Breaking the exponential many body wall using quantum information

Frank Verstraete
Ghent University, University of Vienna

Overview

- Quantum many body problem:
  - Exponential wall
  - Breaking the wall
    - Quantum computing
    - Quantum tensor networks

- Quantum computing:
  - Quantum simulation: dynamics
  - Quantum simulation: statics

- Tensor networks:
  - The entanglement structure of quantum many body systems: area laws
  - Variational quantum tensor networks
    - MPS / PEPS / MERA

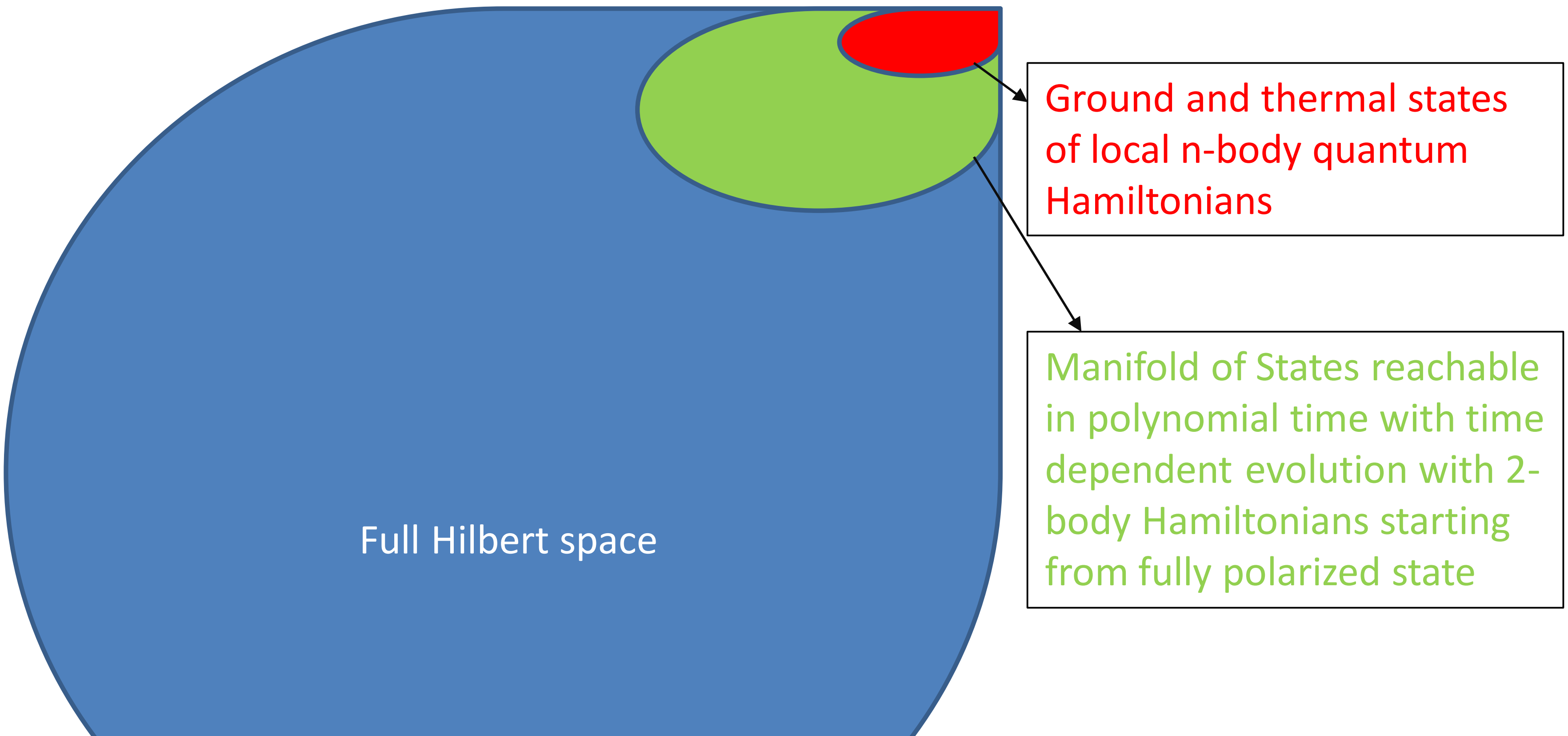
Quantum many body problem

- Central premise: strongly interacting quantum many body systems cannot be described using mean field theory such as Hartree Fock and variants:
  - QFT in the strong coupling regime
  - Quantum chemistry
  - Condensed matter theory: quantum Hall, Hubbard model, …
  - Nuclear physics: effective field theory and/or many body problem
- Exponential wall:
  - Tensor product structure leads to exponential scaling of Hilbert space:

$$|\psi\rangle = \sum_{ijk\ldots} X_{ijk\ldots} |i\rangle |j\rangle \ldots$$

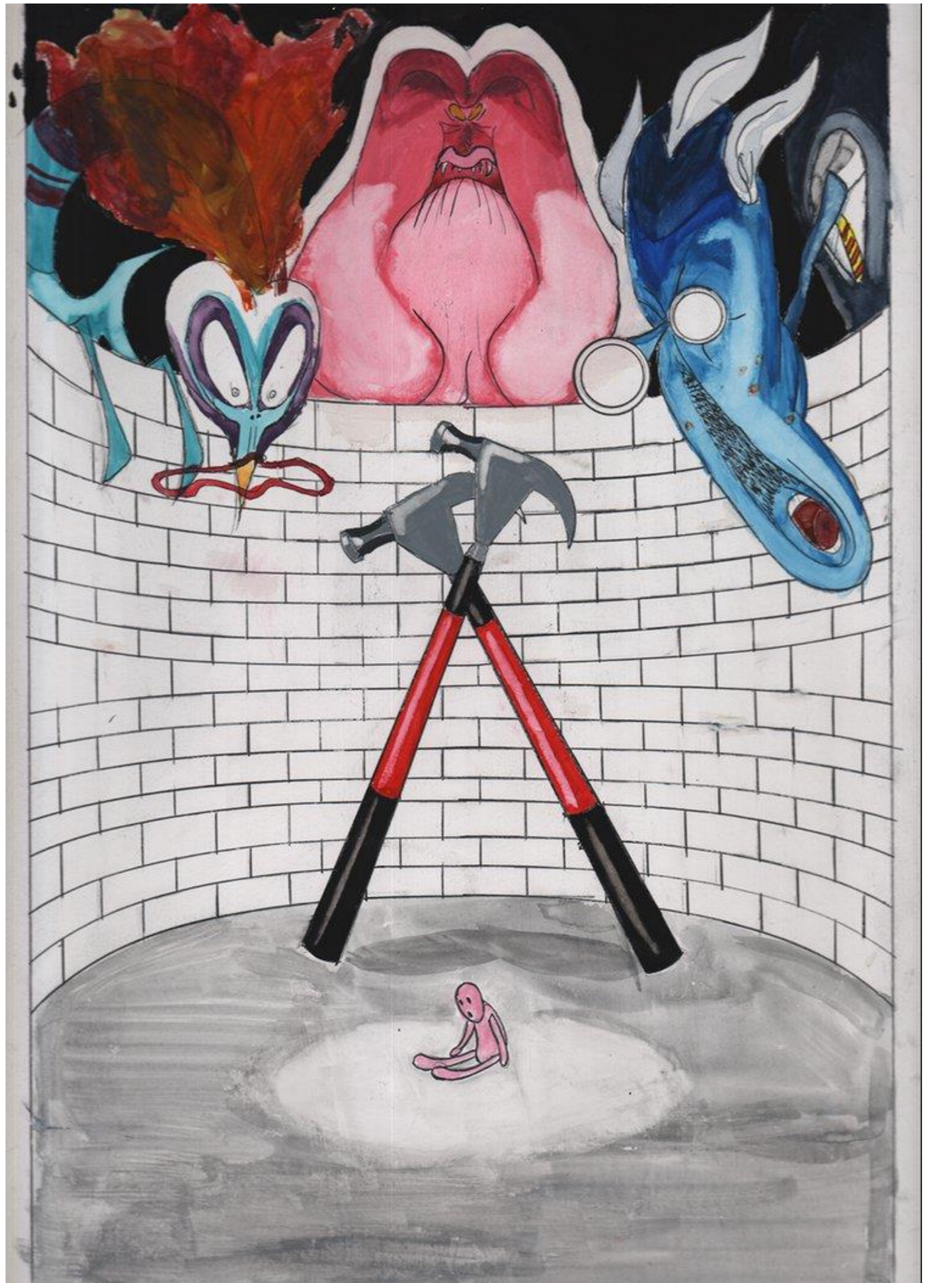
Quantum Simulation of Time-Dependent Hamiltonians and the Convenient Illusion of Hilbert Space

David Poulin,[1] Angie Qarry,[2,3] Rolando Somma,[4] and Frank Verstraete[2]



$$\left(\frac{1}{\epsilon}\right)^{2^N} \gg N^{N^\alpha} \gg N^2$$

Breaking the exponential wall



Breaking the exponential wall: 2 ideas from QIT

J. Preskill, '99

- Use a quantum computer to simulate the low dimensional manifolds: digital quantum simulation
- Use entanglement structure to construct novel classes of variational wavefunctions for ground and thermal states

Digital quantum simulation

- Use a quantum computer to simulate the low dimensional manifolds
  - Central point: work with wavefunctions (Schrodinger picture) as opposed to path integrals: quantum computer can only do unitary time evolution
    - Fock space is encoded using qubits:

$$|\psi\rangle = \sum_{i_1 i_2 i_3 \cdots} \psi_{i_1 i_2 i_3 \ldots} a^\dagger_{i_1} a^\dagger_{i_2} a^\dagger_{i_3} \cdots |\Omega\rangle$$

  - Real time evolution of many body Hamiltonian: digitize / trotterize Hamiltonian evolution (Lloyd '96)

    - Start with a fiducial state, and then evolve quantum state

$$|\psi_t\rangle = e^{-it \sum_{<\alpha,\beta>} \hat{H}_{\alpha\beta}} |\psi_0\rangle \simeq \lim_{N\to\infty} \left(\prod_{<\alpha,\beta>} e^{-i\frac{t}{N}\hat{H}_{\alpha\beta}} \right)^N |\psi_0\rangle$$

