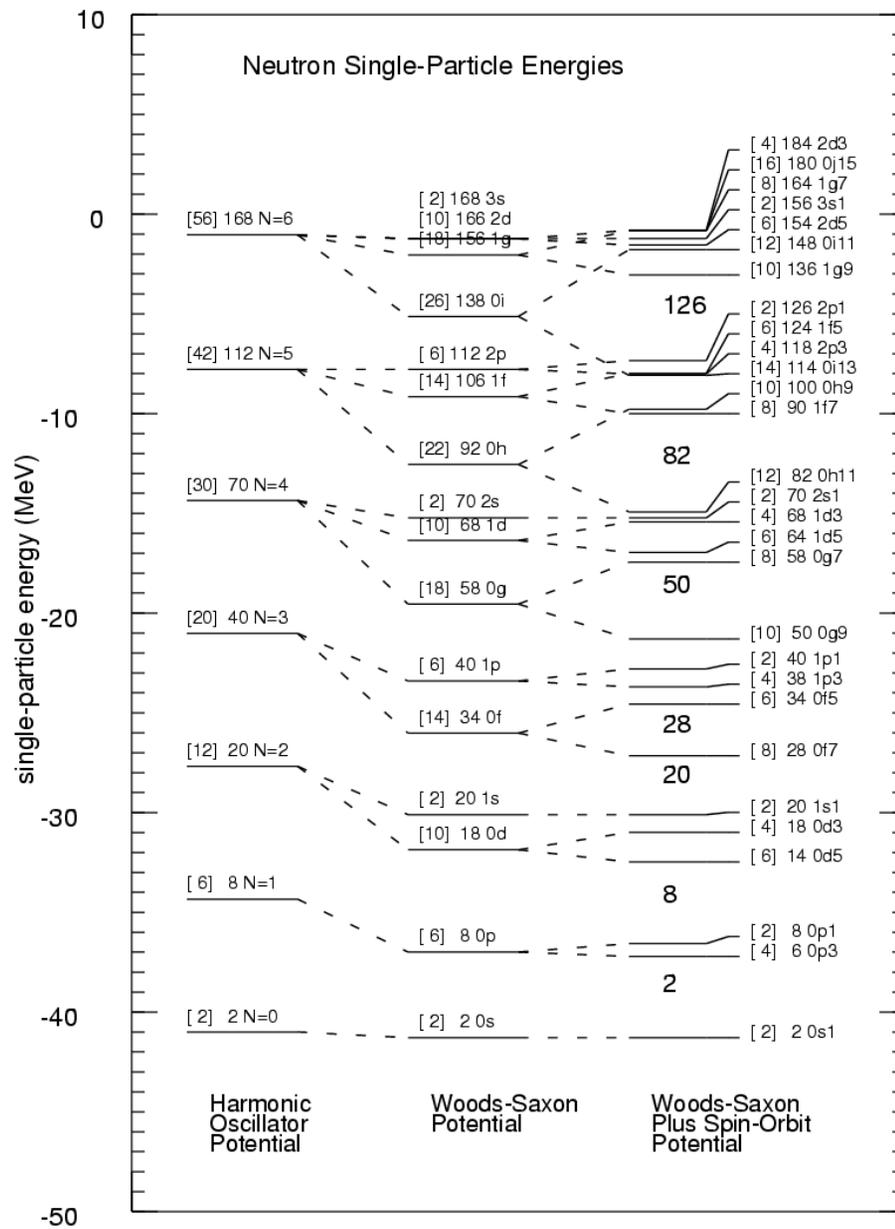


Shell Model Configuration Mixing

Phenomenology and Foundations

Basic Goal

- Treatment of the all configurations for a few orbits near the fermi surface.
- Orbit truncation is based on the observed clustering.
- Goal is to understand the wavefunctions of all low-lying states up to about 5 MeV above the yrast line.
- Use this understanding to calculate the overlaps needed for astrophysics and weak-interaction physics.



pf shell
sd shell
p shell

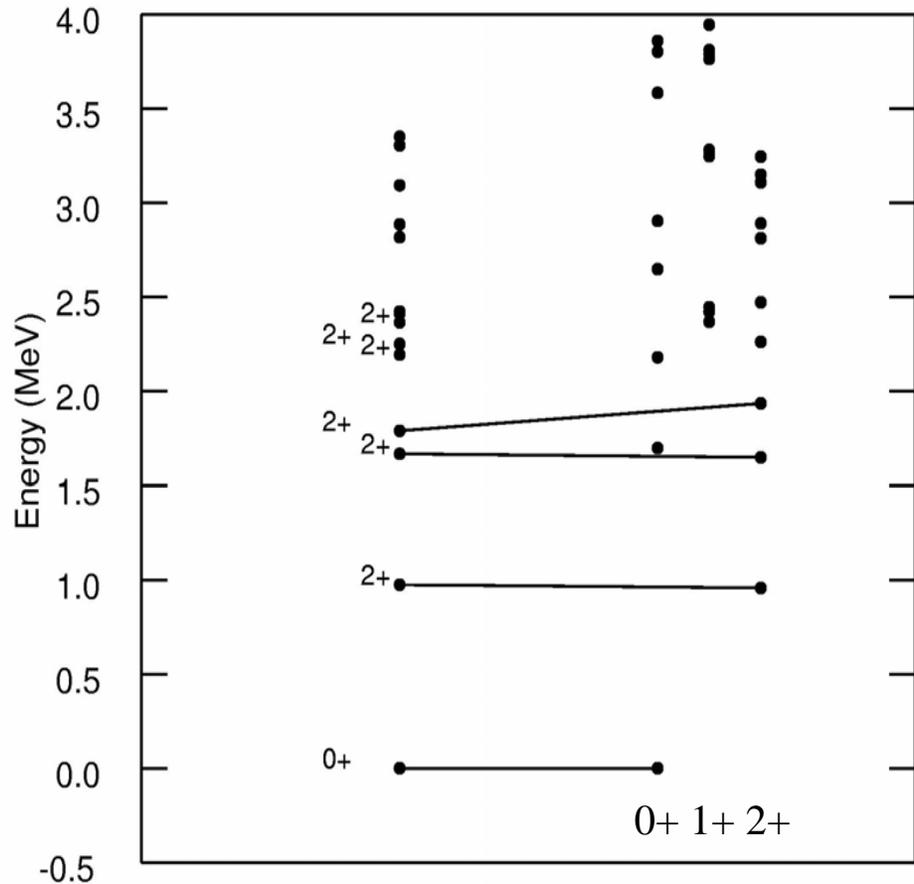
Method and Assumptions

$$\hat{H} = \sum_{\alpha\beta} \langle \alpha | U | \beta \rangle a_{\alpha}^{+} a_{\beta} + \frac{1}{4} \sum_{\alpha\beta\gamma\delta} \langle \alpha\beta | V | \gamma\delta \rangle a_{\alpha}^{+} a_{\beta}^{+} a_{\delta} a_{\gamma}$$

- 1) Exact solution of H within the model space
- 2) Single-particle matrix elements of U from experiment
- 3) Two-body matrix elements of V from NN interaction in an oscillator basis renormalized to the model space
- Works well when the number of valence particles is small and orbits are well bound.

Example for a ^{132}Sn core

- New results for ^{132}Te based on the ^{132}Sn core and CD-Bonn interaction.
- $^{132}\text{Te} = ^{132}\text{Sn} + 2p - 2n$
- Calculation by Grineviciute and Brown
- Compared to new experimental data from Casten et al.



