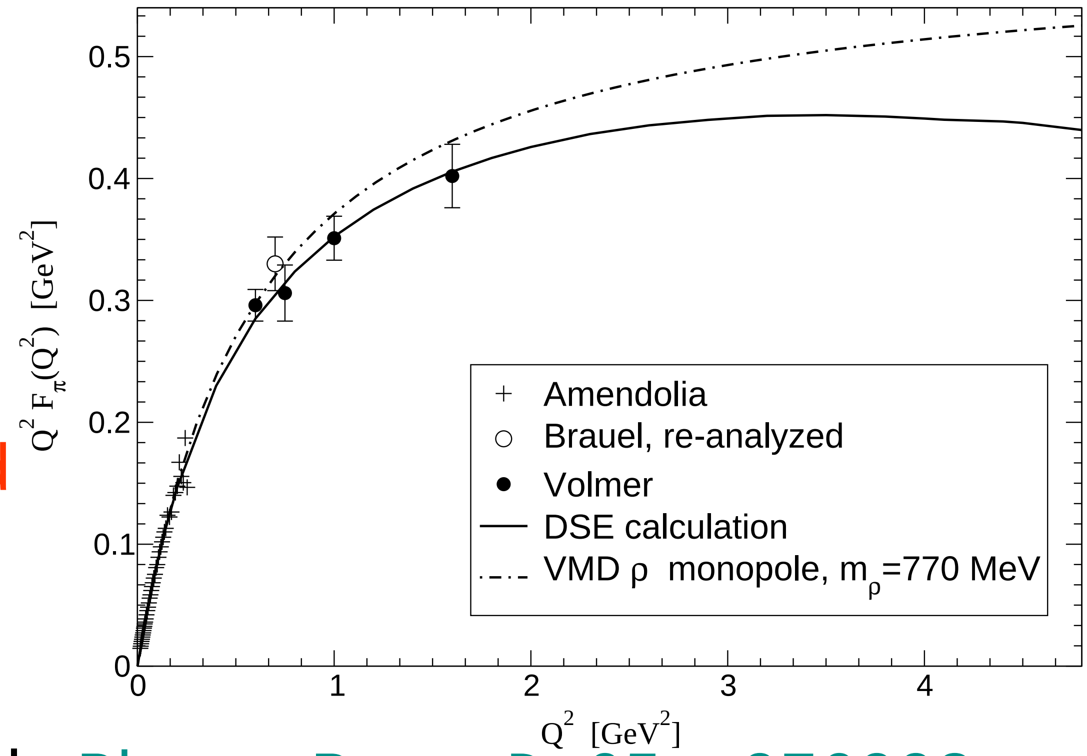
Calculation published in 1999; No Parameters Varied

Data published in 2001

Many subsequent successful applications

. . Again, parameters Fixed

Notably π π Scattering



Maris, et al., Phys. Rev. D 65, 076008

Bicudo, Phys. Rev. C 67, 035201

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Deep-inelastic scattering

Deep-inelastic scattering

- $\boxed{\text{Signature Experiment}}$ for QCD:

  Discovery of Quarks at SLAC

- Cross-section: Interpreted as Measurement of Momentum-Fraction Probability Distribution: $q(x)$, $g(x)$

