NEUTRON MATTER : A SUPERFLUID GAS

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The large number of authors is indicative of the recent interest

Work is in progress, and several results are preliminary

This is a "Progress Report"

CONTENTS

INTERESTS IN NEUTRON MATTER

1. NEUTRON STARS $(\rho > \rho_0)$

The EOS ($E_{PNM}(\rho)$) of Cold Neutron Matter Determines the Gross Structure of Neutron Stars.

2. ASYMMETRIC NUCLEAR MATTER

$(\rho \leq \rho_0)$

$$Define\ Asymmetry\ \ \beta = \frac{N-Z}{A} = \frac{\rho_n - \rho_p}{\rho}$$

SNM : Symmetric Nuclear Matter $N = Z; \ \ \beta = 0$

PNM : Pure Neutron Matter $Z = 0; \ \ \beta = 1$

$$Energy\ per\ nucleon\ \ E(\rho,\beta) = E(\rho) + \beta^2 E_S(\rho) + \beta^4 E_4(\rho) + \ldots$$

Present theory indicates that $E_{n\geq 4}$ are negligibly small.

Neglecting β^4 and higher terms we get:

$$\begin{aligned} E(\rho) &= E_{SNM}(\rho) \\ E_S(\rho) &= E_{PNM}(\rho) - E_{SNM}(\rho) \end{aligned}$$

We know ρ_0, E_0 and compressibility K_0 of $E_{SNM}(\rho)$ From Experiment

$E_{PNM}(\rho)$ and the Symmetry Energy $E_S(\rho)$ are Not Experimentally Known

Use theoretical $E_{PNM}(\rho)$ to get $E_S(\rho)$?

The neutron drip line could be sensitive to $E_S(\rho)$ or equivalently $E_{PNM}(\rho)$

ISSUES IN PHYSICS OF NEUTRON MATTER

$$Definitions \;:\; E_{FG}(\rho) = \frac{3\hbar^2}{10m}k_F^2 \quad and \quad k_F = (3\pi^2\rho)^{1/3}$$

- **The nn $^1S_0$ Scattering Length is Large : $a \sim -18$ fm.**

  Low Density Expansion : $E(\rho) = E_{FG}(\rho)\left[1 + \frac{10}{9\pi}k_F a + \mathcal{O}(k_F a)^2 + \ldots\right]$

  Is not applicable for $|k_F a| > 1$ or $\rho > 10^{-4}\rho_0$

- **The effective range, $r_{nn} \sim 2.8$ fm, is not small**

  At $\rho > 0.1\ \rho_0 \quad r_{nn}/r_0 > 1$: Can not use δ-function approximation

- **v_{nn} contains a repulsive core**

  Must use an appropriate $v_{eff}(\rho)$ in Hartree-Fock-Bogolubov scheme

- **The ^3P_J interaction has equally large**

  central, tensor and spin-orbit terms

  It is necessary to get the correct shell-structure of nuclei

- **$^1S_0$ pairing influences neutron drip in nuclei**

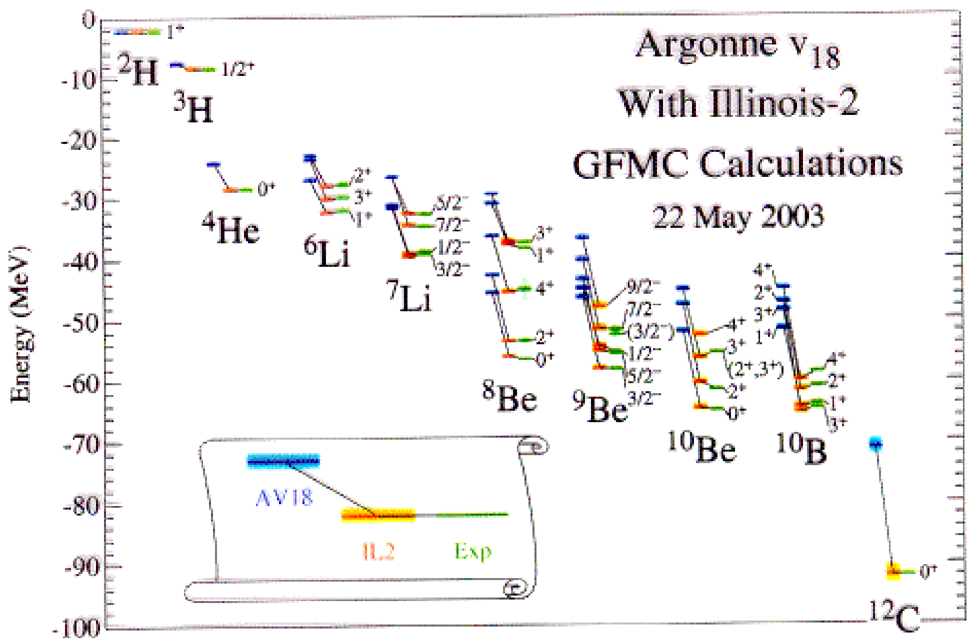
  and dynamics of neutron stars

  Must work with the superfluid phase

All the above can be treated (essentially exactly) with quantum Monte Carlo methods which can NOW do all nuclei with $A \leq 12$

Problem : How to develop a consistent theory for large nuclei

And get better understanding of large nuclei



- AV18: Argonne v_{18} with no NNN potential
  - significantly underpredicts experimental values
  - error increases with increasing size of nucleus
- IL2: Argonne v_{18} and Illinois-2 NNN potential
  - generally very good agreement with experiment
  - note correct ground-state spin for ^{10}B obtained only with NNN potential
- Many other nuclei and levels have been computed
- ^{12}C results are preliminary

THEORY OF SIMPLE SUPERFLUID FERMI GASES

- They have $r_{eff}/r_0 \to 0$ (δ-function like interactions)
- Their properties depend only on the dimensionless parameter ak_F
- Example 1 : Cold ^6Li atomic gas in a trap ($ak_F \sim -7.4$; tunable)
- Example 2 : Neutron gas at $\rho < 10^{-2}\rho_0$

