# New quantum number and selection rules for chiral doublet bands and chiral wobblers

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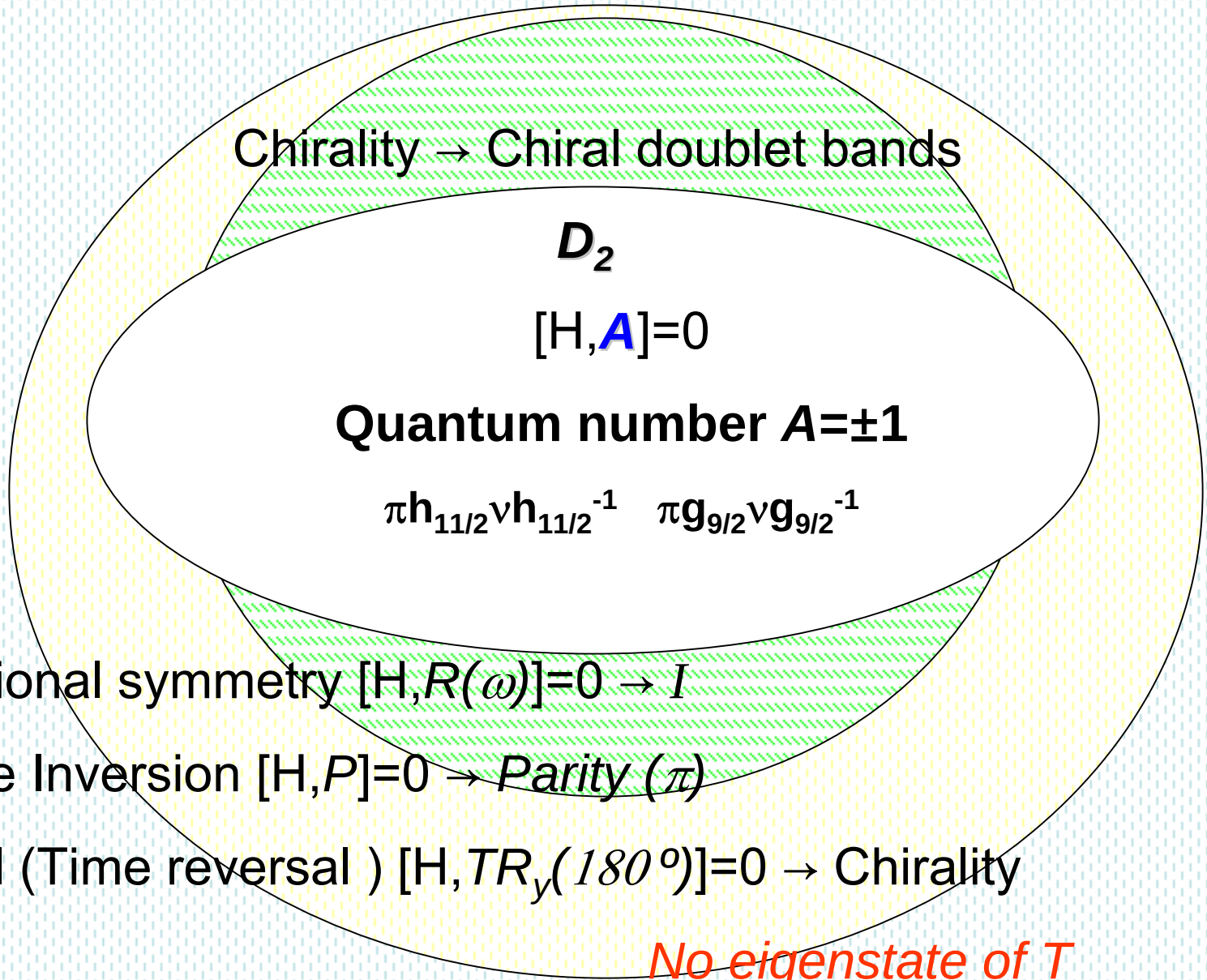



Rotational symmetry [H,$R(\omega)$]=0 → $I$

Space Inversion [H,$P$]=0 → *Parity* ($\pi$)