exp versus theory

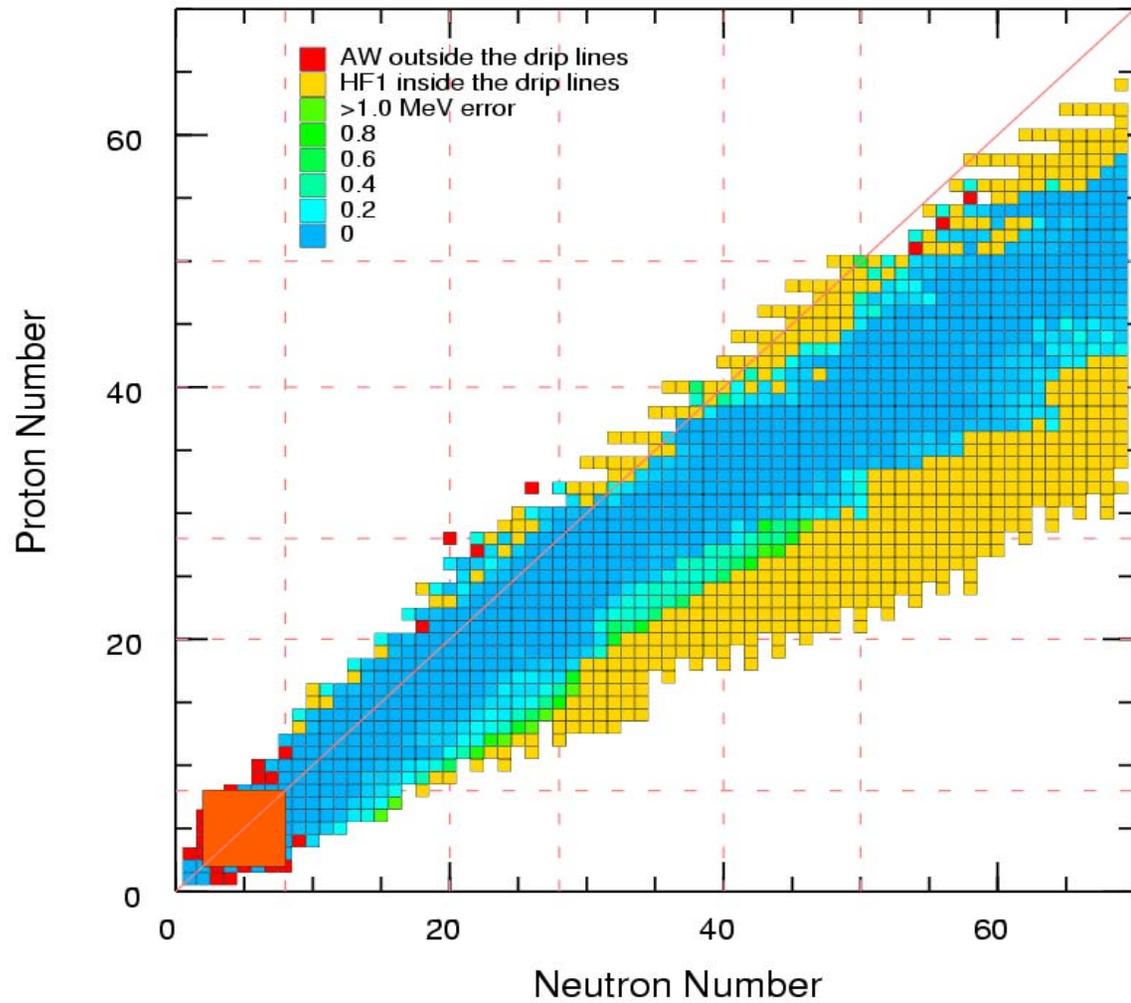
p-shell

Matrix size

10^2

1960's

Nuclear drip lines

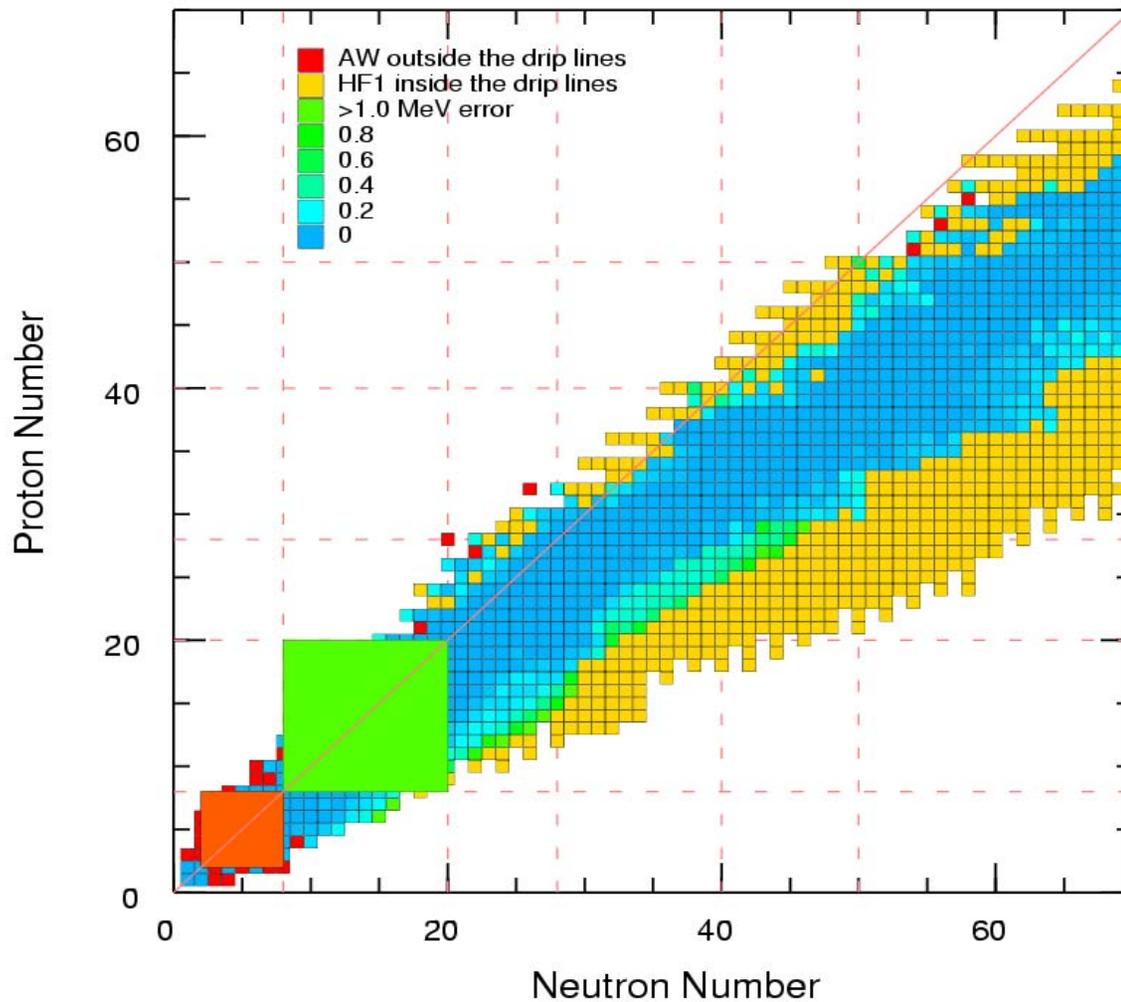


sd-shell

10^5

1980's

Nuclear drip lines

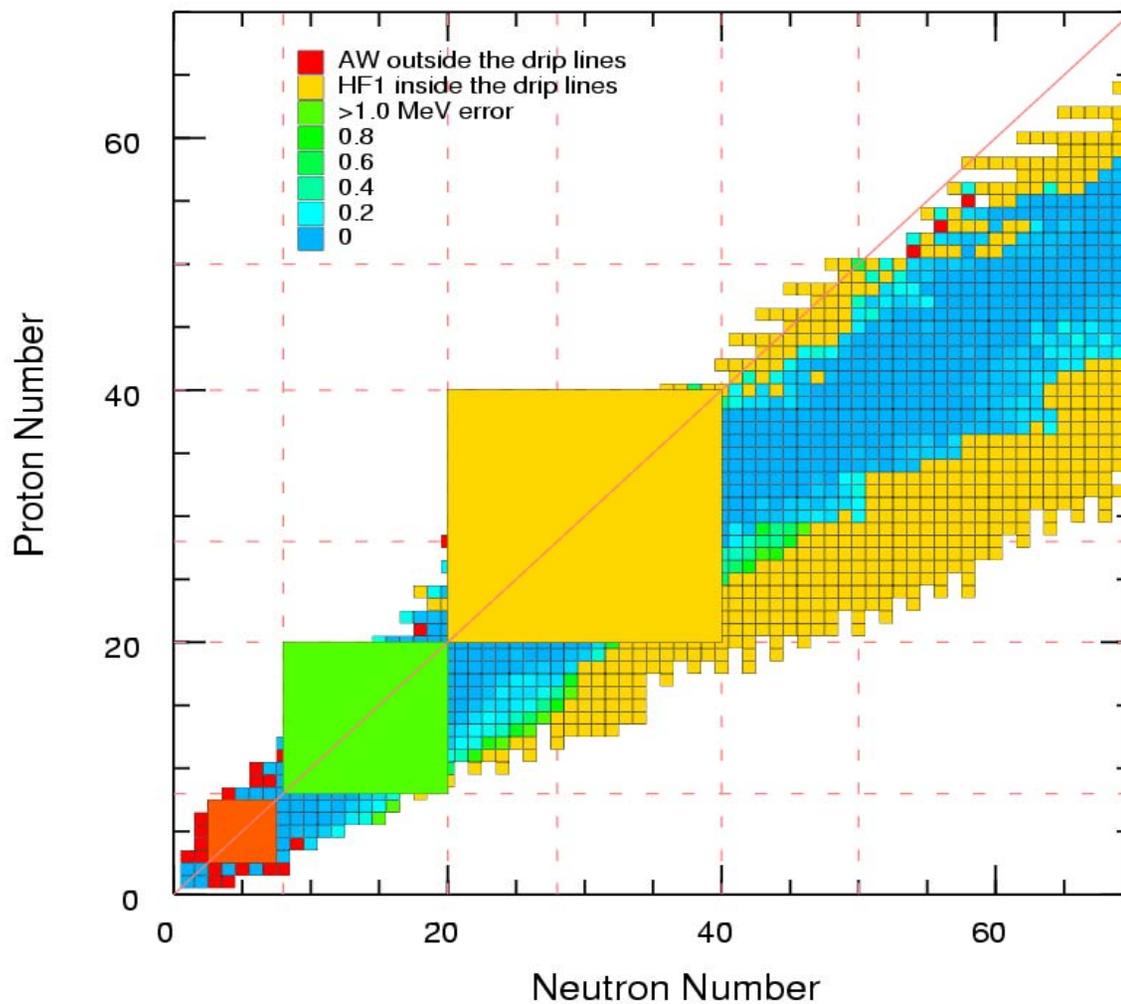


pf-shell

10^{11}

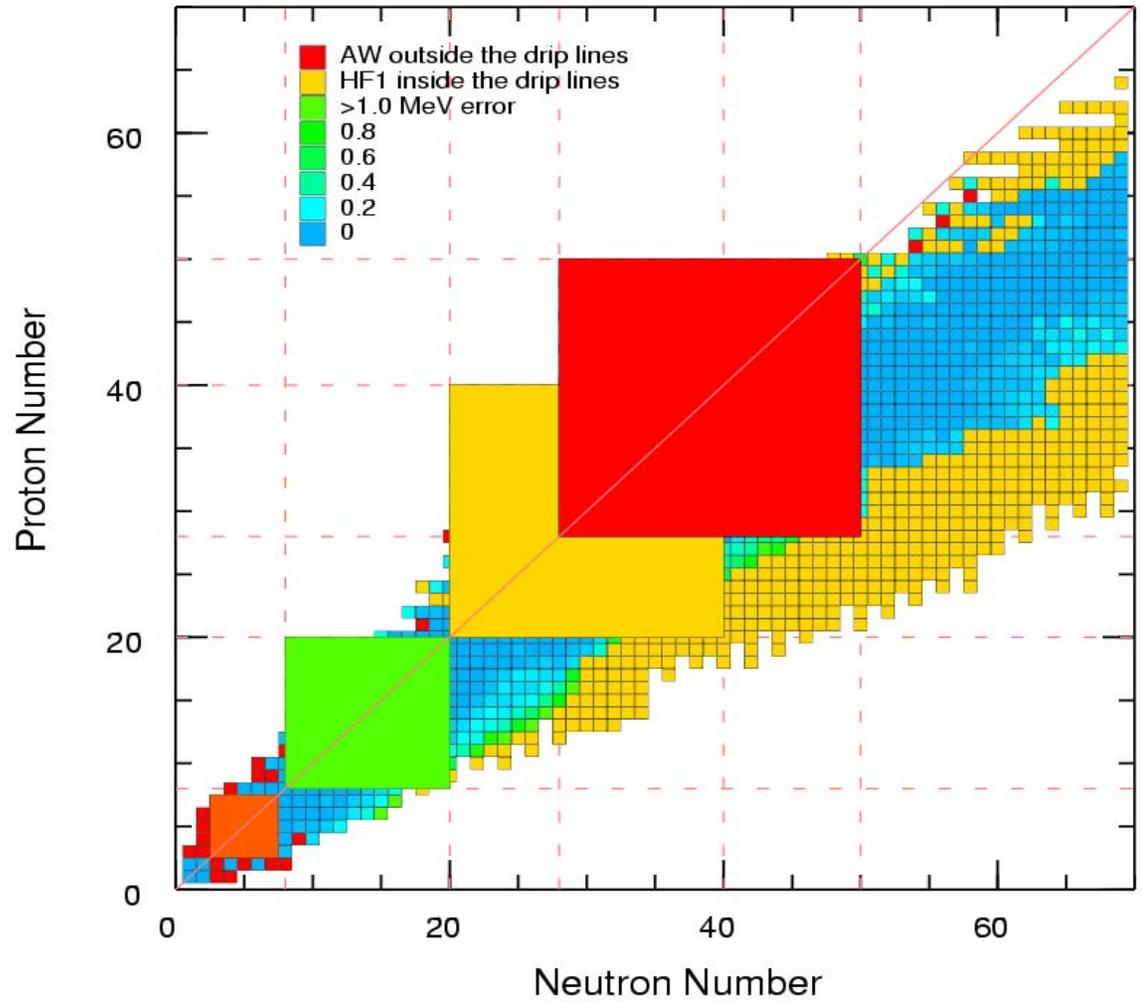
1995-

Nuclear drip lines



Nuclear drip lines

pfg_{9/2}-shell
10¹³
2003-



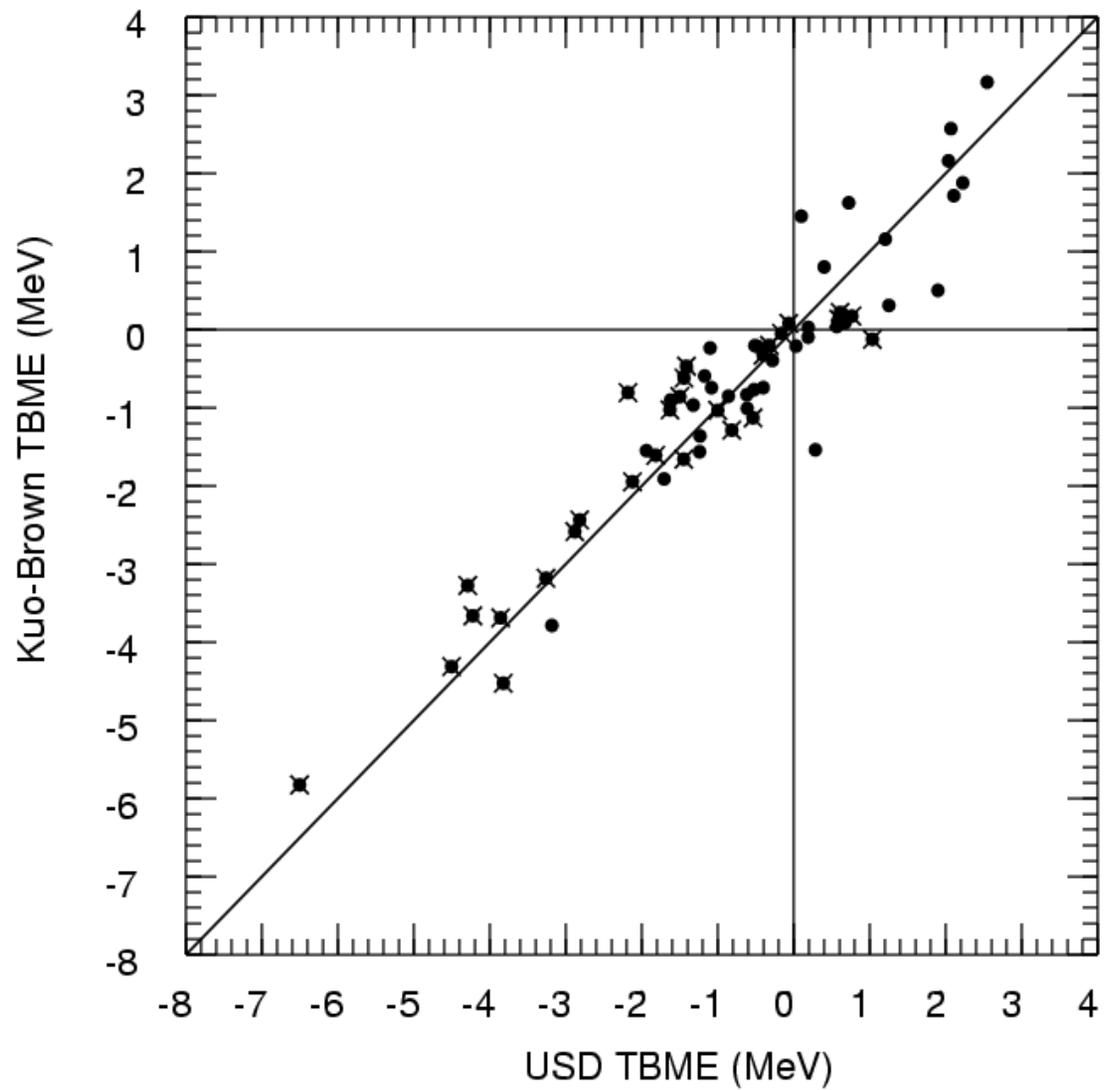
Method and Assumptions

$$\hat{H} = \sum_{\alpha\beta} \langle \alpha | U | \beta \rangle a_{\alpha}^{+} a_{\beta} + \frac{1}{4} \sum_{\alpha\beta\gamma\delta} \langle \alpha\beta | V | \gamma\delta \rangle a_{\alpha}^{+} a_{\beta}^{+} a_{\delta} a_{\gamma}$$

- 1) Exact solution of H within the model space
- 2) Single-particle matrix elements of U from experiment
- 3) Two-body matrix elements of V from NN interaction in a oscillator basis renormalized to the model space
- 4) **Emprical Modification of the TBME**

The sd-shell example

- Results for the sd-shell start with the renormalized G matrix represented by 63 two-body matrix elements (TBME) and the experimental single-particle energies for $A=17$.
- Linear combinations of these TBME are tuned to a set of data – about 50 linear combinations are required – the rest kept at the G matrix values.
- The resulting USD hamiltonian can describe on the order of 1000 energy levels to an accuracy of about 200 keV.
- The only limitation is when intruder states (non sd-shell) become important (always at some excitation energy and sometimes in the ground state – ^{32}Mg region).



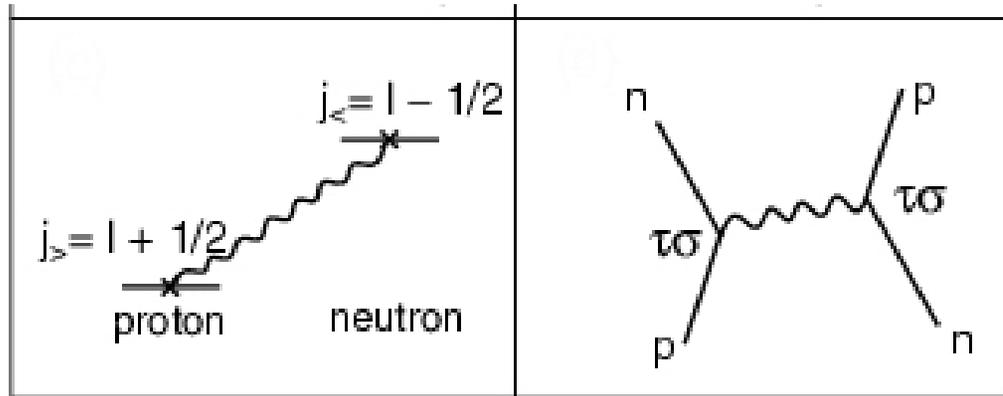
Effective single-particle energies

- The **effective single-particle energies** represented by the mean-field part of the shell model Hamiltonian change with mass.
- This change comes from the “monopole” parts of the two-body interaction.

$$\bar{V}_{j,j'} = \frac{\sum_J (2J+1) \langle j, j'; J | V | j, j'; J \rangle}{\sum_J (2J+1)}$$

Single-particle energies (proton-neutron part)

- The G matrix provides an essential guide for the **proton-neutron** interaction.
- For example as protons are added the neutron shell gaps change (as neutrons as added the proton shell gaps change).
- Talk by Taka Otsuka in the following session.