Digital quantum simulation

- Ground state physics can now be simulated by adiabatic time evolution (Farhi et al.):

$$\hat{H}(t) = (1-t)\hat{H}_0 + t\hat{H}_1$$

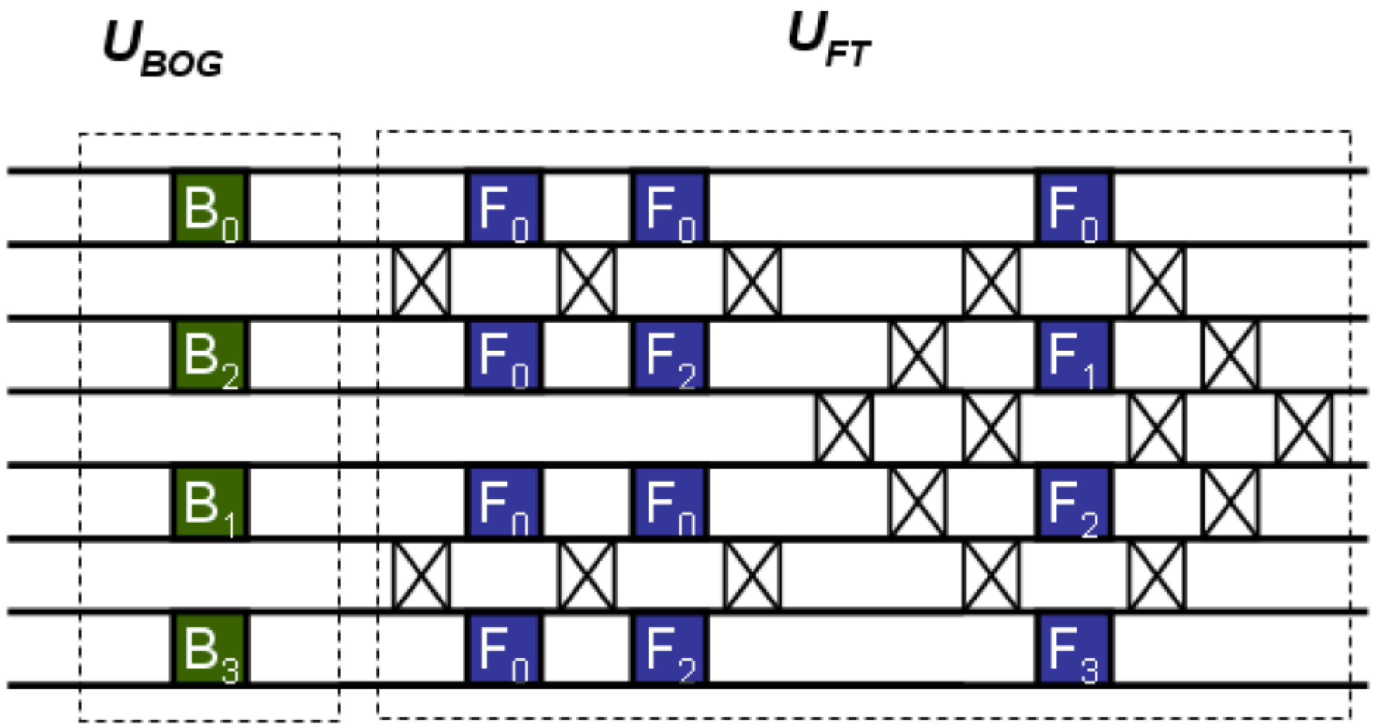
  - Speed at which Hamiltonian can be evolved is inversely proportional to the gap
    - Problem in the continuum limit: the closer to continuum limit (gapless), the slower we have to evolve
    - Problem at quantum phase transition: there cannot be phase transitions along the path!

  - How to prepare initial ground state of H_0?
    - Efficient quantum circuits for Slater determinants

Efficient quantum circuits for Slater determinants

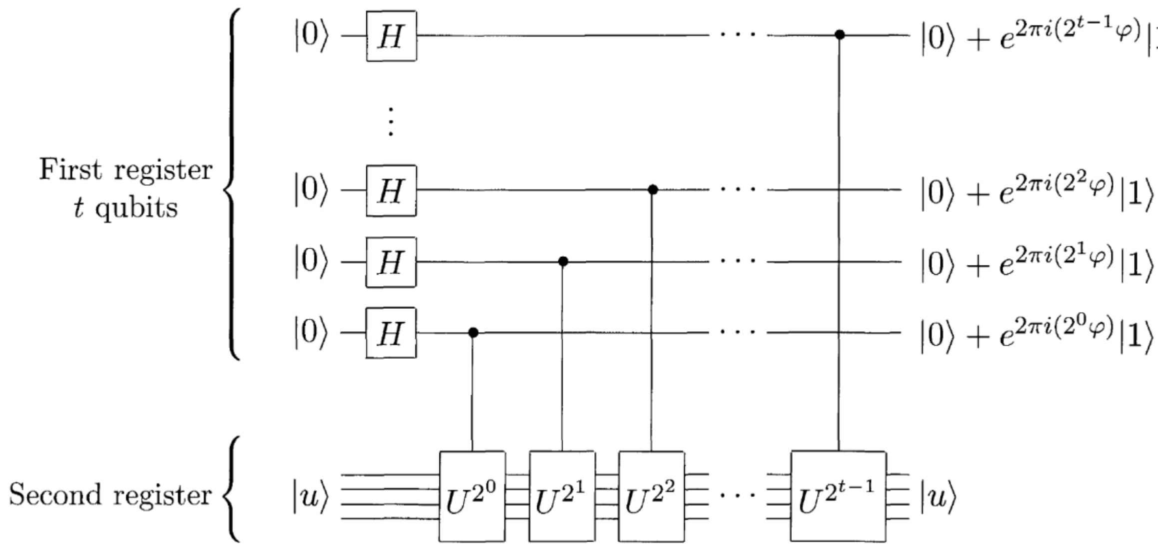
FV, Cirac, Latorre '09

- Any canonical and in particular Bogoliubov and (quantum fast) Fourier transform can be performed on a quantum computer with cost N.logN
  - Note: no need for Jordan Wigner transform; JW just encodes the mapping $$|\psi\rangle = \sum_{i_1 i_2 i_3 \cdots} \psi_{i_1 i_2 i_3 \ldots} a^\dagger_{i_1} a^\dagger_{i_2} a^\dagger_{i_3} \cdots |\Omega\rangle$$



- Alternative (but essentially equivalent) to adiabatic time evolution:

  - Prepare an initial state |u> (e.g. Slater determinant) with a large (poly) overlap with the many body ground state, and then measure the energy with a quantum non-demolition measurement (quantum phase estimation)



  - This method is very useful when the Hartree Fock state has a sizeable overlap with the many body ground state
    - Quantum chemistry
    - Nuclear Schell model?

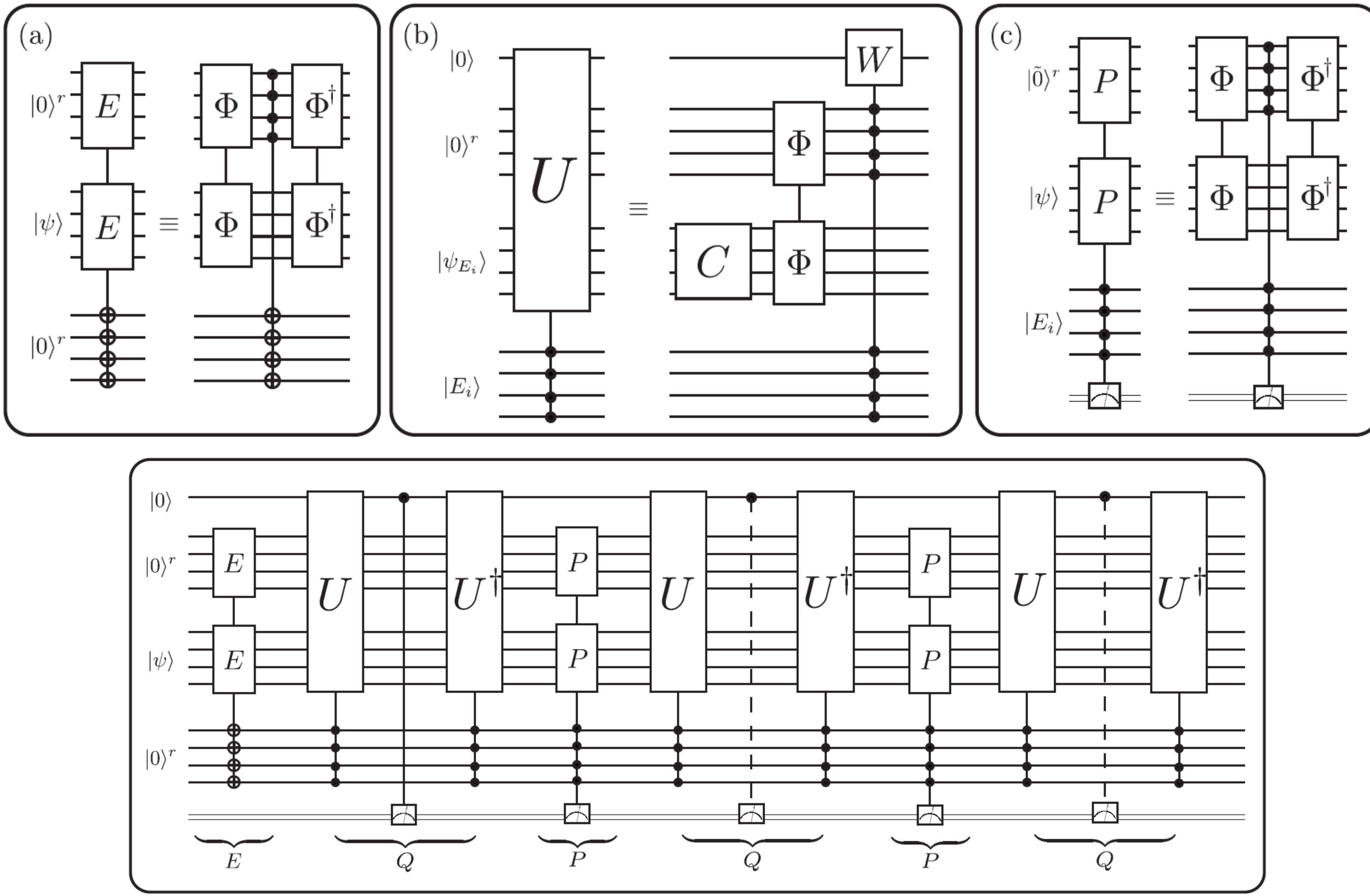
- What if there is a phase transition and/or we want to simulate quantum many body system at finite temperature?
  - Cfr. classical many body problem: has been completely monopolized by Monte Carlo sampling techniques ("We devised a general method to calculate the properties of any substance comprising individual molecules with classical statistics", Metropolis, Rosenbluth, Teller '52)
  - Quantum Metropolis Sampling (Temme, Vollbrecht, Osborne, Poulin, FV Nature '11): use a quantum computer to sample eigenstates of Hamiltonian
    - QC solves the sign problem!
    - Quantum detailed balance criterion ensures convergence to Gibbs state $e^{-\beta \hat{H}}$

Quantum Metropolis Sampling

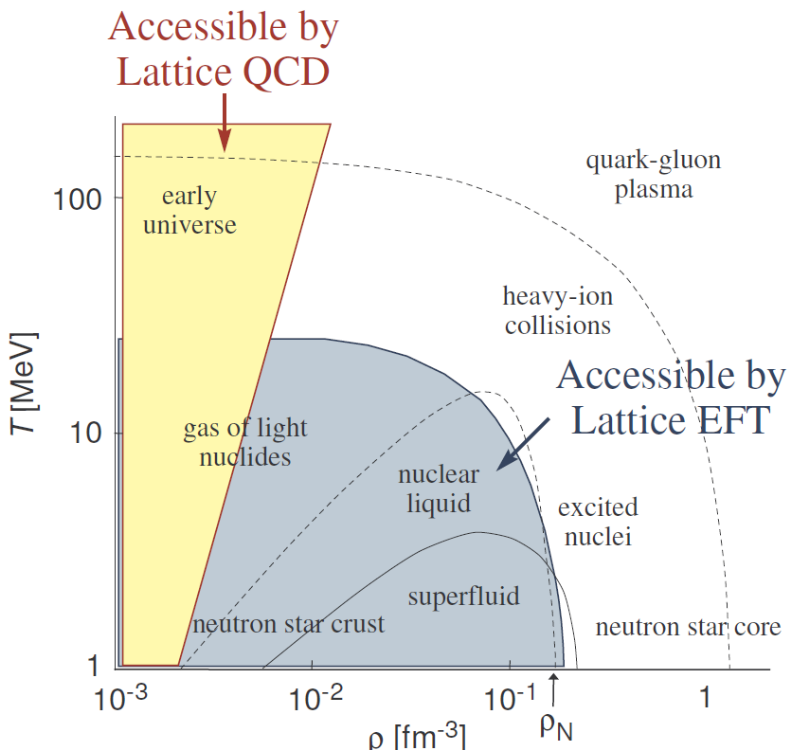
- Three obstacles to overcome:
  - How to prepare eigenstates of an interacting Hamiltonian?
    - Quantum phase estimation
  - How to do local moves?
    - Just apply random 2-body unitary transformations to the state such as to assure ergodicity
  - How to reject a move using Metropolis' criterion (no cloning theorem!)?
    - Most difficult step: we have to do a measurement which reveals whether we want to accept or reject the move, AND does not contain more information than that (otherwise state would be disturbed too much)
    - Can be done using Jordan's lemma and quantum unwiding trick of Marriott and Watrous

