Quantum simulations of models from high energy physics

Christine Muschik

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Quantum Optics Theory

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Quantum Interactions

Postdoc positions available

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How can we we use quantum systems to achieve a **quantum advantage?**

How can this be done **in practice?**

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Quantum Networks

Quantum Simulations

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Quantum Networks

Quantum Simulations

New design concepts for 2D quantum networks

➡ Vision: ‘quantum internet’

Autonomous quantum error correction

Nat. Commun. 8, 1822 (2017).

➡ Vision: self-correcting quantum technology

See also: Work by David Schuster and Eliot Kapit

Quantum Optics Theory

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Quantum Networks

Quantum Simulations

Quantum Simulations
for
High Energy Physics

Use quantum methods to
develop new tools for basic science

We want to understand:

- Why is there more matter than antimatter in the universe?
- What happens inside neutron stars?
- What happened in the early universe?
- What happens in heavy ion collisions in particle accelerators?

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To find answers to these question we need:

New methods for **gauge theories**

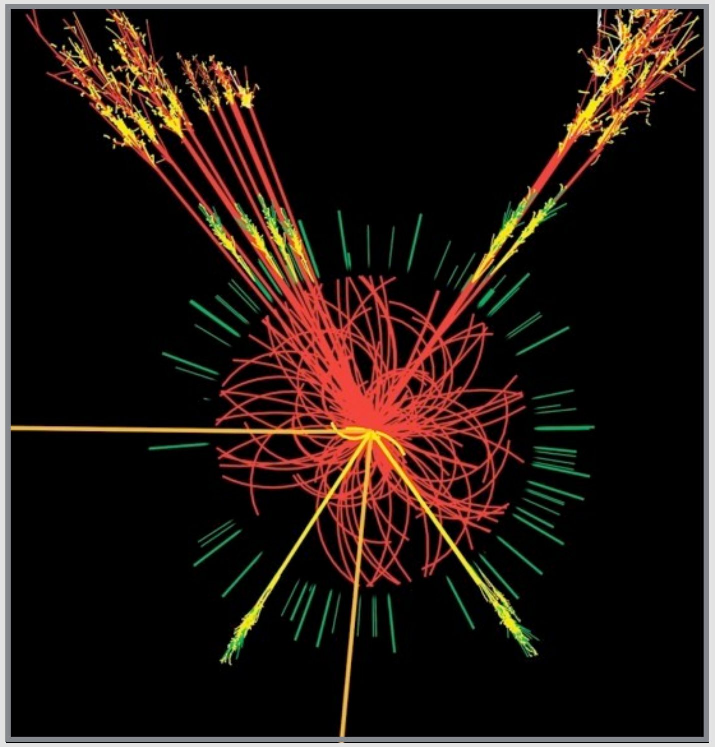
Gauge Theories:

- **underlie our understanding how fundamental particles interact** (for example: Quantum Electrodynamics, Quantum Chromodynamics)
- are the backbone of the **standard model**
- play an important role in many areas of physics, including the description of **condensed matter systems** displaying frustration or topological order

Hard questions in gauge theories
(plagued the sign-problem)

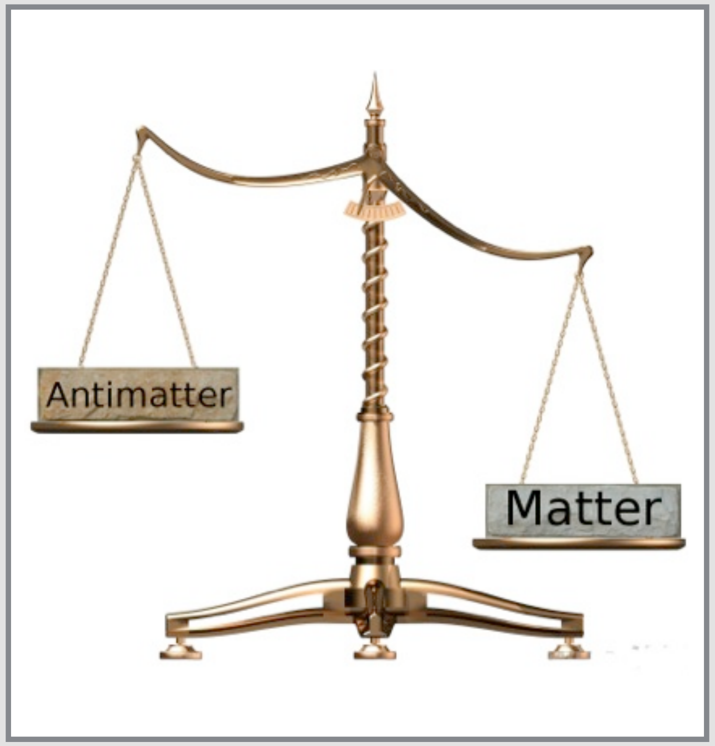
Dynamical problems:
What happens in heavy ion collisions

?



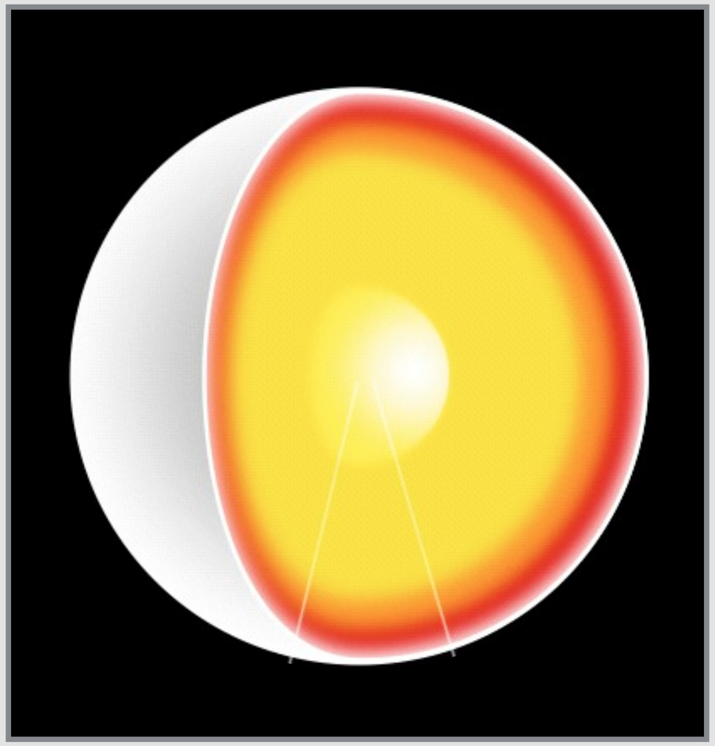
Topological terms:
How can we understand the large degree of CP violation in nature?

?



High baryon density:
What happens inside neutron stars

?



Gauge Theories:

Quest to find sign-problem free methods

- Quantum Simulations
- Numerical methods based on tensor network states

Gauge Theories:

Quest to find sign-problem free methods

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Gauge Theories:

Quest to find sign-problem free methods

- Quantum Simulations
- Numerical methods based on tensor network states

Two routes towards the same goal.
Both paths are actively explored.

This talk: Quantum simulations

Short-term goal:

Develop a new type of
Quantum Simulator

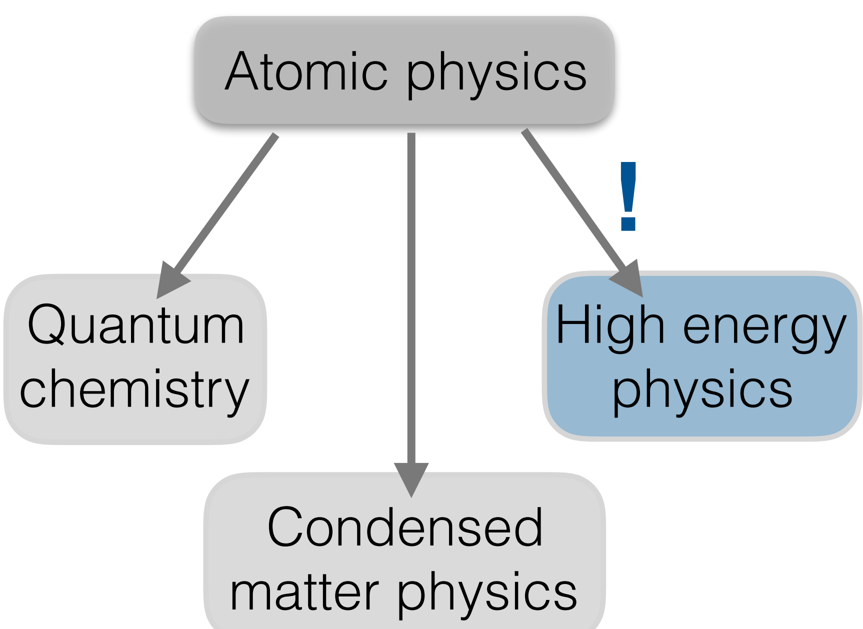
Perform proof-of-concept
Experiments

Long-term vision:

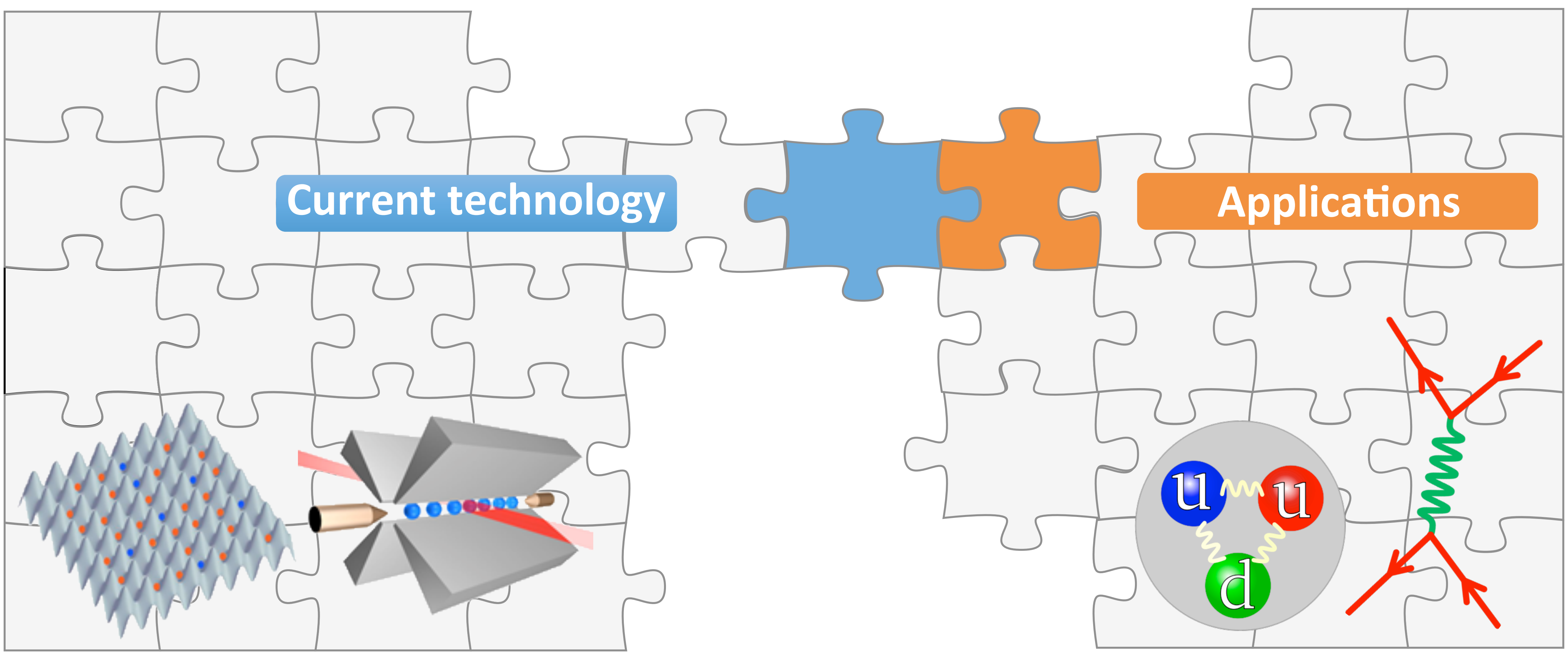
Simulate
Quantum Chromo Dynamics

Answer questions that
can not be tackled
numerically

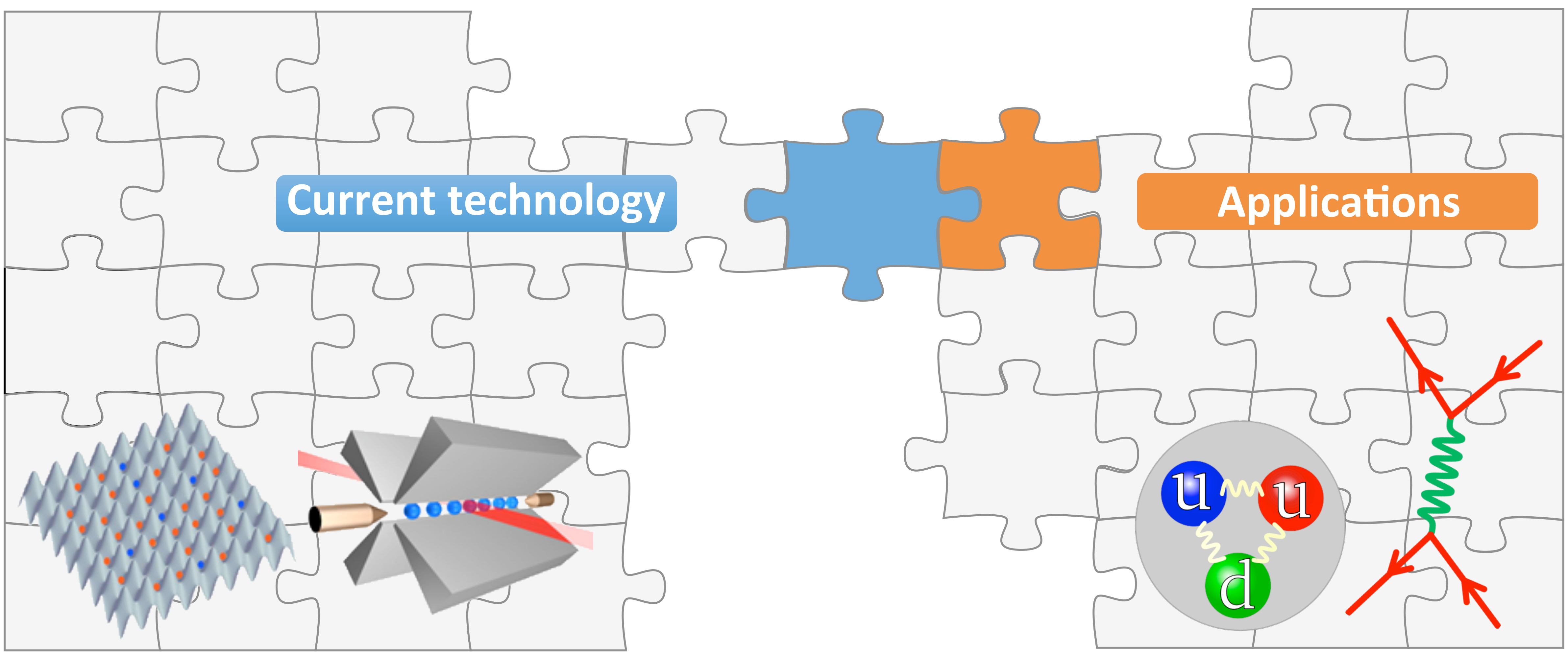
Time



Quantum information science **High energy physics**



Quantum information science **High energy physics**

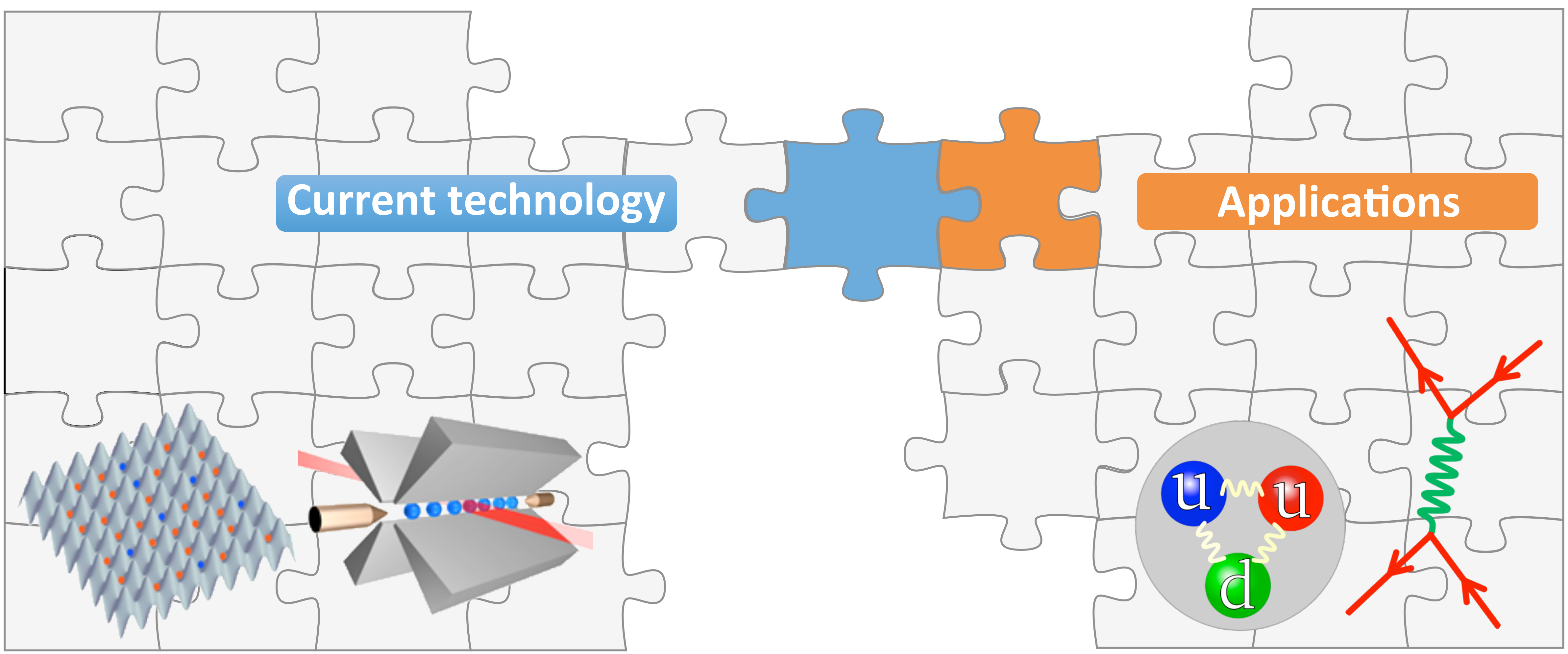


Review by Erez Zohar

Review Articles: Ann. Phys. 525, 777 (2013); Rep. Prog. Phys. 79, 014401 (2016); Contemporary Physics 57 388 (2016).

