Ab Initio Theory

of

Light Nuclei

Solving A-nucleon systems as A-body problems with "bare" interactions that fit elastic NN scattering data.

$$H = \sum_i K_i + \sum_{i<j} v_{ij} + \sum_{i<j<k} V_{ijk}$$

Barrett, Mihaila, Pieper, & Wiringa, Nucl.Phys.News **13**, 17 (2003)

NN interactions

Nijmegen partial-wave analysis (PWA93) obtained $\chi^2/N_{data} = 1$ for 1787 *pp* + 2514 *np* observables in range $E_{lab} = 0 - 350$ MeV.

NN potentials of comparable quality became available in 1990s:

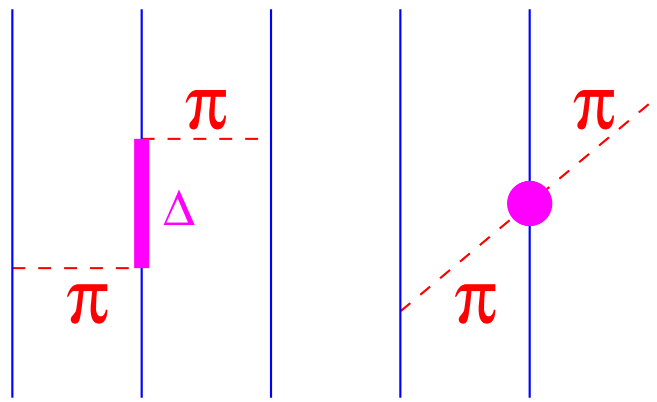
Potential	*pp*	*np*	Structure
Argonne v_{18}	1.10	1.08	Operator CD-CSB-EM local r-space
CD Bonn	1.03	1.03	PW CD OBE nonlocal k-space
Nijmegen I	1.00	1.04	PW CD nonlocal r- & k-space
Nijmegen II	1.00	1.04	PW CD local r- & k-space
Reid 93	1.00	1.04	PW CD r-space

New Nijmegen PWA in progress; new models, e.g., CD Bonn 2000 and various EFT-based potentials, continue to be developed.

Representation of *NN* interaction is already VERY GOOD.

$3N$ interactions

First models constructed by Fujita & Miyazawa in 1950s from p-wave 2π-exchange (TPE) with intermediate Δ resonance.



Most three-body scattering data fit by good NN interactions; some problems remain – three-body PWA needed for progress.

Potential	Fit	Structure
Tucson-Melbourne	πN, A=3	p- & s-wave TPE
TM$'$	πN, A=3	p- & s-wave TPE (chiral corrected)
Urbana IX	$A = 3, 4, \infty$	p-wave TPE + SR repulsion
Illinois 2	$A \leq 8$	p- & s-wave TPE + 3π-rings + SR

New EFT-based models also being produced – promise of closer relation to NN models.

Currents

Consistent charge, current, and axial operators also needed for electroweak properties, either in meson-exchange or EFT.

Benchmark test calculation of a four-nucleon bound state

Kamada et al., Phys.Rev.C **64**, 044001 (2001)
18 theorists + 7 methods solve ^{4}He for AV8$'$ NN interaction.

Method	$\langle T \rangle$	$\langle V \rangle$	E_b	$\sqrt{\langle r^2 \rangle}$
FY	102.39(5)	-128.33(10)	-25.94(5)	1.485(3)
CRCGV	102.25	-128.13	-25.90	1.482
SVM	102.35	-128.27	-25.92	1.486
HH	102.44	-128.34	-25.90(1)	1.483
GFMC	102.3(1.0)	-128.25(1.0)	-25.93(2)	1.490(5)
NCSM	103.35	-129.45	-25.80(20)	1.485
EIHH	100.8(9)	-126.7(9)	-25.944(10)	1.486(1)

Faddeev-Yakubovsky
Coupled-Rearrangement-Channel Gaussian-basis Variational
Stochastic Variational Method
Hyperspherical Harmonic
Green's Function Monte Carlo
No-Core Shell Model
Effective Interaction Hyperspherical Harmonic

Results agree to better than 0.5% !