Green's Function Monte Carlo (GFMC) Calculations

Carlson, Chang, Pandharipande and Schmidt, PRL **91**, (2003), 050401

+ Few More

The commonly known BCS wave function:

$$|BCS\rangle = \prod_{\mathbf{k}} (u_{\mathbf{k}} + v_{\mathbf{k}} a^{\dagger}_{\mathbf{k},\uparrow} a^{\dagger}_{-\mathbf{k},\downarrow})|0\rangle$$

does not conserve number of particles N

Number conserving BCS wave function is:

$$\begin{aligned} \Phi_{BCS} &= \mathcal{A}[\phi(r_{11'})\phi(r_{22'})...\phi(r_{nn'})] \\ \phi(r) &= \sum_{\mathbf{k}} \frac{v_{\mathbf{k}}}{u_{\mathbf{k}}} e^{i\mathbf{k}\cdot\mathbf{r}} \quad : \quad \textit{The Pair Function} \end{aligned}$$

$1, 2, ...n$ have spin $\uparrow$ and $1', 2', ...n'$ have $\downarrow$ and $n = N/2$

The Variational Wave Function

$$\Psi_V = [\mathcal{S} \prod_{i<j} \mathcal{F}_{ij}] \Phi_{BCS}$$

GFMC CALCULATIONS OF FERMI SUPERFLUIDS

$$Exact \;\; \Psi_0 = e^{-(H-E_0)\tau}\Psi_V \;\; lim \;\; \tau \to \infty$$

Not possible to calculate due to the **Fermion Sign Problem**

Fixed Node or Constrained Path GFMC

$$Fixed\ Node \;\; (FN) \; : \; \Psi_{FN} = |e^{-(H-E_0)\tau}|_{FN}\Psi_V \;\; lim \;\; \tau \to \infty$$

Ψ_{FN} is the lowest energy wave-function with nodal surfaces of Ψ_V

Nodal surfaces and the Ψ_{FN} depend upon the pair function $\phi(r)$

Minimize E_{FN}, the energy obtained with Ψ_{FN} to obtain $\phi(r)$

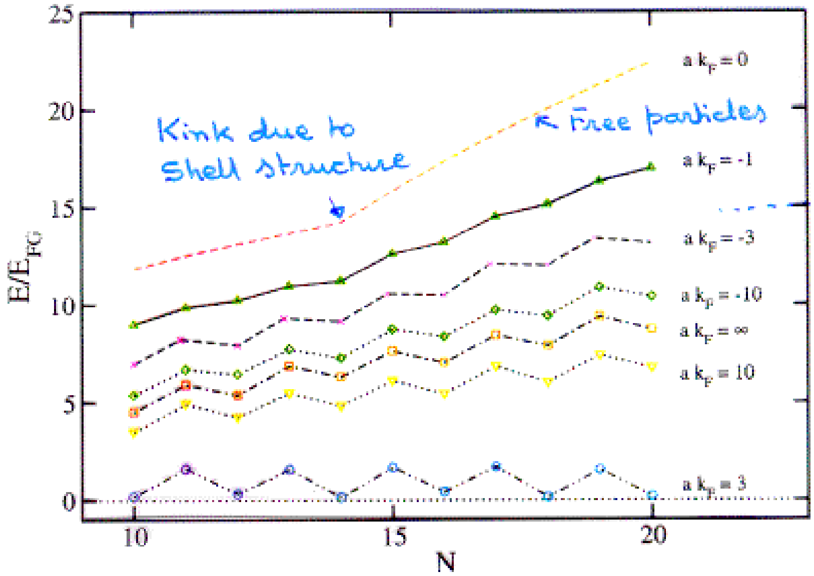
ASSUME that the variation of $\phi(r)$ is sufficient to span the required range of nodal surfaces. **Is it ?** or How accurate it is ?

In GFMC Periodic Boxes are Used to Approx. Uniform Matter

Shell Structure in 3D Oscillator and Cubic Periodic Box (PB)

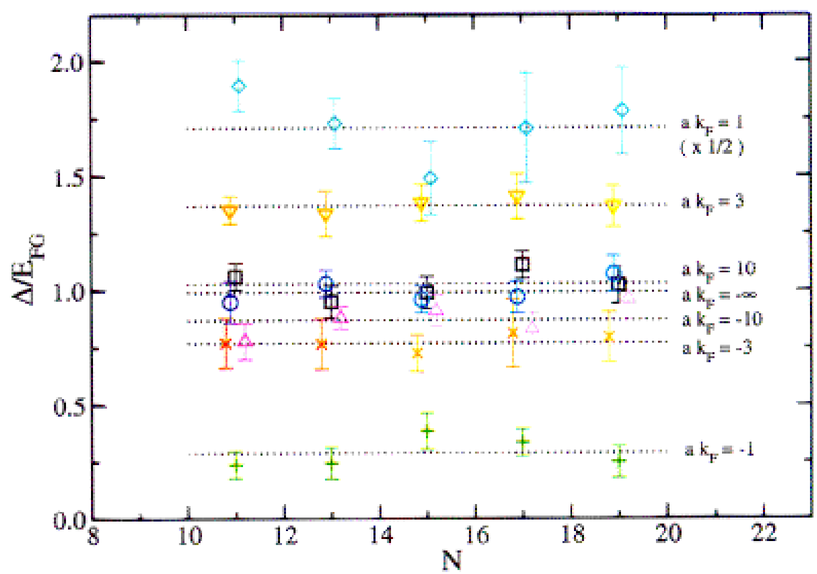
$2\ \hbar\omega$	$---$	[12]	(20)	(38)	[24]	$---$	$2(2\pi^2\hbar^2/mL^2)$
$1\ \hbar\omega$	$---$	[6]	(8)	(14)	[12]	$---$	$1(2\pi^2\hbar^2/mL^2)$
$0\ \hbar\omega$	$---$	[2]	(2)	(2)	[2]	$---$	$0(2\pi^2\hbar^2/mL^2)$
Harmonic Osc.							**Periodic Box**