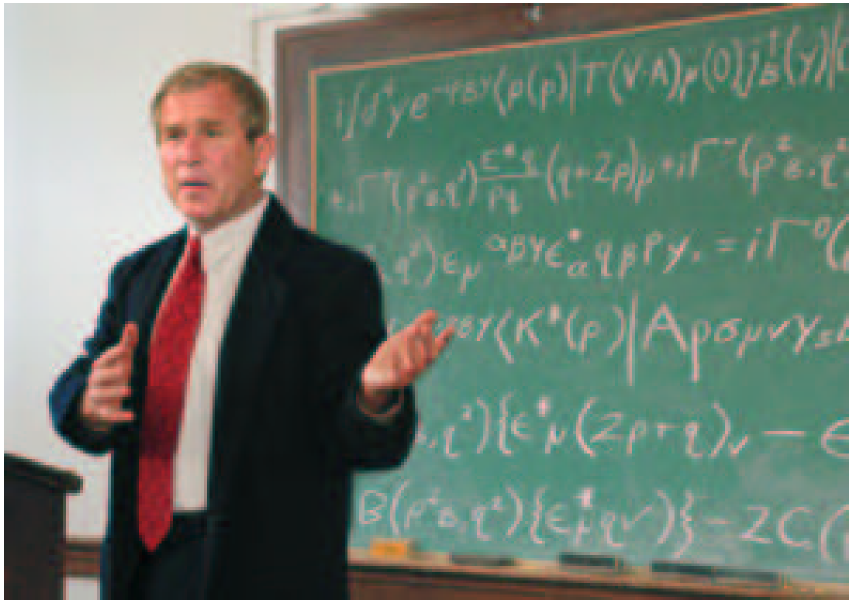
Pion's valence quark distn

Pion's valence quark distn

- π is Two-Body System: "Easiest" Bound State in QCD
- However, NO π Targets!

Pion's valence quark distn

- π is Two-Body System: "Easiest" Bound State in QCD
- However, NO π Targets!
- Proved recently (22/July, ANL)



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Pion's valence quark distn

- π is Two-Body System: "Easiest" Bound State in QCD
- However, NO π Targets!
- Existing Measurement Inferred from Drell-Yan:
  $\pi N \to \mu^+ \mu^- X$

Pion's valence quark distn

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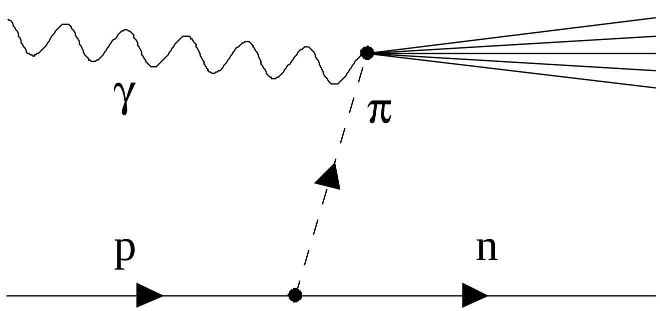
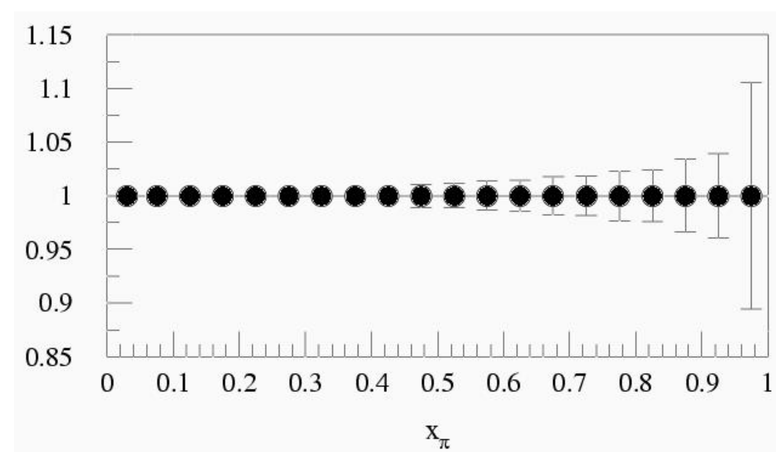
- Proposal (Holt & Reimer, ANL, nu-ex/0010004)

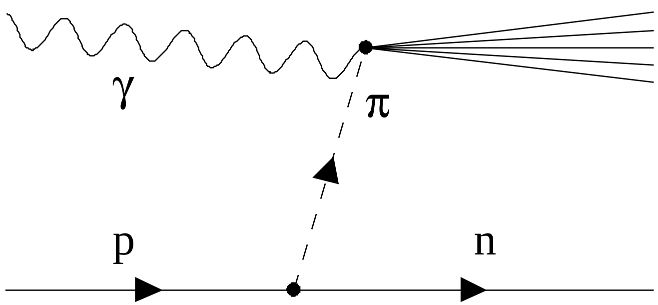
  $e^-_{5\mathrm{GeV}} - p_{25\,\mathrm{GeV}}$ Collider $\to$ Accurate "Measurement"

