Clusters and Shell-Structure in Light Nuclei

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Nuclei at the Limits

Argonne National Laboratory
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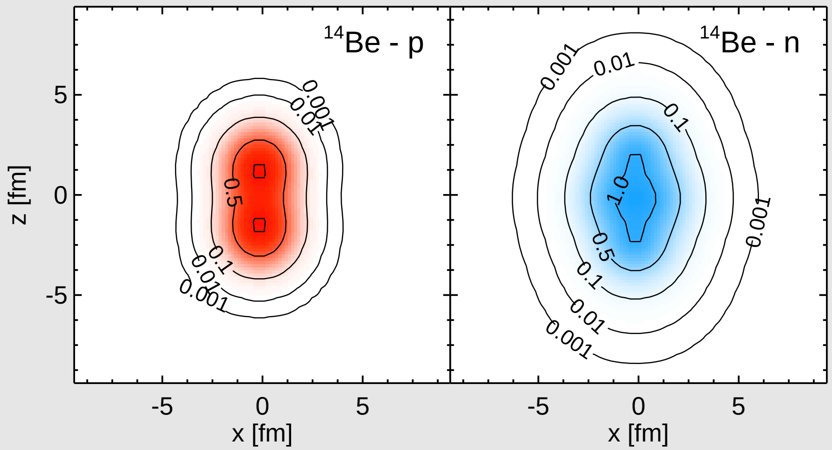


Overview

FMD wave functions

Nucleon Nucleon Interaction

- Short-range central and tensor correlations
- Interaction in momentum space
- Effective correction to two-body interaction

PAV, VAP and Multiconfiguration

Applications

- Helium isotopes
- Beryllium isotopes
- Carbon isotopes, ^{12}C spectrum

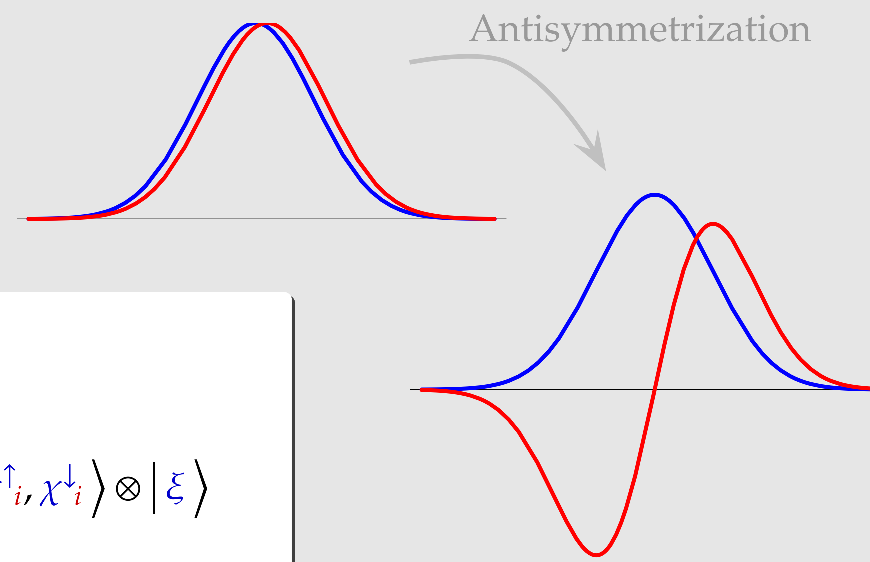
Fermionic Molecular Dynamics Many-Body States

Fermionic

Slater determinant

$$\left| Q \right\rangle = \underset{\sim}{\mathcal{A}}\Big(\left| q_1 \right\rangle \otimes \cdots \otimes \left| q_A \right\rangle \Big)$$

