INTERSECTIONS BETWEEN
NUCLEAR PHYSICS AND QUANTUM INFORMATION
ARGONNE NATIONAL LABORATORY
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QUANTUM SIMULATION OF LATTICE GAUGE THEORIES WITH ULTRACOLD ATOMS

EREZ ZOHAR

Theory Group, Max Planck Institute of Quantum Optics (MPQ)

IN COLLABORATION WITH (ALPHABETICALLY)
J. IGNACIO CIRAC, MPQ
BENNI REZNIK, TAU
ALESSANDRO FARACE, MPQ
JULIAN BENDER, MPQ

Gauge Symmetries

- In the standard model of **high energy physics**, the interactions are carried by vector bosons, which are excitations of gauge (or broken gauge) fields

- In the context of **condensed matter physics**,
  - String nets, Kitaev's toric code, Quantum double models, dimer models, spin liquids, etc.
  - Emergent, effective field theories (High-T_C?)

- Very interesting non-trivial behaviour:
  - Local symmetry → Many conservation laws → Special Hilbert space structure
  - Nonperturbative physics

Gauge Theories

- Still involve many puzzling, non-perturbative phenomena:
  - Mass gap of Yang-Mills (pure gauge) theories, quark confinement
  - Phases of non-Abelian gauge theories with fermionic matter
    - Color superconductivity
    - Quark-gluon Plasma
    - Confinement/deconfinement of **dynamical, fermionic** charges
  - High-T_c superconductivity described by emergent gauge fields?

Lattice Gauge Theories

- Formulations of gauge theories on discrete space or spacetime (Wilson, Kogut-Susskind, Polyakov…)

- Allow for lattice regularization in a gauge invariant way, as well as many extremely successful nonperturbative calculations using Monte-Carlo methods (e.g. the hadronic spectrum)

- Numerical calculations still face several difficulties, due to the use of Euclidean spacetime for Monte-Carlo calculations:
  - The sign problem, for fermions with finite chemical potential
  - No real-time dynamics

Lattice Gauge Theories

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- Still face several difficulties, due to the use of Euclidean spacetime for Monte-Carlo calculations:
  - The sign problem, for fermions with finite chemical potential
  - No real-time dynamics

New approaches are needed:

- **Tensor network methods**
- **Quantum Simulation**

Q. Sim. and TN for LGTs

- An active, rapidly growing research field

- Quantum Simulation (around 7 years):
  - MPQ Garching & Tel Aviv University
  - ICFO, Barcelona (Lewenstein)
  - Innsbruck, Bern, Trieste, IQC (Zoller, Wiese, Blatt, Dalmonte, Muschik)
  - Heidelberg (Oberthaler, Berges)
  - …

- Tensor Networks (around 4 years):
  - MPQ Garching
  - Ghent (Verstraete)
  - ICFO (Lewenstein)
  - IQOQI, Bern, Trieste, Ulm (Zoller, Wiese, …)
  - …

Quantum Simulation

- Take a model, which is either
  - Theoretically unsolvable
  - Numerically problematic
  - Experimentally inaccessible
  - Not known to exist in nature
- Map it to a fully controllable quantum system – quantum simulator

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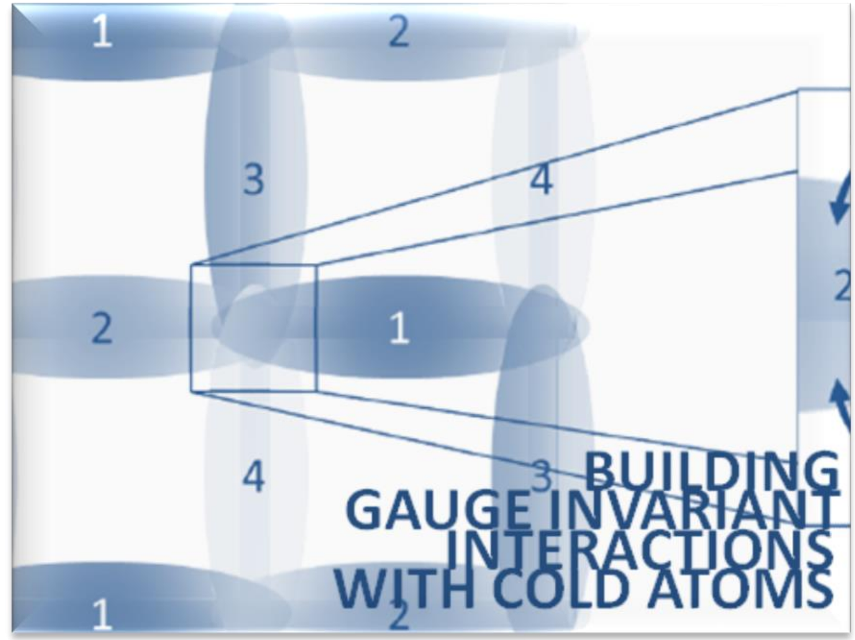
LATTICE
GAUGE
THERORIES:
ESSENTIAL FEATURES


BUILDING
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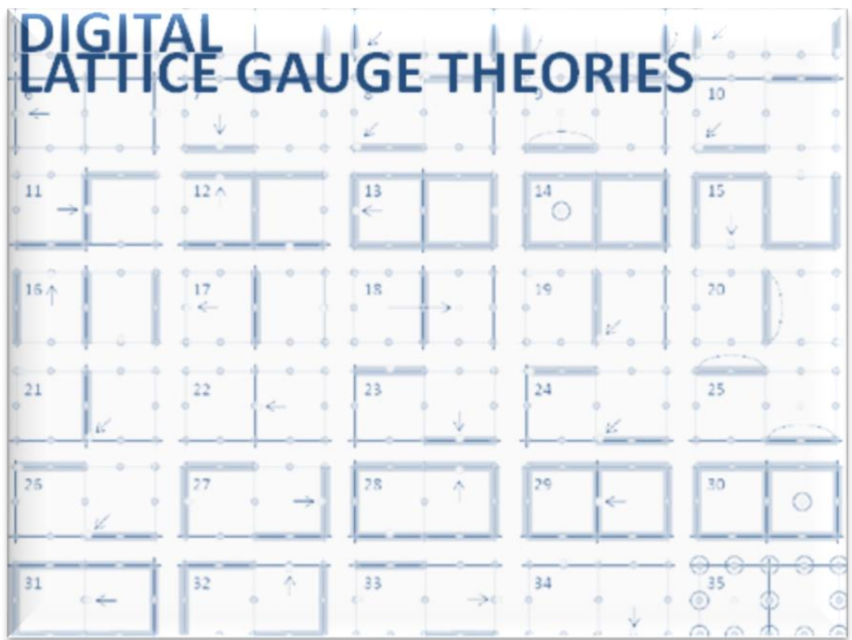

DIGITAL
LATTICE GAUGE THEORIES

LATTICE GAUGE THERORIES: ESSENTIAL FEATURES

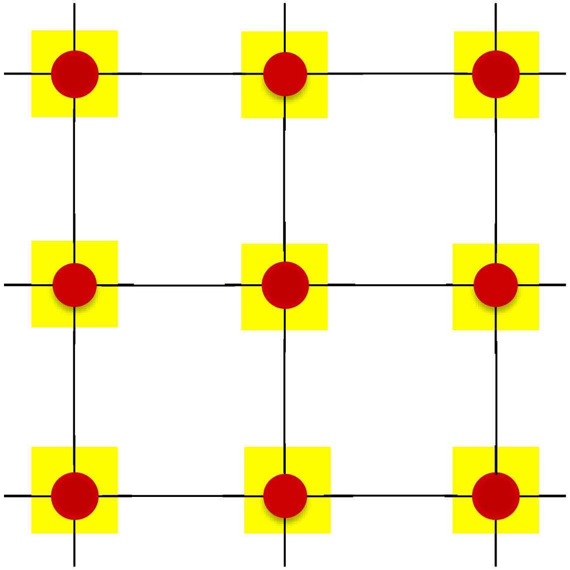
Global Symmetry

- Fermionic hopping

$$H_F = \sum_{\mathbf{x},k} \left(\epsilon\left(k\right)\psi^{\dagger}\left(\mathbf{x}\right)\psi\left(\mathbf{x}+\hat{\mathbf{e}}_k\right)+h.c.\right)+\sum_{\mathbf{x}} M\left(\mathbf{x}\right)\psi^{\dagger}\left(\mathbf{x}\right)\psi\left(\mathbf{x}\right)$$

- Invariant under **global transformations**

$$\psi\left(\mathbf{x}\right)\longrightarrow e^{i\alpha}\psi\left(\mathbf{x}\right)$$



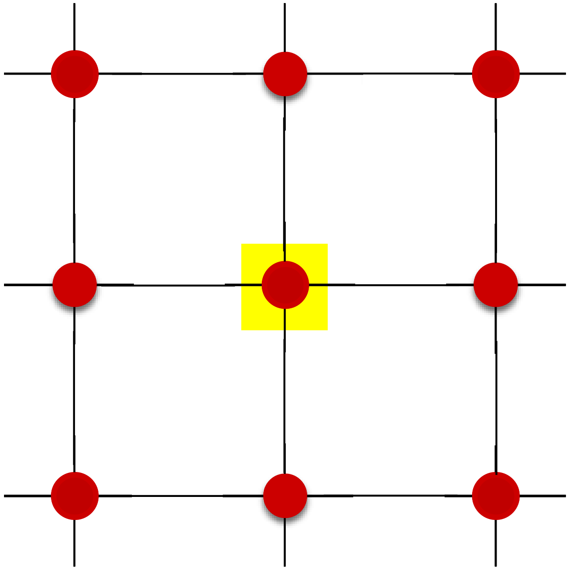
Local Symmetry

- Fermionic hopping

$$H_F = \sum_{\mathbf{x},k} \left(\epsilon\left(k\right) \psi^{\dagger}\left(\mathbf{x}\right) \psi\left(\mathbf{x}+\hat{\mathbf{e}}_k\right) + h.c. \right) + \sum_{\mathbf{x}} M\left(\mathbf{x}\right) \psi^{\dagger}\left(\mathbf{x}\right) \psi\left(\mathbf{x}\right)$$

- Not invariant under **local transformations**

$$\psi\left(\mathbf{x}\right) \longrightarrow e^{i\alpha\left(\mathbf{x}\right)} \psi\left(\mathbf{x}\right)$$

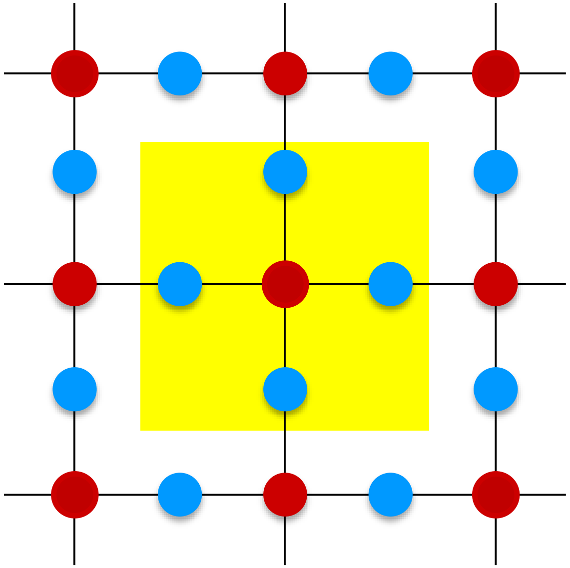


Local Symmetry

- Add another degree of freedom - **connection**

$$H_F = \sum_{\mathbf{x},k} \left(\epsilon\left(k\right) \psi^\dagger\left(\mathbf{x}\right) U\left(\mathbf{x},k\right) \psi\left(\mathbf{x}+\hat{\mathbf{e}}_k\right) + h.c. \right) + \sum_{\mathbf{x}} M\left(\mathbf{x}\right) \psi^\dagger\left(\mathbf{x}\right) \psi\left(\mathbf{x}\right)$$

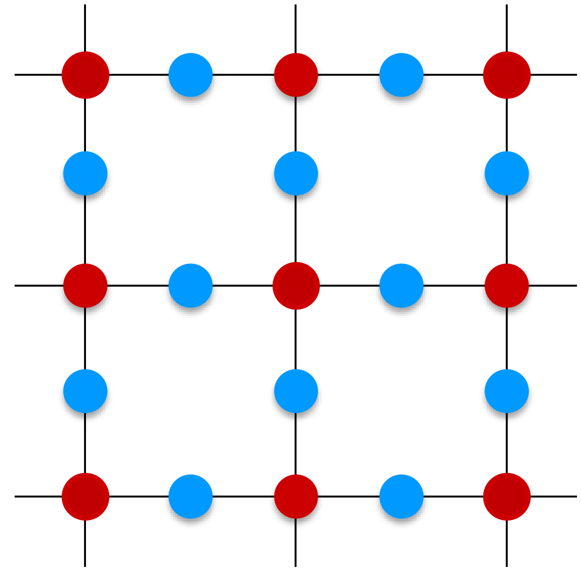
- Invariant under **local** **transformations**



$$\psi\left(\mathbf{x}\right) \longrightarrow e^{i\alpha(\mathbf{x})} \psi\left(\mathbf{x}\right)$$

$$U\left(\mathbf{x},\mathbf{y}\right) \longrightarrow e^{i\alpha(\mathbf{x})} U\left(\mathbf{x},\mathbf{y}\right) e^{-i\alpha(\mathbf{y})}$$

On a lattice, the fermions are located at the vertices (sites), and the connectors are defined along links; e.g., for *U*(1):



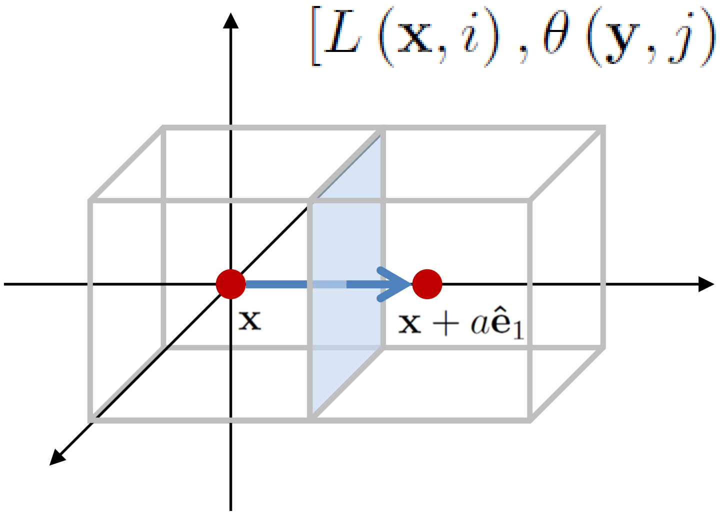
$$U\left(\mathbf{x},\mu\right) = e^{ie\int_{\mathbf{x}}^{\mathbf{x}+\hat{\mathbf{e}}_\mu} dz^\mu A_\mu(\mathbf{z})} \longrightarrow e^{i\theta(\mathbf{x},\mu)}$$

$\theta(\mathbf{x},\mu)$ - **"Lattice vector potential" – compact (angle)**

$$L\left(\mathbf{x},1\right) = e^{-1}\int\int dx^2 dx^3 E_1\left(\mathbf{x}\right)$$

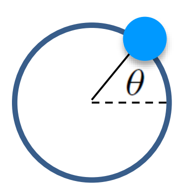
Electric flux (or lattice E field)
- *U*(1) angular momentum - integer

$$\left[L\left(\mathbf{x},i\right),\theta\left(\mathbf{y},j\right)\right] = -i\delta_{ij}\delta_{\mathbf{x},\mathbf{y}}$$



$$\left[L\left(\mathbf{x},i\right),U\left(\mathbf{x},i\right)\right] = U\left(\mathbf{x},i\right)$$

$U\left(\mathbf{x},i\right)$ **- Electric flux raising operator.**



$$L\left|m\right\rangle = m\left|m\right\rangle$$

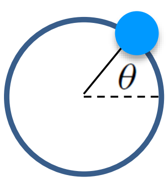
$$U\left|m\right\rangle = \left|m+1\right\rangle$$

Gauge Field Dynamics

$$\left[L\left(\mathbf{x},i\right),\theta\left(\mathbf{y},j\right)\right]=-i\delta_{ij}\delta_{\mathbf{x},\mathbf{y}}$$

$$\left[L\left(\mathbf{x},i\right),U\left(\mathbf{x},i\right)\right]=U\left(\mathbf{x},i\right)$$

$U\left(\mathbf{x},i\right)$ **- Electric flux raising operator.**



$$L\left|m\right\rangle=m\left|m\right\rangle$$

$$U\left|m\right\rangle=\left|m+1\right\rangle$$

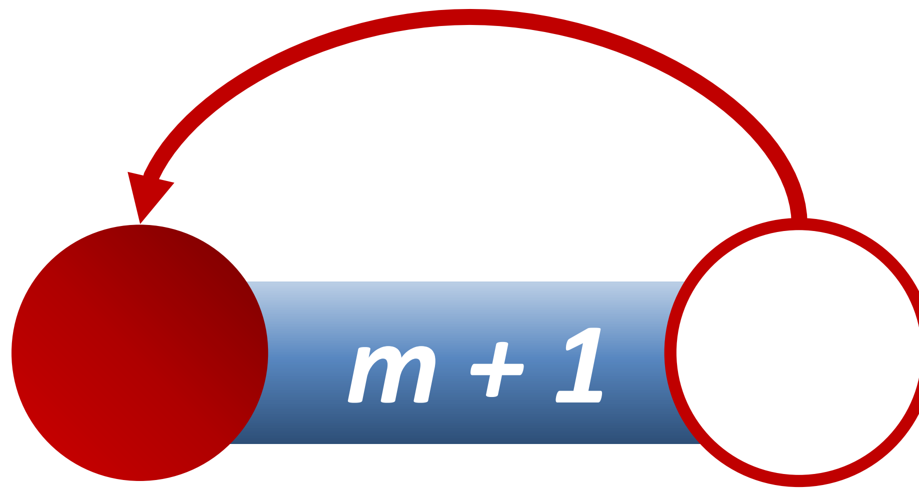
$$\psi^{\dagger}\left(\mathbf{x}\right)U\left(\mathbf{x},k\right)\psi\left(\mathbf{x}+\hat{\mathbf{e}}_k\right)$$

Raises the electric field on a link.