- Should work like a charm, plus no sign problem! Convergence rate will have to be determined empirically (just as in classical case)

Quantum Metropolis Sampling



- Quantum Metropolis should be able to sample in regimes where lattice QCD and Lattice EFT fail:

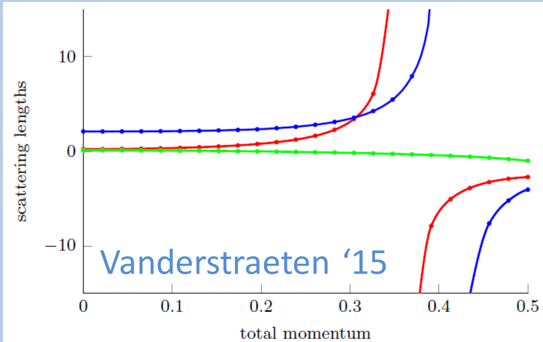


Dean Lee

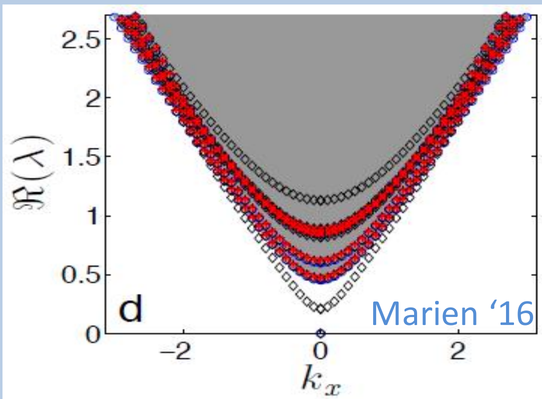
Breaking the Wall II: Tensor Networks

Computational aspects:

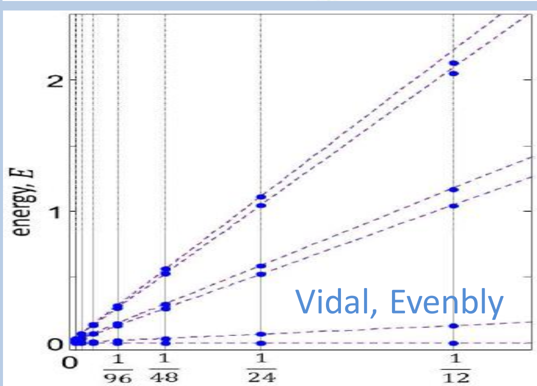
- MPS



- PEPS

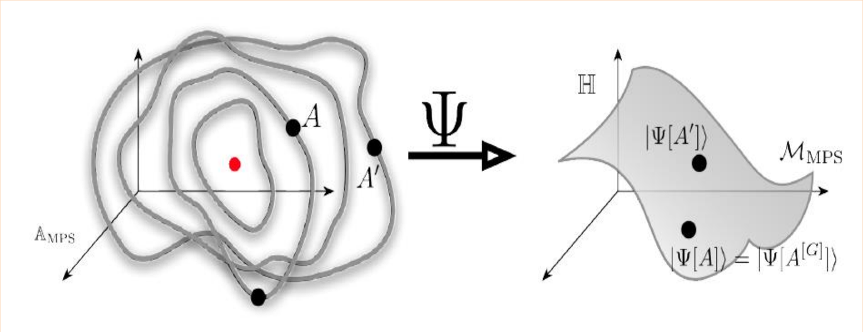


- MERA



Conceptual aspects: *τηε σηαδοω ωορλδ*

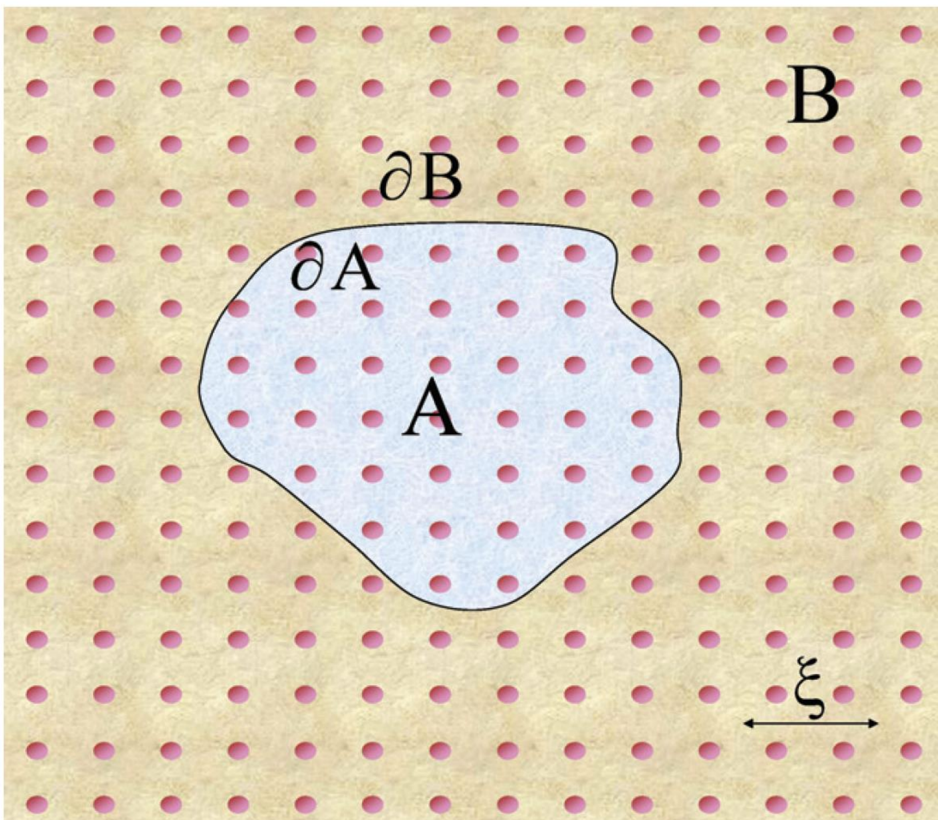
- Area laws and the corner of Hilbert space: manifold of ground states of local Hamiltonians, symplectic structures



- Modelling the entanglement degrees of freedom: symmetries
  - Symmetry fractionalization, classification of SPT phases
  - Gauge theories and topologically ordered matter: holographic Landau type order parameters

Quantum tensor networks

- Ground and Gibbs states of interacting quantum many body Hamiltonians have very peculiar properties
  – Area law for the entanglement entropy (ground states) or for mutual information (Gibbs states)



1. Ground states:

$$S(\rho_A) = c.\partial A$$

Srednicki '93; Hastings '07; …

$$S(\rho_A) = \frac{c}{6}.\log(A/\epsilon)$$

Holzhey, Larsen, Wilczek '94; …

2. Gibbs states:

$$\begin{aligned} I(A,B) &= S(\rho_A) + S(\rho_B) - S(\rho_{AB}) \\ &= c.\partial A \end{aligned}$$

Wolf, Hastings, Cirac, FV '08

Feynman's vision of modelling entanglement degrees of freedom:

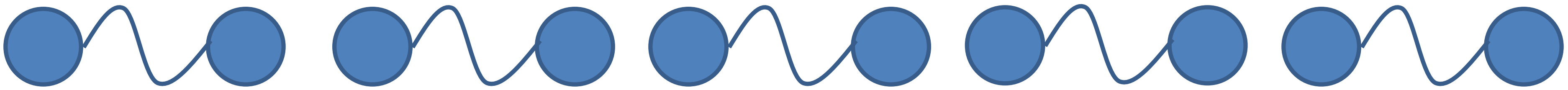
"Now, in field theory, what's going on over here and what's going on over there and all over space is more or less the same. Why do we have to keep track in our functional of all things going on over there while we are looking at the things that are going on over here? . . . It's really quite insane, actually: we are trying to find the energy by taking the expectation value of an operator which is located here and we present ourselves with a functional which is dependent on everything all over the map. That's something wrong. Maybe there is some way to surround the object, or the region where we want to calculate things, by a surface and describe what things are coming in across the surface. It tells us everything that's going on outside."

"I think it should be possible some day to describe field theory in some other way than with the wave functions and amplitudes. It might be something like the density matrices where you concentrate on quantities in a given locality and in order to start to talk about it you don't immediately have to talk about what's going on everywhere else . . . "

Wangerooge 1987, Proceedings,
Variational calculations in quantum field theory

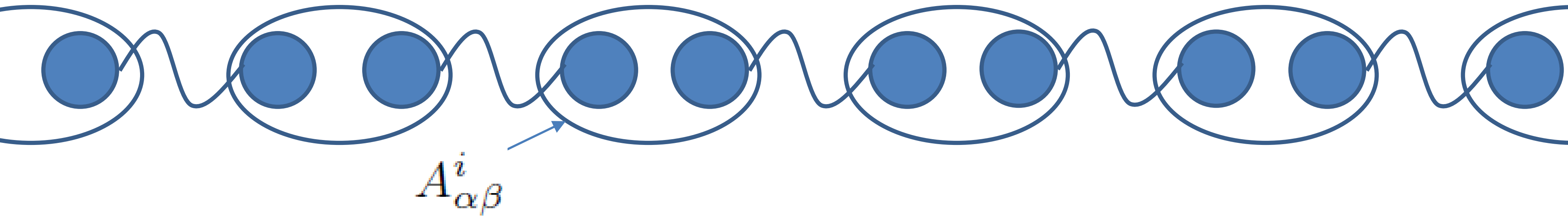
Tensor Calculus for Quantum Spin Chains

- Systematic way of creating states which have extremal local marginals but keep translational invariance: *matrix product states*



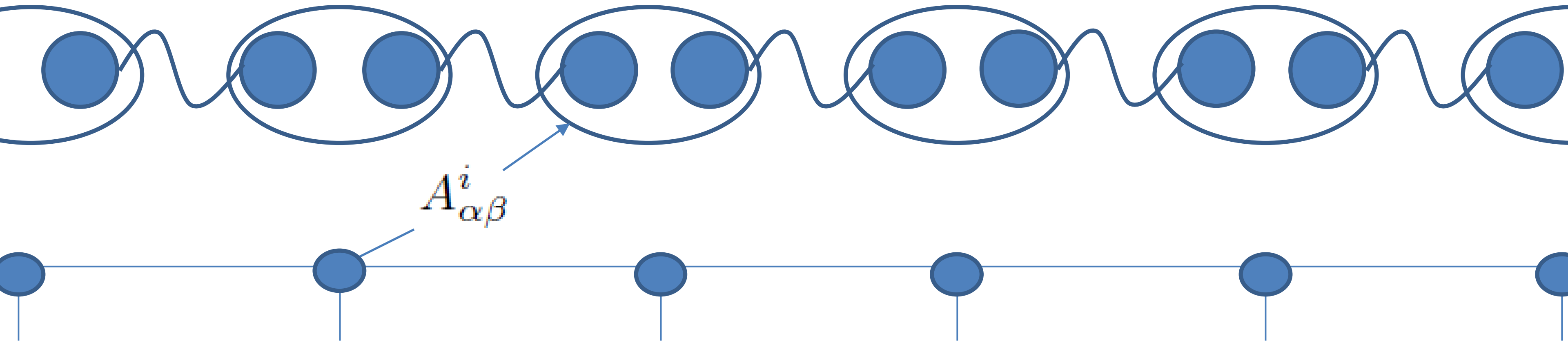
Tensor Calculus for Quantum Spin Chains

- Systematic way of creating states which have extremal local marginals but keep translational invariance: *matrix product states*



