Formulating nonabelian gauge theories for a quantum computer

D. B. Kaplan ~ Argonne Nat'l Lab ~ 3/29/18

Practical implementation of Wilson's formulation of the Feynman path integral on a classical computer:

32^3 x 64 lattice size: millions of degrees of freedom
Hilbert space size ~ e^{millions}. Lattice QFT: sample it!

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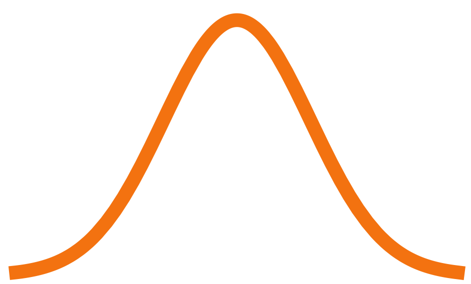
1d wave function



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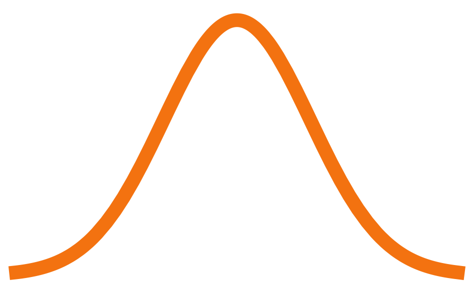

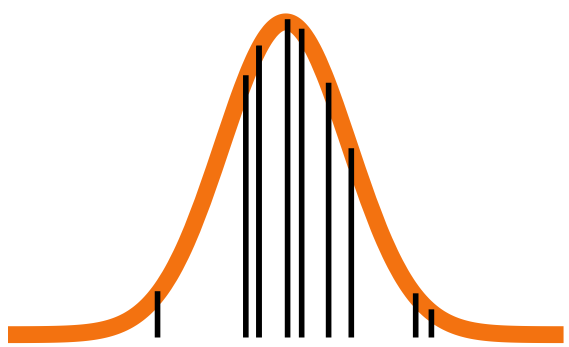

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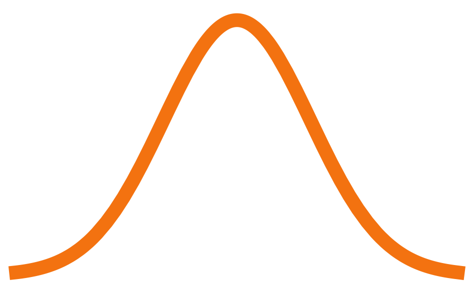
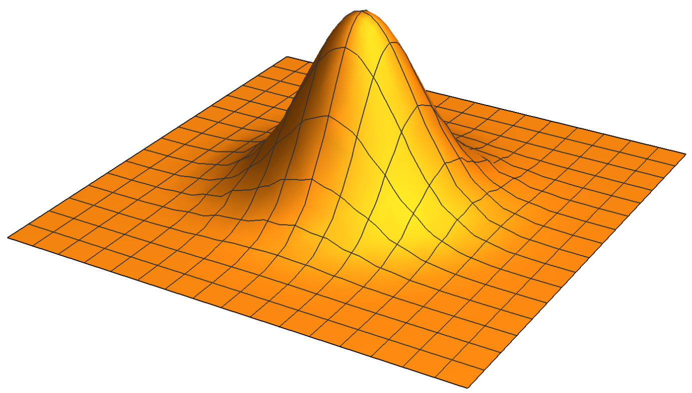
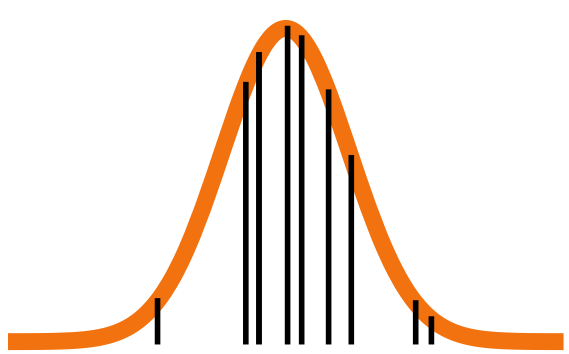


2d wave function





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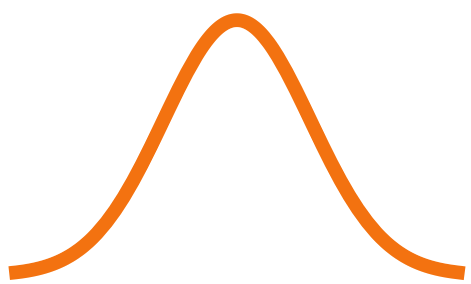

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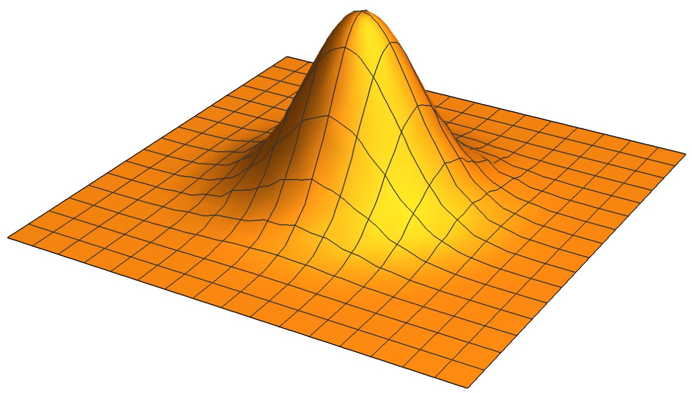

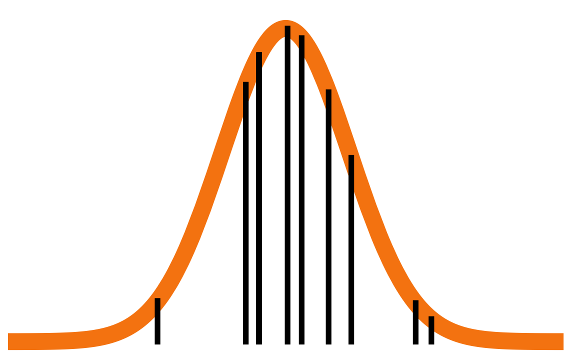

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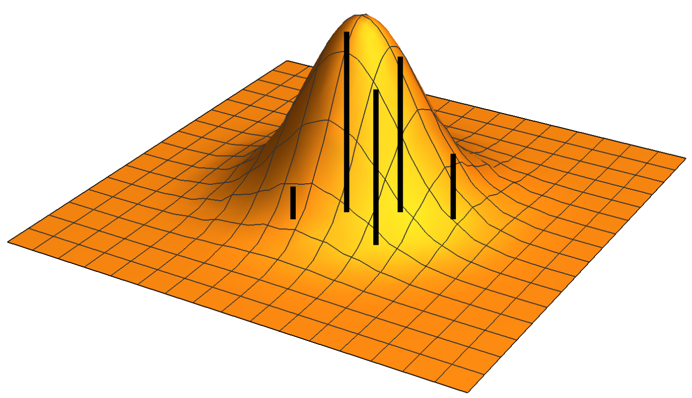

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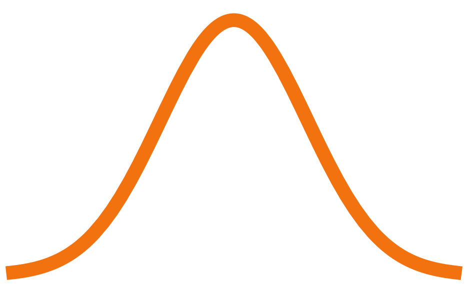

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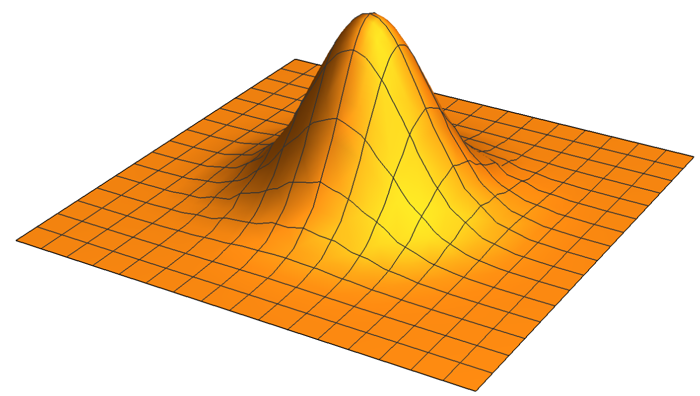
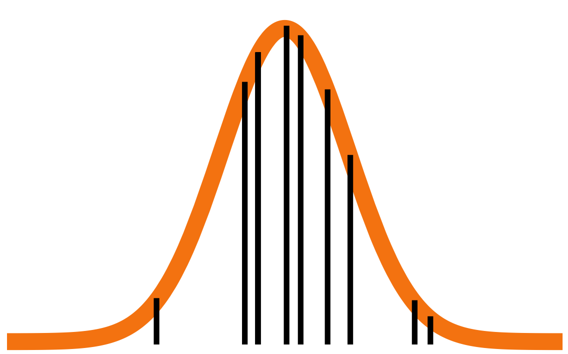


… $e^{\text{“millions”}}$ d wave function



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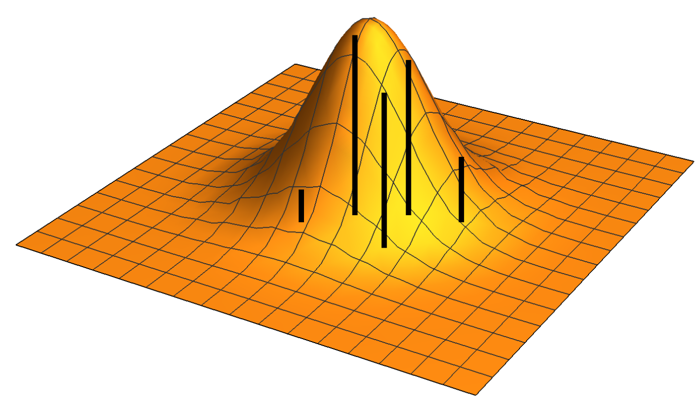

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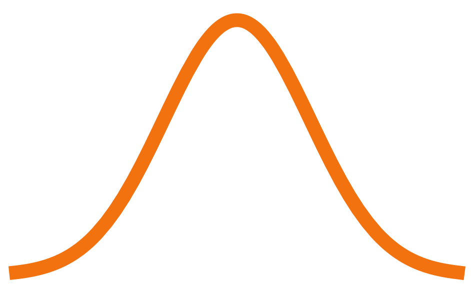

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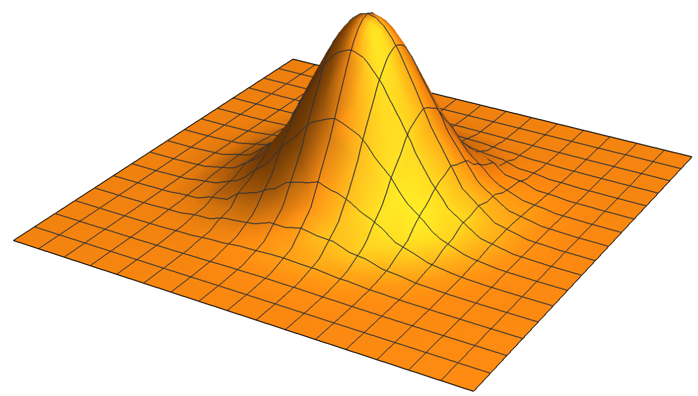

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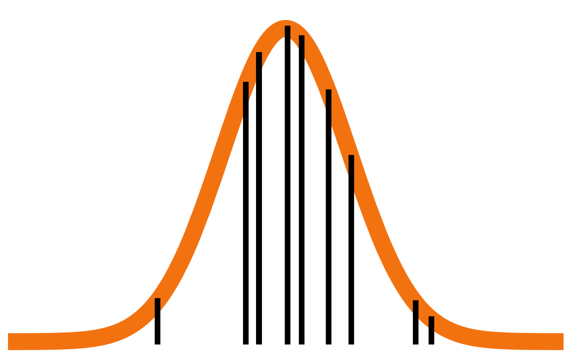

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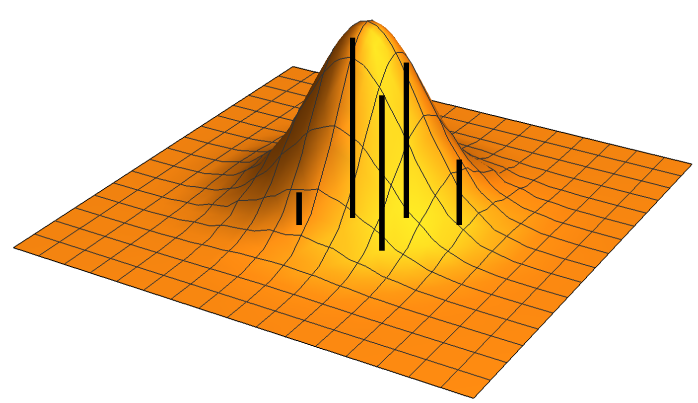
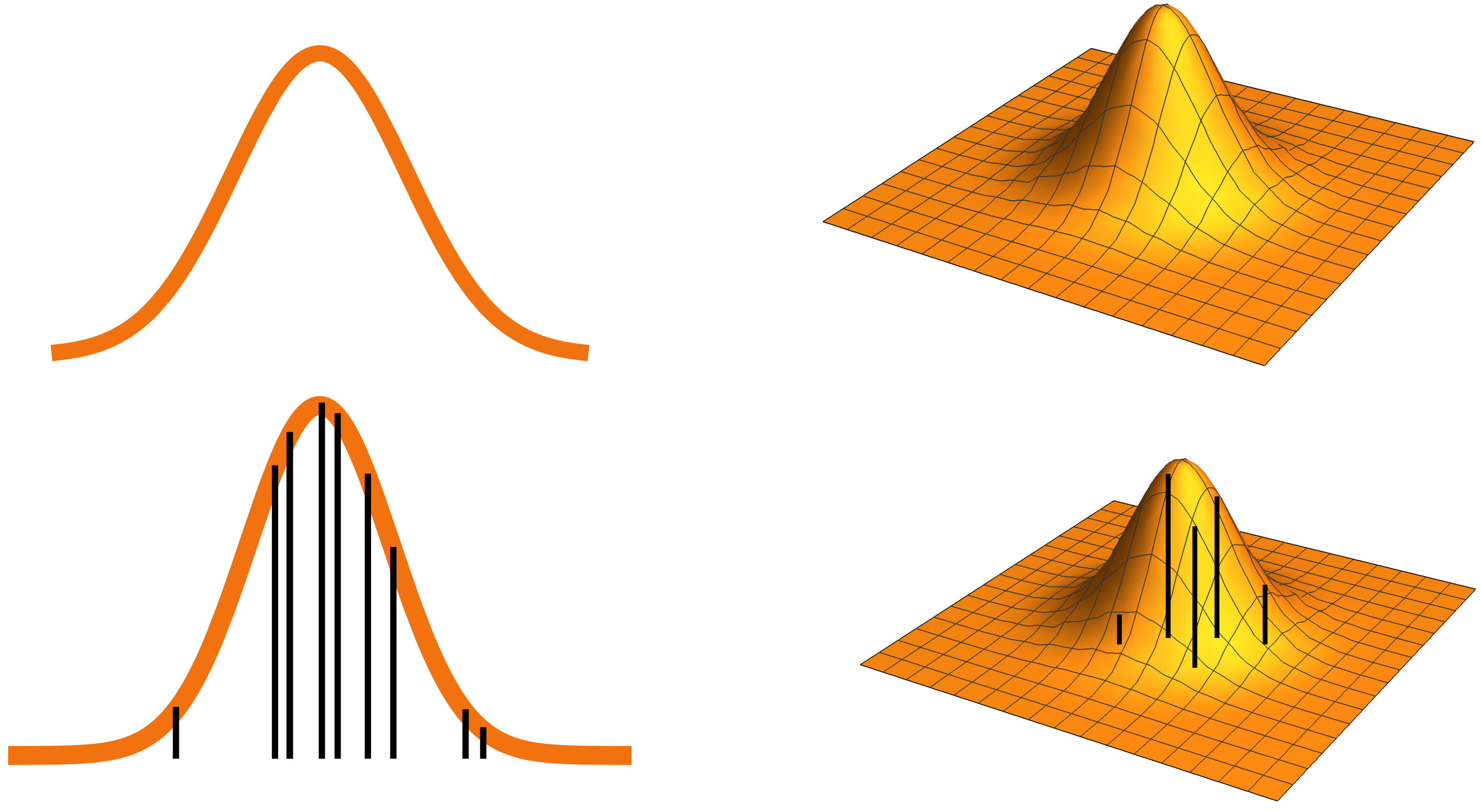