PRL 91 (2003) 050401 has $E(N \le 42)$ for $ak_F \to \infty$

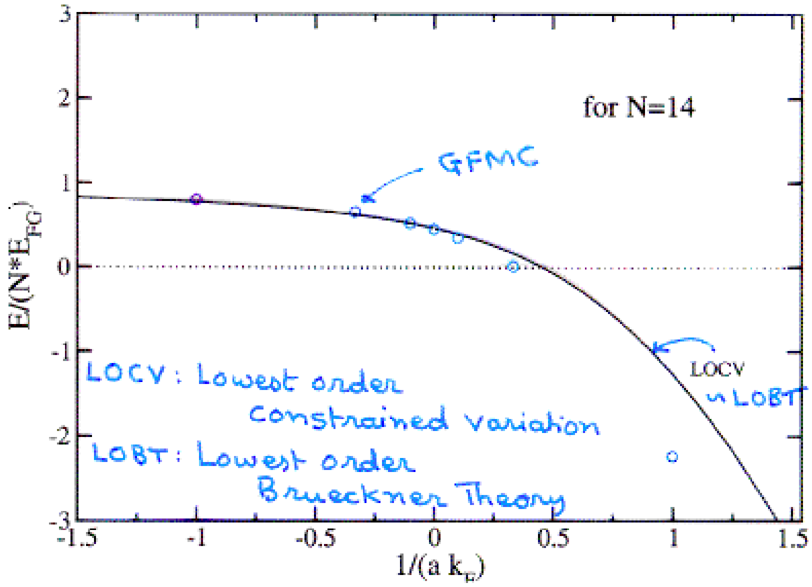


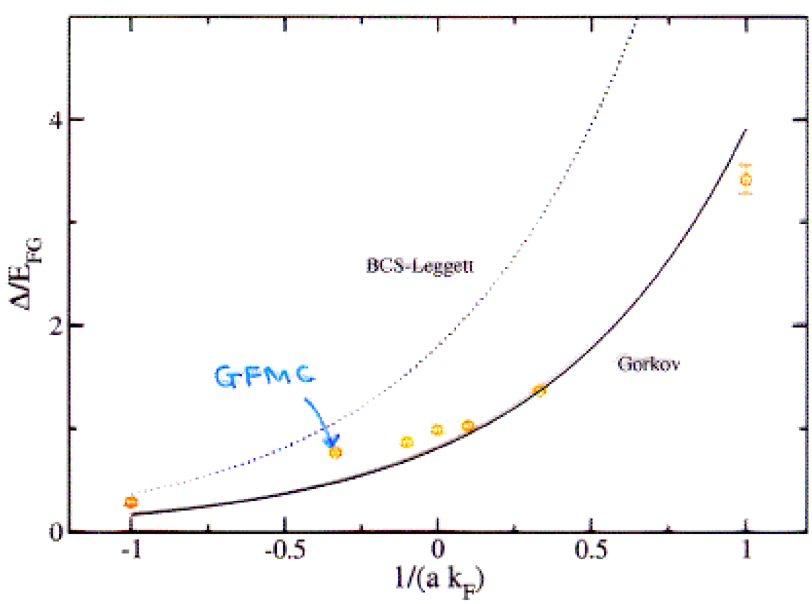
No Shell Kink for $\frac{1}{ak_F} \gtrsim -\frac{1}{3}$

The Gap $\Delta(N=2n+1) = -\frac{1}{2}\left(E(2n)+E(2n+2)\right) + E(2n+1)$



The Gap is fairly independent of N within statistical errors.


for N=14
GFMC
LOCV
LOBT
LOCV: Lowest order constrained variation
LOBT: Lowest order Brueckner Theory
E/(N*E_FG)
1/(a k_F)


BCS-Leggett
Gorkov
GFMC
Δ/E_FG
1/(a k_F)

CONCLUSIONS FROM GFMC STUDIES OF SUPERFLUID FERMI GASES (SFG)

We may soon have final conclusions from studies of cold Fermi gases in atom traps

- Energy of SFG $(ak_F \to \infty) = (0.44 \pm 0.01)\ E_{FG}$

  OK with PNM : All realistic calculations since 1970 give:

$$E_{PNM}(\rho) \sim 0.5 E_{FG}$$

- When $ak_F >> 1$ it is useful to expand in powers of $(ak_F)^{-1}$

  Known : Engelbrecht, Randeria and Sá de Melo, PRB **55**, 15,153 (1997)

- The pairing gap $\Delta \sim E_{FG}$ for $|ak_F| >> 1$ **is very large in SFG**

