Many-body physics on the border of nuclear stability

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This work was done with Vladimir Zelevinsky
Supported by DOE and NSF.

Basic Theory

$|1\rangle$ - set of "internal" A-nucleon many-body states (P-space)

$|c; E\rangle$ set of "external" many-body continuum states (Q-space)

Solve problem:

$$H|\Psi\rangle = E|\Psi\rangle$$

where

$$|\Psi\rangle = \sum_1 x_1 |1\rangle + \sum_c \int dE' \, \chi^c(E') |c; E'\rangle$$

For structure physics solve for internal coefficients x_1

$$\sum_2 \left[\underbrace{\langle 1|H|2\rangle + \sum_c \int dE' \frac{\langle 1|H|c; E'\rangle\langle c; E'|H|2\rangle}{E - E' + i0}}_{\mathcal{H}_{12}(E)} - \delta_{12} E \right] x_2 = 0$$

[1] C. Mahaux and H. Weidenmüller, *Shell-model approach to nuclear reactions*, North-Holland Publishing, Amsterdam 1969

$\langle 1|H|2\rangle$ Usual shell-model Hamiltonian involving intrinsic states

$$\langle 1|H|2\rangle = H^{\circ}_{12} + V_{12}$$

$A^c_1(E') = \langle 1|H|c; E'\rangle$ decay amplitude

$$\sum_c \int dE' \frac{A^c_1 A^{c*}_2}{E - E'} = \underbrace{\sum_{c\,(\text{all})} P \int dE' \frac{A^c_1 A^{c*}_2}{E - E'}}_{\Delta(E)} - i\,\pi \underbrace{\sum_{c\,(\text{open})} A^c_1 A^{c*}_2}_{W(E)/2}$$

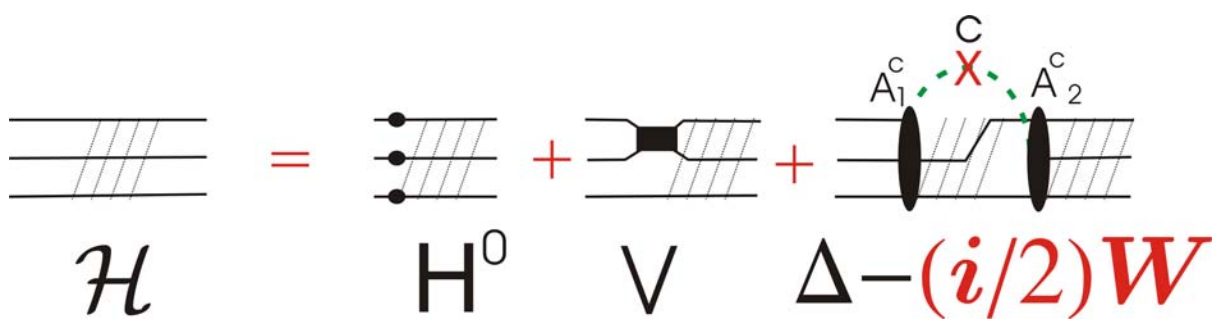
$$\mathcal{H}(E) = H^{\circ} + V + \Delta(E) - \frac{i}{2} W(E)$$

H° s.p energies

Δ interaction via continuum

V residual inteaction

W non-Hermitian - decay



Continuum Shell Model Hamiltonian

- Non-Hermitian and Energy dependent
- PHP – Internal, one-body + two-body
  - “Original” shell model, adjusted, tested… (USD and others)
  - Exact shell model for bound states E<threshold
- QHQ - External, one-body (presently)
  - kinetic energy + long range interaction, (plane waves or Bessel functions, Coulomb functions)
- QHP – Cross space, one-body + two-body
  - Responsible for coupling of spaces

One-body decay

- Continuum channel $|c; E\rangle_N = c_j^\dagger(\epsilon_j)|\alpha; N-1\rangle$
  - State α in N-1 nucleon daughter
  - Particle in continuum state j
  - Energy E=E_α+ε_j

- Transition Amplitude

s.p. decay amplitude

$$A_1^c(E_\alpha + \epsilon_j) = a^j(\epsilon_j)\ \langle \alpha; N-1|b_j\,|1; N\rangle$$

