Open Challenges for QCD at High Temperatures and Densities

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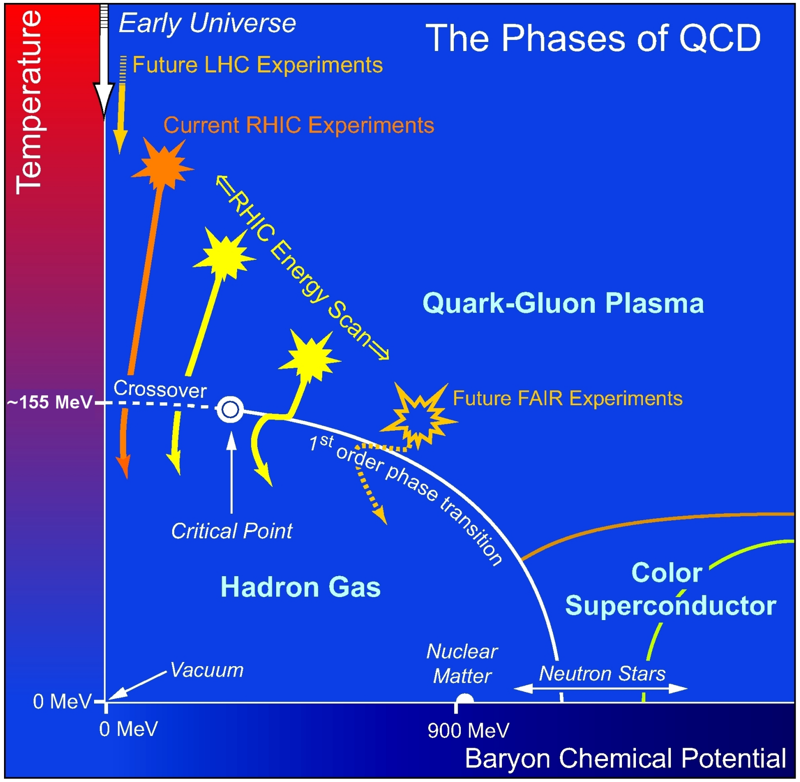


The system created at RHIC behaves like perfect liquid (2005) How does the system thermalize ?

Is there is a critical point on the QCD phase diagram ? (2019-2021)