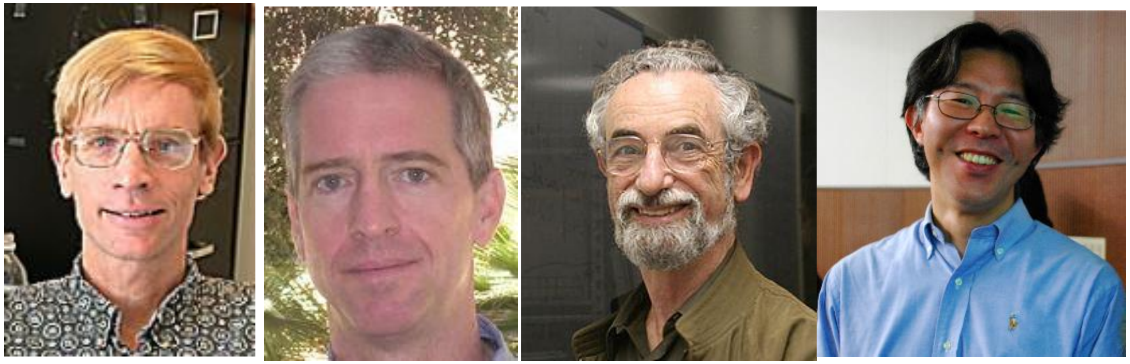
Tensors for Quantum Spin Chains

- Systematic way of creating states which have extremal local marginals but keep translational invariance: *matrix product states (MPS)*



$$|\psi\rangle = \sum_{i_1 i_2 i_3 \cdots} \mathrm{Tr}\left(A^{i_1} A^{i_2} A^{i_3} \cdots\right) |i_1\rangle |i_2\rangle |i_3\rangle \cdots$$

- MPS model the correlations and how the entanglement is distributed



I. Affleck, T. Kennedy,
E. Lieb, H. Tasaki ‘87

M. Fannes, B. Nachtergaele,
R. Werner ‘91

K. Wilson ’70s

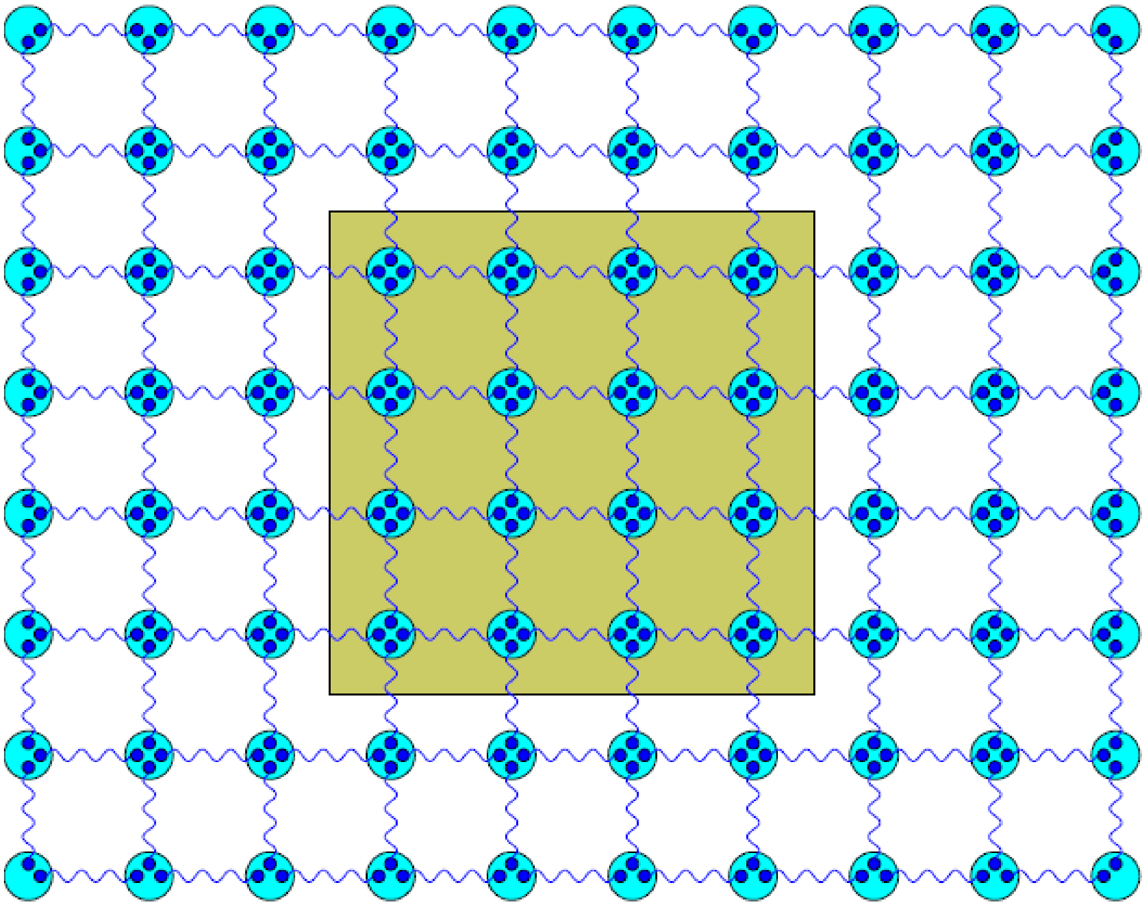
S. White ‘92

I. Cirac ‘04

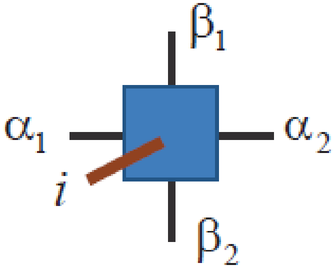
G. Vidal ‘04

Higher dimensions: Projected Entangled Pair States (PEPS)

FV, Cirac '04



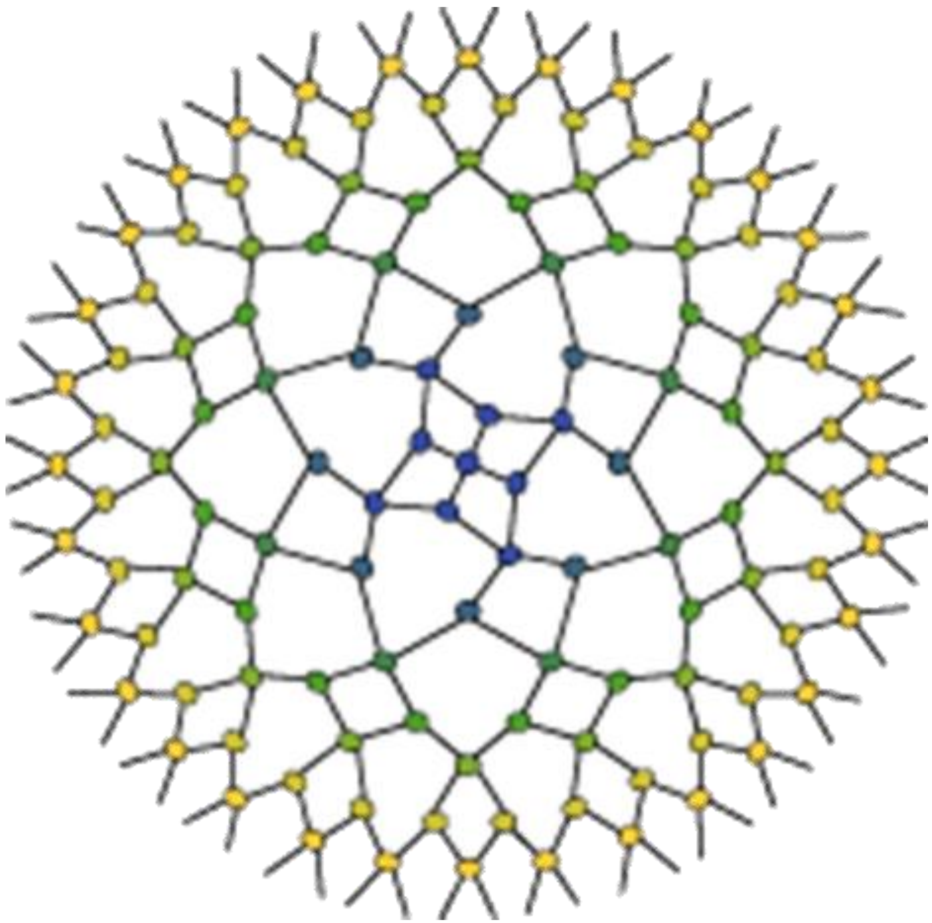
$$A^{i}_{\alpha_1,\alpha_2;\beta_1,\beta_2}$$



Multiscale Entanglement Renormalization Ansatz

G. Vidal ‘07

Scale invariant (critical) systems



- Crucial ideas in tensor networks:
  - Tensors model the entanglement structure: modelling correlations makes much more sense than modelling wavefunction directly
  - Tensor networks can be efficiently contracted due to holographic property: map quantum 3D -> 2D -> 1D -> 0D problems, and this can be done efficiently due to area laws
  - Local tensor contains all global information about quantum many body state
    - different phases of matter can be distinguished by symmetries of those local tensor, including topological phases
    - Tensor networks provide a natural way of dealing with gauge theories: enforcing symmetries

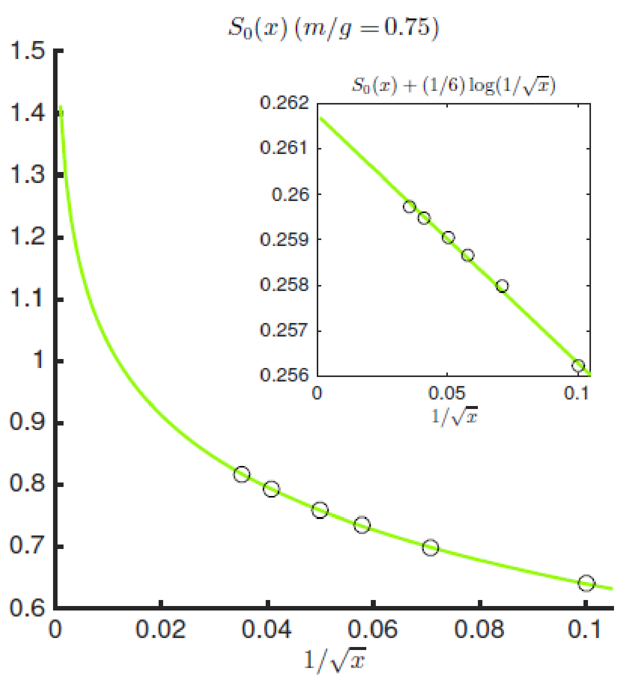
Tensor networks at work: Schwinger model