Suitable “kinetic” energy term:

$$H_E=\frac{e^2}{2}\sum_{\mathbf{x},k}L^2\left(\mathbf{x},k\right)$$

$$\longrightarrow\frac{1}{2}\int d^Dx\mathbf{E}^2$$



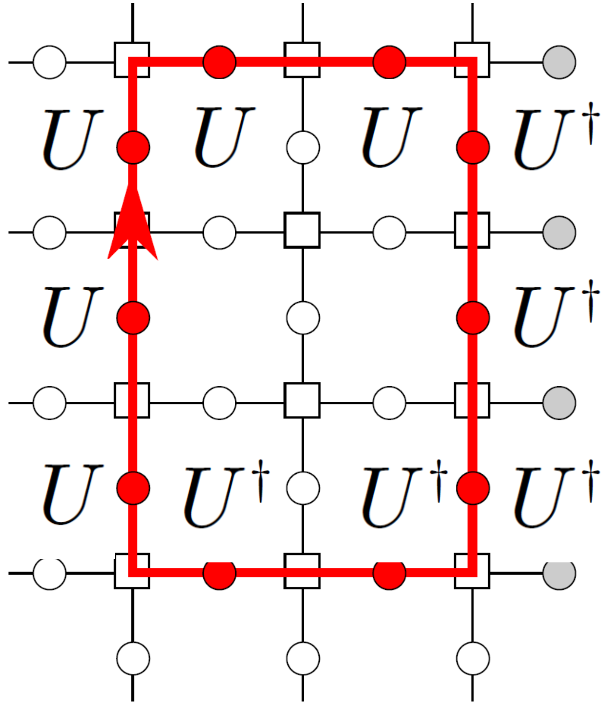
- Electric energy

Other gauge invariant operators are connectors along a closed loop - Wilson loops

$$W(C) = e^{i\oint_C dx^\mu A_\mu(\mathbf{x})}$$

On a lattice – products of oriented link operators (connectors)

$$W(C) = \prod_C U^{(\pm)}$$

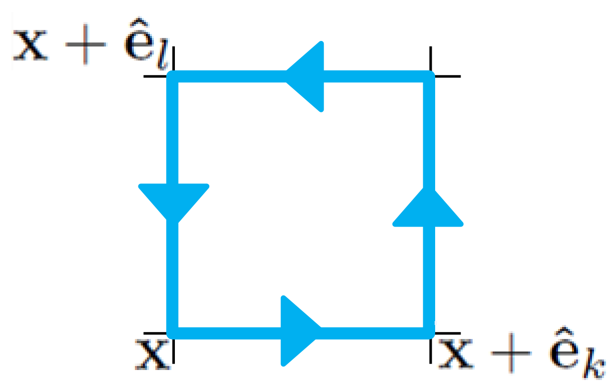


We can use these to add a self interaction term to the Hamiltonian:

$$H_B = -\frac{1}{2e^2}\sum_{\mathbf{x},k,l}\left(U_k\left(\mathbf{x},k\right)U\left(\mathbf{x}+\hat{\mathbf{e}}_k,l\right)U^\dagger\left(\mathbf{x}+\hat{\mathbf{e}},k\right)U^\dagger\left(\mathbf{x},l\right)+h.c.\right)=$$

$$-e^{-2}\sum_{\mathbf{x},k,l}\cos\left(\theta\left(\mathbf{x},k\right)+\theta\left(\mathbf{x}+\hat{\mathbf{e}}_k,l\right)-\theta\left(\mathbf{x}+\hat{\mathbf{e}}_l,k\right)-\theta\left(\mathbf{x},l\right)\right)$$

$$\longrightarrow \frac{1}{2}\int d^Dx\left(\nabla\times\mathbf{A}\right)^2=\frac{1}{2}\int d^Dx\mathbf{B}^2$$



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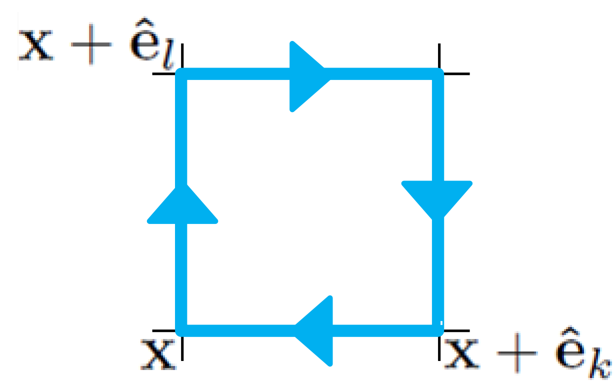


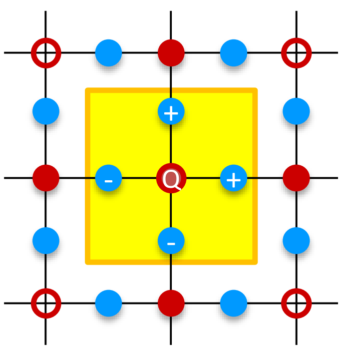
Symmetry → Conserved Charge

- **For the global symmetry, the conserved charge is the total number of fermions.**

- **For the local symmetry, we have local conservation laws, which may be formulated by the Gauss's law:**

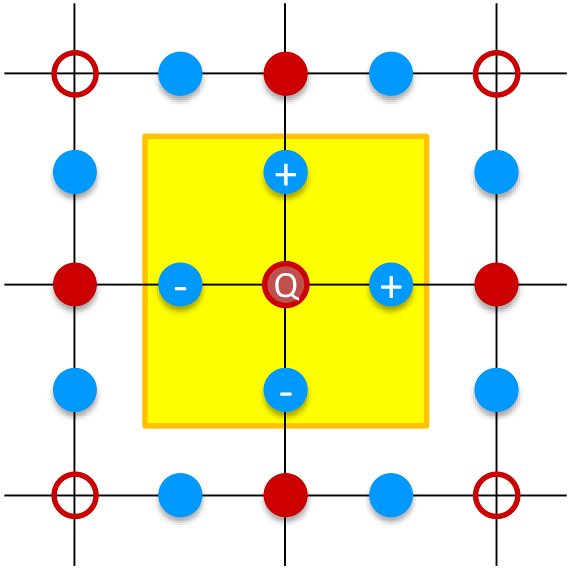
$$G\left(\mathbf{x}\right) = \nabla \cdot \mathbf{E}\left(\mathbf{x}\right) - Q\left(\psi^{\dagger}\left(\mathbf{x}\right)\psi\left(\mathbf{x}\right)\right)$$

$$\longrightarrow \sum_{k}\left(L_k\left(\mathbf{x}\right) - L_k\left(\mathbf{x}-\hat{\mathbf{e}}_k\right)\right) - Q\left(\mathbf{x}\right)$$



$$\left[G\left(\mathbf{x}\right), H\right] = 0 \quad \forall \mathbf{x} \Longleftrightarrow \dot{G}\left(\mathbf{x}\right) = 0 \quad \forall \mathbf{x}$$

Structure of the Hilbert Space



- Generators of gauge transformations (cQED):

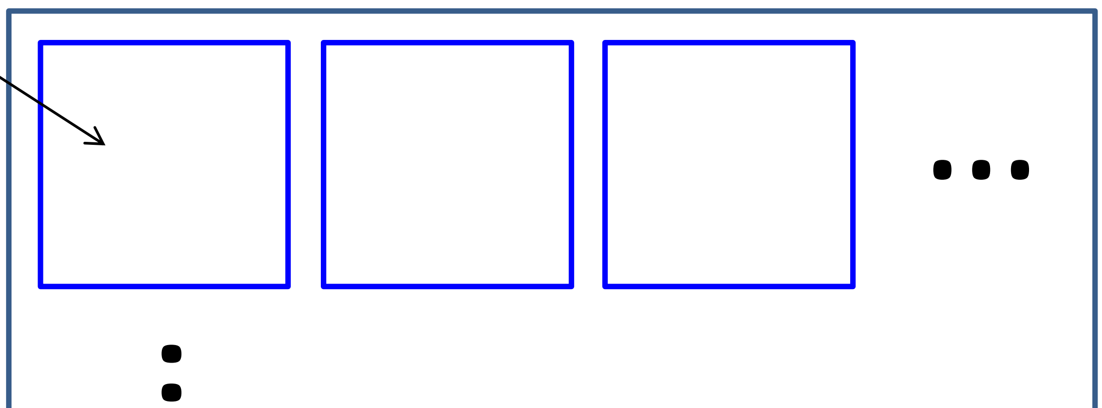
$$G\left(\mathbf{x}\right) = \mathrm{div}L\left(\mathbf{x}\right) - Q\left(\mathbf{x}\right)$$

$$\equiv \sum_k \left(L_k\left(\mathbf{x}\right) - L_k\left(\mathbf{x}-\hat{\mathbf{e}}_k\right)\right) - Q\left(\mathbf{x}\right)$$

Gauss' Law $G\left(\mathbf{x}\right)\left|\psi\right\rangle = q\left(\mathbf{x}\right)\left|\psi\right\rangle$

$$\left[G\left(\mathbf{x}\right), H\right] = 0 \quad \forall \mathbf{x}$$

Sectors with fixed
Static charge
configurations

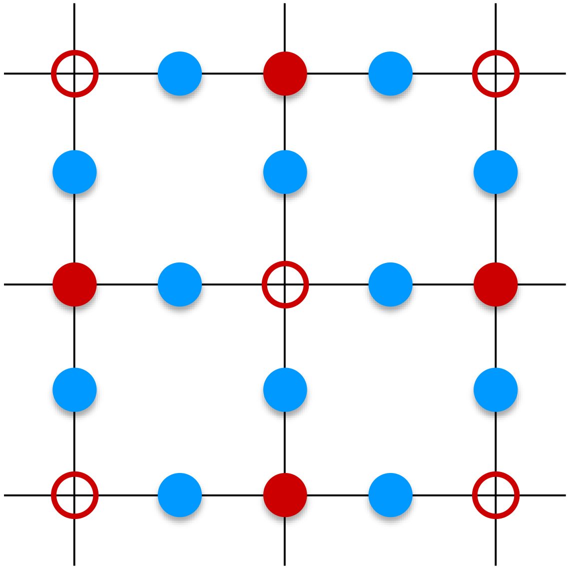


$$\mathcal{H} = \oplus\mathcal{H}\left(\left\{q\left(\mathbf{x}\right)\right\}\right)$$

Degrees of Freedom

- Matter particles (fermions) – on the **vertices** of a lattice.

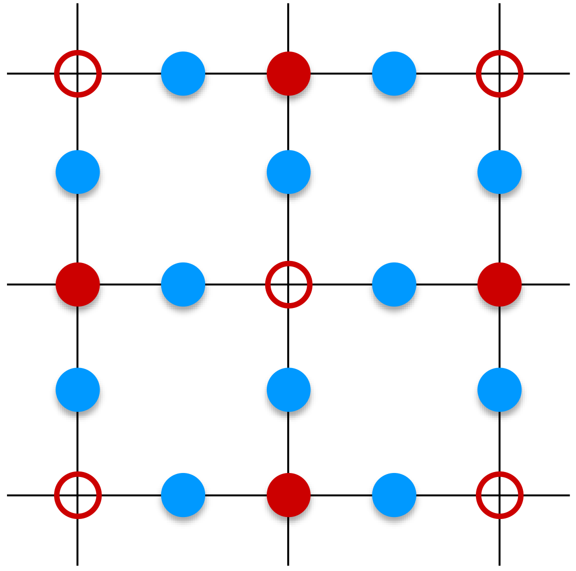
- Gauge fields – on the lattice's **links**
  - Described by local Hilbert spaces, which may be spanned in terms of **Electric Field Eigenstates** (whose exact nature depends on the gauge group) – **connection**.
  - For compact Lie groups, the local Hilbert space on a link is **infinite**.



Gauge Transformations

- Act on both the **matter** and **gauge** degrees of freedom.

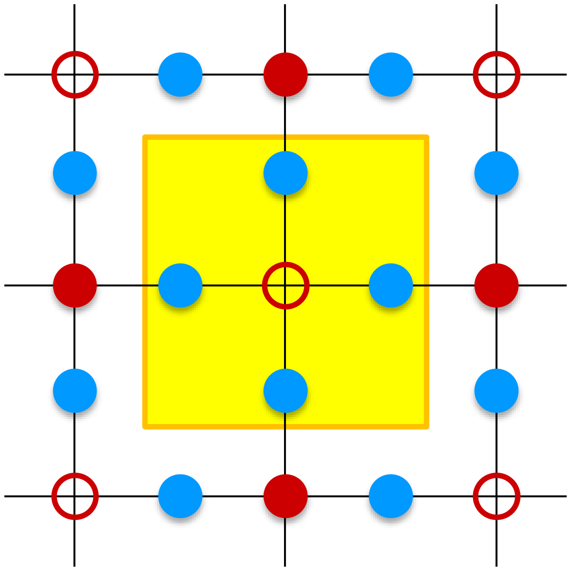
- Local: a unique transformation (depending on a unique element of the gauge group) may be chosen for each site



Gauge Transformations

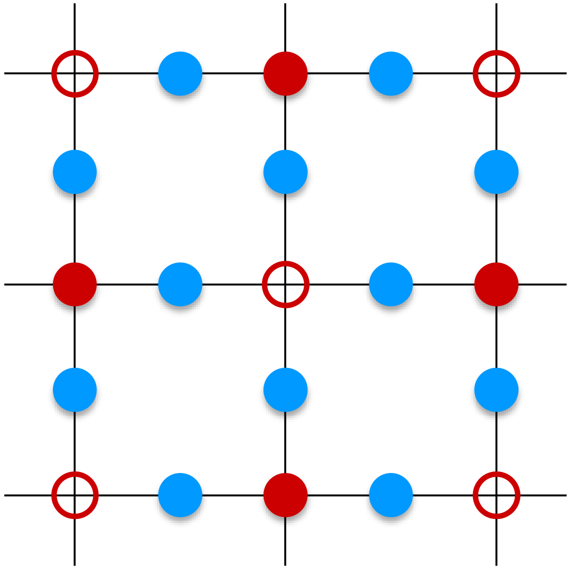
- Act on both the matter and gauge degrees of freedom.

- Local: a unique transformation (depending on a unique element of the gauge group) may be chosen for each site
- This means that the state is invariant under each local transformation separately.



Allowed Interactions

- Must preserve the symmetry – commute with the "Gauss Laws" (generators of symmetry transformations)

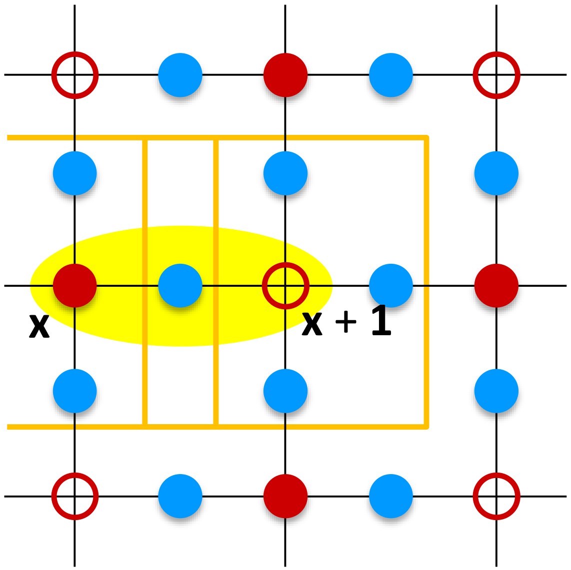


Allowed Interactions

- Must preserve the symmetry – commute with the "Gauss Laws" (generators of symmetry transformations)
- First option: Link (**matter**-**gauge**) interaction:

$$\psi_m^\dagger(\mathbf{x})\, U_{mn}(\mathbf{x},k)\, \psi_n\left(\mathbf{x}+\hat{\mathbf{k}}\right)$$

  – A **fermion** hops to a **neighboring site**, and the **flux on the link in the middle changes** to preserve **Gauss laws on the two relevant sites**

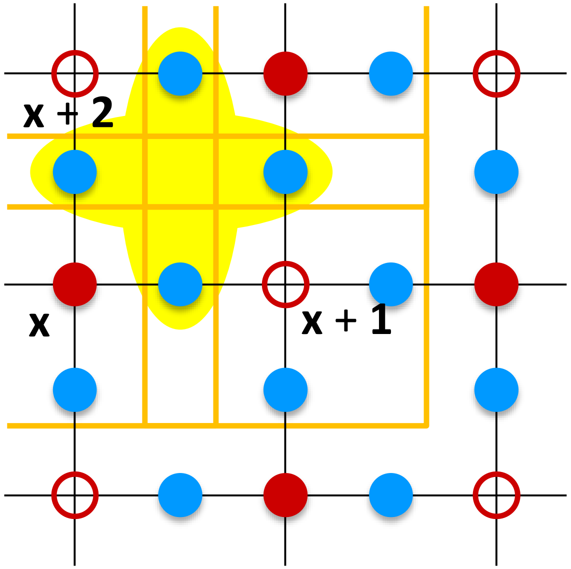


Allowed Interactions

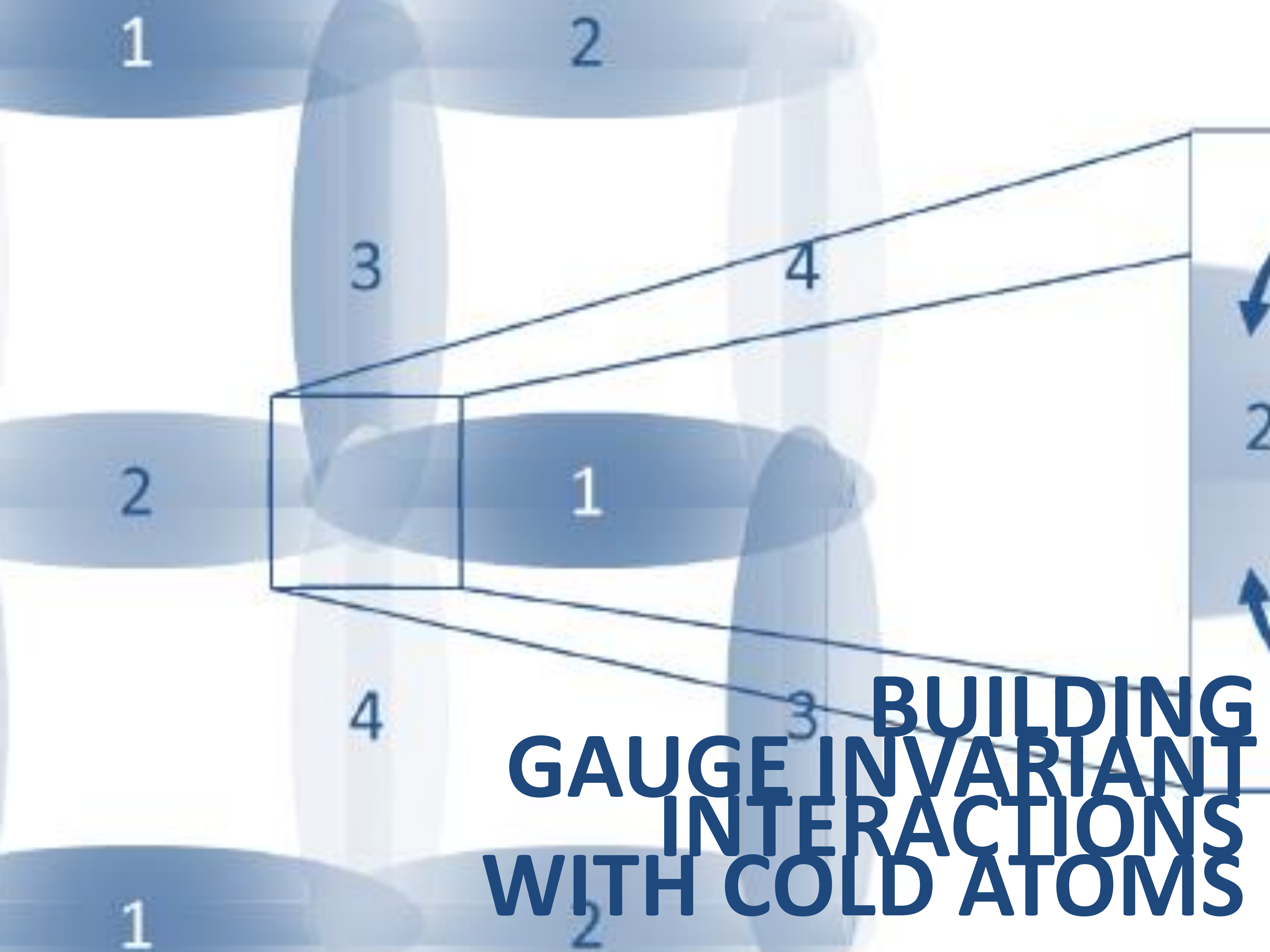
- Must preserve the symmetry – commute with the "Gauss Laws" (generators of symmetry transformations)
- Second option: plaquette interaction:

$$\mathrm{Tr}\left(U(\mathrm{x},1)U(\mathrm{x}+\hat{1},2)U^{\dagger}(\mathrm{x}+\hat{2},1)U^{\dagger}(\mathrm{x},2)\right)$$

  - The **flux on the links of a single plaquette changes** such that the **Gauss laws on the four relevant sites** is preserved.
  - **Magnetic interaction.**



BUILDING GAUGE INVARIANT INTERACTIONS WITH COLD ATOMS



Quantum Simulation of LGT

- Theoretical Proposals:
  - Various gauge groups:
    - Abelian ($U(1)$, Z_N)
    - non-Abelian (SU(N)…)

PRL **107**, 275301 (2011) PHYSICAL REVIEW LETTERS week ending 30 DECEMBER 2011

Confinement and Lattice Quantum-Electrodynamic Electric Flux Tubes Simulated with Ultracold Atoms

Erez Zohar and Benni Reznik

School of Physics and Astronomy, Raymond and Beverly Sackler Faculty of Exact Sciences, Tel-Aviv University, Tel-Aviv 69978, Israel

(Received 7 August 2011; published 27 December 2011)

We propose a method for simulating (2 + 1)D compact lattice quantum-electrodynamics, using ultracold atoms in optical lattices. In our model local Bose-Einstein condensates' (BECs) phases correspond to the electromagnetic vector potential, and the local number operators represent the conjugate electric field. The well-known gauge-invariant Kogut-Susskind Hamiltonian is obtained as an effective low-energy theory. The field is then coupled to external static charges. We show that in the strong coupling limit this gives rise to "electric flux tubes" and to confinement. This can be observed by measuring the local density deviations of the BECs, and is expected to hold even, to some extent, outside the perturbative calculable regime.

LETTER

doi:10.1038/nature18318

Real-time dynamics of lattice gauge theories with a few-qubit quantum computer

Esteban A. Martinez[1*], Christine A. Muschik[2,3*], Philipp Schindler[1], Daniel Nigg[1], Alexander Erhard[1], Markus Heyl[2,4], Philipp Hauke[2,3], Marcello Dalmonte[2,3], Thomas Monz[1], Peter Zoller[2,3] & Rainer Blatt[1,2]

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  - Various simulating systems:
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    - Trapped Ions
    - Superconducting Qubits

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  - Various simulating systems:
    - Ultracold Atoms
    - Trapped Ions
    - Superconducting Qubits
  - Various simulation approaches:
    - Analog
    - Digital

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Quantum Simulation of LGT

- First Experimental Realization (**Christine's talk**):

LETTER

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How to go further?

→ Larger Systems
→ 2+1d and more
→ Fermions
→ Non-Abelian models

PARTICLE PHYSICS

Quantum simulation of fundamental physics

Gauge theories underpin the standard model of particle physics, but are difficult to study using conventional computational methods. An experimental quantum system opens up fresh avenues of investigation. SEE LETTER P.516

EREZ ZOHAR

answer some of those outstanding questions. Theoretical physics often involves problems

Basic Requirements from a LGT Q. Simulator

- Include both fermions (matter) and gauge fields

- Have Lorentz (relativistic) symmetry

- Manifest **Local** (Gauge) Invariance

E. Zohar, J. I. Cirac, B. Reznik, Phys. Rev. Lett. 110, 055302 (2013)
E. Zohar, J. I. Cirac, B. Reznik, Rep. Prog. Phys. 79, 014401 (2016)

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    Use ultracold atoms in optical lattices: both bosonic and fermionic atoms may be trapped and manipulated.