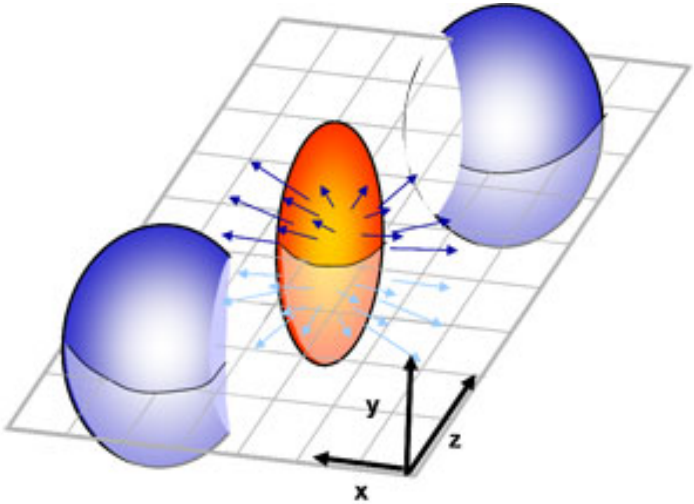
NPQI Workshop, ANL March 28-30, 2018

Viscous hydrodynamics and flow

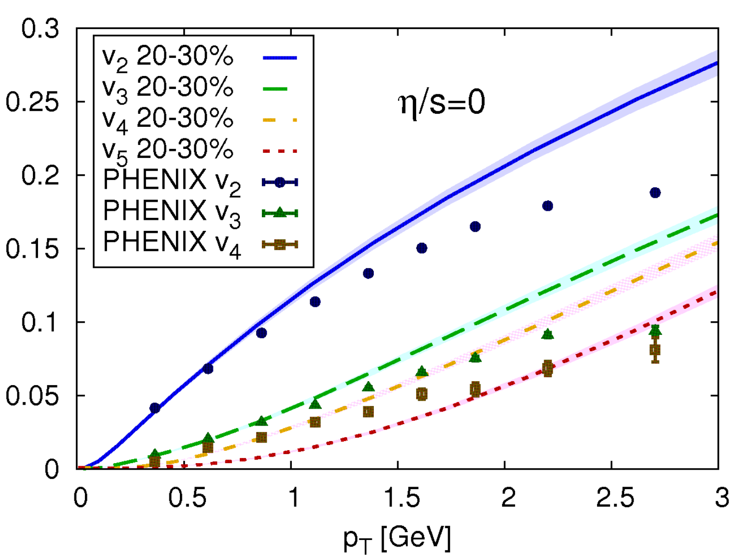
$$\frac{dN}{d\Phi} = v_0\,(1 + 2\,v_1\,\cos(\Phi) + 2\,v_2\,\cos(2\Phi))$$

Assume that a thermal system is created shortly after the collisions that expands hydrodynamically.

To describe the experimental data very small shear viscosity to entropy ratio is needed

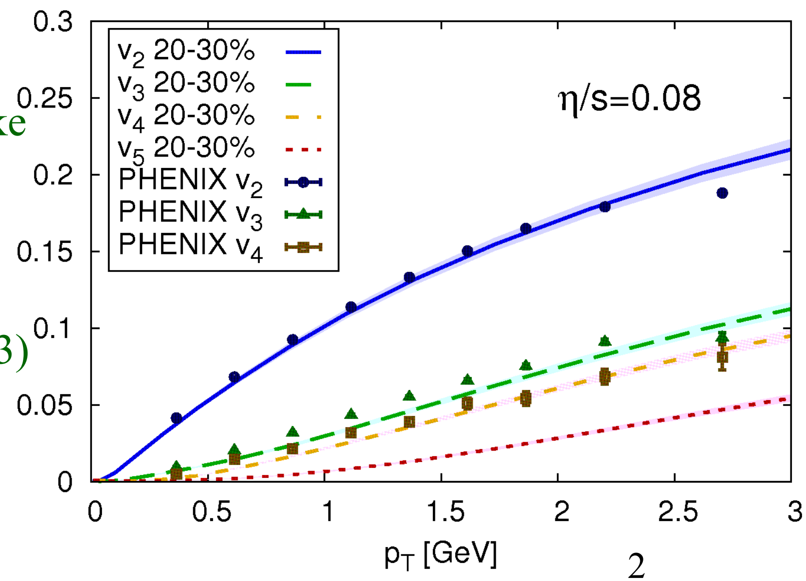


RHIC Scientists Serve Up "Perfect" Liquid, New state of matter more remarkable than predicted -raising many new questions
April 18, 2005



Bjoern Schenke
BNL

Gale et al,
PRL 110 (2013) 012302

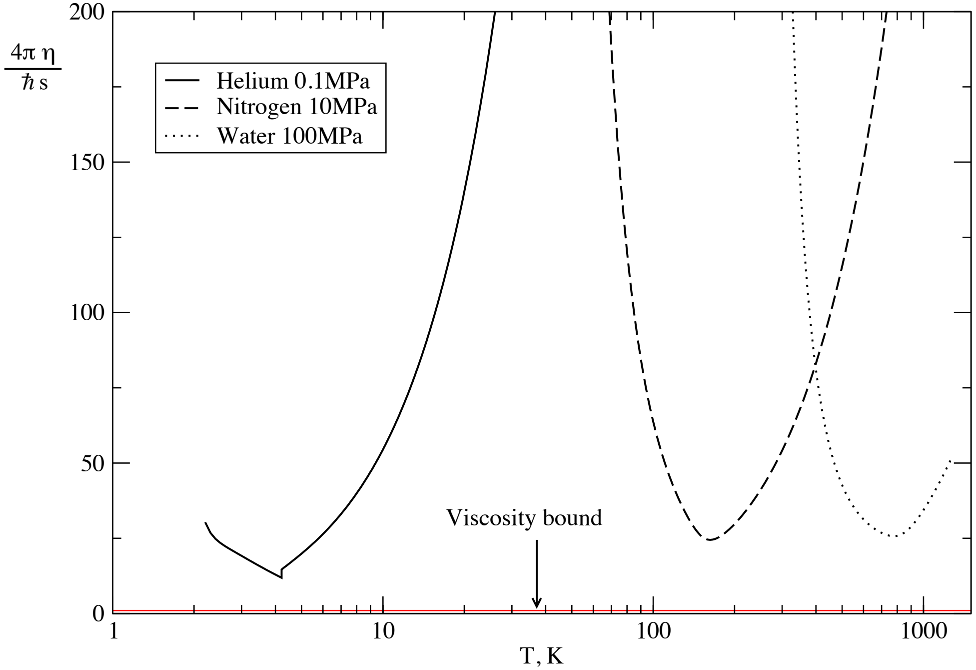


How small is the shear viscosity ?