Quantum information science

High energy physics



E. Martinez et al, Nature 534, 516 (2016).

X. Zhang, et al, Nature Commun. 9, 95 (2018).

N. Klco et al, arXiv:1803.03326 (2018).

New Experiments under way:
Waterloo: Chris Wilson (superconducting qubits)
Heidelberg: Fred Jendrzejewski, Marklus Oberthaler (cold atoms)

Develop a new type of quantum simulator

Review Articles: Ann. Phys. 525, 777 (2013); Rep. Prog. Phys. 79, 014401 (2016); Contemporary Physics 57 388 (2016).

Develop a new type of quantum simulator

Simulated states and dynamics must be gauge-invariant

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Develop a new type of quantum simulator

Simulated states and dynamics must be gauge-invariant

Difficulty for realizing quantum simulations of lattice gauge theories:
Implement a quantum many-body Hamiltonian
and a large set of local constraints (‘Gauss law’, in the case of QED: $\nabla E(r) = \rho(r)$)

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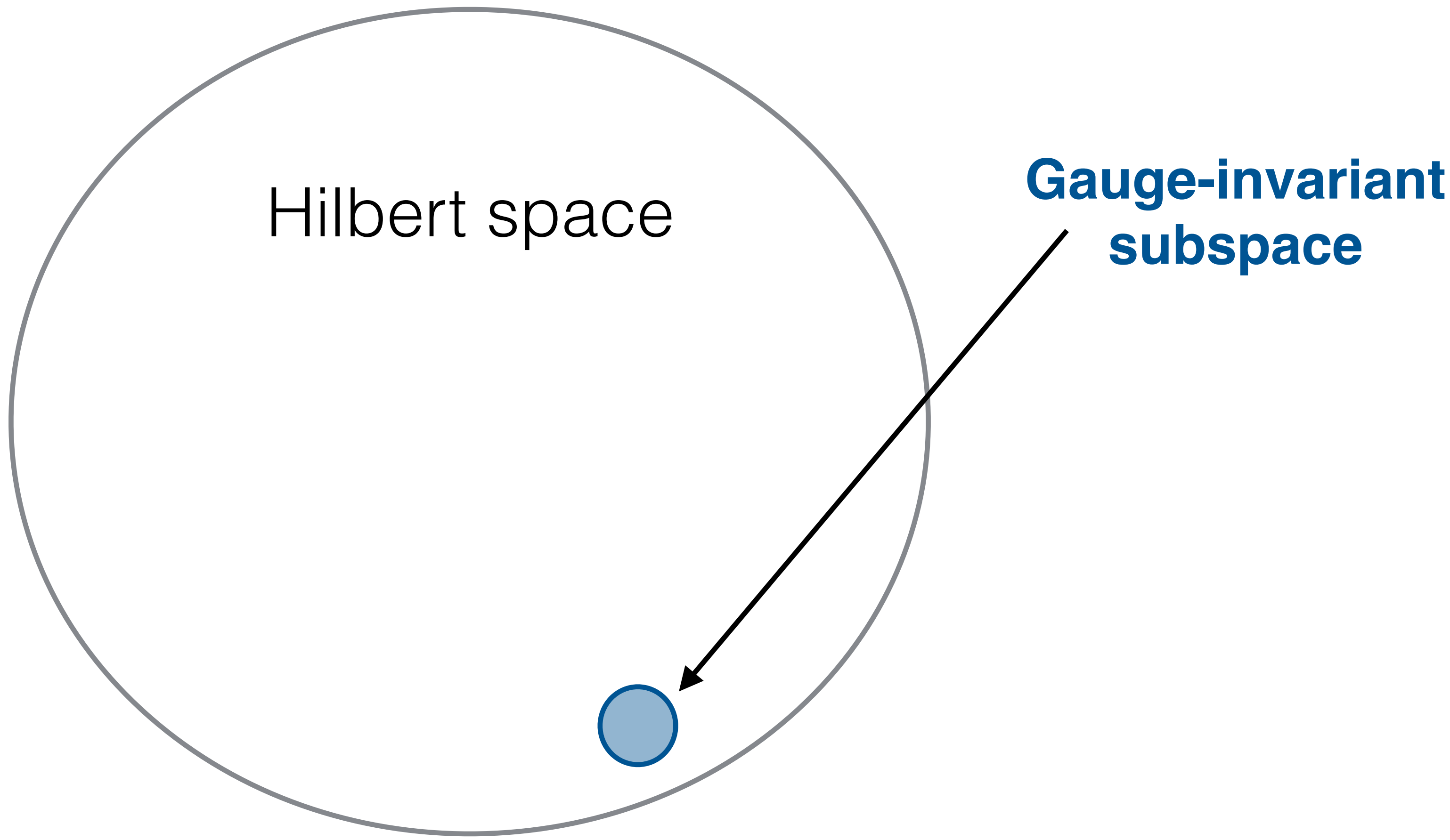


Review Articles: Ann. Phys. 525, 777 (2013); Rep. Prog. Phys. 79, 014401 (2016); Contemporary Physics 57 388 (2016).

QED in (1+1) dimensions

Electromagnetic fields:

Vector potential: $A_0(x), A_1(x)$

Electric field: $E(x) = \partial_0 A_1(x)$

$[E(x), A_1(x')] = -i\delta(x - x')$

Matter fields:

$$\Psi(x) = \begin{pmatrix} \Psi_1(x) \\ \Psi_2(x) \end{pmatrix}$$

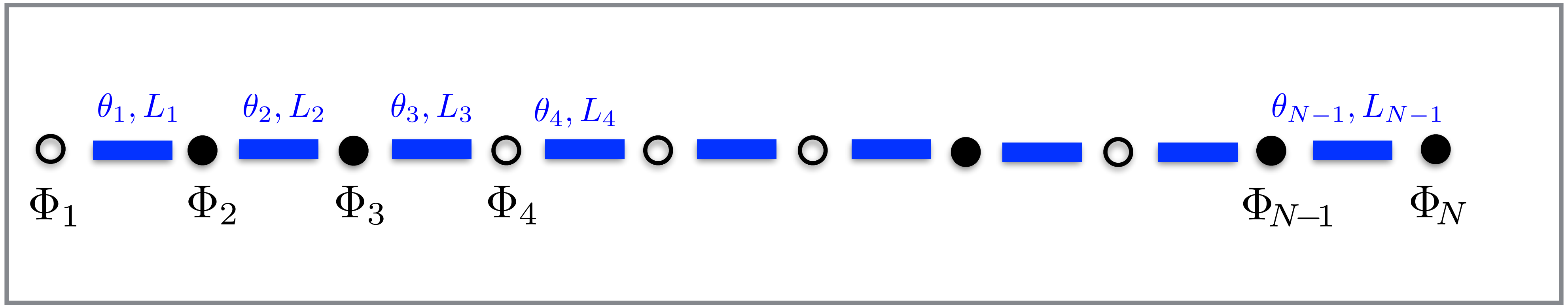
Hamiltonian:

$$H_{\text{cont}} = \int dx \left[-i\Psi^\dagger(x)\gamma^1 \left(\delta_1 - igA_1\right) \Psi(x) + m\Psi^\dagger(x)\Psi(x) + \frac{1}{2}E^2(x) \right]$$

$\gamma_1 = -i\sigma_y$ coupling strength (charge) Fermion mass

The lattice Schwinger Model

The lattice Schwinger Model



Continuum

Vector potential $A_1(x)$

Electric field $E(x)$

$[E(x), A_1(x')] = -i\delta(x - x')$

Dirac spinor

$$\Psi(x) = \begin{pmatrix} \Psi_1(x) \\ \Psi_2(x) \end{pmatrix}$$

Lattice

$\theta_n = agA_1(x_n)$

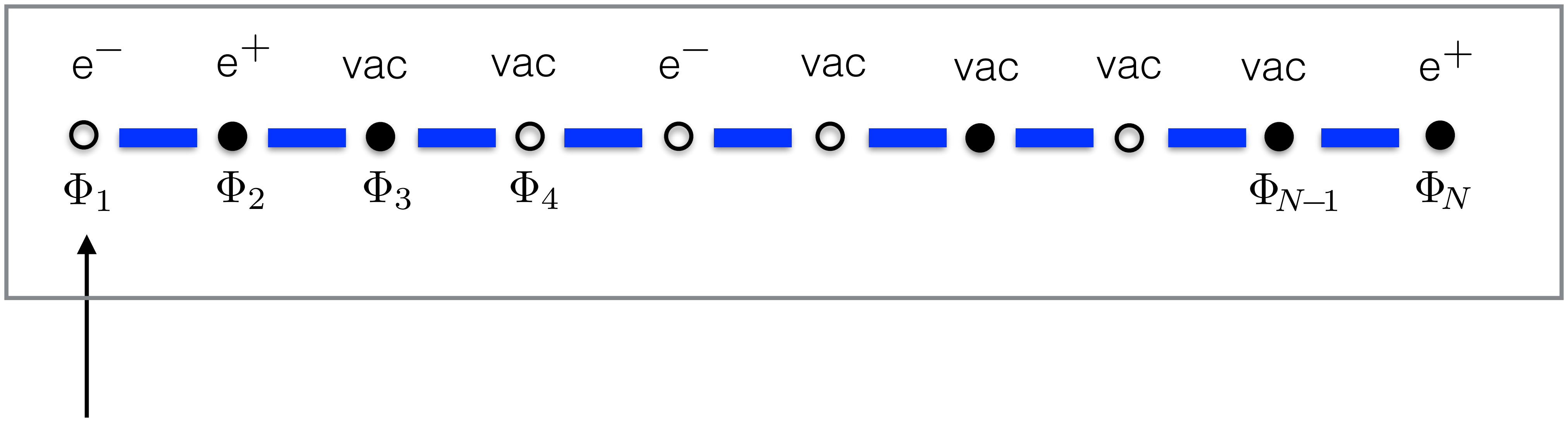
$L_n = \dfrac{1}{g}E(x_n)$

$[\theta_n, L_m] = i\delta_{n,m}$

odd lattice sites:
$\Phi_n = \sqrt{a}\Psi_1(x_n)$

even lattice sites:
$\Phi_n = \sqrt{a}\Psi_2(x_n)$

Wilson's staggered Fermions



one-component fermion fields

<u>odd sites:</u>

● $\cong$ vac

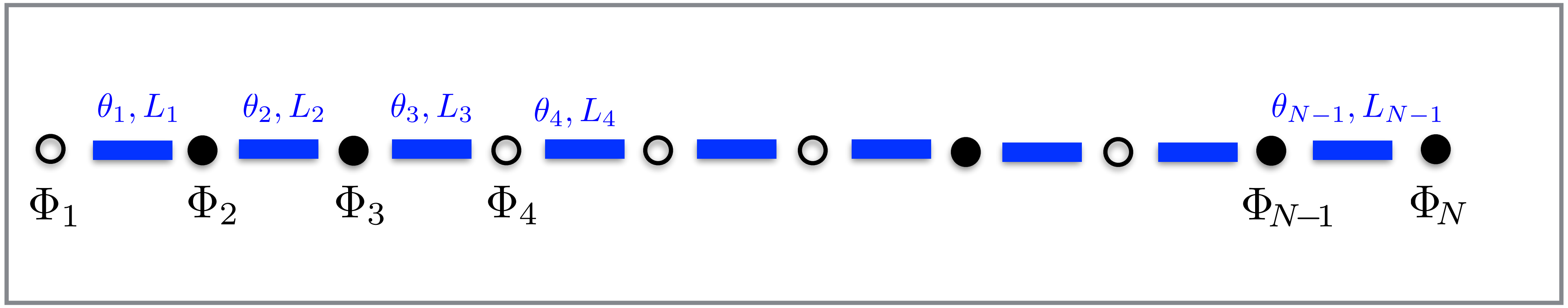
○ $\cong$ e$^-$

<u>even sites:</u>

● $\cong$ e$^+$

○ $\cong$ vac

The lattice Schwinger Model



Continuum

$$H_{\text{cont}} = \int dx \left[-i\Psi^\dagger(x)\gamma^1 \left(\delta_1 - igA_1\right)\Psi(x) + m\Psi^\dagger(x)\Psi(x) + \frac{1}{2}E^2(x) \right]$$

Lattice

$$H_{\text{lat}} = \quad -iw \sum_{n=1}^{N-1} \left[\Phi_n^\dagger e^{i\theta_n} \Phi_{n+1} - H.C. \right] + m \sum_{n=1}^{N} (-1)^n \Phi_n^\dagger \Phi_n + J \sum_{n=1}^{N-1} L_n^2$$

$$w = \frac{1}{2a} \qquad\qquad J = \frac{g^2 a}{2}$$

J. Kogut and L. Susskind, Phys. Rev. D 11, 395 (1975).

Hamiltonian formulation of the Schwinger model:

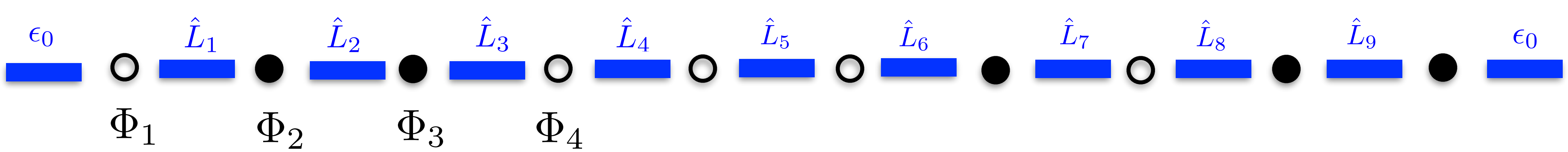
J. Kogut and L. Susskind, Phys. Rev. D 11, 395 (1975).

$$\hat{H} = -iw \sum_{n=1}^{N-1} \left[\hat{\Phi}_n^\dagger e^{i\hat{\theta}_n} \hat{\Phi}_{n+1} - \text{H.c.}\right] + J \sum_{n=1}^{N-1} \hat{L}_n^2 + m \sum_{n=1}^{N} (-1)^n \hat{\Phi}_n^\dagger \hat{\Phi}_n$$

The dynamics is constraint by the Gauss law:

In the continuum in 3D: $\nabla E = \rho$

Here: $\hat{L}_n - \hat{L}_{n-1} = \hat{\Phi}_n^\dagger \hat{\Phi} - \frac{1}{2}\left[1 - (-1)^n\right]$



Local (gauge) symmetries

Local symmetry generators: $\{G_n\}$

The Hamiltonian is invariant under gauge transformations of the form:

$$H' = \left(\Pi_n e^{i\alpha_n G_n}\right)\ H\ \left(\Pi_n e^{-i\alpha_n G_n}\right) \qquad [H, G_n] = 0$$

For 1D QED: $G_n = L_n - L_{n-1} - \Phi^\dagger \Phi - \frac{1}{2}\left[1 - (-1)^n\right]$

The Hamiltonian does not mix eigenstates of G_n with different eigenvalues λ_n.

In the following, we restrict ourselves to the zero-charge subsector: $\lambda_{G_n} = 0,\ \forall n$
(# of particles = # of antiparticles).

$$G_n |\Psi_{\text{physical}}\rangle = 0 \qquad \forall n$$

Real time dynamics in lattice gauge theories with a trapped ion computer

Theory:
C. Muschik, M. Heyl, M. Dalmonte, P. Hauke, and P. Zoller

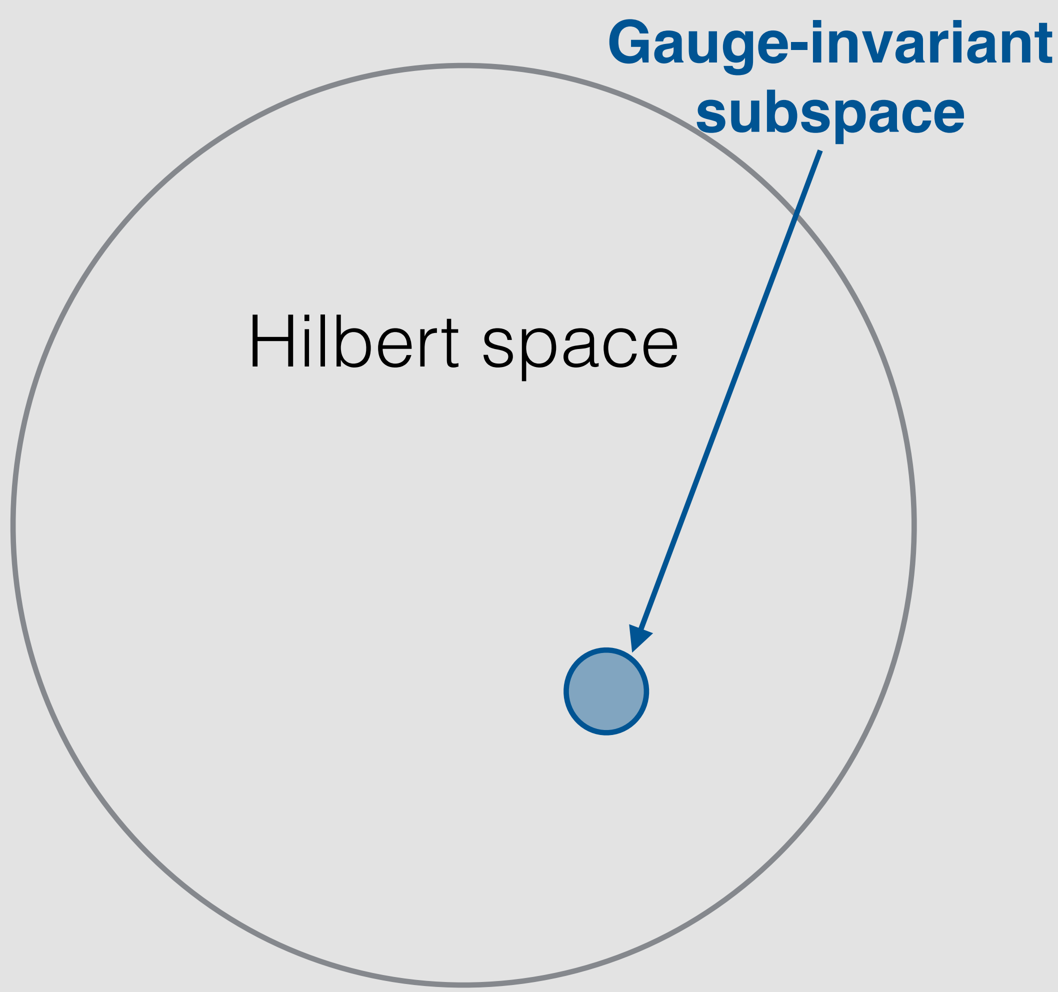
Experiment:
E. Martinez, P. Schindler, D. Nigg, A. Erhard, T. Monz, and R. Blatt

Nature 534, 516-519 (2016).

NJP 19, 103020 (2017).