  NOT TRUE IN NUCLEI

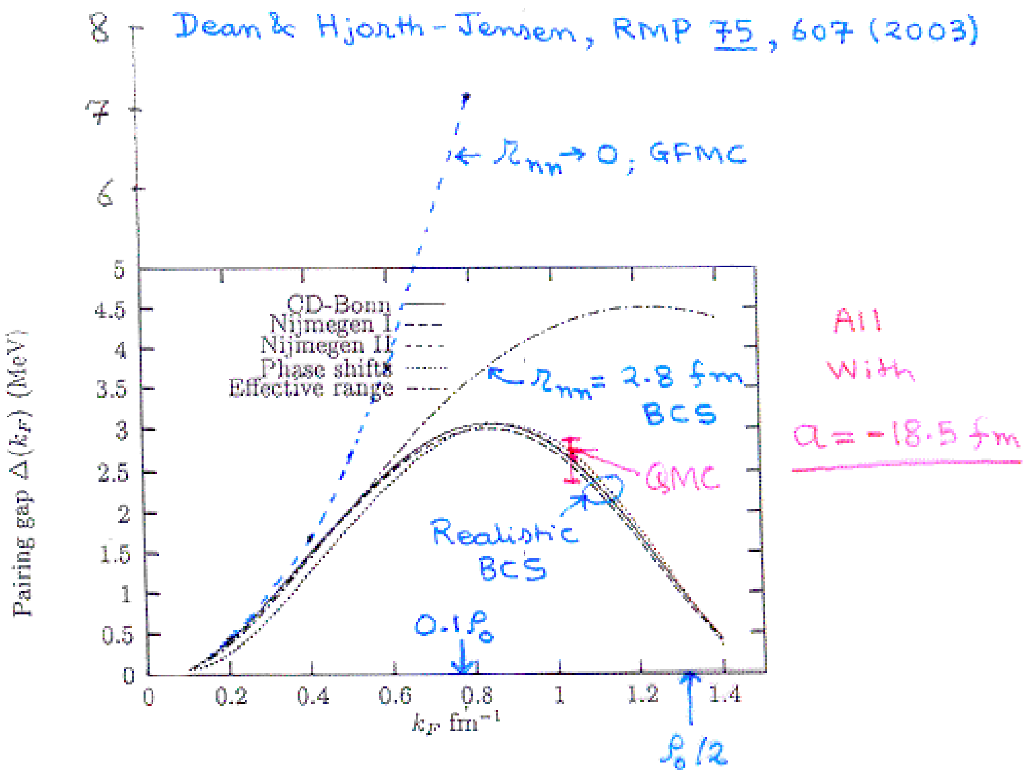
- When $(ak_F)^{-1} > -0.3$ the shell structure is overcome by pairing

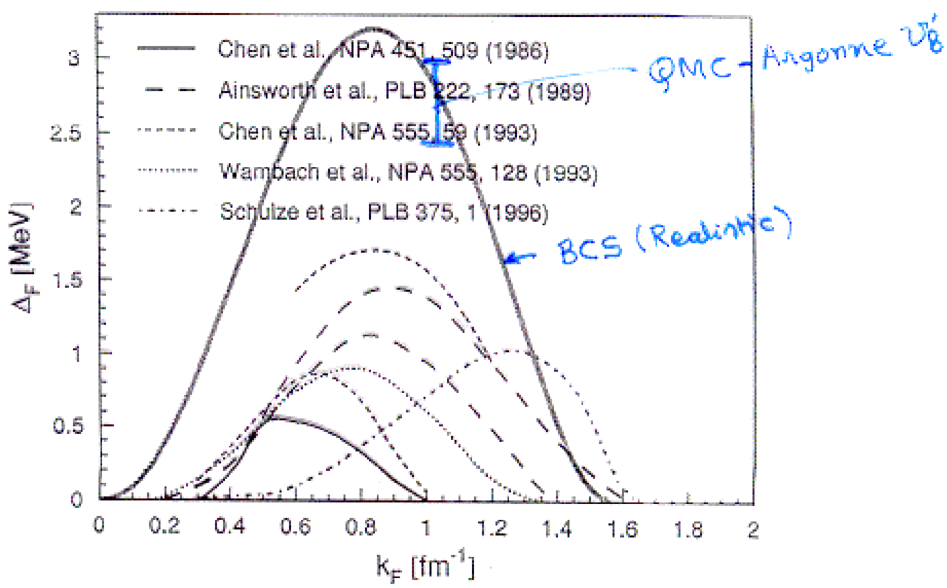
  NOT TRUE IN NUCLEI

- **THE EFFECTIVE RANGE AND PROBABLY EVEN THE REPULSIVE CORE IN THE v_{nn} IS IMPORTANT**

  Was indicated by studies of pairing gaps with the BCS equation

  But results of BCS equation with bare interaction were not trusted


Dean & Hjorth-Jensen, RMP 75, 607 (2003)
$r_{nn} \to 0$; GFMC
CD-Bonn
Nijmegen I
Nijmegen II
Phase shifts
Effective range
$r_{nn} = 2.8$ fm
BCS
All with
$a = -18.5$ fm
QMC
Realistic BCS
$0.1\rho_0$
$\rho_0/2$
Pairing gap $\Delta(k_F)$ (MeV)
k_F fm^{-1}


Chen et al., NPA 451, 509 (1986)
Ainsworth et al., PLB 222, 173 (1989)
Chen et al., NPA 555, 59 (1993)
Wambach et al., NPA 555, 128 (1993)
Schulze et al., PLB 375, 1 (1996)
QMC - Argonne v_8'
BCS (Realistic)
Δ_F [MeV]
k_F [fm^{-1}]

QUANTUM MONTE CARLO CALCULATIONS OF NEUTRON MATTER WITH REALISTIC INTERACTION

Carlson, Morales, Pandharipande and Ravenhall, PRC, **68**, 025802 (2003).

- Use periodic box with 14 neutrons
- Use Argonne v_8' NN-interaction. $(v_{18} - v_8')$ is a "perturbation"
- Truncate $v_{NN}(r)$ at $r = L/2$: This limits the density range
- Estimate "Box correction" and $v(r > L/2)$ contribution with cluster expansions
- Neglect the V_{ijk}, we need to learn how to box three-body interactions.
- **RESULTS**

ρ	$\rho_0/4$	$\rho_0/2$	ρ_0	$3\rho_0/2$	
E^{PB}_{GFMC}	6.3	9.5	17.0	28.4	MeV
Box Corr.	−0.2	−0.4	−0.6	−0.8	MeV
$\langle v(r > L/2)\rangle$	−0.1	−0.7	−4.5	−10.7	MeV
$E(\rho)$	6.0	8.4	12.0	16.9	MeV
$E(\rho)/E_{FG}(\rho)$	0.43	0.38	0.34	0.37	

Conclusions

- At small ρ, $E(\rho)/E_{FG}(\rho) \to 0.44$ (old result improved and confirmed)
- At $\rho \geq \rho_0$ the corrections to GFMC are large
- At $\rho \leq \rho_0/2$ the present results are close to exact