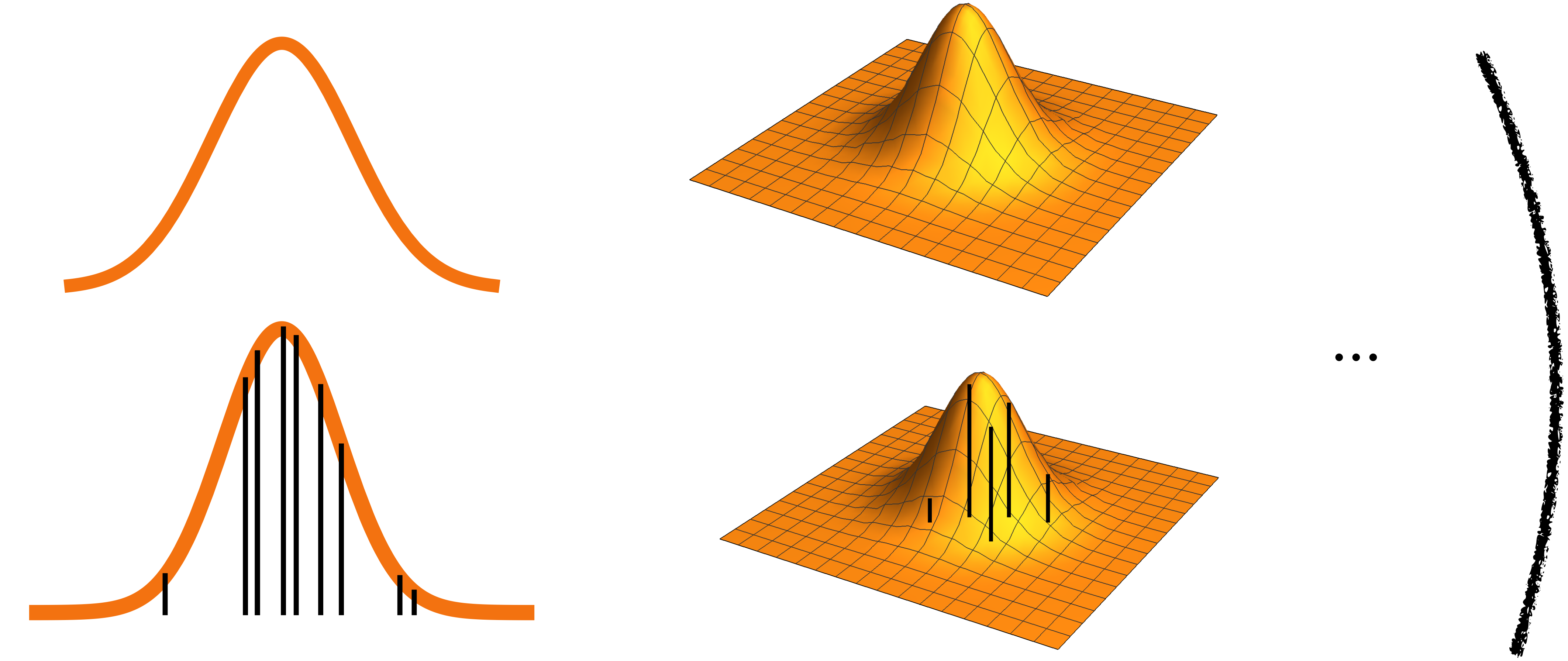
2d wave sampling

<u>Amazingly</u>, the ground state wave function of glue + quark/antiquark pairs is possible to sample effectively



...

at zero
baryon
number



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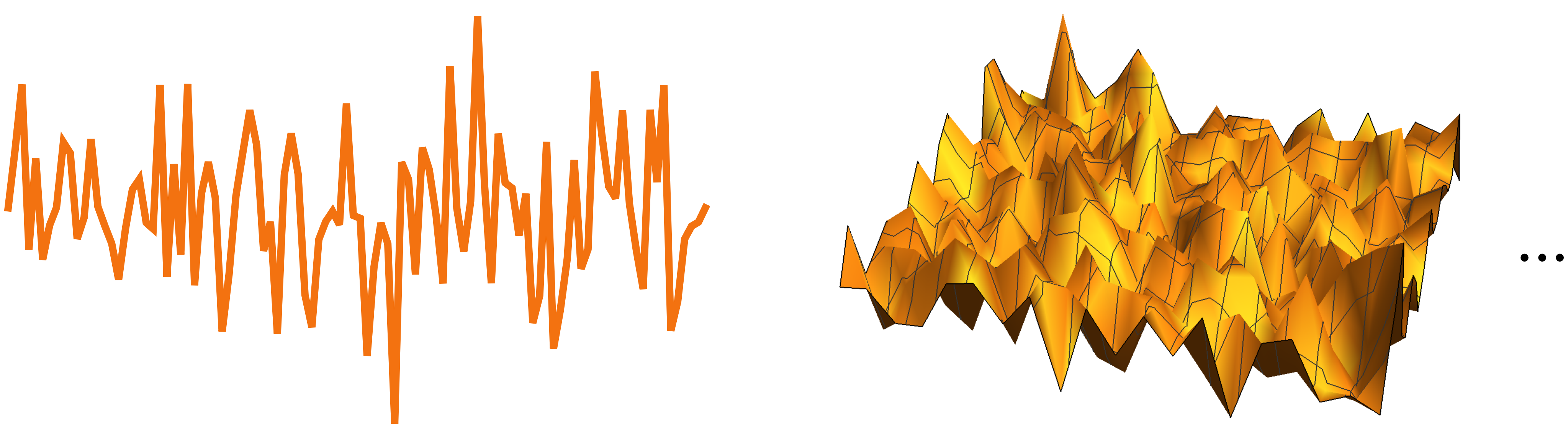

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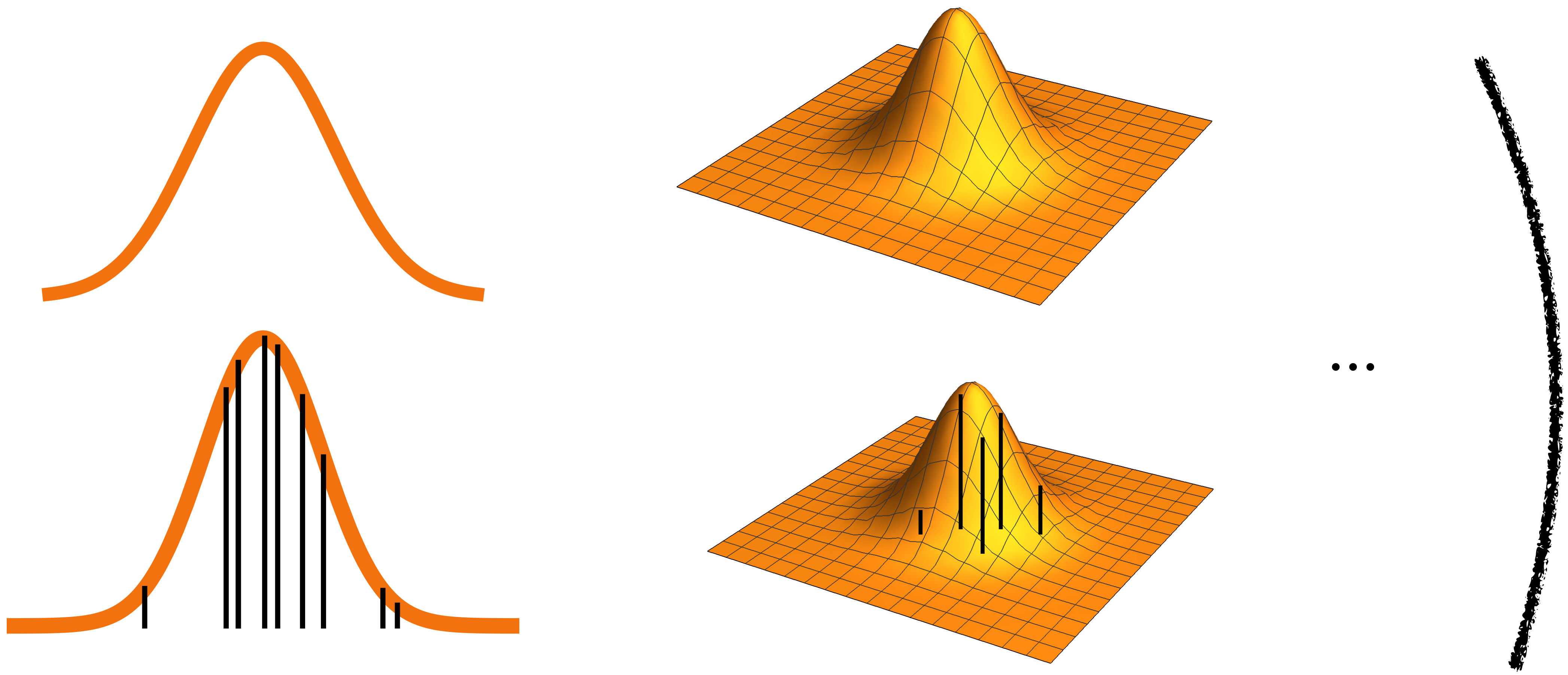

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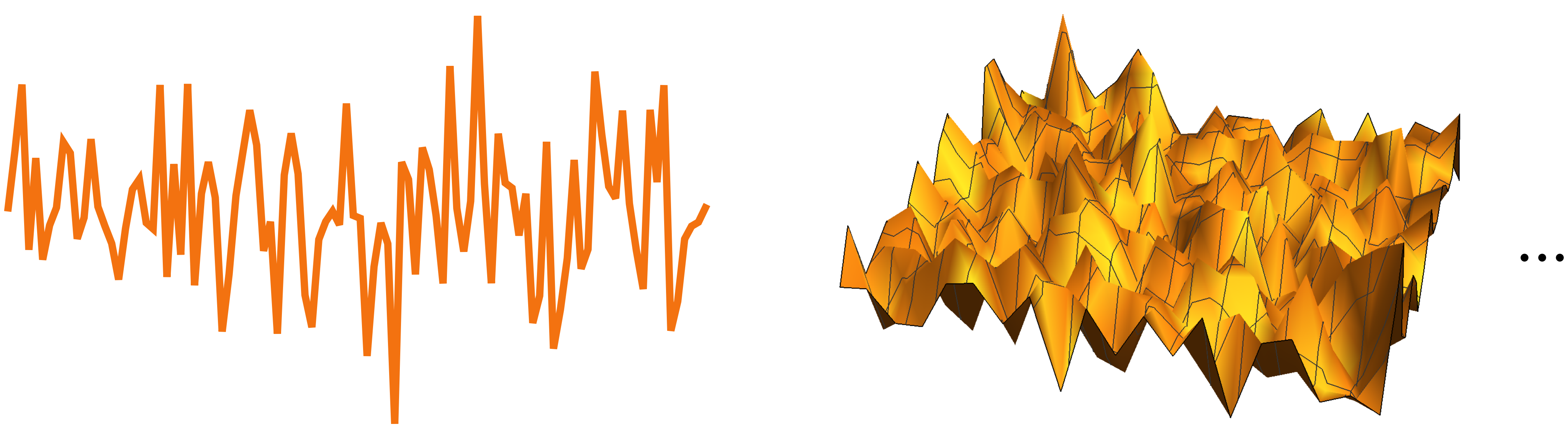

Problem
at nonzero
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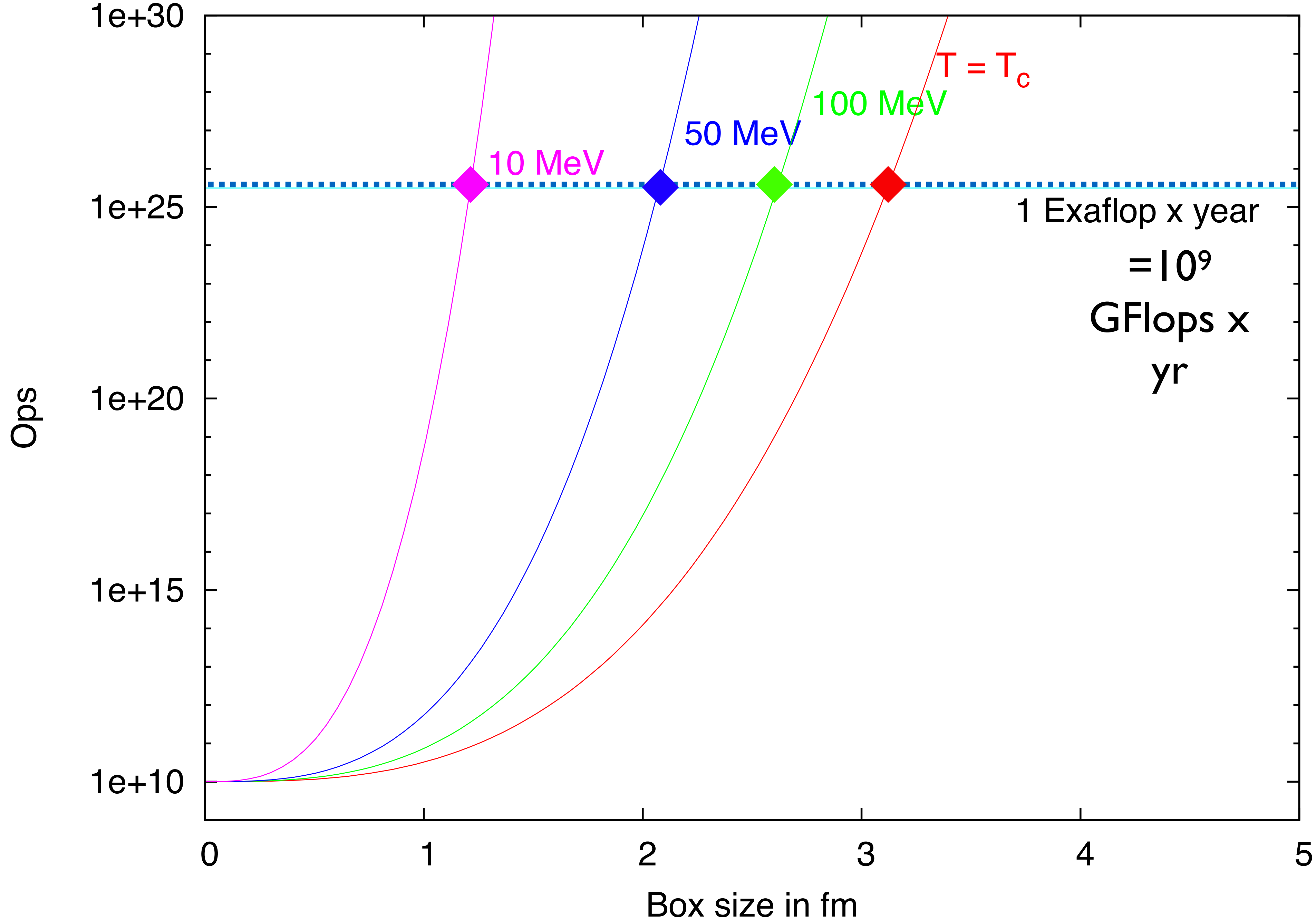
number!

One needs an exponentially large number of samples to approximate the wave function

Problem at nonzero baryon number: how hard is the sign problem?