Previous approaches:

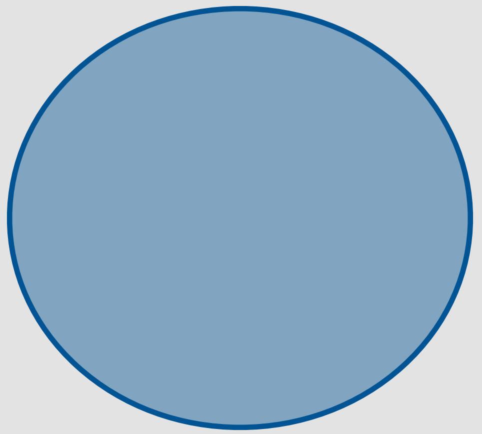
Introduce the full Hilbert space [matter + gauge fields] and enforce constraints



Encoding approach:

Schwinger model: A given matter configuration and choice of background field completely determines the gauge degrees of freedom.

Elimination of the gauge fields results in a pure matter model with long-range interactions



Ideal case: exact gauge invariance by construction (on all energy scales).

C. Hamer, Z. Weihong, and J. Oitmaa, Phys. Rev. D **56** 55 (1997).

Encoding

Elimination of the gauge fields $\longrightarrow$ **Pure spin model with long-range interactions**
(+ Jordan Wigner transformation)

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Elimination of the gauge fields ⟶ **Pure spin model with long-range interactions**
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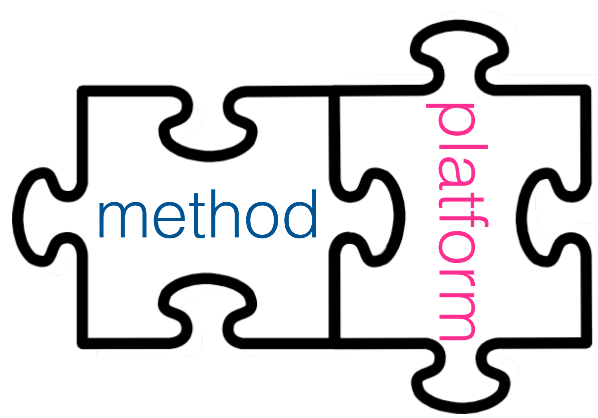
The gauge fields don't appear explicitly in the encoded description. Instead, they act in the form of a non-local interaction.

The required long-range interactions can be realised efficiently in a robust digital scheme in a trapped ion quantum computer.



Efficient implementation

Two simple transformations:

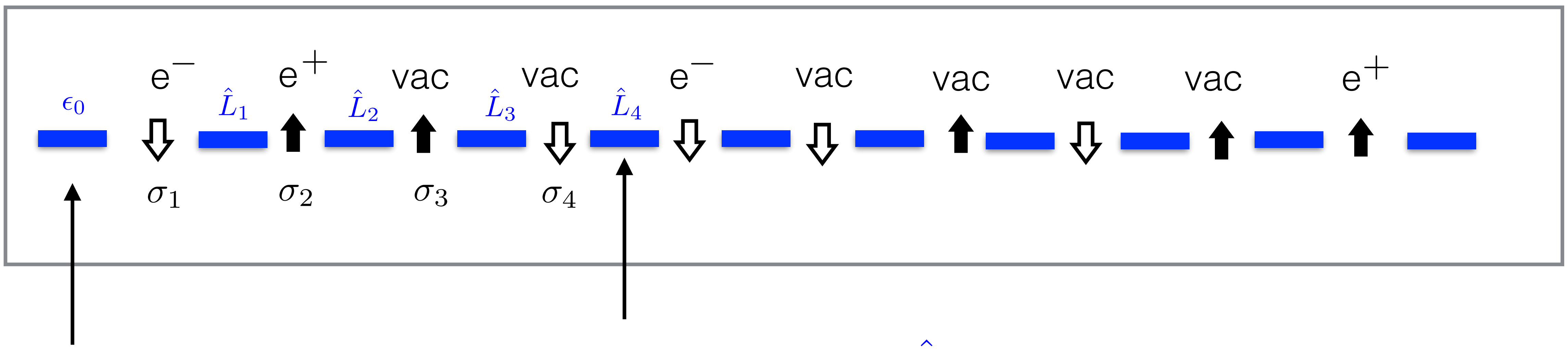
(1) Fermions —> spins $\quad \Phi_n = \prod_{l<n} [i\sigma_l^z]\, \sigma_n^-$

(2) Elimination of $\hat{\theta}_n$ $\quad \hat{\sigma}_n^- \rightarrow \prod_{l<n} \left[e^{-i\hat{\theta}_l}\right] \hat{\sigma}_n^-$

Hamiltonian in terms of spins and electric fields

Transformed Hamiltonian:

$$\hat{H} = w \sum_{n=1}^{N-1} \left[\hat{\sigma}_n^+ \hat{\sigma}_{n+1}^- + \mathrm{H.c.} \right] + J \sum_{n=1}^{N-1} \hat{L}_n^2 + m \sum_{n=1}^{N} (-1)^n \hat{\sigma}_n^z$$



background field

The operators $\hat{L}_n$ represent the electric fields on the links.
They take eigenvalues $\hat{L}_n = 0, \pm 1, \pm 2, \pm 3 ...$

Odd lattice sites:

$\underset{n}{\bullet} \cong \underset{n}{\uparrow} \cong$ vac $\quad L_n = L_{n-1}$

$\underset{n}{\circ} \cong \underset{n}{\Downarrow} \cong$ e$^-$ $\quad L_n = L_{n-1} - 1$

Even lattice sites:

$\underset{n}{\bullet} \cong \underset{n}{\uparrow} \cong$ e$^+$ $\quad L_n = L_{n-1} + 1$

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Transformed Hamiltonian:

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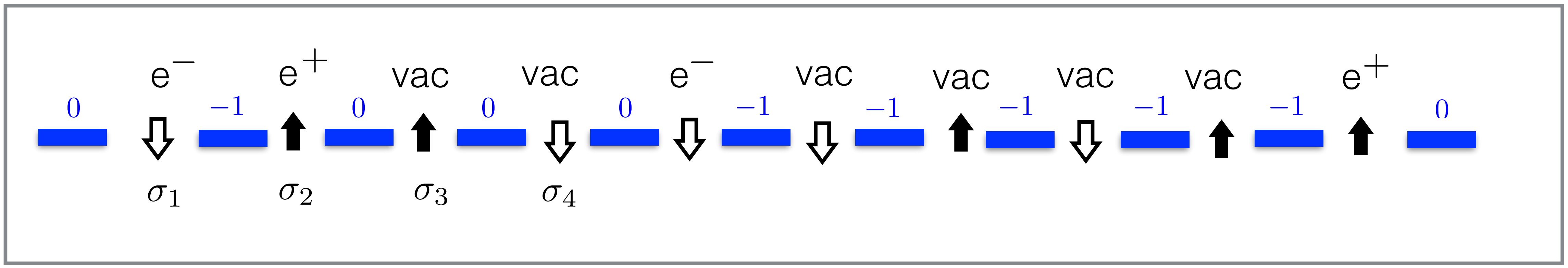

A given configuration of spins and choice of background field completely determines the gauge degrees of freedom.

Odd lattice sites:

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Transformed Gauss law:

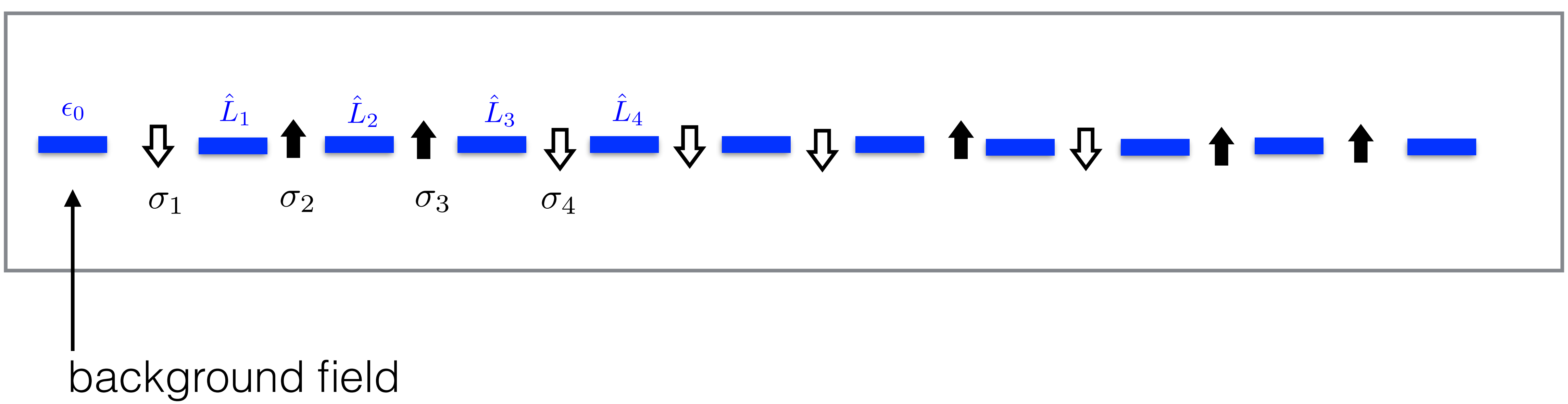
$$\hat{L}_n - \hat{L}_{n-1} = \frac{1}{2}\left[\hat{\sigma}_n^z + (-1)^n\right]$$

Transformed Hamiltonian:

$$\hat{H} = w \sum_{n=1}^{N-1} \left[\hat{\sigma}_n^+ \hat{\sigma}_{n+1}^- + \mathrm{H.c.}\right] + \boxed{J \sum_{n=1}^{N-1} \hat{L}_n^2} + m \sum_{n=1}^{N} (-1)^n \hat{\sigma}_n^z$$

$$\boxed{+J \sum_{n=1}^{N-1} \left[\epsilon_0 + \frac{1}{2} \sum_{m=1}^{n} \left[\hat{\sigma}_m^z + (-1)^m\right]\right]^2}$$

$$\epsilon_0 = 0 \qquad \hat{L}_n - \hat{L}_{n-1} = \frac{1}{2}\left[\hat{\sigma}_n^z + (-1)^n\right]$$



Elimination of the gauge fields $\Rightarrow$ **Pure spin model with long-range interactions**

The gauge fields don't appear explicitly in the encoded description. Instead, they act in the form of a non-local interaction that corresponds to the Coulomb-interaction between the simulated charged particles.

The Schwinger model as exotic spin model

$$\hat{H}_S = w\sum_i (\hat{\sigma}_i^+\hat{\sigma}_{i+1}^- + \hat{\sigma}_{i+1}^+\hat{\sigma}_i^-)$$

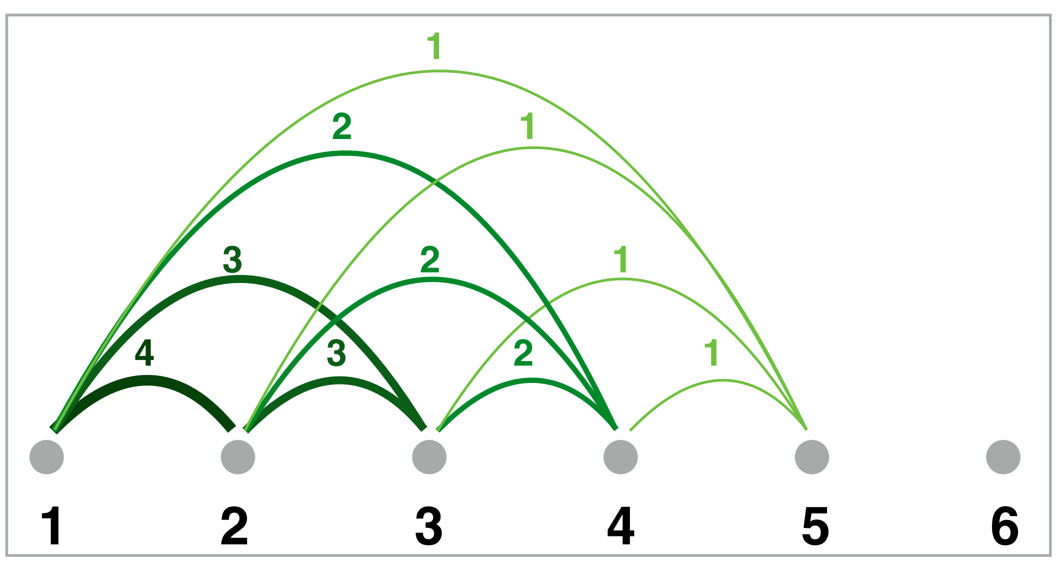
particle - antiparticle creation/annihilation

$$+ \boxed{J\sum_{i<j} c_{ij}\hat{\sigma}_i^z\hat{\sigma}_j^z}$$

long - range interaction

$$+ m\sum_i c_i\hat{\sigma}_i^z + J\sum_i \tilde{c}_i\hat{\sigma}_i^z$$

effective particle masses



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effective particle masses

- Efficient implementation on an ion-quantum computer
- N spins simulate N matter fields and N-1 gauge fields

Quantum simulation of 1+1-dimensional QED on a lattice

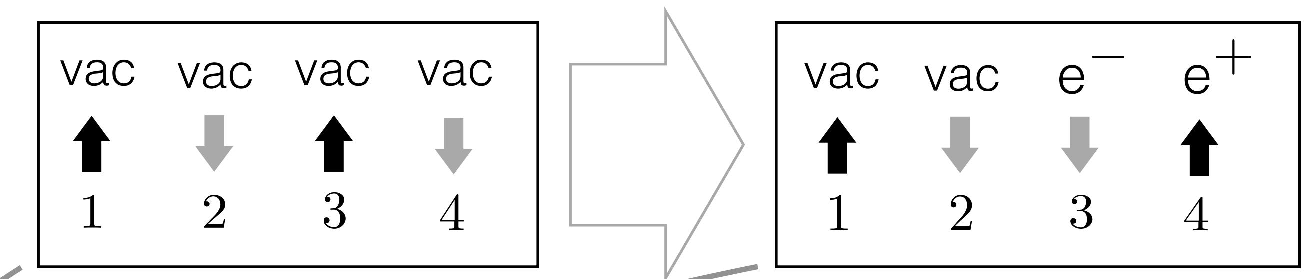
We explore:

- Coherent real-time dynamics of particle-antiparticle creation
- Entanglement generation during pair creation

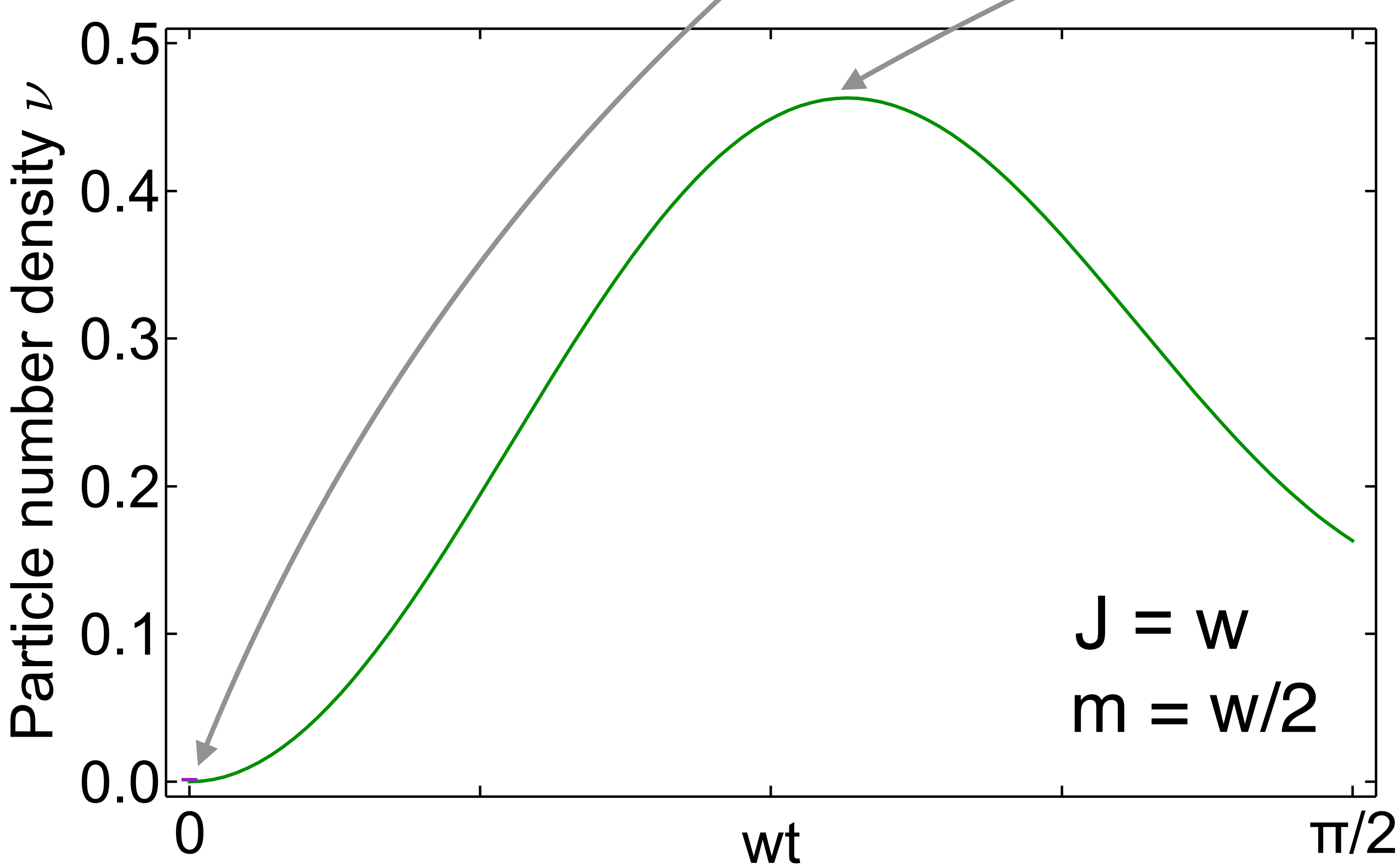
Schwinger Mechanism

Particle number density: $\nu(t) = \frac{1}{N}\sum_{n=1}^{N}\langle(-1)^n\sigma_n^z(t)+1\rangle$

Creation of a particle antiparticle pair:



In the ideal case (N=4):



$$\hat{H}_S = w\sum_i\left(\hat{\sigma}_i^+\hat{\sigma}_{i+1}^- + \hat{\sigma}_{i+1}^+\hat{\sigma}_i^-\right)$$

particle - antiparticle creation/annihilation

$$+\ J\sum_{i<j} c_{ij}\hat{\sigma}_i^z\hat{\sigma}_j^z$$

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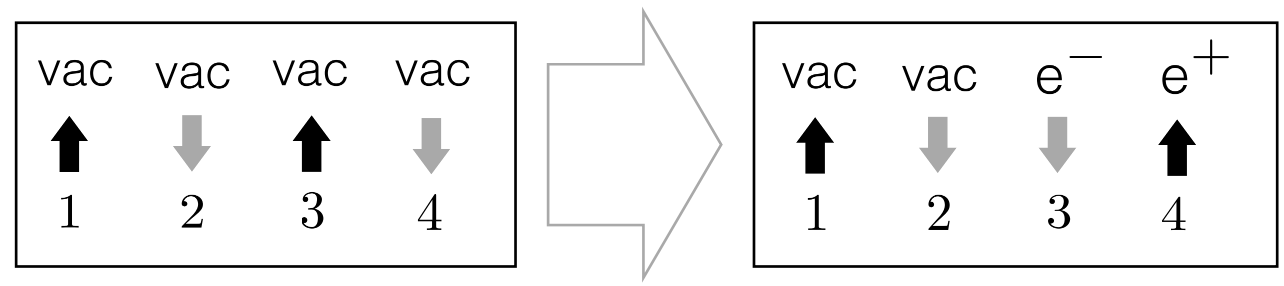
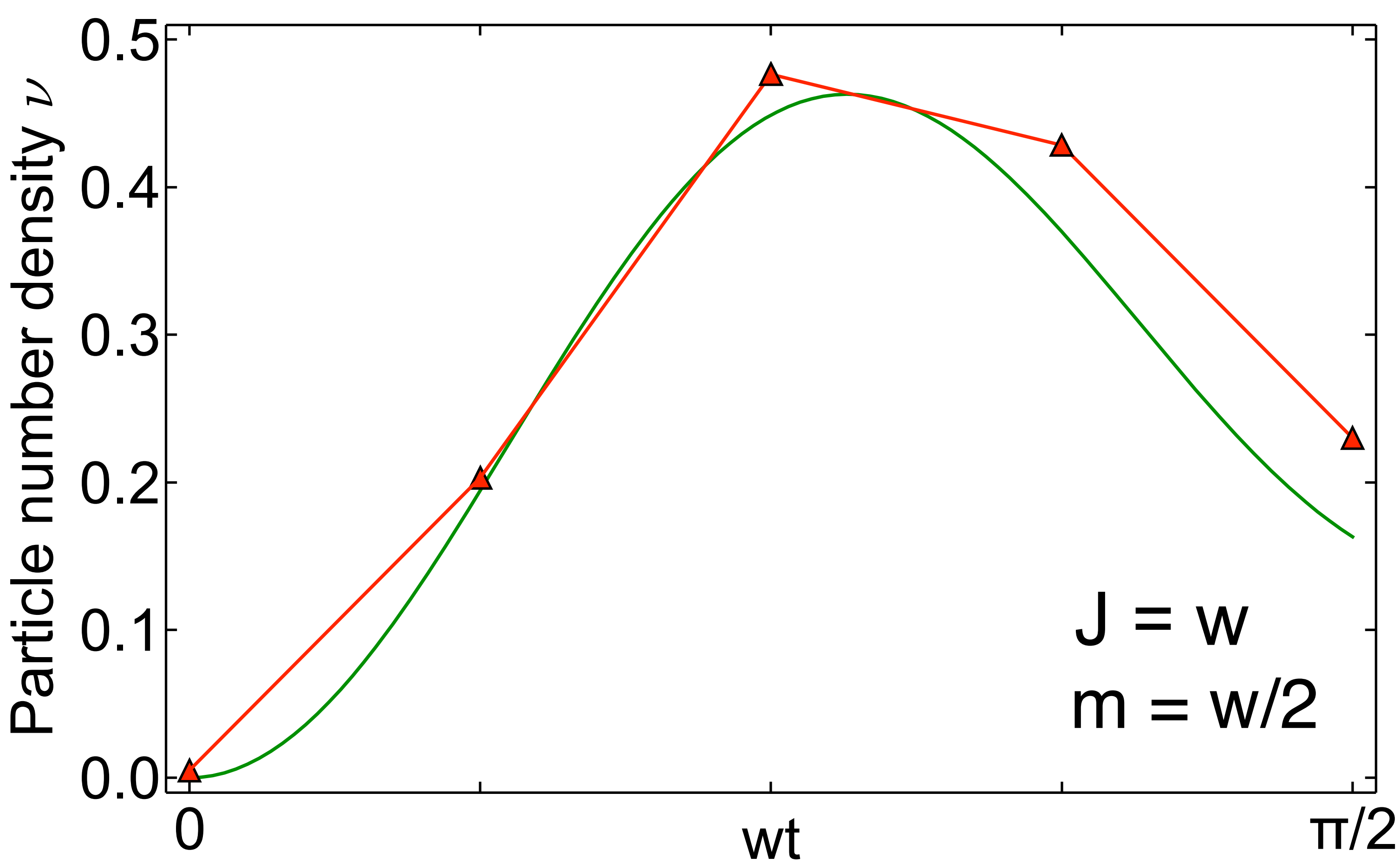


Including discretisation errors (N=4):



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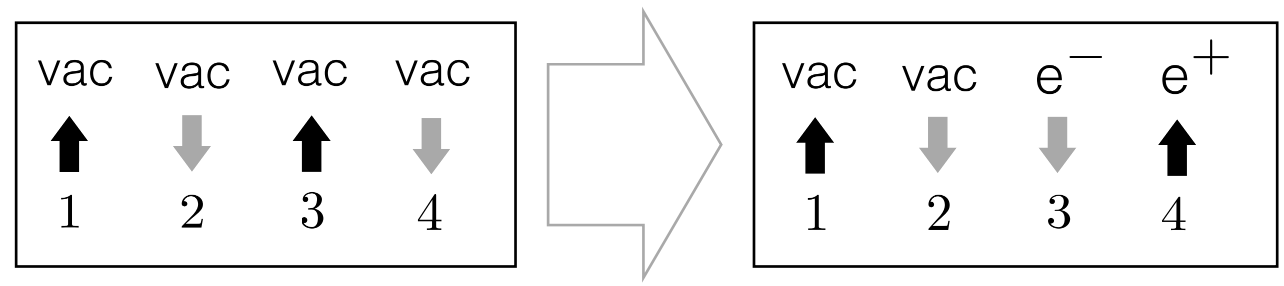
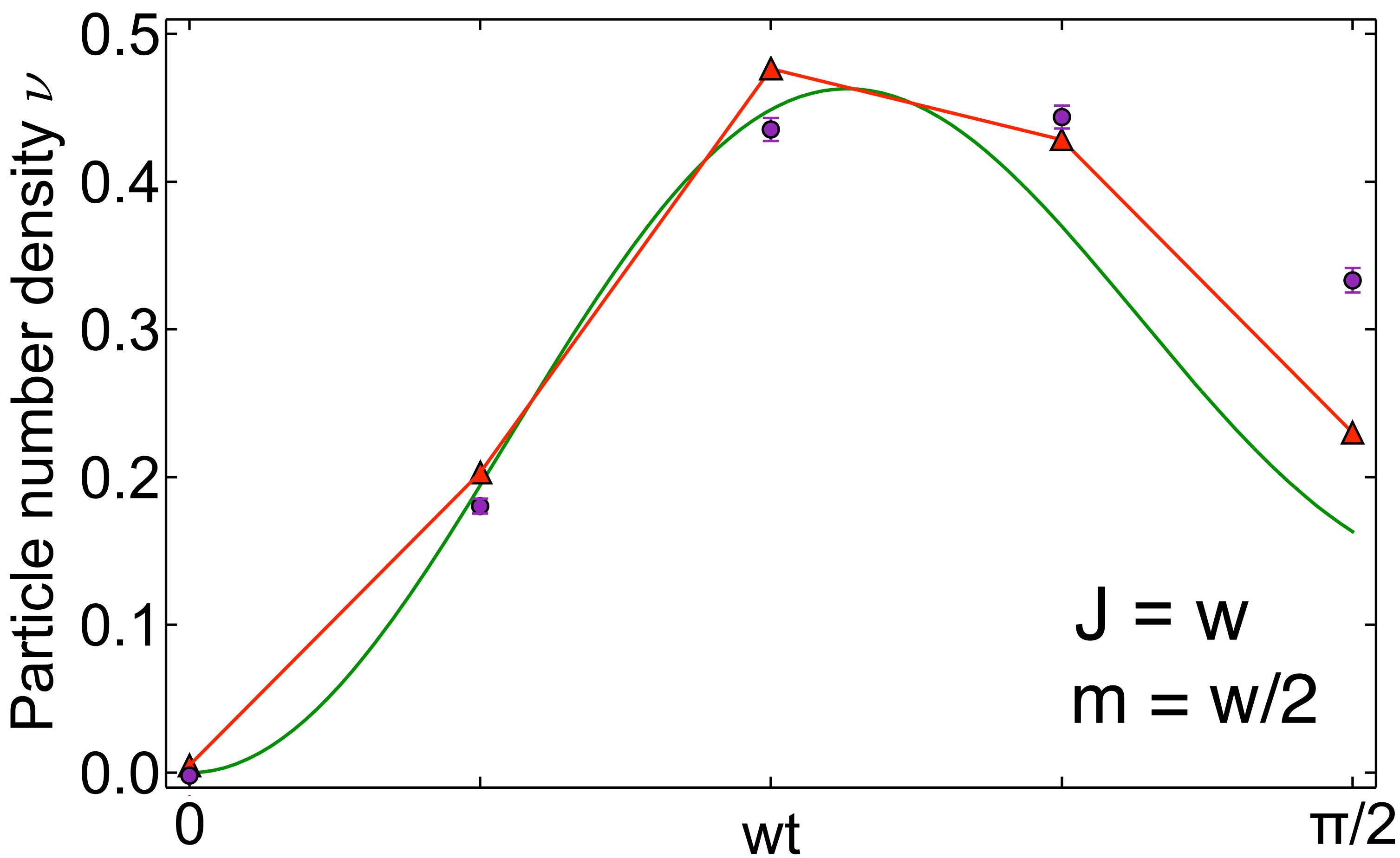


Experimental data (after postselection):



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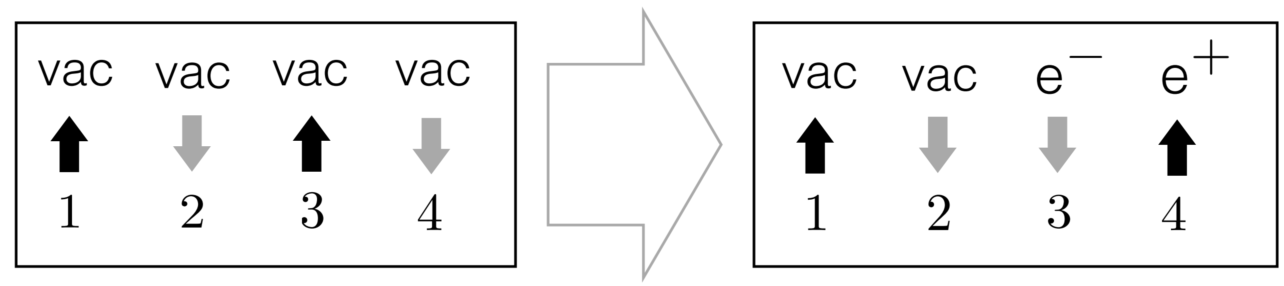
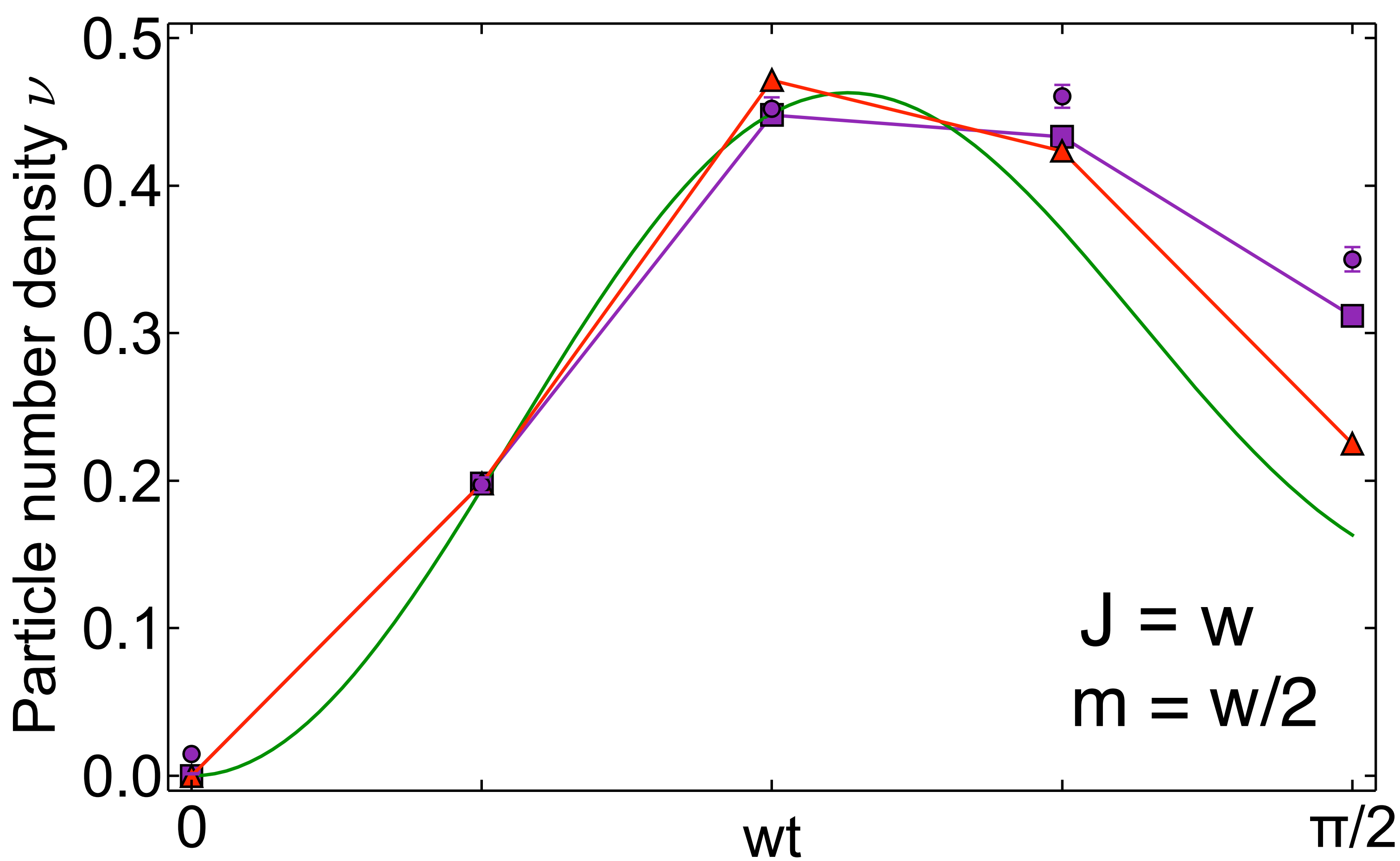
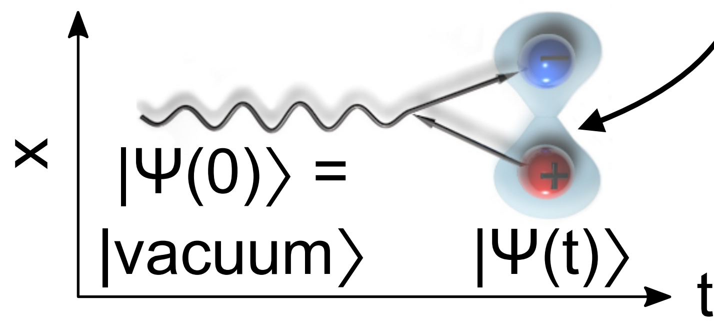


Simple error model (uncorrelated dephasing):

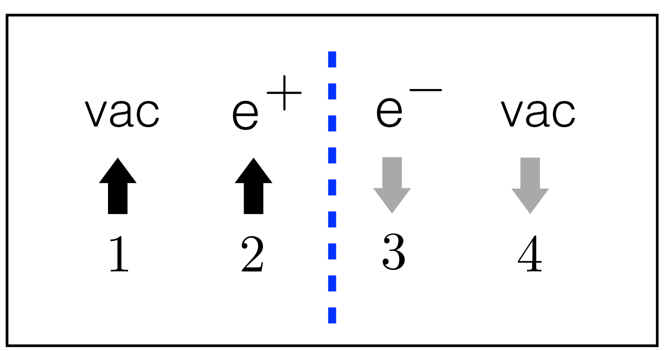


Entanglement in the Schwinger mechanism

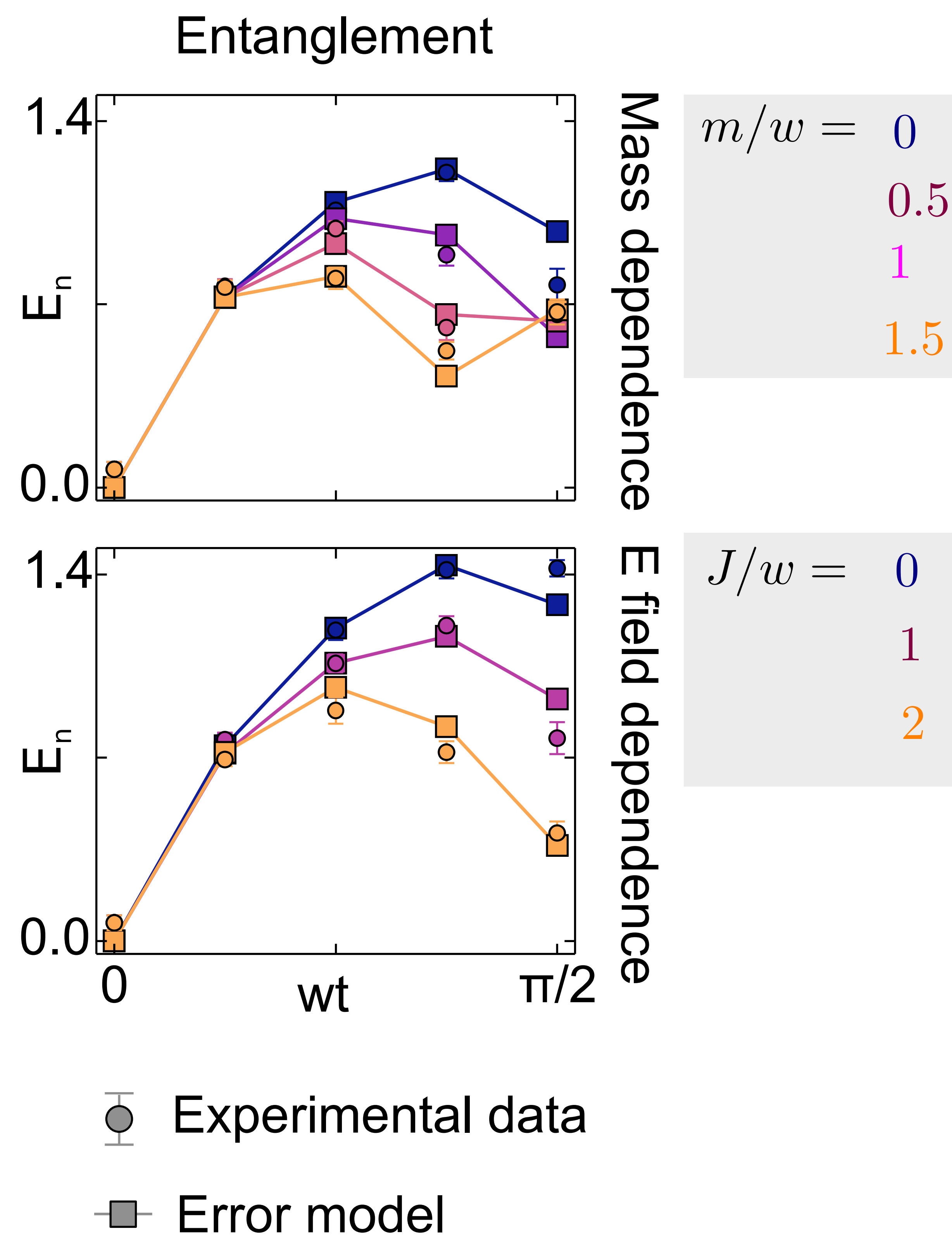


State tomography:
access to the full density matrix

E_n : logarithmic negativity
evaluated with respect to this bipartition:



Entanglement between the two halves of the system.