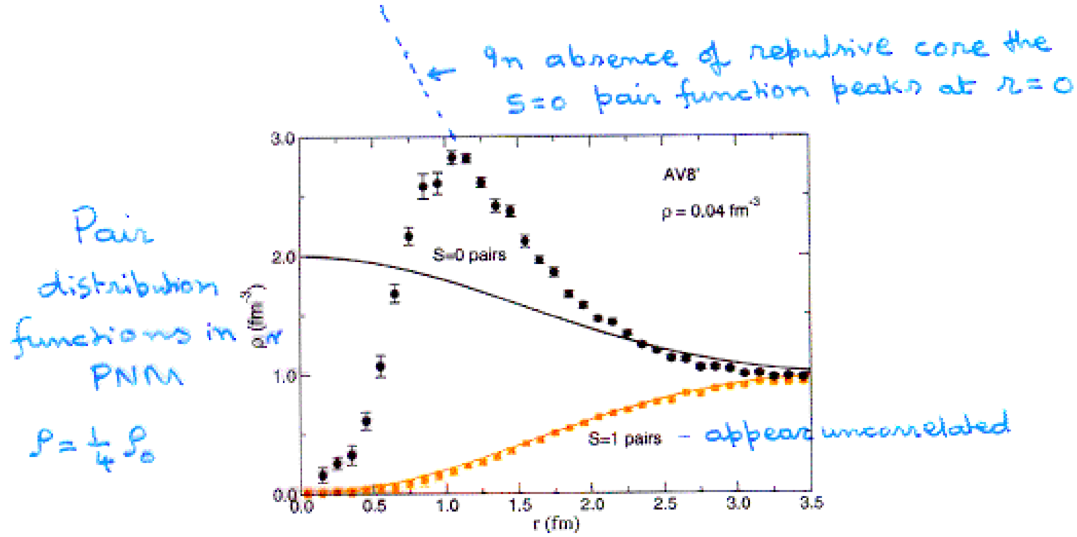


FIG. 8: Pair distribution functions for spin 0 and spin 1 pairs at $\rho = 0.04$ fm^{-3}; results of unconstrained GFMC calculations are compared to distributions in noninteracting FG shown by solid lines.

PRC 68 (2003) 025802

Energy of Low density PNM

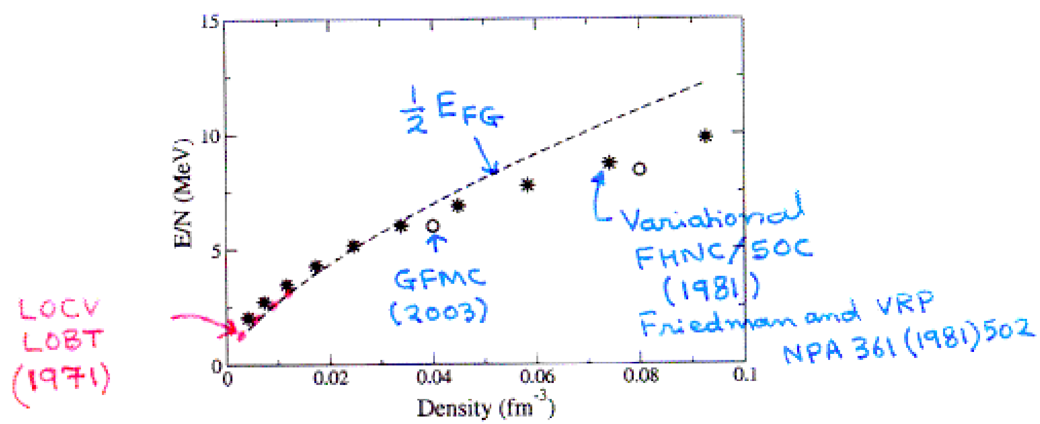


FIG. 9: The energy of neutron matter at low densities. The stars give the results of Ref.[30] and the circles of the present calculation. The dashed line shows 0.5 E_{FG}.

Exact Calculations of SNM - Not Yet Possible

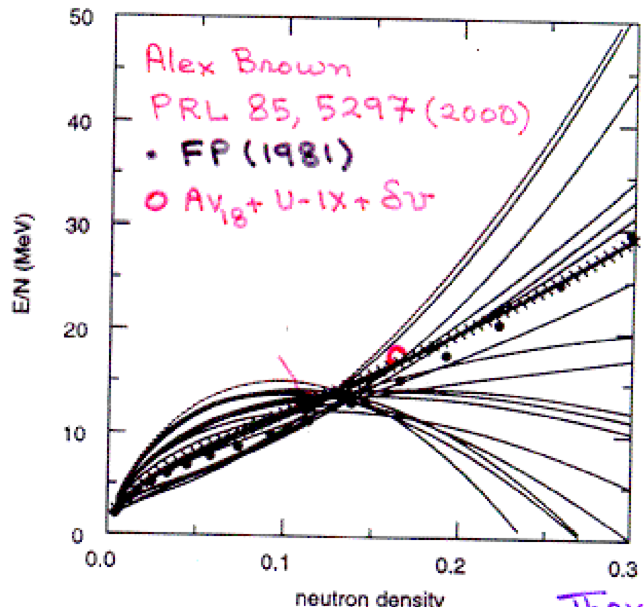
Exact Calculations of PNM - Becoming Possible

2002 Urbana Calculations (Variational) of Symmetric Nuclear Matter with Argonne v_{ij} and Urbana V_{ijk}, $E(\rho)$ in MeV/A.