CPU effort to study matter at nuclear density in a box of given size
Give or take a few powers of 10…

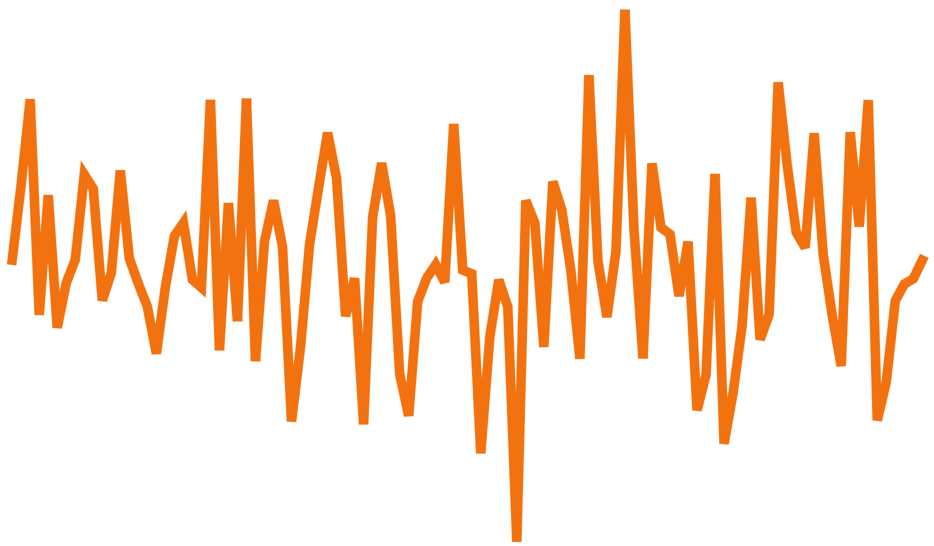


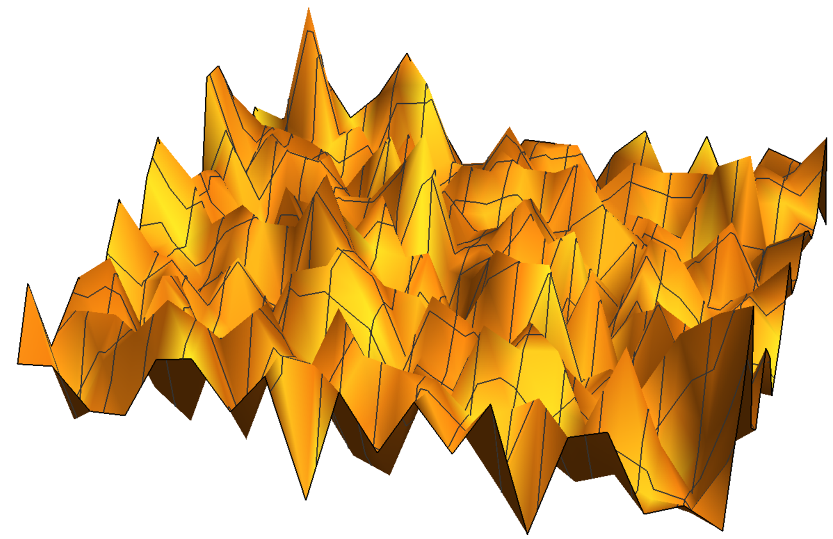
<u>radius</u> of ^{238}U

X

neutron stars will take a little while….

Apparently at significant baryon number density the wave function explores a large Hilbert space.





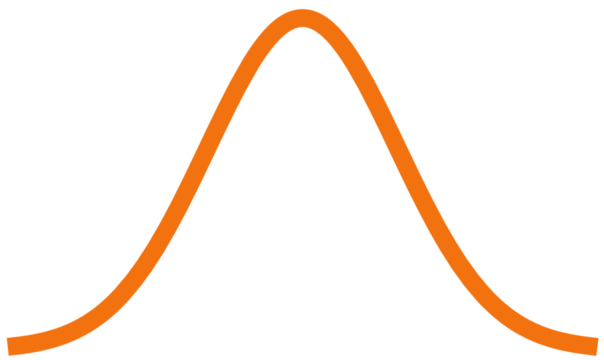
… Problem at nonzero baryon number!

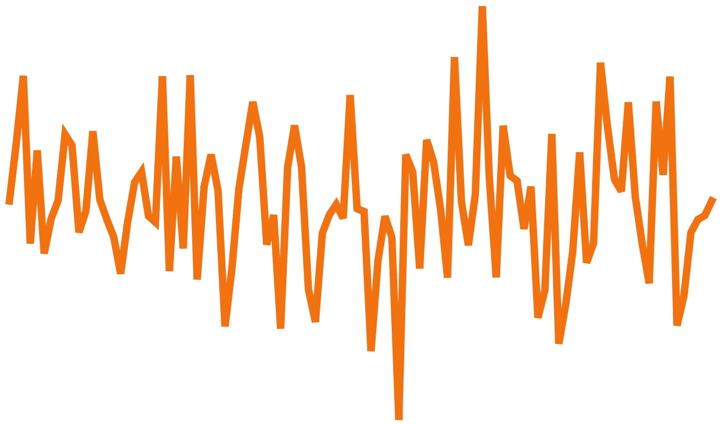
Hilbert space is VERY BIG, growing exponentially with the number of particles.

Classical computers are ill-equipped for searching this space when you don't know where to start.

Can a quantum computer help?

What do these cartoons mean?





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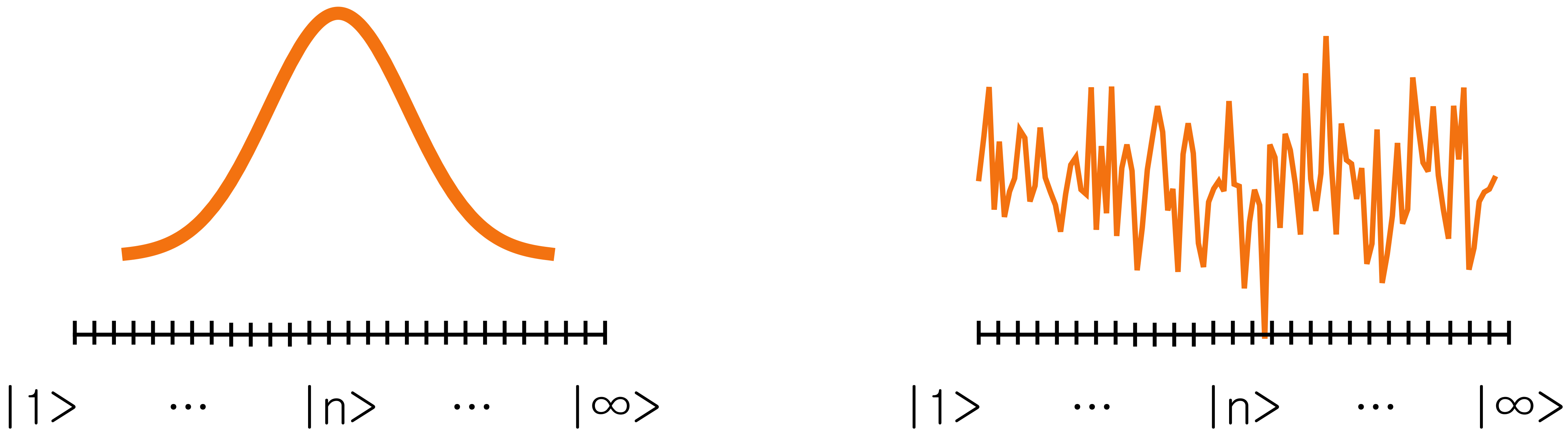

Just a problem with a bad basis for expanding our wave functions?

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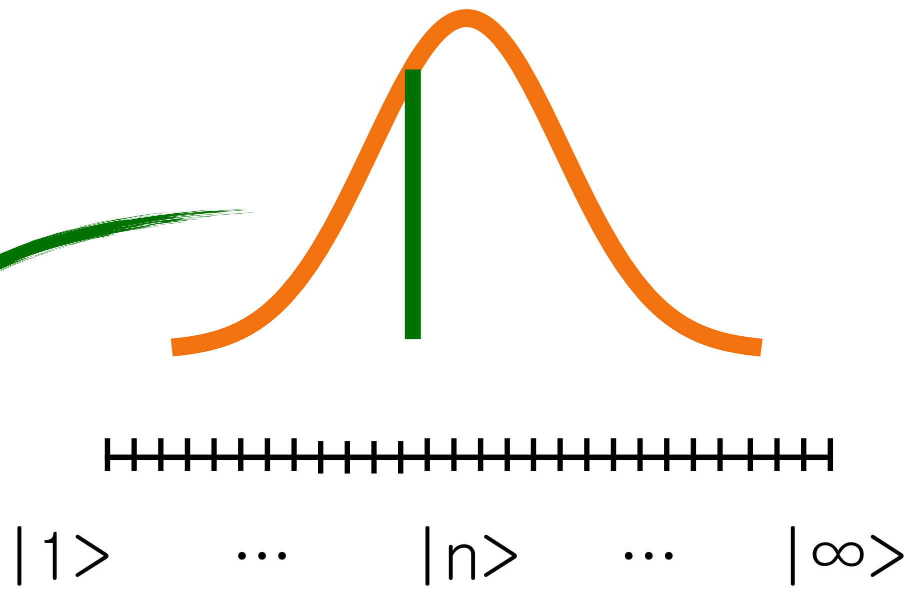

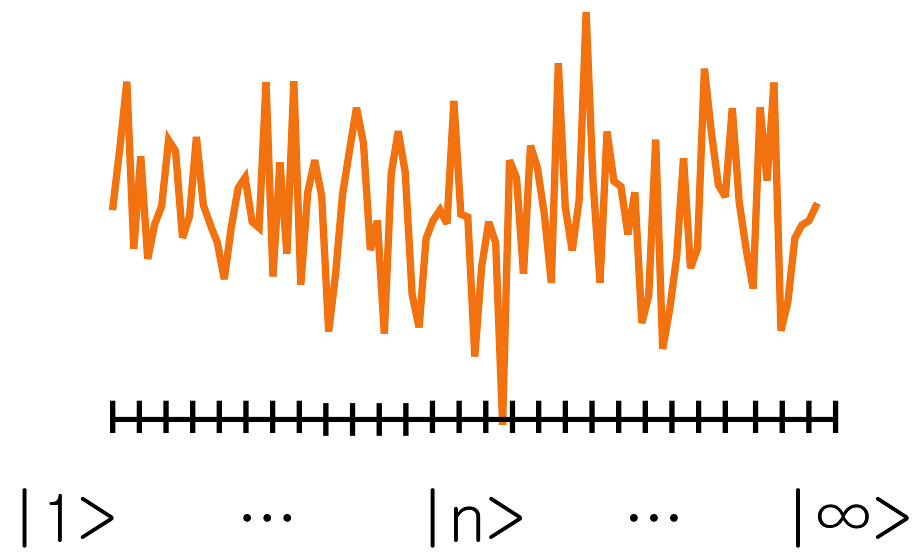

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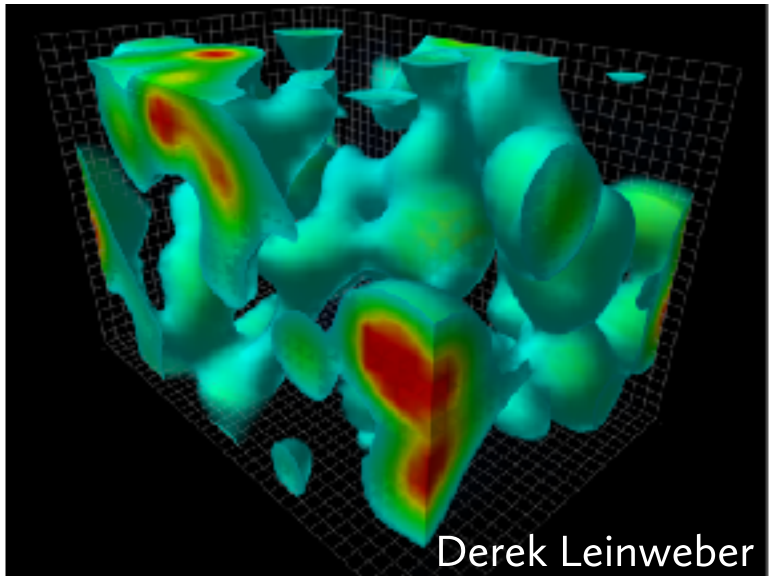
A typical contribution to the QCD vacuum...complicated, but simply generated from local Wilson Yang-Mills action



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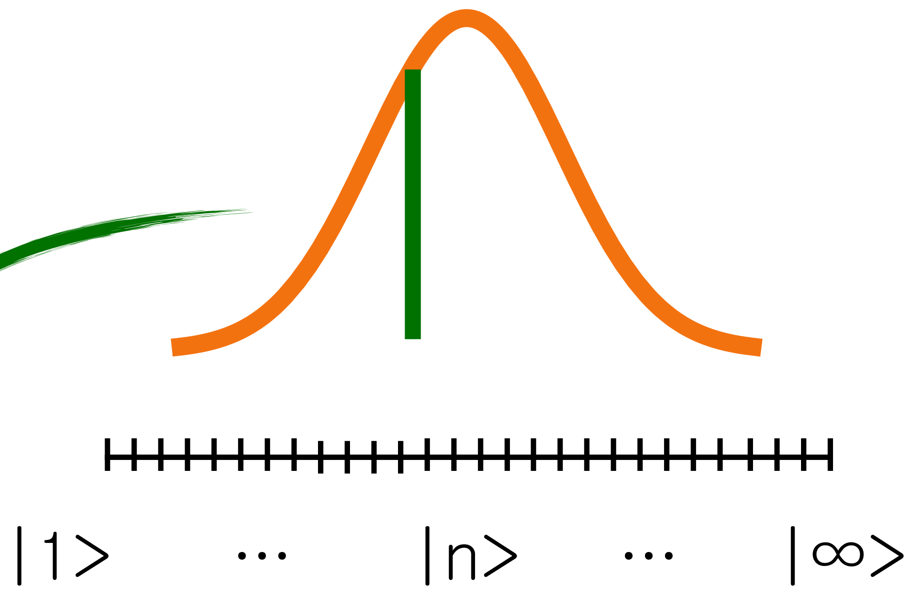

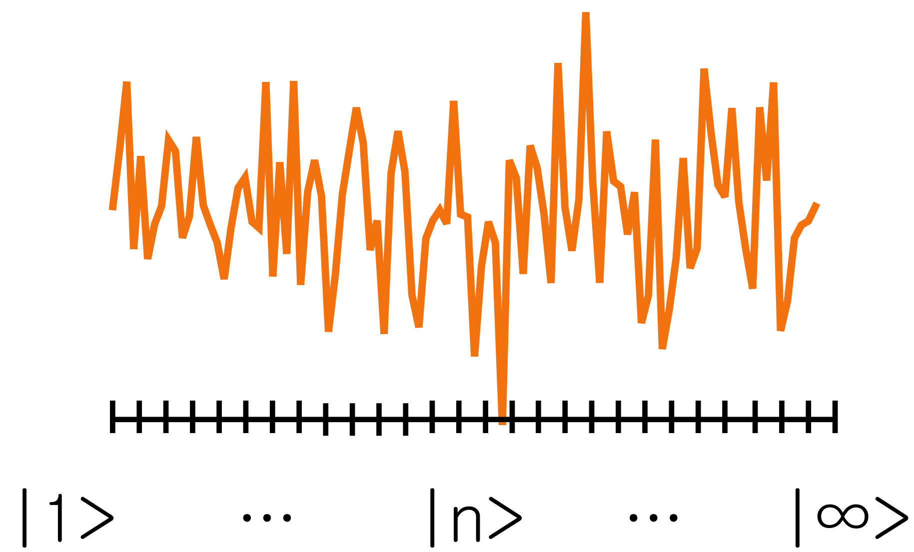

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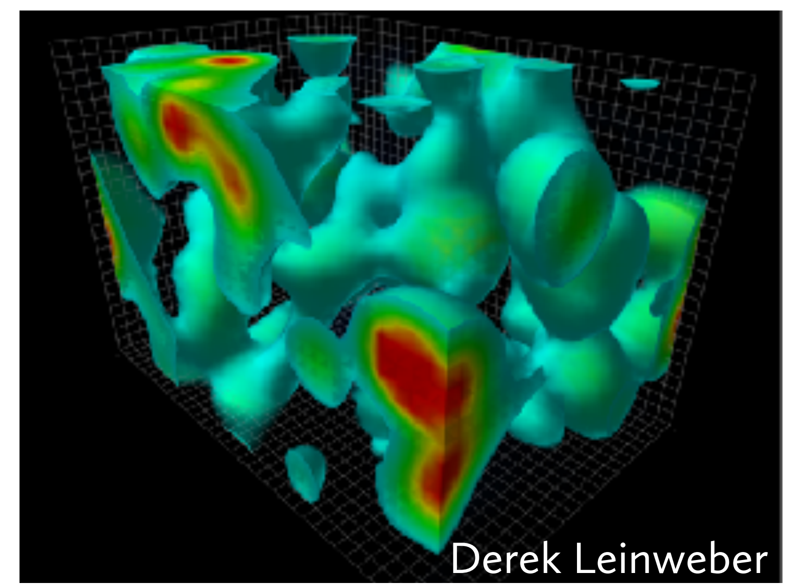

Yes! The "good" basis will exhibit complex entanglement in space, color, spin relative to "theory basis" which is based on principles of locality, gauge invariance, statistics