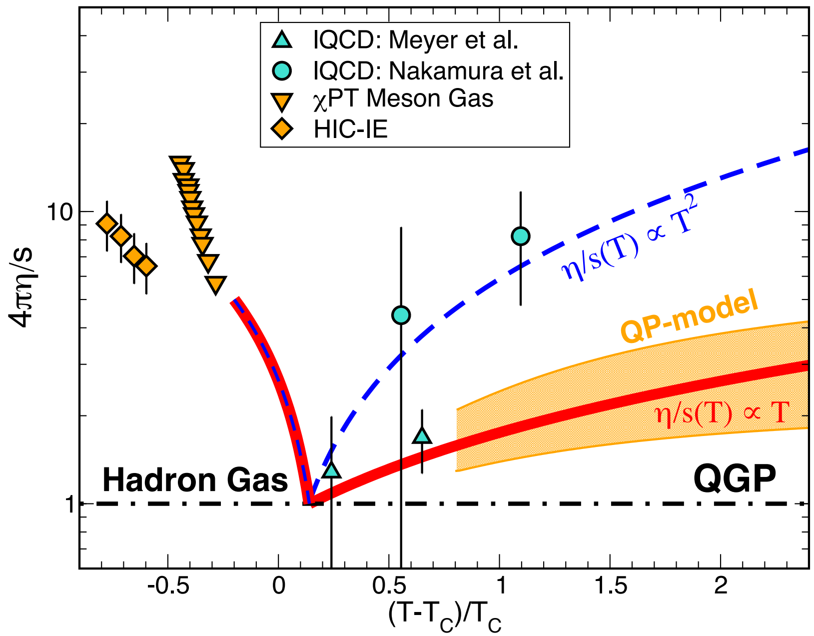
Validity of the hydrodynamics is governed by η/s
Hadron gas and QGP at very high temperature have large value η/s

Super-symmetric gauge theories at strong coupling have small η/s with lower bound dictated by quantum mechanics $\eta/s>1/(4\ \pi)$ (Kovtun, Son Starinets 2005)
$\Rightarrow$ QGP near the transition temperature T_c has close to minimal η/s