➨ antisymmetrized A-body state



Molecular

single-particle states

$$\left\langle \mathbf{x} \middle| q \right\rangle = \sum_i c_i \exp\left\{ -\frac{(\mathbf{x}-\mathbf{b}_i)^2}{2a_i} \right\} \otimes \left| \chi^{\uparrow}_i, \chi^{\downarrow}_i \right\rangle \otimes \left| \xi \right\rangle$$

➨ Gaussian wave-packets in phase-space, spin is free, isospin is fixed

➨ basis states can describe **shell model**, **intrinsically deformed**, **cluster** and **halo** states on the same footing

Dynamics

Time-dependent variational principle

$$\delta \int dt \frac{\left\langle Q \middle| i\frac{d}{dt} - \underset{\sim}{\hat{H}} \middle| Q \right\rangle}{\left\langle Q \middle| Q \right\rangle} = 0$$

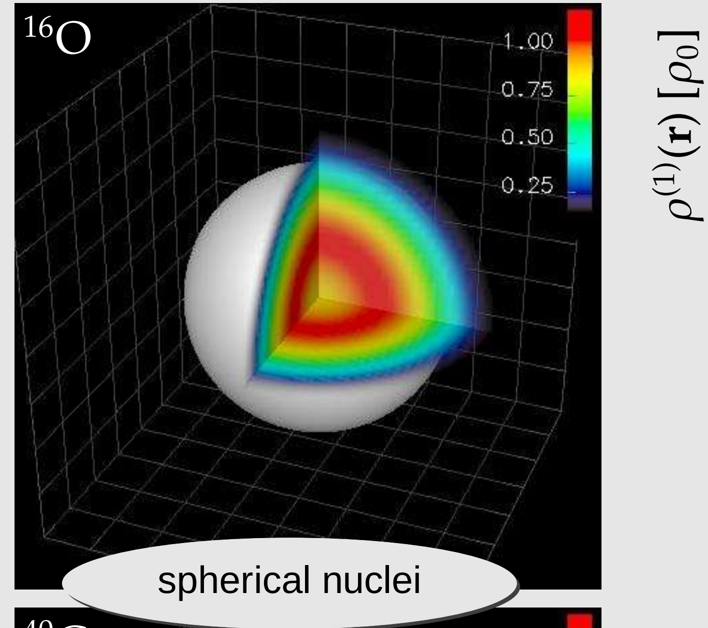
Perform Variation

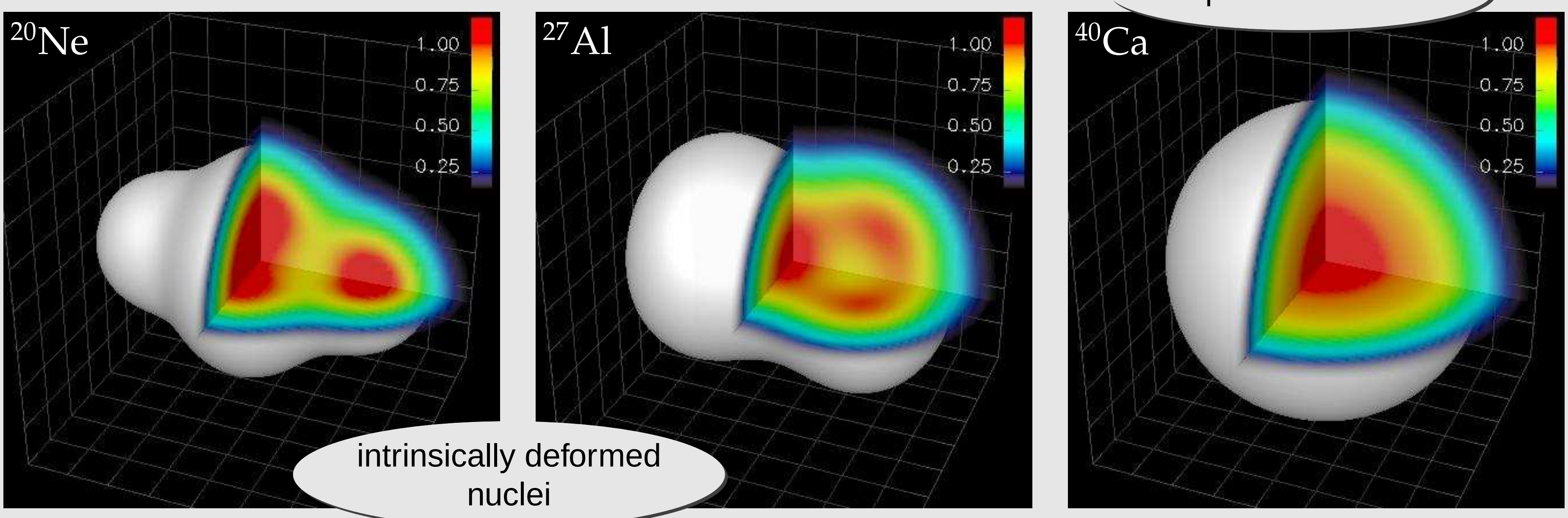
Minimization

- **minimize Energy** with respect to all single-particle parameters q_k

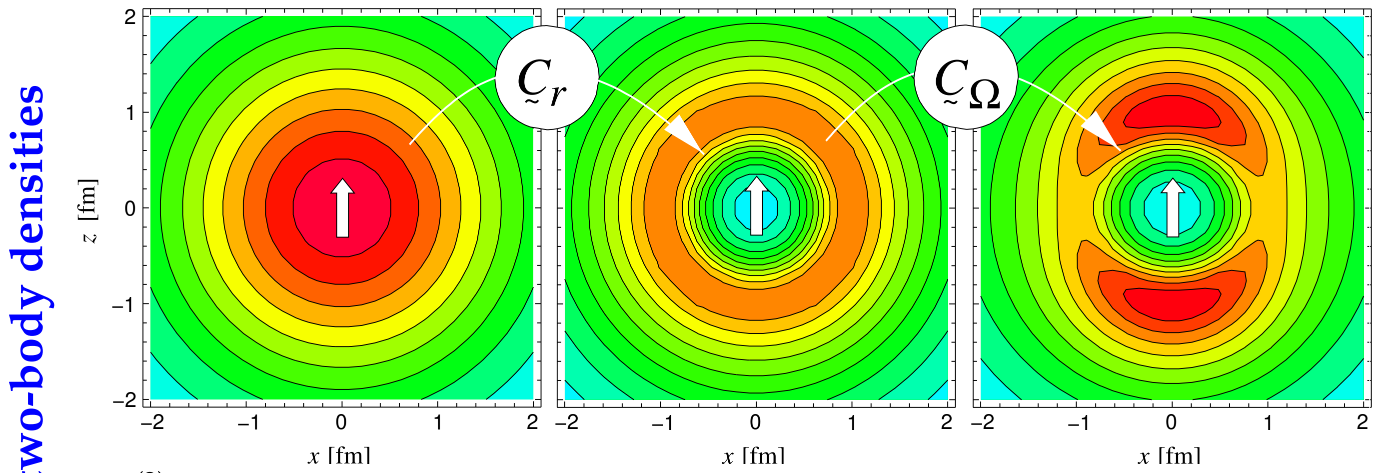
$$\min_{\{q_k\}} \frac{\left\langle Q \middle| \underset{\sim}{\hat{H}} - \underset{\sim}{T}_{cm} \middle| Q \right\rangle}{\left\langle Q \middle| Q \right\rangle}$$

- this is a **Hartree-Fock** calculation in our particular single-particle basis
- the mean-field may **break the symmetries** of the Hamiltonian