Green's function Monte Carlo

Carlson, Nollett, Pandharipande, Pieper, Schiavilla, Wiringa

- Generate orthogonal set of $\Psi_T(J^\pi)$ from VMC calculation
- $\Psi_T(\mathbf{R})$ is vector with $2^A \times \binom{A}{Z}$ spin-isospin components
- Remove excited state contamination from Ψ_T by propagation with Green's function in imaginary time:

$$\Psi(\tau) = \{\exp[-(H - E_0)\Delta\tau]\}^n \Psi_T = G^n(\Delta\tau)\Psi_T$$

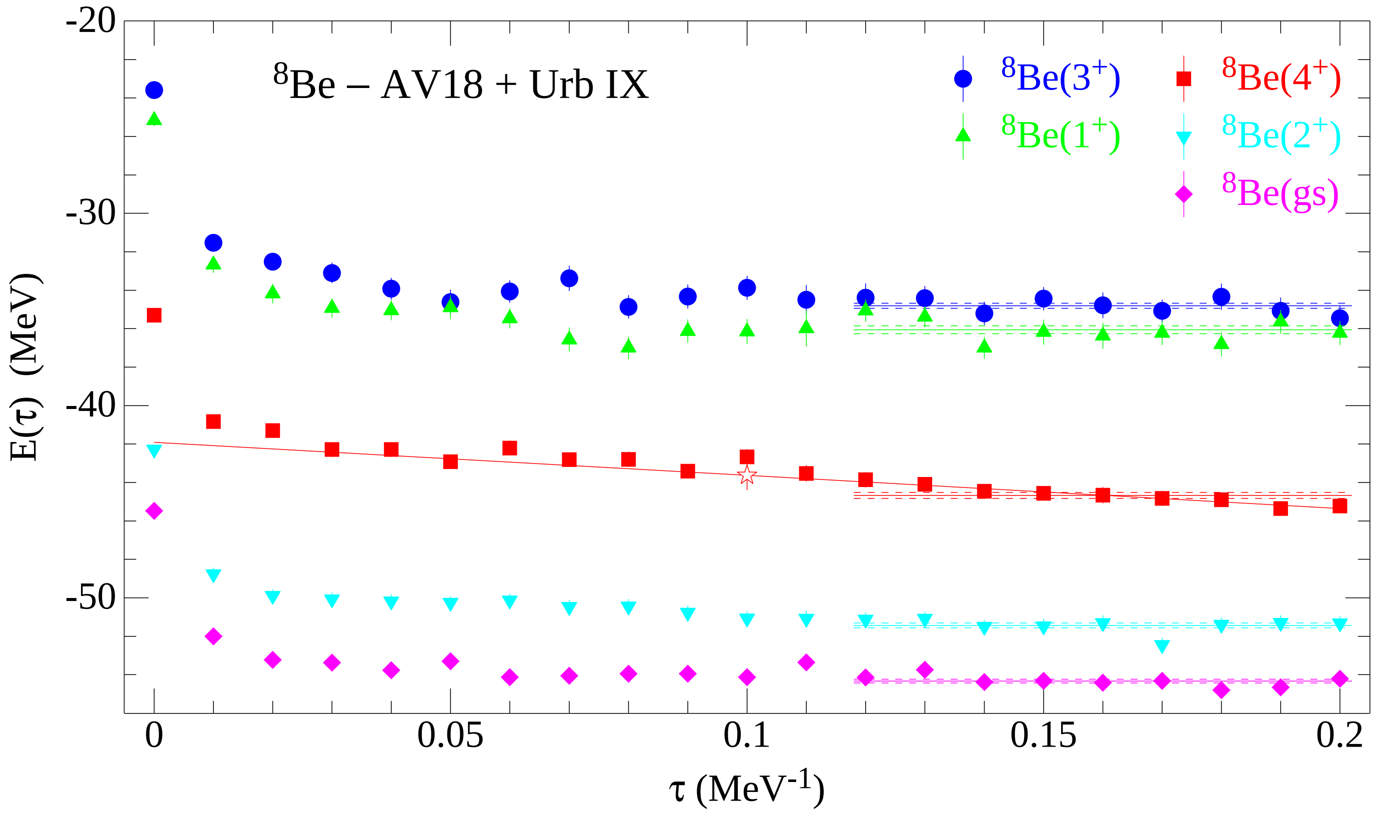
- Constrained paths needed to overcome Fermion sign problem
- Evaluate mixed energy and look for convergence in τ