Next challenges:

- Realisation of 2D models
- Simulate increasingly complex dynamics
- Realisation of non-Abelian theories
- …

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CANADA FIRST RESEARCH EXCELLENCE FUND

ARL

Thank you very much
for your attention!

Variational Quantum Simulation

Controllable
Quantum
System

Mode A:
Trotter Simulator

Time evolutions

$e^{-iH_T t}$

Quench dynamics
Real-time dynamics

Mode B:
Variational Simulator

Ground state preparation

$|\Psi_0\rangle_{H_T}$

Equilibrium physics
Ground state properties

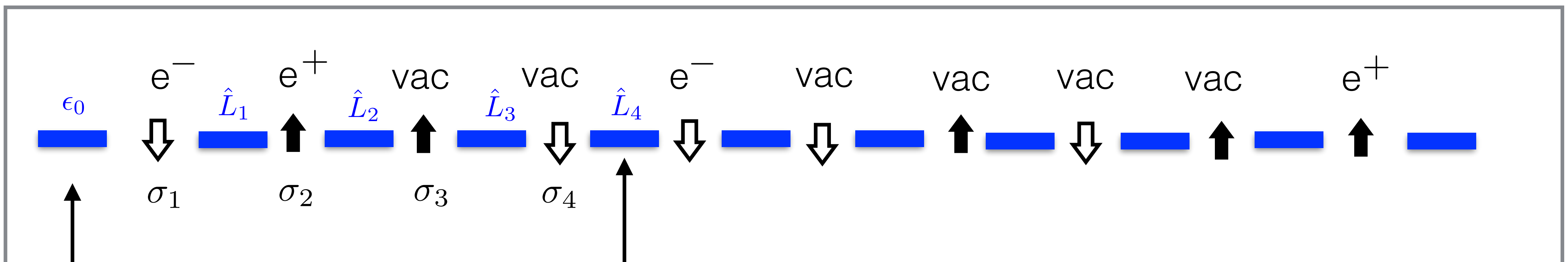
Combined application, e.g.:

(1) Prepare the true vacuum of the Schwinger mode A

(2) Perform real-time dynamics in mode B

Transformed Hamiltonian:

$$\hat{H} = w \sum_{n=1}^{N-1} \left[\hat{\sigma}_n^+ \hat{\sigma}_{n+1}^- + \mathrm{H.c.} \right] + J \sum_{n=1}^{N-1} \hat{L}_n^2 + m \sum_{n=1}^{N} (-1)^n \hat{\sigma}_n^z$$



background field

The operators $\hat{L}_n$ represent the electric fields on the links.
They take eigenvalues $\hat{L}_n = 0, \pm 1, \pm 2, \pm 3 ...$

Odd lattice sites:

$\underset{n}{\bullet} \cong \underset{n}{\uparrow} \cong$ vac $\quad L_n = L_{n-1}$

$\underset{n}{\circ} \cong \underset{n}{\Downarrow} \cong$ e$^-$ $\quad L_n = L_{n-1} - 1$

Even lattice sites:

$\underset{n}{\bullet} \cong \underset{n}{\uparrow} \cong$ e$^+$ $\quad L_n = L_{n-1} + 1$

$\underset{n}{\circ} \cong \underset{n}{\Downarrow} \cong$ vac $\quad L_n = L_{n-1}$

Transformed Hamiltonian:

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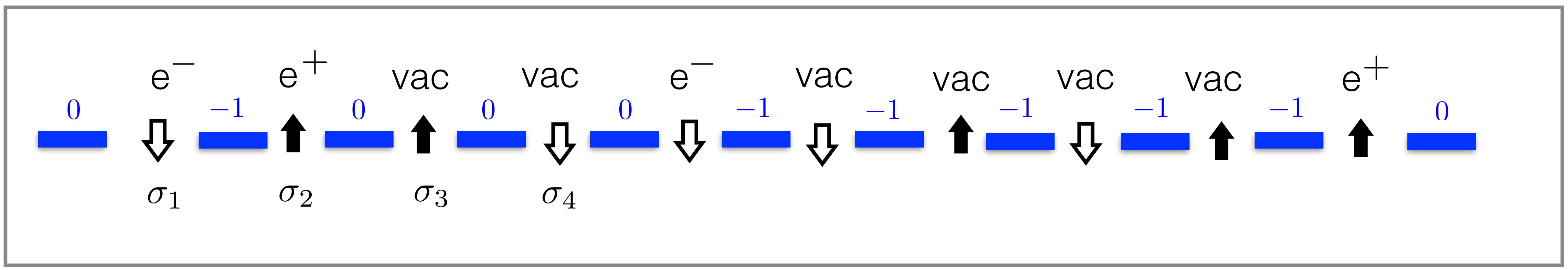

A given configuration of spins and choice of background field completely determines the gauge degrees of freedom.

Odd lattice sites:

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Transformed Gauss law:

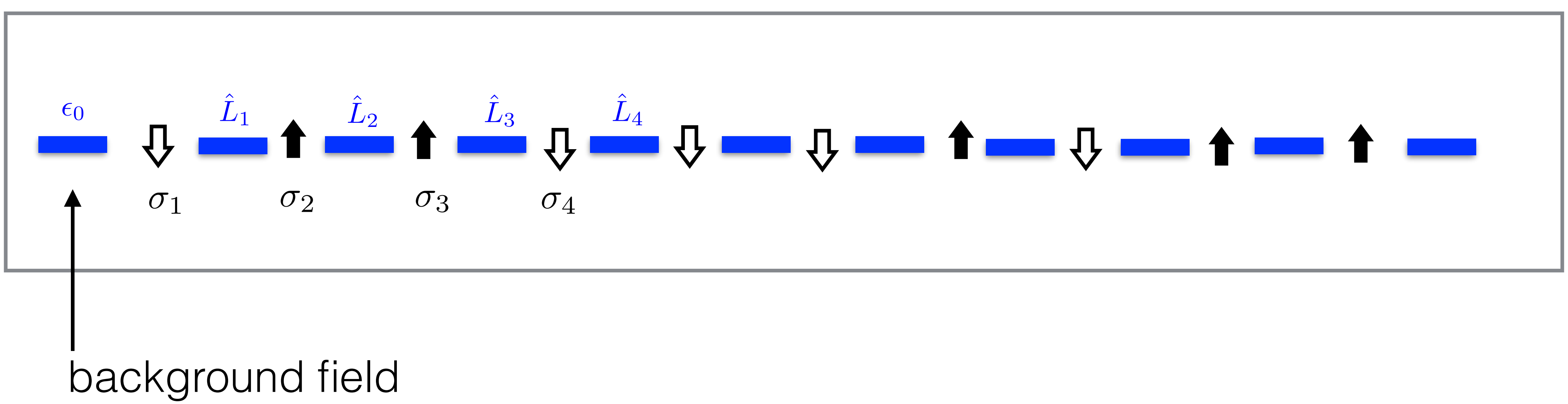
$$\hat{L}_n - \hat{L}_{n-1} = \frac{1}{2} \left[\hat{\sigma}_n^z + (-1)^n \right]$$

Transformed Hamiltonian:

$$\hat{H} = w \sum_{n=1}^{N-1} \left[\hat{\sigma}_n^+ \hat{\sigma}_{n+1}^- + \text{H.c.} \right] + J \sum_{n=1}^{N-1} \hat{L}_n^2 + m \sum_{n=1}^{N} (-1)^n \hat{\sigma}_n^z$$

$$+J \sum_{n=1}^{N-1} \left[\epsilon_0 + \frac{1}{2} \sum_{m=1}^{n} [\hat{\sigma}_m^z + (-1)^m] \right]^2$$

$$\epsilon_0 = 0 \qquad \hat{L}_n - \hat{L}_{n-1} = \frac{1}{2} [\hat{\sigma}_n^z + (-1)^n]$$



Elimination of the gauge fields $\rightarrow$ **Pure spin model with long-range interactions**

The gauge fields don't appear explicitly in the encoded description. Instead, they act in the form of a non-local interaction that corresponds to the Coulomb-interaction between the simulated charged particles.

The Schwinger model as exotic spin model

$$\hat{H}_S = w\sum_i(\hat{\sigma}_i^+\hat{\sigma}_{i+1}^- + \hat{\sigma}_{i+1}^+\hat{\sigma}_i^-)$$

particle - antiparticle creation/annihilation

$$+\ J\sum_{i<j} c_{ij}\hat{\sigma}_i^z\hat{\sigma}_j^z$$

long - range interaction

$$+\ m\sum_i c_i\hat{\sigma}_i^z + J\sum_i \tilde{c}_i\hat{\sigma}_i^z$$

effective particle masses

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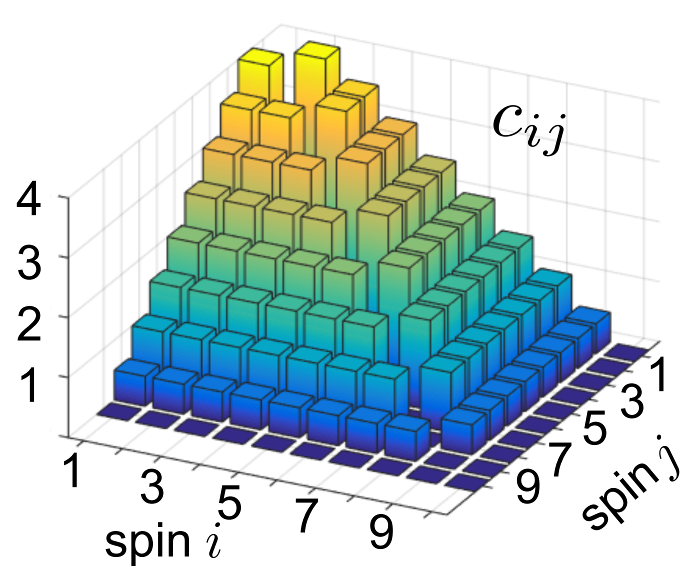

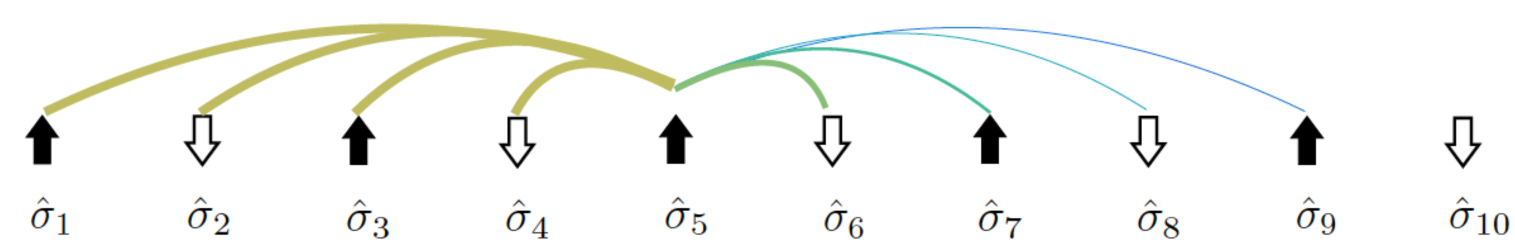

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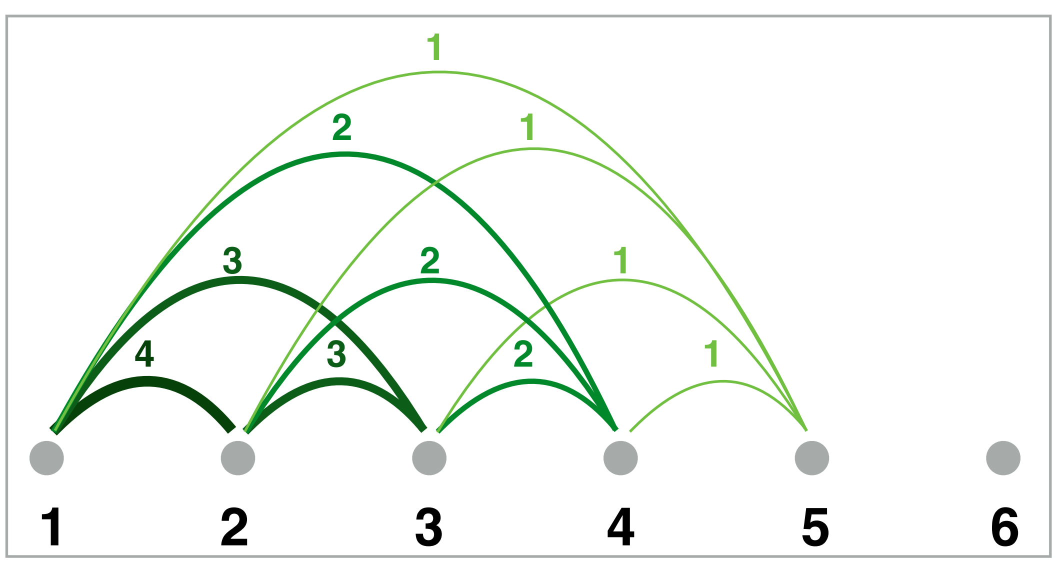

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effective particle masses

- Efficient implementation on an ion-quantum computer
- N spins simulate N matter fields and N-1 gauge fields
- Exotic spin interactions can be simulated efficiently: Digital scheme

Digital quantum simulation

Approximate time evolution by a stroboscopic sequence of gates

The evolution under a desired Hamiltonian is realised on a coarse-grained time scale

$$H = H_1 + H_2$$

$$U(t) \equiv e^{-iHt/\hbar} = e^{-iH\Delta t_n/\hbar} \ldots {}^{-iH\Delta t_1/\hbar}$$

Trotter expansion:

$$e^{-iH\Delta t/\hbar} \simeq \underbrace{e^{-iH_1\Delta t/\hbar}}_{\text{first term}} \underbrace{e^{-iH_2\Delta t/\hbar}}_{\text{second term}} \underbrace{e^{\frac{1}{2}\frac{(\Delta t)^2}{\hbar^2}[H_1, H_2]}}_{\substack{\text{Trotter errors for} \\ \text{non-commuting terms}}}$$

S. Lloyd, Science 273, 1073 (1996).

Digital quantum simulation

Approximate time evolution by a stroboscopic sequence of gates

The evolution under a desired Hamiltonian is realised on a coarse-grained time scale

$$U_{\mathrm{S}} \;=\; e^{-i\hat{H}_{\mathrm{S}}t}$$

$$U_{\mathrm{sim}} = \left(e^{-iH_1 t/n} \ldots e^{-iH_n t/n}\right)^n$$

Operations that can be performed straightforwardly

Trotter error: $U_{\mathrm{S}} - U_{\mathrm{sim}} = \dfrac{t^2}{2n} \sum_{i,j} [H_i, H_j] + \epsilon$

This scheme: Trotter errors do not violate gauge invariance

Our toolbox

Ion trap quantum computers:

- Fast and accurate single qubit operations
- Entangling gates: Mølmer-Sørensen interaction

All-to-all 2-body interaction: $H_0 = J_0 \sum_{i,j} \sigma_i^x \sigma_j^x$

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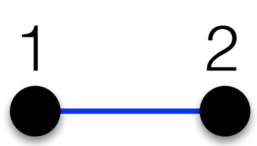
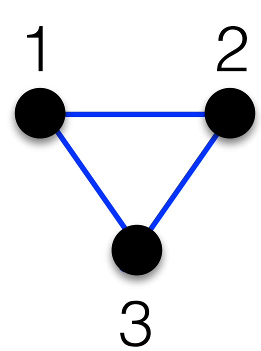
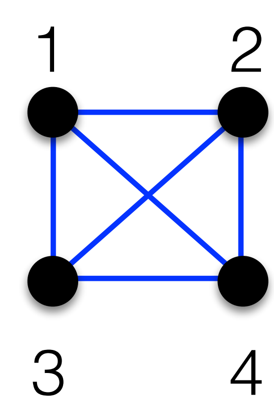