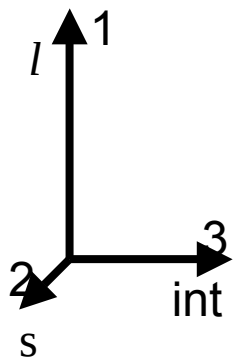
Chiral (Time reversal ) [H,$TR_y(180^o)$]=0 → Chirality

*No eigenstate of T*

# SYMMETRIC rotor having a TRIAXIAL mass distribution

Particle Rotor Model: $H\left(\gamma = 90^{\circ}\right) = H_{rot} + V_{sp}^{\pi} + V_{sp}^{\nu}$

*Irrot. flow moment of inertia*




**Core** $H_{rot} = \sum_{k}^{3} \frac{\hbar^2}{2J_k} R_k^{\ 2} \Rightarrow \frac{\hbar^2}{8J_0}(R_3^{\ 2} + 4(R_1^2 + R_2^2))$

$D_2$ symmetry → $R_3$ = 0,±2,±4,±6,…..

**Single proton particle in unique parity *j* (=$h_{11/2}$) shell**

$$V_{sp}^{\pi} \propto \beta(Y_{22} + Y_{2-2}) \Rightarrow (j_{\pi 1}^2 - j_{\pi 2}^2)$$

**Single neutron hole in unique parity *j* (=$h_{11/2}$) shell**

$$V_{sp}^{\nu} \propto \beta(Y_{22} + Y_{2-2}) \Rightarrow -(j_{\nu 1}^2 - j_{\nu 2}^2)$$

# Invariance properties of $H=H_{rot}+V_{sp}^{\pi}+V_{sp}^{\nu}$

$$H=\frac{\hbar^2}{8J_0}\underline{(R_3{}^2+4(R_1^2+R_2^2))}+\alpha\underline{[(j_{\pi 1}^2-j_{\pi 2}^2)-(j_{\nu 1}^2-j_{\nu 2}^2)]}$$

Symmetric by rotation around the 3-axis

$\tilde{A}$

$\tilde{C}$:

Exchange between valence proton and neutron

+

**Rotation by 90º or 270º** :

Exchange between 1- and 2-axis

$\tilde{A}=\tilde{C}\exp(i\pi/2R_3)$

$[H,\tilde{A}]=0$

$\tilde{A}^2=I$ (identity)

Quantum number $A=\pm 1$

# Operator $\tilde{A}$

***Rotation***

$$r \equiv \exp(i\frac{\pi}{2}R_3) \quad \text{OR} \quad \exp(i\frac{3\pi}{2}R_3)$$

$r$ = 1, for $R_3$ = 0,±4, ±8, ±12…

$r$ = -1, for $R_3$ = ±2, ±6, ±10…

***Exchange symmetry between valence proton and neutron***

$\tilde{C}$ : $\pi \leftrightarrow \nu$

$c$ = +1 for symmetric

$c$ = -1 for anti-symmetric

| $A$ | $c$ | $r$ |
|---|---|---|
| +1 | +1 | +1 |
| | -1 | -1 |
| -1 | +1 | -1 |
| | -1 | +1 |

# Quantum number *A* & Selection rule

**$[H,\tilde{A}]=0$**

Quantum number A=±1

| $A$ | $c$ | $r$ |
|---|---|---|
| +1 | +1 | +1 |
| | -1 | -1 |
| -1 | +1 | -1 |
| | -1 | +1 |