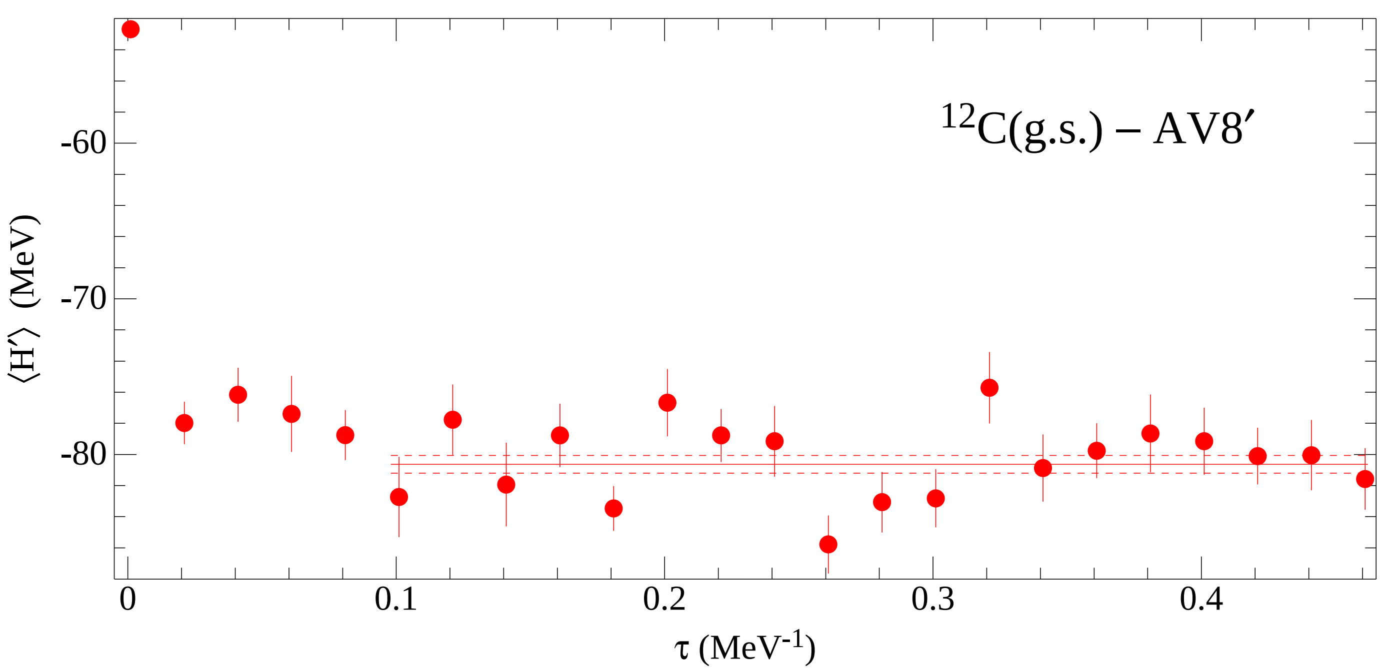
$$E(\tau) = \langle\Psi_T|H|\Psi(\tau)\rangle / \langle\Psi_T|\Psi(\tau)\rangle \geq E_0$$