Buyens, Van Acoleyen, FV '13-'17

- Kogut-Susskind staggered formulation with $x = 1/g^2a^2$

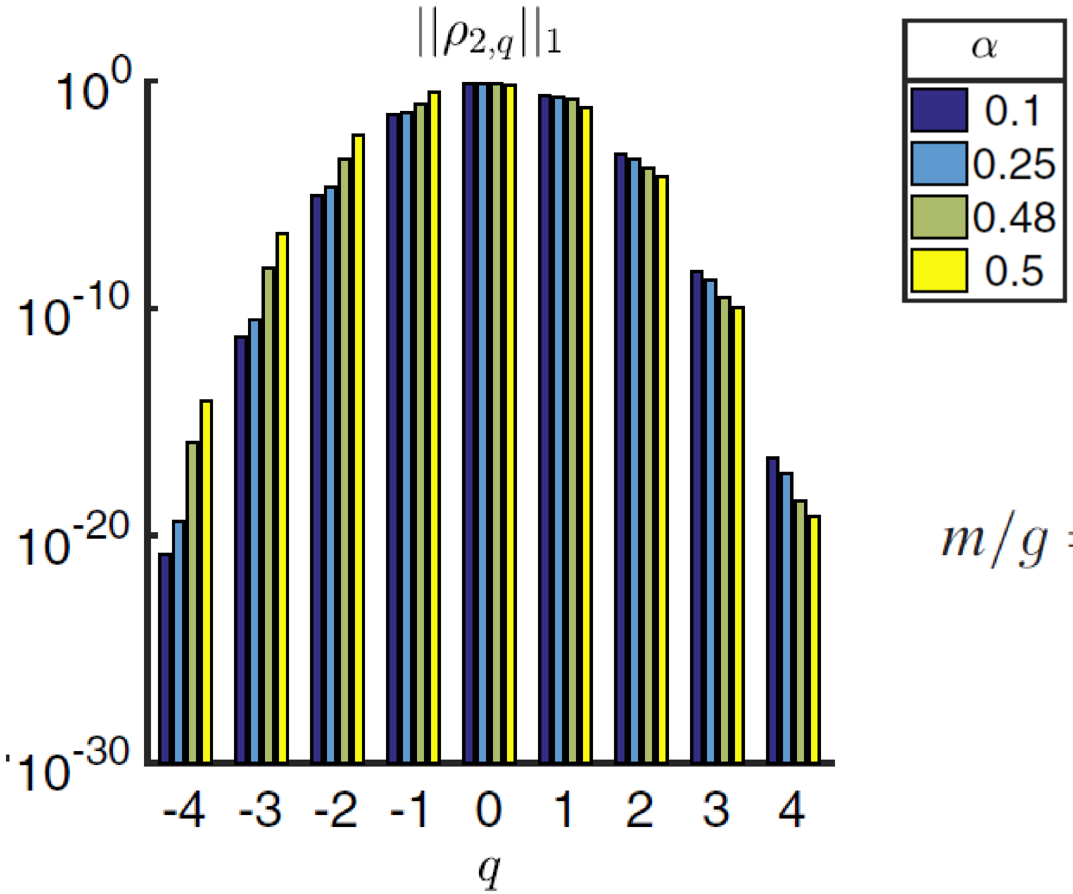
$$\mathcal{H} = \frac{g}{2\sqrt{x}}\left(\sum_{n=1}^{2N}[L(n)+\alpha(n)]^2 + \frac{\sqrt{x}}{g}m\sum_{n=1}^{2N}(-1)^n\left(\sigma_z(n)+(-1)^n\right) \quad + x\sum_{n=1}^{2N-1}(\sigma^+(n)e^{i\theta(n)}\sigma^-(n+1)+h.c.)\right)$$

- Entanglement spectrum in continuum limit



$$f_1(x) = A_0 + B_0 \log\left(\frac{1}{\sqrt{x}}\right) + C_0\frac{1}{\sqrt{x}}$$

- Cutting off the electrical field:

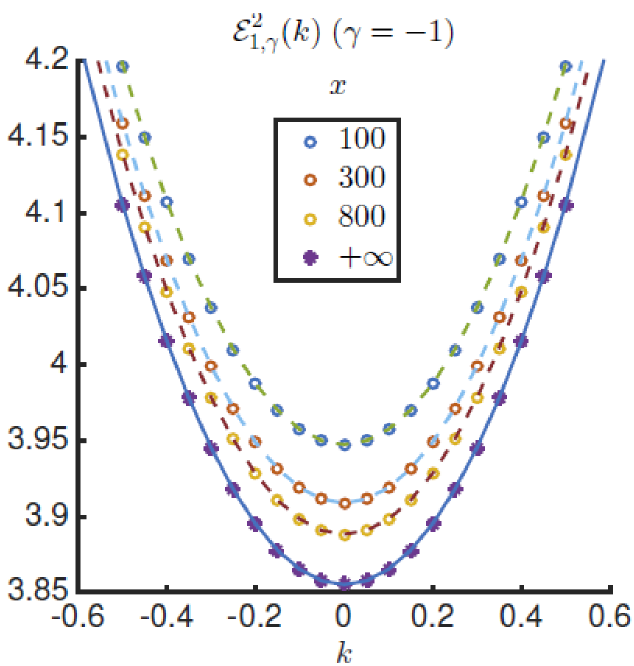


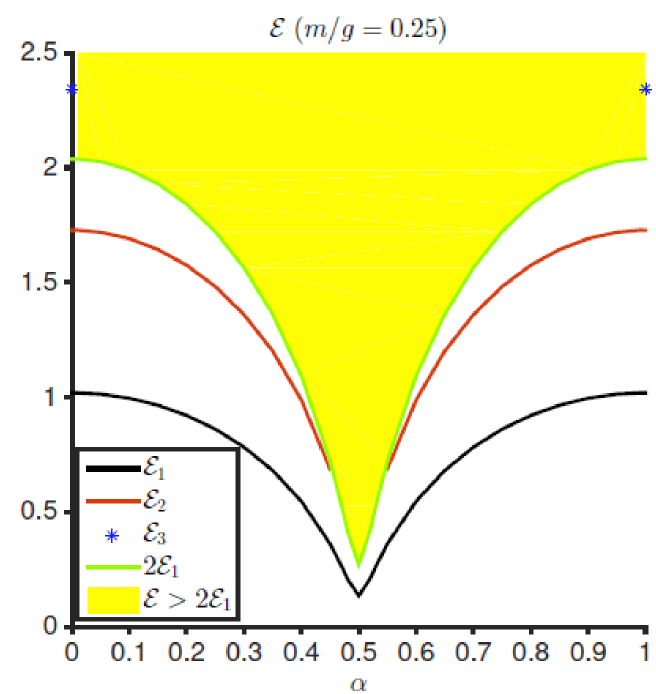
$m/g = 0.3,\ x = 100$

  - This justifies “qubit” approach to Schwinger model

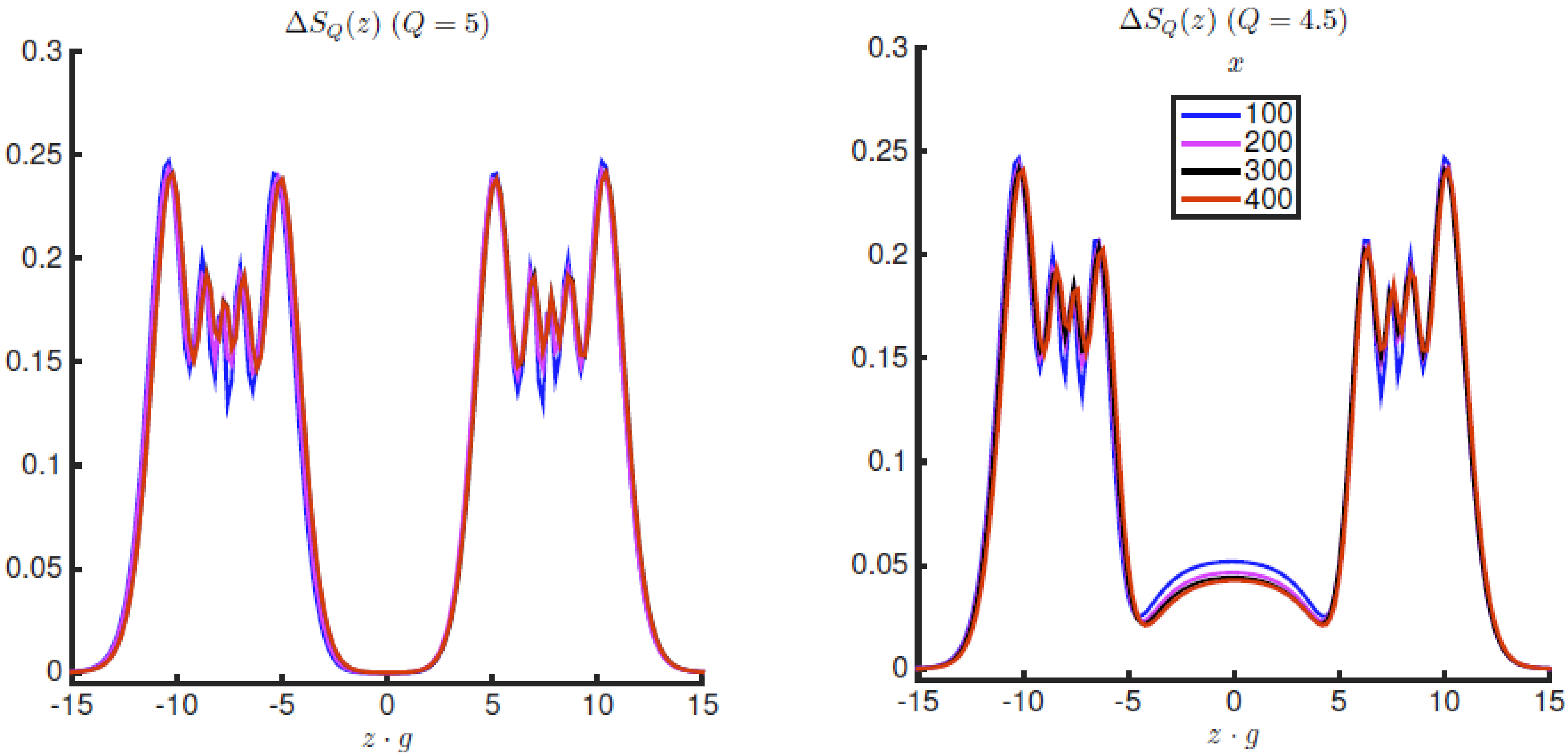
- Excitation spectrum: continuum limit

m/g	ω_0	$\mathcal{E}_{1,v}$	$\mathcal{E}_{1,s}$	$\mathcal{E}_{2,v}$
0	-0.318320(4)	0.56418(2)		
0.125	-0.318319(4)	0.789491 (8)	1.472 (4)	2.10 (2)
0.25	-0.318316(3)	1.01917 (2)	1.7282 (4)	2.339(3)
0.3	-0.318316(3)	1.11210 (8)	1.82547 (3)	2.4285 (3)
0.5	-0.318305(2)	1.487473 (7)	2.2004 (1)	2.778 (2)
0.75	-0.318285 (9)	1.96347 (3)	2.658943(6)	3.2043(2)
1	-0.31826 (2)	2.44441 (1)	3.1182 (1)	3.640(4)