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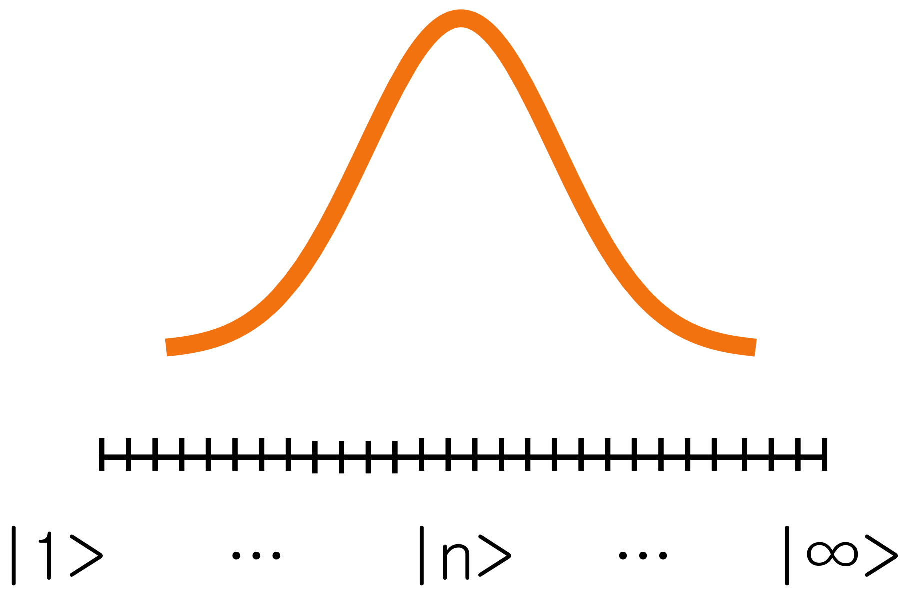

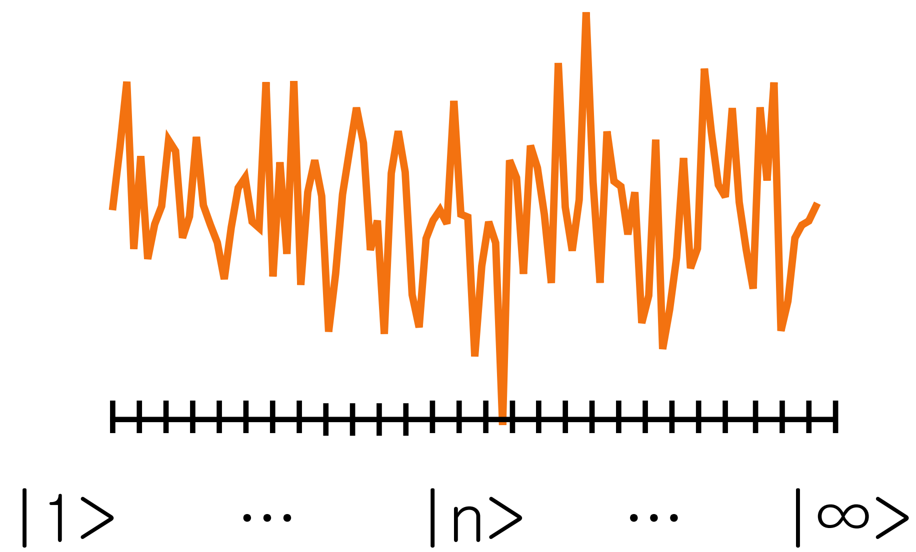

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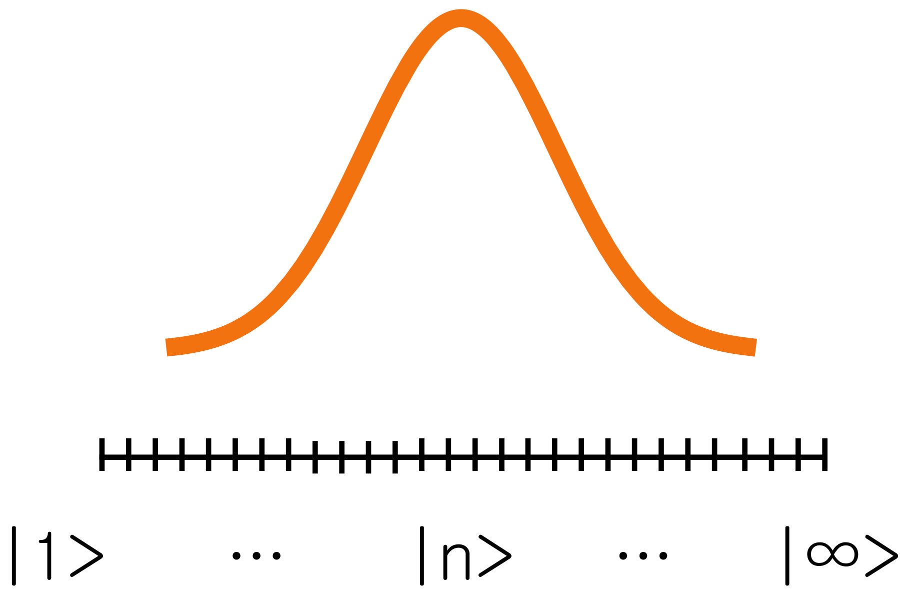

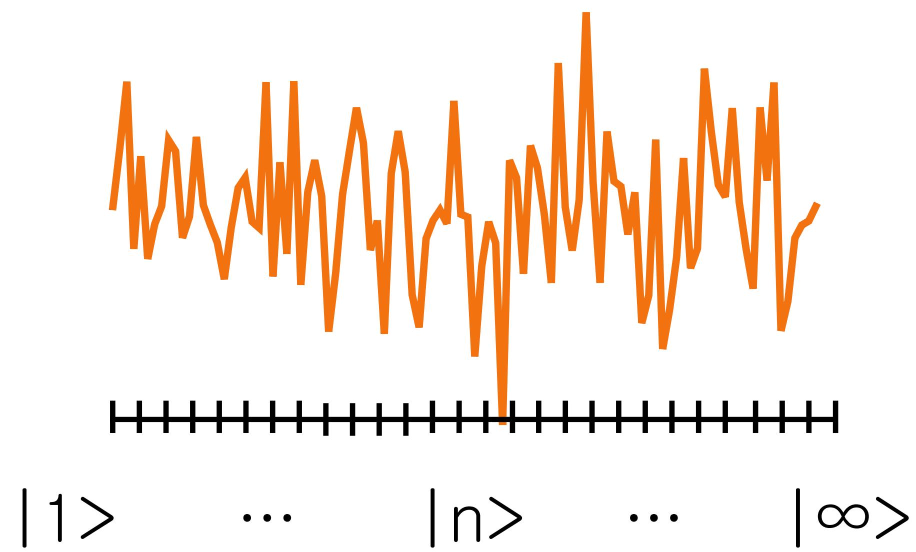

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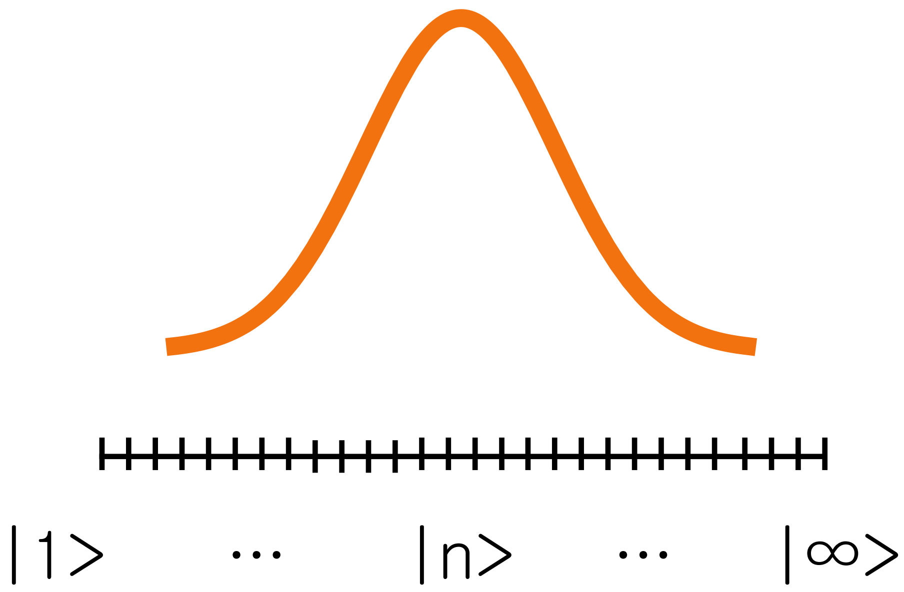

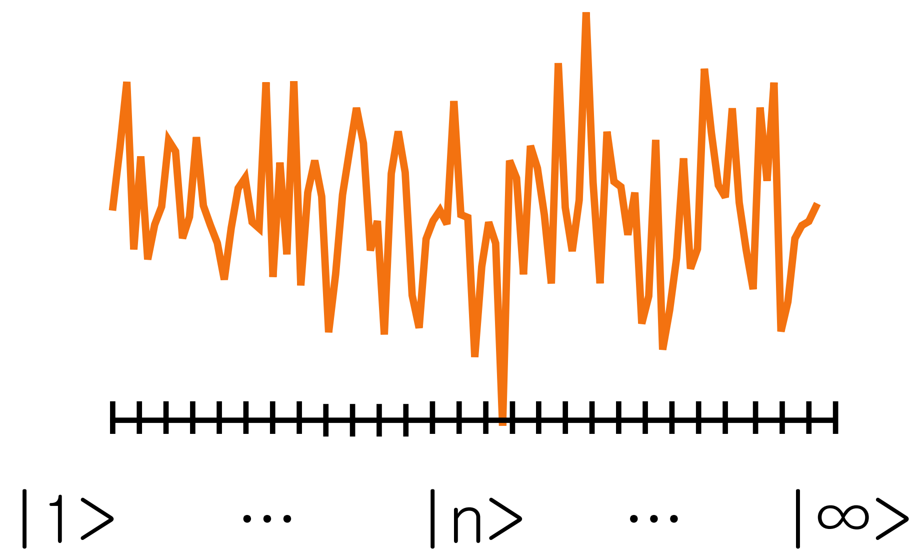

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Quantum circuits are an efficient way to create highly entangled nonlocal states from few-qubit interactions…maybe they can help!

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- Finite baryon density
- Real time dynamics
- Nontrivial topology
- N=4 SUSY, matrix models for quantum gravity
- Chiral gauge theory…

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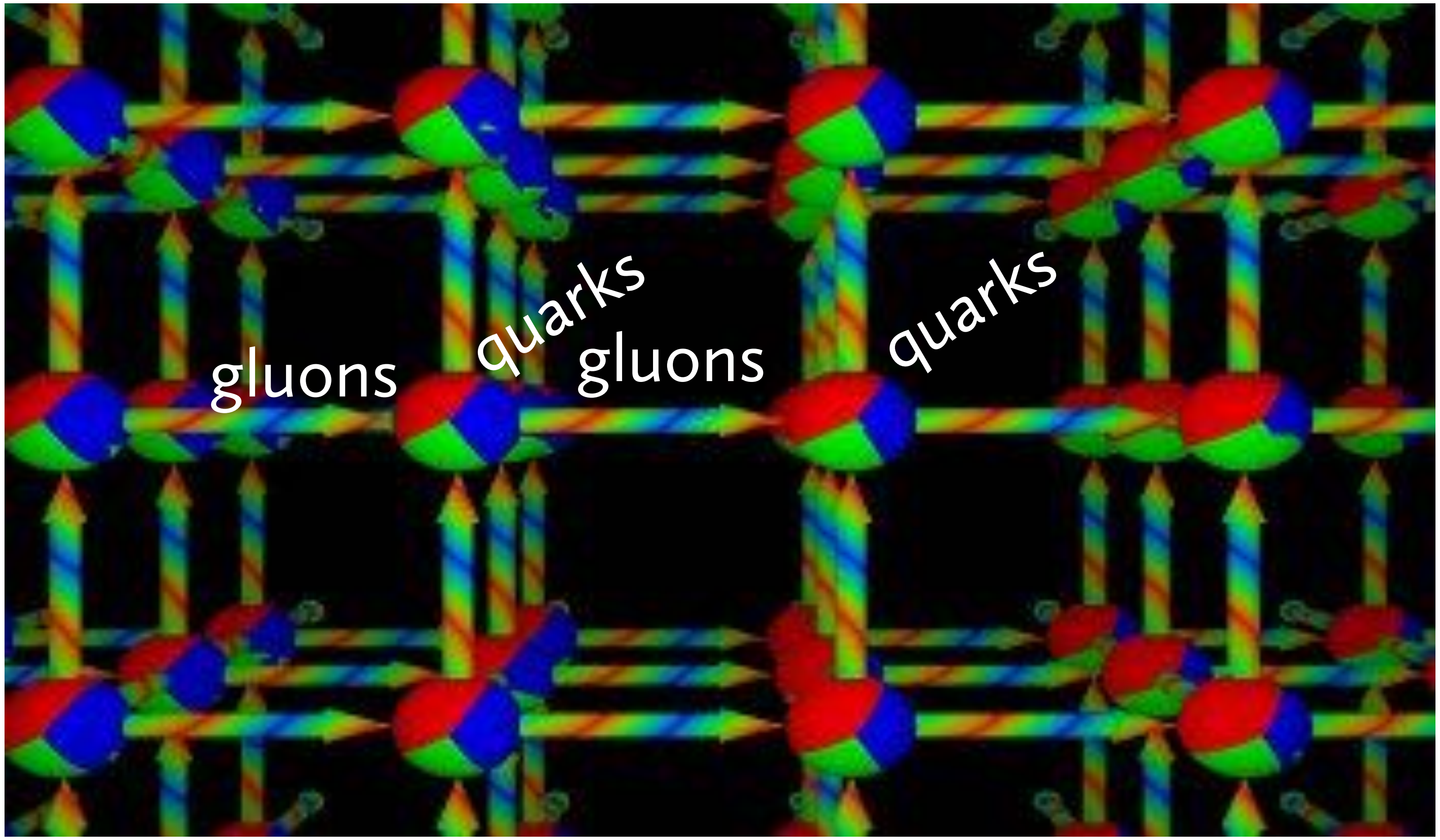
This talk: a couple comments on Yang-Mills theory

✦ Truncation in Hamiltonian formulation

✦ gauge invariance

Starting point: the Hamiltonian derived from Wilson's lattice gauge theory (Kogut & Susskind, 1975)



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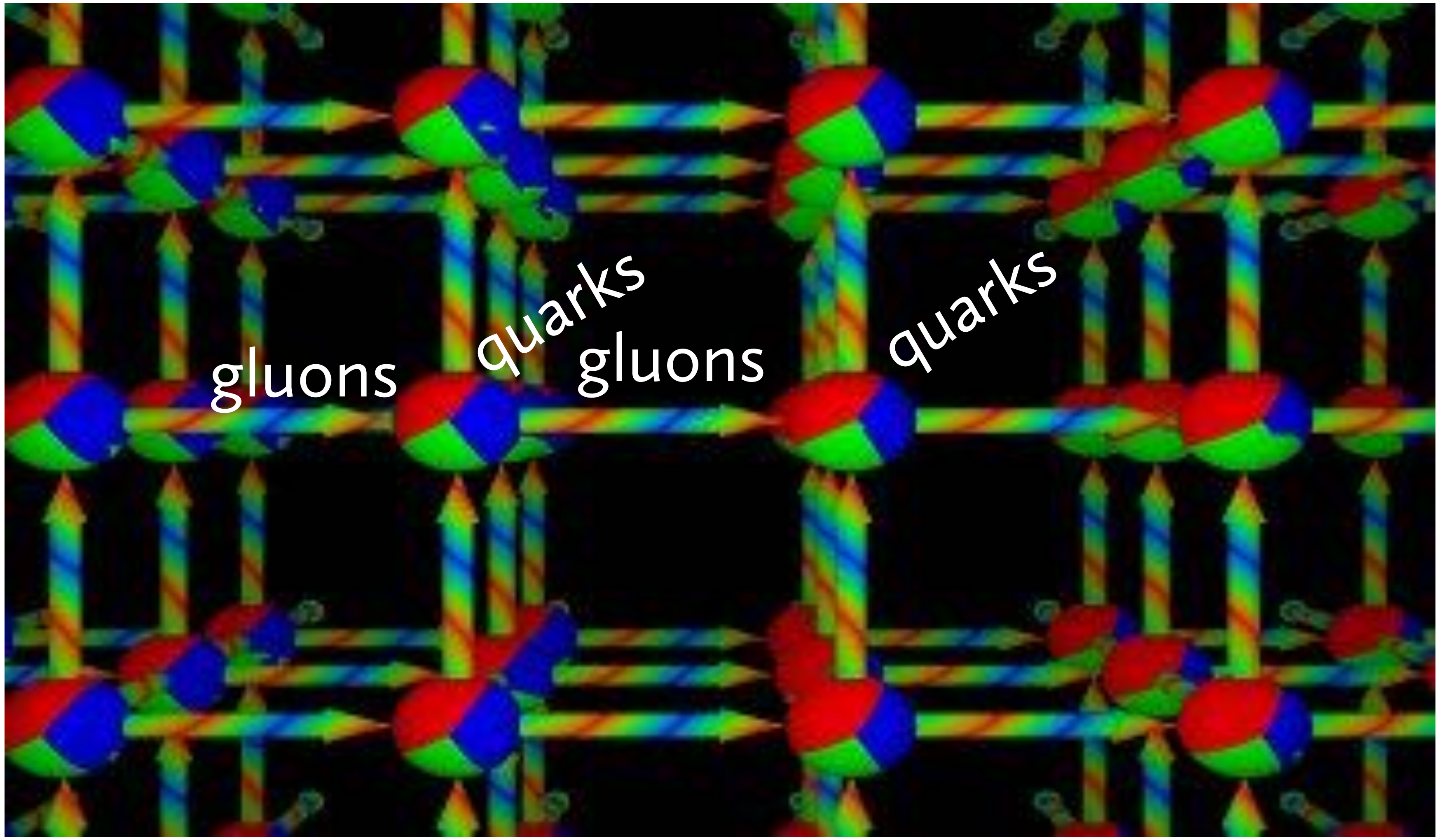