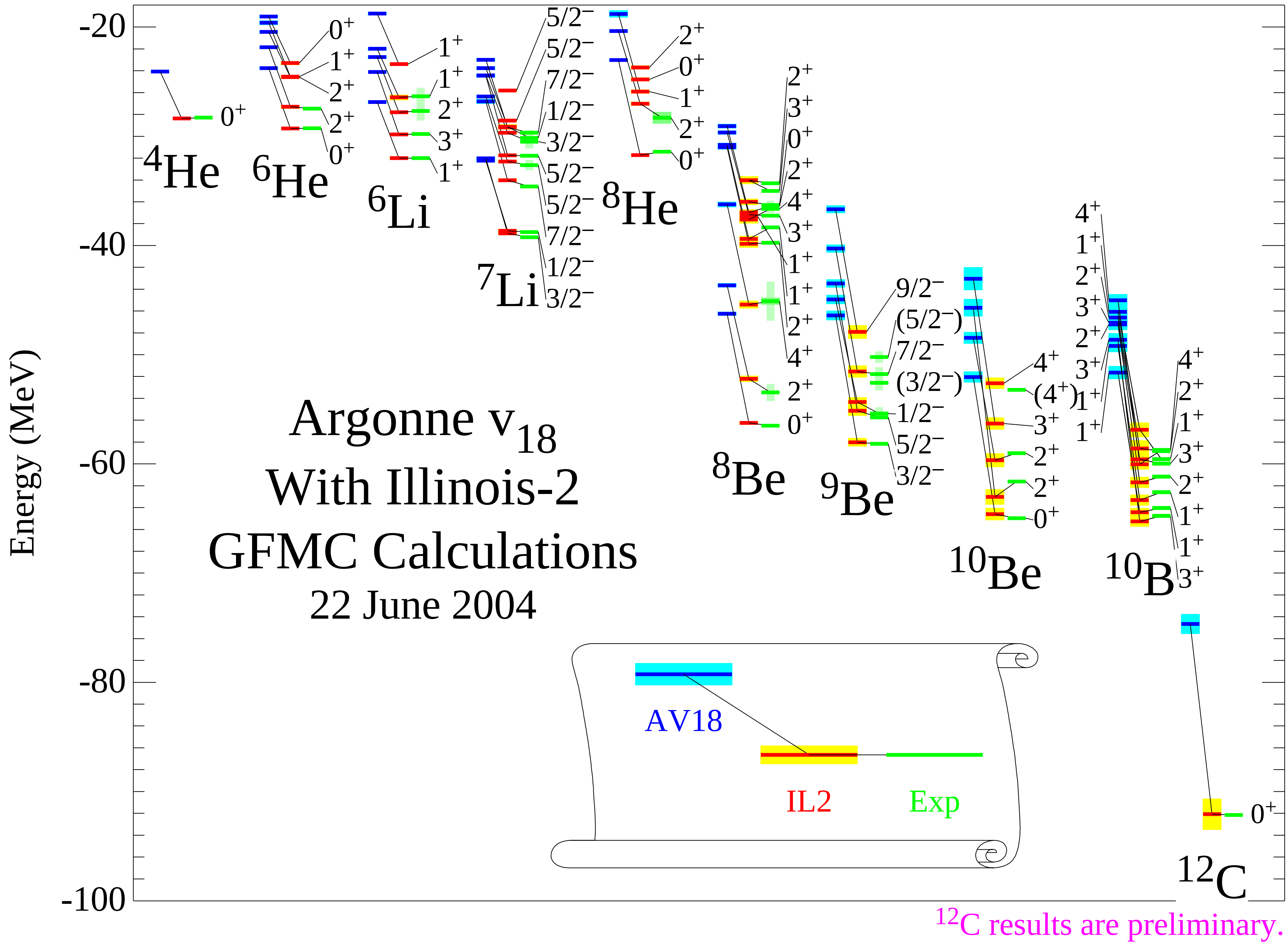
- Statistical & systematic errors $\approx$ 1-2% for $6 \leq A \leq 10$
- Each J^π state done separately
- Bare electroweak operators for transitions – VMC calculations done for form factors, weak decays, inelastic pion scattering, $(e,e'p)$, (α,γ) reactions
- CPU time grows exponentially; current limit $A \leq 12$