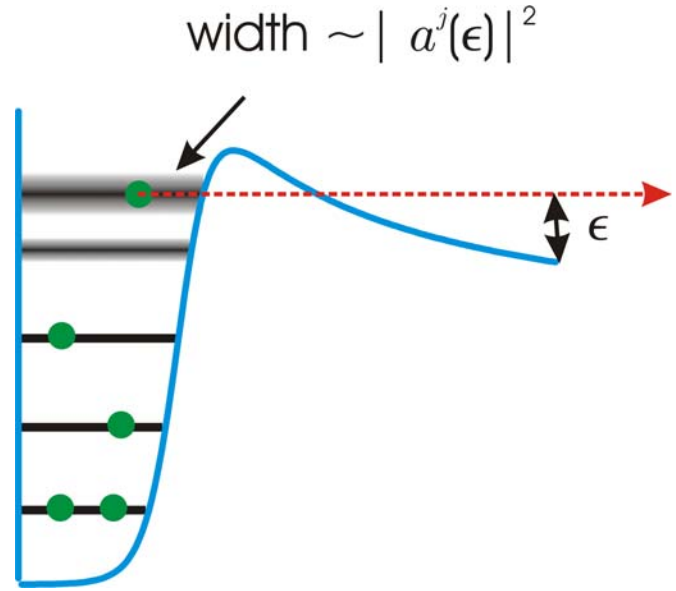
Shell model s.p. transition

Single-particle scattering problem

The same non-Hermitian eigenvalue problem

$$hu_l = \frac{1}{2\mu}\left\{-\frac{d^2}{dr^2} + \frac{l(l+1)}{r^2} + 2\mu\left[V(r) + \alpha\frac{Zz}{r}\right]\right\} u_l(r) = \epsilon u_l(r),$$

Internal states: $u_l(r)$



External states: $\epsilon = \frac{k^2}{2\mu}$

$$F_l(r) = krj_l(kr)$$

$$G_l(r) = krn_l(kr)$$

Single-particle decay amplitude

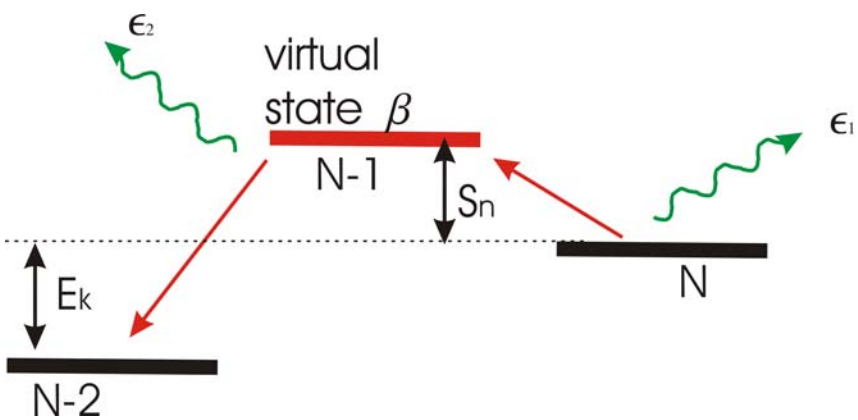
$$a^j(\epsilon) = \langle 0|c_j(\epsilon)Vb_j^\dagger|0\rangle = \sqrt{\frac{2\mu}{\pi k}}\int_0^\infty dr F_l(r)\,V(r)\,u_l(r)$$

Single particle decay width: (requires definition and solution for resonance energy)

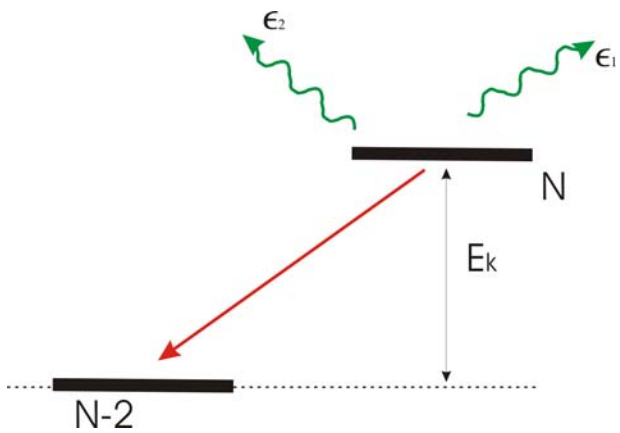
$$\gamma_j(\epsilon) = 2\pi\left|a^j(\epsilon)\right|^2$$