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Basic Requirements from a LGT Q. Simulator

- Include both fermions (matter) and gauge fields

  Use ultracold atoms in optical lattices: both bosonic and fermionic atoms may be trapped and manipulated.

- Have Lorentz (relativistic) symmetry

  Simulate lattice gauge theory. Symmetry may be restored in an appropriate continuum limit.

- Manifest **Local** (Gauge) Invariance

E. Zohar, J. I. Cirac, B. Reznik, Phys. Rev. Lett. 110, 055302 (2013)

E. Zohar, J. I. Cirac, B. Reznik, Rep. Prog. Phys. 79, 014401 (2016)

Basic Requirements from a LGT Q. Simulator

- Include both fermions (matter) and gauge fields

  Use ultracold atoms in optical lattices: both bosonic and fermionic atoms may be trapped and manipulated.

- Have Lorentz (relativistic) symmetry

  Simulate lattice gauge theory. Symmetry may be restored in an appropriate continuum limit.

- Manifest **Local** (Gauge) Invariance on top of the natural global atomic symmetries (number conservation)

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  Simulate lattice gauge theory. Symmetry may be restored in an appropriate continuum limit.

- Manifest **Local** (Gauge) Invariance on top of the natural global atomic symmetries (number conservation)

  Local (gauge) symmetries may be introduced to the atomic simulator using several methods.

E. Zohar, J. I. Cirac, B. Reznik, Phys. Rev. Lett. 110, 055302 (2013)

E. Zohar, J. I. Cirac, B. Reznik, Rep. Prog. Phys. 79, 014401 (2016)

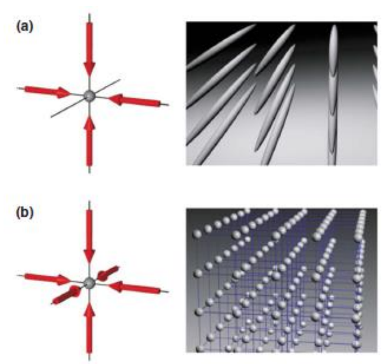
Ultracold Atoms in Optical Lattices

- Atoms are cooled and trapped in periodic potentials created by laser beams.
- Highly controllable systems:
  - Tuning the laser beams → shape of the potential
  - Tunable interactions (S-wave collisions among atoms in the ultracold limit tunable with Feshbach resonances, external Raman lasers)
  - Use of several atomic species →
    different internal (hyperfine) levels $\mathbf{F} = \mathbf{I} + \mathbf{L} + \mathbf{S}$ may be used, experiencing different optical potentials
  - Easy to measure, address and manipulate

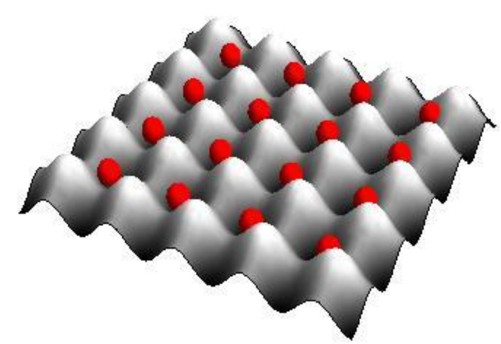
VOLUME 81, NUMBER 15 PHYSICAL REVIEW LETTERS 12 OCTOBER 1998

Cold Bosonic Atoms in Optical Lattices

D. Jaksch,[1,2] C. Bruder,[1,3] J. I. Cirac,[1,2] C. W. Gardiner,[1,4] and P. Zoller[1,2]
[1] *Institute for Theoretical Physics, University of Santa Barbara, Santa Barbara, California 93106-4030*
[2] *Institut für Theoretische Physik, Universität Innsbruck, A-6020 Innsbruck, Austria*
[3] *Institut für Theoretische Festkörperphysik, Universität Karlsruhe, D-76128 Karlsruhe, Germany*
[4] *School of Chemical and Physical Sciences, Victoria University, Wellington, New Zealand*
(Received 26 May 1998)

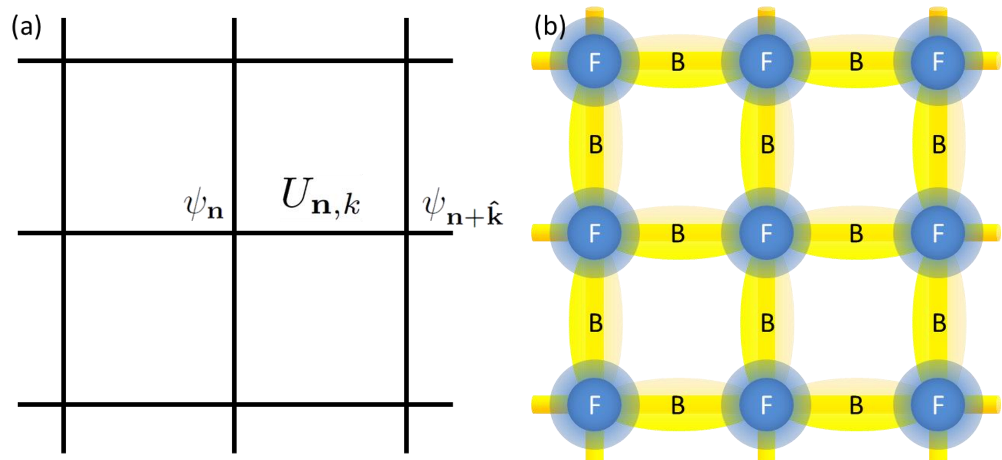


RMP 80, 885 (2008)

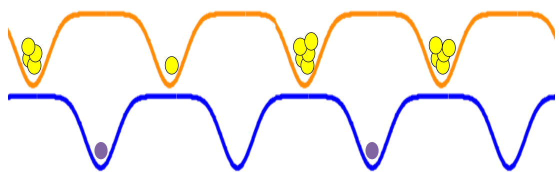


QS of LGTs with Ultracold Atoms in Optical Lattices

- **Fermionic** matter fields
- (Bosonic) gauge fields



Super-lattice:



Atomic internal (**hyperfine**) levels

0 (ultracold)

$$\mathbf{F} = \mathbf{I} + \cancel{\mathbf{L}} + \mathbf{S} \qquad \mathbf{F}^2|F, m_\mathrm{F}\rangle = F(F+1)|F, m_\mathrm{F}\rangle \qquad F_z|F, m_\mathrm{F}\rangle = m_\mathrm{F}|F, m_\mathrm{F}\rangle$$

$$\mathcal{H} = \sum_{\alpha,\beta} \Phi_\alpha^\dagger(\mathbf{x}) \left(\delta^{\alpha\beta} \left(-\frac{\nabla^2}{2m} + V_{\mathrm{op}}^{\alpha}(\mathbf{x}) + V_\mathrm{T}(\mathbf{x}) \right) + \Omega^{\alpha\beta}(\mathbf{x}) \right) \Phi_\beta(\mathbf{x})$$
$$+ \sum_{\alpha,\beta,\gamma,\delta} \int \mathrm{d}^3x' \Phi_\alpha^\dagger(\mathbf{x}') \Phi_\beta^\dagger(\mathbf{x}) V_{\alpha\beta\gamma\delta}(\mathbf{x} - \mathbf{x}') \Phi_\gamma(\mathbf{x}) \Phi_\delta(\mathbf{x}')$$

The atomic Hamiltonian (Hubbard) has a global symmetry

General form (after "overlap" Wannier integrations)

$$H = \sum_{\mathrm{m,n}} J_{\mathrm{m,n}} a^\dagger_{\mathrm{m}} a_{\mathrm{n}} + \sum_{\mathrm{m,n,k,l}} U_{\mathrm{m,n,k,l}} a^\dagger_{\mathrm{m}} a^\dagger_{\mathrm{n}} a_{\mathrm{k}} a_{\mathrm{l}}$$

Assuming nearest neighbor interactions

$$H = J \sum_{\langle \mathrm{m,n} \rangle} a^\dagger_{\mathrm{m}} a_{\mathrm{n}} + U \sum_{\mathrm{m}} N_{\mathrm{m}} \left(N_{\mathrm{m}} - 1 \right)$$

For many species

$$H = \sum_{\mathrm{m,n},\alpha,\beta} J^{\alpha,\beta}_{\mathrm{m,n}} a^\dagger_{\mathrm{m},\alpha} a_{\mathrm{n},\beta} + \sum_{\mathrm{m,n,k,l}} U^{\alpha,\beta,\gamma,\delta}_{\mathrm{m,n,k,l}} a^\dagger_{\mathrm{m},\alpha} a^\dagger_{\mathrm{n},\beta} a_{\mathrm{k},\gamma} a_{\mathrm{l},\delta}$$

Total number of particles is conserved (global symmetry): no apparent local symmetry

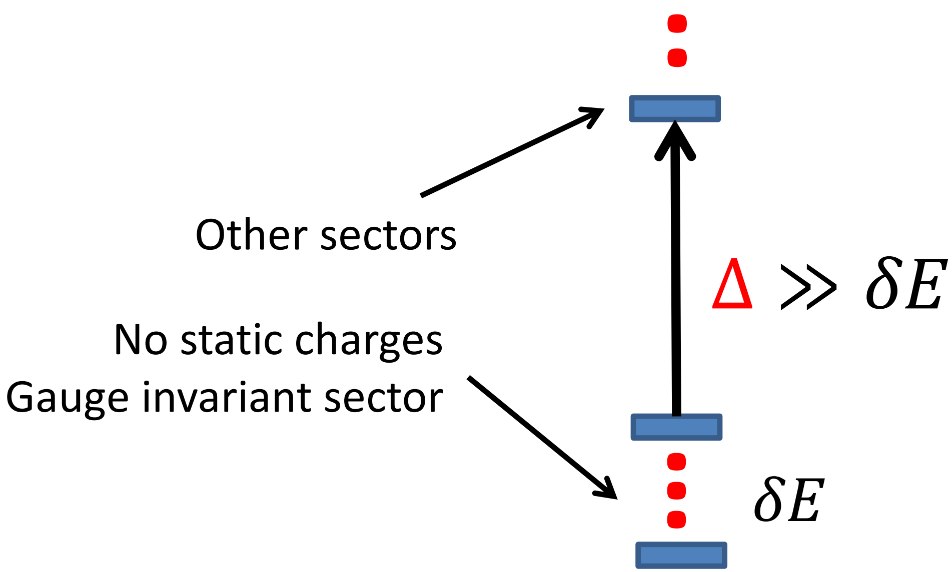
Analog Approach I: Effective Local Gauge Invariance

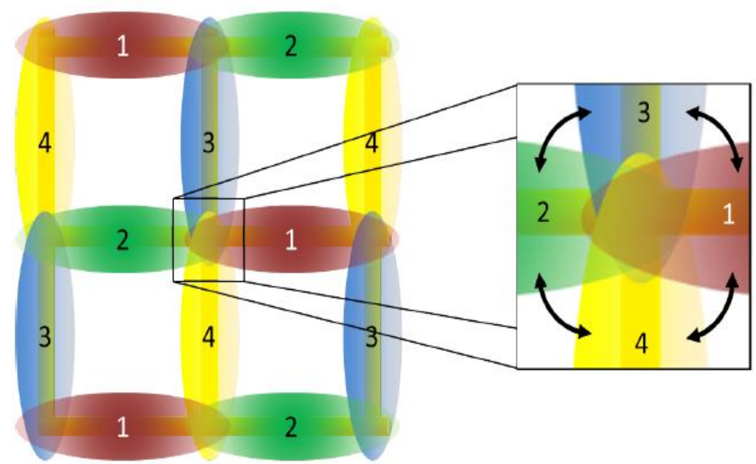
Gauss law is added to the Hamiltonian as a constraint (penalty term).

Leaving a gauge invariant sector of Hilbert space costs too much Energy.

Low energy sector with an effective gauge invariant Hamiltonian.

Emerging plaquette interactions (second order perturbation theory).





E. Zohar, B. Reznik, Phys. Rev. Lett. 107, 275301 (2011)

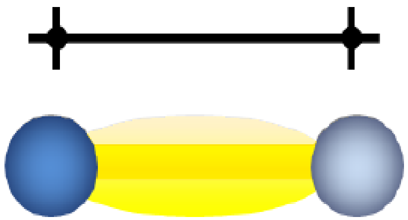
E. Zohar, J. I. Cirac, B. Reznik, Phys. Rev. Lett. 109, 125302 (2012)

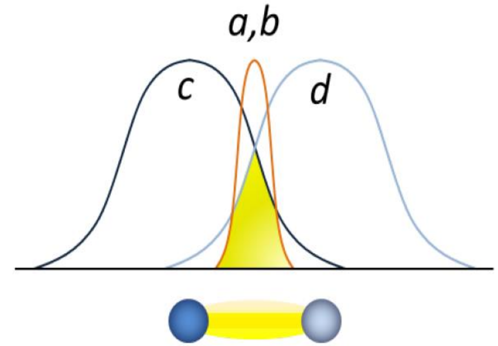
E. Zohar, J. I. Cirac, B. Reznik, Phys. Rev. Lett. 110, 055302 (2013)

E. Zohar, J. I. Cirac, B. Reznik, Rep. Prog. Phys. 79, 014401 (2016)

Analog Approach II:
Atomic Symmetries → Local Gauge Invariance

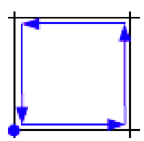
$$\psi_L^\dagger U \psi_R$$

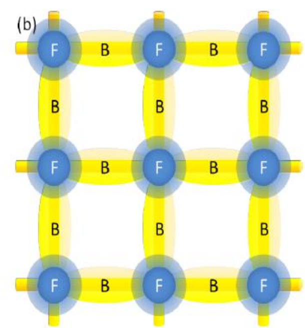




- Links $\leftrightarrow$ atomic scattering : gauge invariance is a <u>fundamental</u> symmetry

$$\sum_{\text{plaquettes}} \left(\mathrm{Tr}\left(U_1 U_2 U_3^\dagger U_4^\dagger \right) + h.c. \right)$$





- Plaquettes $\leftrightarrow$ gauge invariant links $\leftrightarrow$ virtual loops of ancillary fermions.

E. Zohar, J. I. Cirac, B. Reznik, Phys. Rev. Lett. 110, 125304 (2013)

E. Zohar, J. I. Cirac, B. Reznik, Phys. Rev. A 88 023617 (2013)

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D. González Cuadra, E. Zohar, J. I. Cirac, New J. Phys. 19 063038 (2017)

Realization of Link Interactions

$$\psi_L^\dagger U \psi_R$$

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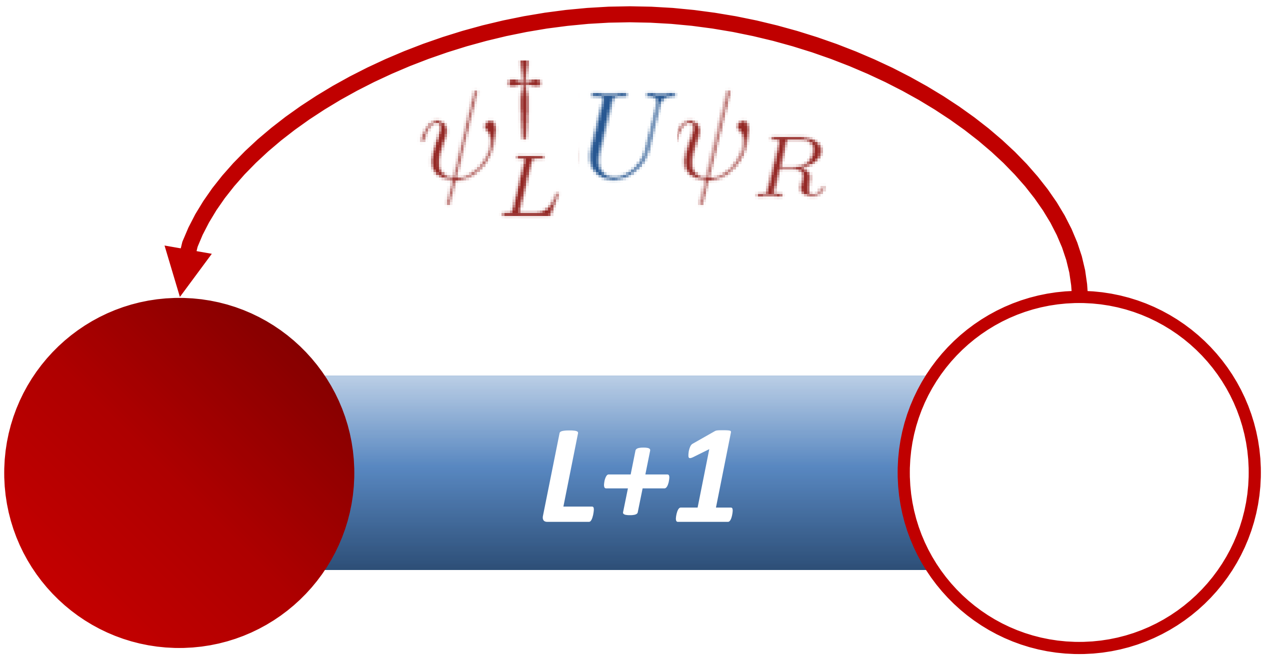

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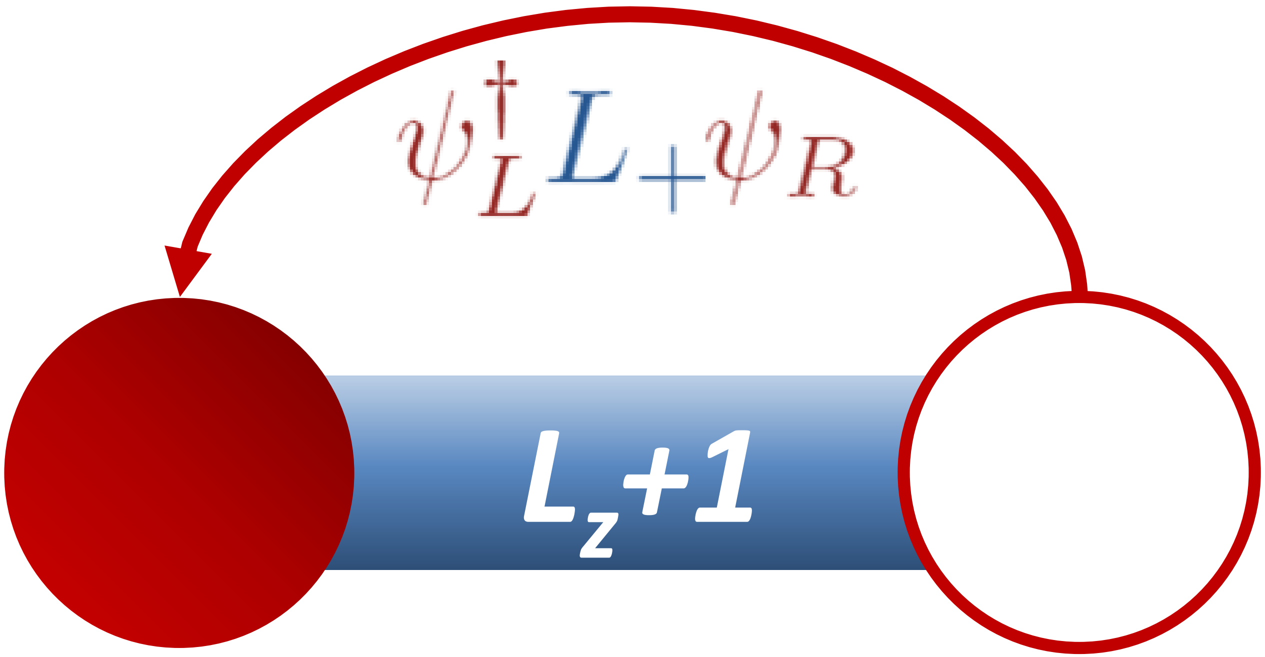

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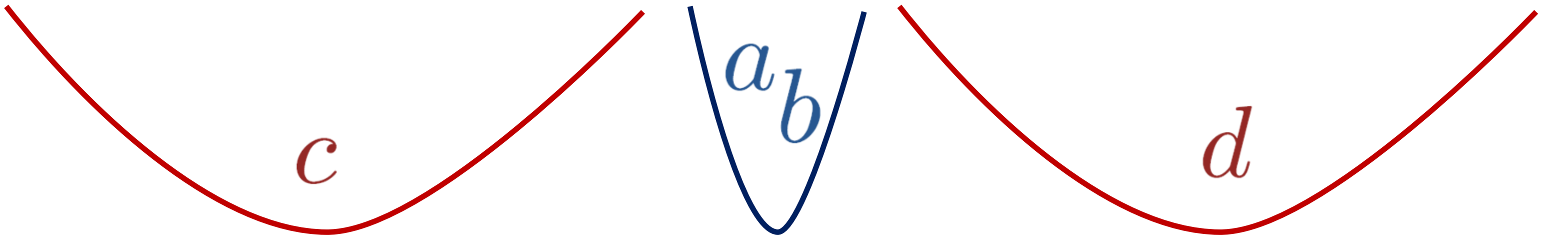
Superlattice Structure

$$\psi_L^\dagger U \psi_R \sim \psi_L^\dagger L_+ \psi_R = c^\dagger a^\dagger b d$$