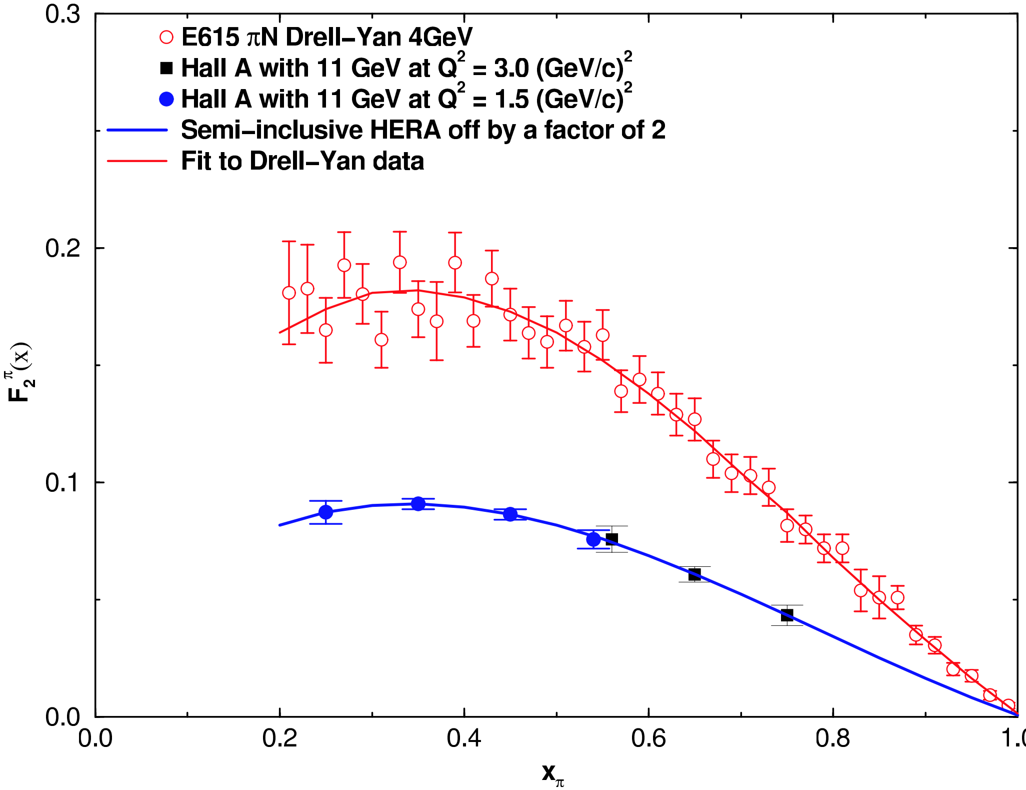




Pion's valence quark distn

- Proposal at JLab
  (Holt, Reimer, Wijesooriya, et al., JLab at 12 GeV)





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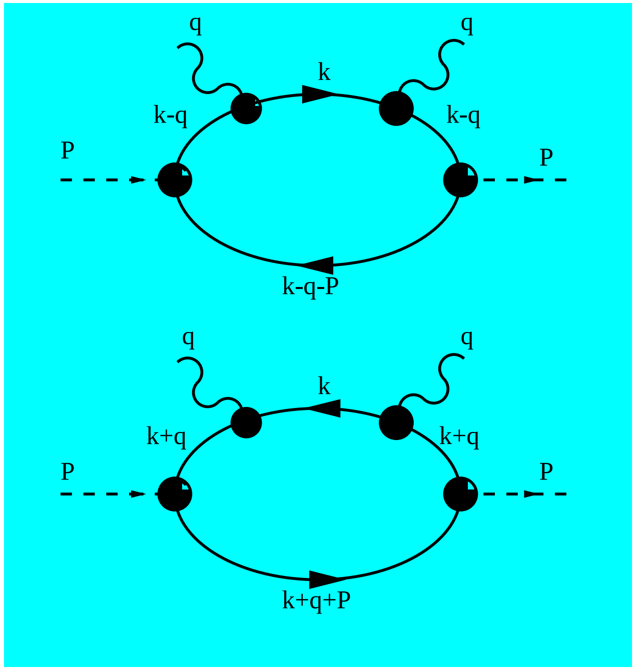
Handbag diagrams