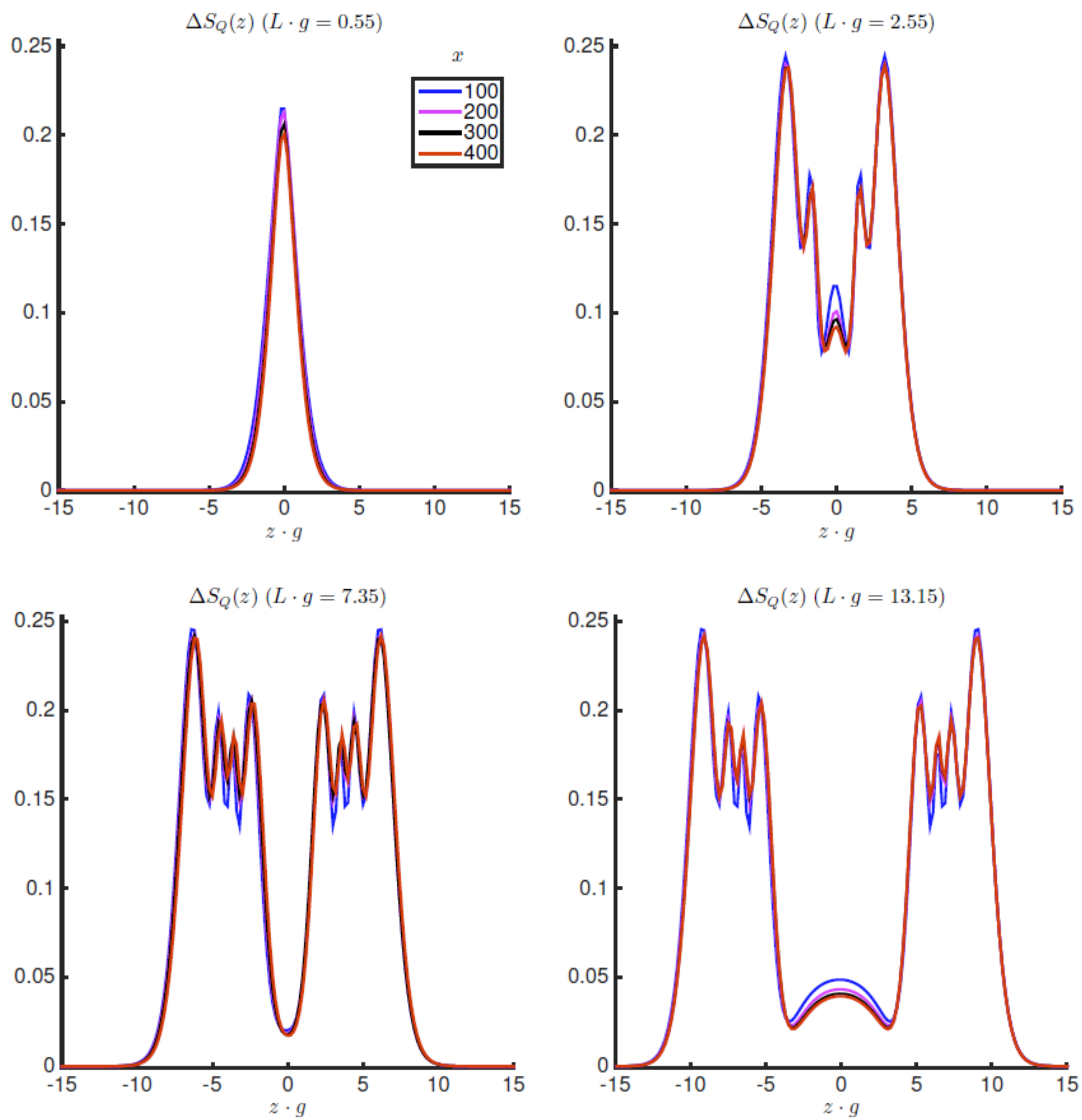




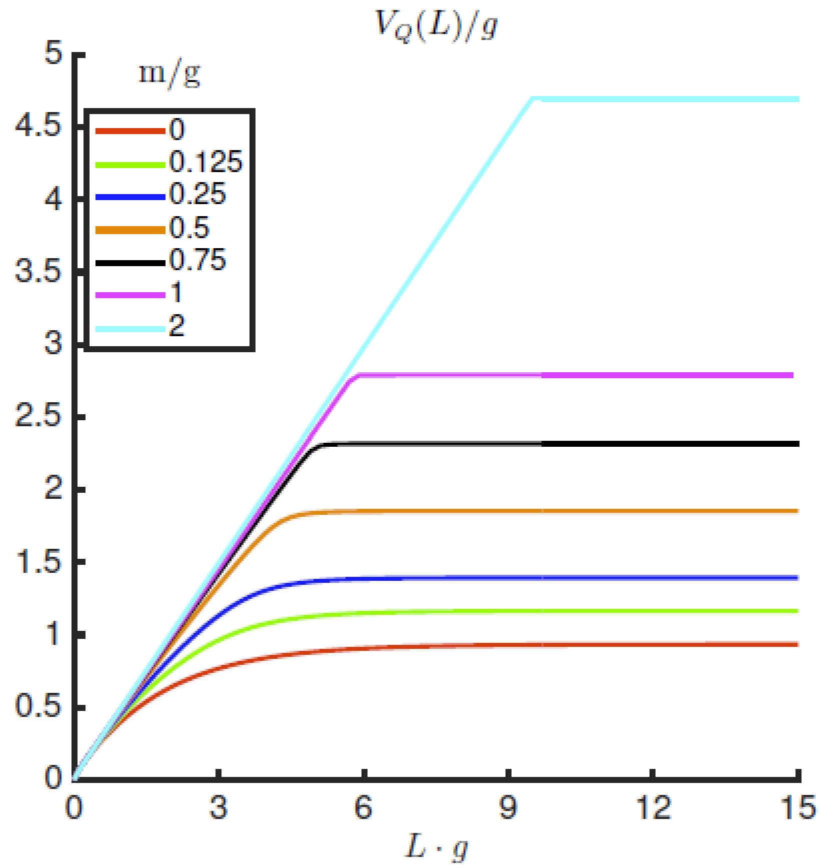
- Entanglement entropy in presence of test charges Q at distance L.g=15: string breaking in the meson states



- Entanglement entropy in presence of test charges Q=4.5 as function of L



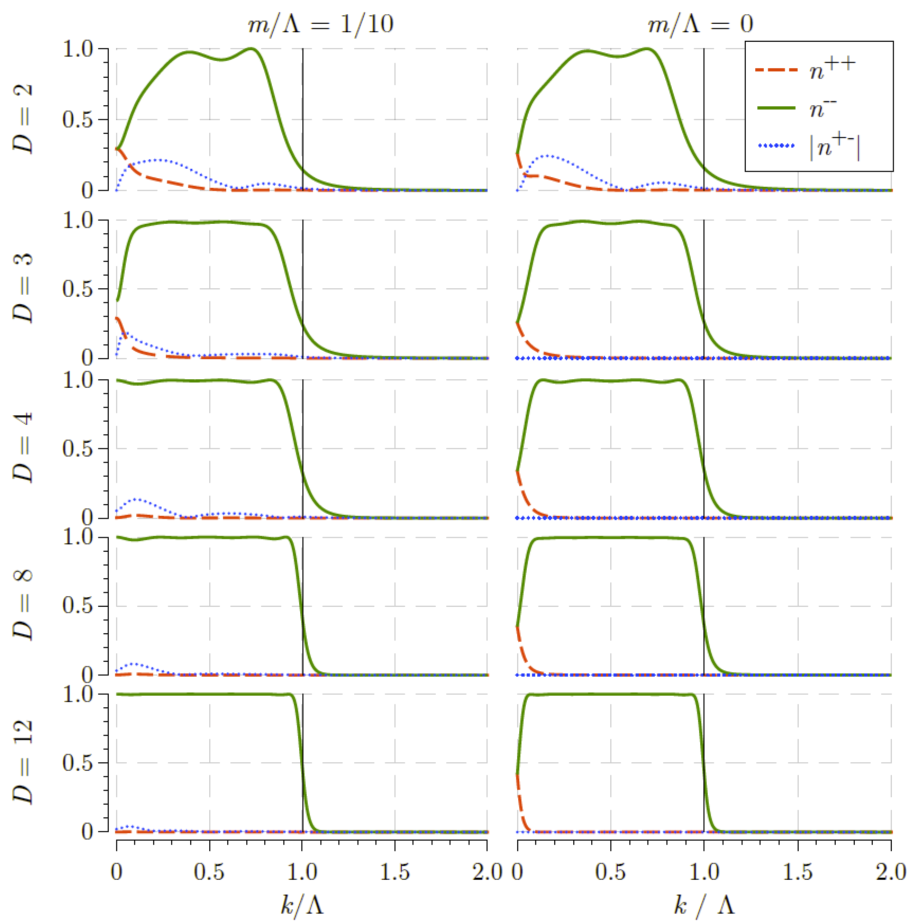
- Quark-andiquark potential for Q=1

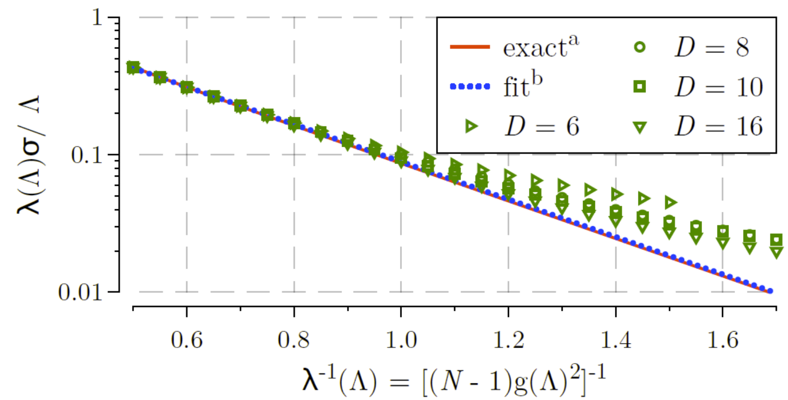


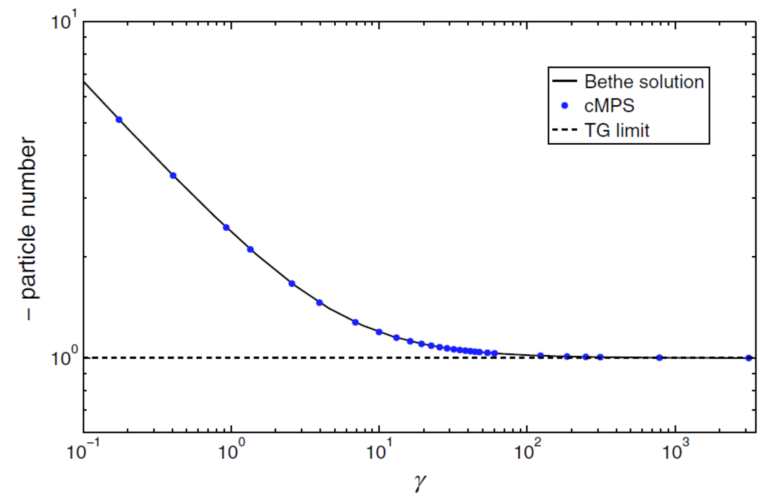
Continuous MPS: variational methods in the continuum

FV, Cirac ’10; Haegeman, FV ’11; …

$$|\Psi\rangle = \mathrm{Tr}_{\mathrm{aux}}\left[\mathscr{P} e^{\int_{-\infty}^{+\infty} \mathrm{d}x\, Q \otimes \mathbb{1} + \sum_\alpha R_\alpha \otimes \hat{\psi}_\alpha^\dagger(x)}\right] |\Omega\rangle$$