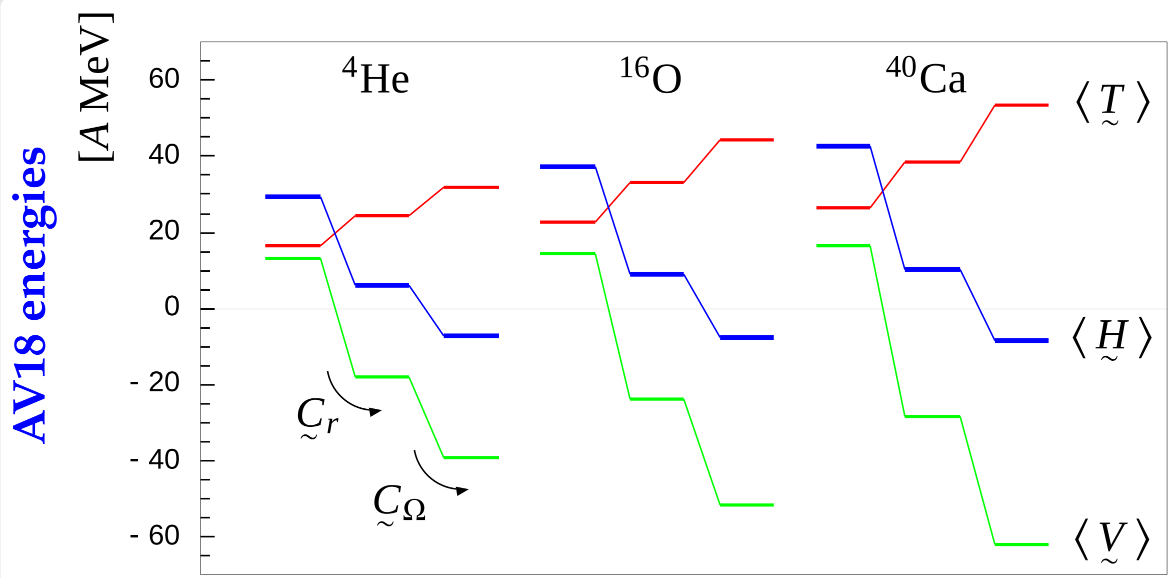
Correlated Realistic Interaction V_{UCOM}



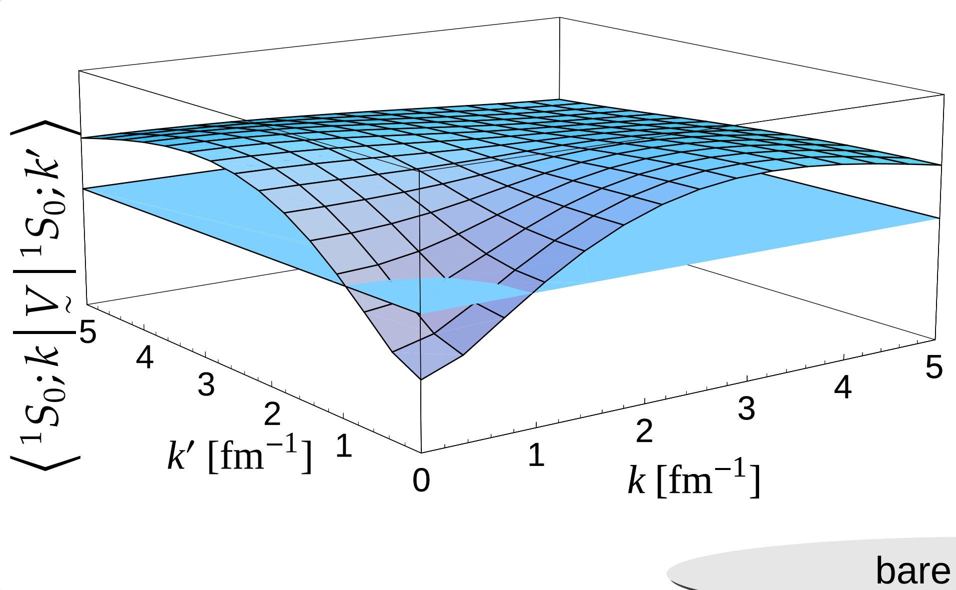
$\rho^{(2)}_{S,T}(\mathbf{r}_1 - \mathbf{r}_2) \quad S = 1, M_S = 1, T = 0$