Two-body decays

Sequential process
via one-body interaction



Direct process
via two-body interaction



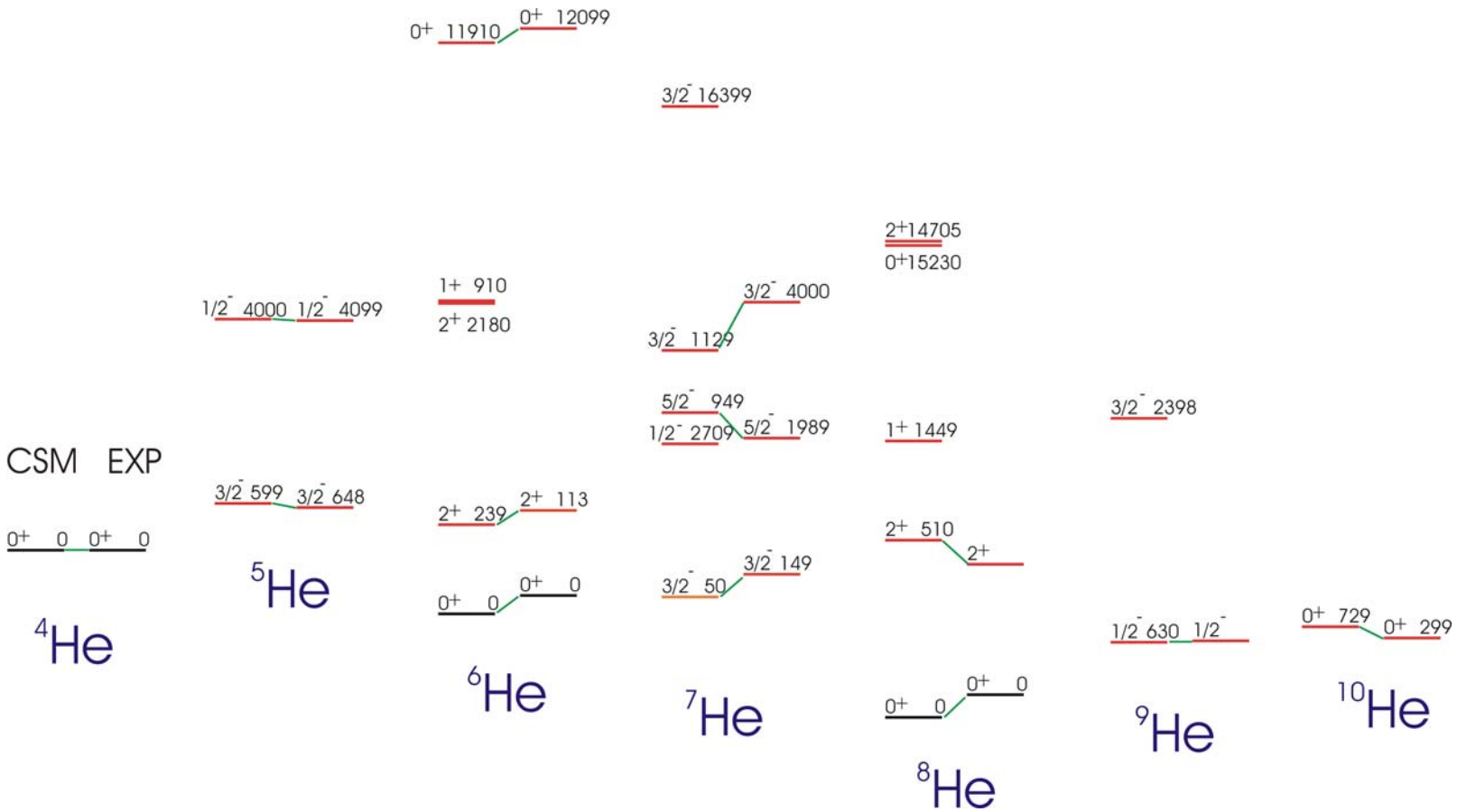
$$A^c(E) = \langle c, \epsilon_1, \epsilon_2 | H_{s.p.} | 1; N \rangle$$

$$A^c(E) = \langle c, \epsilon_1, \epsilon_2 | H_{\mathrm{2body}} | 1; N \rangle$$