Pieper & Wiringa, Annu.Rev.Nucl.Part.Sci. **51**, 53 (2001)

$E(\tau)$ convergence







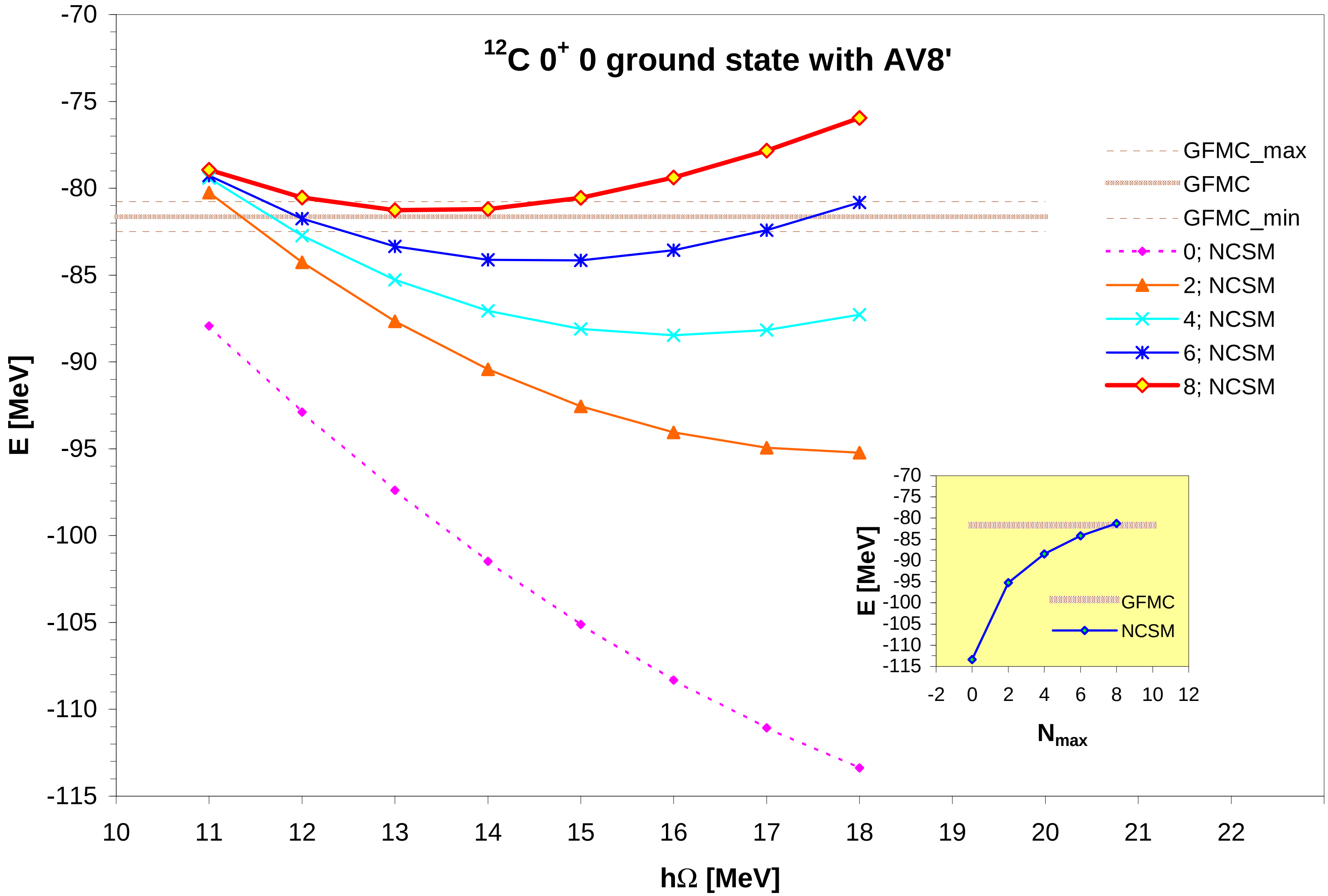
Energy (MeV)
-20
-40
-60
-80
-100
4He
6He
6Li
7Li
8He
8Be
9Be
10Be
10B
12C
Argonne v18
With Illinois-2
GFMC Calculations
22 June 2004
AV18
IL2
Exp
12C results are preliminary.