Schwinger algebra

$$\ell = \frac{1}{2}\left(a^\dagger a + b^\dagger b\right)$$

$$L_+ = a^\dagger b \qquad L_- = b^\dagger a \qquad L_z = \frac{1}{2}\left(a^\dagger a - b^\dagger b\right)$$

E. Zohar, J. I. Cirac, B. Reznik, Phys. Rev. Lett. 110, 125304 (2013)
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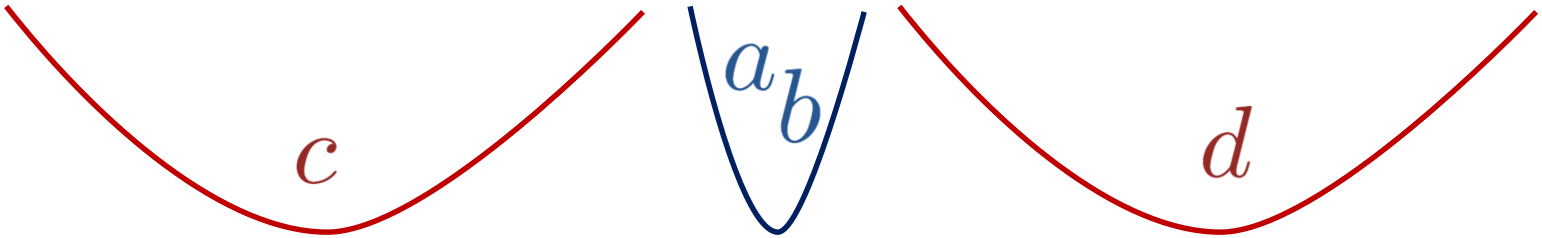

Schwinger algebra

$$L_+ = a^\dagger b \sim e^{i(\phi_a - \phi_b)} \equiv e^{i\phi} = U$$

For large ℓ , $m \ll \ell$

E. Zohar, J. I. Cirac, B. Reznik, Phys. Rev. Lett. 110, 125304 (2013)

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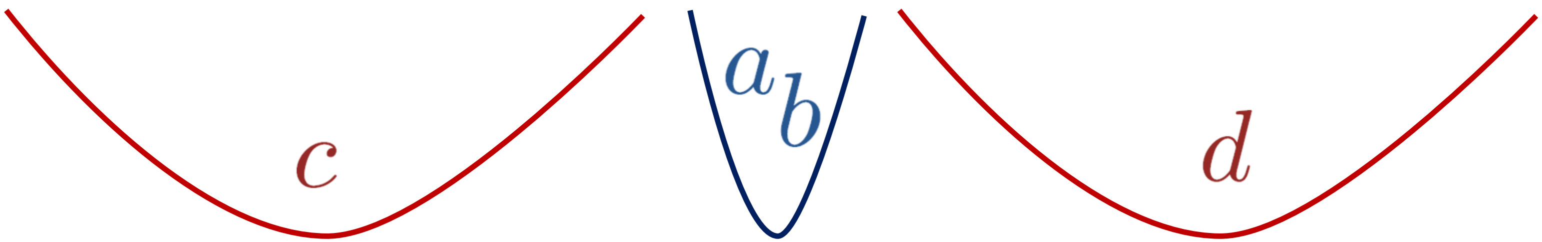
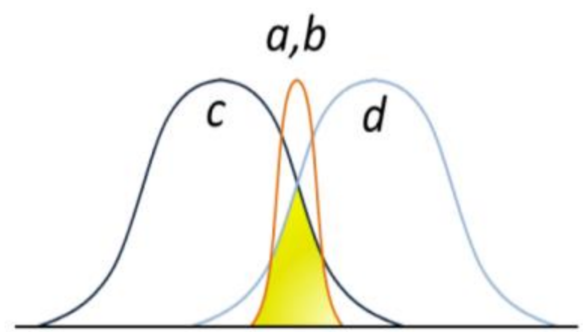


- Narrow, deep bosonic wells → no tunneling, fixed number on link → fixed Schwinger representation: $\ell = \frac{1}{2}\left(a^\dagger a + b^\dagger b\right)$
- Staggered fermionic wells → no tunneling
- Only possible Hamiltonian terms: Scattering on the link: B-F – interaction, B-B – electric energy



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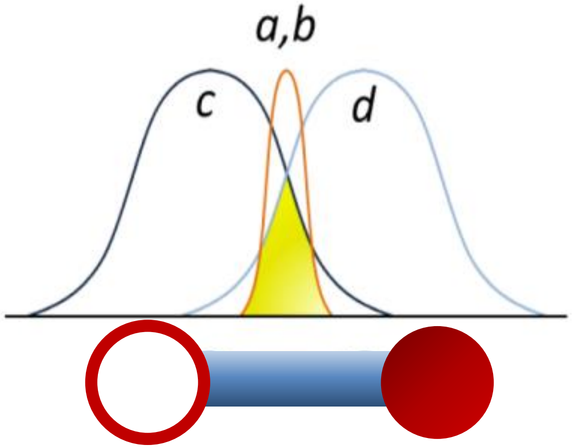
D. González Cuadra, E. Zohar, J. I. Cirac, New J. Phys. 19 063038 (2017)

Atomic symmetry → Gauge Invariance

$$\sum_{\alpha,\beta,\gamma,\delta}\int \mathrm{d}^3x\mathrm{d}^3x'\Phi^\dagger_\alpha(\mathbf{x}')\Phi^\dagger_\beta(\mathbf{x})V_{\alpha\beta\gamma\delta}(\mathbf{x}-\mathbf{x}')\Phi_\gamma(\mathbf{x})\Phi_\delta(\mathbf{x}')$$

$$V_{\alpha,\beta,\gamma,\delta}(\mathbf{x}-\mathbf{x}') = \delta^{(3)}(\mathbf{x}-\mathbf{x}')\sum_{F_\mathrm{T}} C_{F_\mathrm{T}}\langle F_1, m_{F1}=\alpha; F_2, m_{F2}=\beta | F_\mathrm{T}, M_\mathrm{F}\rangle \times \langle F_\mathrm{T}, M_\mathrm{F} | F_1, m_{F1}=\gamma; F_2, m_{F2}=\delta\rangle$$



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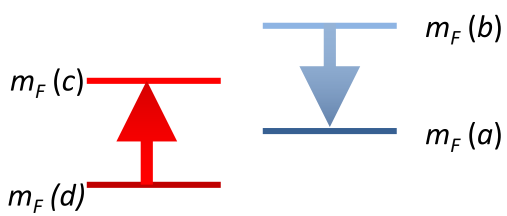
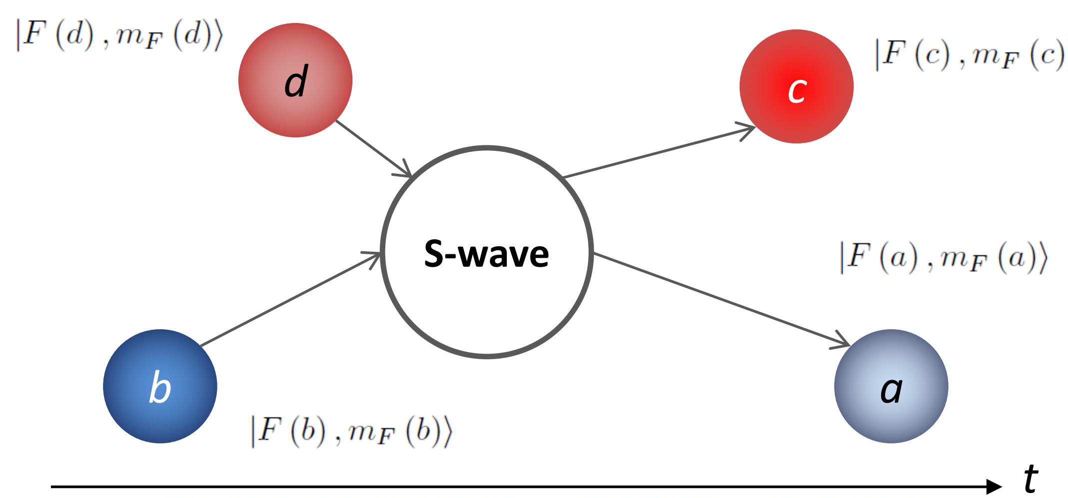


Fermionic atoms - matter

Bosonic atoms - Gauge field



$$m_F(a) + m_F(c) = m_F(b) + m_F(d)$$

β α γ δ

E. Zohar, J. I. Cirac, B. Reznik, Phys. Rev. Lett. 110, 125304 (2013)

E. Zohar, J. I. Cirac, B. Reznik, Phys. Rev. A 88 023617 (2013)

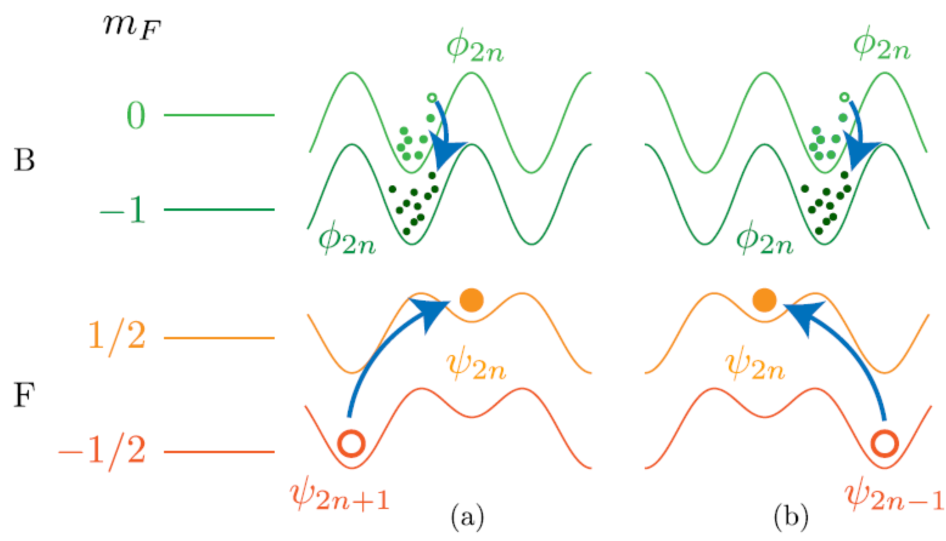
E. Zohar, J. I. Cirac, B. Reznik, Rep. Prog. Phys. 79, 014401 (2016)

D. González Cuadra, E. Zohar, J. I. Cirac, New J. Phys. 19 063038 (2017)

Realization in Heidelberg

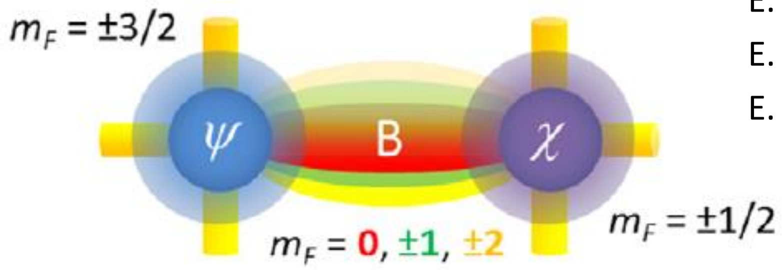
- Calculations applying our scheme towards an experiment: **Kasper, Hebenstreit, Jendrzejewski, Oberthaler, Berges**, NJP 19 023030 (2017) – very exciting results

- Matter: F = ½ ^{6}Li atoms
- Gauge field: F = 1 ^{23}Na atoms
- No Feshbach resonance!
- On the links, around 100 atomic bosons – very high electric field truncation (±50)
- More dimensions?



Generalizations

- Valid for any gauge group, including non-Abelian



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E. Zohar, J. I. Cirac, B. Reznik, Rep. Prog. Phys. 79, 014401 (2016)

- Truncation schemes for general groups (analogous to the Schwinger representation used in the abelian case)– possible as well)

$$U^j_{mm'} = \sum_{J,K} \sqrt{\frac{\dim(J)}{\dim(K)}} \langle JM\, jm | KN \rangle \langle KN' | JM'\, jm' \rangle\, a^{\dagger K}_{NN'} a^J_{MM'}$$

$$U^{j=1/2}_{mn} = \frac{1}{\sqrt{2}} \begin{pmatrix} |{+}{+}\rangle\langle 0| + |0\rangle\langle{-}{-}| & |{+}{-}\rangle\langle 0| - |0\rangle\langle{-}{+}| \\ |{-}{+}\rangle\langle 0| - |0\rangle\langle{+}{-}| & |0\rangle\langle{+}{+}| + |{-}{-}\rangle\langle 0| \end{pmatrix}$$

E. Zohar, M. Burrello, Phys. Rev. D. 91, 054506 (2015)

Further Dimensions → Plaquette Interactions

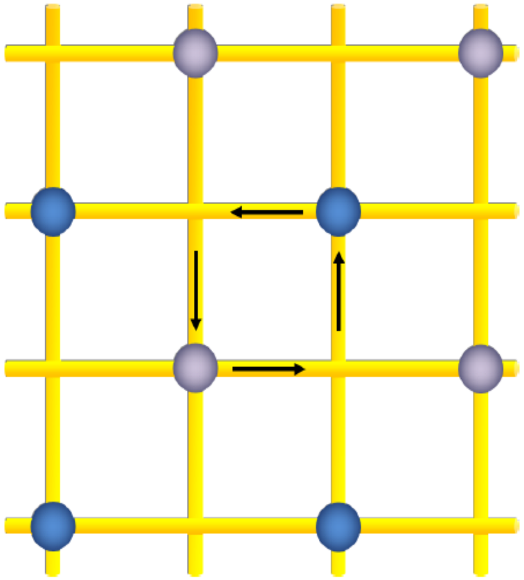
$$\sum_{\text{plaquettes}} \left(\mathrm{Tr} \left(U_1 U_2 U_3^\dagger U_4^\dagger \right) + h.c. \right)$$

1d elementary link interactions are **already gauge invariant**

Auxiliary fermions:

Heavy,
constrained to “sit”
on special vertices

- Virtual processes
- Valid for any gauge group, once the link interactions are realized



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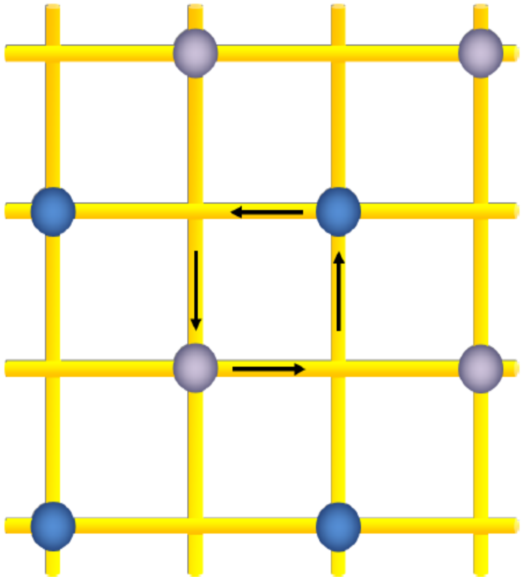
D. González Cuadra, E. Zohar, J. I. Cirac, New J. Phys. 19 063038 (2017)

Further Dimensions → Plaquette Interactions

$$\sum_{\text{plaquettes}} \left(\mathrm{Tr}\left(U_1 U_2 U_3^\dagger U_4^\dagger \right) + h.c. \right)$$

Perturbative interactions!

- Practically, second order and not fourth, since only even orders contribute to the perturbative series.
- However, it's still weak, so why not do it without perturbation theory at all?



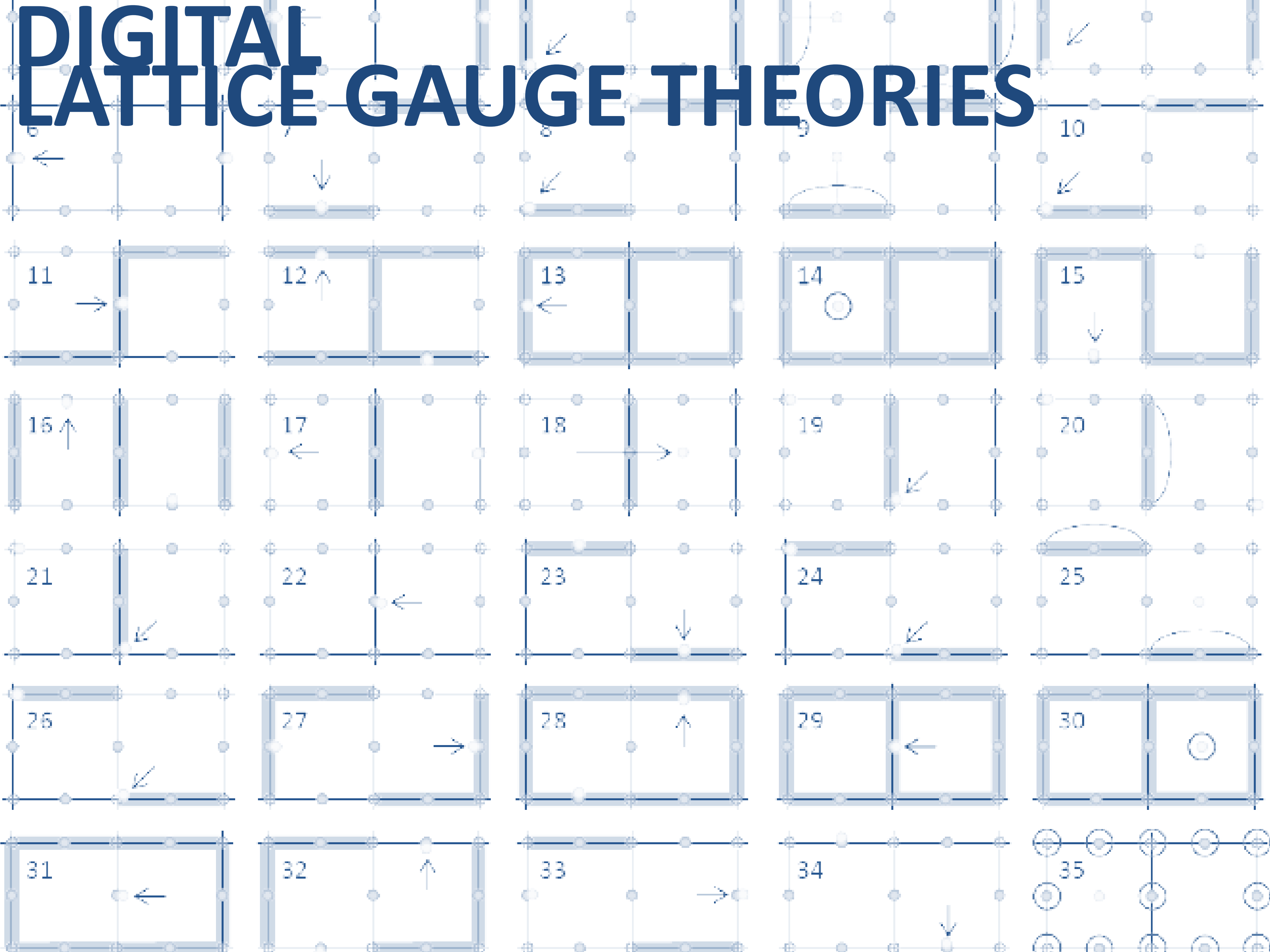
E. Zohar, J. I. Cirac, B. Reznik, Phys. Rev. Lett. **110**, 125304 (2013)