Otsuka et al., Phys. Rev. Lett. 87, 082502 (2001)

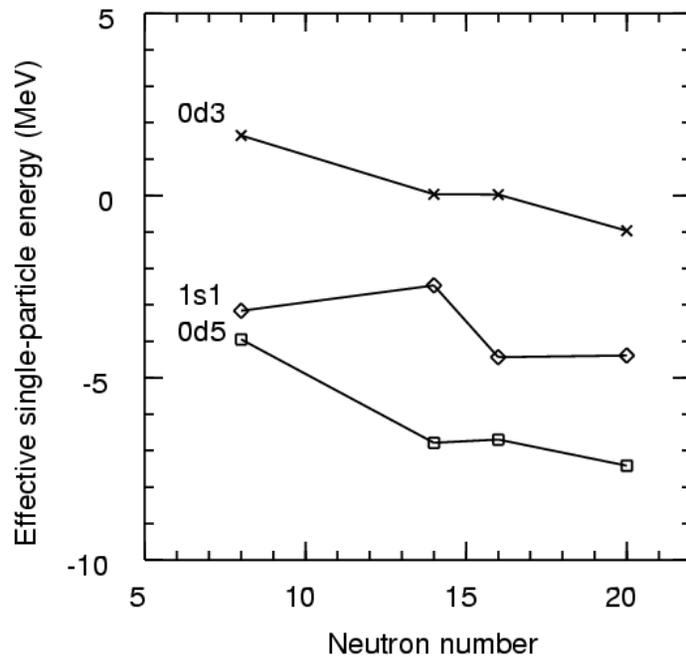
Single-particle energies (T=1 part)

- There is something missing for the T=1 part – e.g. the **neutron-neutron** renormalized G matrix.
- This determines how the neutron shell gaps change as neutrons are added.
- Empirical modifications greatly improve the T=1 part.

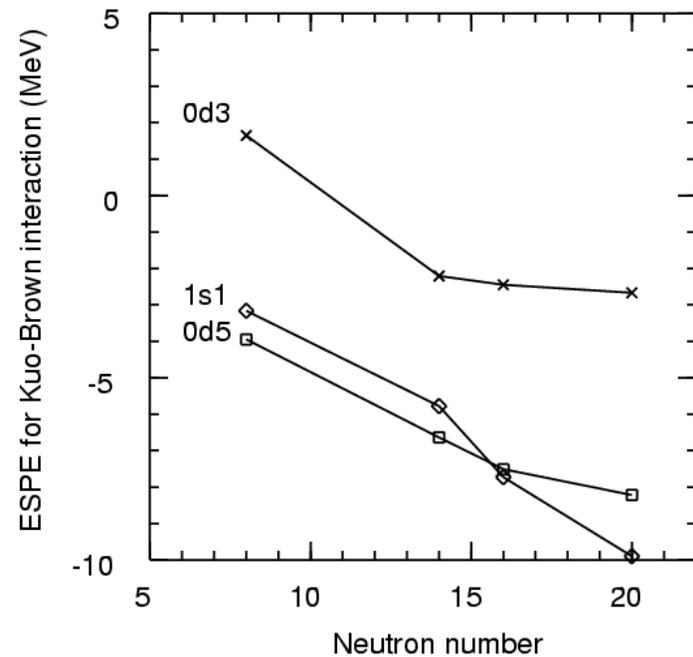
	$\beta^+ = 100\%$	$\beta^+ = 100\%$	$\beta^+ = 100\%$	$\beta^+ = 100\%$	Abundance=78.99%	Abundance=10.00%	Abundance=11.01%	$\beta^- = 100\%$						
	19 Na $p = 100\%$	20 Na $\beta^+ = 100\%$	21 Na $\beta^+ = 100\%$	22 Na $\beta^+ = 100\%$	23 Na Abundance=100%	24 Na $\beta^- = 100\%$	25 Na $\beta^- = 100\%$	26 Na $\beta^- = 100\%$	27 Na $\beta^- = 100\%$	28 Na $\beta^- = 100\%$	29 Na $\beta^- = 100\%$	30 Na $\beta^- = 100\%$	31 Na $\beta^- = 100\%$	32 Na $\beta^- = 100\%$
	18 Ne $\beta^+ = 100\%$	19 Ne $\beta^+ = 100\%$	20 Ne Abundance=90.48%	21 Ne Abundance=0.27%	22 Ne Abundance=9.25%	23 Ne $\beta^- = 100\%$	24 Ne $\beta^- = 100\%$	25 Ne $\beta^- = 100\%$	26 Ne $\beta^- = 100\%$	27 Ne $\beta^- = 100\%$	28 Ne $\beta^- = 100\%$	29 Ne $\beta^- = 100\%$	30 Ne $\beta^- = 100\%$	31 Ne $\beta^- = 100\%$
	17 F $\beta^+ = 100\%$	18 F $\beta^- = 100\%$	19 F Abundance=100%	20 F $\beta^- = 100\%$	21 F $\beta^- = 100\%$	22 F $\beta^- = 100\%$	23 F $\beta^- = 100\%$	24 F $\beta^- = 100\%$	25 F $\beta^- = 100\%$	26 F $\beta^- = 100\%$	27 F $\beta^- = 100\%$	28 F n?	29 F $\beta^- = 100\%$	30 F $\beta^- = 100\%$
	16 O Abundance=99.76%	17 O Abundance=0.038%	18 O Abundance=0.205%	19 O $\beta^- = 100\%$	20 O $\beta^- = 100\%$	21 O $\beta^- = 100\%$	22 O $\beta^- = 100\%$	23 O $\beta^- = 100\%$	24 O $\beta^- = 100\%$	25 O n?	26 O 2n?	27 O n?	28 O n?	29 O n?
	15 N Abundance=0.368%	16 N $\beta^- = 100\%$	17 N $\beta^- = 100\%$	18 N $\beta^- = 100\%$	19 N $\beta^- = 100\%$	20 N $\beta^- = 100\%$	21 N $\beta^- = 100\%$	22 N $\beta^- = 100\%$	23 N $\beta^- = 100\%$	24 N n?	25 N n?	26 N n?	27 N n?	28 N n?
	14 C $\beta^- = 100\%$	15 C $\beta^- = 100\%$	16 C $\beta^- = 100\%$	17 C $\beta^- = 100\%$	18 C $\beta^- = 100\%$	19 C $\beta^- = 100\%$	20 C $\beta^- = 100\%$	21 C n?	22 C $\beta^- = 100\%$	23 C $\beta^- = 100\%$	24 C $\beta^- = 100\%$	25 C $\beta^- = 100\%$	26 C $\beta^- = 100\%$	27 C $\beta^- = 100\%$
	13 B $\beta^- = 100\%$	14 B $\beta^- = 100\%$	15 B $\beta^- = 100\%$	16 B n?	17 B $\beta^- = 100\%$	18 B n?	19 B $\beta^- = 100\%$	20 B $\beta^- = 100\%$	21 B $\beta^- = 100\%$	22 B $\beta^- = 100\%$	23 B $\beta^- = 100\%$	24 B $\beta^- = 100\%$	25 B $\beta^- = 100\%$	26 B $\beta^- = 100\%$
	12 Be $\beta^- = 100\%$	13 Be n?	14 Be $\beta^- = 100\%$	15 Be n?	16 Be 2n?	17 Be $\beta^- = 100\%$	18 Be $\beta^- = 100\%$	19 Be $\beta^- = 100\%$	20 Be $\beta^- = 100\%$	21 Be $\beta^- = 100\%$	22 Be $\beta^- = 100\%$	23 Be $\beta^- = 100\%$	24 Be $\beta^- = 100\%$	25 Be $\beta^- = 100\%$