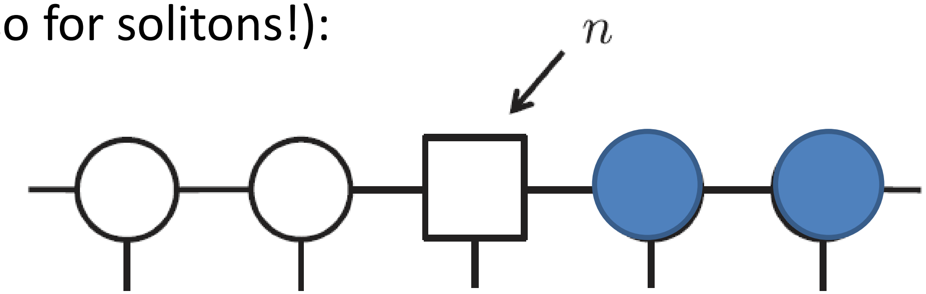


Scattering elementary particles and solitons

Vanderstraeten et al. '14

- Single particle ansatz (also for solitons!):

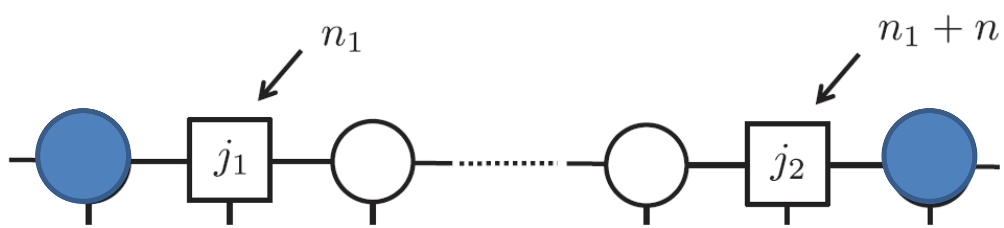
$$|\Phi_\kappa[B]\rangle = \sum_n \mathrm{e}^{i\kappa n}$$

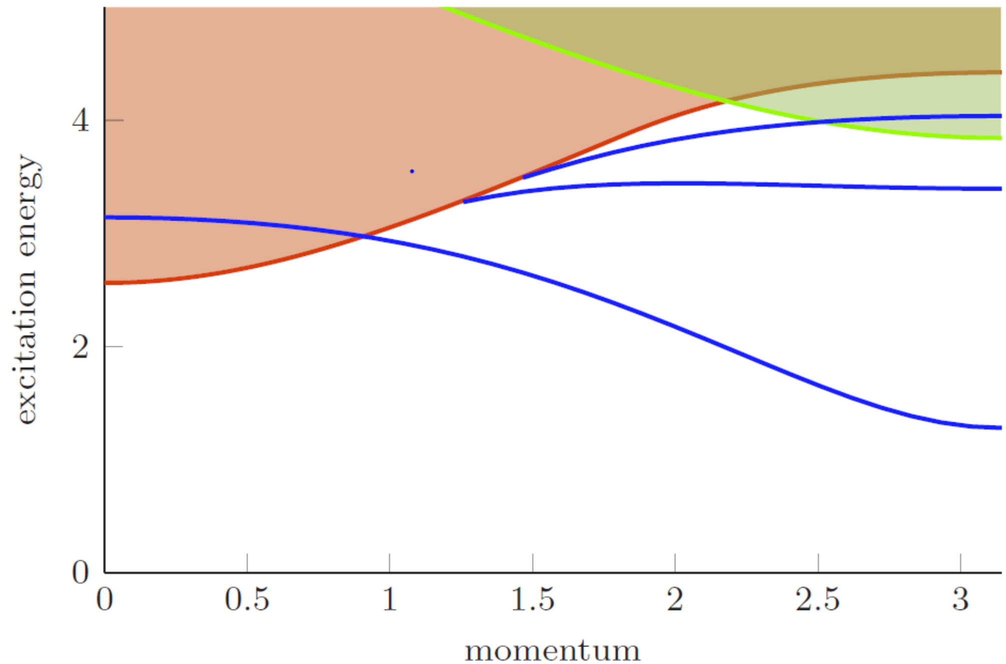


- Scattering ansatz:

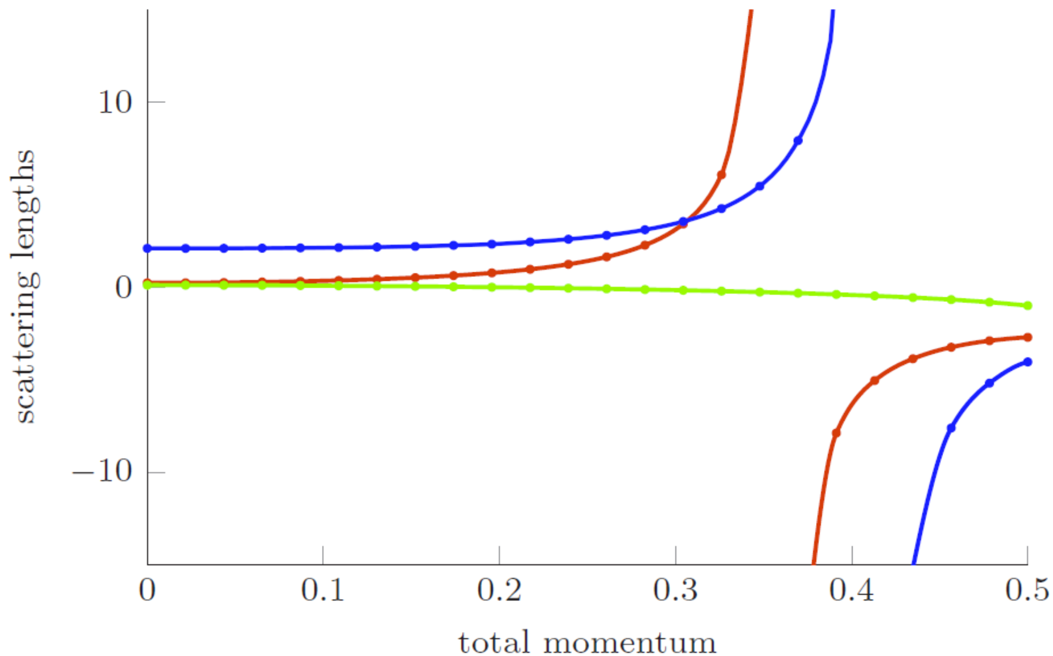
$$|\Upsilon(K)\rangle = \sum_{n=0}^{+\infty} \sum_{j=1}^{L_n} c^j(n) |\chi_{K,j}(n)\rangle$$

$$|\chi_{(j_1,j_2),K}(n)\rangle = \sum_{n_1} \mathrm{e}^{iKn_1}$$





Dispersion relation in a Heisenberg spin ½ ladder

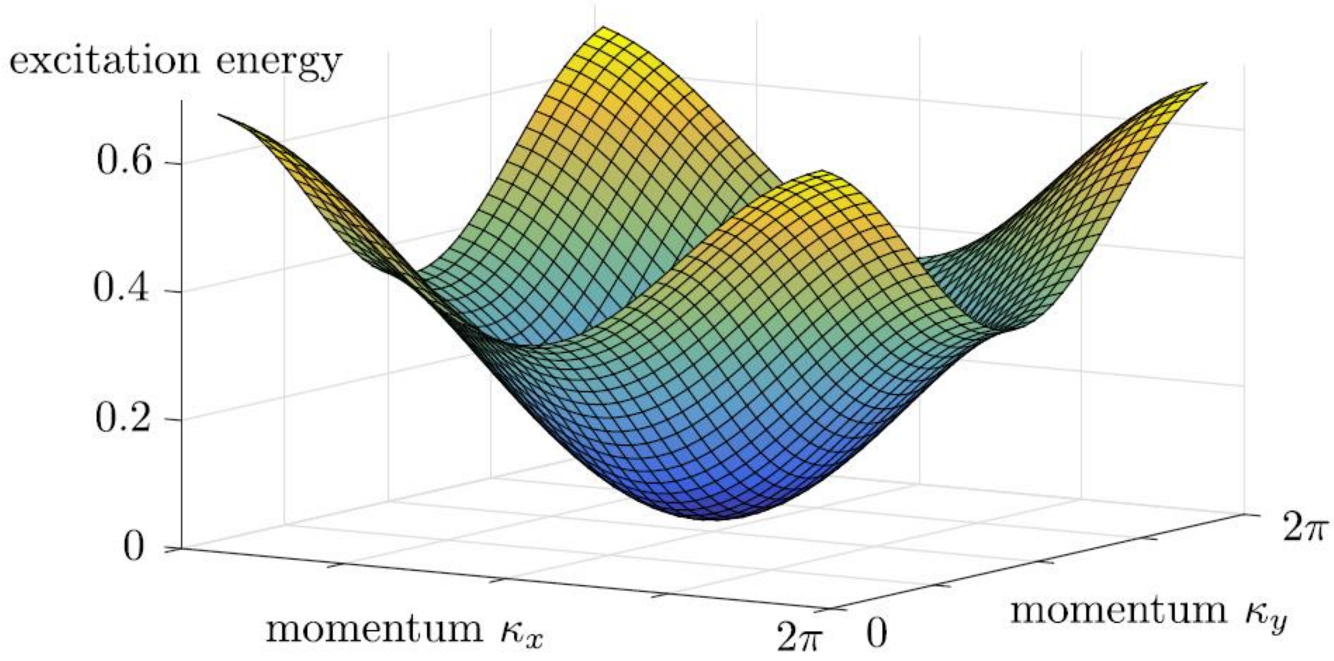


Scattering lengths in Heisenberg spin ½ ladder; the divergences in the spin 0 and 1 sector indicate the formation of bound states