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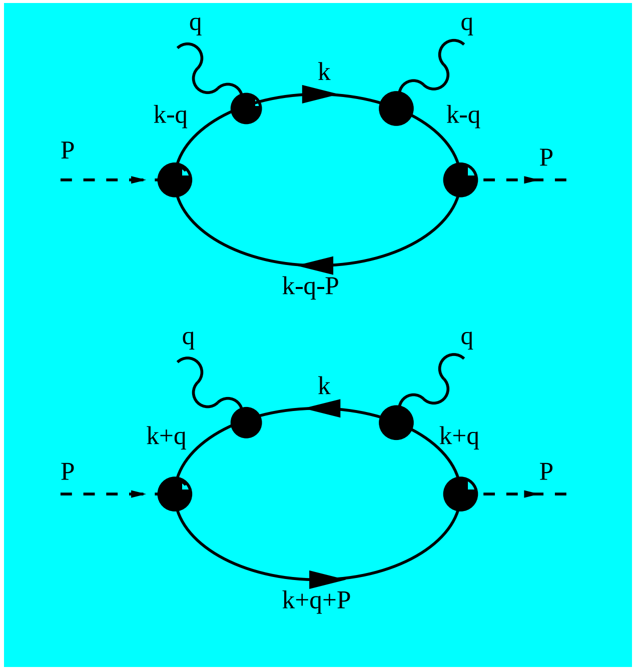


$$W_{\mu\nu}(q;P) = \frac{1}{2\pi}\mathrm{Im}\left[T^{+}_{\mu\nu}(q;P) + T^{-}_{\mu\nu}(q;P)\right]$$

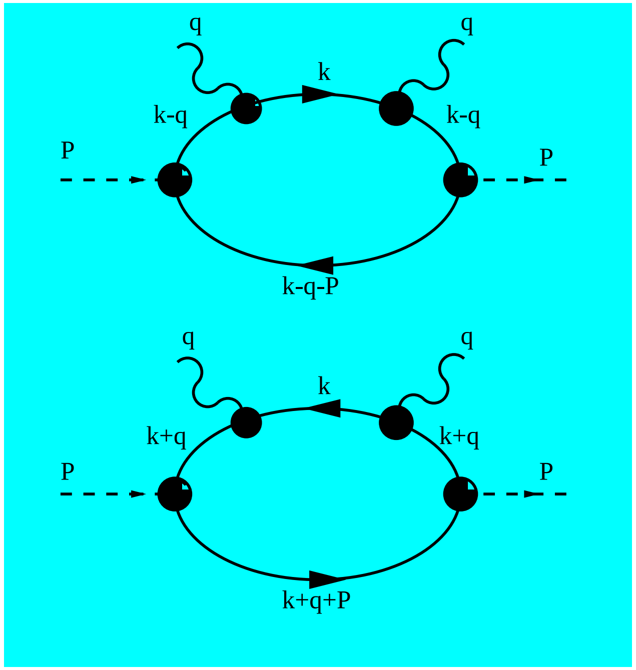
$$T^{+}_{\mu\nu}(q,P) = \mathrm{tr}\int \frac{d^4k}{(2\pi)^4}\,\tau_-\bar{\Gamma}_\pi(k_{-\frac{1}{2}};-P)\,S(k_{-0})\,ieQ\Gamma_\nu(k_{-0},k)$$
$$\times\, S(k)\,ieQ\Gamma_\mu(k,k_{-0})\,S(k_{-0})\tau_+\Gamma_\pi(k_{-\frac{1}{2}};P)\,S(k_{--})$$

Handbag diagrams

Bjorken Limit: $q^2 \to \infty\,, \quad P\cdot q \to -\infty$

but $x := -\dfrac{q^2}{2P\cdot q}$ fixed.