Effective single-particle energies for the oxygen isotopes

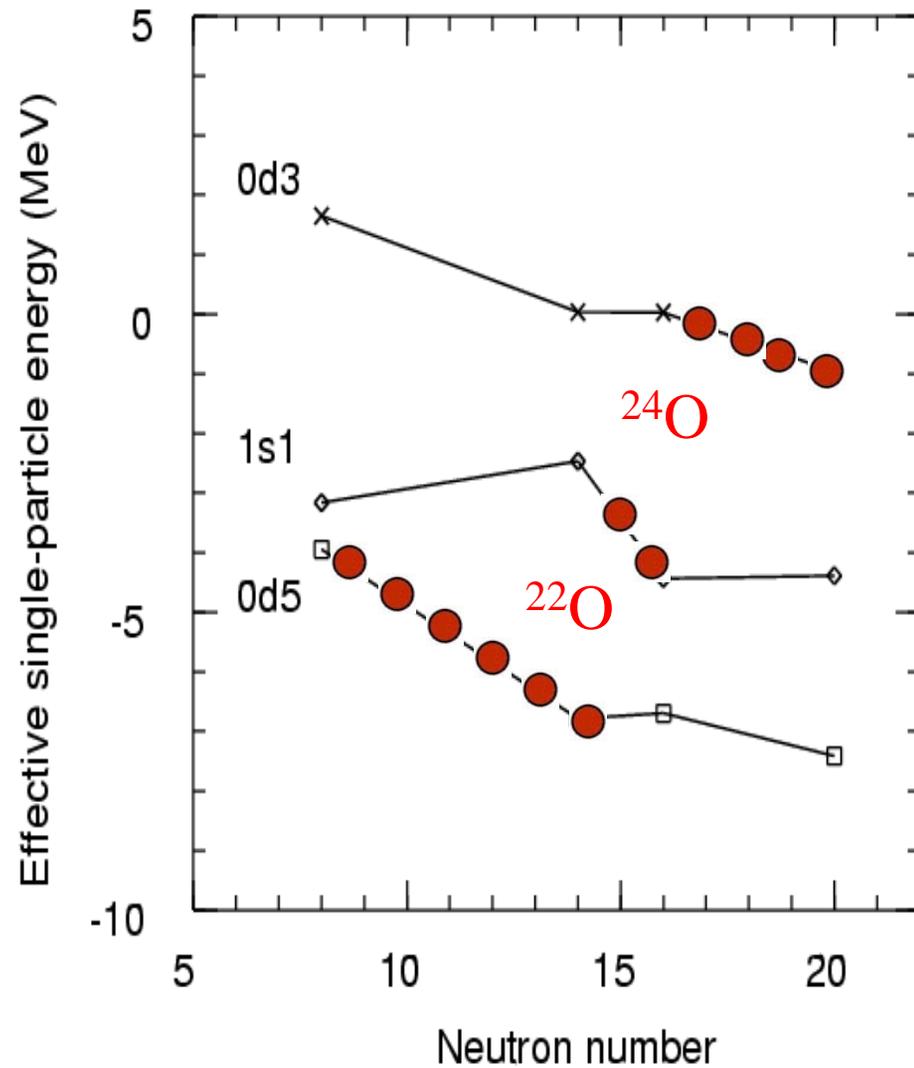
USD



G matrix

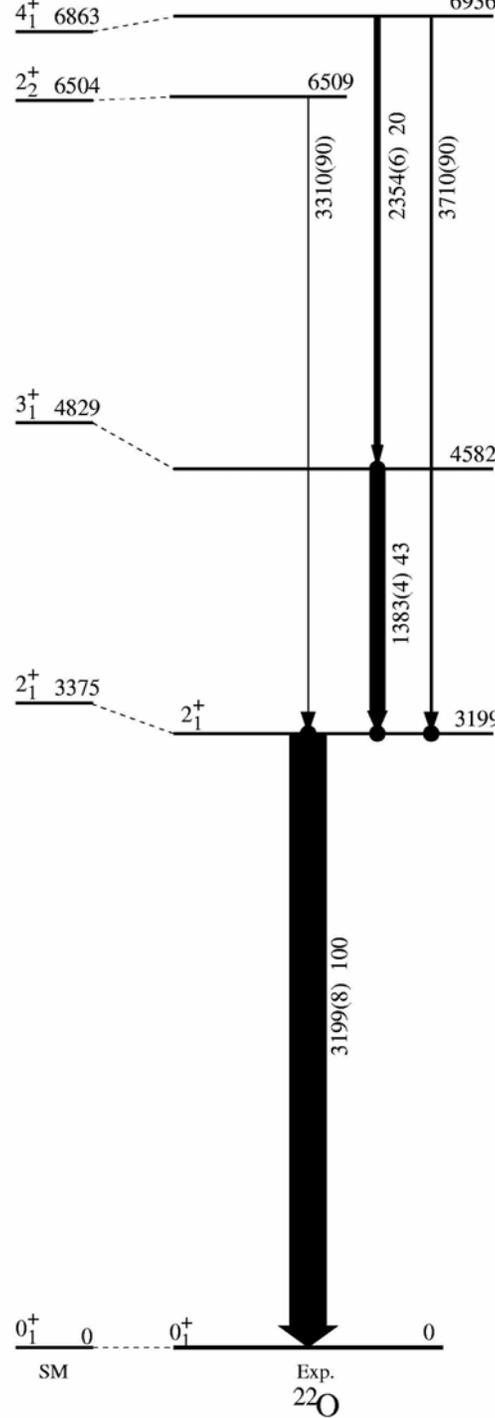


Effective single-particle energies based on USD hamiltonian



$$S_n=0$$

GANIL
results for
 ^{22}O

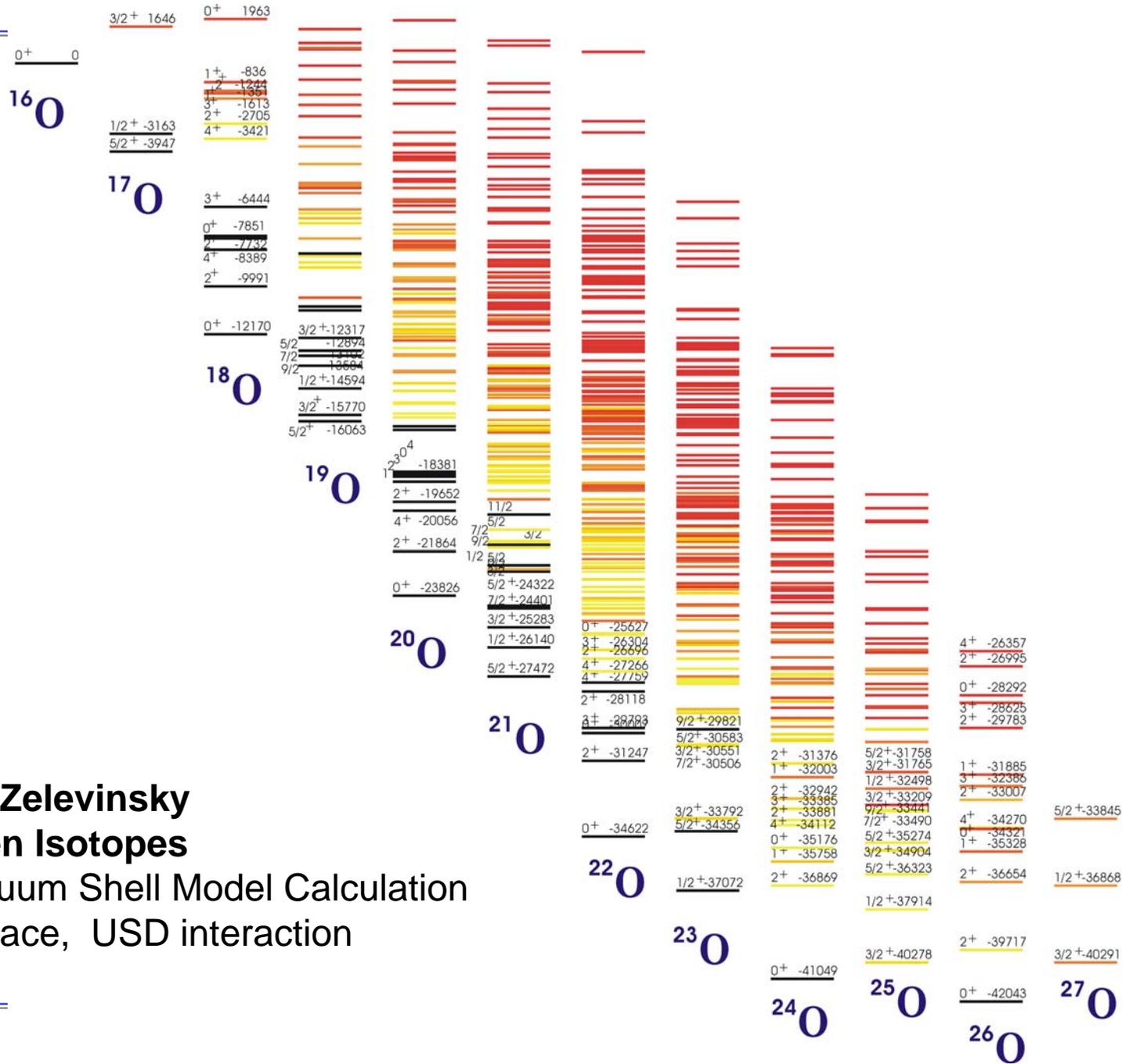


4_+
 2_+ $2p-2h$

3_+
 $1p-1h$

2_+

0_+ $0p-0h$



Volya-Zelevinsky
Oxygen Isotopes
 Continuum Shell Model Calculation
 • sd space, USD interaction

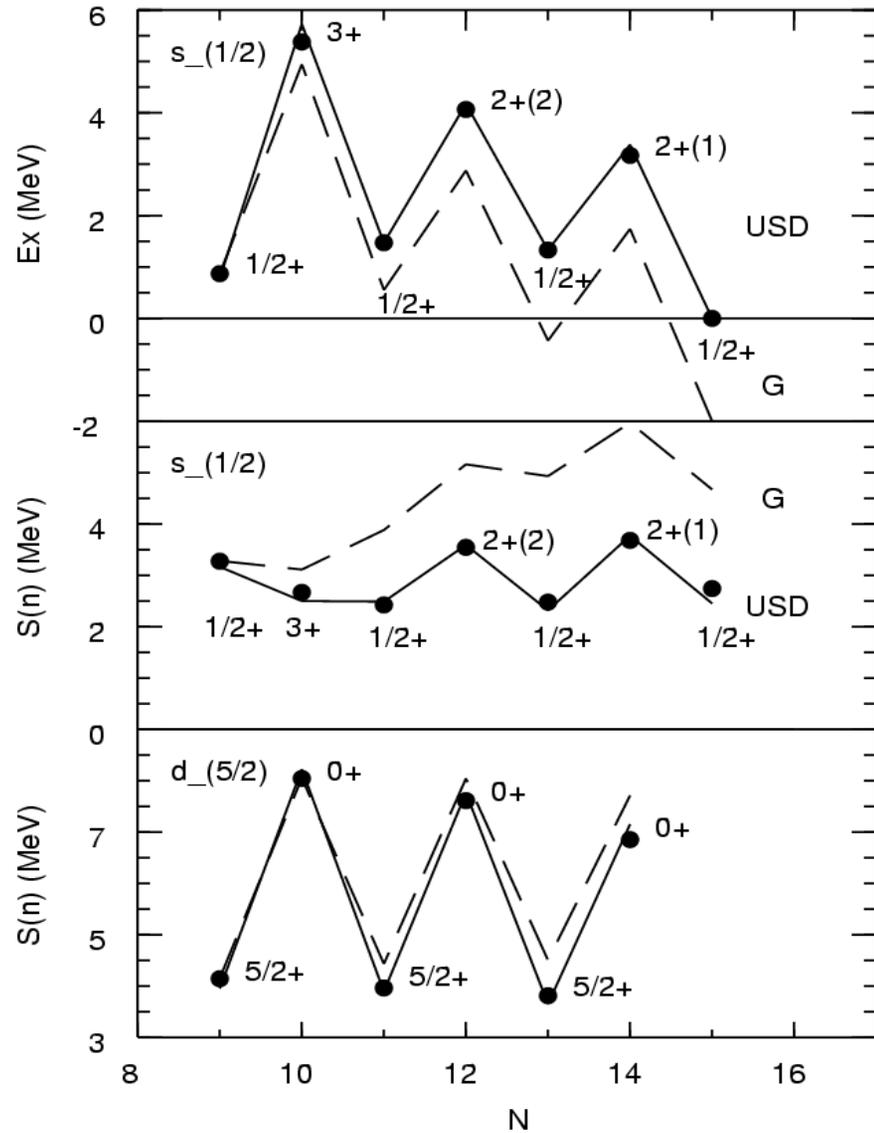
What determines the extrapolation?

- Properties of **excited states near stability** which are related to orbitals which become ground states away from stability can be used to determine the effective two-body matrix elements.

Excitation energy of $s_{1/2}$ states.

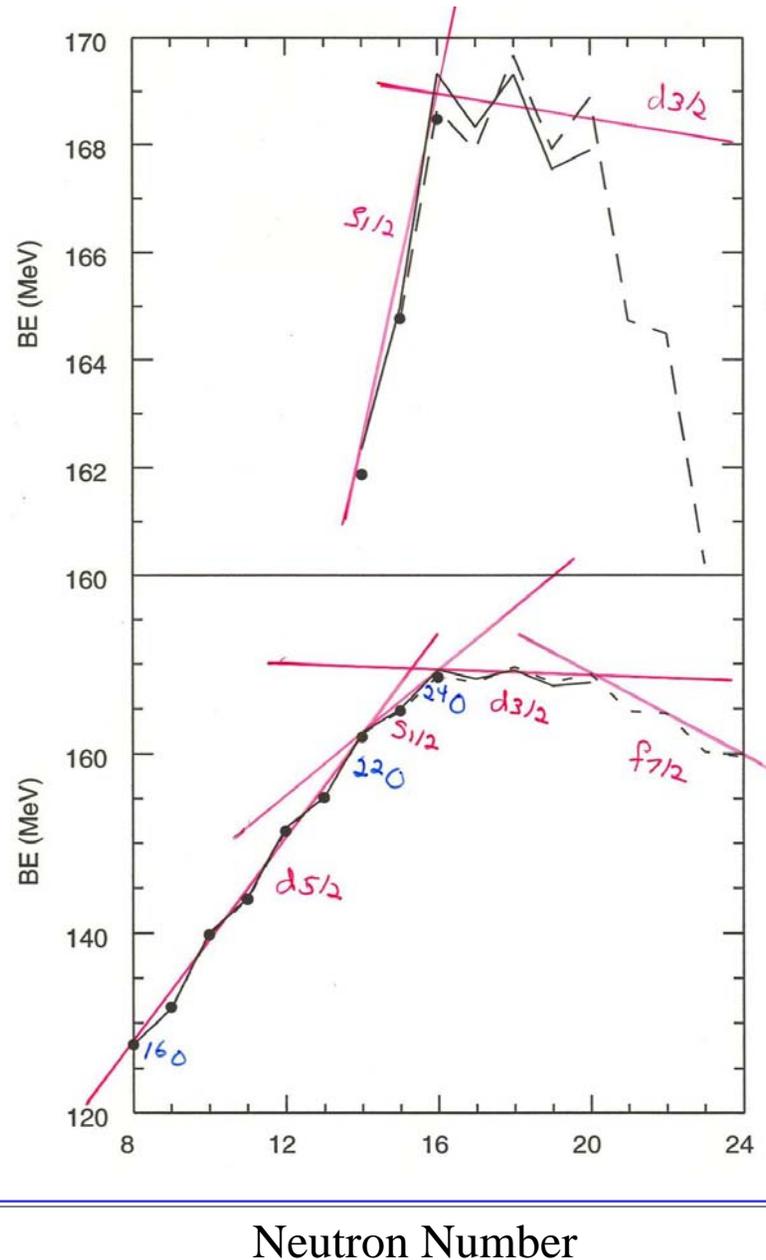
Separation energy of $s_{1/2}$ states

Separation energy of $d_{5/2}$ states



Ground-state binding energy data for the oxygen isotopes

Changes of slope due to closed shells

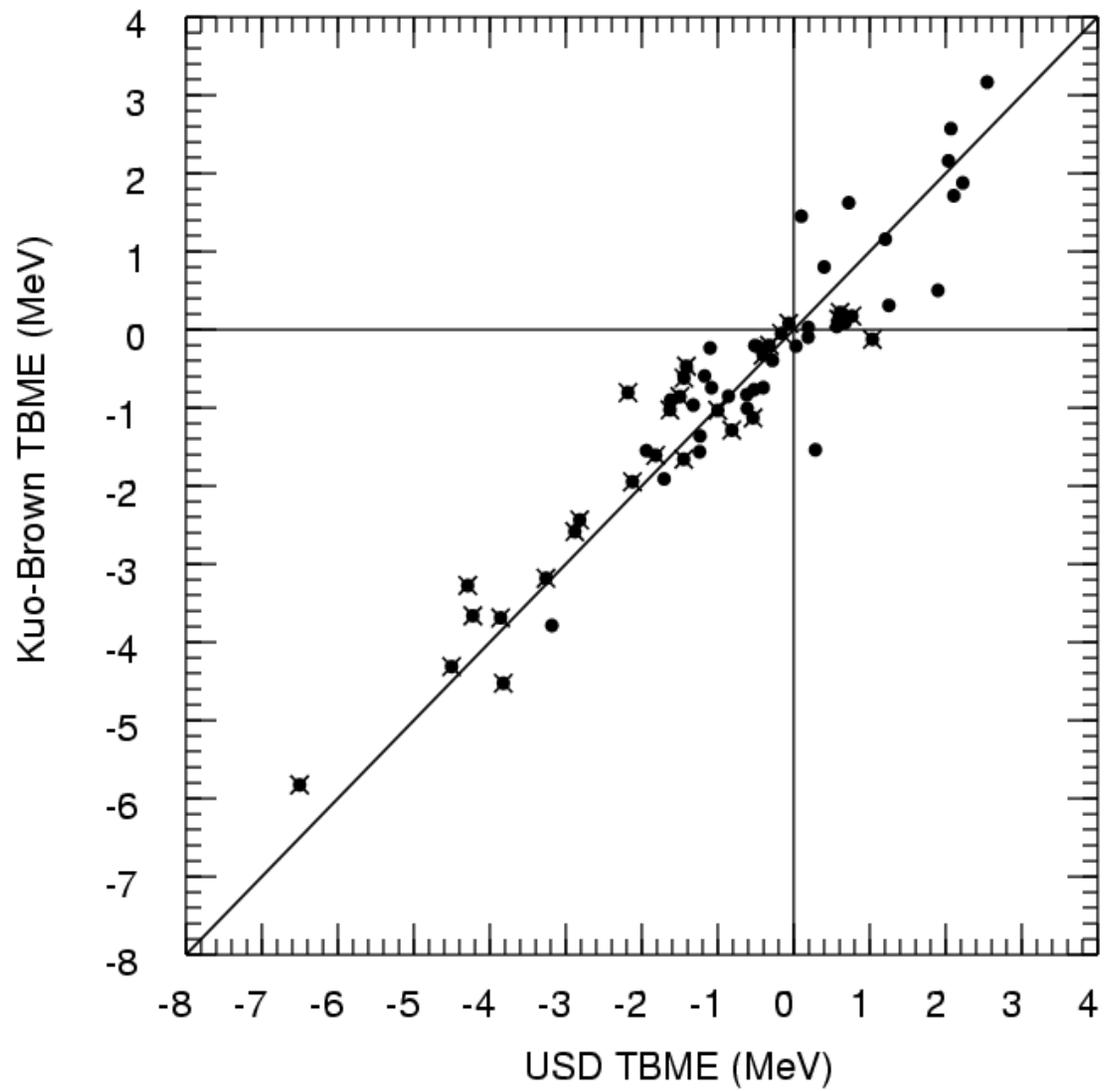


Effective interactions

- For ^{22}O the important difference between USD and G matrix is mainly due to only two matrix $T=1$ elements. The USD values were deduced in 1980 from the properties of excited states in stable nuclei (up to ^{20}O)

- USD (G)
 $\langle d_{5/2} s_{1/2} J=3 | V | d_{5/2} s_{1/2} J=3 \rangle = 0.76 \text{ (0.17) MeV}$
 $\langle d_{5/2} s_{1/2} J=2 | V | d_{5/2} s_{1/2} J=2 \rangle = -0.82 \text{ (-1.19) MeV}$

- Mainly a monopole shift.
- What is the origin of this shift?