No-Core Shell Model

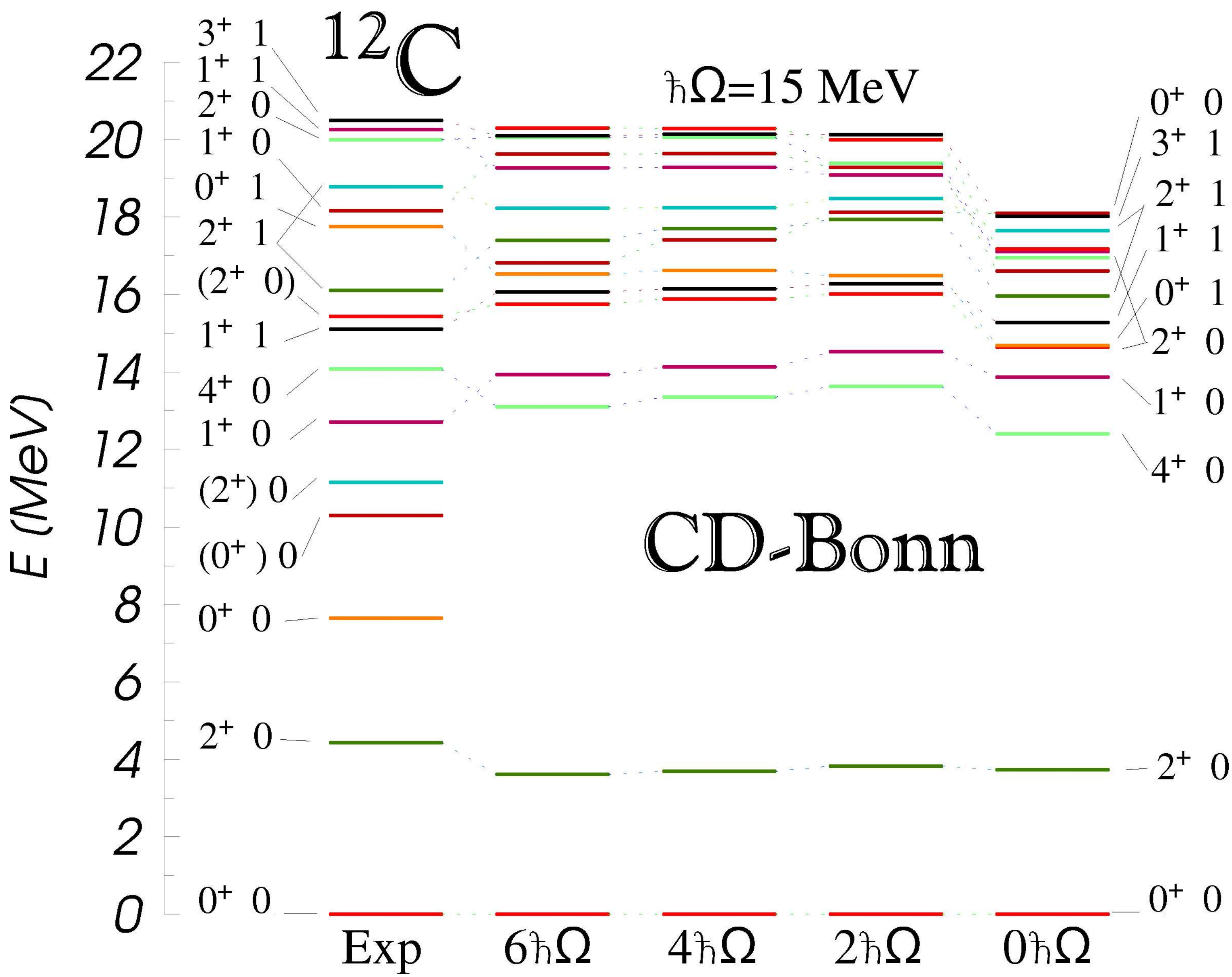
Barrett, Caurier, Navrátil, Nogga, Ormand, Vary