Numerous algebraic simplifications



$$W_{\mu\nu}(q;P) = \frac{1}{2\pi}\mathrm{Im}\left[T^+_{\mu\nu}(q;P) + T^-_{\mu\nu}(q;P)\right]$$

$$T^+_{\mu\nu}(q,P) = \mathrm{tr}\int\frac{d^4k}{(2\pi)^4}\,\tau_-\bar{\Gamma}_\pi(k_{-\frac{1}{2}};-P)\,S(k_{-0})\,ieQ\Gamma_\nu(k_{-0},k)$$
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Calc. $u_V(x)$ cf. Drell-Yan data

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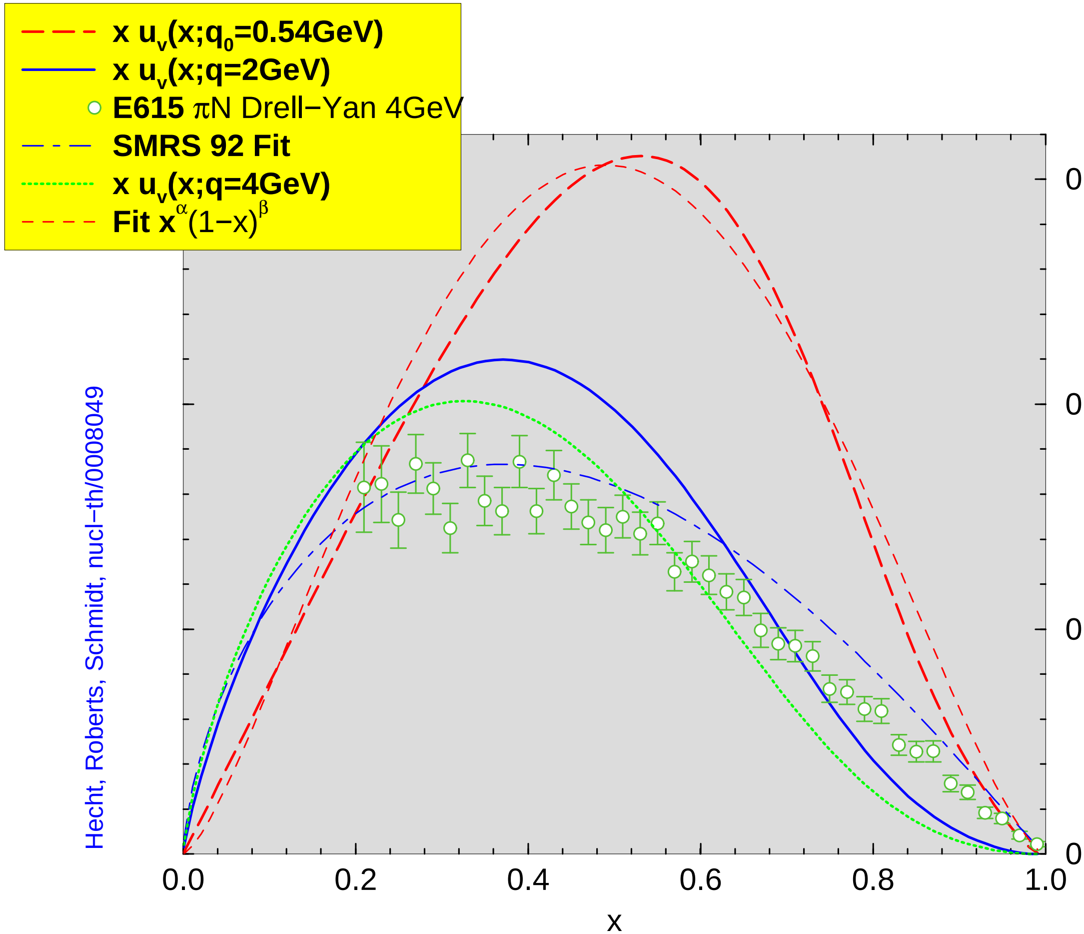