E. Zohar, J. I. Cirac, B. Reznik, Phys. Rev. A **88** 023617 (2013)

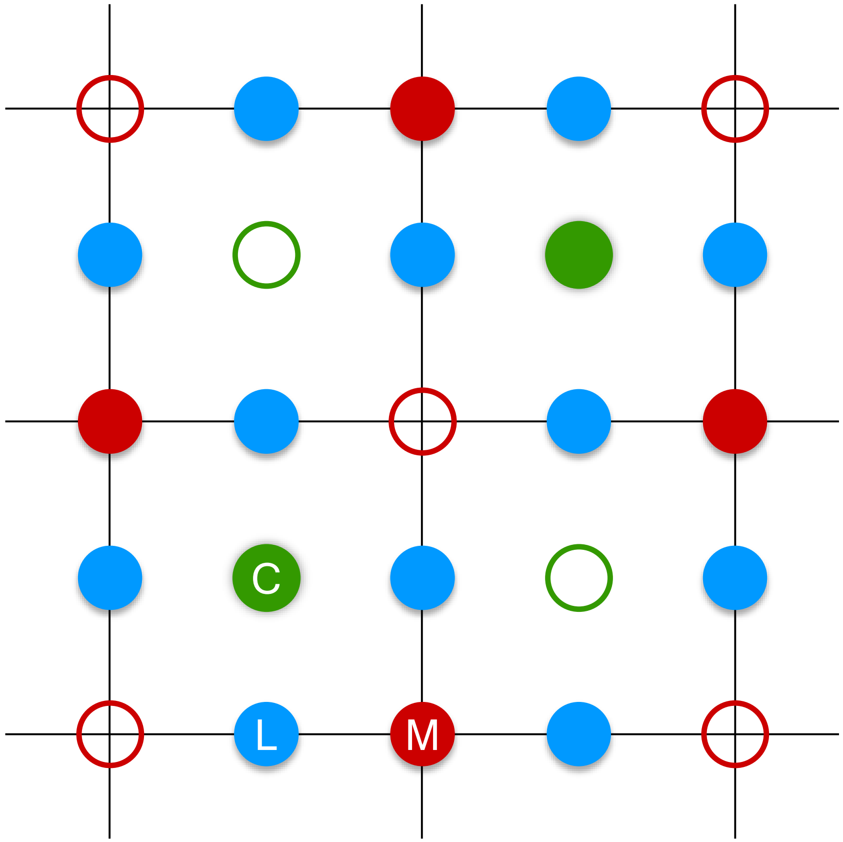
E. Zohar, J. I. Cirac, B. Reznik, Rep. Prog. Phys. **79**, 014401 (2016)

D. González Cuadra, **E. Zohar**, J. I. Cirac, New J. Phys. **19** 063038 (2017)

DIGITAL LATTICE GAUGE THEORIES



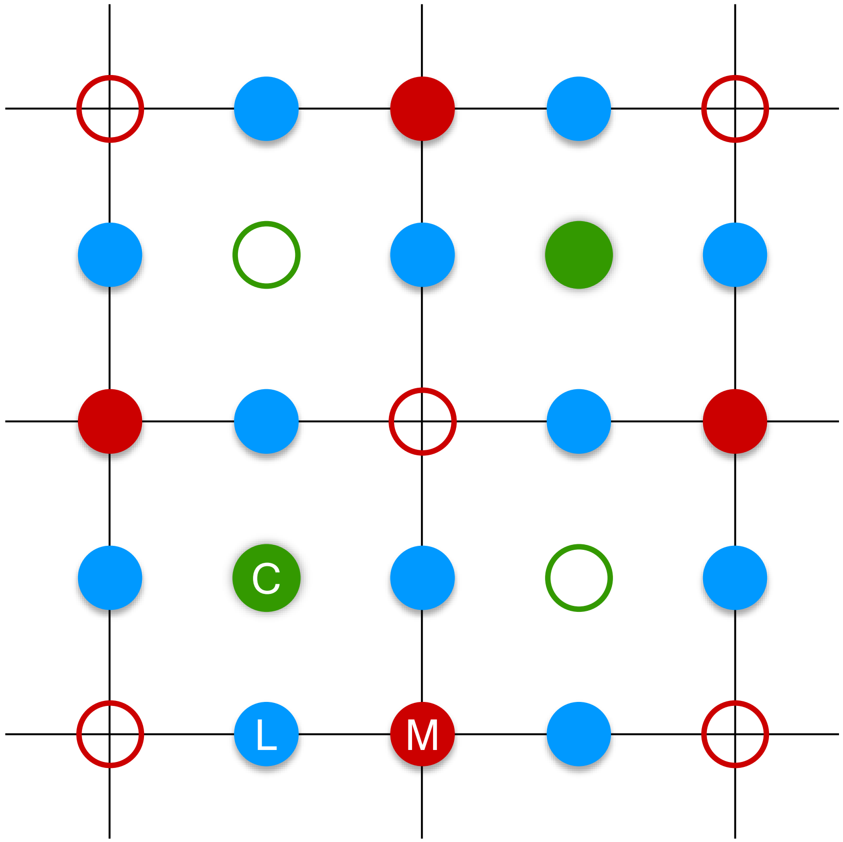
Digital Lattice Gauge Theories



Matter Fermions
Link (Gauge) degrees of freedom
Control degrees of freedom

E. Zohar, A. Farace, B. Reznik, J. I. Cirac, Phys. Rev. Lett. 118 070501 (2017)
E. Zohar, A. Farace, B. Reznik, J. I. Cirac, Phys. Rev. A. 95 023604 (2017)

Digital Lattice Gauge Theories

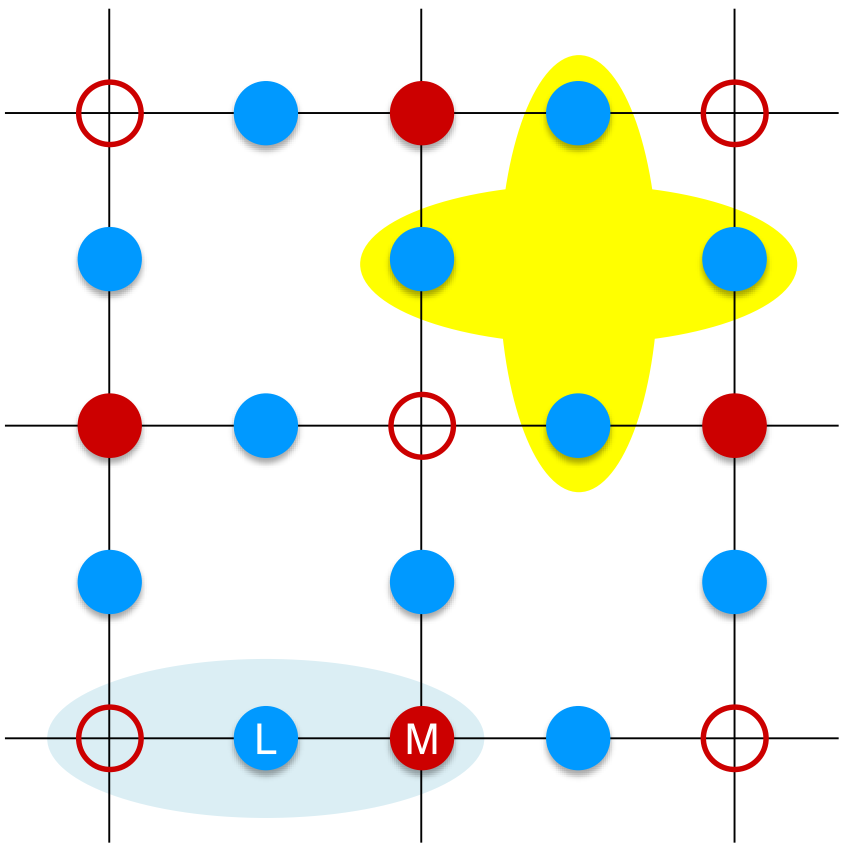


Matter Fermions
Link (Gauge) degrees of freedom
Control degrees of freedom

Entanglement is created and undone between the control and the physical degrees of freedom.

E. Zohar, A. Farace, B. Reznik, J. I. Cirac, Phys. Rev. Lett. 118 070501 (2017)
E. Zohar, A. Farace, B. Reznik, J. I. Cirac, Phys. Rev. A. 95 023604 (2017)

Digital Lattice Gauge Theories



The Z_N example:

- Plaquette interactions

$$Q(\mathbf{x},1)Q(\mathbf{x}+\hat{1},2)Q^{\dagger}(\mathbf{x}+\hat{2},1)Q^{\dagger}(\mathbf{x},2)+\mathrm{H.c.}$$

- Link interactions

$$\psi^{\dagger}(\mathbf{x})Q(\mathbf{x},k)\psi(\mathbf{x}+\hat{\mathbf{k}})$$

$$P^N = Q^N = 1,$$
$$PQP^{\dagger} = e^{i(2\pi/N)}Q,$$
$$Q|m\rangle = |m+1\rangle \text{ (cyclically)},$$
$$P|m\rangle = e^{i(2\pi/N)m}|m\rangle.$$

E. Zohar, A. Farace, B. Reznik, J. I. Cirac, Phys. Rev. Lett. 118 070501 (2017)
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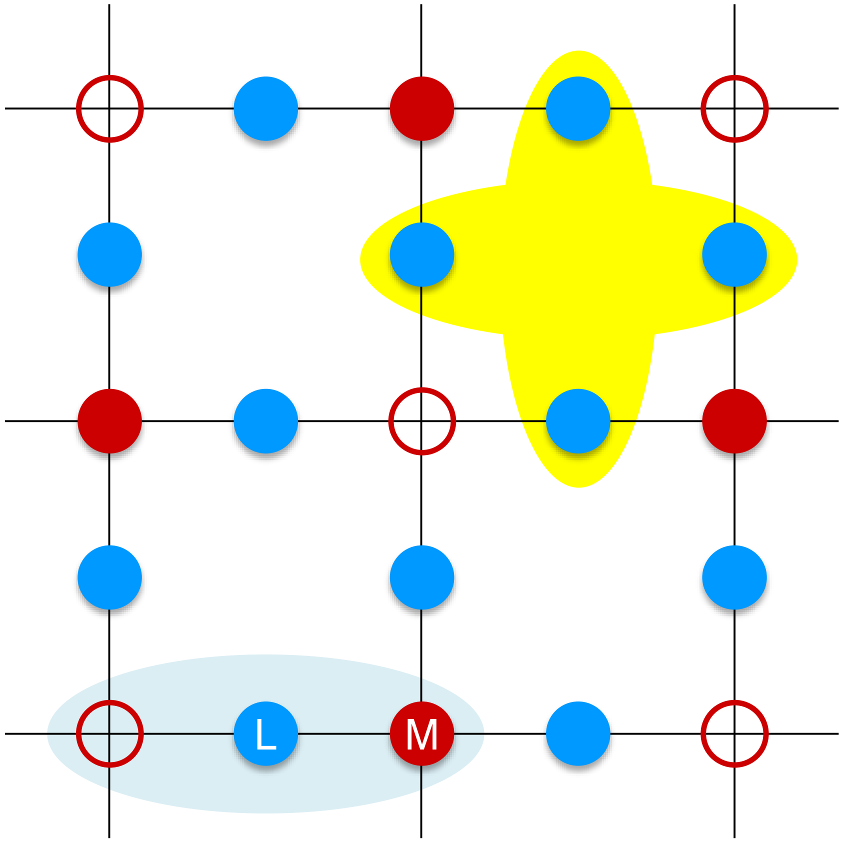
Digital Lattice Gauge Theories



The Z_2 example:

- Plaquette interactions

$$\sigma_x(\mathbf{x},1)\,\sigma_x(\mathbf{x}+\hat{\mathbf{1}},2)\,\sigma_x(\mathbf{x}+\hat{\mathbf{2}},1)\,\sigma_x(\mathbf{x},2)$$

- Link interactions

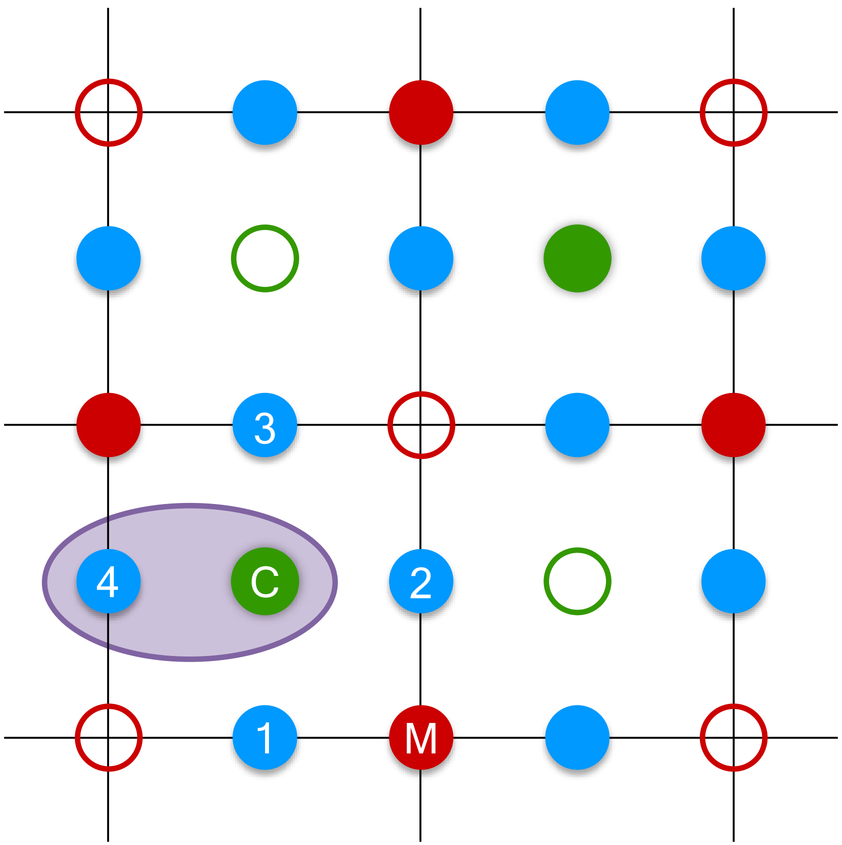
$$\psi^\dagger(\mathbf{x})\sigma_x(\mathbf{x},k)\psi(\mathbf{x}+\hat{\mathbf{k}})$$

E. Zohar, A. Farace, B. Reznik, J. I. Cirac, Phys. Rev. Lett. 118 070501 (2017)

Plaquettes: Four-body Interactions

Two-body interactions → four-body interactions

$$\mathcal{U} = \mathcal{U}^\dagger = \left|\tilde{\uparrow}\right\rangle\left\langle\tilde{\uparrow}\right| + \sigma^x \otimes \left|\tilde{\downarrow}\right\rangle\left\langle\tilde{\downarrow}\right|$$



$$\left|\tilde{in}\right\rangle = \frac{1}{\sqrt{2}}\left(\left|\tilde{\uparrow}\right\rangle + \left|\tilde{\downarrow}\right\rangle\right)$$

$$\mathcal{U}_4^\dagger\left|\tilde{in}\right\rangle = \frac{1}{\sqrt{2}}\left(\left|\tilde{\uparrow}\right\rangle + \sigma_4^x \otimes \left|\tilde{\downarrow}\right\rangle\right)$$

E. Zohar, A. Farace, B. Reznik, J. I. Cirac, Phys. Rev. Lett. 118 070501 (2017)

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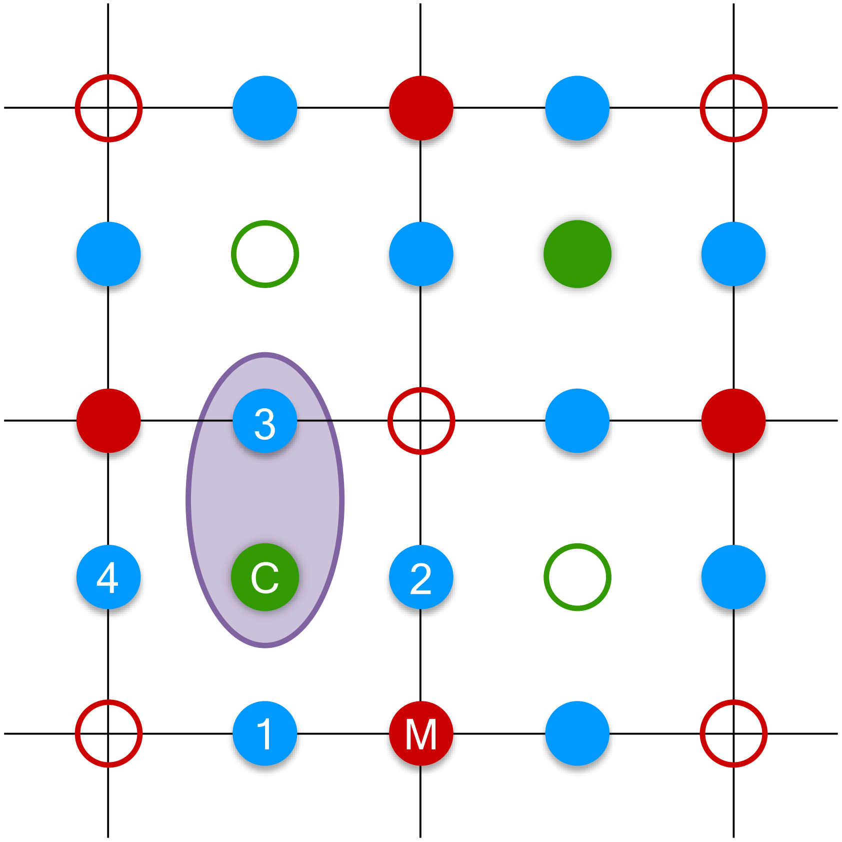

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E. Zohar, A. Farace, B. Reznik, J. I. Cirac, Phys. Rev. Lett. 118 070501 (2017)

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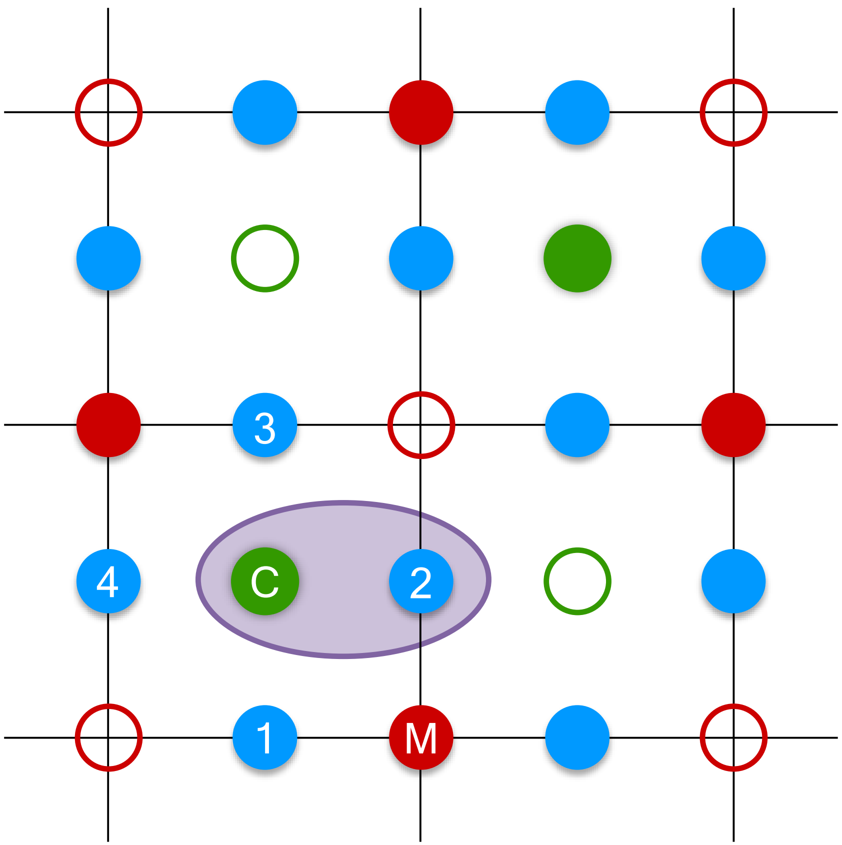

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$$\mathcal{U}_2\mathcal{U}_3^\dagger\mathcal{U}_4^\dagger|\tilde{in}\rangle = \frac{1}{\sqrt{2}}\left(|\tilde{\uparrow}\rangle + \sigma_2^x\sigma_3^x\sigma_4^x \otimes |\tilde{\downarrow}\rangle\right)$$

E. Zohar, A. Farace, B. Reznik, J. I. Cirac, Phys. Rev. Lett. 118 070501 (2017)

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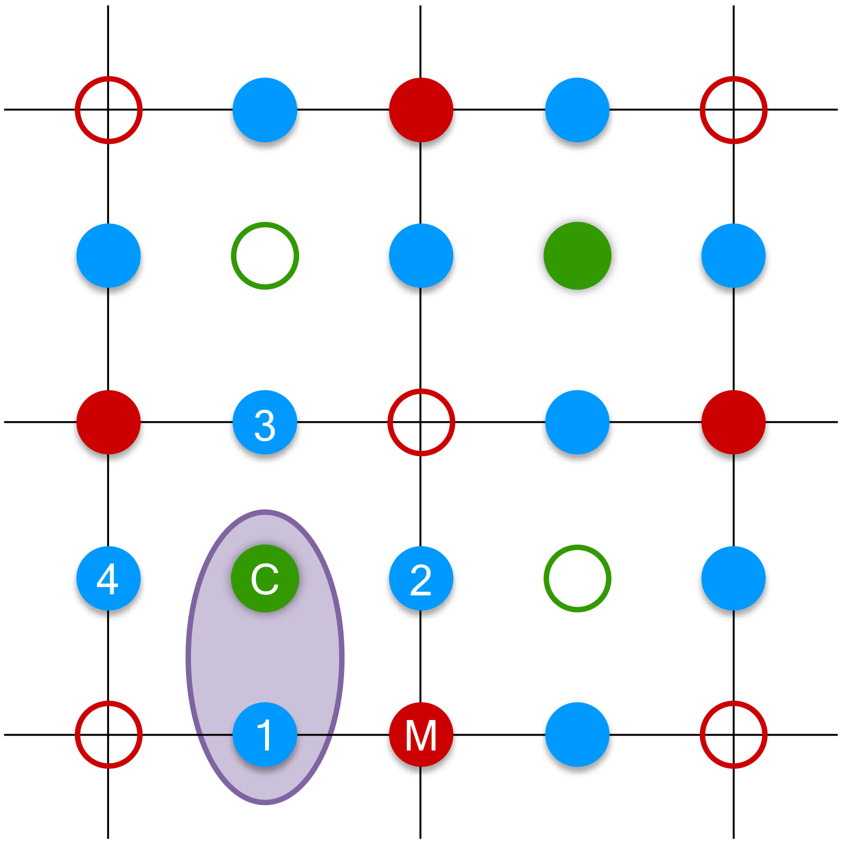