	Density [$\mathbf{fm^{-3}}$]	0.08	0.16	0.24	
	1-b T_{FG}	13.9	22.1	29.1	Exact
	2-b all	−25.9	−43.7	−56.2	"
	3-b static	4.9	10.9	19.1	"
	3-b $(\mathbf{L}\cdot\mathbf{S}) + \geq$ 4-b all	−2.2	−1.7	+0.8	Approx.
	$(E_0 - E_V)$ Pert.	−0.6	−1.8	−3.3	"
Do Not Use $\Rightarrow$	**Calculated** $E_0(\rho)$	−9.9	−14.2	−10.6	Total
Use $\Rightarrow$	Empirical $E_0(\rho)$	−12.1	−16.0	−12.9	"

We have a 15% underbinding

$E(\rho \sim \rho_0)$ of Pure neutron matter



Use Theory

They all fit a set of nuclear data including $E(^{132}Sn) - E(^{100}Sn)$

FIG. 2. The neutron EOS for $\boxed{\text{18 Skyrme}}$ parameter sets. The filled circles are the Friedman-Pandharipande (FP) variational calculations and the crosses are SkX. The neutron density is in units of neutron/fm^3.

QMC Calculations of the gap in neutron matter with Argonne v_8' Interaction and Periodic Box at $\rho = \rho_0/4$

- **Carlson's GFMC** : Uses $\mathcal{F}_{ij} = f_c(r_{ij}) + f_\sigma(r_{ij})\sigma_i \cdot \sigma_j + f_t(r_{ij})S_{ij}$
  Exactly sums over all the 2^N spin states of the N neutrons
  $\Psi_V = [\mathcal{S} \prod \mathcal{F}_{ij}]\Phi_{BCS}$ presumably provides realistic constraints
  for the $|e^{-(H-E_0)\tau}|_{CP}$ propagation.

Present Limit $N \leq 16$

- **Schmidt's AFDMC** : Uses the simple Jastrow $\mathcal{F}_{ij} = f_c(r_{ij})$
  Relies on the $e^{-(H-E_0)\tau}$ propagation to generate noncentral correlations
  The Jastrow $\Psi_V = [\prod f_c(r_{ij})]\Phi_{BCS}$ provides less accurate constraints
  for the $|e^{-(H-E_0)\tau}|_{CP}$ propagation
  Samples all the N-neutron spin states; does not scale with 2^N

Present Limit $N \leq 100$

- Both calculations rely on the $e^{-(H-E_0)\tau}$ propagation to generate spin-orbit correlations (not a big effect at $\rho_0/4$). In GFMC the constraint is released for a limited τ range. Provides a useful check.
- The Ground state $E(N)$ is calculated by both methods, and **The Gap**:

$$\Delta(2n+1) = E(2n+1) - \frac{1}{2}[E(2n) + E(2n+2)]$$

- Need less than 0.25 % statistical error to get gaps with $\sim$ 10 % error

Preliminary Results at $\rho = \rho_0/4$ with Argonne v_8'

	GFMC	GFMC	AFDMC	AFDMC
N	$E(N)$	$\Delta(N)$	$E(N)$	$\Delta(N)$
10	82.8(2)			
11	86.8(2)	2.65(30)		
12	85.5(3)		89.4(4)	
13	89.7(4)	2.6(5)	94.0(4)	2.6(5)
14	88.7(4)		93.4(4)	
15			110.1(3)	3.3(4)
16			120.2(2)	

Big surprise : The gap is as large as predicted using the *bare* v_{nn} in the BCS equation

Obvious shell-effect at $N = 14$

$$shell\ effect \equiv \frac{[E(16) - E(14)] - [E(14) - E(12)]}{[E(16) - E(14)] + [E(14) - E(12)]} = 0.74$$

TO DO : Study how the shell-effect goes to zero as $\rho \to 0$

neutron matter with $r_0 > r_{nn}$ should not have shell effects

NEUTRON DROPS IN EXTERNAL WELL

- A simple approximation to neutron rich nuclei.
- Provide **"Psuedo-Data"** for construction of effective interactions in inhomogeneous neutron matter.

Pieper's Woods-Saxon well to mimic oxygen isotopes

$$
\begin{aligned}
V_1(r) &= \frac{-35.5\ MeV}{1+exp[(r-3\ fm)/1.1]} \\
v_{nn} &= Argonne\ v_{18} \\
V_{nnn} &= Illinois\ Model\ 2 \\
H &= \sum_i \left(-\frac{\hbar^2}{2m}\nabla_i^2 + V_1(r_i)\right) + \sum_{i<j} v_{ij} + \sum_{i<j<k} V_{ijk}
\end{aligned}
$$