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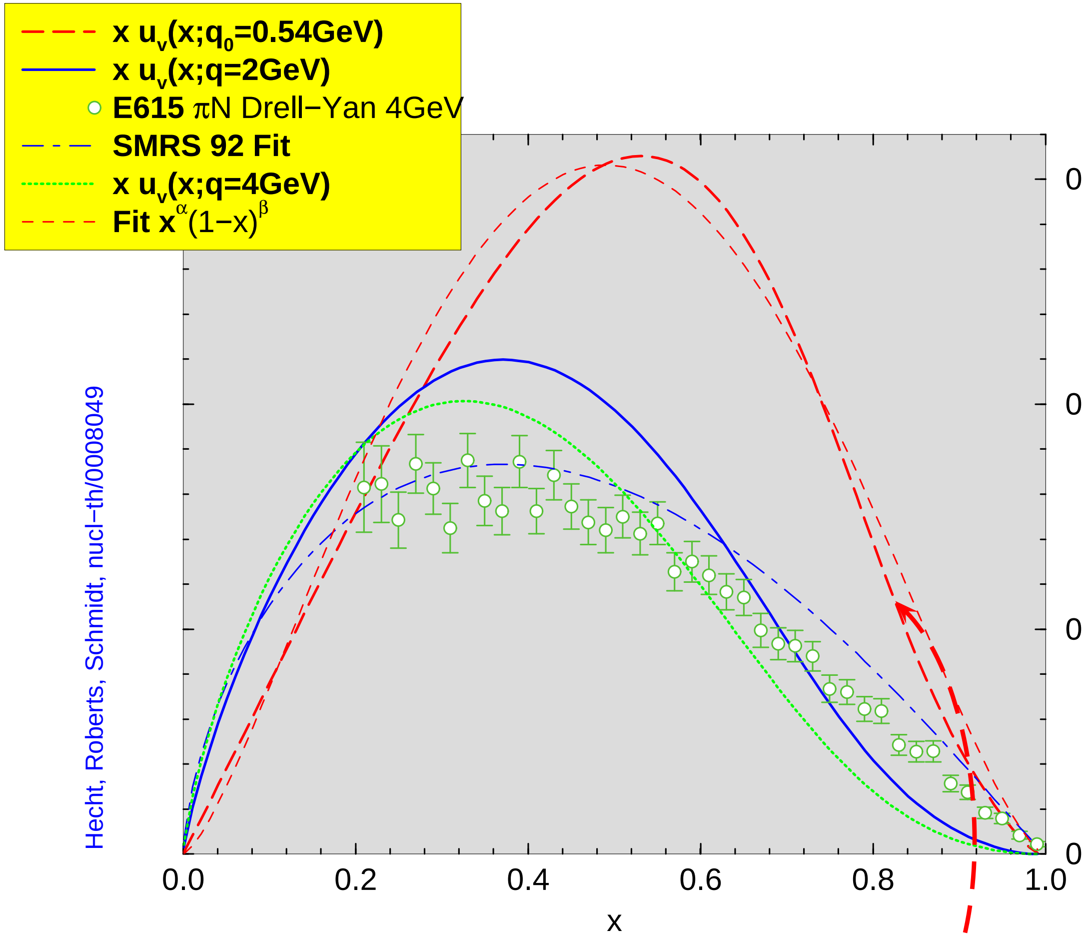
Calc. $u_V(x)$ cf. Drell-Yan data