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$$\mathcal{U}_1\mathcal{U}_2\mathcal{U}_3^{\dagger}\mathcal{U}_4^{\dagger}|\tilde{in}\rangle = \frac{1}{\sqrt{2}}\left(|\tilde{\uparrow}\rangle + \sigma_1^x\sigma_2^x\sigma_3^x\sigma_4^x \otimes |\tilde{\downarrow}\rangle\right)$$

E. Zohar, A. Farace, B. Reznik, J. I. Cirac, Phys. Rev. Lett. 118 070501 (2017)

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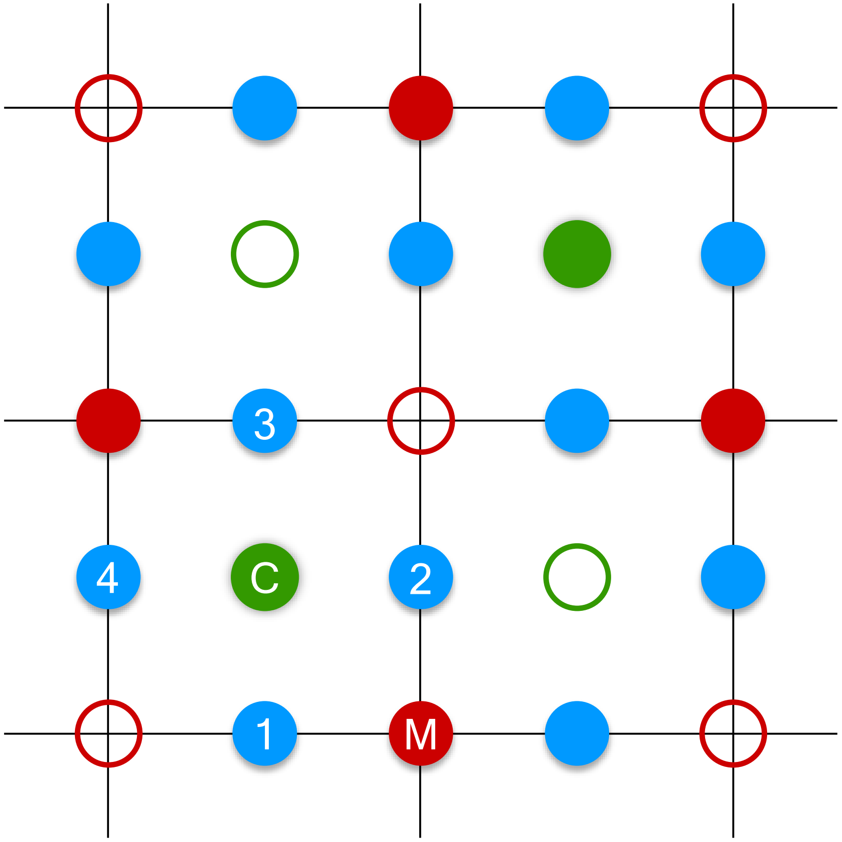

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$$\mathcal{U}_1 \mathcal{U}_2 \mathcal{U}_3^\dagger \mathcal{U}_4^\dagger |\widetilde{in}\rangle = \frac{1}{\sqrt{2}}\left(|\tilde{\uparrow}\rangle + \sigma_1^x \sigma_2^x \sigma_3^x \sigma_4^x \otimes |\tilde{\downarrow}\rangle\right)$$

$$S_\square = \frac{1}{\sqrt{2}}\left(|\tilde{\uparrow}\rangle + \sigma_\square^x \otimes |\tilde{\downarrow}\rangle\right)$$

E. Zohar, A. Farace, B. Reznik, J. I. Cirac, Phys. Rev. Lett. 118 070501 (2017)

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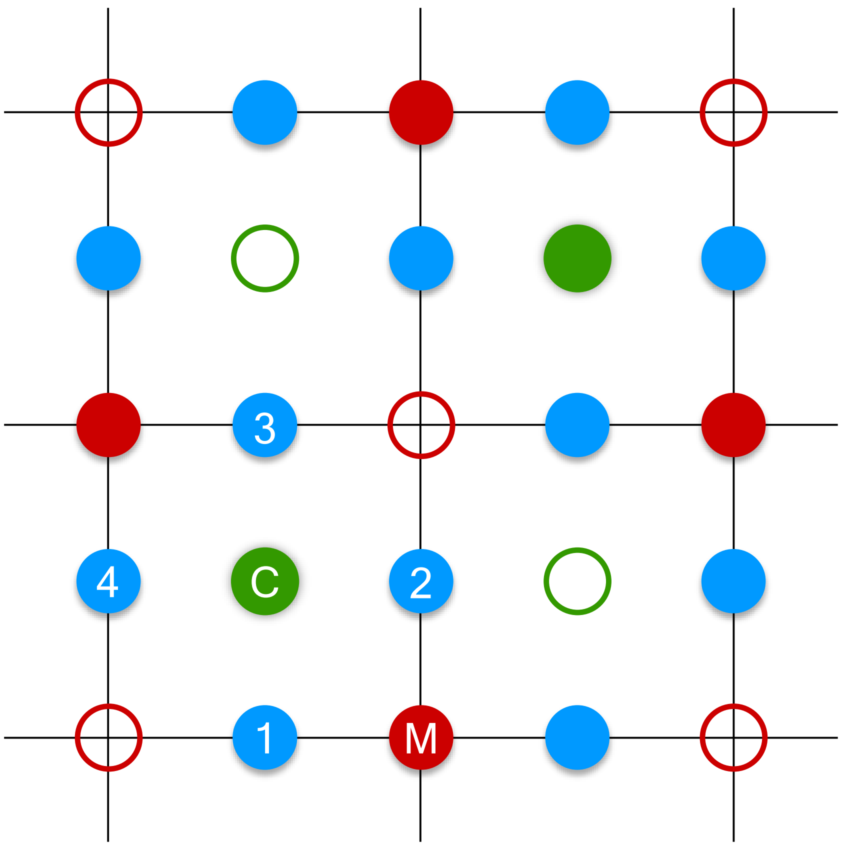


$$S_\square = \frac{1}{\sqrt{2}}\left(\left|\widetilde{\uparrow}\right\rangle + \sigma^x_\square \otimes \left|\widetilde{\downarrow}\right\rangle\right)$$

$$\widetilde{\sigma}^x S_\square = S_\square \sigma^x_\square$$

$$e^{-i\lambda\widetilde{\sigma}^x\tau} S_\square = S_\square e^{-i\lambda\sigma^x_\square\tau}$$

$$\mathcal{U}_4\mathcal{U}_3\mathcal{U}_2^\dagger\mathcal{U}_1^\dagger e^{-i\lambda\widetilde{\sigma}^x\tau}\mathcal{U}_1\mathcal{U}_2\mathcal{U}_3^\dagger\mathcal{U}_4^\dagger\left|\widetilde{in}\right\rangle = \left|\widetilde{in}\right\rangle e^{-i\lambda\sigma^x_\square\tau}$$

- A "Stator" (state-operator)

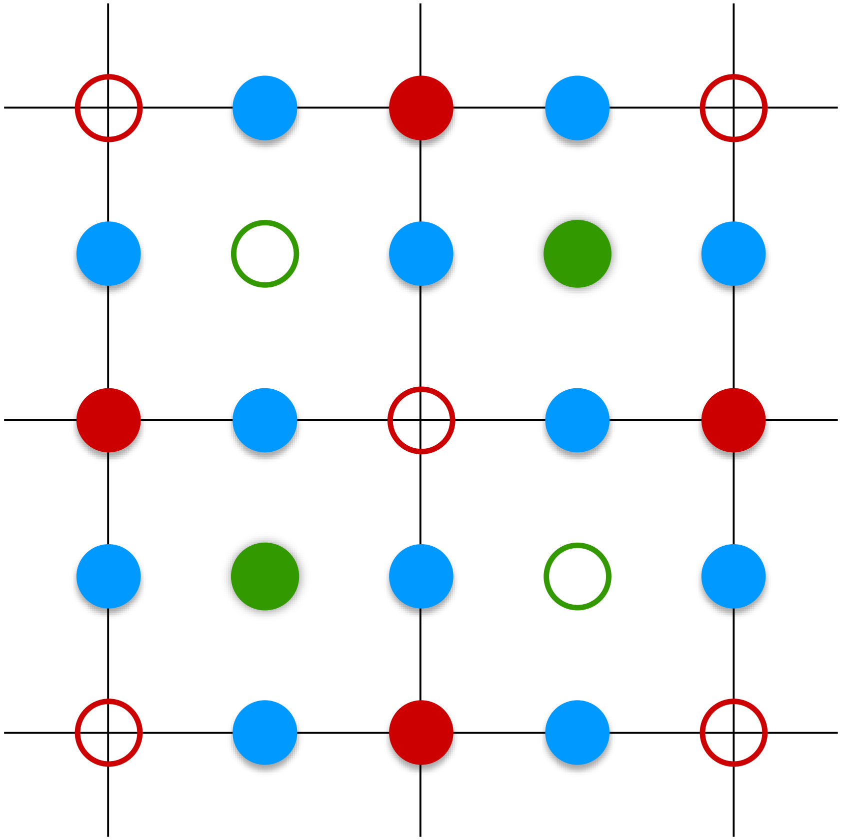
B. Reznik, Y. Aharonov, B. Groisman, Phys. Rev. A 6 032312 (2002)

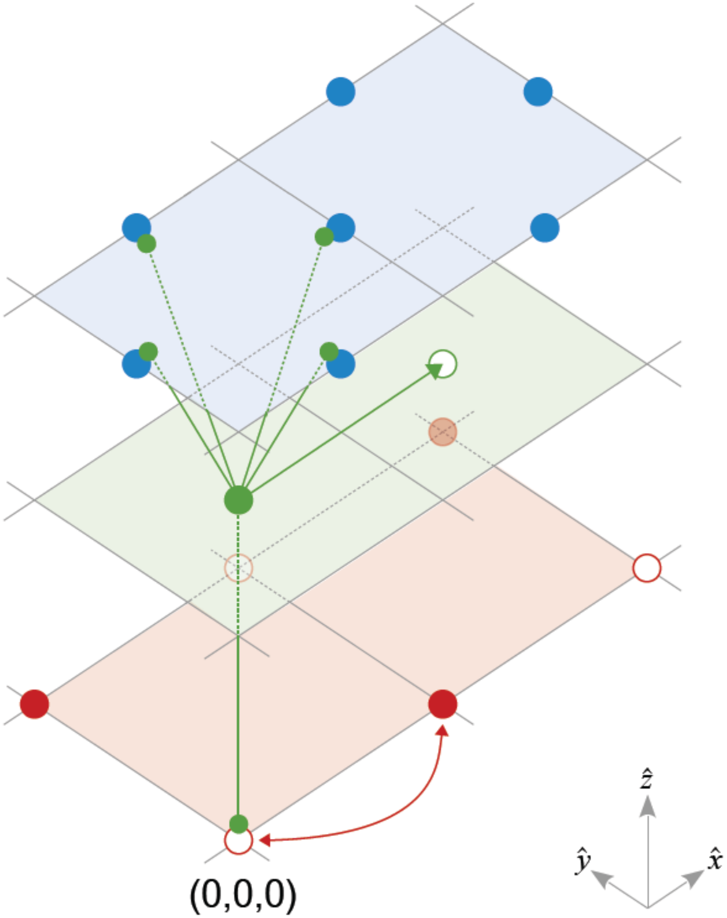
E. Zohar, J. Phys. A. 50 085301 (2017)

E. Zohar, A. Farace, B. Reznik, J. I. Cirac, Phys. Rev. Lett. 118 070501 (2017)

Realization

Three atomic layers:
The control atoms are movable

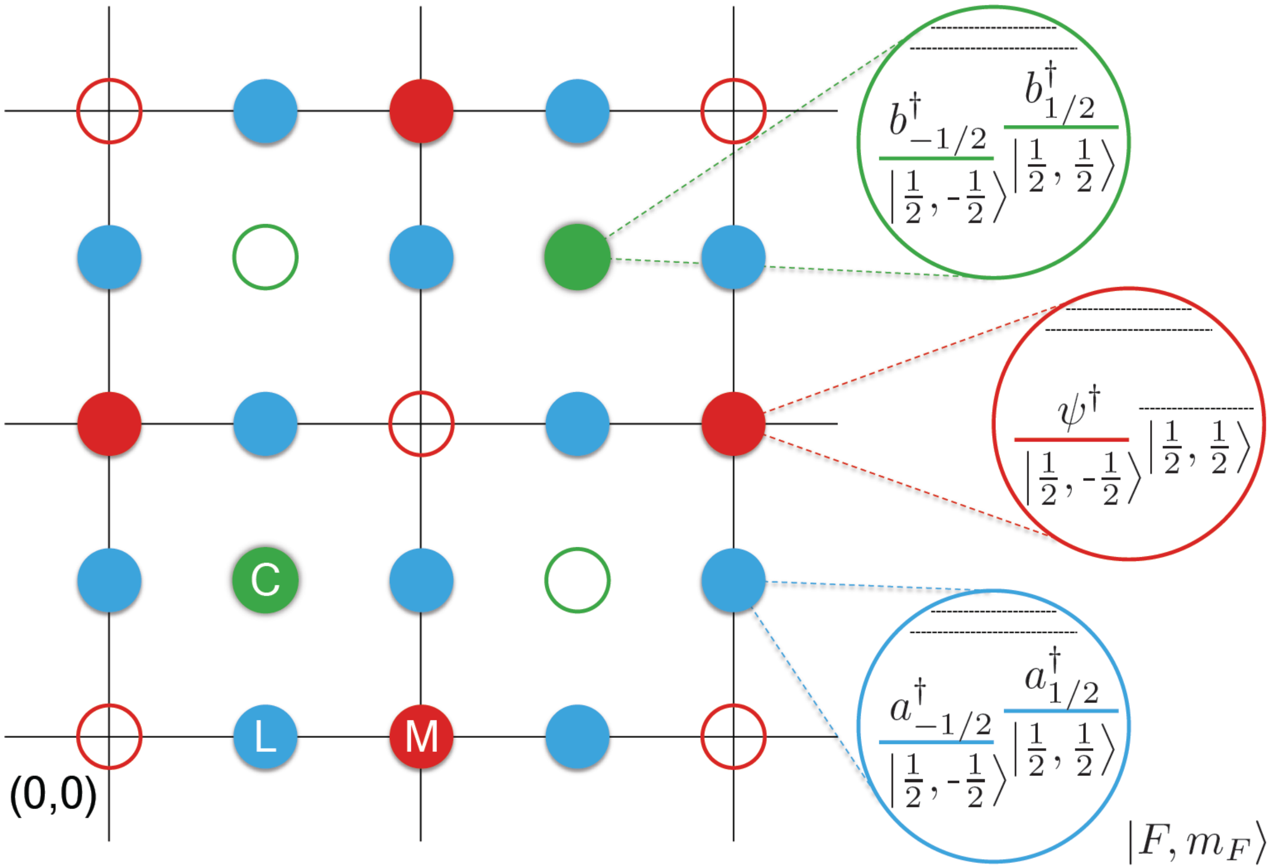




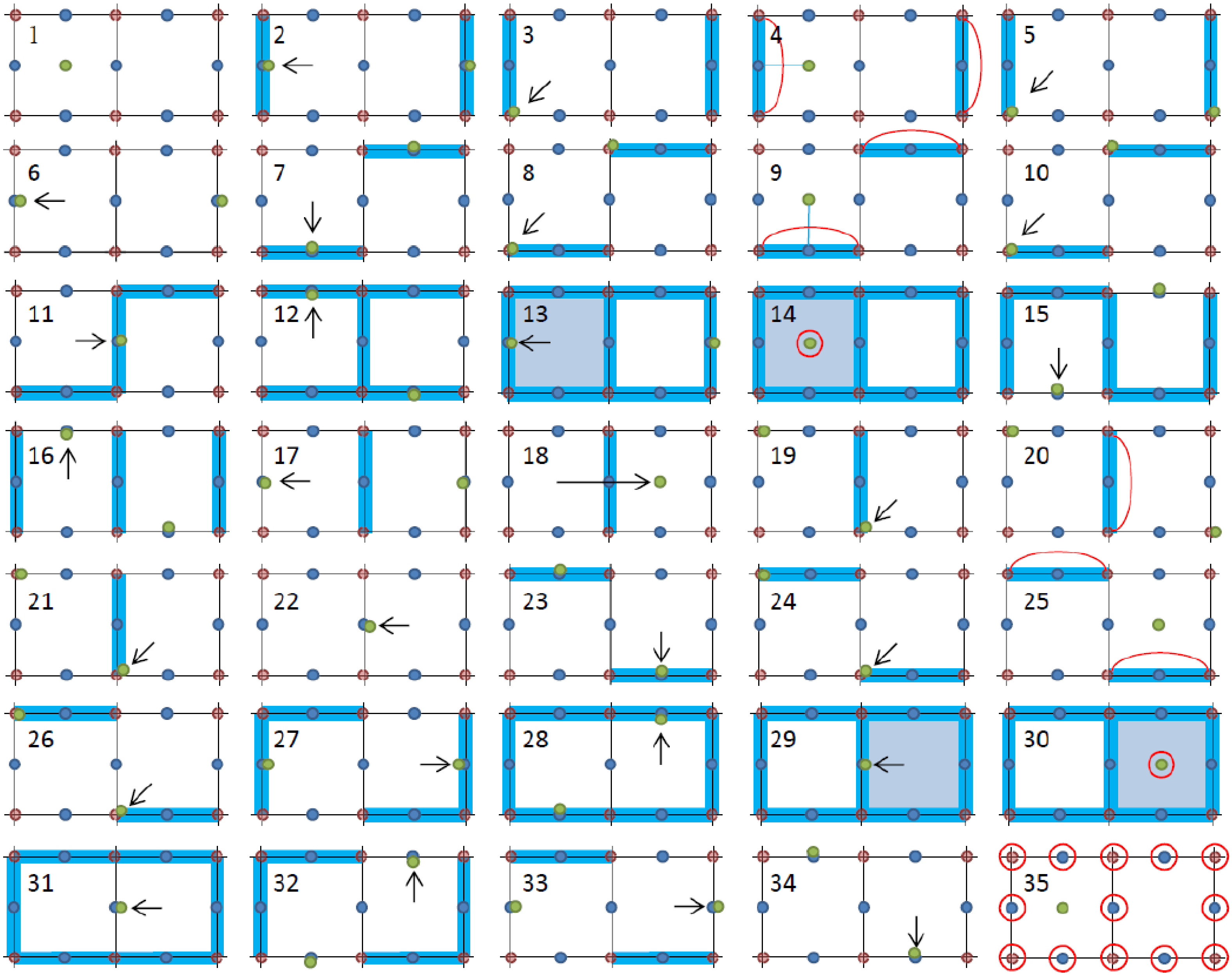
E. Zohar, A. Farace, B. Reznik, J. I. Cirac, Phys. Rev. Lett. 118 070501 (2017)

Realization

Use of hyperfine structure



E. Zohar, A. Farace, B. Reznik, J. I. Cirac, Phys. Rev. Lett. 118 070501 (2017)



E. Zohar, A. Farace, B. Reznik, J. I. Cirac, Phys. Rev. A. 95 023604 (2017)

Realization

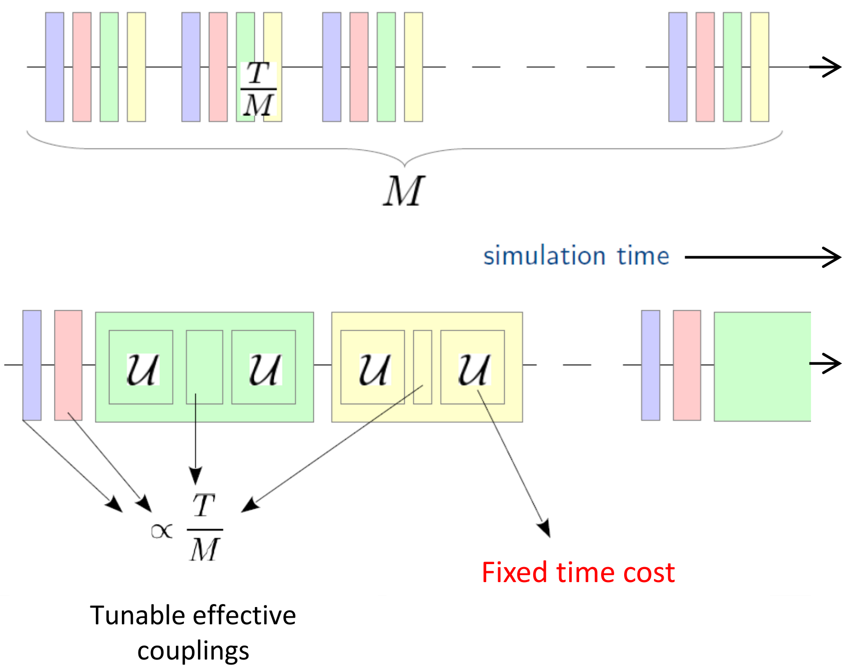
A bipartite single time step (two sublattices)