**Allowed B(E2;$I_i \rightarrow I_f$) $A_i \neq A_f$**

Core contribution only

$\Delta C=0$

$\Delta R_3=\pm 2$ for $\gamma=90^o \rightarrow \Delta r \neq 0$

**[B(M1;$I_i \rightarrow I_f$) with $A_i \neq A_f$] >>[B(M1;$I_i \rightarrow I_f$) with $A_i=A_f$]**

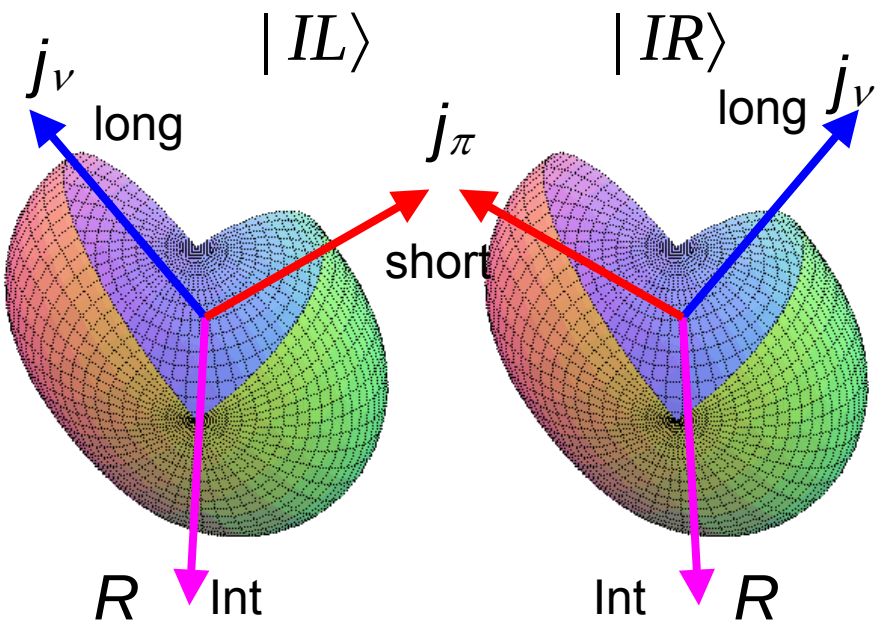
$\langle M1\rangle_n \approx -\langle M1\rangle_p$

$\rightarrow \langle M1\rangle_{total} \approx 0$ & B(M1;$Ii \rightarrow If$)≈0 for $\Delta C=0$