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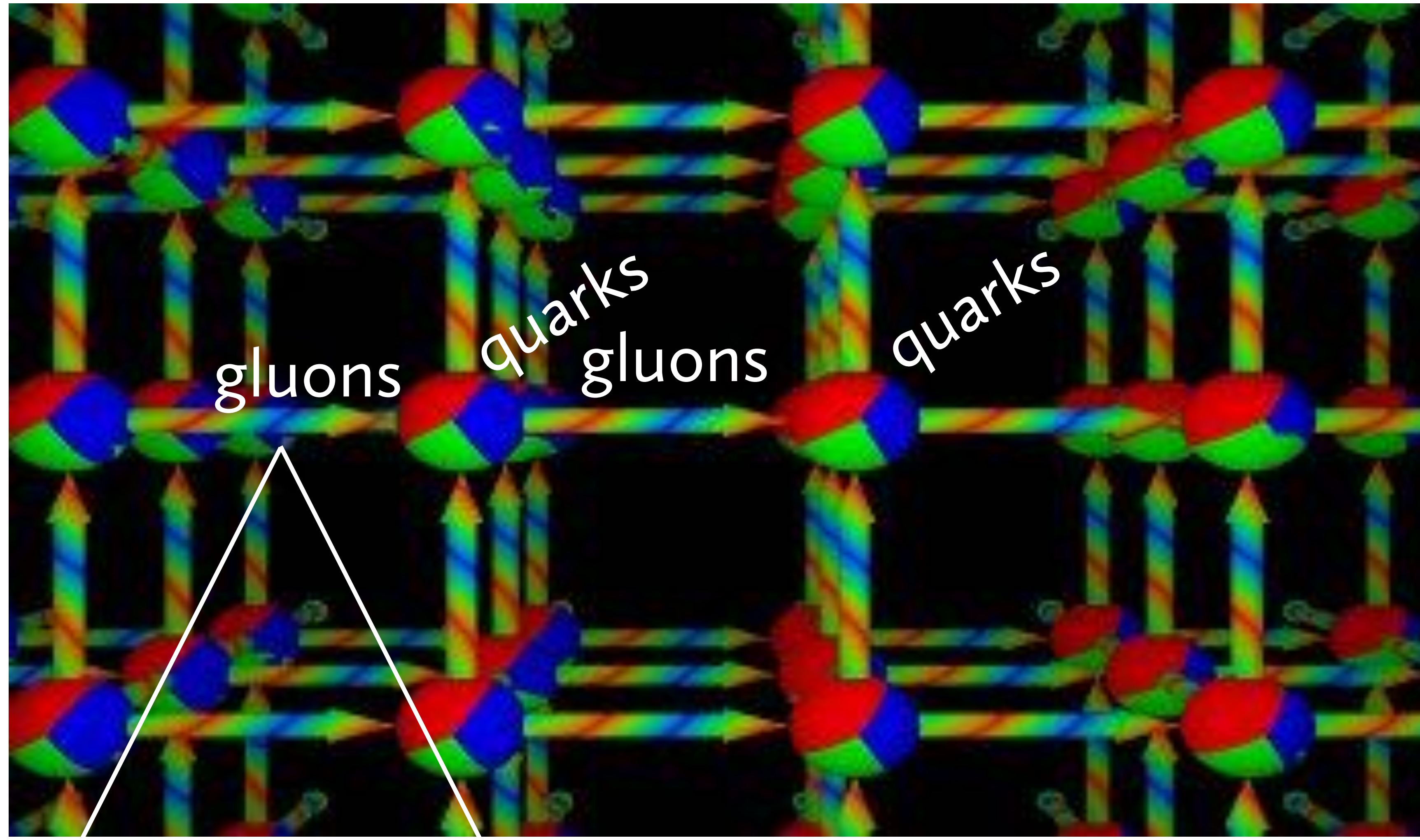

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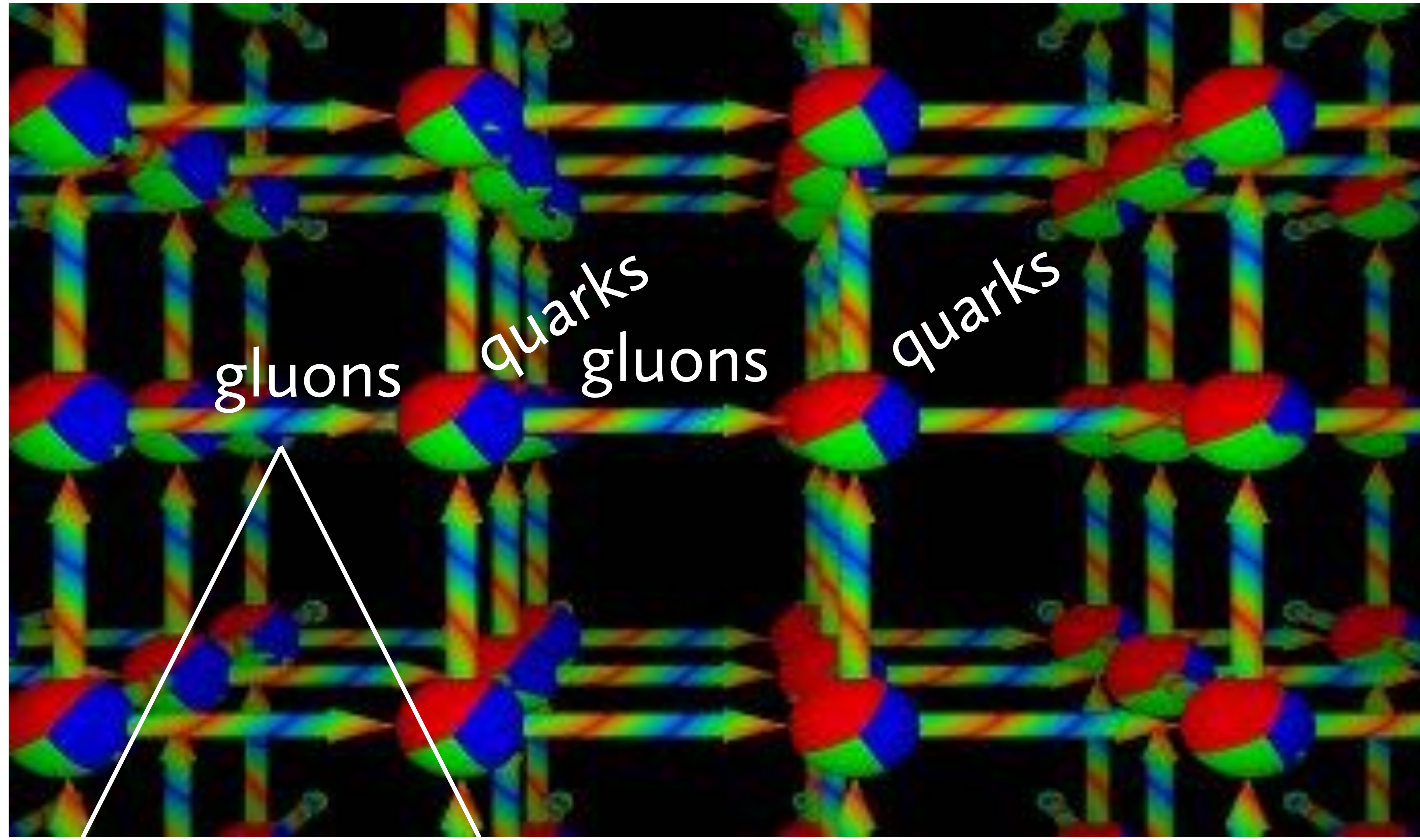

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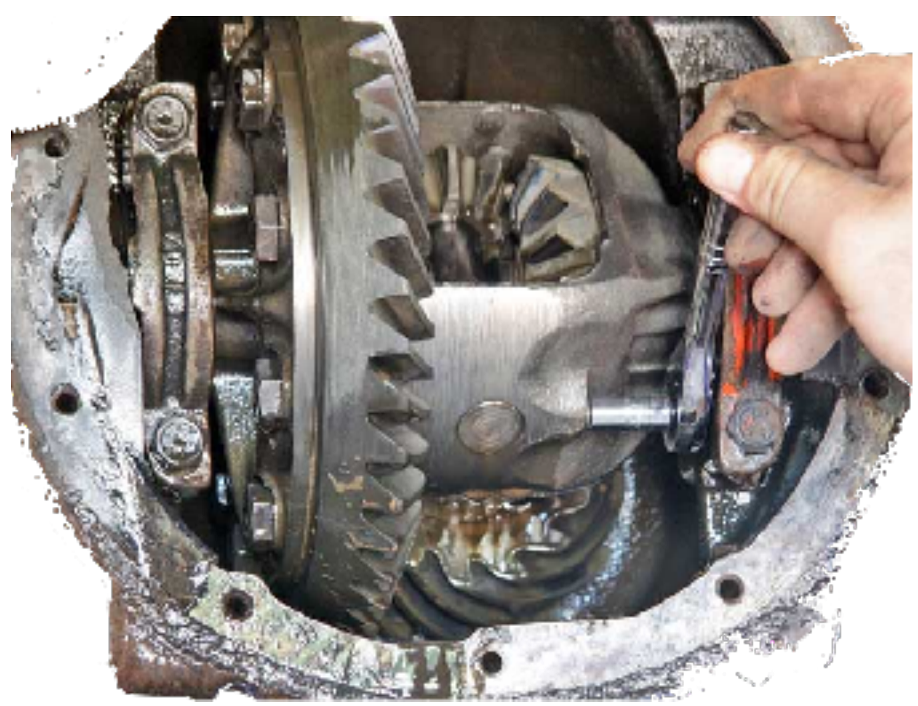

- On every link:
- $U \in G$ (matrix in gauge group)
- $U \to L\, U\, R^{\dagger}$
- $L \in G$, $R \in G$.

The continuum Yang-Mills Hamiltonian (no quarks):

- Fix A_0=0 gauge
- $H = \frac{1}{2}\left(g^2\vec{E}_a\vec{E}_a + \frac{1}{g^2}\vec{B}_a\vec{B}_a\right) \;,\qquad \left[A^i_a, E^j_b\right] = i\hbar\,\delta^{ij}\,\delta_{ab}$
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Kogut-Susskind (lattice) Yang-Mills Hamiltonian:

- Fix U=1 gauge on temporal links, U on spatial links ▸ operators
- $\vec{B}_a \vec{B}_a \to -\mathrm{Re}\,\mathrm{Tr}\,\hat{U}_\Box$ (product of U's around plaquette)
- $\vec{E}_a \vec{E}_a \to \hat{\ell}^2_a = \hat{r}^2_a$ (Casimir operator)
- $\left[\hat{\ell}_a, \hat{U}\right] = -T_a \hat{U} , \qquad \left[\hat{r}_a, \hat{U}\right] = \hat{U} T_a$
- Impose at each site: $\sum(\hat{\ell}_a + \hat{r}_a)|\psi\rangle = 0$

The Hilbert space: the link operators are coordinates in the gauge group, the ℓ_a, r_a operators are their conjugates

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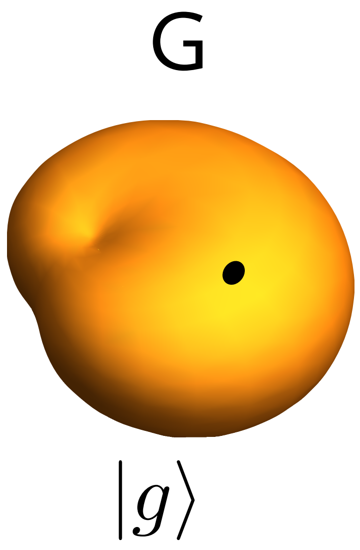
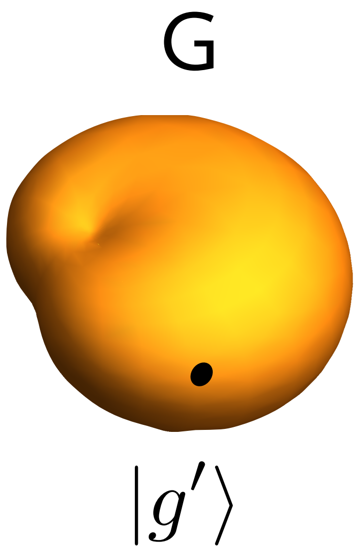
"coordinate" basis:

$$\langle g|g'\rangle = \delta(g - g')\,, \qquad \int dg\, |g\rangle\langle g| = \mathbf{1}$$





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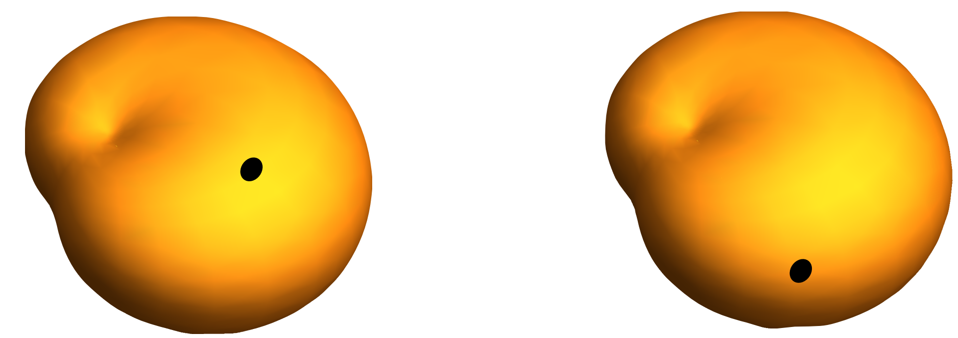
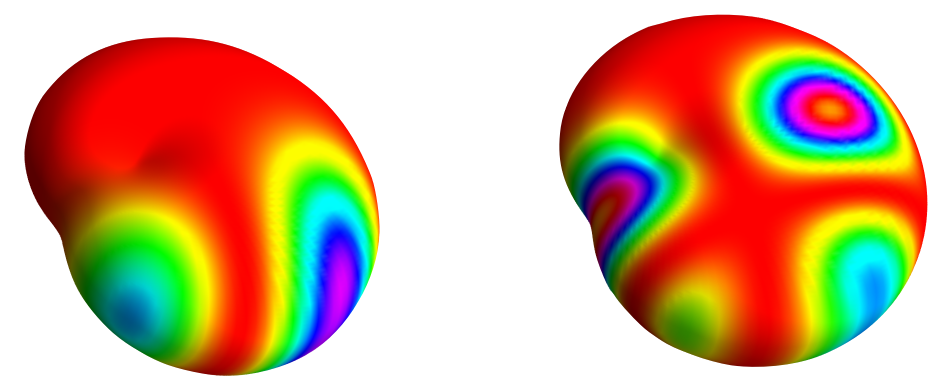


$|g\rangle$ $|g'\rangle$

"momentum" basis:

$$\langle Rab|R'a'b'\rangle = \delta_{RR'}\delta_{aa'}\delta_{bb'}\,, \qquad \sum_{Rab} |Rab\rangle\langle Rab| = \mathbf{1}$$