Method and Assumptions

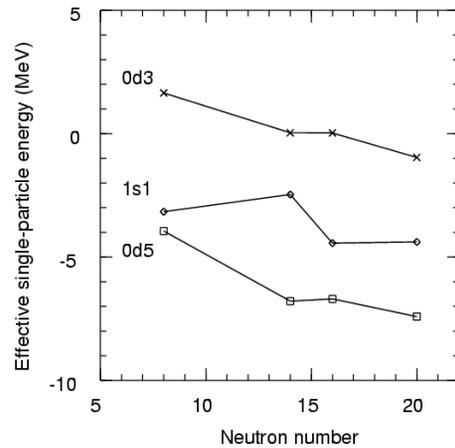
$$\hat{H} = \sum_{\alpha\beta} \langle \alpha | U | \beta \rangle a_{\alpha}^{+} a_{\beta} + \frac{1}{4} \sum_{\alpha\beta\gamma\delta} \langle \alpha\beta | V | \gamma\delta \rangle a_{\alpha}^{+} a_{\beta}^{+} a_{\delta} a_{\gamma}$$

- Exact solution of H within the model spaces
- Single-particle matrix elements of U from experiment
- Two-body matrix elements of V from NN interaction in a oscillator basis renormalized to the model space

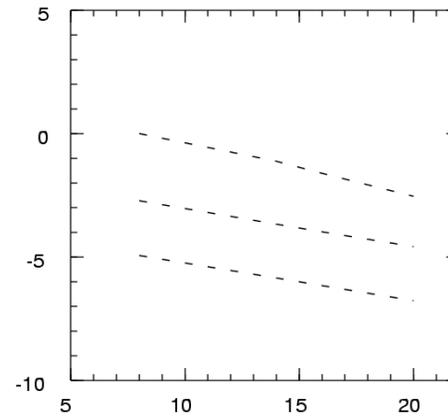
Why do the two-body matrix elements need to be modified from the G matrix?

- Oscillator basis is not good enough.
- Effective three-body interactions – e.g. not enough to derive a renormalized G matrix for ^{16}O and then apply it all the way through to ^{24}O .
- Treating ^{16}O as a closed core is not good enough – e.g. do core-excitations (or alpha clustering) modify the spe near ^{16}O ?
- Real three-body forces.

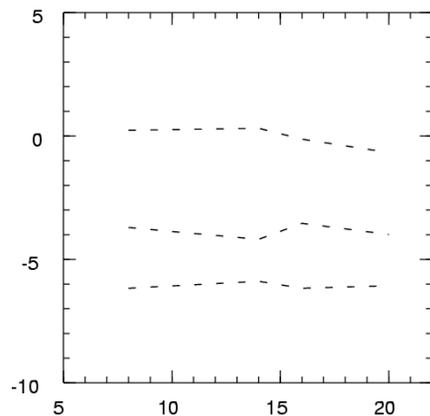
Finite well effect?



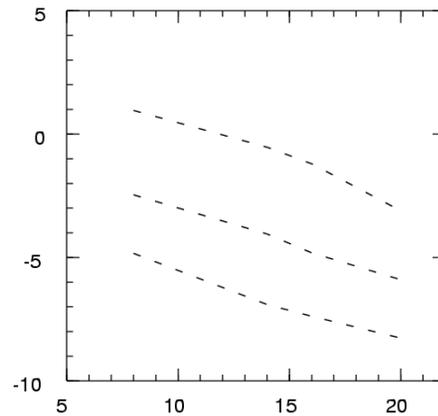
“Experiment”



Woods-Saxon



Skyrme HF



Relativistic HF

Very exotic nuclei – like ^{78}Ni

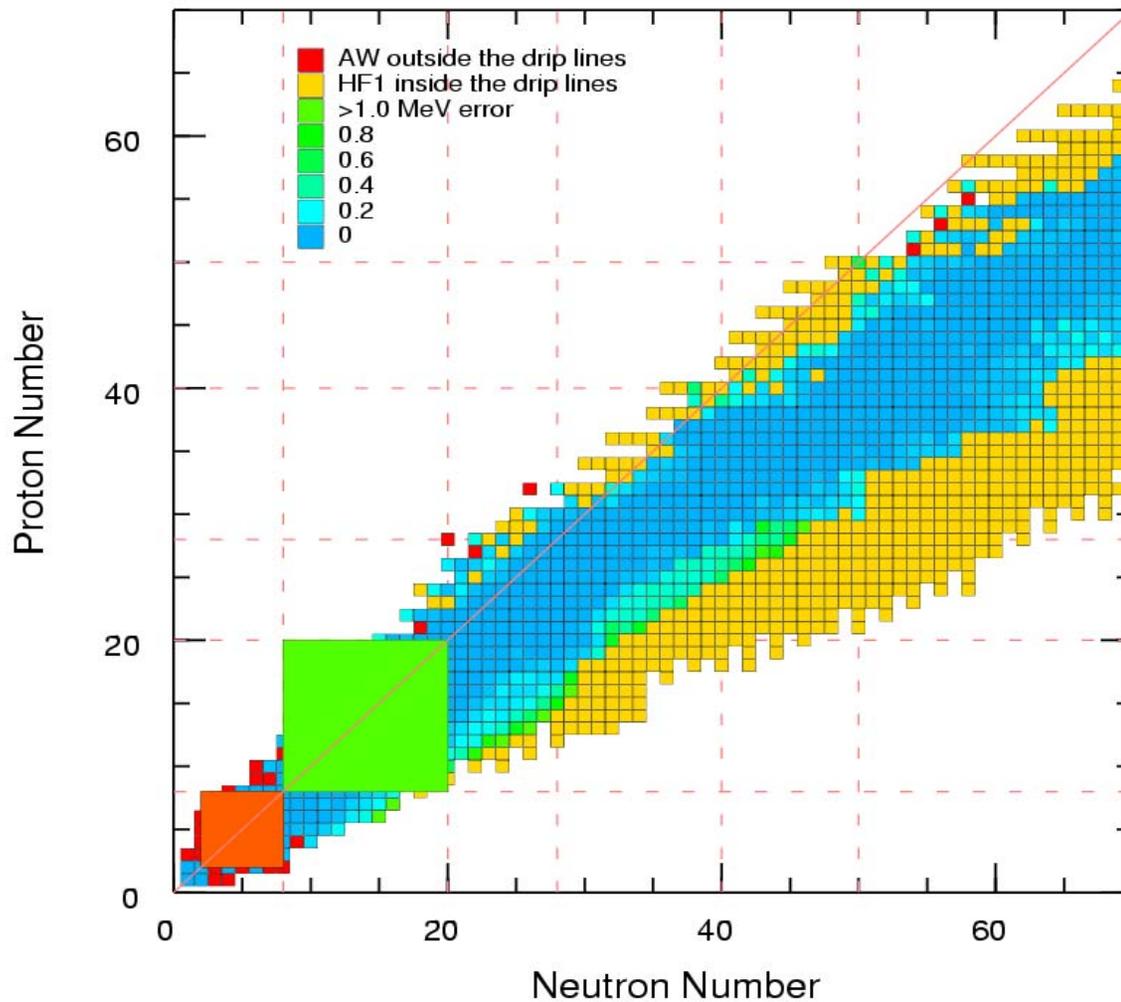
- The experimental single-particle energies are not known.
- Can we rely on Hartree-Fock calculations as a start? – SKX was an attempt to fit spe – its OK but not good enough.
- There are some essential things missing from Skyrme Hartree-Fock such as the tensor interaction (contained in the G matrix).
- Near the drip lines the oscillator basis becomes a poor approximation. Can we evaluate the renormalized G matrix in a finite-well potential (including the continuum)?