$\rightarrow \langle M1\rangle_{total}$ *is enhanced* for $\Delta C \neq 0$

$|\Delta R_3| < 2 \rightarrow \Delta r=0$

# EM properties of chiral geometry in general




$$|I+\rangle = \frac{1}{\sqrt{2}}(|IL\rangle + |IR\rangle)$$

$$|I-\rangle = \frac{i}{\sqrt{2}}(|IL\rangle - |IR\rangle)$$

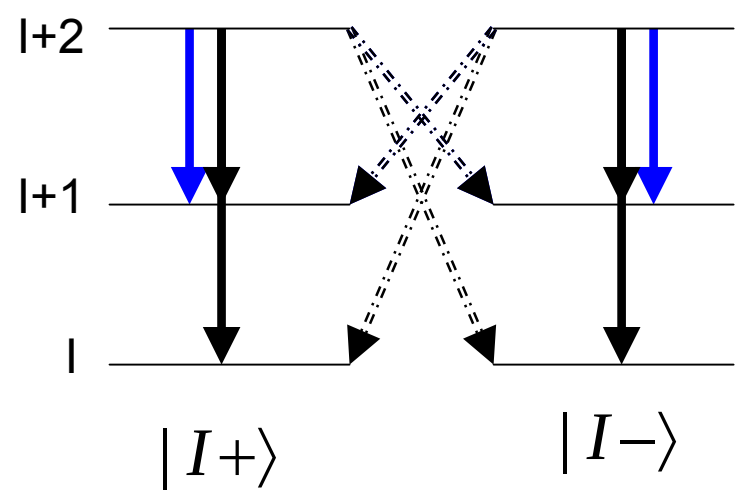
*I>>1*

$$\langle IL|E2|IR\rangle \approx 0$$

$$\langle IL|M1|IR\rangle \approx 0$$

$$B(EM;I_i+\rightarrow I_f+) \approx B(EM;I_i-\rightarrow I_f-)$$

$$B(EM;I_i+\rightarrow I_f-) \approx B(EM;I_i-\rightarrow I_f+)$$



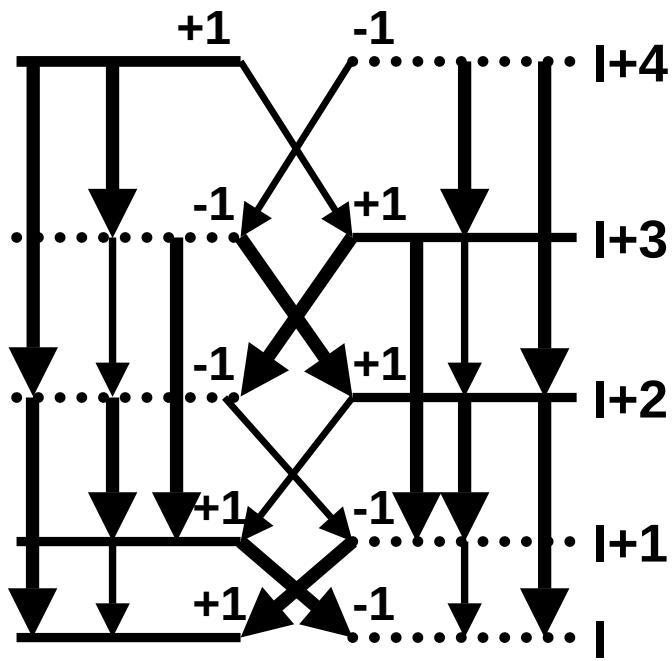
# EM properties of chiral geometry with *A* symmetry

$$\tilde{C}\,|\,IL\rangle \rightarrow |\,IR\rangle \qquad \exp(i\theta R_3)\,|\,IL\rangle \rightarrow |\,IL\rangle$$

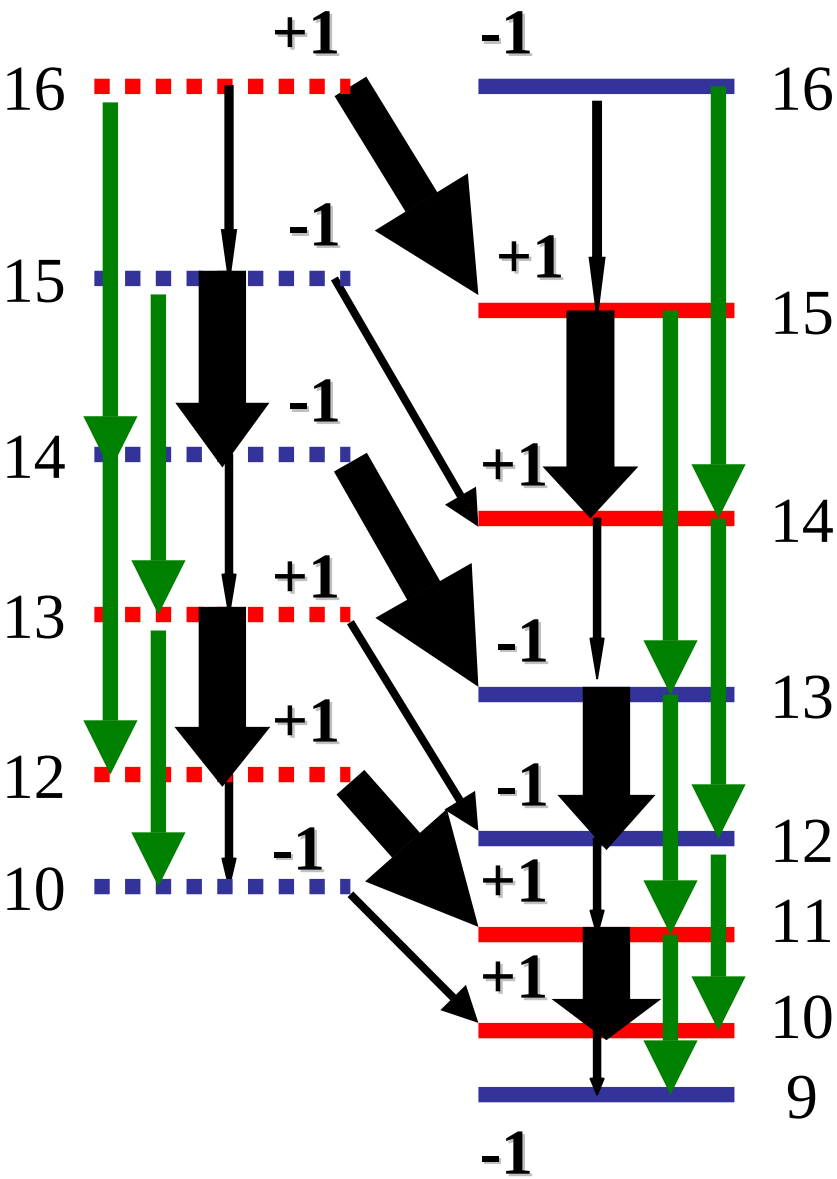
$$\tilde{C}\,|\,IR\rangle \rightarrow |\,IL\rangle \qquad \exp(i\theta R_3)\,|\,IR\rangle \rightarrow |\,IR\rangle$$