$|Rab\rangle$ $|R'\,a'\,b'\rangle$

$$\langle Rab|g\rangle \equiv \sqrt{\frac{d_R}{|G|}}\, D^{(R)}_{ab}(g)$$

Irreducible representations of G

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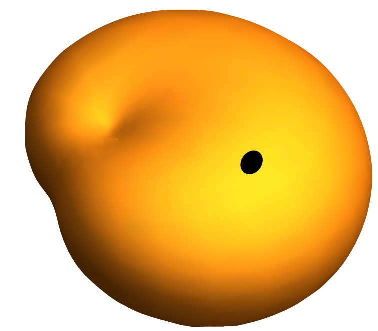

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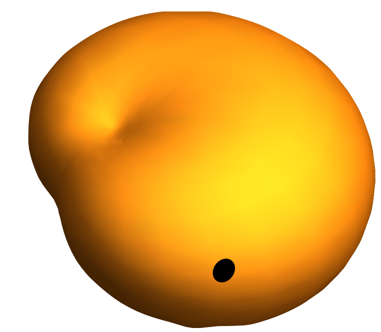

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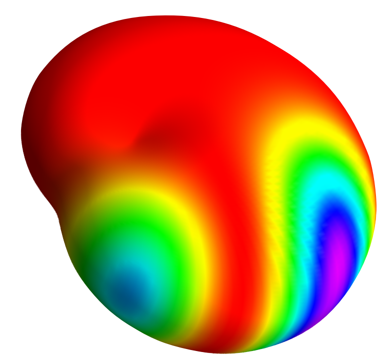
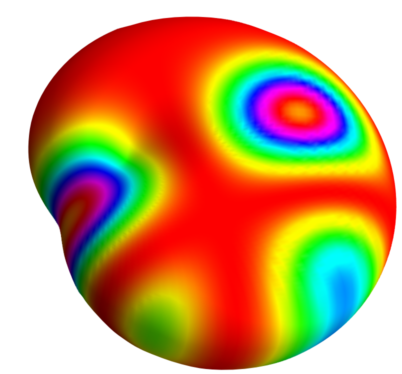


$|Rab\rangle$



$|R'\,a'\,b'\rangle$

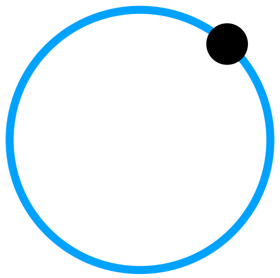
$$\langle\, Rab | g \,\rangle \equiv \sqrt{\frac{d_R}{|G|}}\, D^{(R)}_{ab}(g)$$

Irreducible representations of G

A Formulation of Lattice Gauge Theories for Quantum Simulations

Erez Zohar and Michele Burrello, Phys. Rev. D ***91****, 054506*

E.g. U(1): particle on a circle



$$|g\rangle \to |\phi\rangle \ , \qquad \phi \in [0, 2\pi)$$

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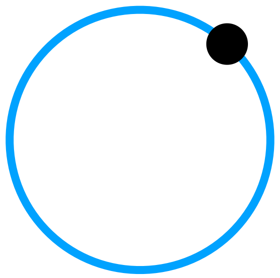

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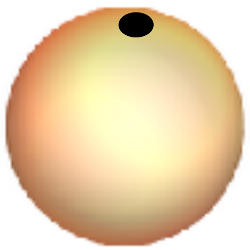

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(Wigner d-matrices)

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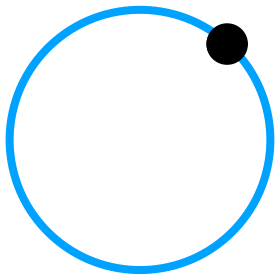

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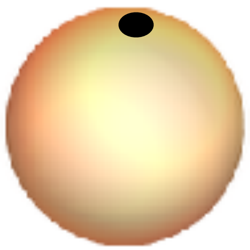

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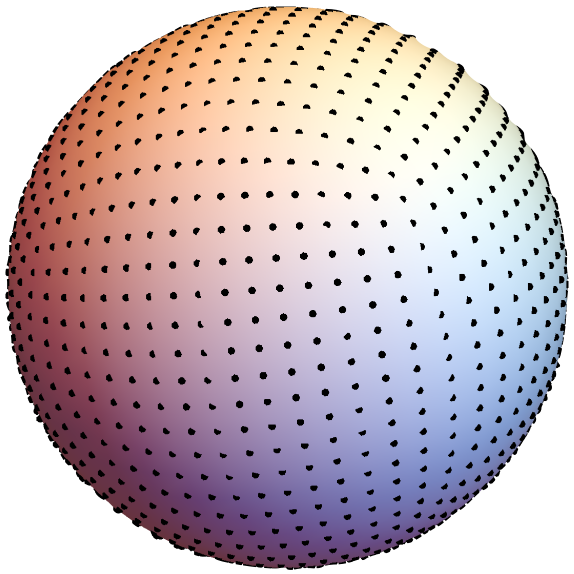
Even with spatial lattice, we have an infinite-dimension Hilbert space:

- The |g> states take continuous values
- The |Rab> states are discrete, but there are ∞ of them

“Latticize” G?



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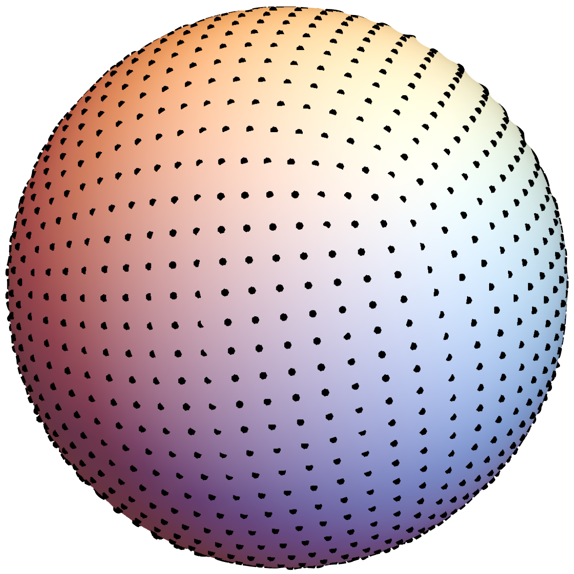

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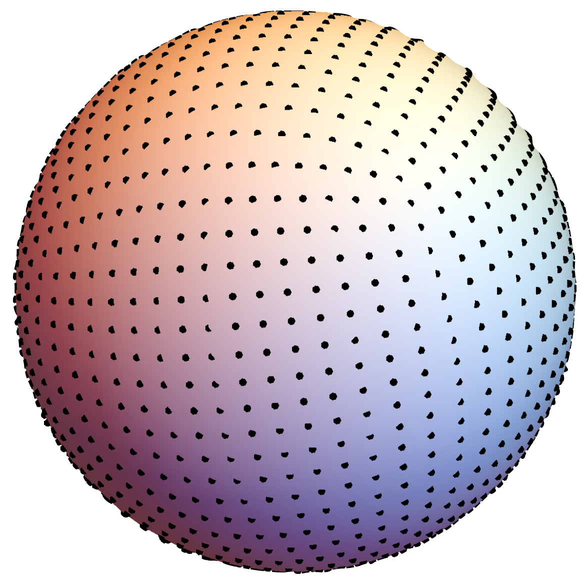

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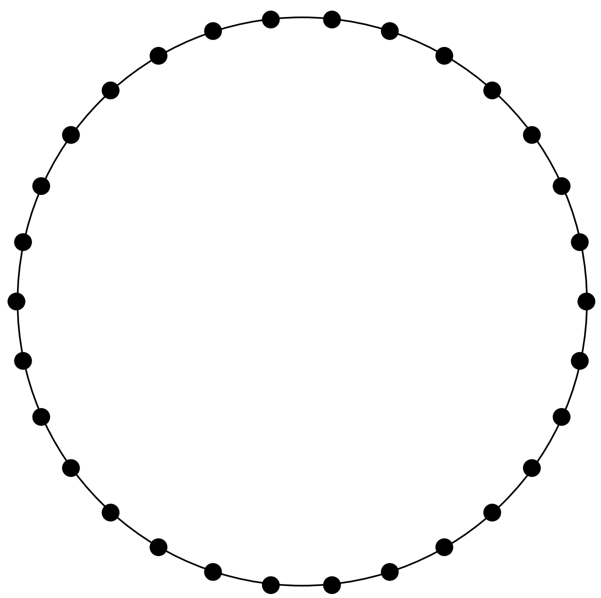
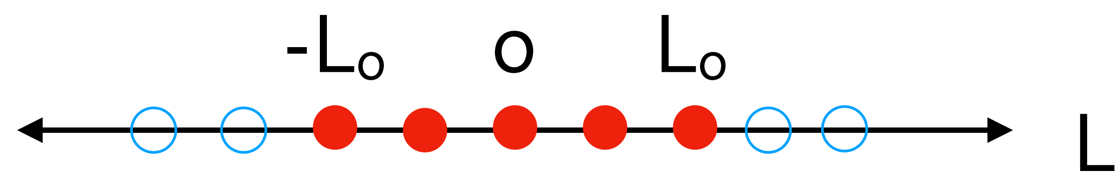
.... except for $Z_N \in U(1)$



Cutoff on |Rab> states (canonical momentum cutoff)?

E.g. U(1), cutoff on L



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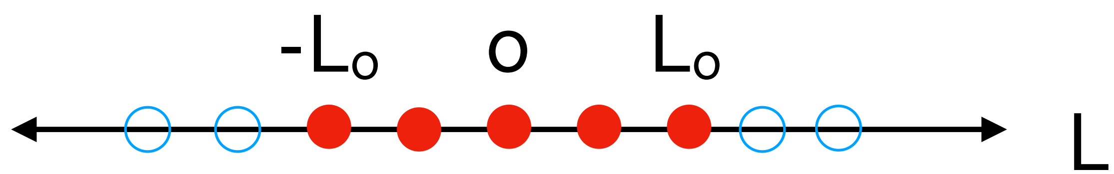
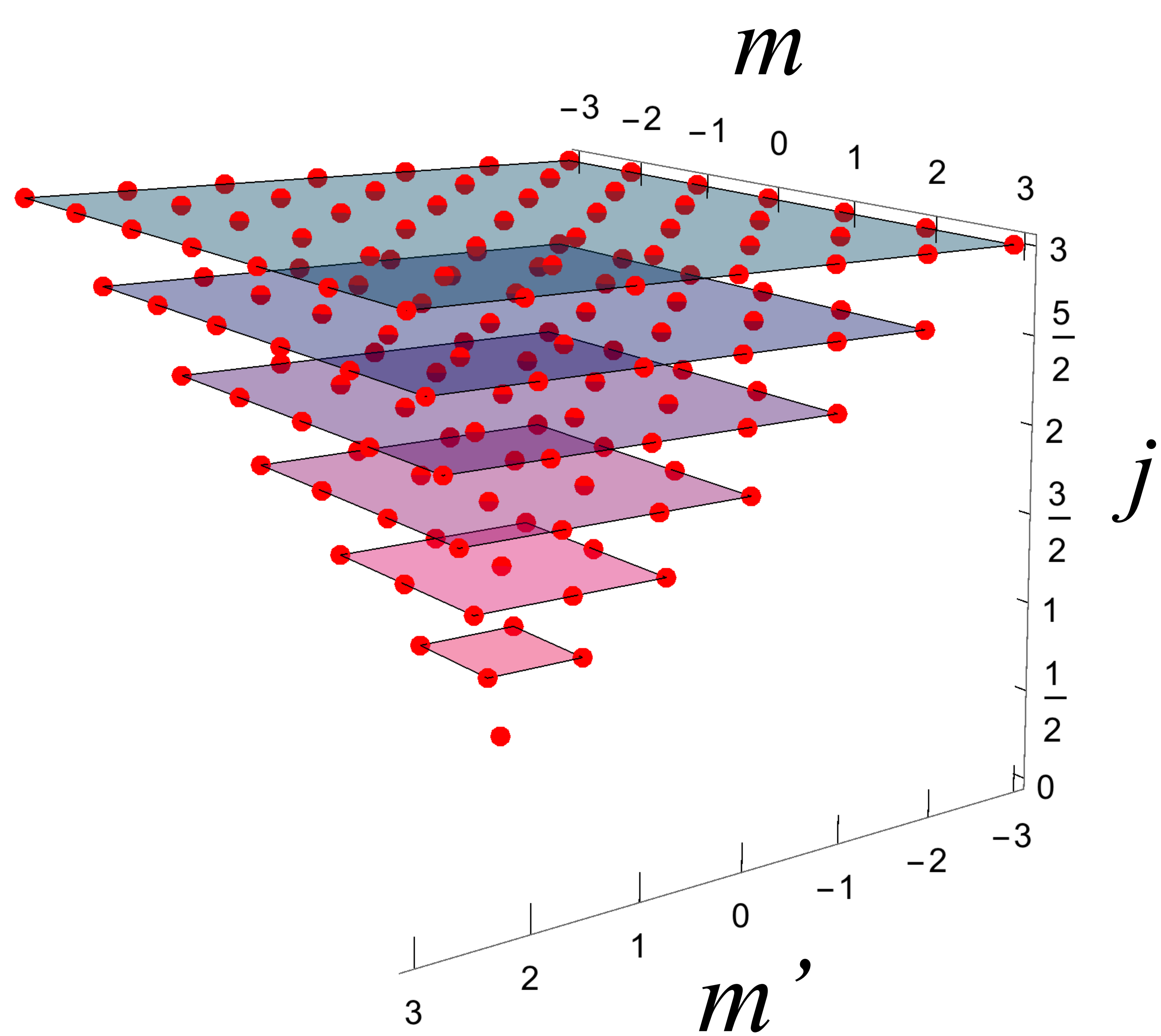


E.g. SU(2), cutoff on j:



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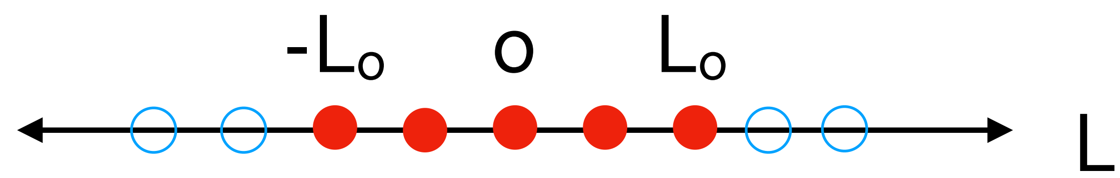

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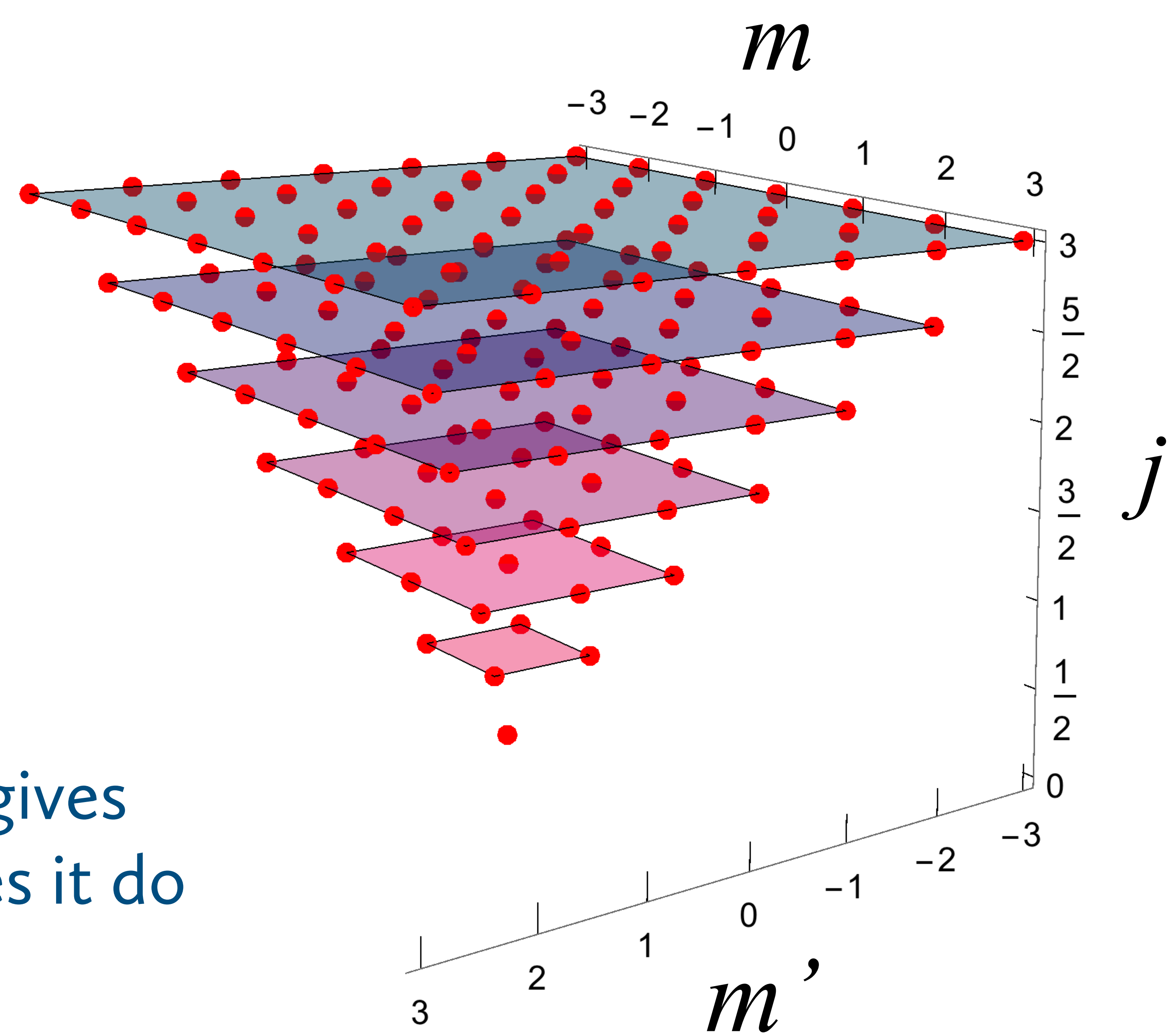

This maintains gauge symmetry, gives finite Hilbert space, but what does it do to the physics?