sd-shell

10^5

1980's

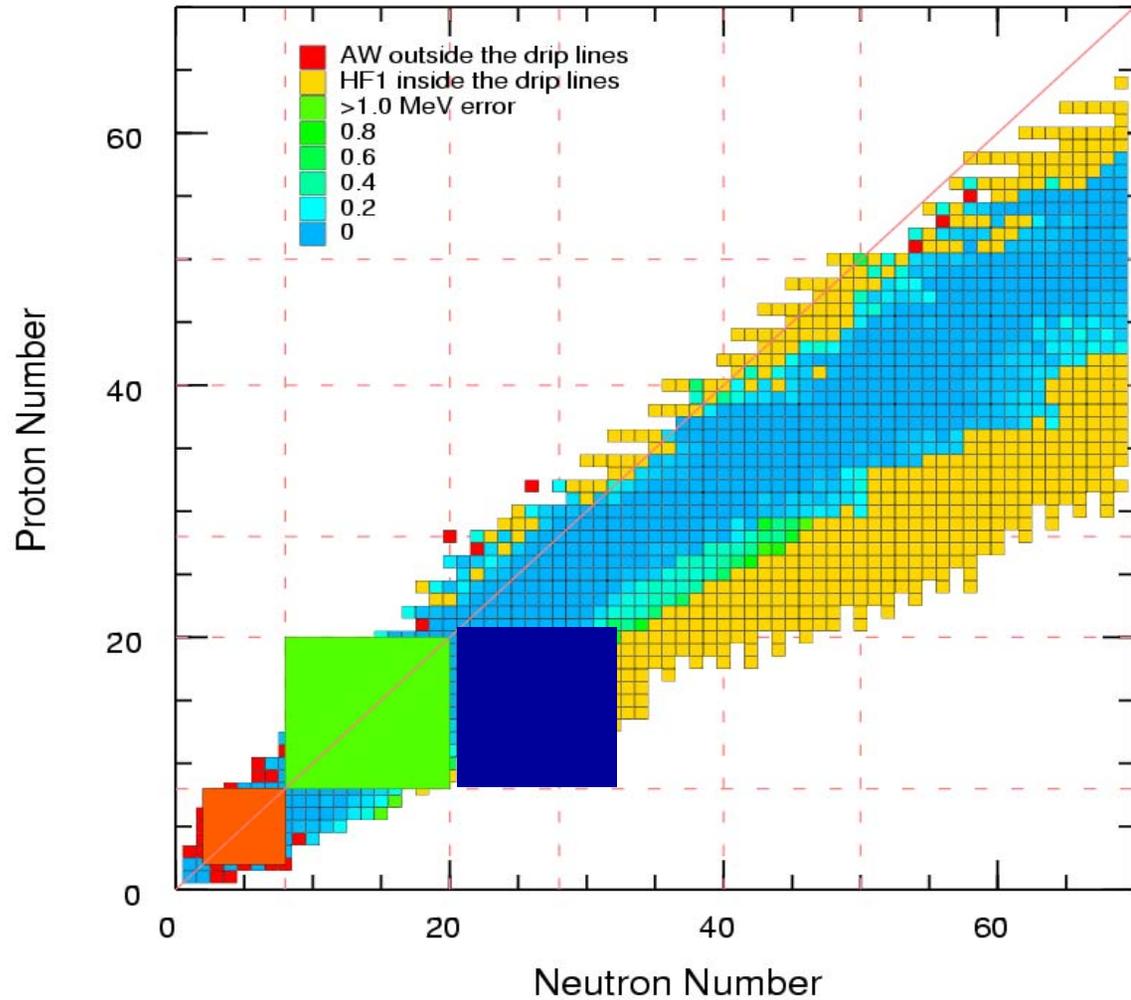
Nuclear drip lines



sd-pf

See the talk by
Campbell in the
following session

Nuclear drip lines

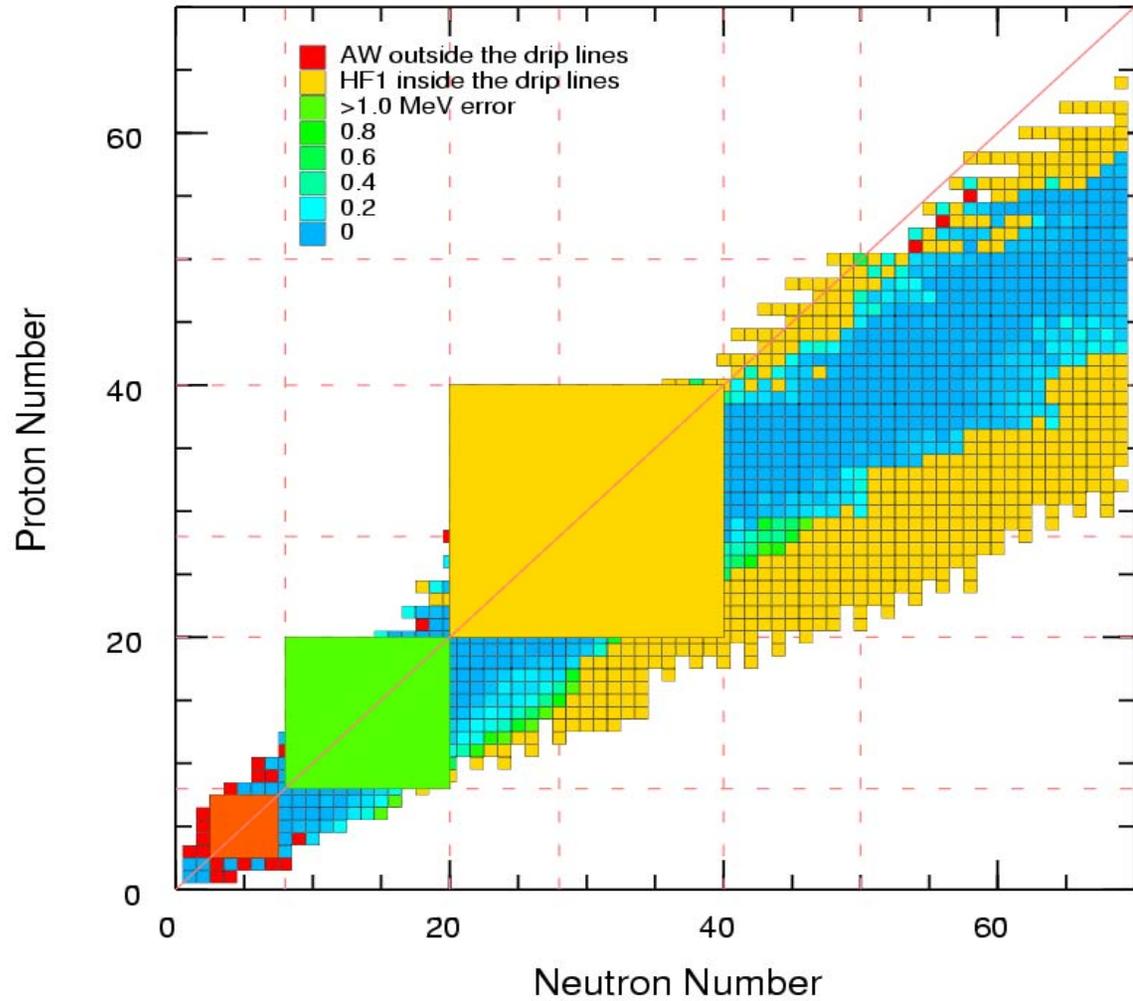


pf-shell

10^{11}

See the talk by
Dinca in the
following session

Nuclear drip lines

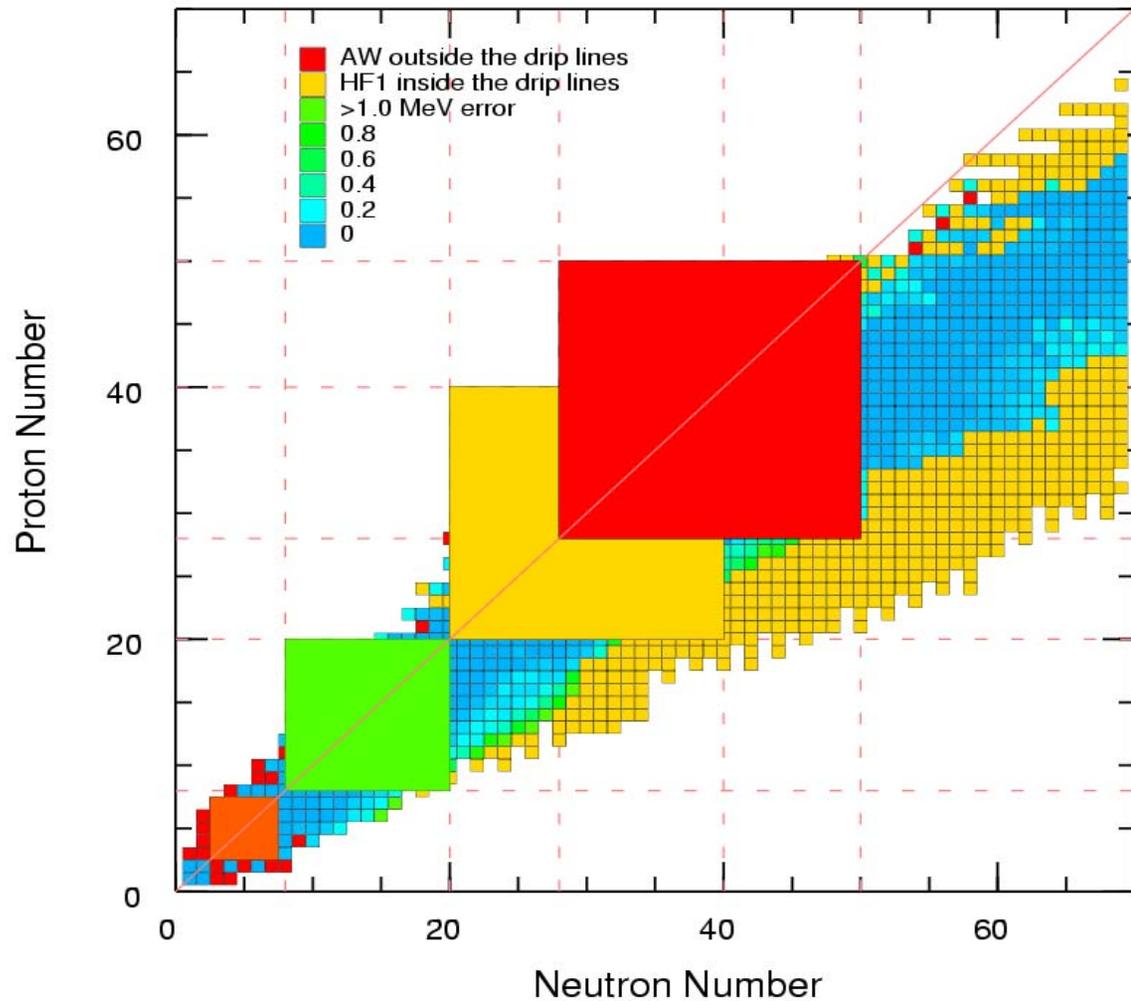


$\text{pfg}_{9/2}$ -shell

10^{13}

See the talk
by Horoi in
the afternoon
session

Nuclear drip lines

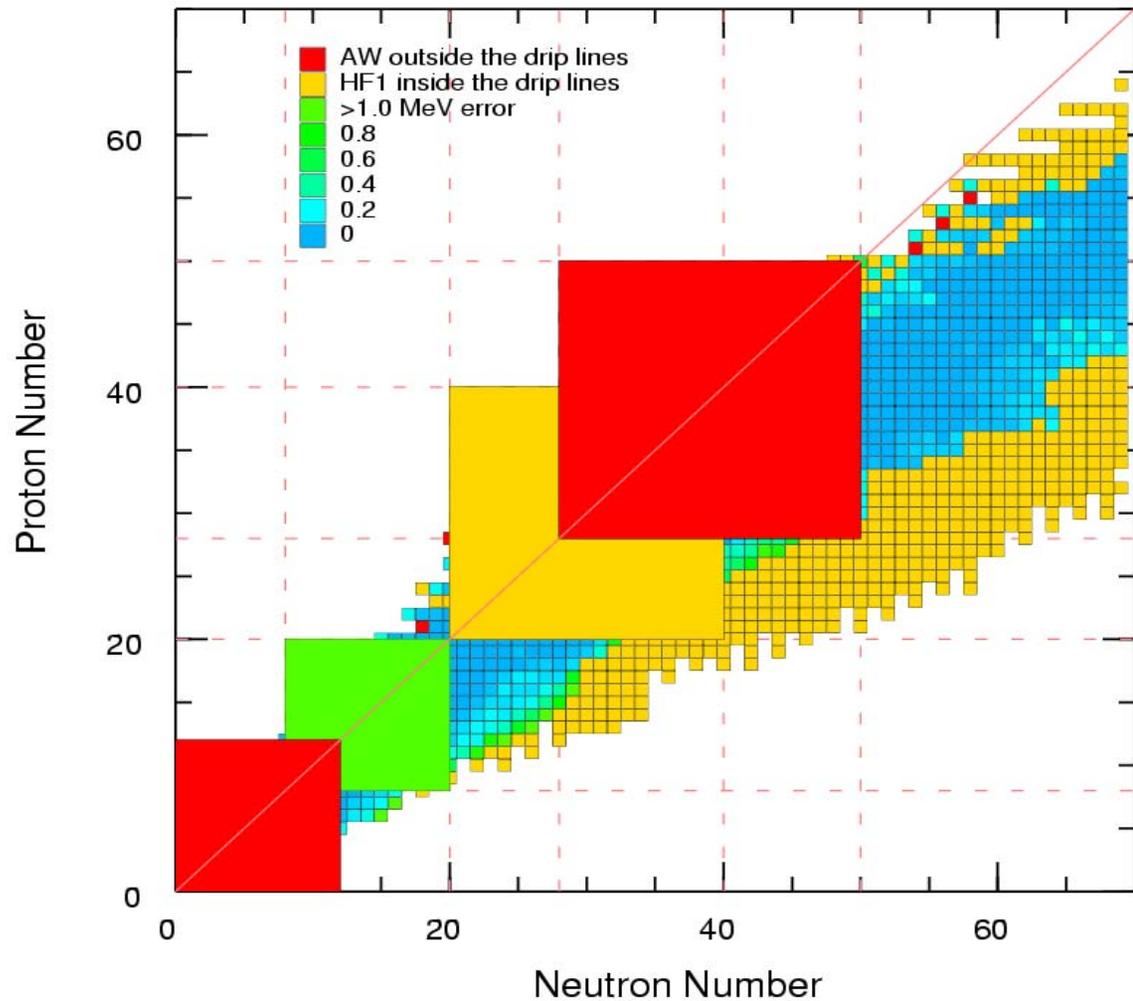


No-core and
GFMC for light
nuclei.

Arbitrarily large
dimensions

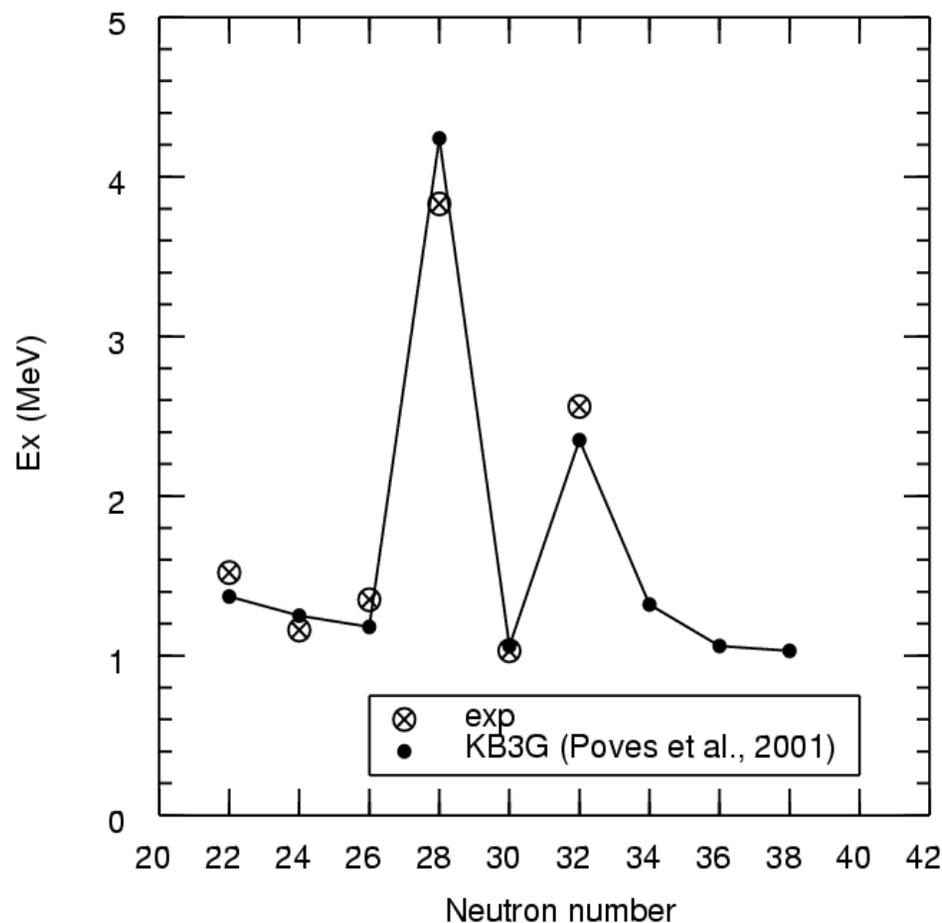
1995-

Nuclear drip lines



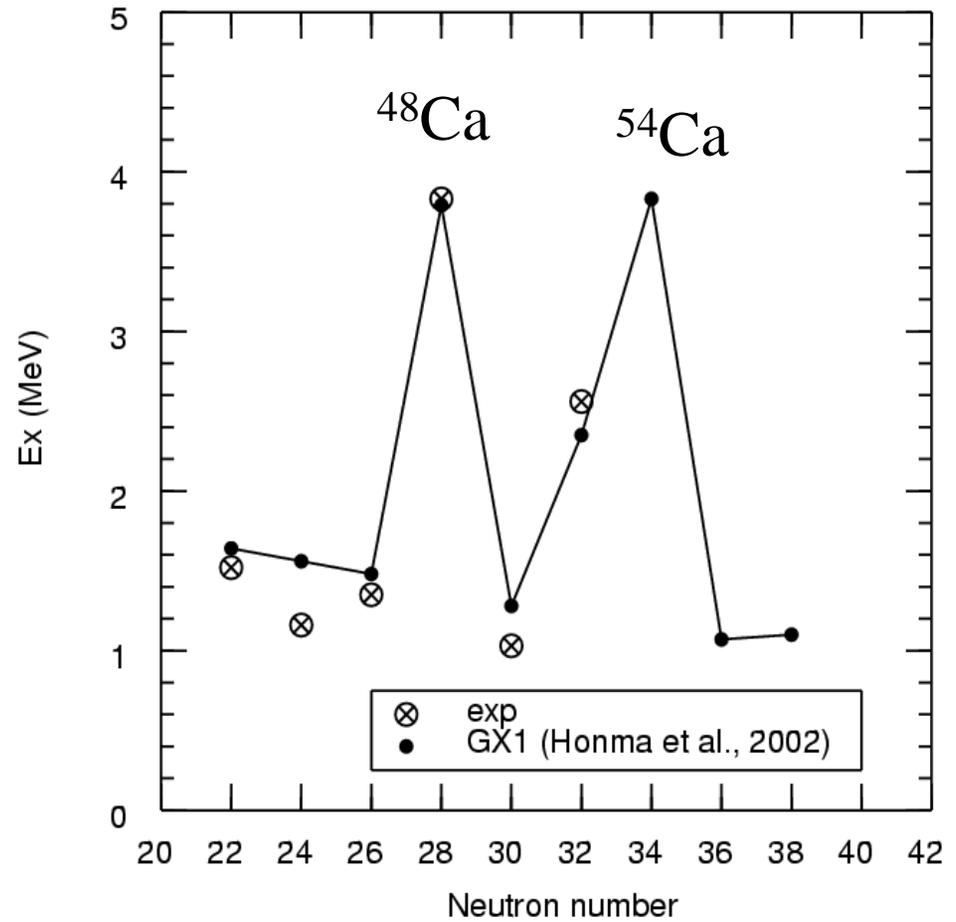
Experiment vs Theory for Ca 2⁺ energies (2001)

- Single-particle energies from ⁴¹Ca
- (KB3G) Renormalized G matrix with some modified diagonal two-body matrix elements



Experiment vs Theory for Ca 2⁺ energies (2002)

- Single-particle energies from ⁴¹Ca
- (GX1) Renormalized G matrix, with some modified linear combinations of two-body matrix elements
- Otsuka, Honma, Mizusaki, Brown (2002)

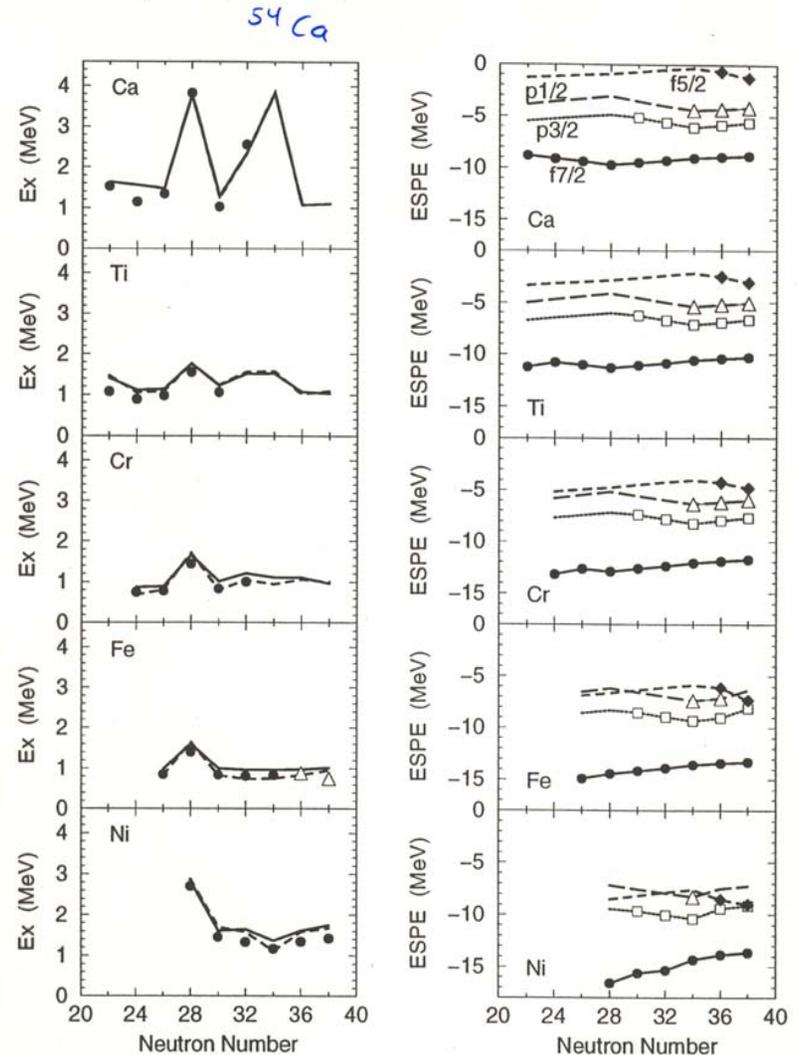


New pf shell interaction – GXPF1

•M. Honma et al., Phys. Rev. C 65, 061301(R) (2002).