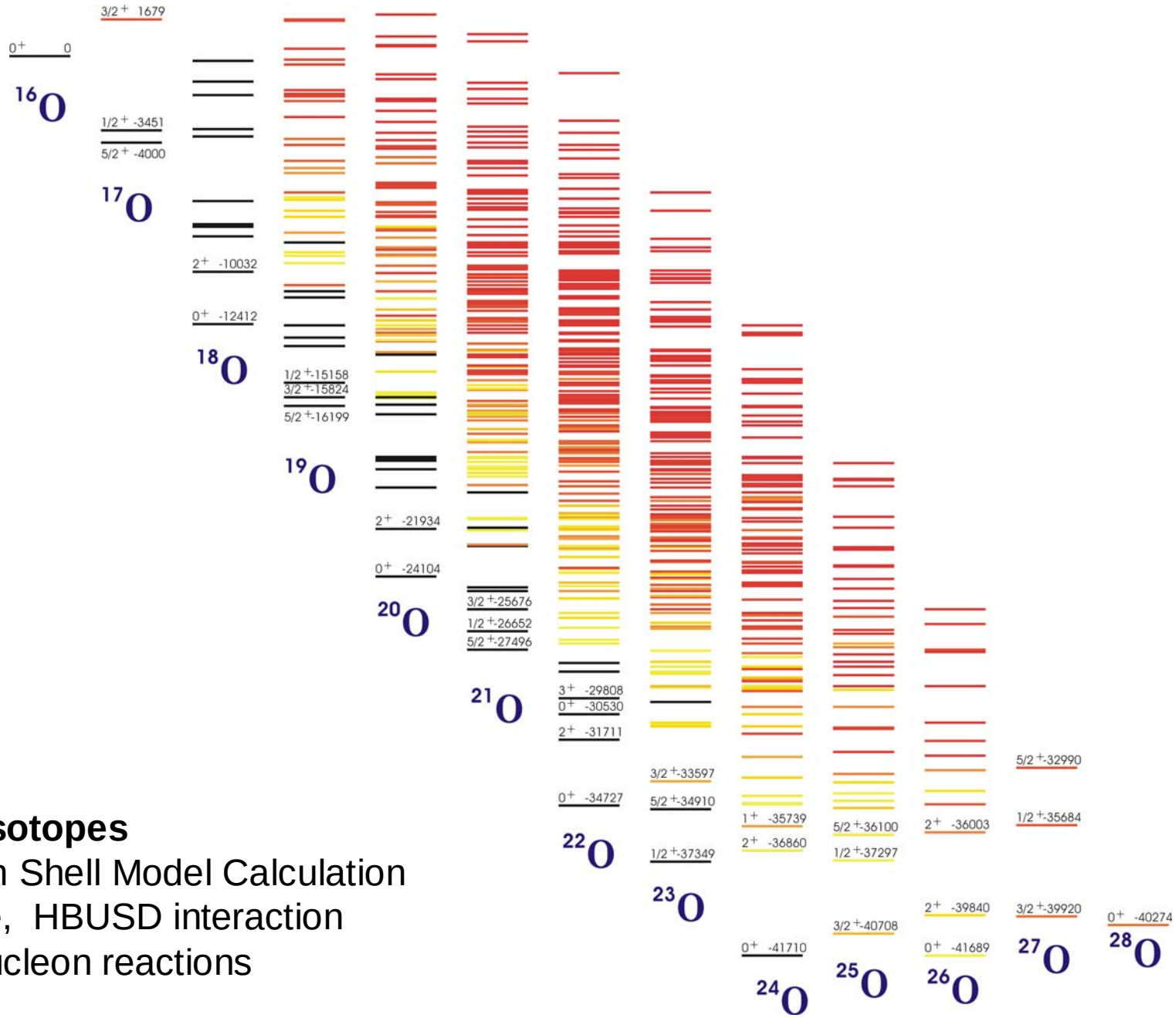
Realistic Shell Model Example

- Interaction
  - PHP – Shell model Hamiltonian, USD or other interaction
  - Assume that shell model interaction includes Hermitian Δ
  - QHQ and QHP one-body Woods-Saxon potential
  - QHP two-body phenomenological parameterization
- Solution
  - From ^{4}He up to ^{10}He
  - From ^{16}O up to ^{28}O
  - Given guess energy E for state α, W(E) is constructed by considering all open channels.
  - Non-Hermitian Hamiltonian is solved for iteratively for new E (Breit-Wigner resonance condition)