- Add V_{HO} ($\hbar\Omega$) to A-body H (c.m. subtracted later)
- Use similarity transformation to generate $V_{2\text{eff}}$ ($V_{3\text{eff}}$) in restricted model space ($N_{\max}\hbar\Omega$)
- Standard shell model techniques to find minimum $E(\hbar\Omega)$
- Seek convergence as $N_{\max}$ increases
  - $18\hbar\Omega$ for $A = 3, 4$ – complete convergence
  - $16\hbar\Omega$ for $A = 6$ & $V_{2\text{eff}}$
  - $8\hbar\Omega$ for $A = 12, 16$ & $V_{2\text{eff}}$ (E not converged, E^* is)
  - $4\hbar\Omega$ for $A = 22$ & $V_{2\text{eff}}$ being tested
  - $6\hbar\Omega$ for $A = 6, 7$ & $V_{3\text{eff}}$ (E not converged, E^* is)
  - $4\hbar\Omega$ for $A = 8 - 13$ & $V_{3\text{eff}}$ (E not converged, E^* is)
- Full energy spectrum at each step
- Effective operators needed for other matrix elements
- CPU requirements modest; large memory needed

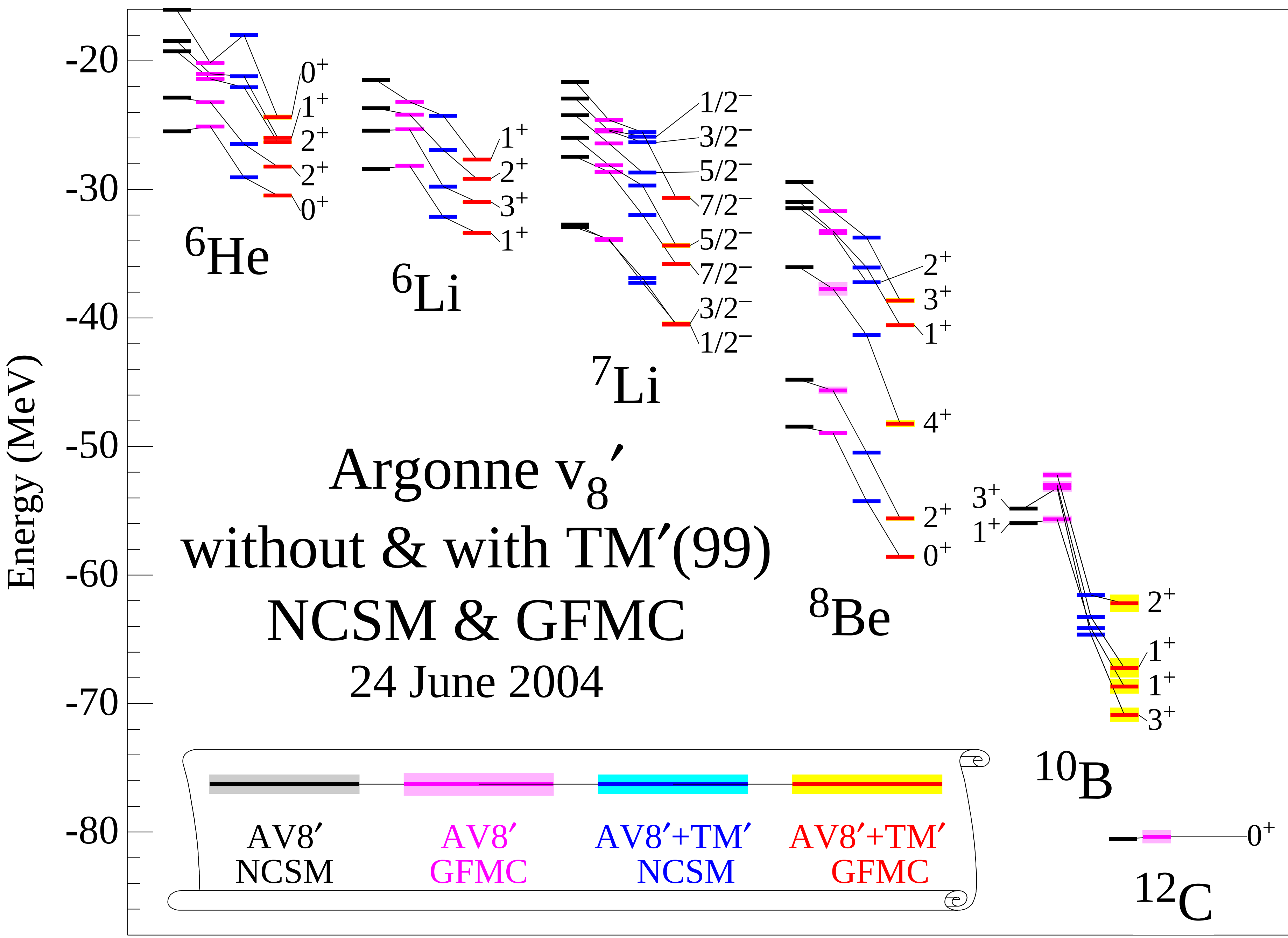
Navrátil & Ormand, Phys.Rev.C **68** 034305 (2003)