Kovtun, Son Starinets, 2005



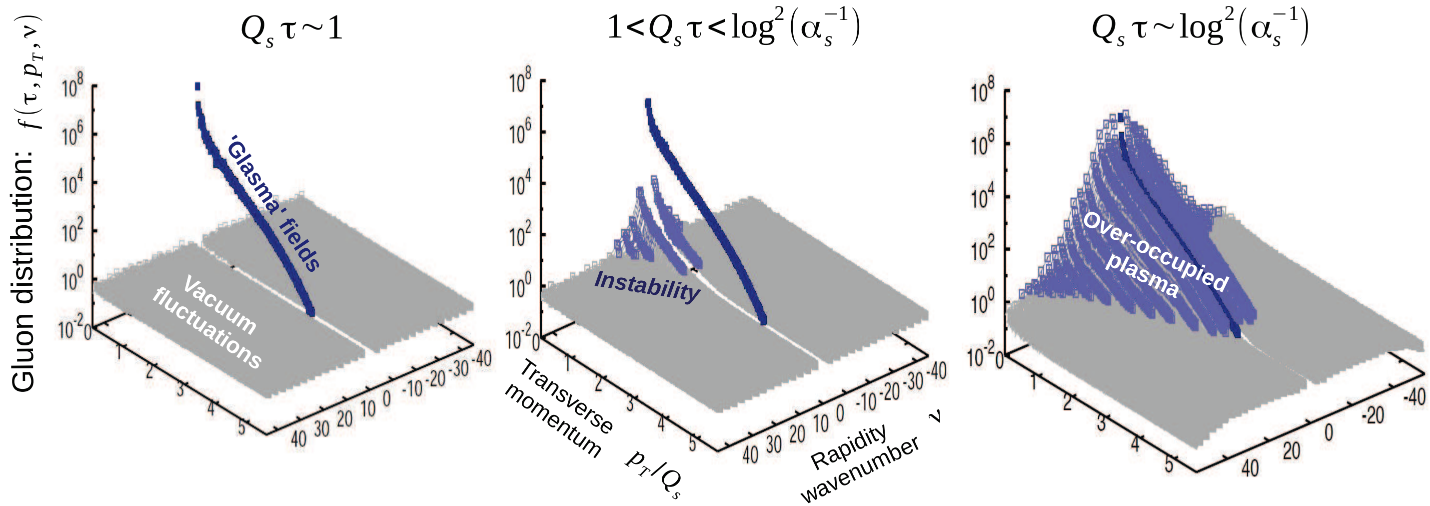
Csernai et al, 2013



Initial time dynamics and thermalization in heavy ion collisions

Classical-statistical calculations of gluon distribution at early times (large gluon occupation numbers)

Berges Schenke, Schlichting, Venugopalan, Nucl. Phys. A 931 (2014) 348



The gluon occupation number decreases at later times reaching *O(1)*, the system becomes quantum and strongly coupled

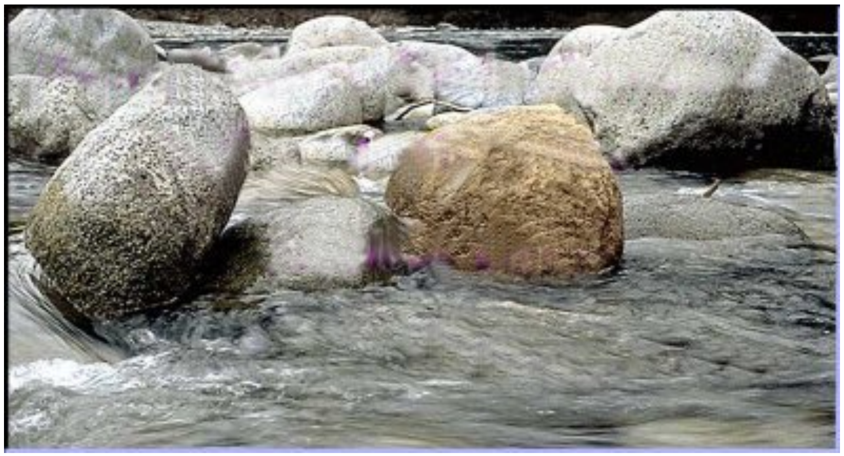
➡ quantum simulations are needed

Early time dynamics is important event-by-event fluctuations in *AA*, and high multiplicity *pA* and *AA* collisions