CSM calculations for He



Interaction in PP space CKIHE PRC37, 2220 (1988).

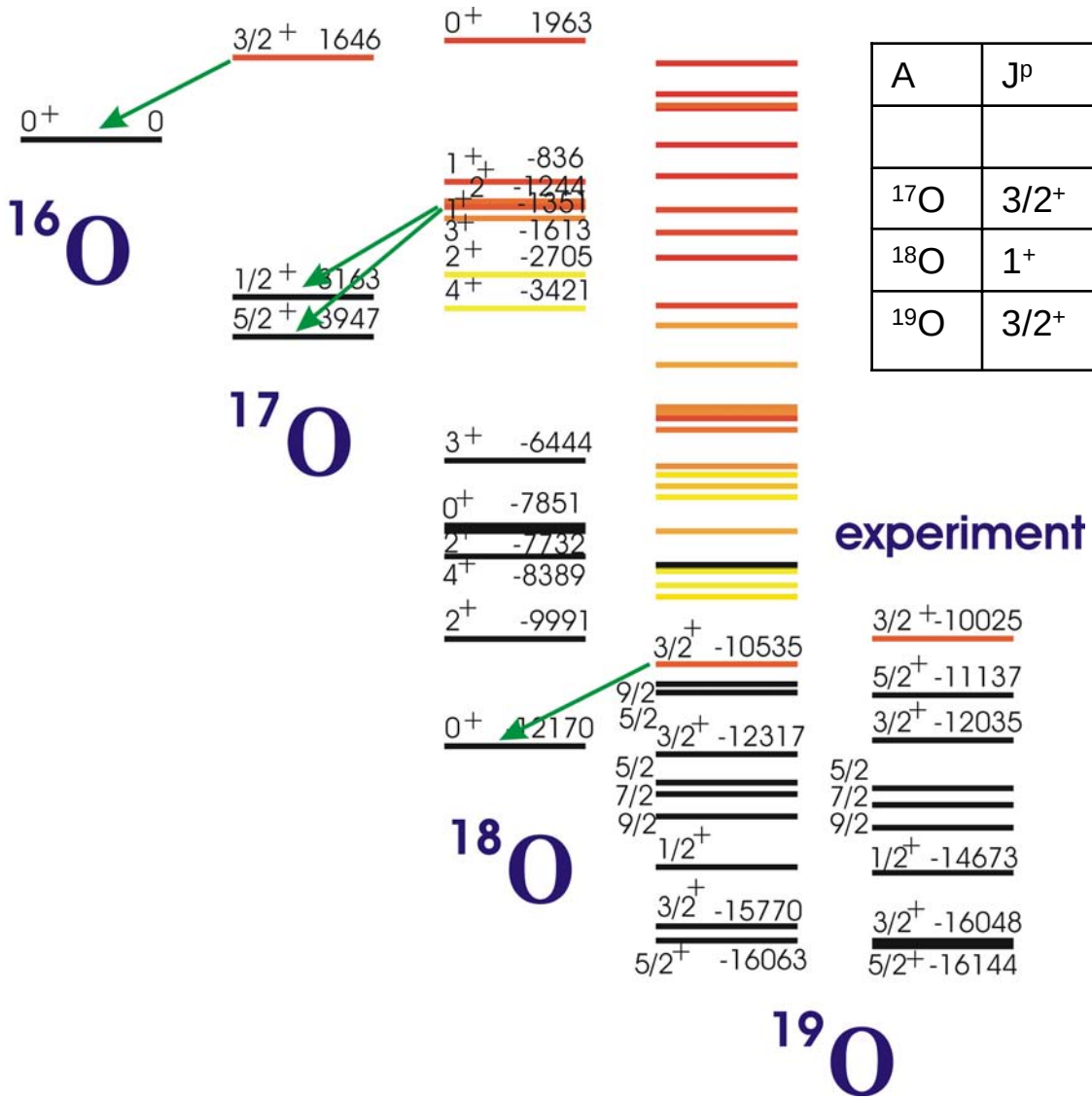


Oxygen Isotopes

Continuum Shell Model Calculation

- sd space, HBUSD interaction
- single-nucleon reactions

Comparison with experiment



A	J^p	Experiment		CSM calculation	
		E(MeV)	Γ (keV)	E(MeV)	Γ (keV)
^{17}O	$3/2^+$	1.646	96	1.646	102
^{18}O	1^+	8.817	70	10.820	86
^{19}O	$3/2^+$	6.119	110	5.530	90

Features

- "No free parameters"
- Bound states identical to SM
- Decaying states shift and acquire width

Features of Continuum Shell Model

- Extension of traditional shell model