$^{12}C\ 0^+\ 0$ ground state with AV8'
E [MeV]
hΩ [MeV]
GFMC_max
GFMC
GFMC_min
0; NCSM
2; NCSM
4; NCSM
6; NCSM
8; NCSM
-70
-75
-80
-85
-90
-95
-100
-105
-110
-115
10
11
12
13
14
15
16
17
18
19
20
21
22
E [MeV]
N_{max}
GFMC
NCSM
-2
0
2
4
6
8
10
12



^{12}C
ħΩ=15 MeV
CD-Bonn
E (MeV)
Exp
6ħΩ
4ħΩ
2ħΩ
0ħΩ



Energy (MeV)
-20
-30
-40
-50
-60
-70
-80
6He
0+
1+
2+
2+
0+
6Li
1+
2+
3+
1+
7Li
1/2−
3/2−
5/2−
7/2−
5/2−
7/2−
3/2−
1/2−
8Be
2+
3+
1+
4+
2+
0+
10B
3+
1+
2+
1+
1+
3+
12C
0+
Argonne v8′
without & with TM′(99)
NCSM & GFMC
24 June 2004
AV8′ NCSM
AV8′ GFMC
AV8′+TM′ NCSM
AV8′+TM′ GFMC

Coupled Cluster Expansion

(1) Heisenberg, Mihaila

(2) Dean, Hjorth-Jensen, Kowalski, Papenbrock, Piecuch

- Developed for closed-shell systems such as ^{4}He, ^{16}O, ^{40}Ca
- Ground state $|\Psi\rangle = \exp(S)|\Phi\rangle$ with $|\Phi\rangle$ the physical vacuum
  S is cluster operator built from ph-creation operators

$$E = \langle\Phi|\exp(S)H\exp(-S)|\Phi\rangle$$

- G-matrix procedure necessary to regularize v_{ij}
- Excited states in closed-shell system now calculable
- Challenge is to go to open-shell systems

- Heisenberg & Mihaila obtain 7.54 MeV/nucleon for ^{16}O using AV18/UIX compared to experimental 8 MeV/nucleon
- see Dean's talk Thursday for latest results from his group