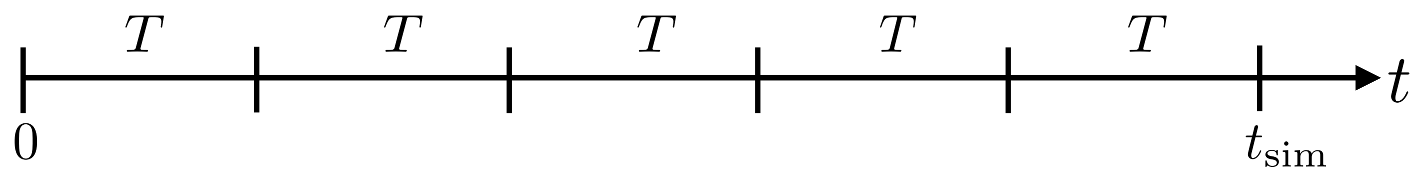
$\sigma_1^x \sigma_2^x$



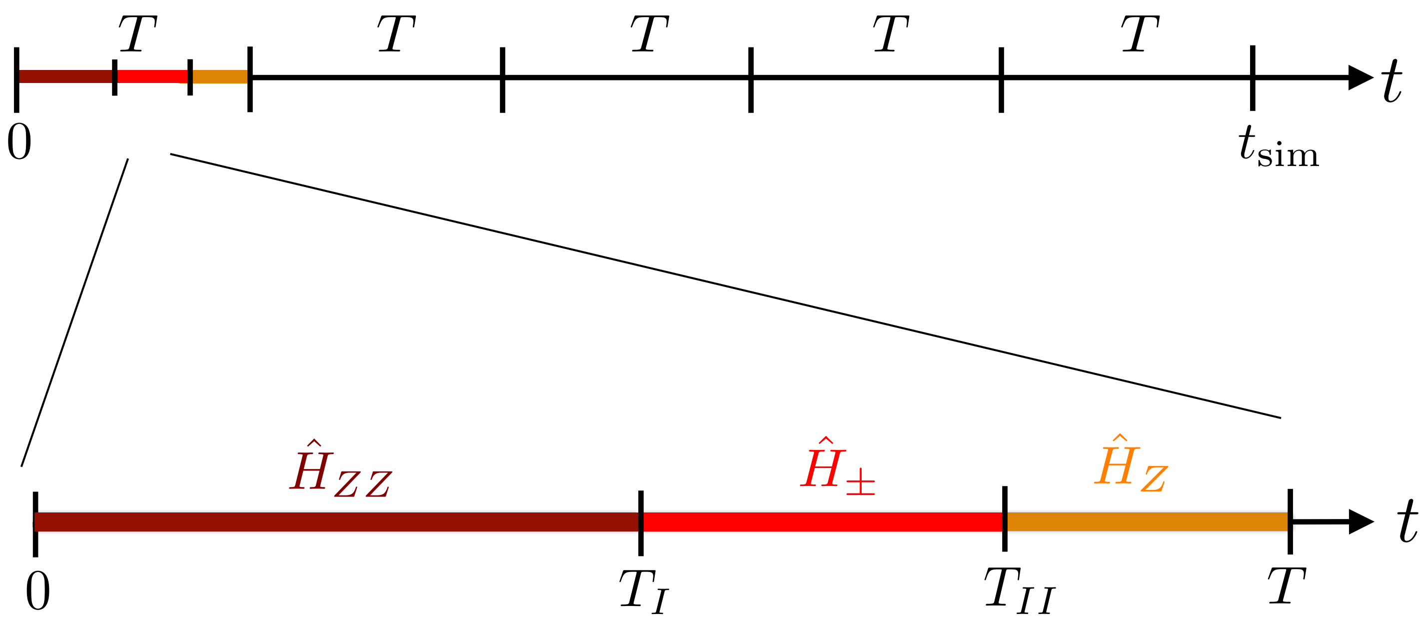
$\sigma_1^x \sigma_2^x + \sigma_2^x \sigma_3^x + \sigma_1^x \sigma_3^x$



$\sigma_1^x \sigma_2^x + \sigma_1^x \sigma_3^x + \sigma_1^x \sigma_4^x + \sigma_2^x \sigma_3^x + \sigma_2^x \sigma_4^x + \sigma_3^x \sigma_4^x$



Time coarse graining



Time coarse graining

$$\hat{H}_S = J \sum_{i<j} c_{ij} \hat{\sigma}_i^z \hat{\sigma}_j^z$$

long - range interaction

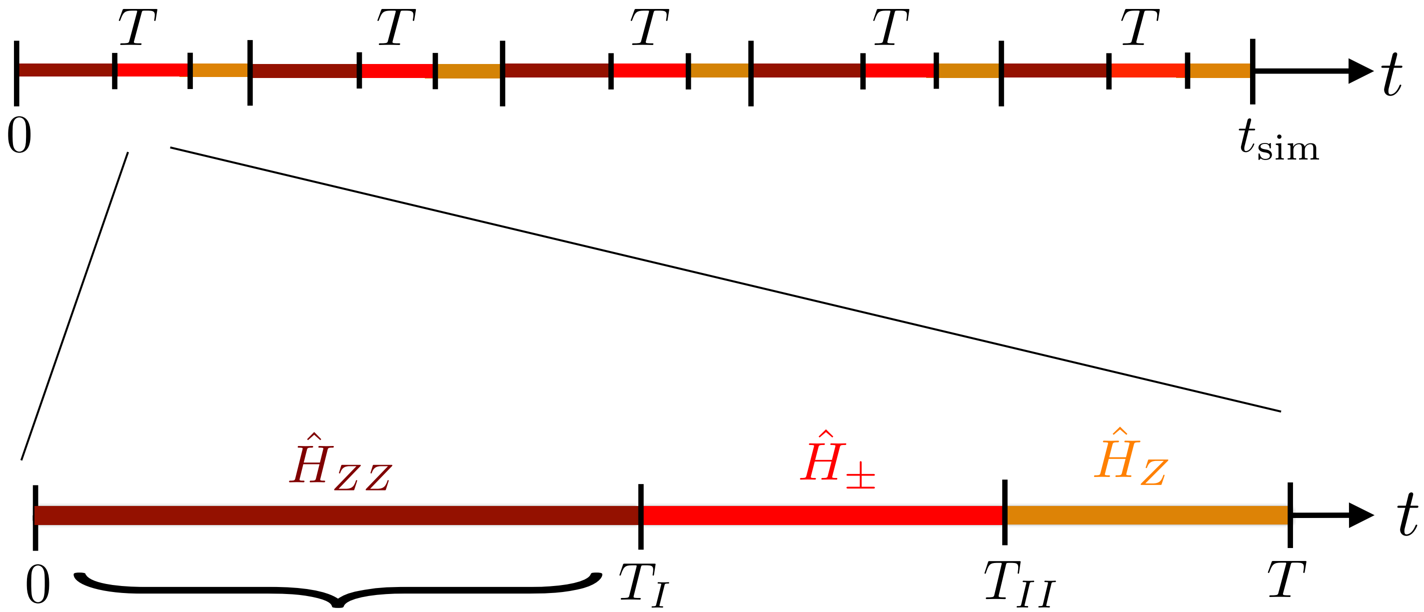
$$+ \; w \sum_i \left(\hat{\sigma}_i^+ \hat{\sigma}_{i+1}^- + \hat{\sigma}_{i+1}^+ \hat{\sigma}_i^- \right)$$

particle - antiparticle creation/annihilation

$$+ \; m \sum_i c_i \hat{\sigma}_i^z + J \sum_i \tilde{c}_i \hat{\sigma}_i^z$$

effective particle masses

Time coarse graining



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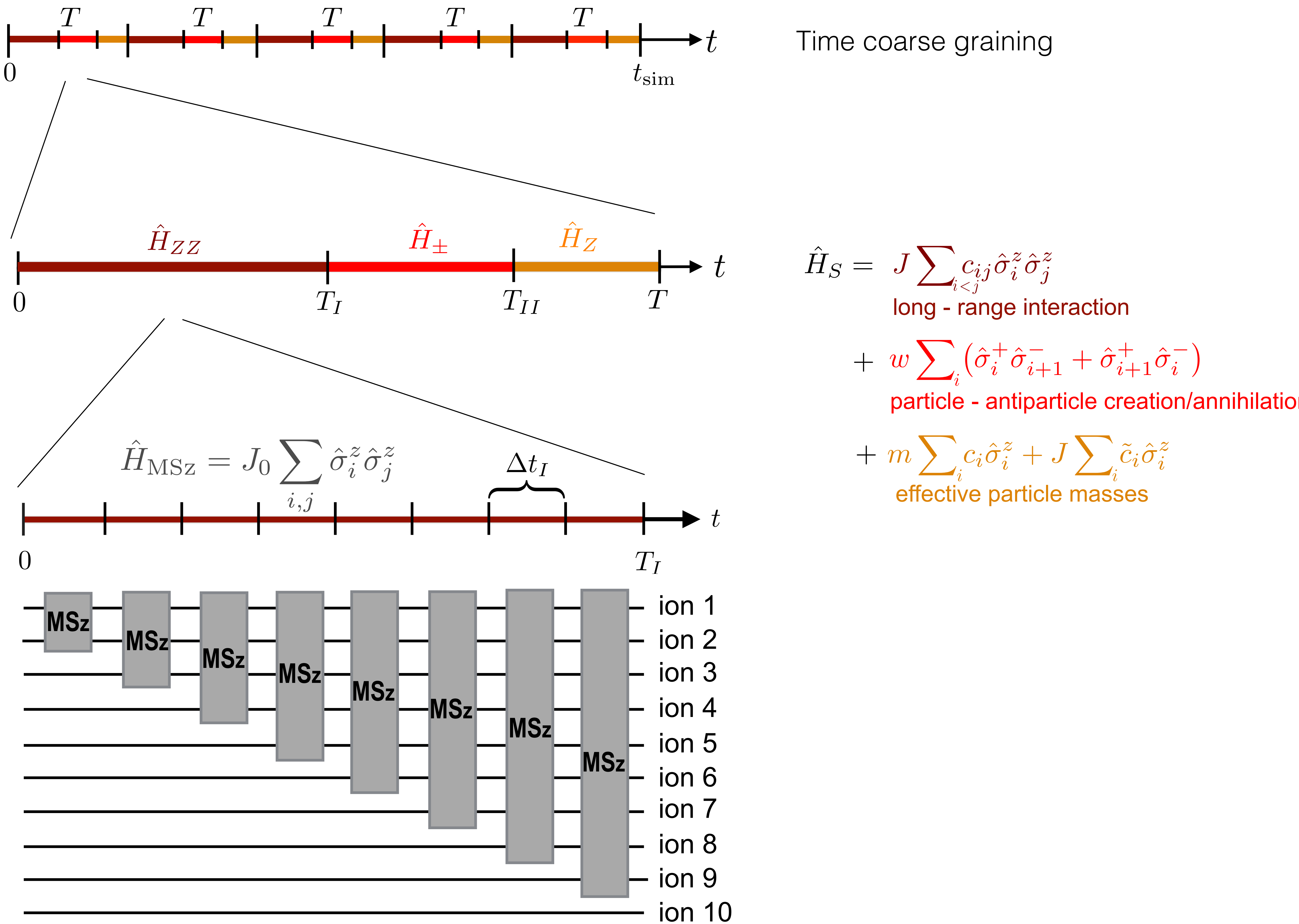

Time coarse graining
T
0
t_{sim}
t
$\hat{H}_{ZZ}$
$\hat{H}_{\pm}$
$\hat{H}_{Z}$
T_I
T_{II}
$\hat{H}_S = J \sum_{i<j} c_{ij} \hat{\sigma}_i^z \hat{\sigma}_j^z$
long - range interaction
$+ w \sum_i \left(\hat{\sigma}_i^+ \hat{\sigma}_{i+1}^- + \hat{\sigma}_{i+1}^+ \hat{\sigma}_i^- \right)$
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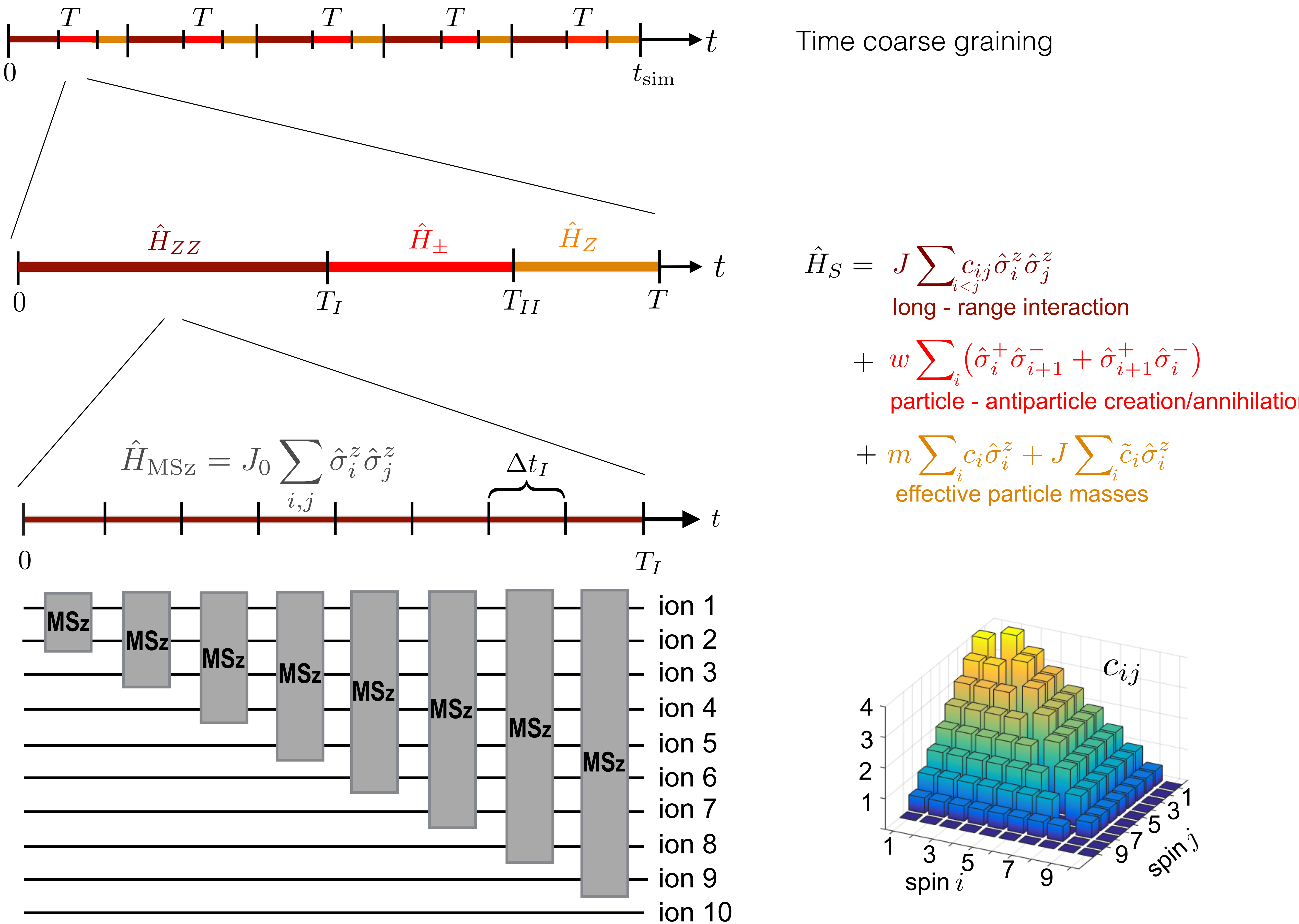
effective particle masses
$\hat{H}_{\mathrm{MSz}} = J_0 \sum_{i,j} \hat{\sigma}_i^z \hat{\sigma}_j^z$
Δt_I
MSz
ion 1
ion 2
ion 3
ion 4
ion 5
ion 6
ion 7
ion 8
ion 9
ion 10



Time coarse graining
T T T T T
0
t_{sim}
t
$\hat{H}_{ZZ}$
$\hat{H}_{\pm}$
$\hat{H}_{Z}$
0
T_I
T_{II}
T
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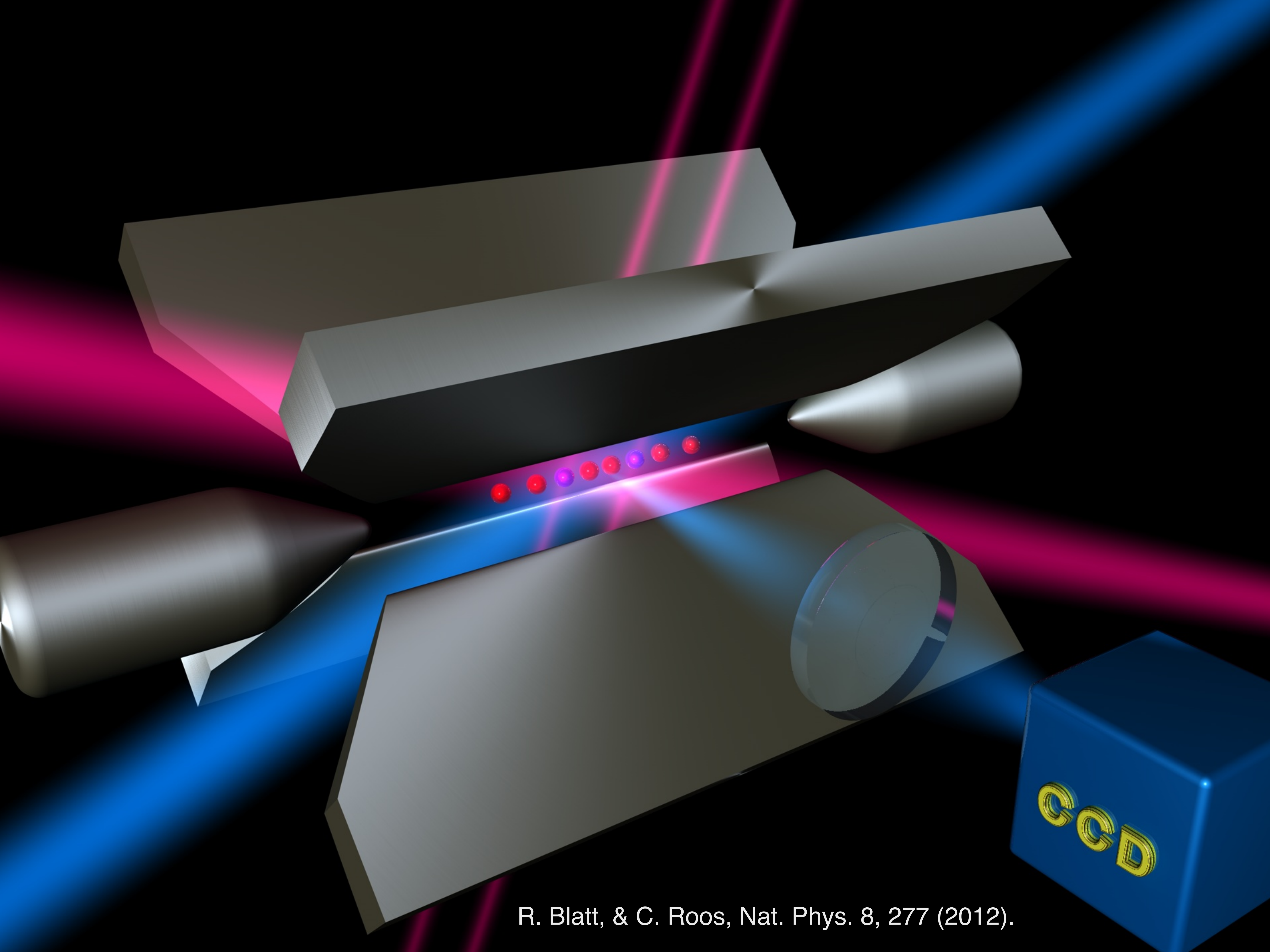
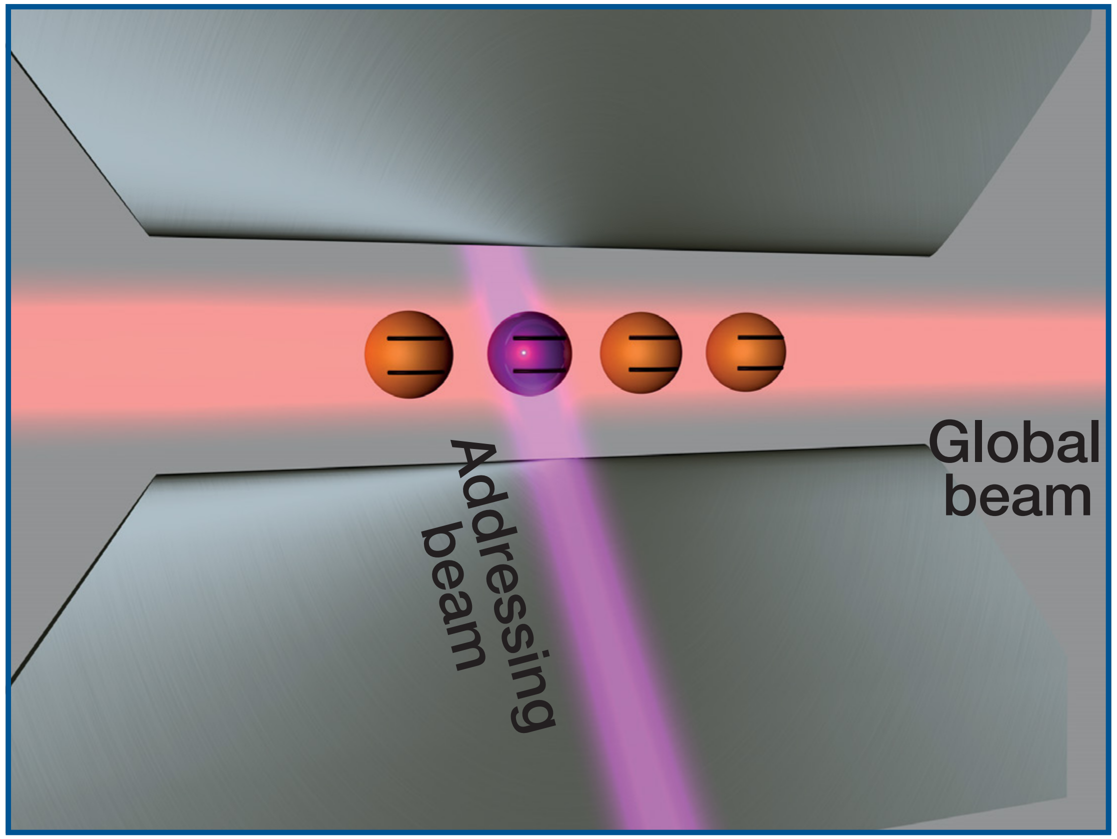
Δt_I
0
T_I
MSz
ion 1
ion 2
ion 3
ion 4
ion 5
ion 6
ion 7
ion 8
ion 9
ion 10
c_{ij}
spin i
spin j