- Non-Hermitian and energy-dependent effective Hamiltonian: Self-consistency
  - Decay chains
  - Open channels
  - Many-body structure
  - Reaction theory
- New phenomena
  - Superradiance and narrow resonances

Interplay of collectivities

Definitions

n - labels particle-hole state

ε_n – excitation energy of state n

d_n - dipole operator

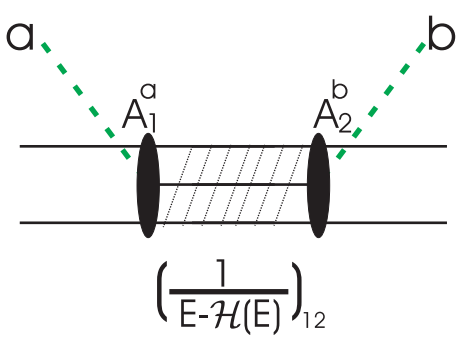
A_n – decay amplitude of n

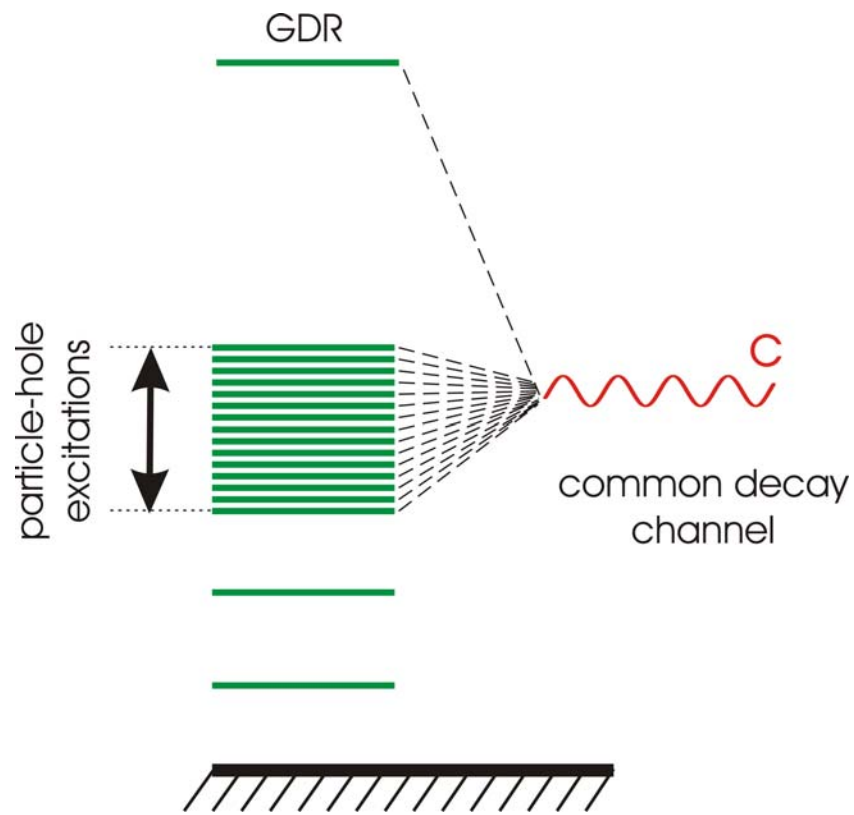
Model Hamiltonian

$$\mathcal{H}_{nn'} = \epsilon_n \delta_{nn'} + \lambda d_n d_{n'} - \frac{i}{2} A_n A_{n'},$$