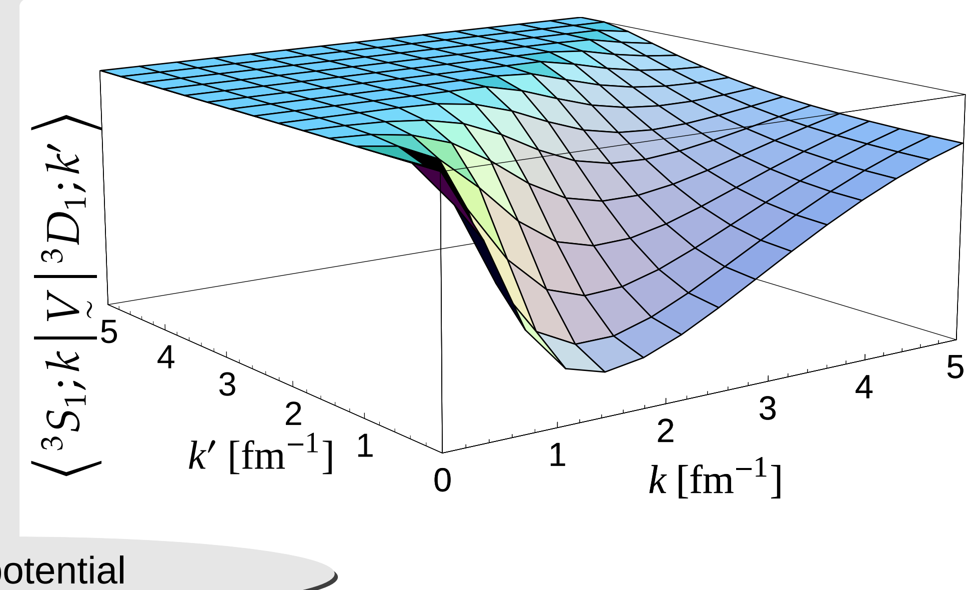
- **central correlator C_r** shifts density out of the repulsive core
- **tensor correlator C_Ω** aligns density with spin orientation

- results in common **low-momentum interaction** very similar to the $V_{low\,k}$ by Kuo, Bogner, Schwenk