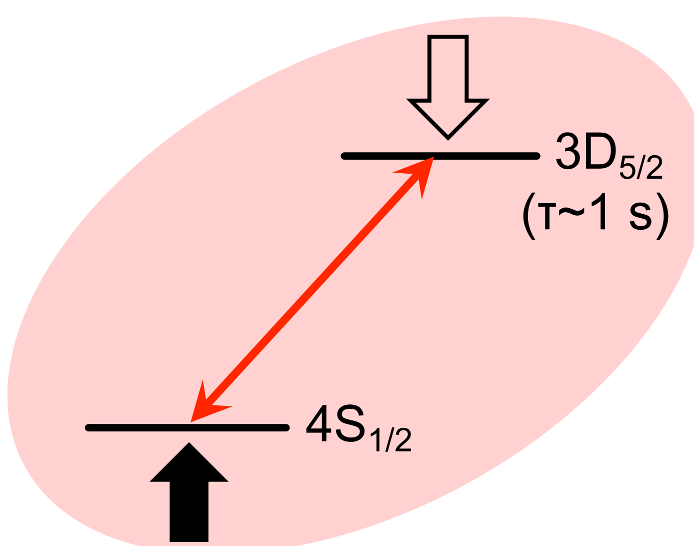
R. Blatt, & C. Roos, Nat. Phys. 8, 277 (2012).

Experiment

E. Martinez, P. Schindler, D. Nigg, A. Erhard, T. Monz, and R. Blatt



Qubit



Tools for universal digital quantum simulation are available:

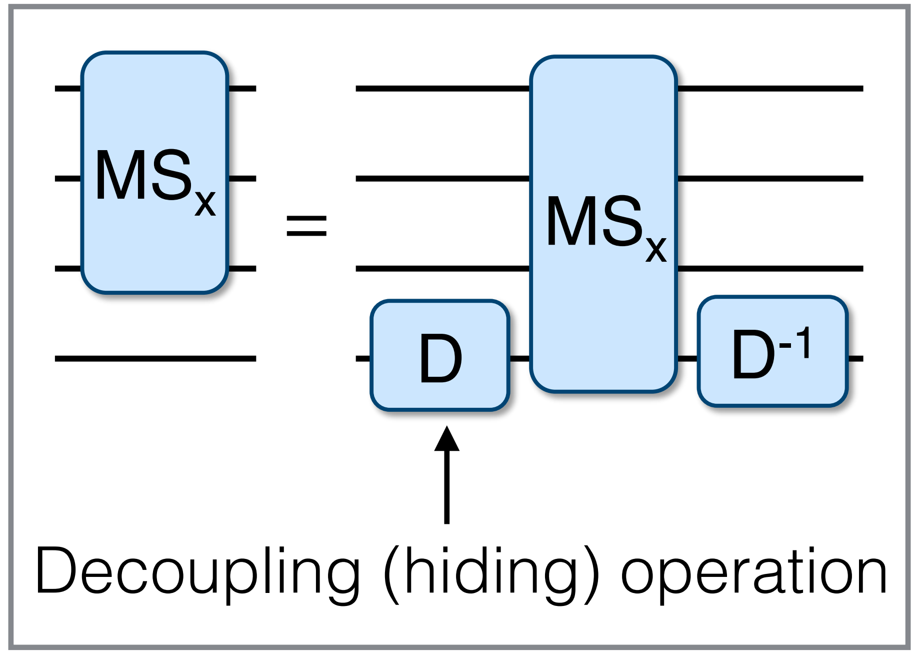
B. Lanyon, et al. Science 334, 57 (2011).

- High fidelity local rotations ✓
- Entangling gates ✓

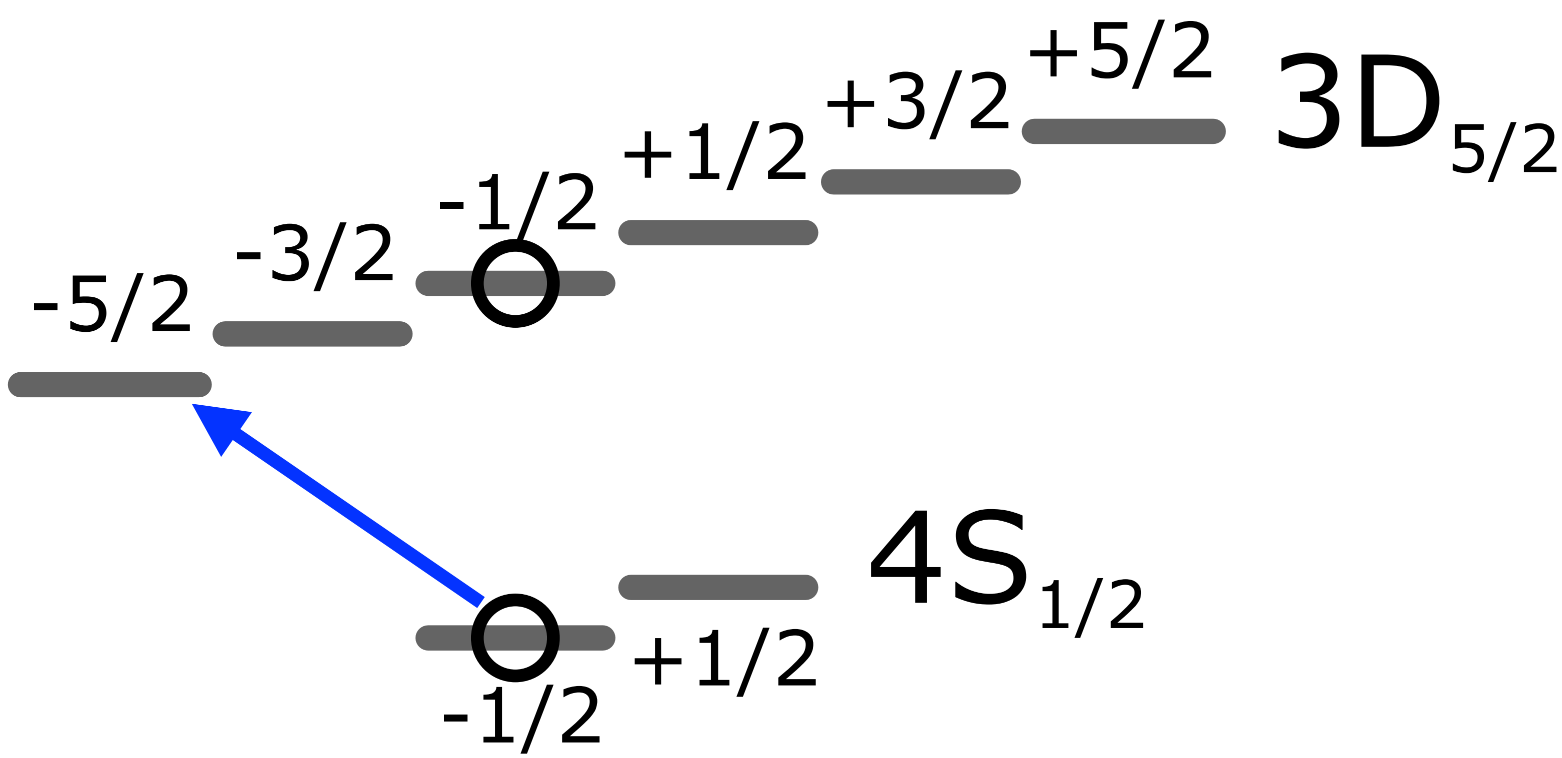
Mølmer-Sørensen interaction

$$H_0 = J_0 \sum_{i,j} \sigma_i^x \sigma_j^x$$

Decoupling of individual ions



Ions are selectively decoupled from the MS interaction by transferring their population to off-resonant Zeeman levels:



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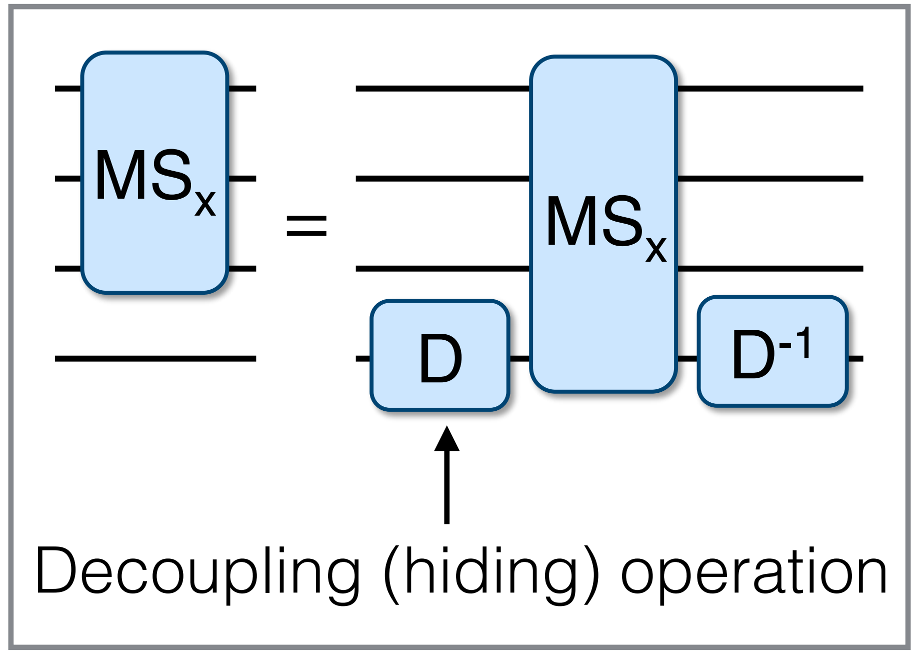

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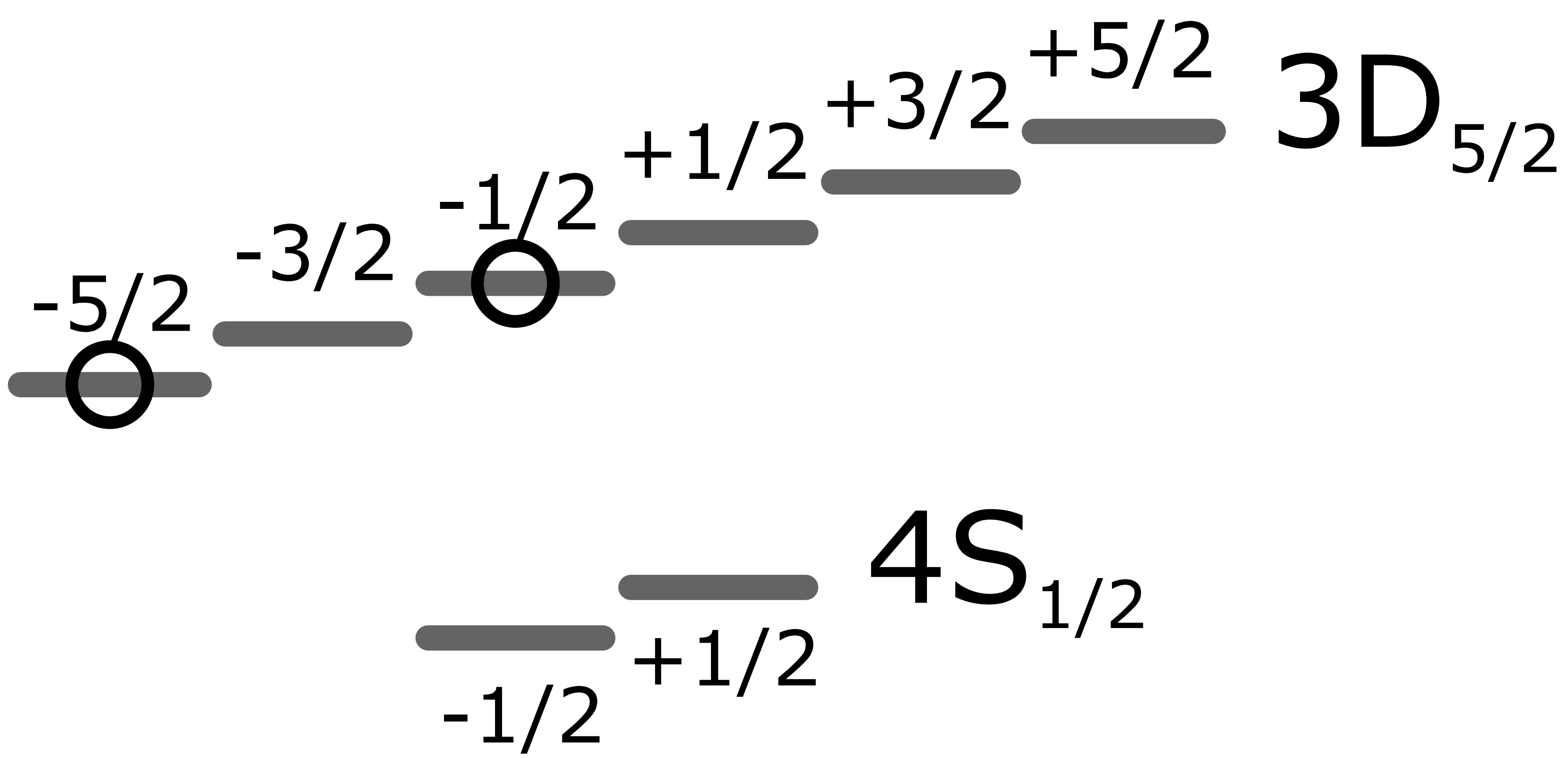

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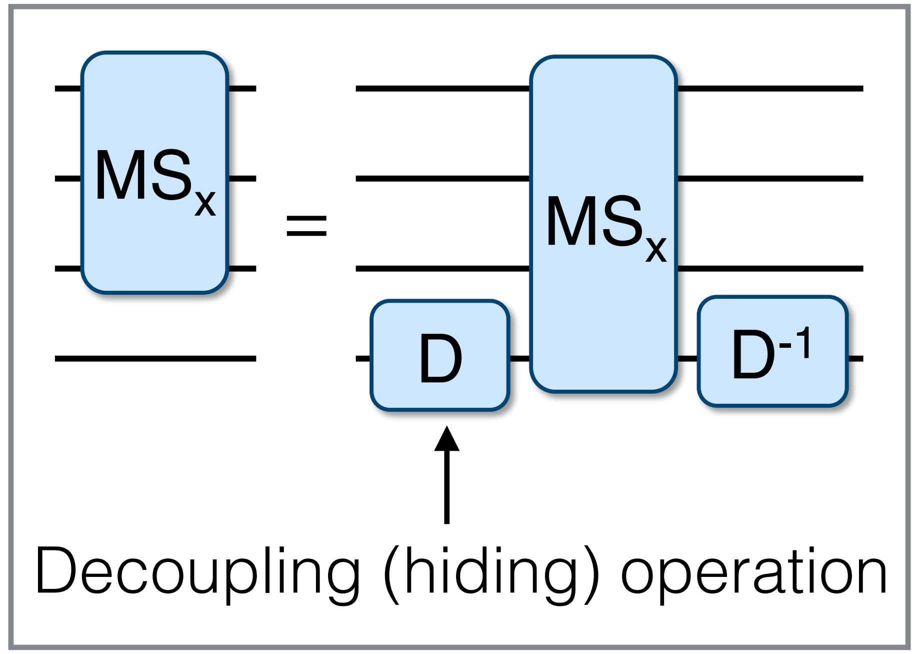