Resolving Scale: $q_0 = 0.54\,\text{GeV} = 1/(0.37\,\text{fm})$

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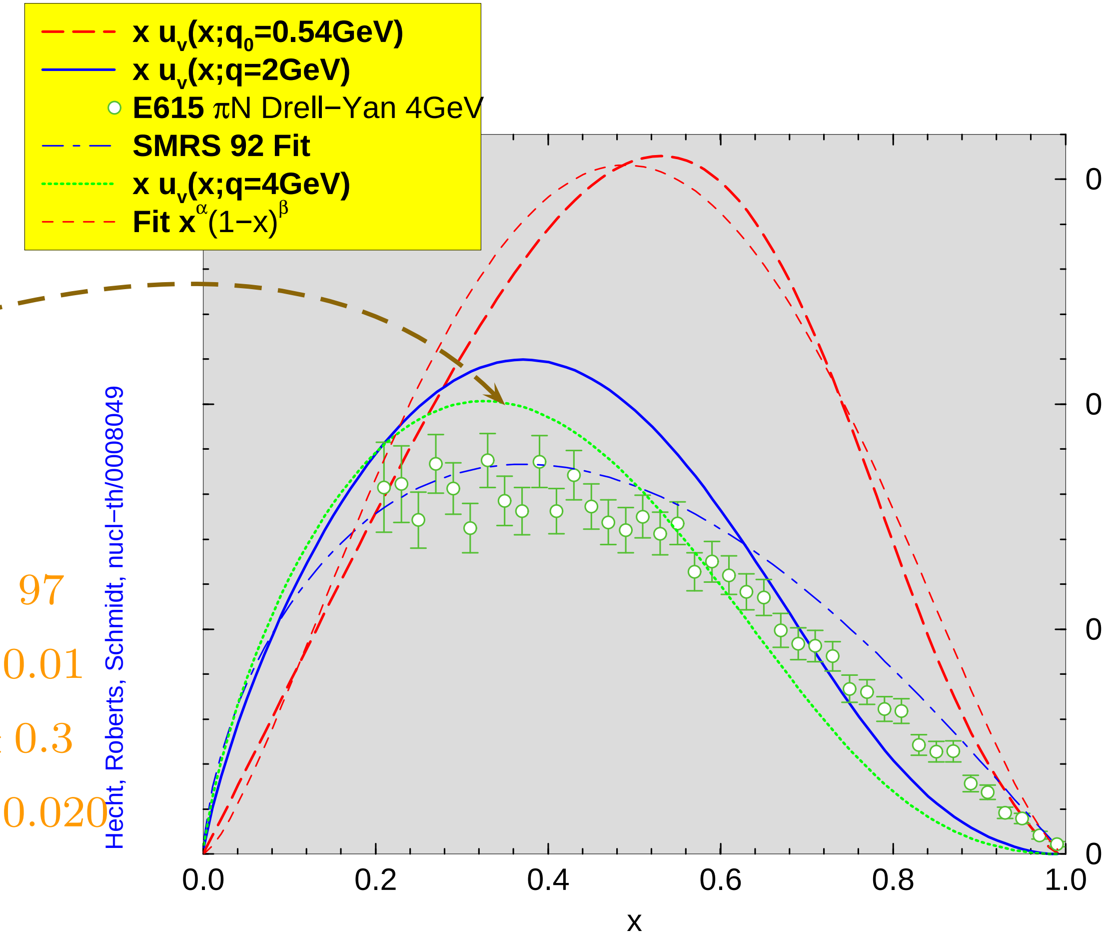
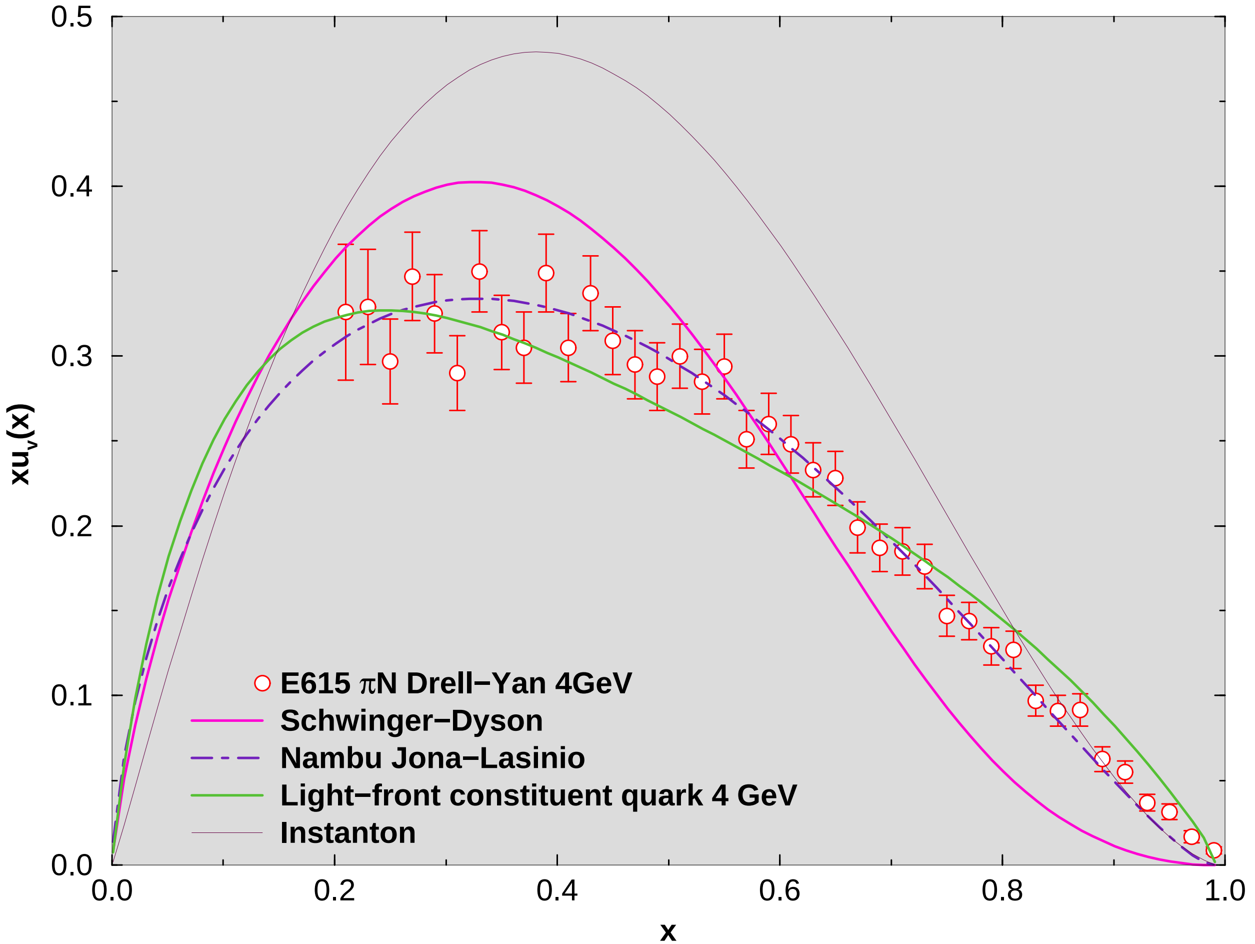
Calc. $u_V(x)$ cf. Drell-Yan data