•The pf-shell equivalent of USD for the sd-shell.

- About 40 parameters
- About 600 data
- A=46-66

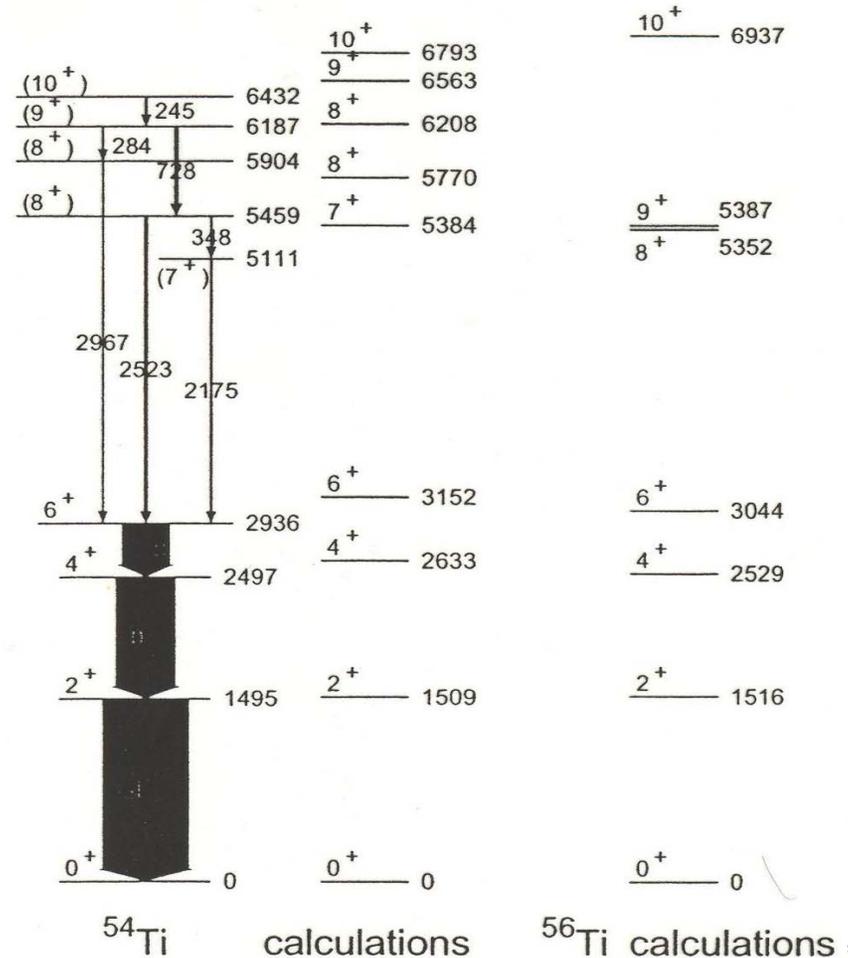


What determines the extrapolation?

- Properties of **excited states near stability** which are related to orbitals which become ground states away from stability
- A nice recent example is for the excited states in ^{54}Ti determined in a unique collaboration
 - NSCL beta-decay experiment which found the 2^+ energy
 - Gammasphere at Argonne which found the high-spin states connected to this.
 - R.F.V. Janssens et al, Phys. Lett. B546, 55 (2002).

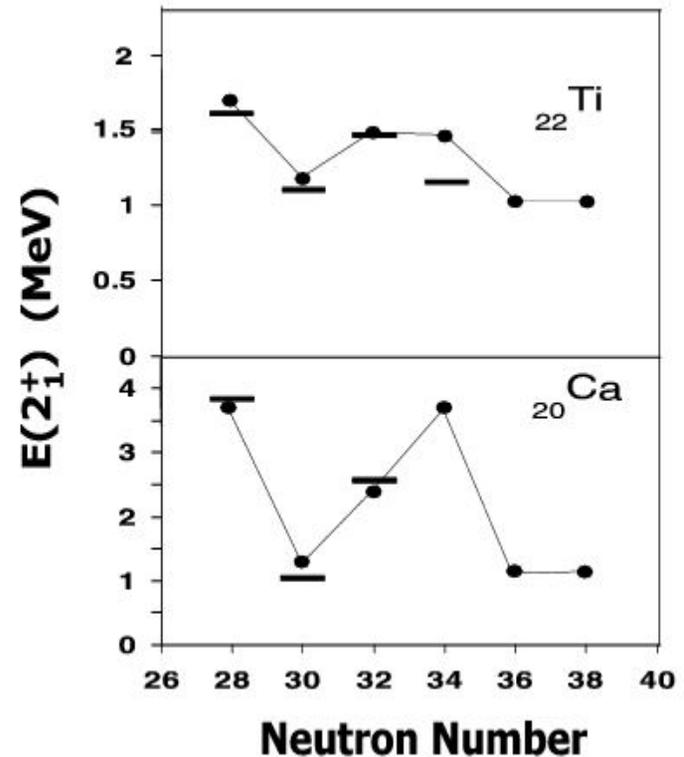
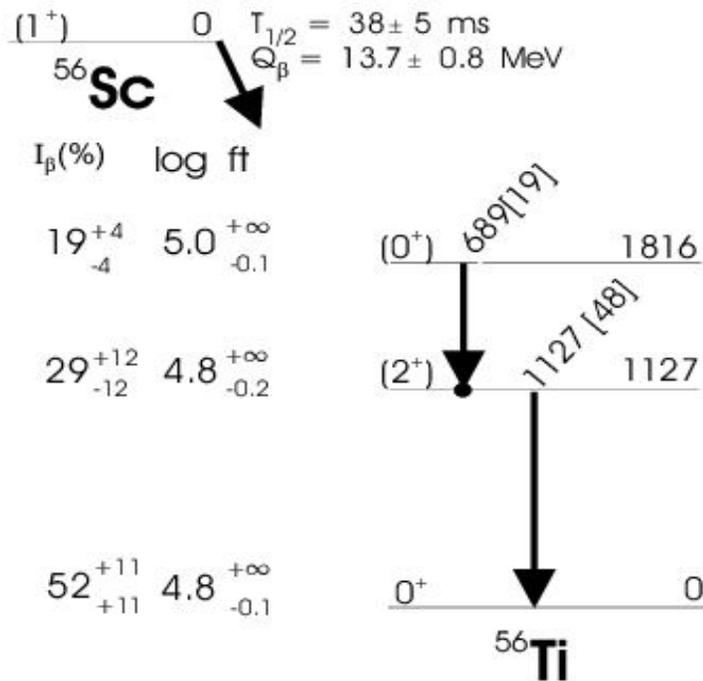
^{54}Ti comparison and ^{56}Ti prediction

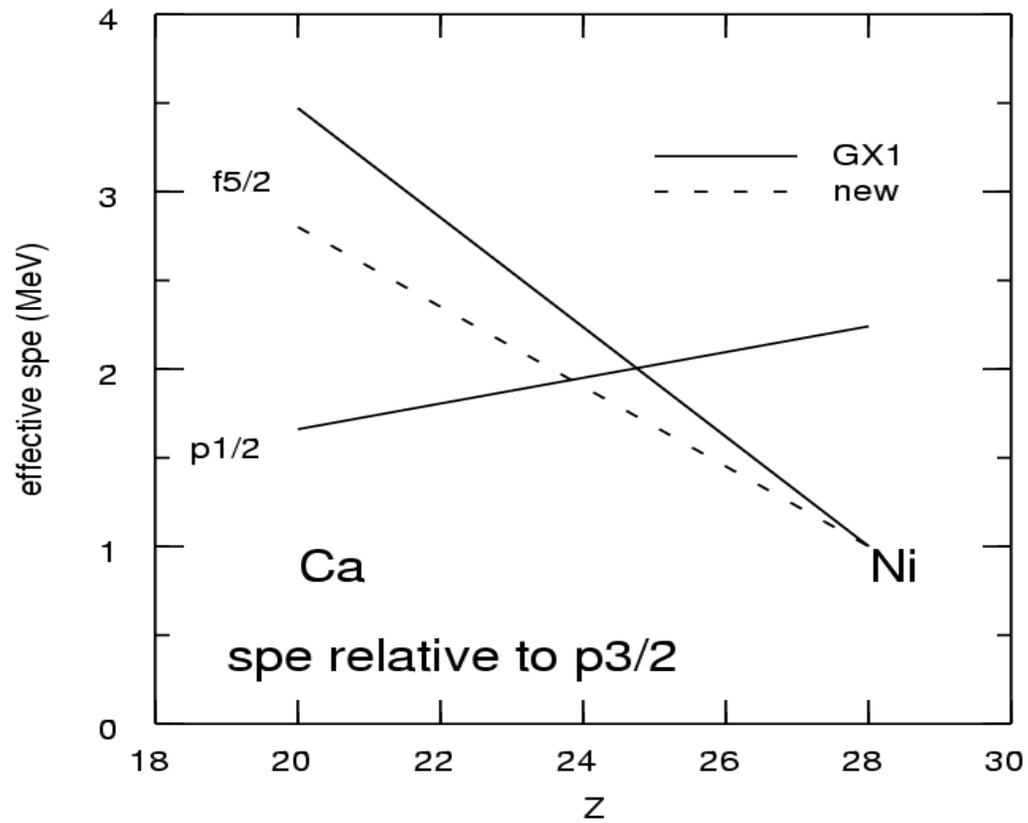
- Experimental results for ^{54}Ti compared to the GX1 predictions
- The 7^+ to 10^+ states are determined by the the $1p_{1/2}$ and $0f_{5/2}$ effective single-particle energies
- The agreement points to ^{54}Ca being a new magic nucleus



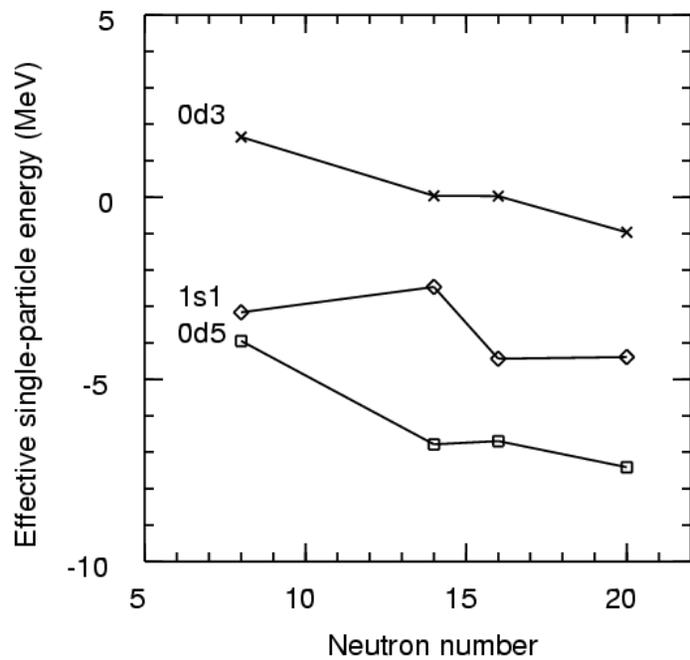
Latest results from the NSCL

S.N. Liddick et al.