Strongly coupled QGP and heavy quarks

Heavy quarks ($M_c \sim 1.5\ GeV$) flow in the strongly coupled QGP

Analogy from Jamie Nagle

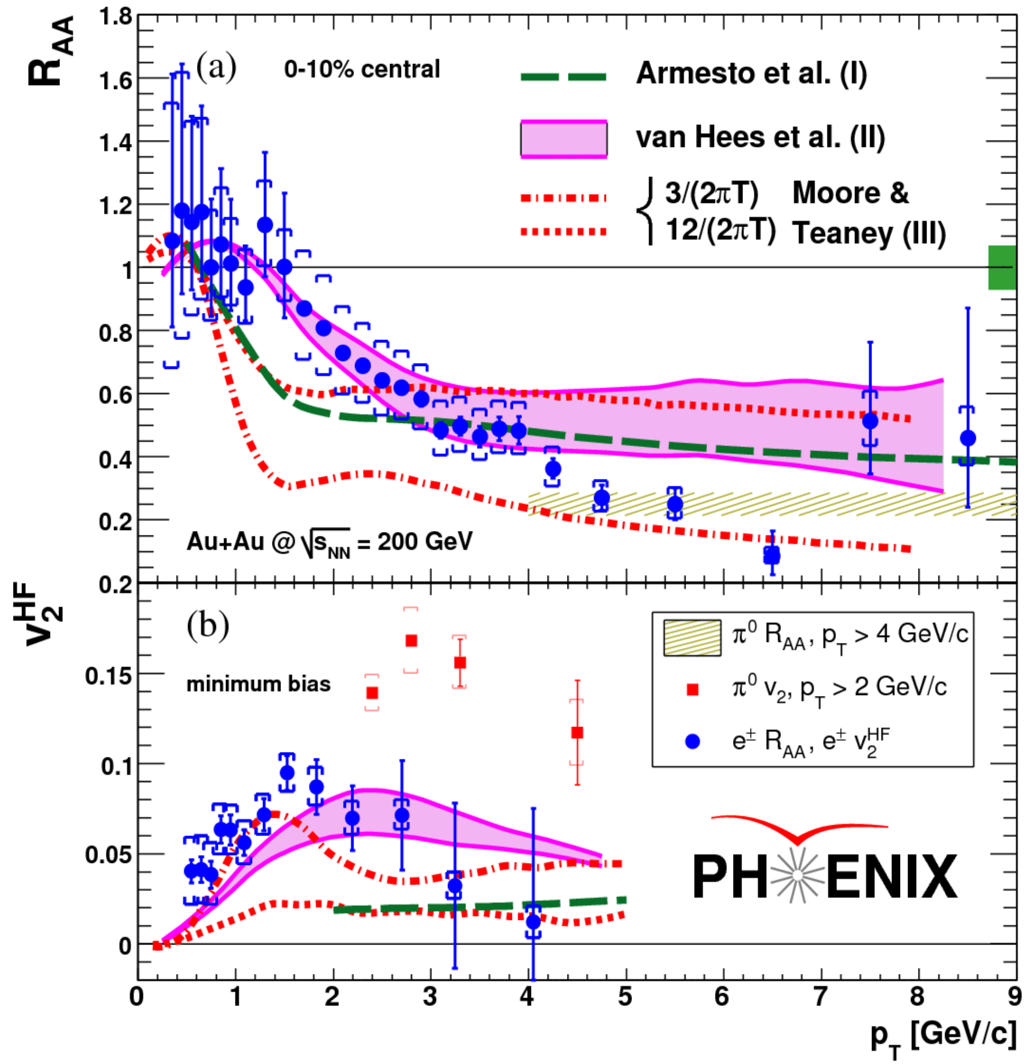


$$t_{rel}^{heavy} \sim \frac{M_c}{T} t_{rel}^{light} \Rightarrow \text{Langevin dynamics:}$$

$$\frac{dx^i}{dt} = \frac{p^i}{M}, \frac{dp^i}{dt} = \xi^i(t) - \eta p^i,$$

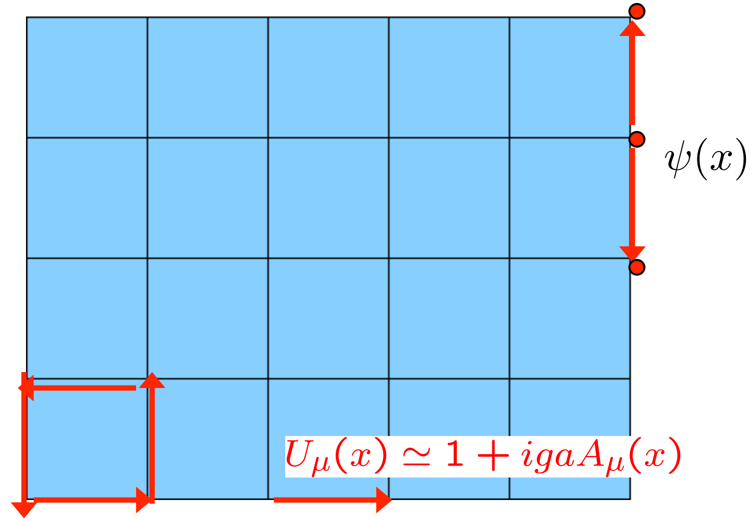
$$\langle \xi^i(t) \xi^j(t') \rangle = \kappa \delta^{ij} \delta(t - t')$$

$$\eta = \frac{\kappa}{2MT}, \ \ D = \frac{T}{M\eta}$$



Finite Temperature QCD and its Lattice Formulation

$$\langle O \rangle = \mathrm{Tr} O e^{-\beta H - \mu N} \qquad \beta = 1/T$$



$$\langle O \rangle = \int \mathcal{D}A_\mu \mathcal{D}\psi \mathcal{D}\bar{\psi} O e^{-\int_0^\beta d\tau d^3x \mathcal{L}_{QCD}}$$

$$A_\mu(0,\mathbf{x}) = A_\mu(\beta,\mathbf{x}) \quad \psi(0,\mathbf{x}) = -\psi(\beta,\mathbf{x})$$