$q =$ 2 GeV	Calc.	Fit, 92	Latt., 97
$\langle x\rangle_q$	0.24	0.24 ± 0.01	0.27 ± 0.01
$\langle x^2\rangle_q$	0.10	0.10 ± 0.01	0.11 ± 0.3
$\langle x^3\rangle_q$	0.050	0.058 ± 0.004	0.048 ± 0.020

Extant theory vs. experiment

Krishni Wijersooriya



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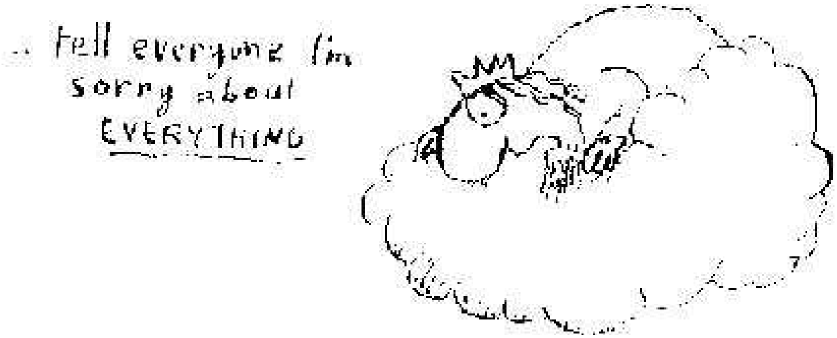
Epilogue



Epilogue

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Epilogue

- Dynamical Chiral Symmetry Breaking $\Leftrightarrow$:

  π is quark-antiquark Bound State

  AND QCD's Goldstone Mode

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  . . . Systematic & NonPerturbative Truncation

Epilogue

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  Bound State Effects Evident at $x > 0.5$

  Discrepancy Theory cf. Experiment

  ... Modern Challenge

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