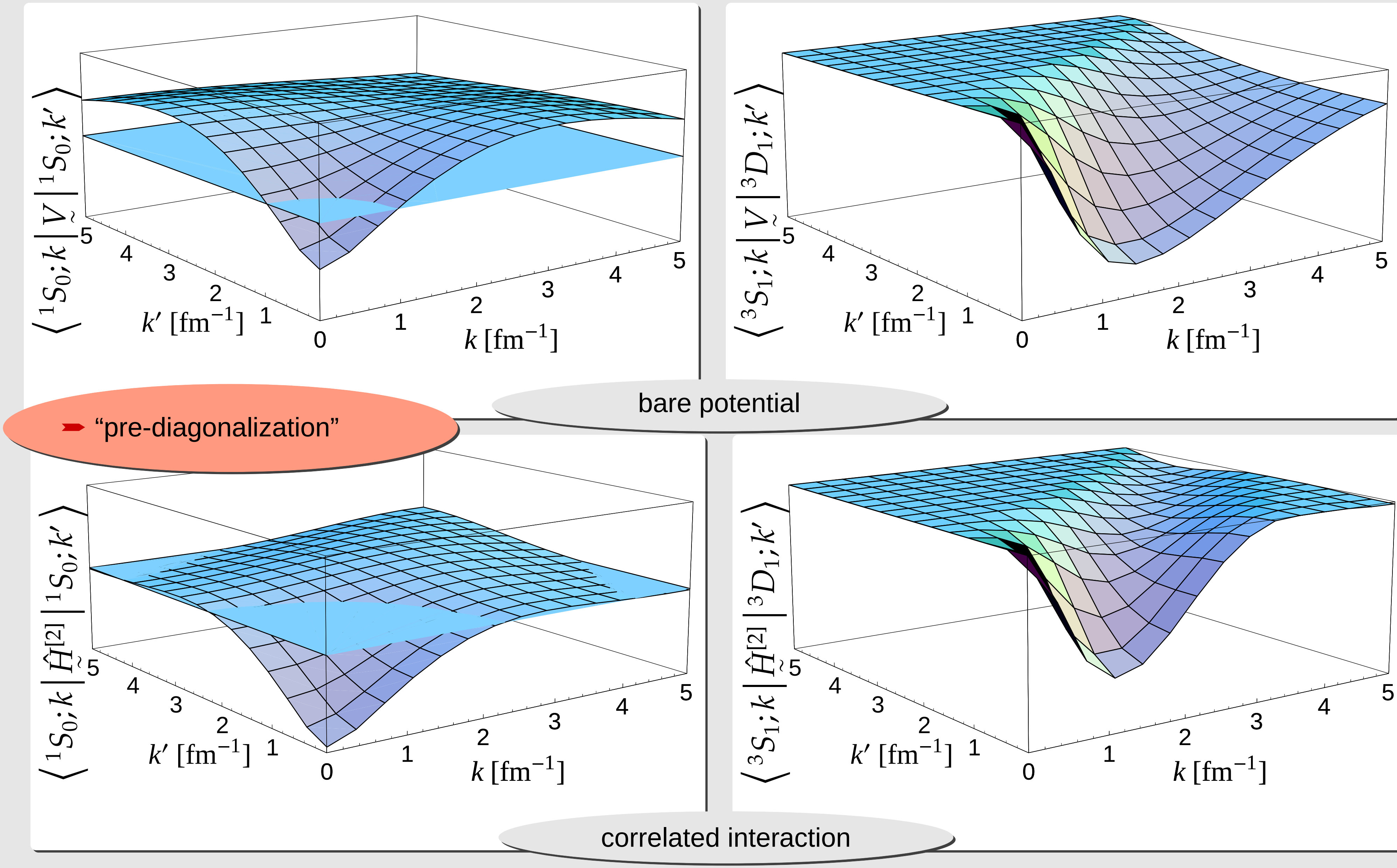
Off-diagonal Matrixelements





bare potential

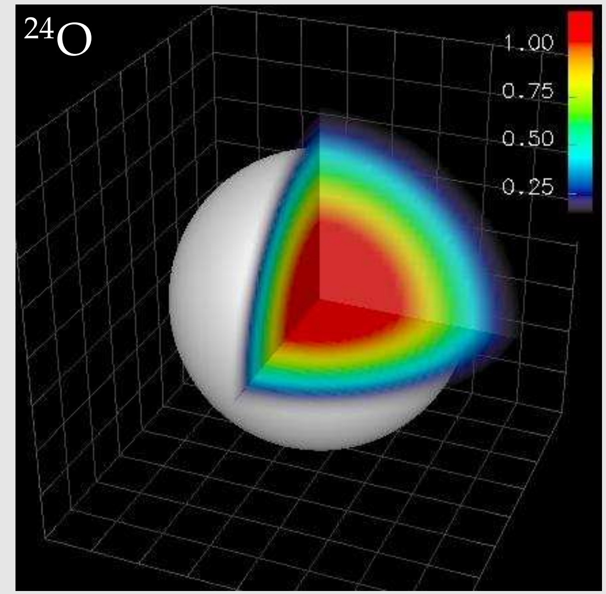
Off-diagonal Matrixelements



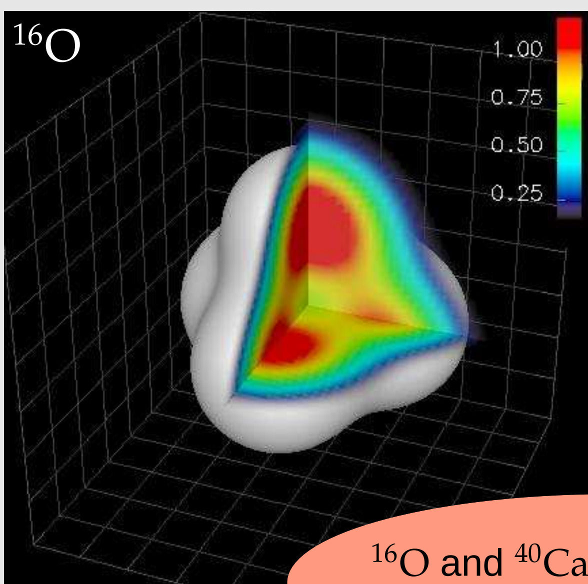
Effective Correction to the Interaction

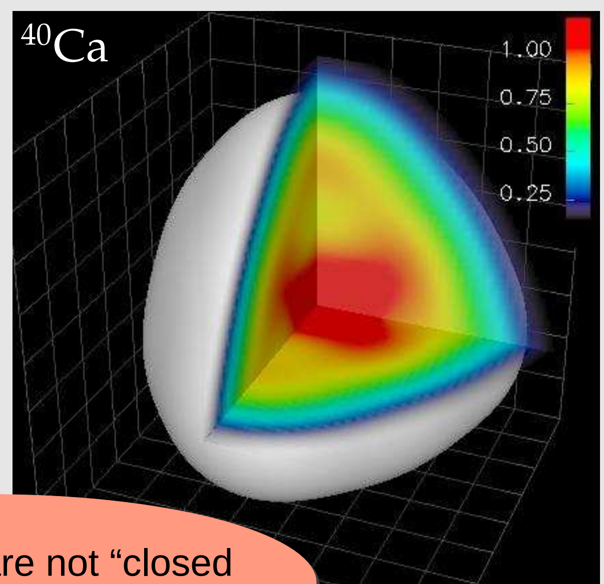
Effective two-body interaction

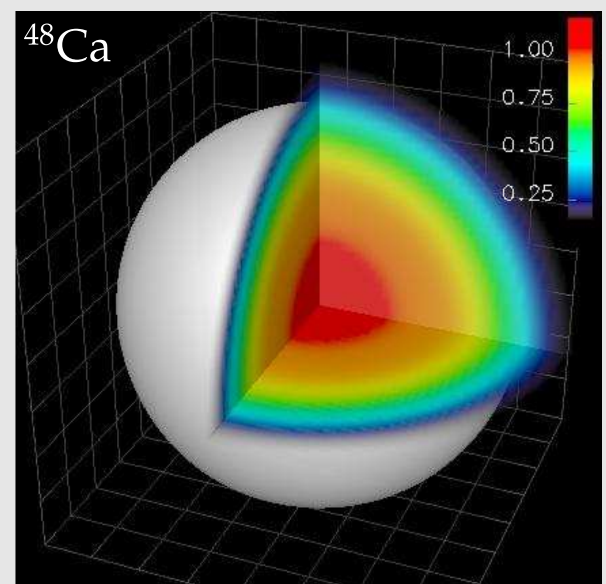
- correlated two-body interaction is lacking **three-body correlations** and genuine **three-body forces**
- explicit tensor parts of correlated interaction are omitted
- simulate missing parts by **momentum-dependend** central and (isospin-dependend) **spin-orbit** two-body correction term
- fit correction term to binding energies and radii of “closed-shell” nuclei (^{4}He, ^{16}O, ^{40}Ca), (^{24}O, ^{34}Si, ^{48}Ca)
- altogether about a **15%** correction to the *ab-initio* potential