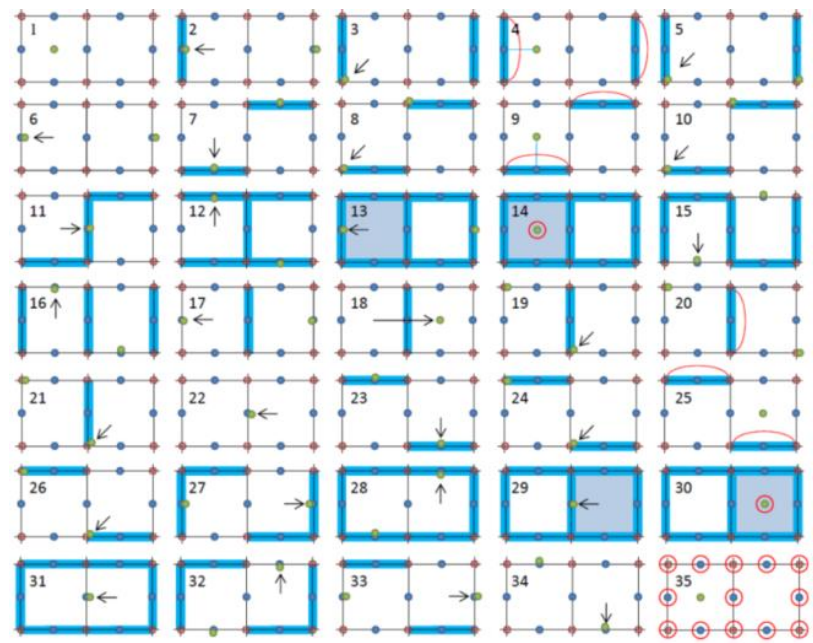
All plaquettes of a given parity are realized at once

Trotterized time evolution, of already gauge invariant pieces

(implementation errors can break the symmetry)

$$e^{-i\Sigma_j H_j T} = \lim_{M\to\infty}\left(\Pi_j e^{-iH_j \frac{T}{M}}\right)^M \qquad M \geqslant \frac{60L^3\lambda_{\max}^{3/2}T^{3/2}}{\sqrt{\epsilon}}$$





E. Zohar, A. Farace, B. Reznik, J. I. Cirac, Phys. Rev. Lett. 118 070501 (2017)

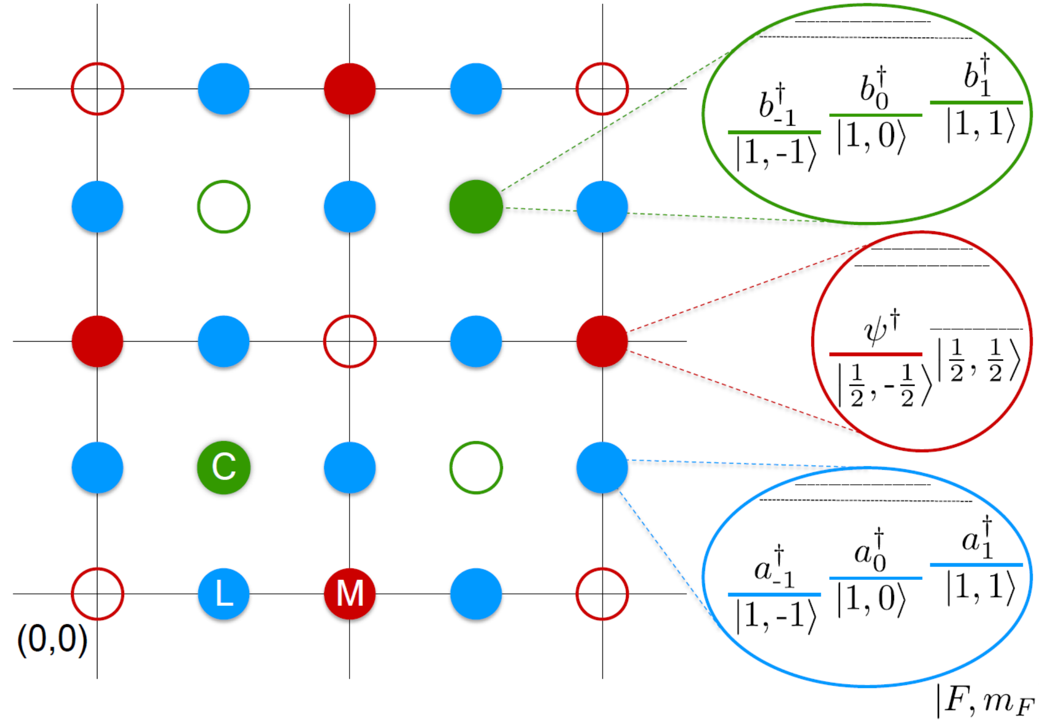
E. Zohar, A. Farace, B. Reznik, J. I. Cirac, Phys. Rev. A. 95 023604 (2017)

3D generalization - J. Bender, E. Zohar, A. Farace, J. I. Cirac, in preparation

First generalization: Z_3

- Larger Hilbert spaces, more complicated interactions

$$P^3 = Q^3 = 1,$$
$$P\,Q\,P^\dagger = e^{i(2\pi/3)}Q,$$
$$Q|m\rangle = |m+1\rangle \text{ (cyclically)},$$
$$P|m\rangle = e^{i(2\pi/3)m}|m\rangle.$$



$$|\widetilde{in}\rangle = \frac{1}{\sqrt{3}}\sum_{m=-1}^{1}|\widetilde{m}\rangle$$

$$\mathcal{U}_i = e^{i(3/2\pi)\ln Q_i \ln \widetilde{P}}$$

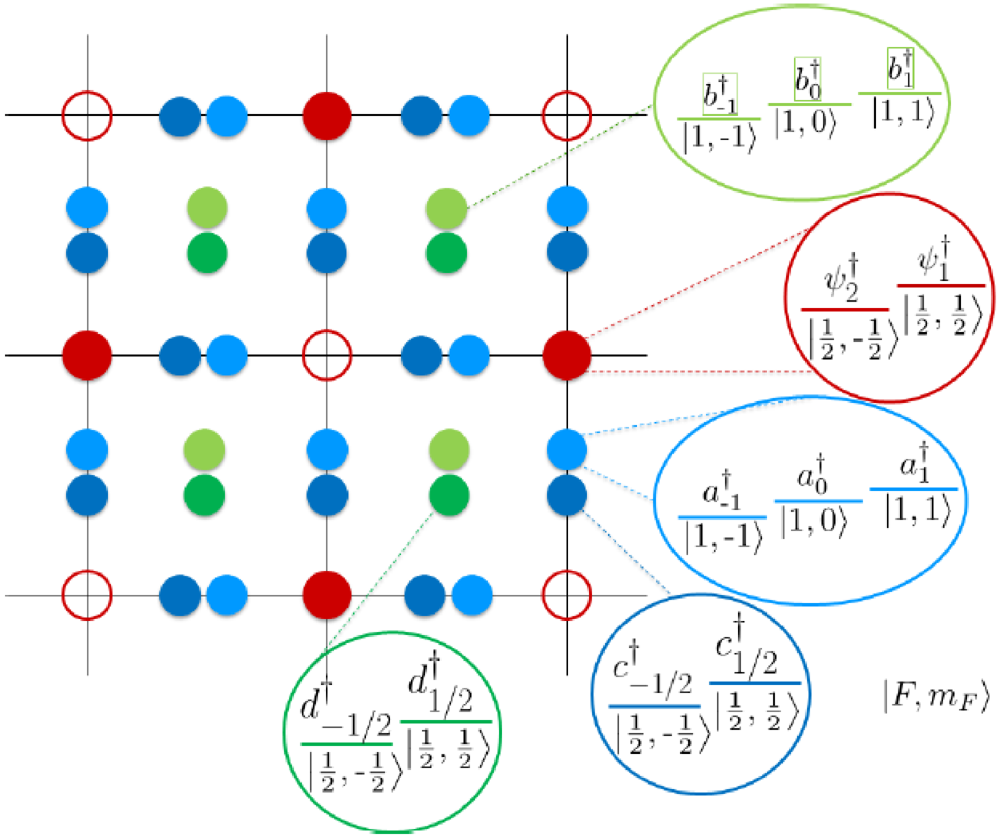
$$\mathcal{U}'_i(\mathbf{x}) = e^{i(3/2\pi)\ln P_i(\mathbf{x})\ln \widetilde{P}(\mathbf{x})}$$

$$S_{Q,i} \equiv \mathcal{U}_i|\widetilde{in}\rangle = \frac{1}{\sqrt{3}}\sum_{m=-1}^{1} Q_i^m \otimes |\widetilde{m}\rangle$$

$$F^z = -\frac{3i}{2\pi}\ln P \;\rightarrow\; \mathcal{U}' = e^{-i(2\pi/3)F_z\tilde{F}_z}$$

E. Zohar, A. Farace, B. Reznik, J. I. Cirac, Phys. Rev. A. 95 023604 (2017)

Second generalization: D_3



Dihedral group D_N:
$D_N = \{\theta^p r^m | p \in (0,1,2,..,N-1), m \in (0,1)\}$ with θ rotations around $\frac{2\pi}{N}$ and r reflection

D_3: symmetry group of the triangle, rotations around multiples of $\frac{2\pi}{3}$ and refelction $\rightarrow$ 6 elements

J. Bender, A. Farace, J. I. Cirac, E. Zohar, in preparation

Further generalization

Any gauge group

$$S = \int dg \left|g_A\right\rangle \left\langle g_A\right| \otimes \left|g_B\right\rangle$$

$$\left(U^j_{mn}\right)_B S = S \left(U^j_{mn}\right)_A$$

$$S_\Box = \mathcal{U}_\Box \left|\widetilde{in}\right\rangle \equiv \mathcal{U}_1 \mathcal{U}_2 \mathcal{U}_3^\dagger \mathcal{U}_4^\dagger \left|\widetilde{in}\right\rangle$$

$$\mathrm{Tr}\left(\widetilde{U^j} + \widetilde{U}^{j\dagger}\right) S_\Box = S_\Box \mathrm{Tr}\left(U_1^j U_2^j U_3^{j\dagger} U_4^{j\dagger} + H.c.\right)$$

Feasible for finite or truncated infinite groups

E. Zohar, J. Phys. A. 50 085301 (2017)
E. Zohar, A. Farace, B. Reznik, J. I. Cirac, Phys. Rev. A. 95 023604 (2017)

Summary

- **Lattice gauge theories** may be simulated by **ultracold atoms** in optical lattices. **Gauge invariance** may be obtained in several methods.

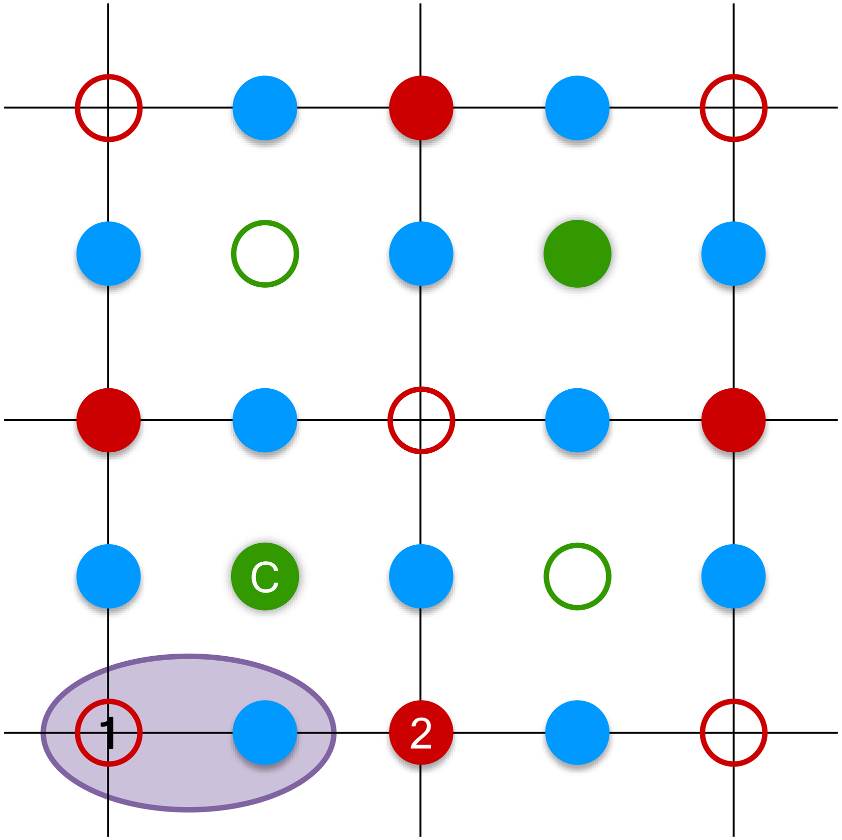
- Atomic interactions may be **mapped exactly to a gauge symmetry** in the ultracold limit, making the gauge invariance fundamental in some sense. This allows to realize a 1+1d q. sim. (work in progress in Heidelberg), and generalizations to more dimensions exits.

- Lattice gauge theories may be formulated in a **digital way**: two body interactions with **ancillary atoms may induce four body interactions**. This can be done with **ultracold atoms in a layered structure**; By doing that, plaquette interactions are **possibly stronger** than in the analog, perturbative approach.

Thank you

Link Interactions

$$\mathcal{U}_W^\dagger = e^{-\psi^\dagger \psi \log \sigma^x}$$



$$\mathcal{U}_W^\dagger$$

E. Zohar, A. Farace, B. Reznik, J. I. Cirac, Phys. Rev. Lett. 118 070501 (2017)

Link Interactions

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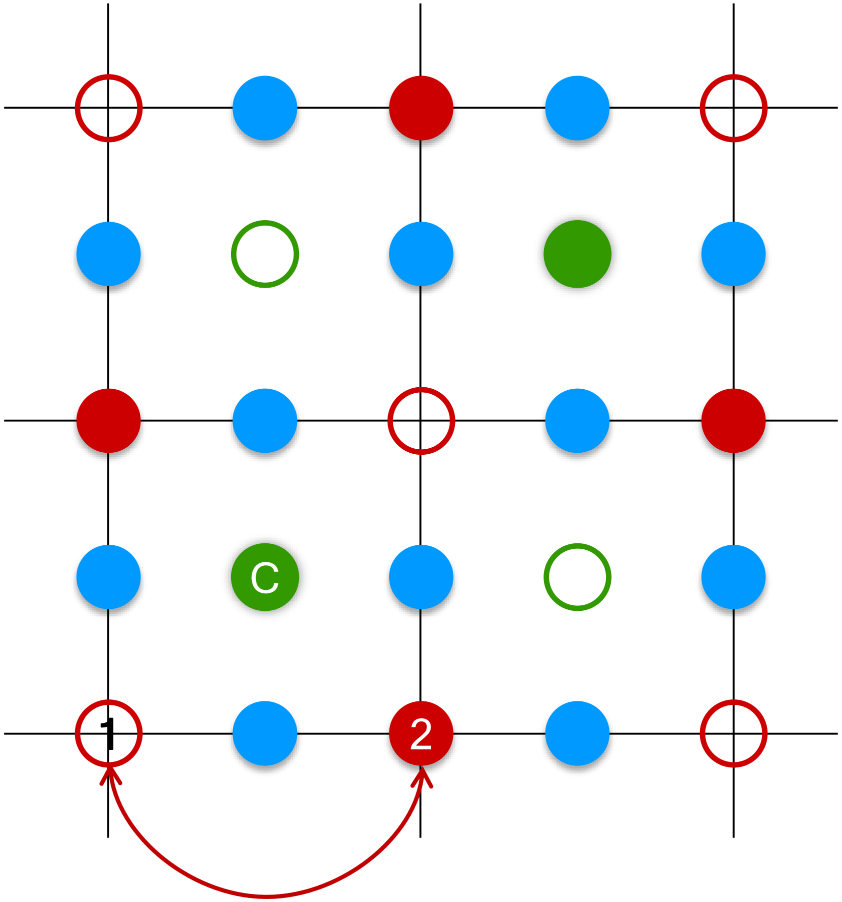

$$e^{-i\epsilon\left(\psi_1^\dagger \psi_2 + \psi_2^\dagger \psi_1\right)\tau} \mathcal{U}_W^\dagger$$

E. Zohar, A. Farace, B. Reznik, J. I. Cirac, Phys. Rev. Lett. 118 070501 (2017)

Link Interactions

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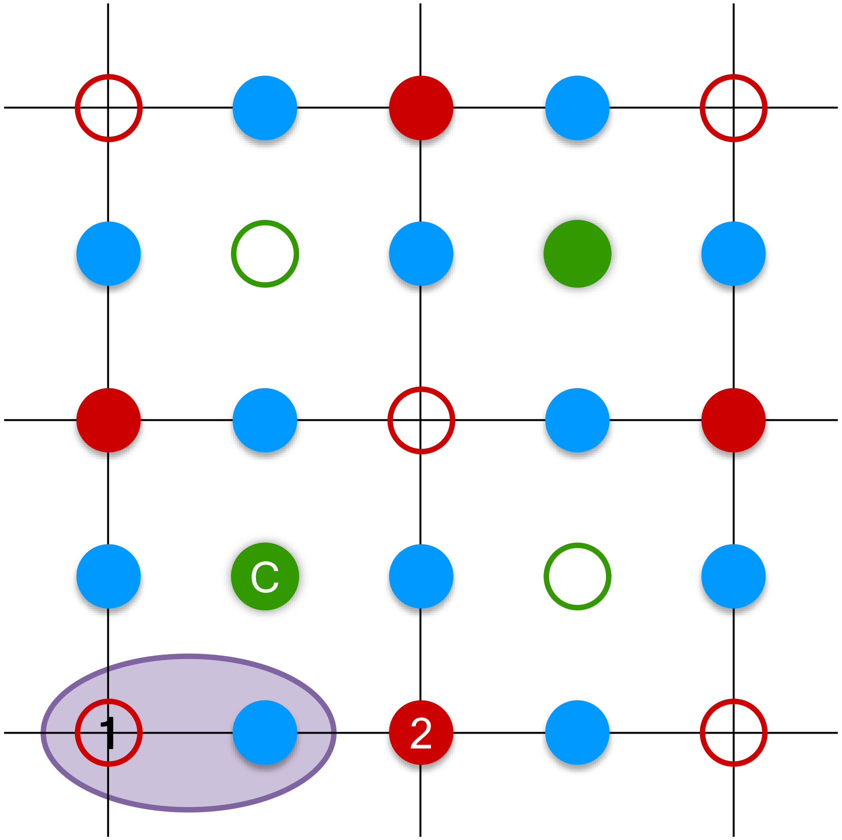

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E. Zohar, A. Farace, B. Reznik, J. I. Cirac, Phys. Rev. Lett. 118 070501 (2017)