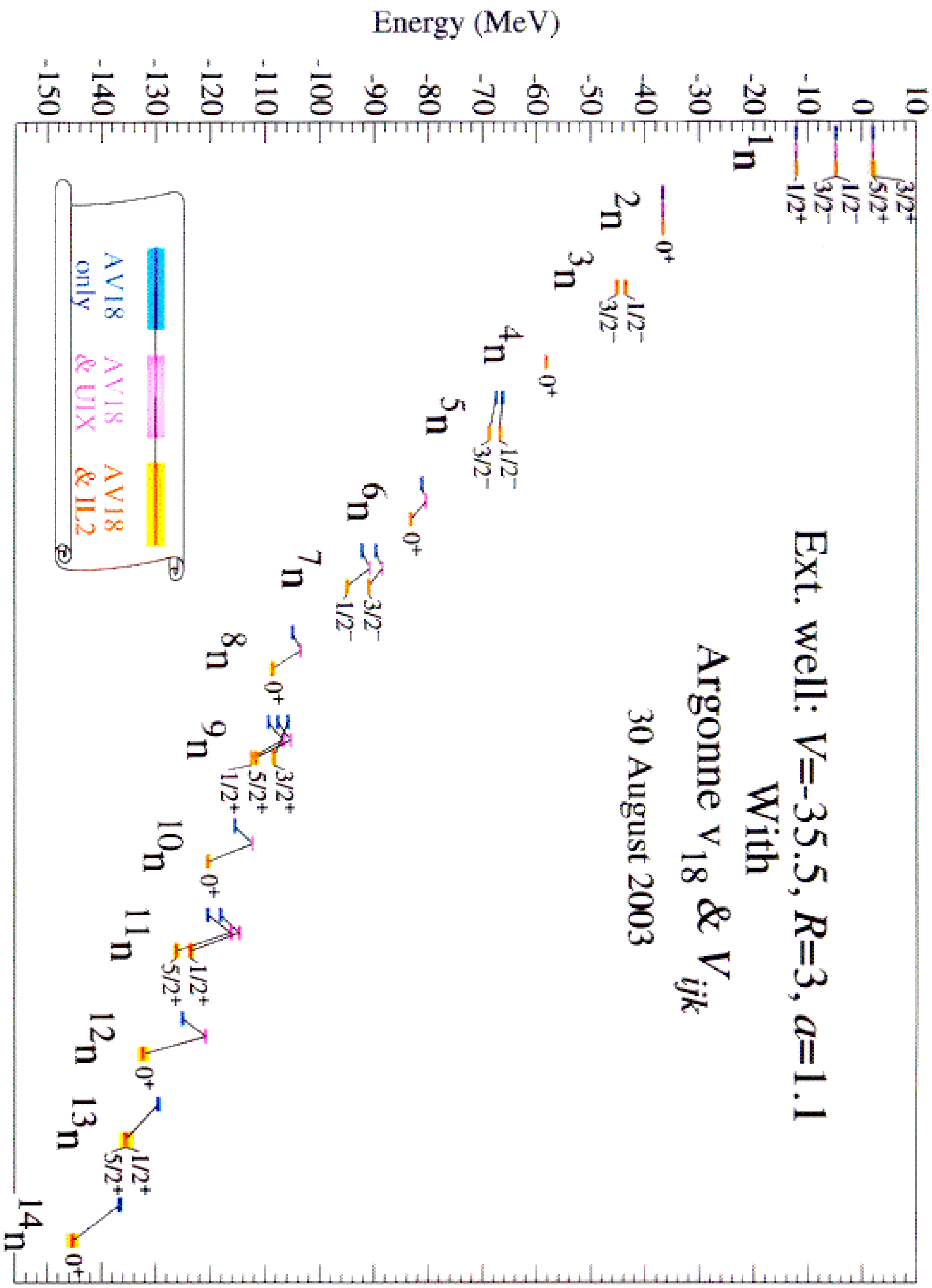
Calculates ground state energies for $N = 1$ *to* 14 neutrons with GFMC

Shell-Model Calculations of $^{17-24}O$

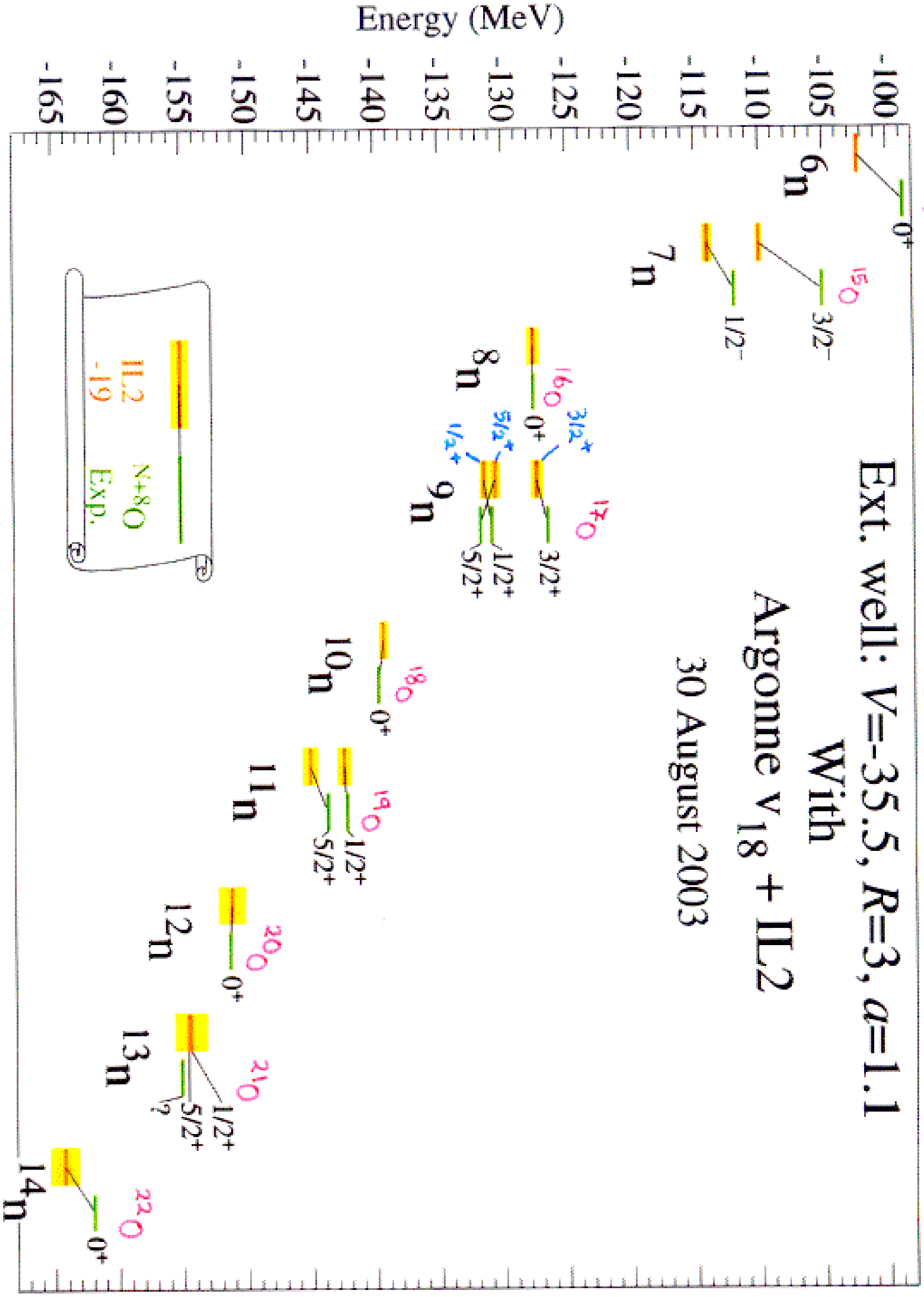
- Use V_1 with an additional spin-orbit term.
- Use G-matrix in place of the bare v_{nn} and a model space
- Include "core-polarization" correction to the nn interaction
- Neglect the V_{ijk}