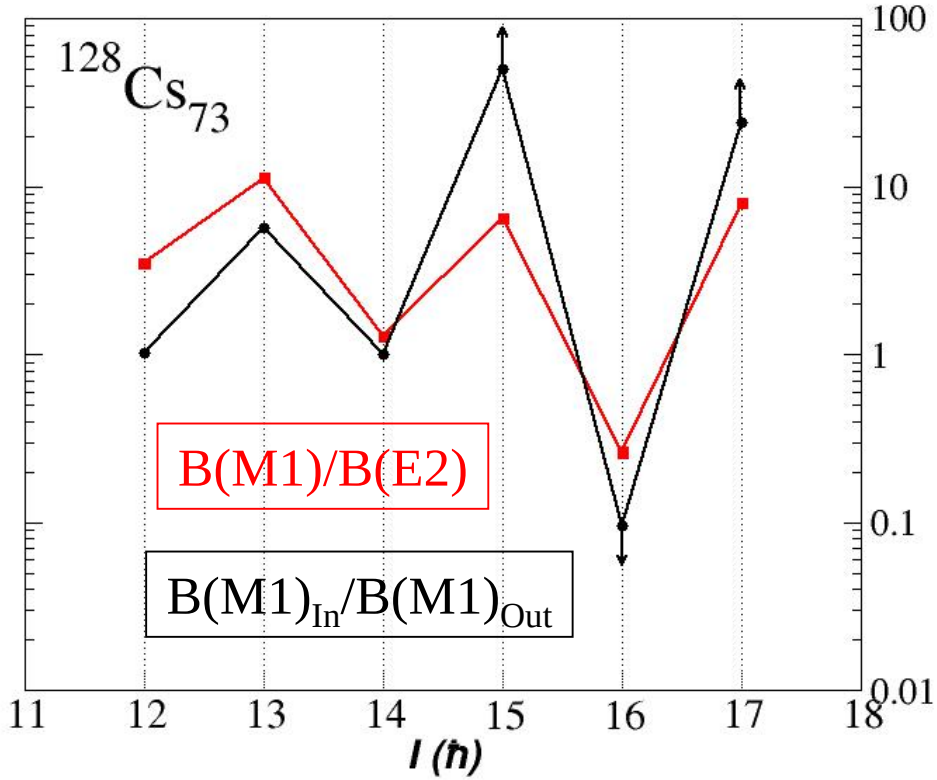
$$A = \tilde{C}\exp(i\theta R_3) \quad \text{where} \quad \theta = \frac{\pi}{2}, \frac{3\pi}{2}$$

$$A\,|\,I+\rangle = \pm 1\,|\,I+\rangle \leftrightarrow A\,|\,I-\rangle = \mp 1\,|\,I-\rangle$$



# Possible observation of the *A* symmetry




Limits04

# Conclusion and future

- A model as simple as possible that describes uniquely triaxial features with a quantum number
  - **Selection rules**
  - ***Chiral doublet bands***
  - ***Chiral wobbling mode***
- Derived selection rules are valid irrespective of whether or not the chiral geometry is achieved
  - **Possibility of evidence for triaxiality at low spin near a bandhead of particular configuration in odd-odd nuclei**
- A good **starting** point to investigate nuclear triaxiality in more complex real nuclei
  - Does *A* symmetry exist in real nuclei?

# In loving memory of

**David B. Fossan**

1934-July 27,2003