A cutoff on E ("p") gives dispersion in B ("x")

Photons or gluons have minimum B.B energy contribution…will give a mass gap depending inversely on the cutoff on E.

- Can this be quantified?
- Is there a "Symanzik action", RG group for the effects of this cutoff?

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U(1) example:

With cutoff on L: $$\hat{U}_{LL'} = \begin{pmatrix} 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

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NEEDS INVESTIGATION

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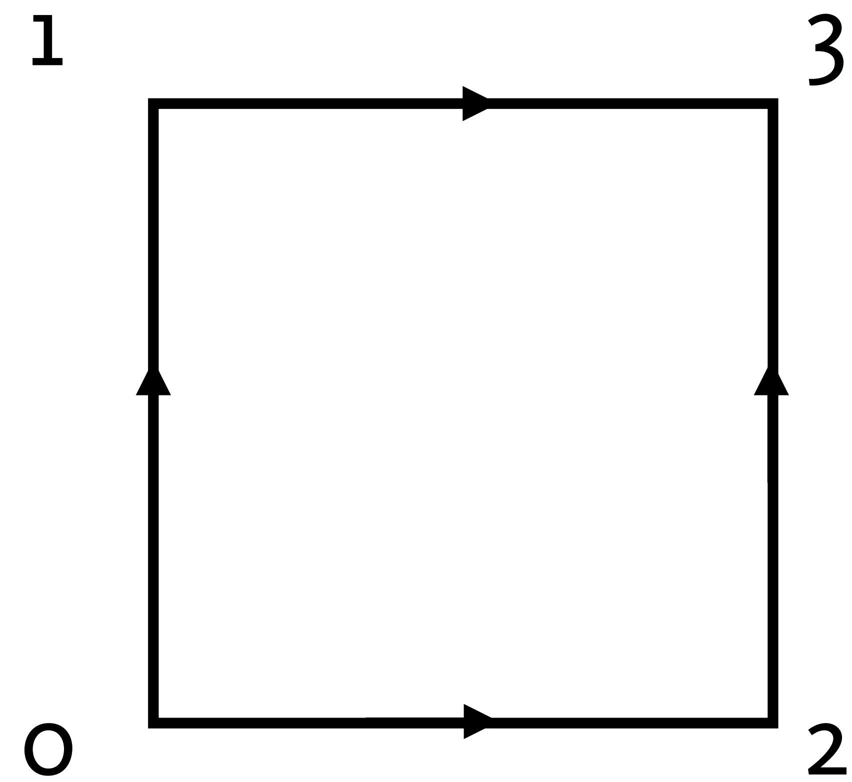
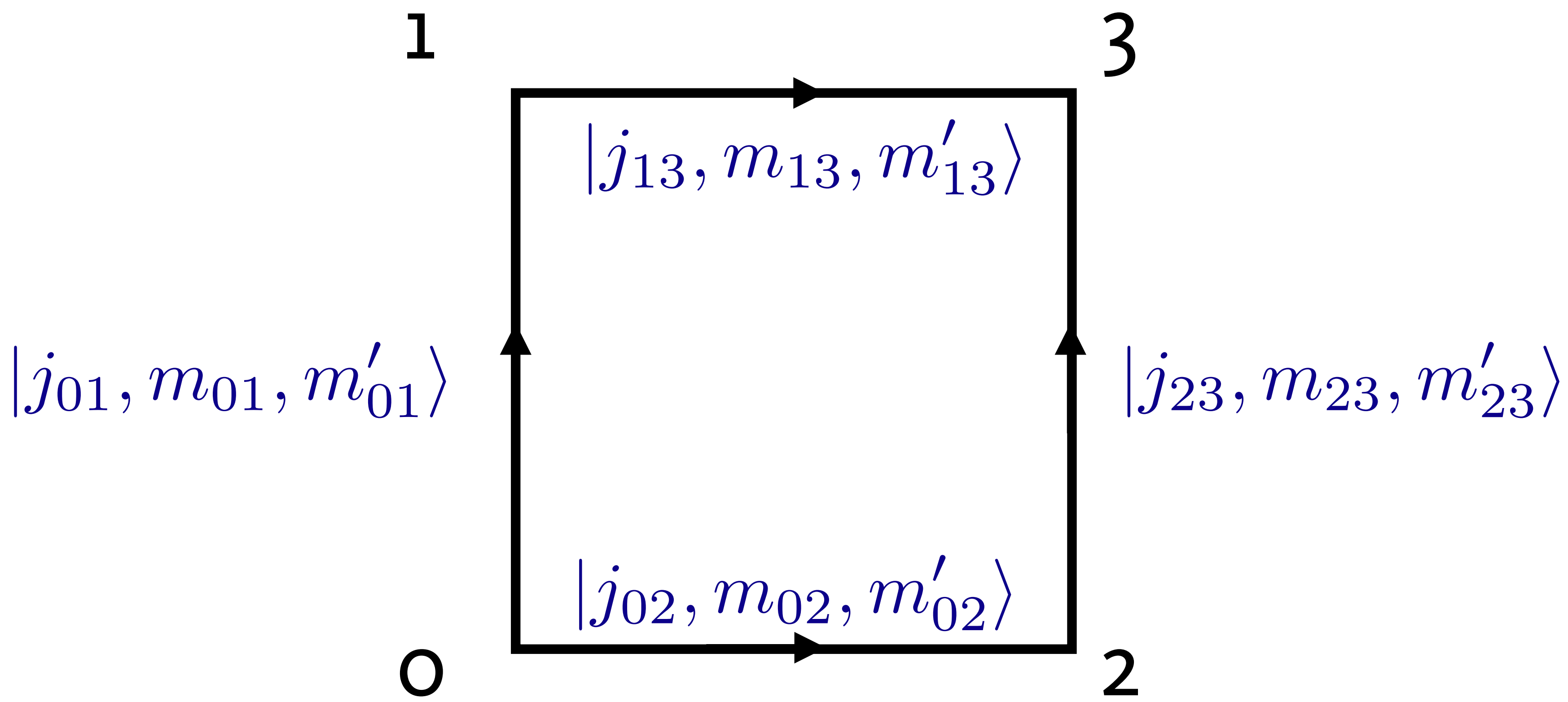
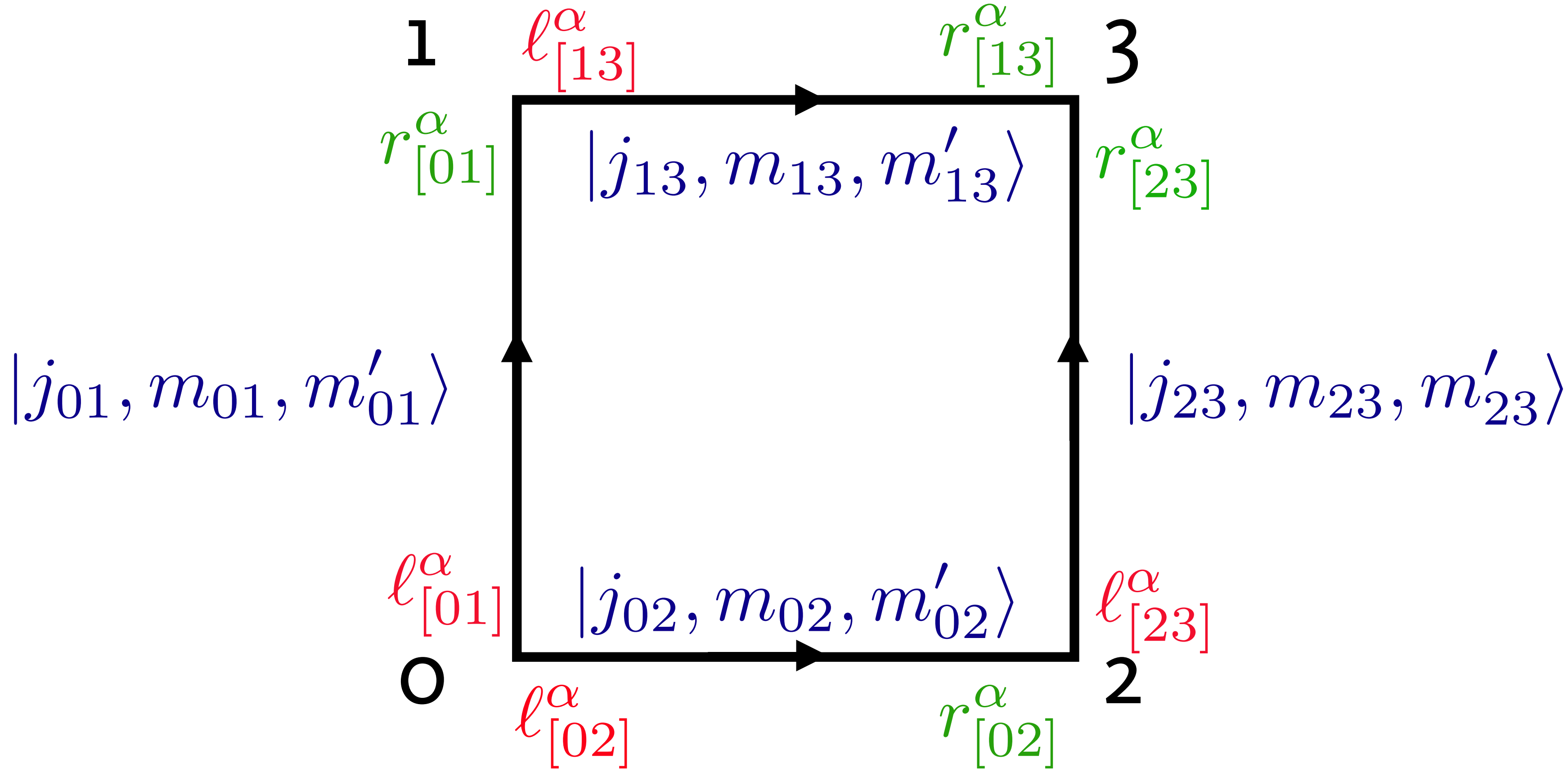
Example: "glueballs" in SU(2), 2+1 dimensions, four lattice sites.

minimal:

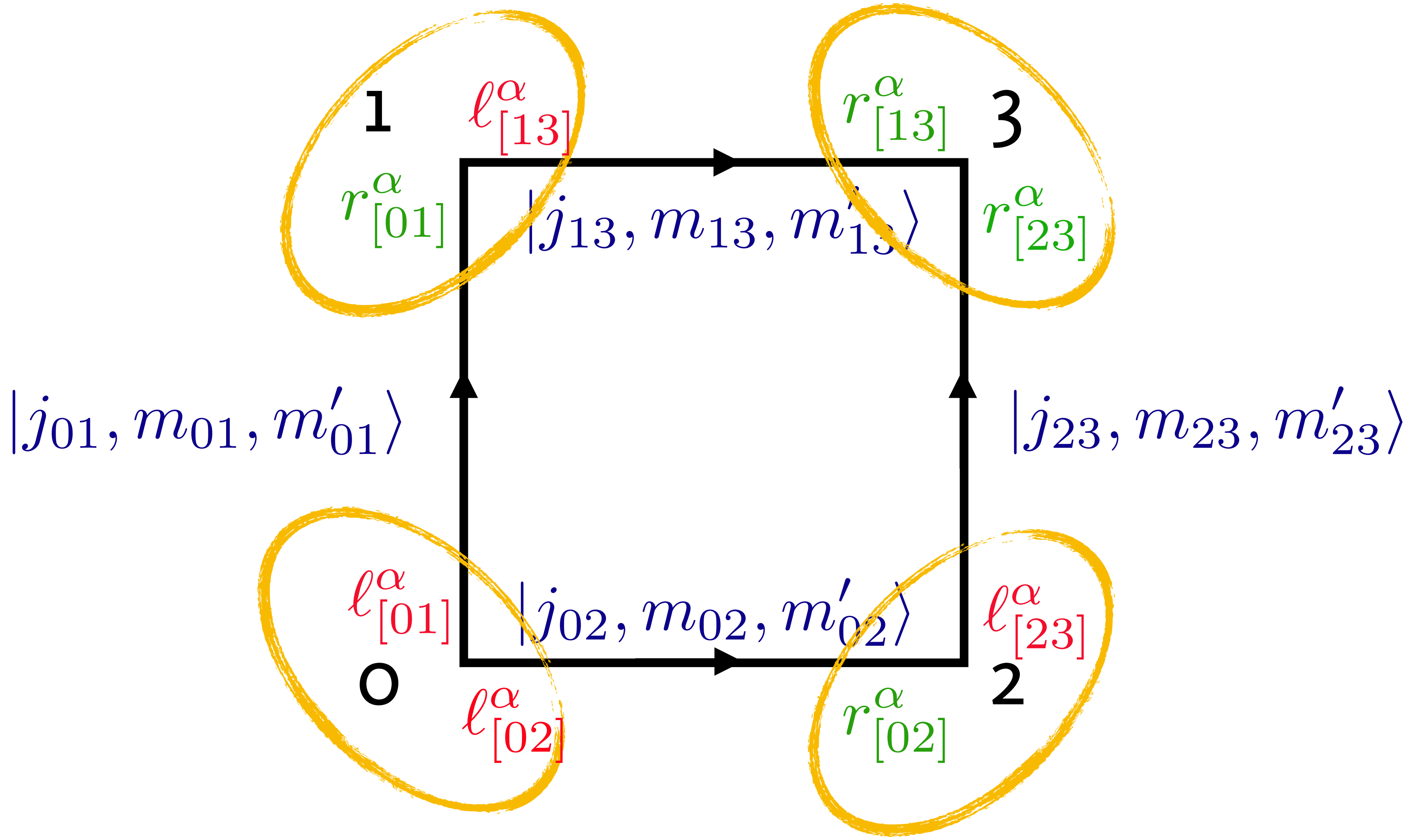
- no glueballs in 1+1 dimensions
- no bluebells in 2+1 with less than 1 plaquette