A more realistic neutron-drop model of $^{17-24}O$ should

- Include the spin-orbit term in V_1
- Include "core polarization" corrections to v_{nn}



Energy (MeV)
10
0
-10
-20
-30
-40
-50
-60
-70
-80
-90
-100
-110
-120
-130
-140
-150
Ext. well: V=-35.5, R=3, a=1.1
With
Argonne v18 & Vijk
30 August 2003
1n
2n
3n
4n
5n
6n
7n
8n
9n
10n
11n
12n
13n
14n
3/2+
5/2+
1/2−
3/2−
1/2+
0+
AV18 only
AV18 & UIX
AV18 & IL2


Ext. well: V=-35.5, R=3, a=1.1
With
Argonne v18 + IL2
30 August 2003
Energy (MeV)
-100
-105
-110
-115
-120
-125
-130
-135
-140
-145
-150
-155
-160
-165
6n
7n
8n
9n
10n
11n
12n
13n
14n
14O
15O
16O
17O
18O
19O
20O
21O
22O
0+
3/2−
1/2−
3/2+
5/2+
1/2+
5/2+
?
IL2
-19
N+8O
Exp.

RESULTS OF PIEPER'S NEUTRON DROP CALCULATIONS

- The interaction contributions to the energies of these neutron drops are large. Without nn interactions the well can hold only 8 neutrons.

- The approximation is better for oxygen isotopes with $N > Z$ than with $N < Z$. The proton core changes more with N when $N < Z$.

- In ^{17}O the interactions between neutrons give only $\sim 2/3$ of the $d_{3/2} - d_{5/2}$ spin orbit splitting. V_{ijk} contributes $\sim$ half of it. The rest of the spin-orbit splitting must come from interactions with the protons.

- **Pairing Energies** (Gaps) : $E(2n+1) - \frac{1}{2}(E(2n) + E(2n+2))$

N	Oxygen	Neut. Drops	
9	1.9	2.1±0.4	MeV
11	1.8	0.3±0.8	MeV
13	1.5	3.2±1.3	MeV
15	0.5		

Need to reduce the statistical errors

& Add spin-orbit to V_1

SKYRME TYPE EFFECTIVE INTERACTIONS

Energy Densities $\mathcal{E}(\rho)$

$$
\begin{aligned}
Noninteracting\ neutrons\ &:\ \mathcal{E}(\rho) = \rho E_{FG}(\rho) \sim \alpha\rho^{5/3} \\
Interacting\ neutron\ gas\ &:\ \mathcal{E}(\rho) = 0.44\rho E_{FG}(\rho) + terms\ of\ order\ \rho^2\ \&\ higher \\
Most\ Skyrme\ Models\ &:\ \mathcal{E}(\rho) = \rho E_{FG}(\rho) + terms\ of\ order\ \rho^2\ \&\ higher
\end{aligned}
$$

Can not reproduce realistic neutron matter energy density.

An effective interaction, $v_{eff} = b\ \delta(r)$, in Hartree-Fock approximation gives:

$$\mathcal{E}(\rho) = \rho\ E_{FG} + \frac{1}{4}b\rho^2$$

To obtain the realistic neutron gas $\mathcal{E}(\rho)$ we need:

$$v_{eff} = \left(B\rho^{-1/3} + b + higher\ terms\right)\delta(r)$$

This is necessary because $|a_{nn}| >>$ than the range of v_{nn}

In Brueckner theory ($v\psi = G\phi$) ignoring the repulsive core

$$
\begin{aligned}
At\ low\ densities\ the\ s-wave\ \psi &\sim \left(1 - \frac{a_{eff}}{r}\right) \\
G(r) &\sim \left(1 - \frac{a_{eff}}{r}\right) v(r) \\
&\sim -\frac{a_{eff}}{r} v(r)
\end{aligned}
$$

In matter, $a_{eff} \sim r_0 \sim \rho^{-1/3}$, and when $r_0 >$ the range of v we get an effective interaction with $\rho^{-1/3}$ density dependence.

The neutron-proton effective interaction has an even more severe ρ^{-1} divergence at small ρ, due to the deuteron, but we ignore it because matter with neutrons and protons is unstable and forms clusters.

Ravenhall-Skyrme Interaction

Pandharipande and Ravenhall; Les Houches Winter School (1989); NATO ASI Series B vol. 205, Ed. M. Soyeur *et.al.*

This effective interaction has ρ^{-1} terms. Used mostly to study nuclei in inner crust of neutron stars surrounded by neutron gas.

Ravenhall has obtained a new interaction with $\rho^{-1/3}$ and ρ^{-1} terms.

The Problem to Solve

Effective interaction to calculate pairing and odd-even energy differences

- A δ-function approximation can be used only within a restricted model space.

  How to define a model space in model independent way ?

- A finite range, density dependent pairing interaction is more realistic, but more difficult to calculate with.

PLEASE HELP , THAKS

Other Possibilities

I. With AFDMC we can study nuclei - neutron rich - with $A \sim 100$ using neutrons in a well approximation.

II. Present $\Phi_{BCS} = \mathcal{A}\left[\phi(r_{11'})\,\phi(r_{22'})\cdots\phi(r_{nn'})\right]$ is an "interacting - boson" state. Can we use it/connect with IBM ?