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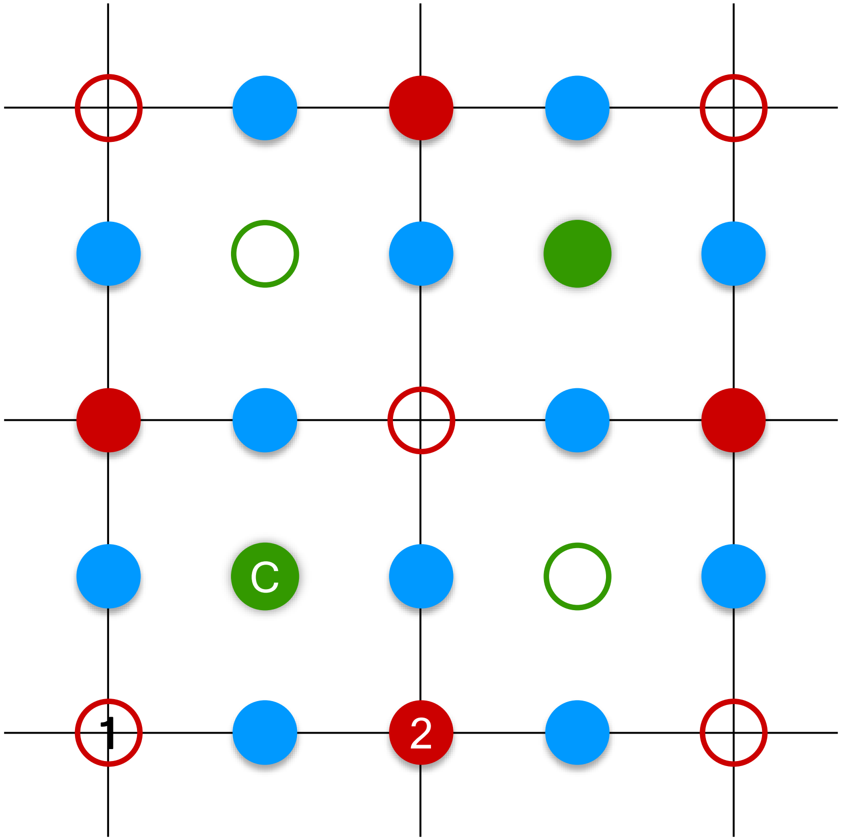
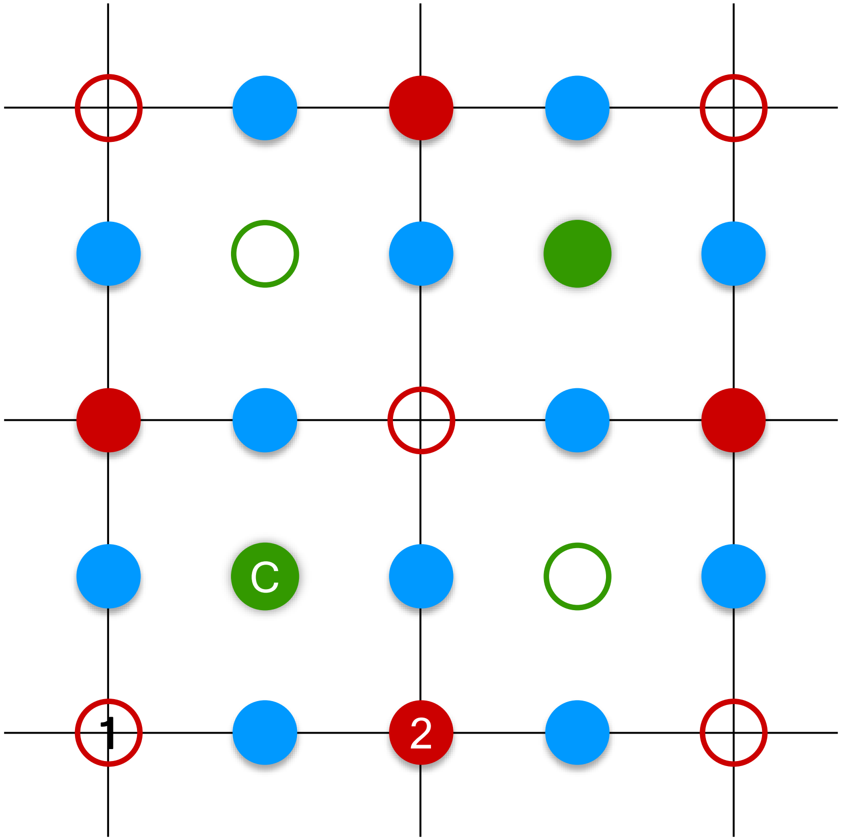


$$\mathcal{U}_W e^{-i\epsilon\left(\psi_1^\dagger \psi_2 + \psi_2^\dagger \psi_1\right)\tau} \mathcal{U}_W^\dagger$$

$$= e^{-i\epsilon\left(\psi_1^\dagger \sigma^x \psi_2 + \psi_2^\dagger \sigma^x \psi_1\right)\tau}$$

E. Zohar, A. Farace, B. Reznik, J. I. Cirac, Phys. Rev. Lett. 118 070501 (2017)

Link Interactions



$$\mathcal{U}_W e^{-i\epsilon\left(\psi_1^\dagger\psi_2+\psi_2^\dagger\psi_1\right)\tau}\mathcal{U}_W^\dagger$$

$$= e^{-i\epsilon\left(\psi_1^\dagger\sigma^x\psi_2+\psi_2^\dagger\sigma^x\psi_1\right)\tau}$$

Global tunneling becomes
Locally Gauge invariant interaction

E. Zohar, A. Farace, B. Reznik, J. I. Cirac, Phys. Rev. Lett. 118 070501 (2017)

Realization

- Local operations – Raman lasers

$$V_{\mathbf{n}}(\phi) = e^{-i\phi \sum_{\mathbf{x}} \mathbf{n}\cdot\sigma(\mathbf{x})}$$
$$\tilde{V}_{\mathbf{n}}(\phi) = e^{-i\phi \sum_{\mathbf{x}} \mathbf{n}\cdot\tilde{\sigma}(\mathbf{x})}$$

- Interactions – S-wave scattering, when the wavefunctions overlap

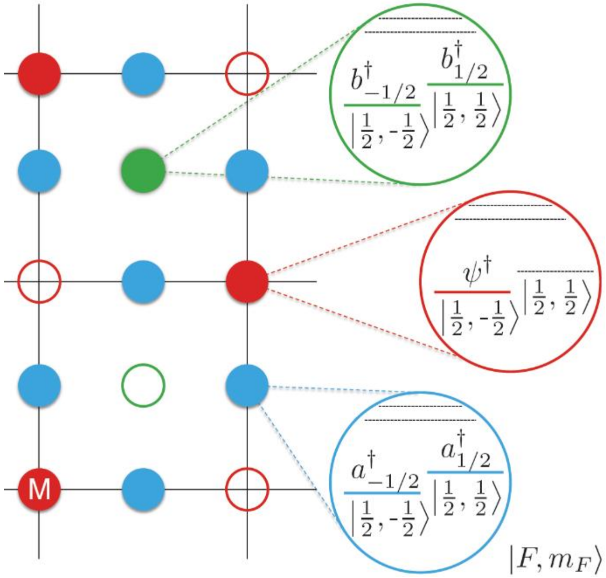
$$H_{ab} = f_0(t)\left(g_0 \sum_{m,n} a_m^\dagger a_m b_n^\dagger b_n + g_1 \mathbf{F}\cdot\tilde{\mathbf{F}}\right)$$

$$H_{b\psi} = f_0'(t)\left(g_0' \psi^\dagger \psi \sum_m b_m^\dagger b_m + g_1' \psi^\dagger \psi \tilde{\sigma}_z\right)$$

In both cases, two channels: ½ x ½ = 0 + 1

$$g_0 = \pi(a_0 + 3a_1)/2\mu,\ g_1 = 2\pi(a_1 - a_0)/\mu$$

- Constraints:
  - Magnetic field in ***z*** direction + RWA
  - $\sum_m a_m^\dagger a_m = \sum_m b_m^\dagger b_m = 1$
- Careful design of the control movement (adiabaticity, overlap)



$$F^\alpha(\mathbf{x}, k) = \frac{1}{2}\sigma^\alpha(\mathbf{x}, k) = \frac{1}{2}a_m^\dagger(\mathbf{x}, k)\,\sigma_{mn}^\alpha a_n(\mathbf{x}, k)$$
$$\tilde{F}^\alpha(\mathbf{x}) = \frac{1}{2}\tilde{\sigma}^\alpha(\mathbf{x}) = \frac{1}{2}b_m^\dagger(\mathbf{x})\,\sigma_{mn}^\alpha b_n(\mathbf{x})$$

E. Zohar, A. Farace, B. Reznik, J. I. Cirac, Phys. Rev. Lett. 118 070501 (2017)

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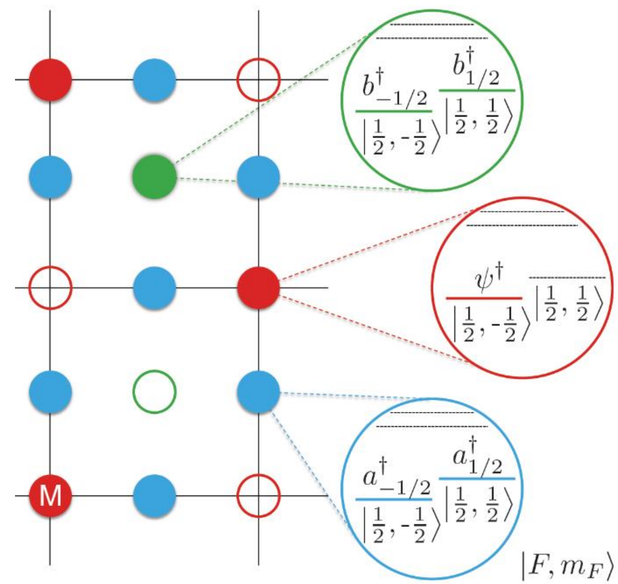
$$\tilde{V}_{\mathbf{n}}(\phi) = e^{-i\phi \sum_{\mathbf{x}} \mathbf{n}\cdot\tilde{\sigma}(\mathbf{x})}$$

  - Realize the local (non-interacting) terms of the Hamiltonian
  - Auxiliary operations (basis changes etc.)

- Interactions – S-wave scattering, when the wavefunctions overlap

$$\mathcal{U}_{\mathrm{ab}}(\phi) = e^{-4i\phi F_z \tilde{F}_z} = e^{-i\phi \sigma_z \tilde{\sigma}_z}$$
$$\mathcal{U}_{b\psi}(\phi) = e^{-i\phi'(\phi)\psi^\dagger\psi} e^{(-\phi/\pi)\psi^\dagger\psi \log \tilde{\sigma}_z}$$



$$F^\alpha(\mathbf{x},k) = \tfrac{1}{2}\sigma^\alpha(\mathbf{x},k) = \tfrac{1}{2}a^\dagger_m(\mathbf{x},k)\,\sigma^\alpha_{mn}a_n(\mathbf{x},k)$$
$$\tilde{F}^\alpha(\mathbf{x}) = \tfrac{1}{2}\tilde{\sigma}^\alpha(\mathbf{x}) = \tfrac{1}{2}b^\dagger_m(\mathbf{x})\,\sigma^\alpha_{mn}b_n(\mathbf{x})$$

E. Zohar, A. Farace, B. Reznik, J. I. Cirac, Phys. Rev. Lett. 118 070501 (2017)

Realization – Plaquettes

Atomic collisions → Interactions

1. Move **all controls** to **link 4**
2. Move **all controls** to **link 3**
3. Move **all controls** to **link 2**
4. Move **all controls** to **link 1**

$$\left.\begin{aligned}\mathcal{U}_{\mathrm{ab}}(\phi) &= e^{-4i\phi F_z \tilde{F}_z} = e^{-i\phi\sigma_z\tilde{\sigma}_z}\\ V_{\mathbf{n}}(\phi) &= e^{-i\phi\sum_{\mathbf{x}}\mathbf{n}\cdot\sigma(\mathbf{x})}\end{aligned}\right] \quad \begin{aligned}\mathcal{U} = \mathcal{U}^\dagger = \\ \left|\tilde{\uparrow}\right\rangle\left\langle\tilde{\uparrow}\right| + \sigma^x \otimes \left|\tilde{\downarrow}\right\rangle\left\langle\tilde{\downarrow}\right|\end{aligned}$$

$$V_y^\dagger\left(\frac{\pi}{4}\right)\mathcal{U}_{\mathrm{a_1b}}\left(\frac{\pi}{4}\right)\mathcal{U}_{\mathrm{a_2b}}\left(\frac{\pi}{4}\right)\mathcal{U}_{\mathrm{a_3b}}\left(\frac{\pi}{4}\right)\mathcal{U}_{\mathrm{a_4b}}\left(\frac{\pi}{4}\right)V_y\left(\frac{\pi}{4}\right)V_x\left(\frac{\pi}{4}\right)\tilde{V}_z\left(\frac{\pi}{4}\right)$$

5. Act locally on **all controls**

$$\tilde{V}_B = \tilde{V}_x\left(2\lambda_B\tau\right)$$

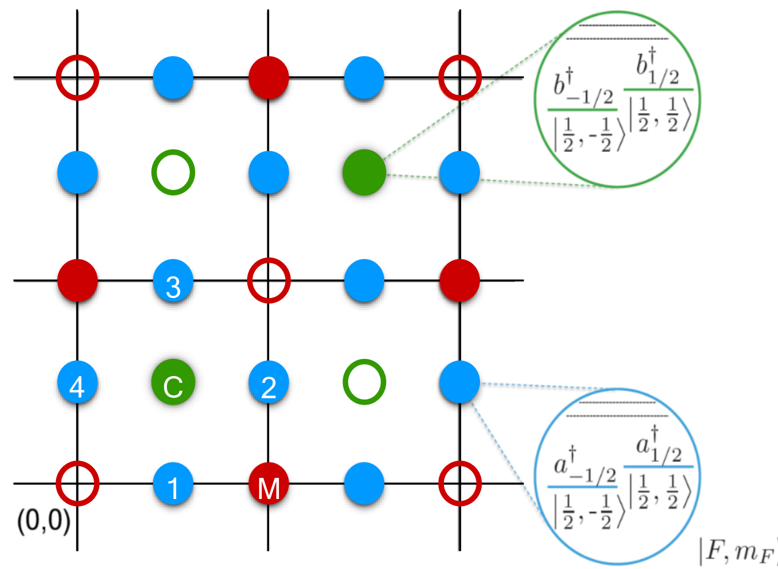
6. Undo steps 1-4, go to the other sublattice

$$S_\square = \frac{1}{\sqrt{2}}\left(\left|\tilde{\uparrow}\right\rangle + \sigma^x_\square \otimes \left|\tilde{\downarrow}\right\rangle\right)$$

$$\tilde{\sigma}^x S_\square = S_\square \sigma^x_\square$$

$$e^{-i\lambda\tilde{\sigma}^x\tau} S_\square = S_\square e^{-i\lambda\sigma^x_\square\tau}$$

$$\mathcal{U}_4\mathcal{U}_3\mathcal{U}_2^\dagger\mathcal{U}_1^\dagger e^{-i\lambda\tilde{\sigma}^x\tau}\mathcal{U}_1\mathcal{U}_2\mathcal{U}_3^\dagger\mathcal{U}_4^\dagger\left|\widetilde{in}\right\rangle = \left|\widetilde{in}\right\rangle e^{-i\lambda\sigma^x_\square\tau}$$



E. Zohar, A. Farace, B. Reznik, J. I. Cirac, Phys. Rev. Lett. 118 070501 (2017)

Realization – Links

Atomic collisions → Interactions

1. Move the **control** to the **link**
2. Move the **control** to the **left fermion**
3. Allow **fermions** to tunnel (reducing the potential barrier along the link)
4. Undo step 2
5. Undo step 1

$$\mathcal{U}_{b\psi}(\phi) = e^{-i\phi'(\phi)\psi^\dagger\psi} e^{(-\phi/\pi)\psi^\dagger\psi \log \tilde{\sigma}_z}$$

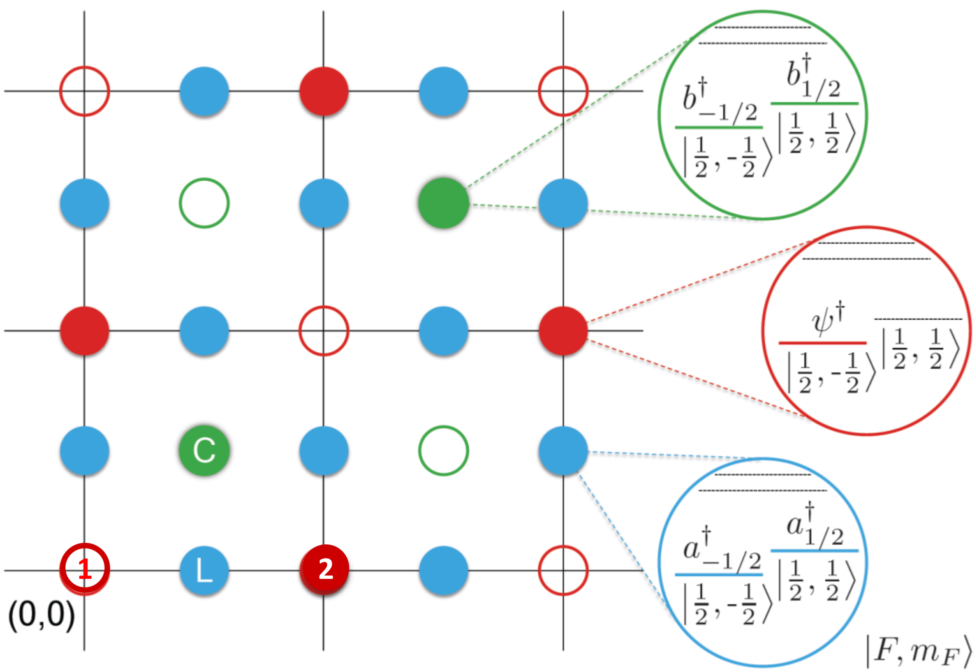
$$\tilde{V}_{\mathbf{n}}(\phi) = e^{-i\phi \sum_{\mathbf{x}} \mathbf{n}\cdot\tilde{\sigma}(\mathbf{x})}$$

$$\tilde{V}_y\left(\frac{\pi}{4}\right)\mathcal{U}_{b\psi}(\pi)\,\tilde{V}_y^\dagger\left(\frac{\pi}{4}\right) = e^{-i\phi'\psi^\dagger\psi}\mathcal{U}_W^\dagger = \mathcal{U}_W^\dagger e^{-i\phi'\psi^\dagger\psi}$$

$$\mathcal{U}_W^\dagger = e^{-\psi^\dagger\psi \log \sigma^x}$$

$$\mathcal{U}_W e^{-i\epsilon\left(\psi_1^\dagger\psi_2 + \psi_2^\dagger\psi_1\right)\tau}\mathcal{U}_W^\dagger$$

$$= e^{-i\epsilon\left(\psi_1^\dagger\sigma^x\psi_2 + \psi_2^\dagger\sigma^x\psi_1\right)\tau}$$



"Rotate" **fermions** with respect to **gauge field** (The rotation parameter is an operator)

Method we apply for tensor constructions as well.

E. Zohar, A. Farace, B. Reznik, J. I. Cirac, Phys. Rev. Lett. 118 070501 (2017)

First generalization: Z_3

- More scattering channels for the a-b interaction

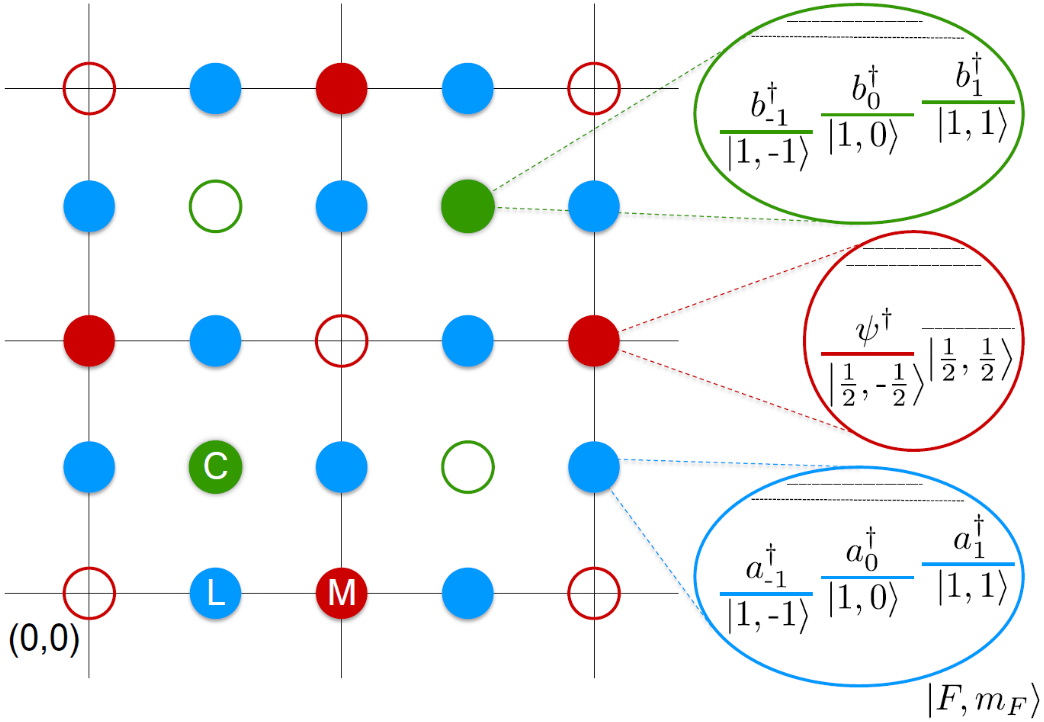
$$V_{\text{scat}}(\mathbf{x}) = \frac{2\pi}{\mu}\delta(\mathbf{x})\sum_{j=0}^{2} g_j(\vec{F}\cdot\vec{\tilde{F}})^j$$

$$g_0 = \tfrac{1}{3}(a_2 + 3a_1 - a_0)$$

$$g_1 = \tfrac{1}{2}(a_2 - a_1)$$

$$g_2 = \tfrac{1}{6}(a_2 - 3a_1 + 2a_0)$$

$$\mathcal{U}_{\text{scat}} = e^{-i\alpha\sum_{j=0}^{2} g_j(\vec{F}\cdot\vec{\tilde{F}})^j}$$



giving rise to undesired interactions – eliminated by using a magnetic field gradient which allows spatial separation of different levels.

- Interaction with the matter fermions – similar.

E. Zohar, A. Farace, B. Reznik, J. I. Cirac, Phys. Rev. A. 95 023604 (2017)