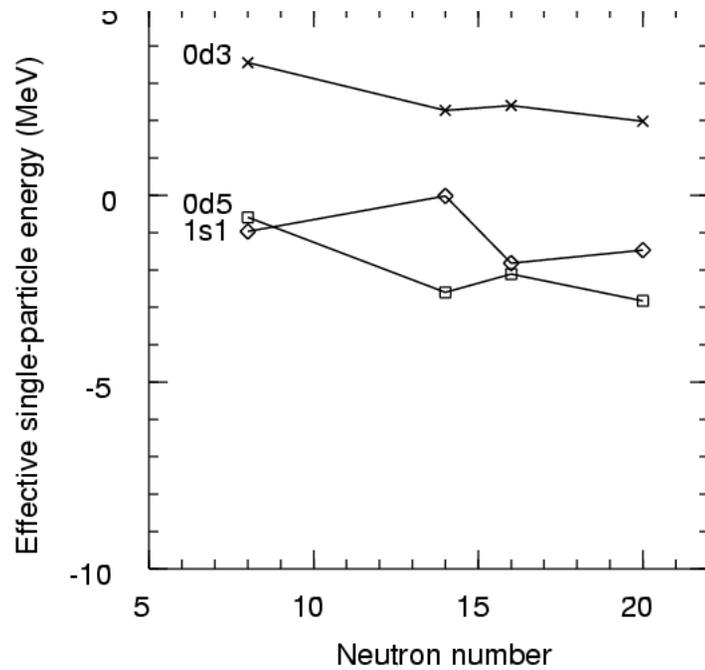




oxygen

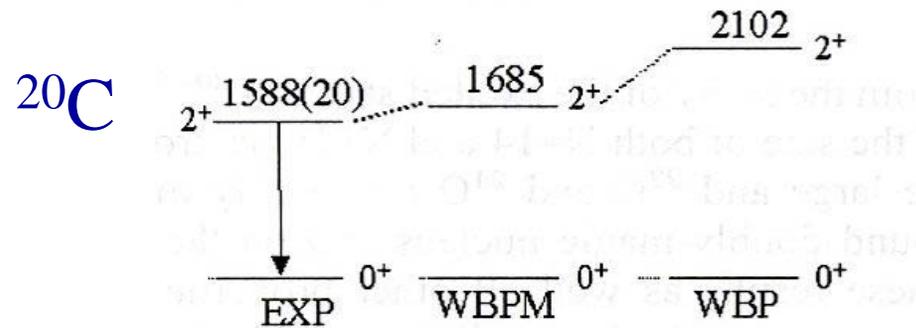
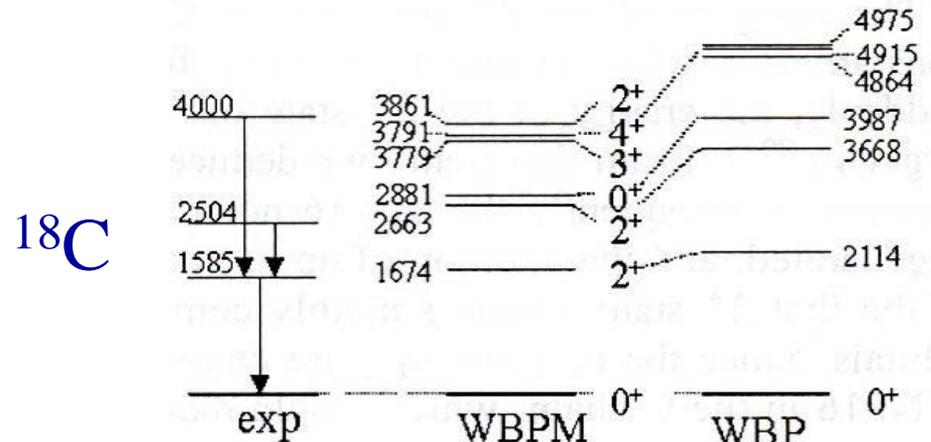


carbon



Comparison of oxygen and carbon

Implication is that the difference in neutron binding between $Z=8$ and $Z=6$ reduces the effective neutron-neutron interaction by 30%



New Magic Nuclei Towards the Drip Lines

Nuclear properties near magic nuclei

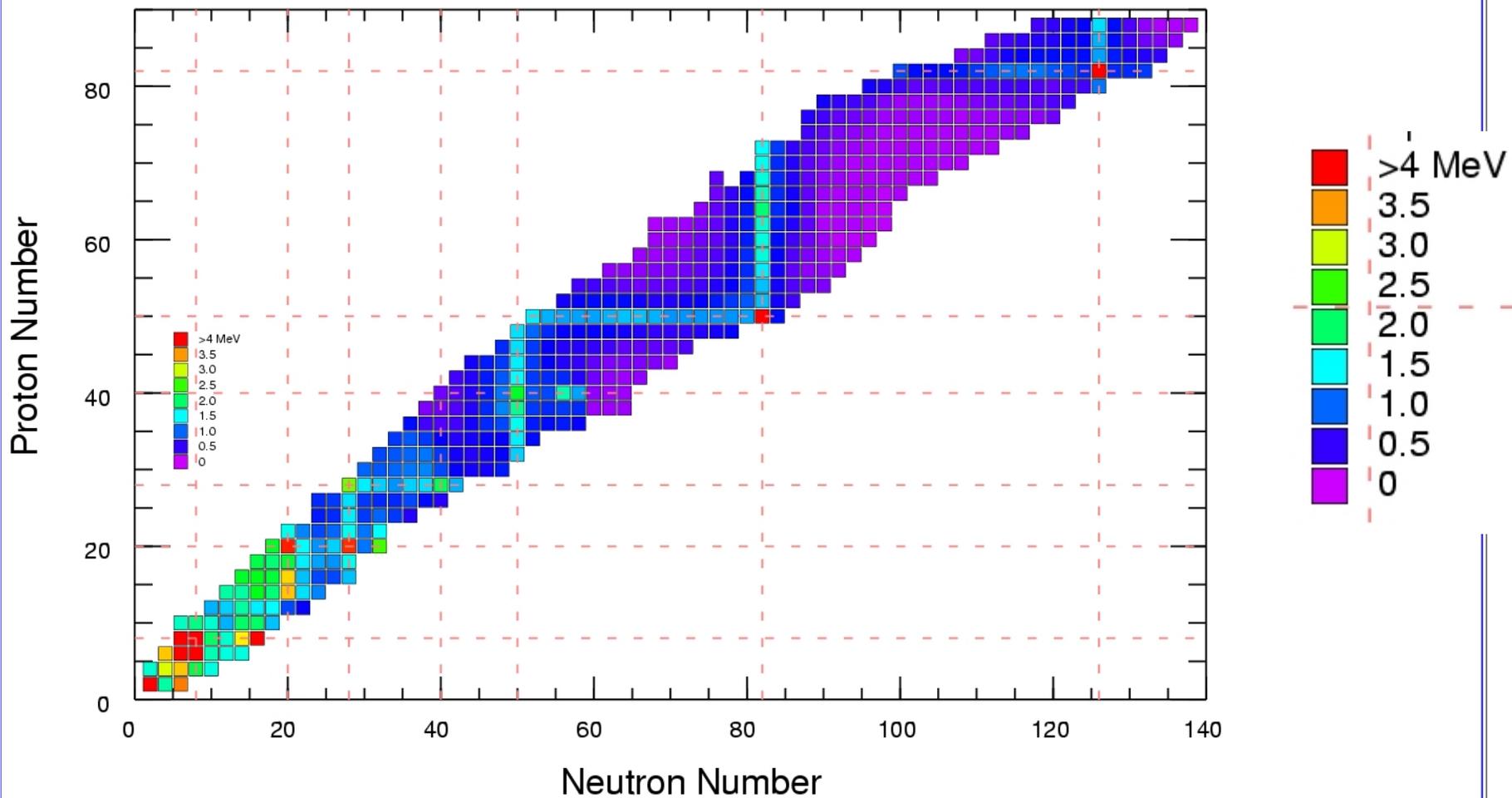
- High excited 2^+ states in the even-even magic nucleus
- Change in slope of separation energies

The underlying cause is a gap in the single-particle energy spectrum relative to the strength of pairing

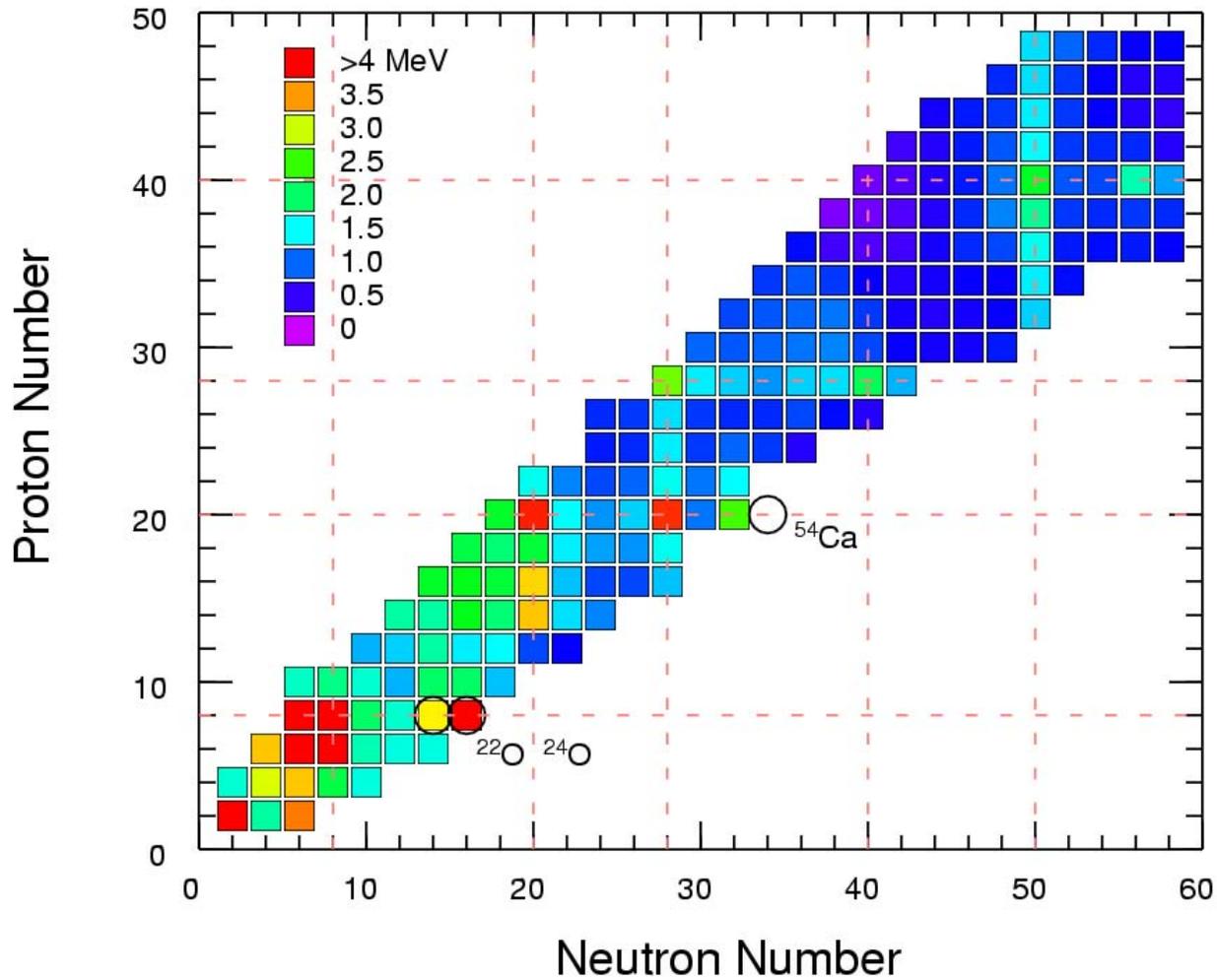
Why are magic nuclei important?

- Closed-shell configurations of the magic nuclei provide a “vacuum” from which to start configuration mixing calculations.
- In each magic nucleus the vacuum is “reset”. This gives us many independent systems from which to start.
- Results can be systematically improved upon using perturbation theory.
- We try to link the properties of closed shells in Hartree-Fock theory (Skyrme or RMF), but this has been only partly successful.

Energy of First Excited States in Even-Even Nuclei

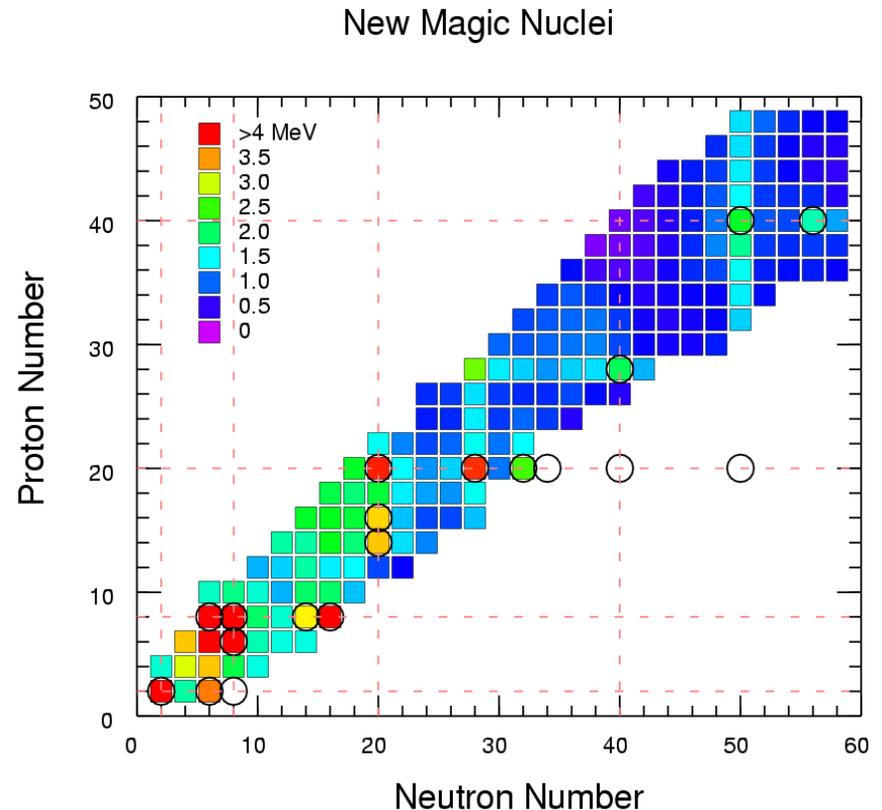


Energy of First Excited States in Even-Even Nuclei



New Magic Nuclei - Surprise

- Taken together with previous observations, one finds a new “rule” for the observed magic numbers:
- If there is an oscillator magic number (2, 8, 20 or 40) for one kind of nucleon, then the other kind of nucleon has a magic number for the filling of every possible (n,l,j) value.
- This rule accounts for 18 doubly-magic nuclei shown by the circles.
- There are no exceptions to the rule.
- New magic nuclei are predicted ^{54}Ca , ^{60}Ca and ^{70}Ca .
- What is behind this? – Three-body interactions...?



New Magic Nuclei - Surprise

	2	8	20	40
0s _{1/2}	⁴ He			
0p _{3/2}	⁸ He	¹⁴ C, ¹⁴ O		
0p _{1/2}		¹⁶ O		
0d _{5/2}		²² Si, ²² O	³⁴ Si	
1s _{1/2}		²⁴ O	³⁶ S, ³⁶ Ca	
0d _{3/2}			⁴⁰ Ca	
0f _{7/2}			⁴⁸ Ni, ⁴⁸ Ca	⁶⁸ Ni
1p _{3/2}			⁵² Ca	
1p _{1/2}			⁵⁴ Ca	
0f _{5/2}			⁶⁰ Ca	
0g _{9/2}			⁷⁰ Ca	⁹⁰ Zr
1d _{5/2}				⁹⁶ Zr

All others

⁵⁶Ni

⁷⁸Ni

¹⁰⁰Sn

¹³²Sn

²⁰⁸Pb