1
3
0
2

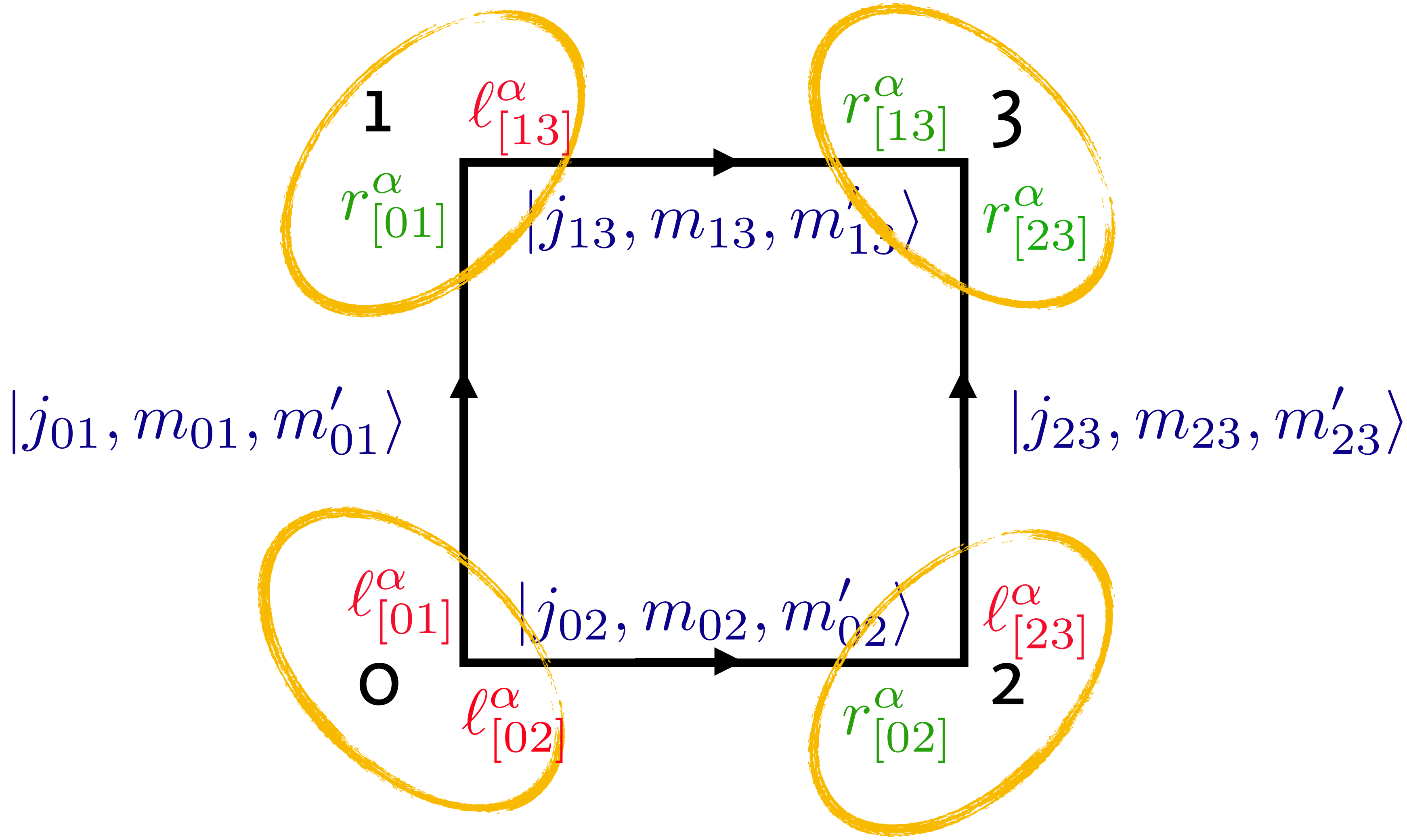

1
3
$|j_{13}, m_{13}, m'_{13}\rangle$
$|j_{01}, m_{01}, m'_{01}\rangle$
$|j_{23}, m_{23}, m'_{23}\rangle$
$|j_{02}, m_{02}, m'_{02}\rangle$
0
2



$$\ell^{\alpha}, r^{\alpha} \in \mathfrak{su}(2)$$



Gauge invariance constraint at each vertex $\quad \ell^\alpha, r^\alpha \in \mathfrak{su}(2)$



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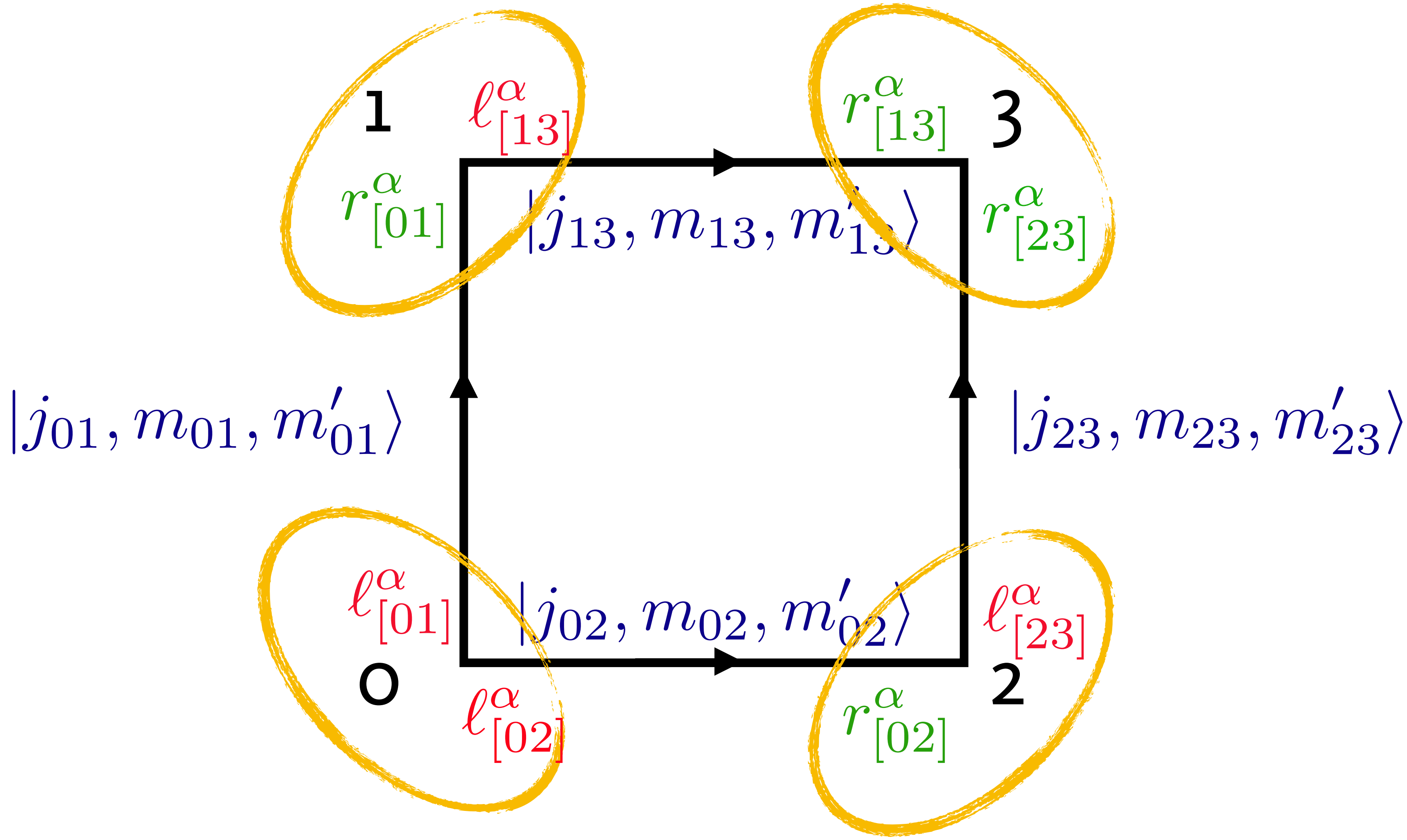
general state:

$$|\psi\rangle = |j_{01}, m_{01}, m'_{01}\rangle |j_{13}, m_{13}, m'_{13}\rangle |j_{23}, m_{23}, m'_{23}\rangle |j_{02}, m_{02}, m'_{02}\rangle$$



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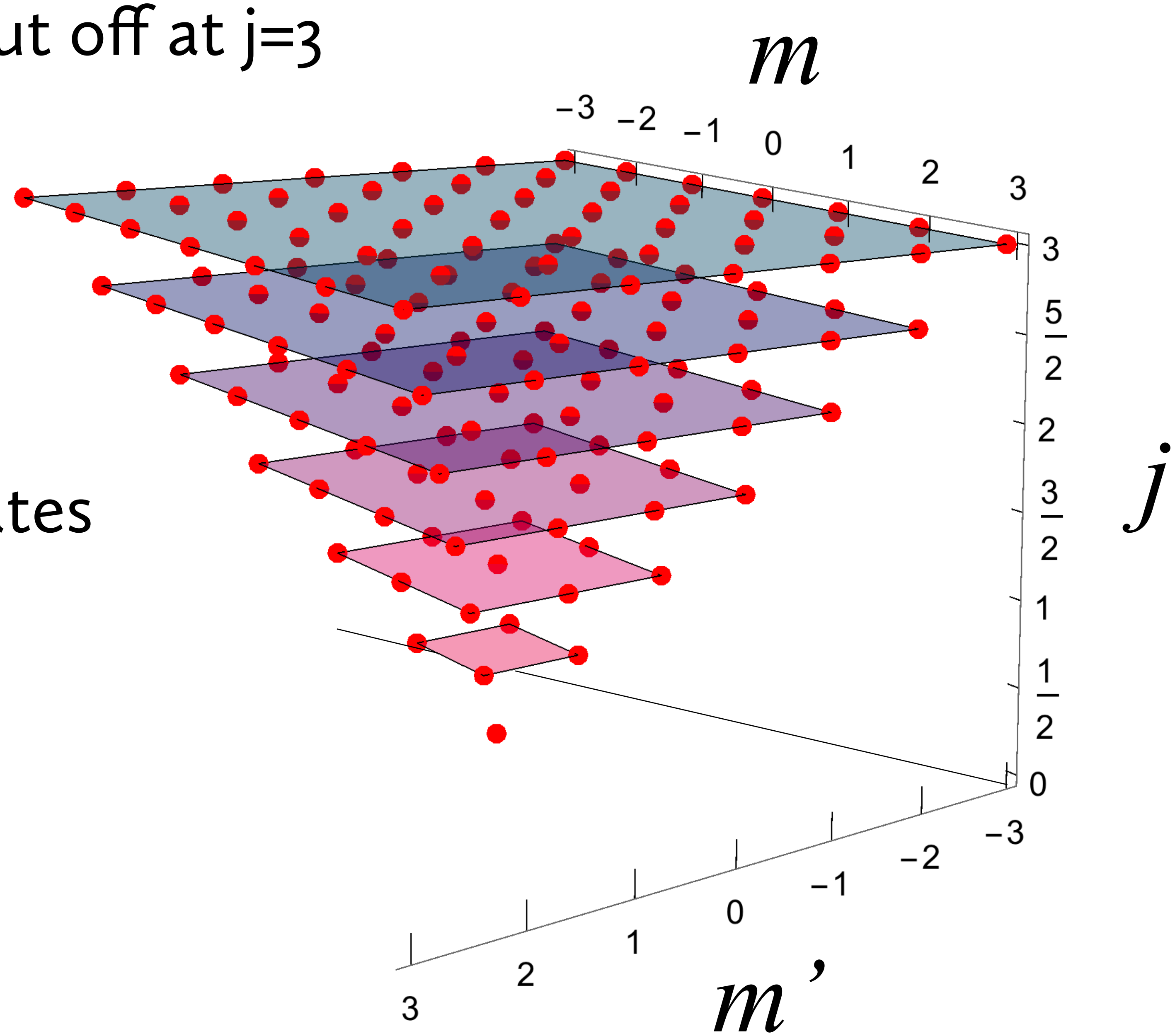
gauge invariant state:

$$|\mathcal{J}\rangle = \frac{1}{(2j+1)^2} \sum_{m_i=-j}^{j} (-1)^{-(m_0+m_3)} |j, m_0, m_1\rangle_{[01]} |j, -m_0, m_2\rangle_{[02]} |j, m_1, m_3\rangle_{[13]} |j, m_2, -m_3\rangle_{[23]}$$

SU(2) Hilbert space for one link, cut off at j=3

140 states



Hilbert space dimension for L links, cutoff J:

$$\left[\sum_{j=0}^{J}(2j+1)^2\right]^L = \left[\frac{(1+J)(1+2J)(3+4J)}{3}\right]^L$$

4-link SU(2) model:

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dimension of Hilbert space with cutoff J:

$$\mathcal{D} = \left[\frac{(1+J)(1+2J)(3+4J)}{3}\right]^4$$

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Same j on all links; all m's summed

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$$\mathcal{D}_{\text{inv}} = 2J+1$$

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dimension of gauge invariant subspace with cutoff J:

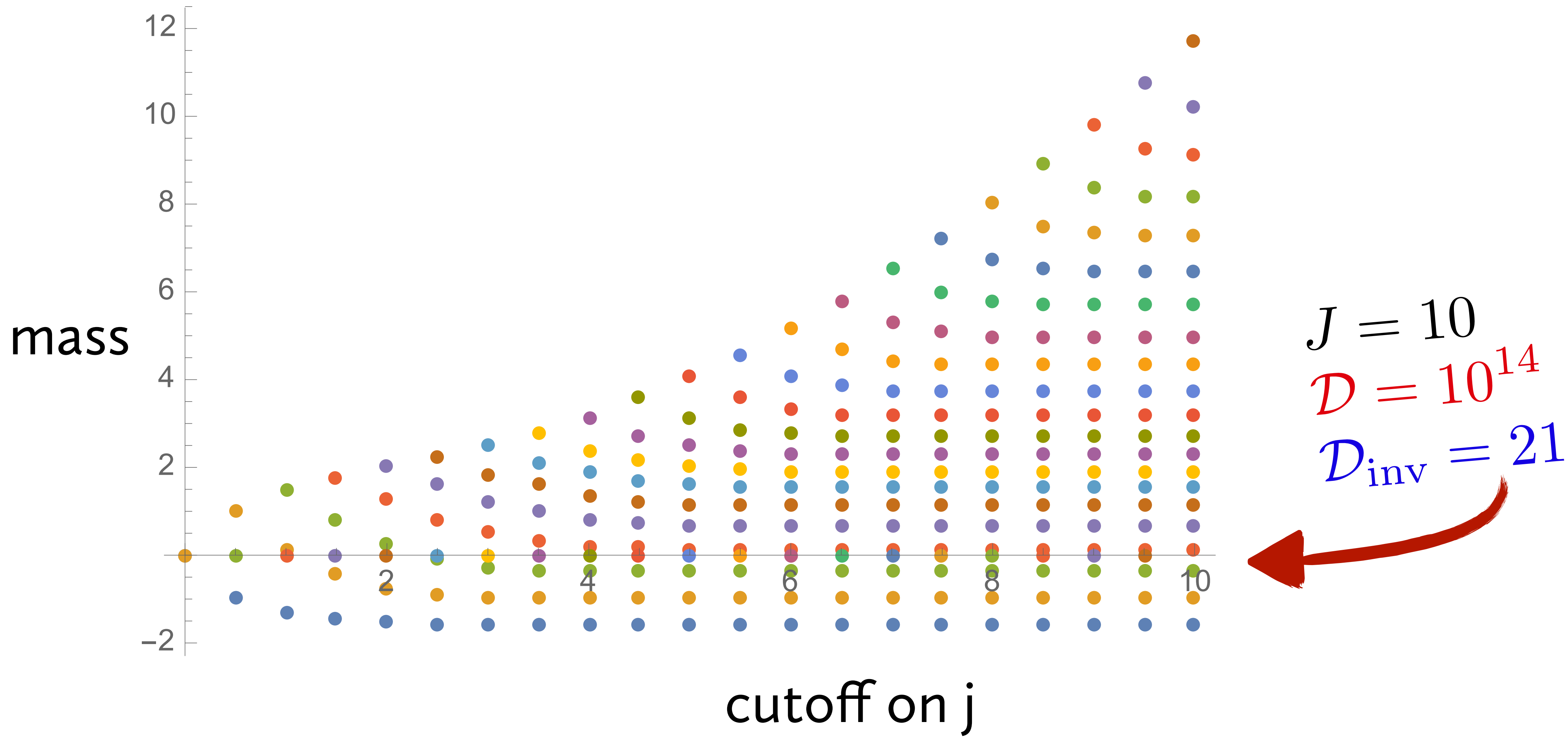
$$\mathcal{D}_{\rm inv} = 2J + 1$$

e.g.: J=3: $\mathcal{D} = 384,160,000$ $\mathcal{D}_{\rm inv} = 7$

≥ 29 qubits ≥ 3 qubits


If you aren’t shocked by gauge invariance, you haven’t understood it!

The SU(2) glue ball spectrum can be calculated quickly (Mathematica) for this simple system (because gauge invariance can be imposed analytically):



For low cutoff, can this be simulated on an existing quantum computer? Stay tuned.

Conclusions:

- Small scale nonabelian gauge theories, far from the continuum limit, can likely be simulated in on a digital quantum computer in the near term (like U(1) Schwinger model, M. Savage talk)
- There exists a straightforward formalism for representing gauge theories with a finite Hilbert space, suitable for computation
- Theorists need to better understand the physics of the cutoff on the Hilbert space for eventual large scale computations
- A vast majority of states simulated are unphysical unless the Hamiltonian can be projected onto the gauge invariant states... E.g. by using dual gauge fields? $\vec{B} = \vec{\nabla} \times \vec{a}\ , \qquad \vec{E} = \vec{\nabla} \times \vec{b}$

"Looks like a fair amount of overtime might be called for"