Lattice

integral with very large dimensions

$$\langle O \rangle = \int \prod_x dU_\mu(x) O(detD_q[U,m,\mu]) e^{-\sum_x S_G[U(x)]}, U_\mu(x) = e^{igaA_\mu(x)}$$

$\mu = 0$ → Monte-Carlo Methods

cost ~ *1/a*^{7}

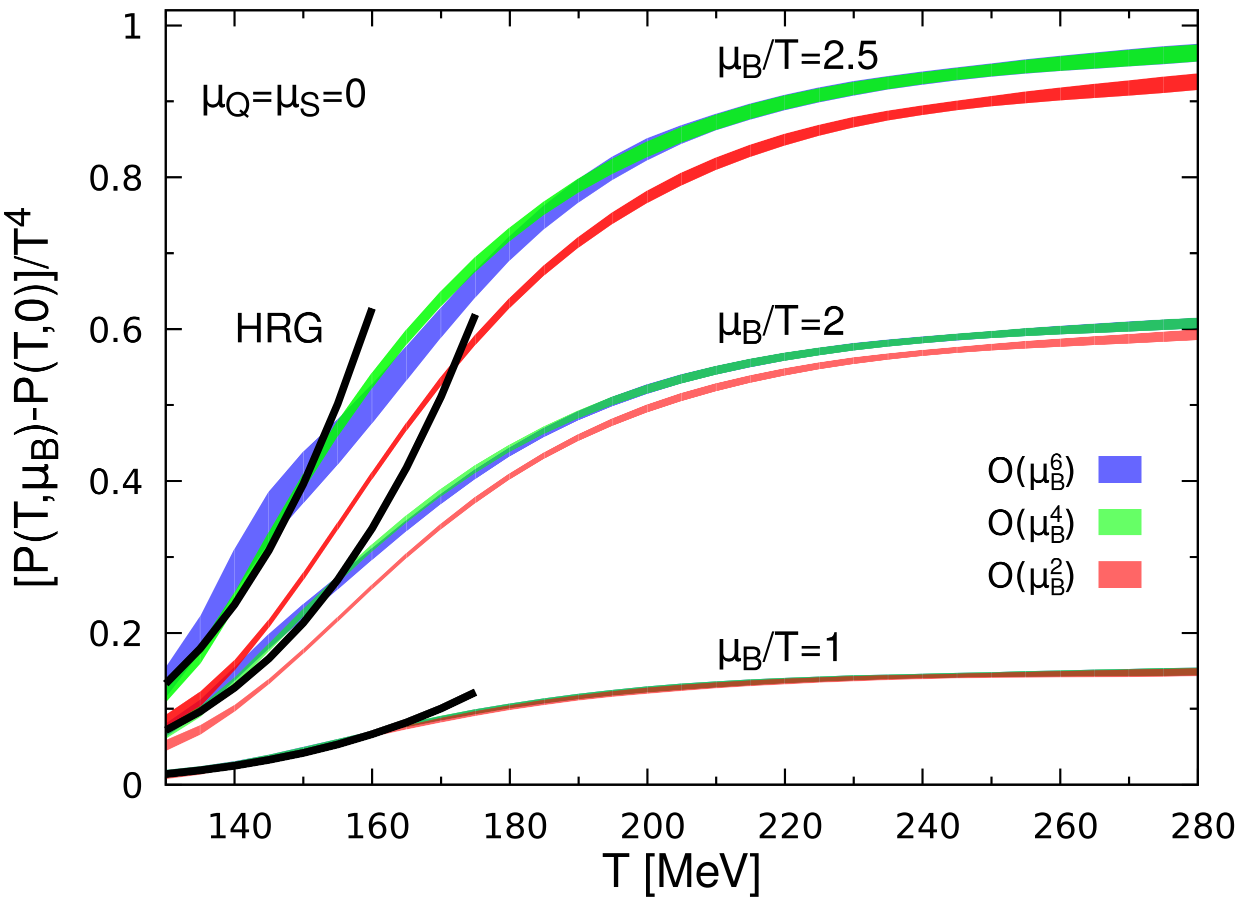
$\mu \neq 0$: $detD_q(U,m,\mu)$ complex → sign problem → Taylor expansion for not too large μ