Driving GDR externally

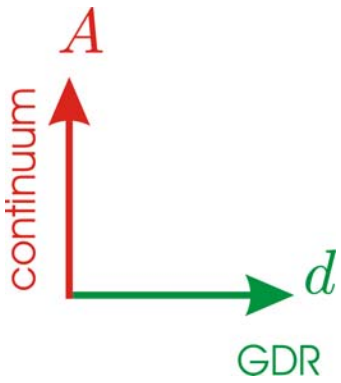
(doing scattering)

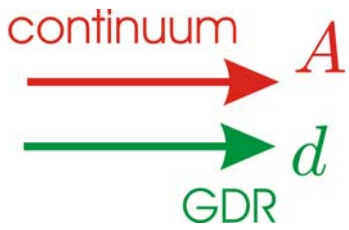


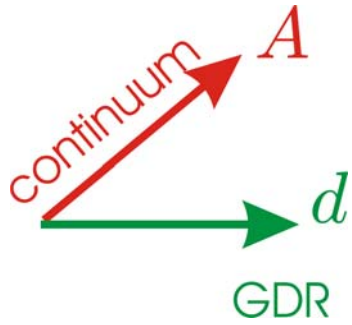


Everything depends on
angle between multi dimensional vectors
A and d

Pigmy resonance





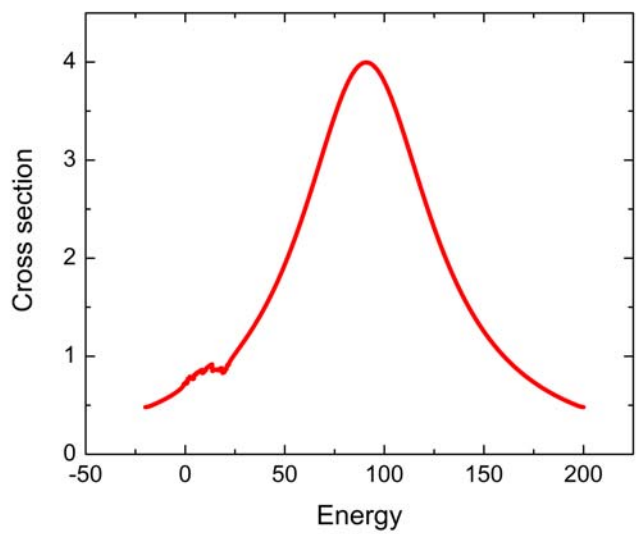


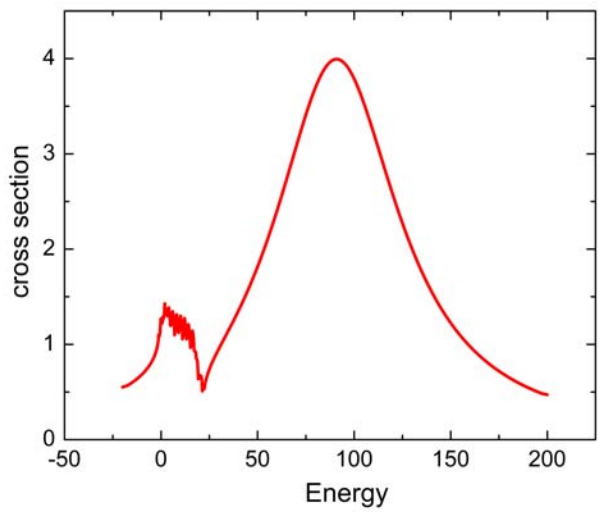
Orthogonal:
GDR not seen

Parallel:
Most effective excitation of GDR from continuum

At angle:
excitation of GDR and pigmy

A model of 20 equally distant levels is used





Conclusions

- The era of new theoretical methods
- Uniting structure and reactions
  - Internal structure depends on external “world”
  - Chains on nuclei united by reactions
- Beyond single-particle decays

- Questions for future
  - More applications
  - Interactions

References

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