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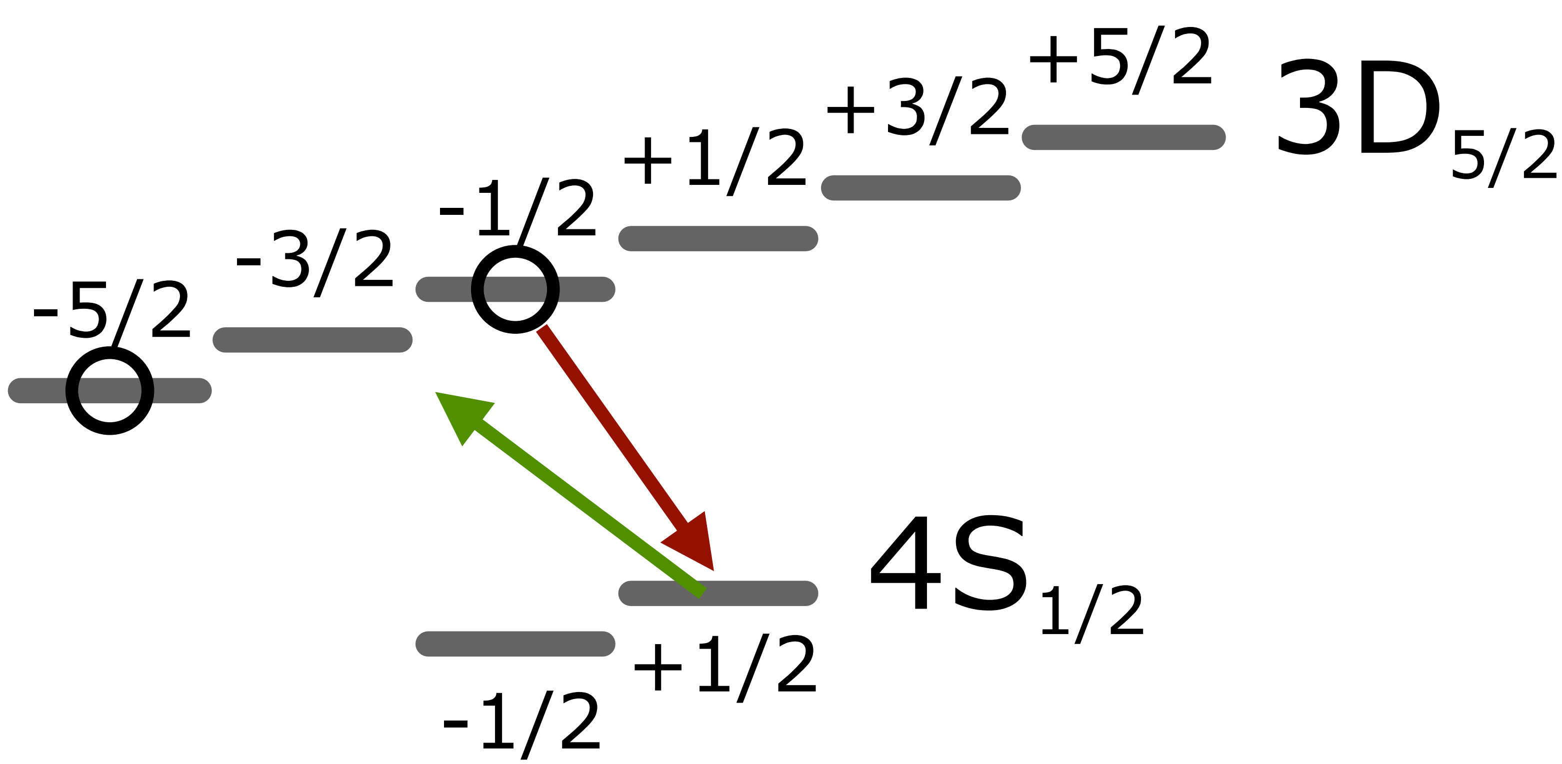

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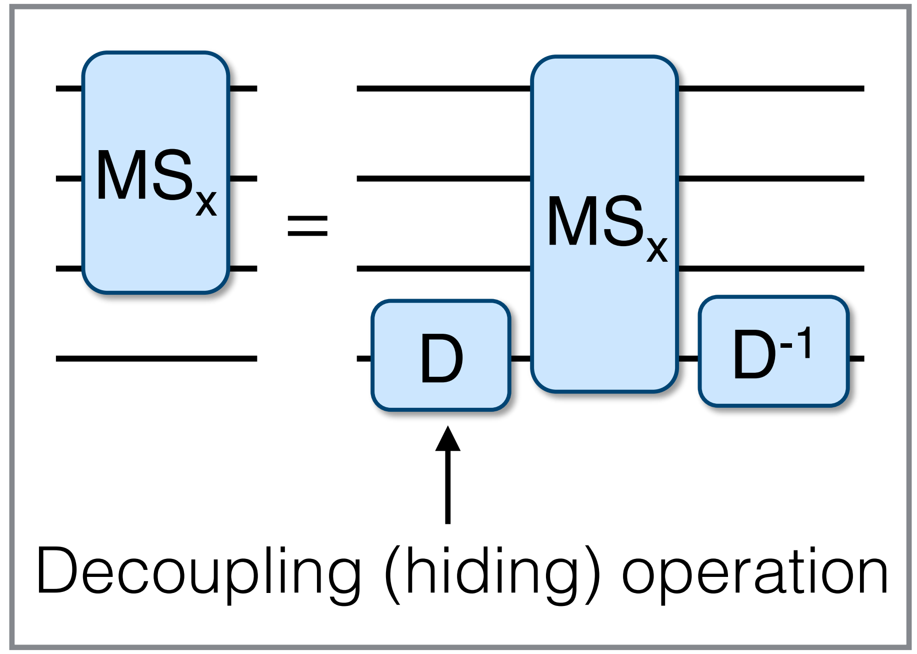

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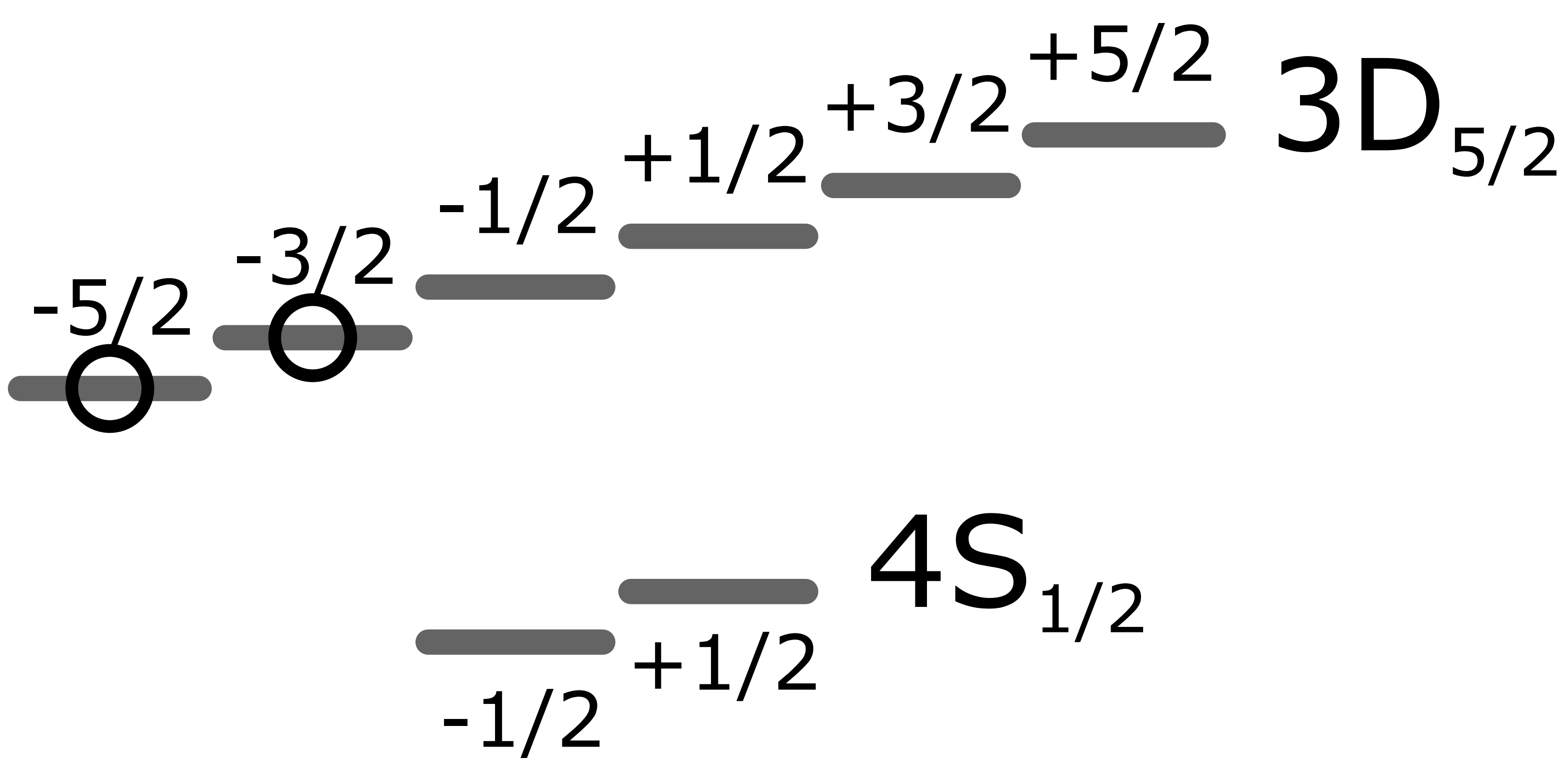

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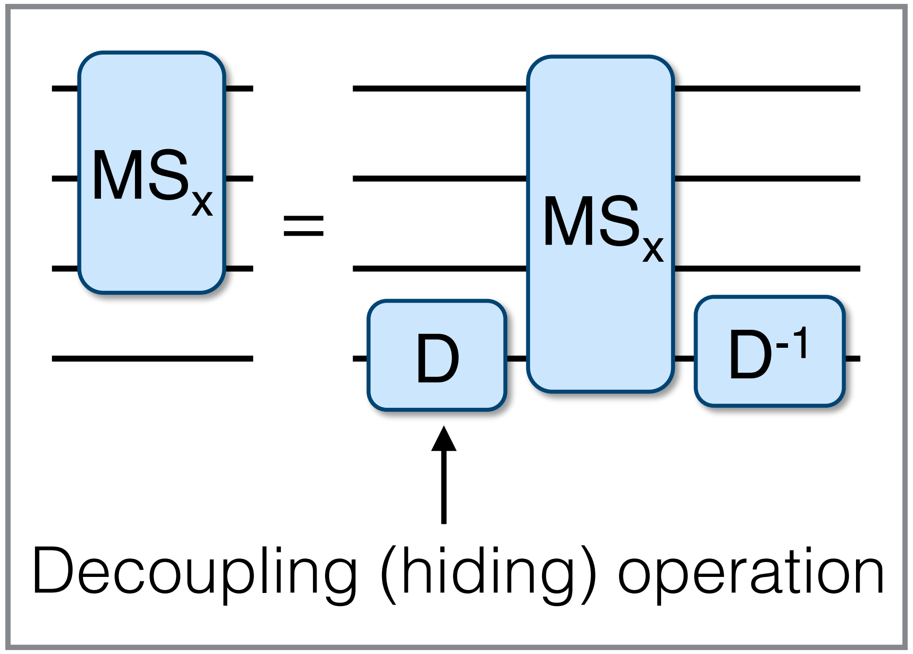

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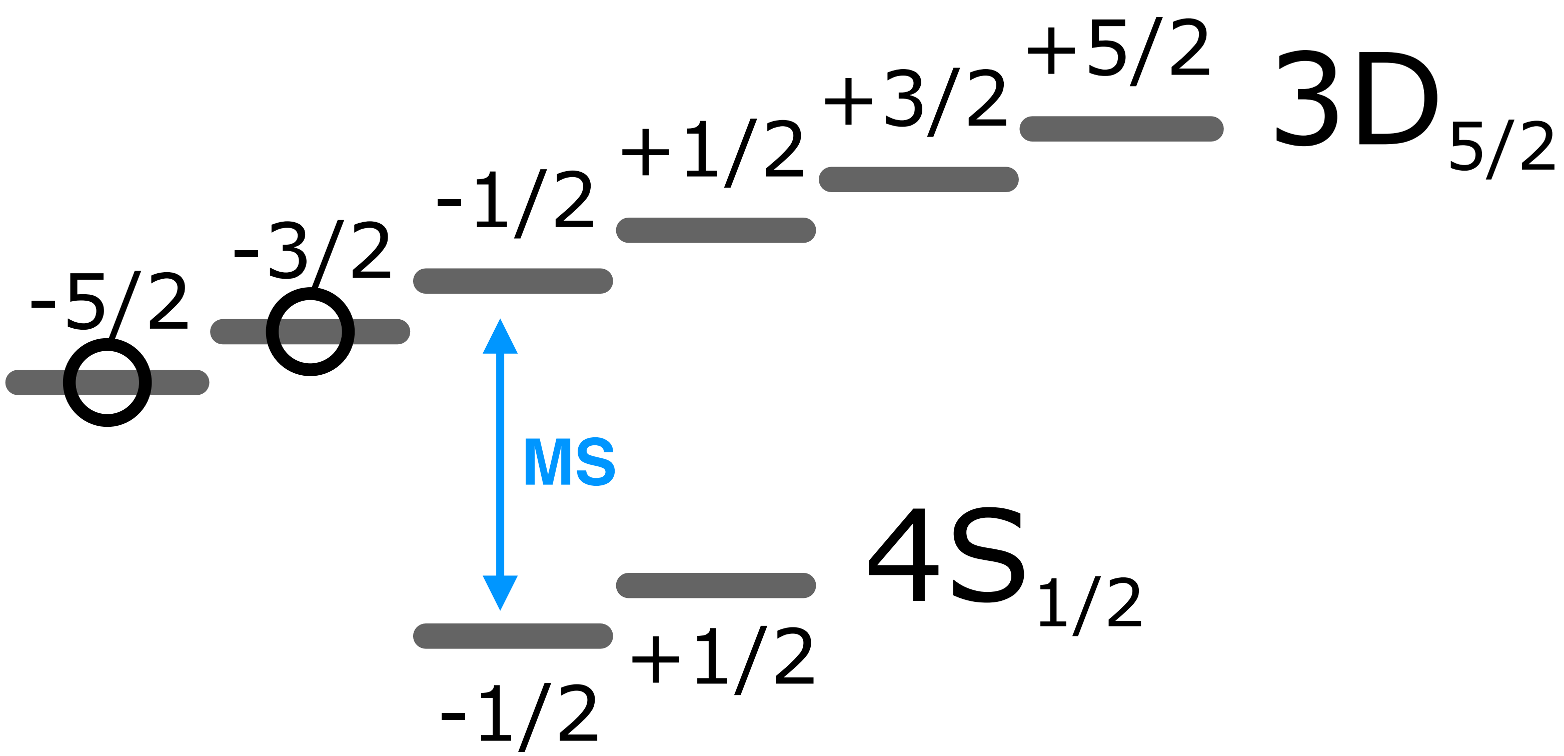
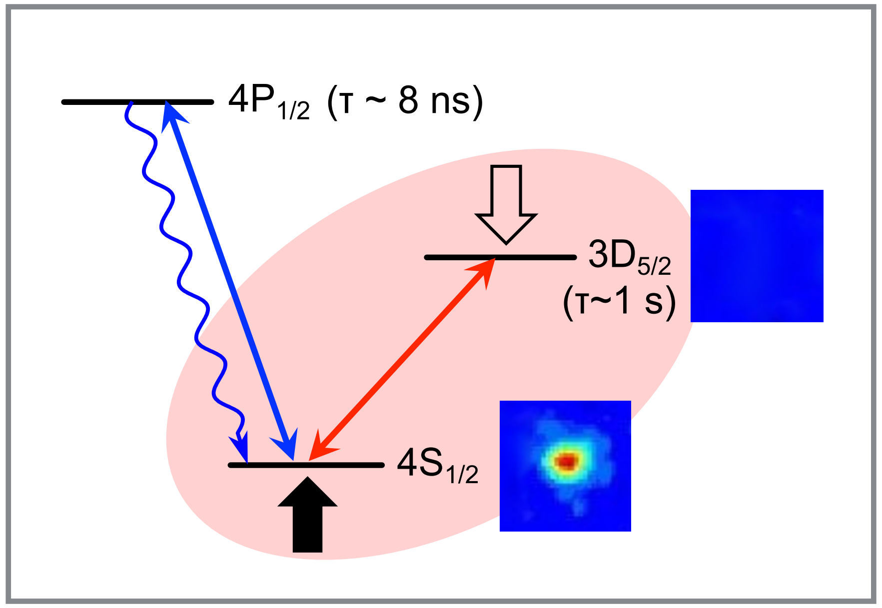
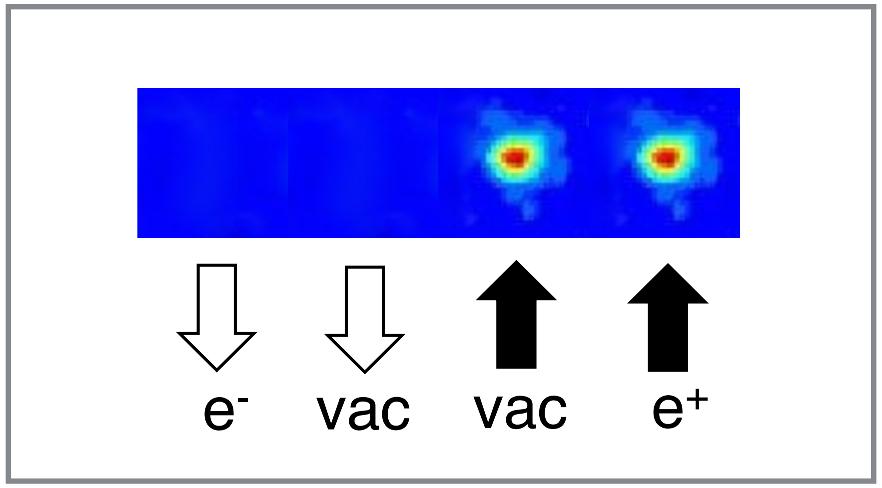


Measurements

- Electron shelving technique (projective measurement in the z-basis)



- Imaging of the whole ion chain on a CCD camera

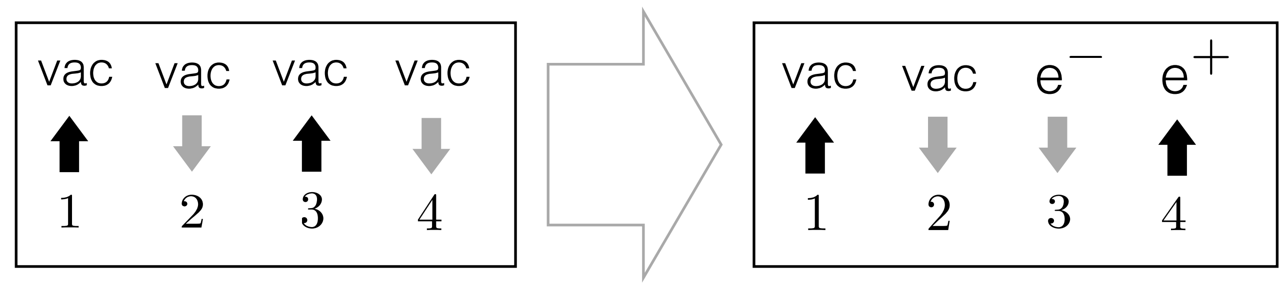


- Change of the measurement basis: full state tomography

Quantum Simulation of pair creation

Particle number density: $$\nu(t) = \frac{1}{N}\sum_{n=1}^{N}\langle(-1)^n\sigma_n^z(t)+1\rangle$$

Creation of a particle antiparticle pair:



$\nu = 0$ $\nu = 0.5$