$$\frac{p(T,\mu)}{T^4} = \sum_{n=1}^{\infty} \frac{1}{(2n)!} \chi_{2n}(T) \mu^{2n}$$

Calculable in LQCD but the computational difficulty increases with *n* !
(noise problem vs. sign problem)
Current calculations exist only to *n=3*.

Thermodynamics at non-zero net baryon density

6th order Taylor expansion, Bazavov et al, PRD 95 (2017) 054504



Truncation errors of the 6th order Taylor expansions are small for $\mu_B/T<2.5$

Critical point is strongly disfavored for $\mu_B/T<2.0$

Correlation functions and transport coeffocients

Transport coefficients are encoded in the spectral functions:

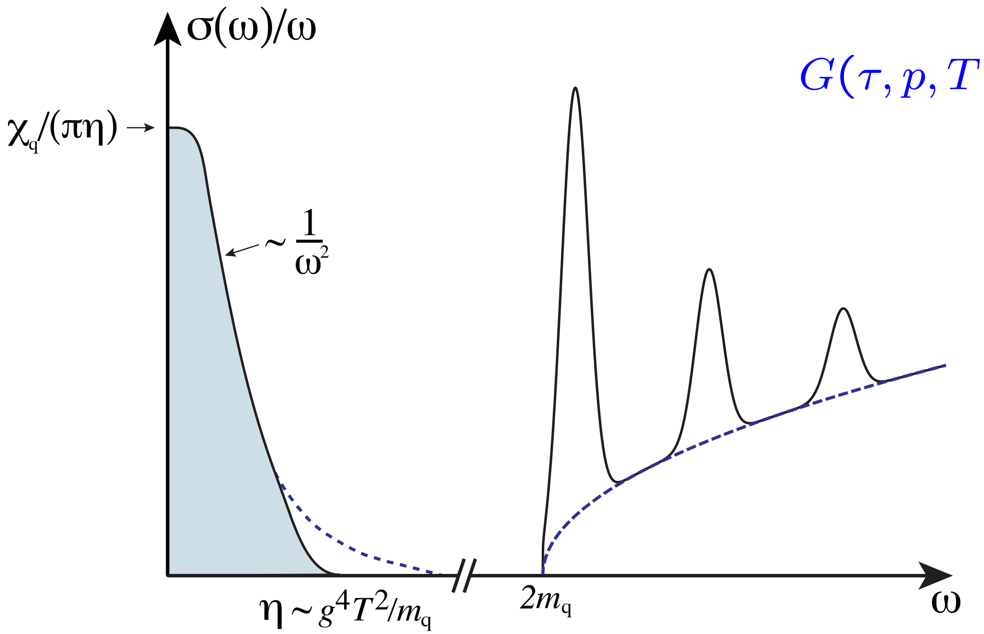
$$\sigma(\omega,p,T) = \frac{1}{2\pi}\,\mathrm{Im}\int_{-\infty}^{\infty} dt e^{i\omega t}\int d^3x e^{ipx}\langle[J(x,t),J(x,0)]\rangle_T$$

In LQCD one can calculated the Euclidean time

$$\text{transport coefficients} = \lim_{\omega\to 0}\frac{\sigma(\omega)}{\omega}$$

$$G(\tau,p,T) = \int d^3x e^{ipx}\langle J(x,-i\tau),J(x,0)\rangle_T$$

Due to analytic continuation $\boxed{G(\tau,T) = D^{>}(-i\tau)}$



$$G(\tau,p,T) = \int_0^{\infty} d\omega\sigma(\omega,p,T)\frac{\cosh(\omega\cdot(\tau-\frac{1}{2T}))}{\sinh(\omega/(2T))}$$