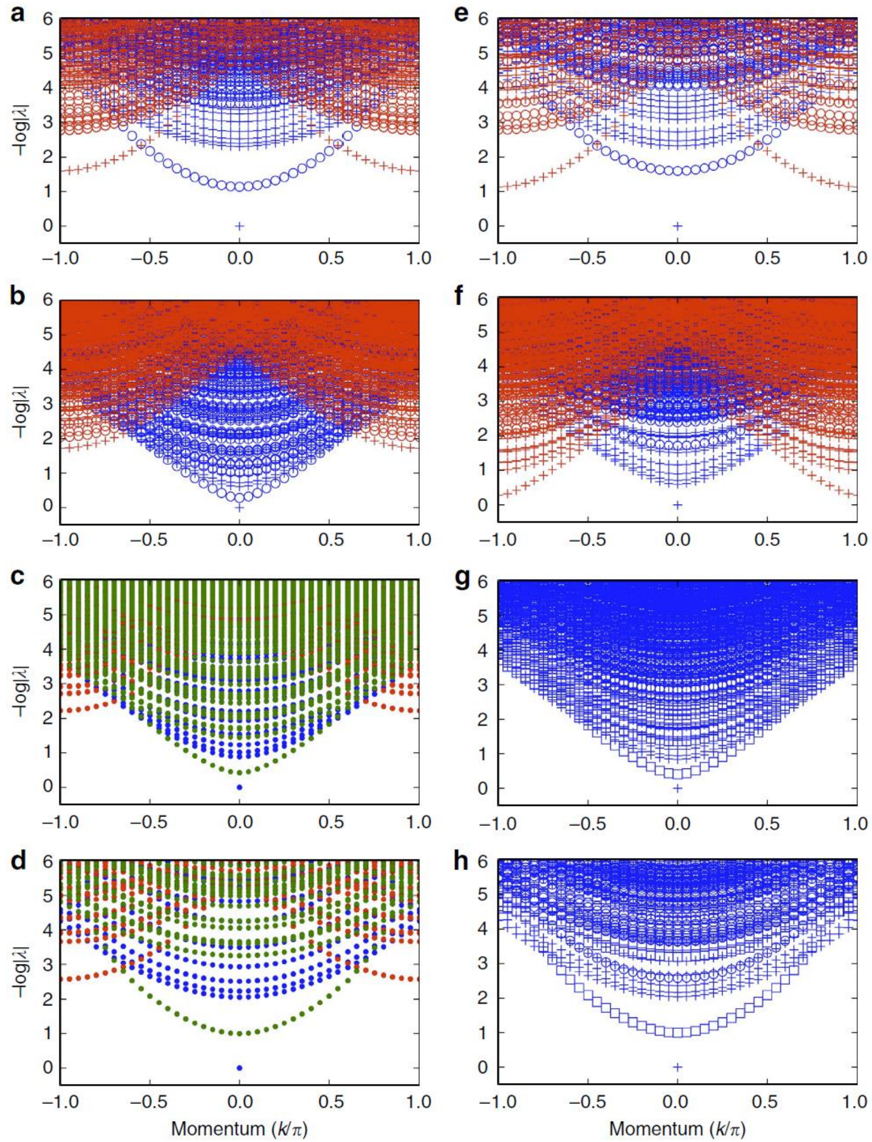
Vanderstraeten et al. '15

Tensor networks in 2+1D

- Dispersion relations for 2D Heisenberg (AKLT) model

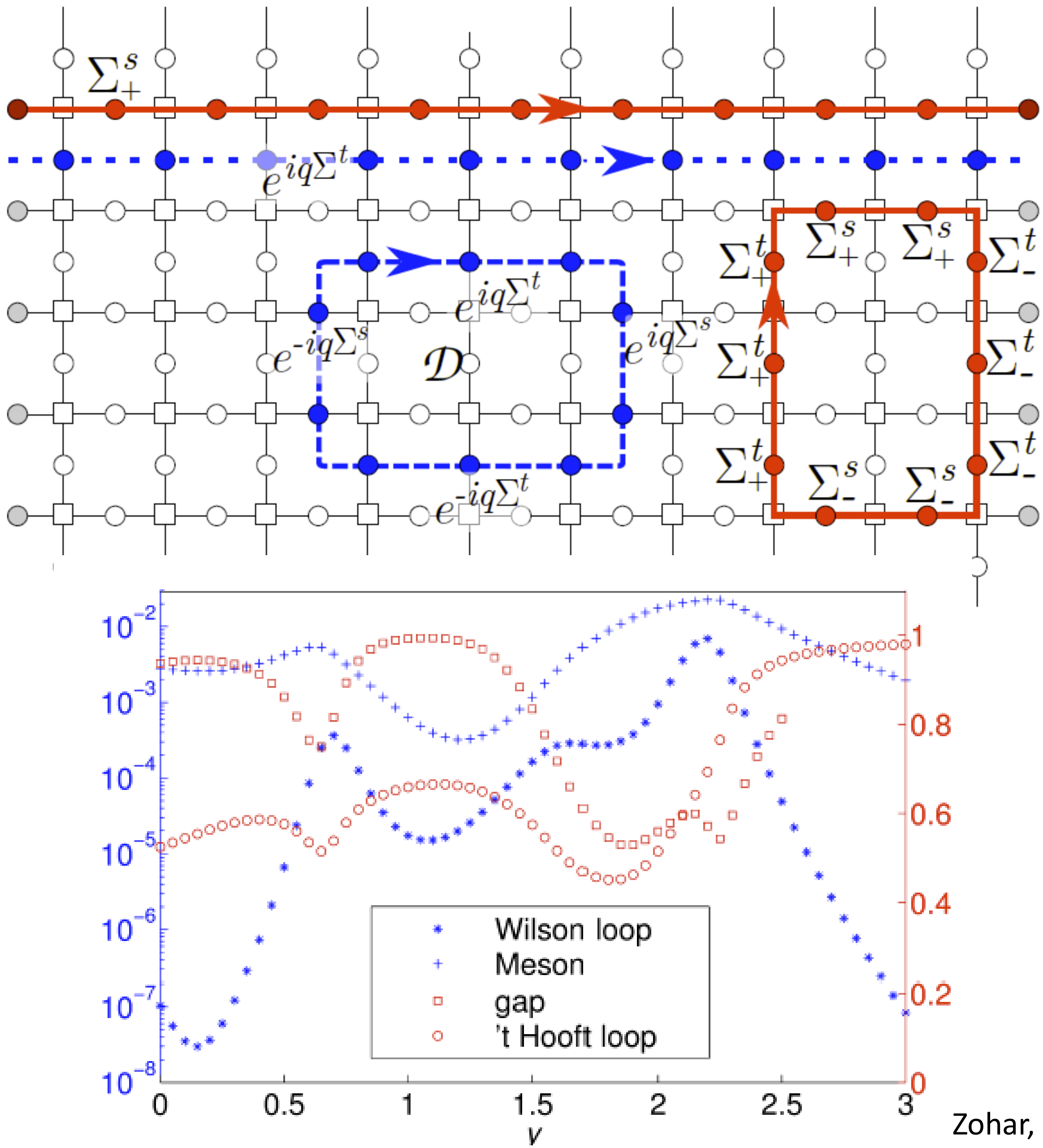


- Entanglement spectrum and confinement/deconfinement phase transition by anyon condensation in a Z2 gauge theory (Shenker/Fradkin == toric code with string tension)



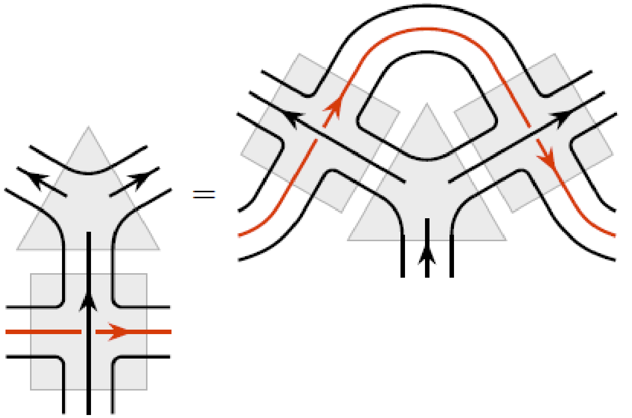
Haegeman, Zauner, Schuch, FV '15

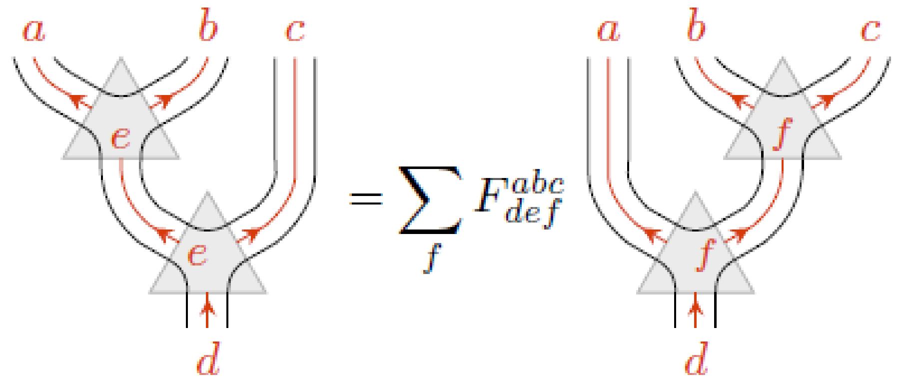
QED in 2+1 D

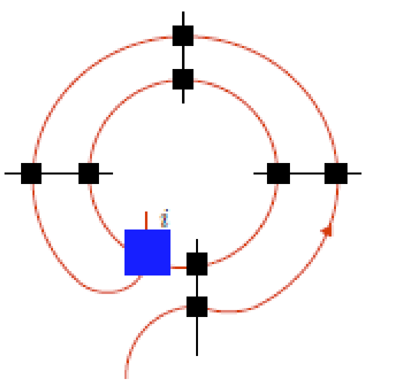


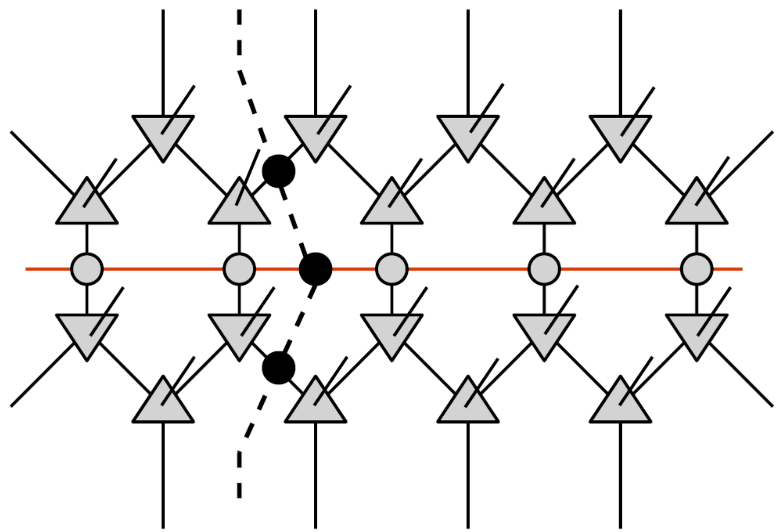
Zohar, Burrello, Wahl, Cirac '16

TQFT in 2+1 and CFT in 2+0: MPO symmetries









Sahinoglu, Williamson, Bultinck, Marien, Haegeman, FV ‘14-’18

Entanglement Matters

Quantum Computation

Projected Entangled Pair States

Multiscale Entanglement Renormalization Ansatz

Matrix Product States

Lieb-Robinson bounds

Entanglement

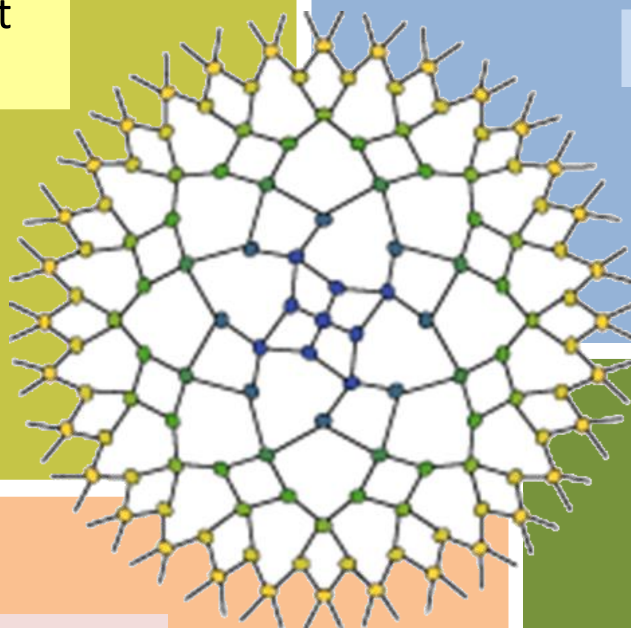
Lattice Gauge Theories

Anyon Condensation

Holographic Principle

Quantum Topological Order

Renormalization Group



Quantum Quenches

Cold Atomic Gases

Quasi-Particles

Quantum Phase Transitions

Non-Commutative Gross-Pitaevskii

Fractional Quantum Hall

Bosonic SPT phases

Hubbard Model

(Virtual) Order Parameter

Quantum Spin Liquids

References

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Physical Review A 79 (3), 032316

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F Verstraete, V Murg, JI Cirac
Advances in Physics 57 (2), 143-224

Diagonalizing transfer matrices and matrix product operators: a medley of exact and computational methods
J Haegeman, F Verstraete
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Matrix product states for gauge field theories
B Buyens, J Haegeman, K Van Acoleyen, H Verschelde, F Verstraete
Physical review letters 113 (9), 091601

S matrix from matrix product states
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