Challenge: resolve a potentially narrow transport peak at zero energy

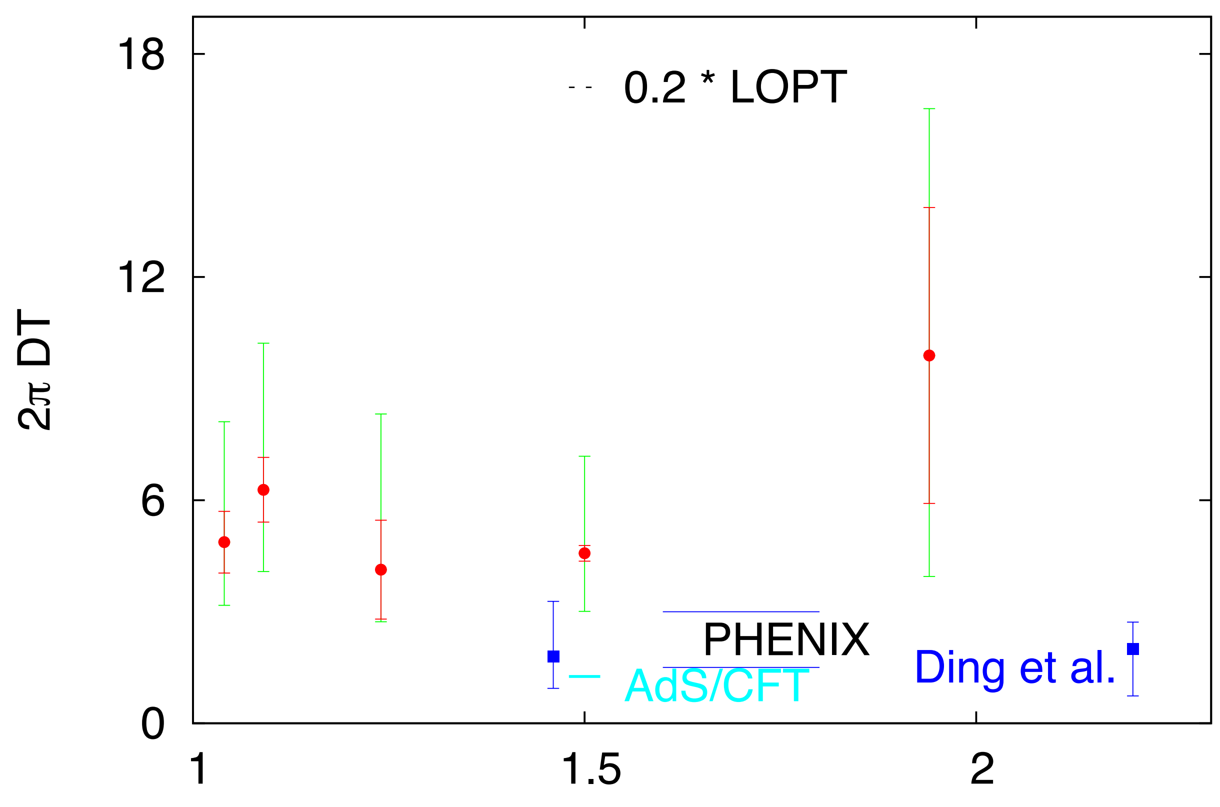
with temporal extent in Euclidean time that is limited by *1/T*

Heavy quark diffusion constant from quenched LQCD

Direct method: determine the width of the transport peak,
Ding et al, arXiv:1204:4954, quenched $128^3 \times N_\tau$ lattices, N_τ=24-48

Integrate out the heavy quark fields: $< J_i(\tau)\, J_i(0) > \Rightarrow <E_i^a(\tau)\, E_i^a\,(0)>$
Banarjee et al, arXiv:1109.5738, Kaczmarek et al, arXiv:1109:3941, N_τ=16-24

Lattice find values of D consistent with experiment and sQGP scenario



the width of
the transport peak is
potentially
overestimated

Summary

- The are compelling questions in hot QCD that require quantum computations:

1) What is the QCD phase diagram at high baryon density ? Is there critical point ?
2) How does thermalization in ultra-relativistic heavy ion collisions happen ?
3) What are the QCD transport coefficients ?

- Quantum simulations using optical lattices might provide an avenue addressing these questions but many open challenges remain