$\rho^{(1)}(\mathbf{r})\ [\rho_0]$

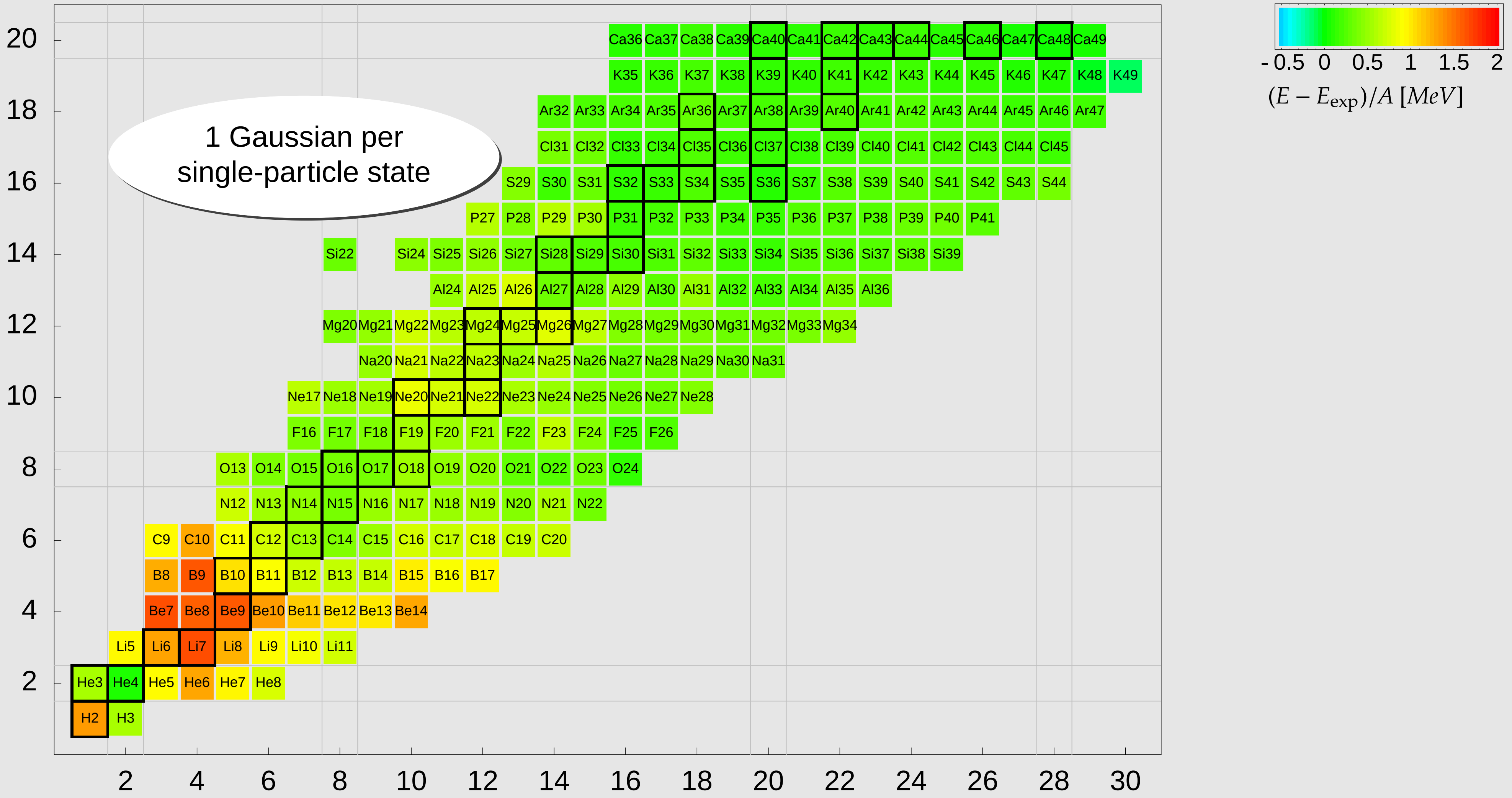






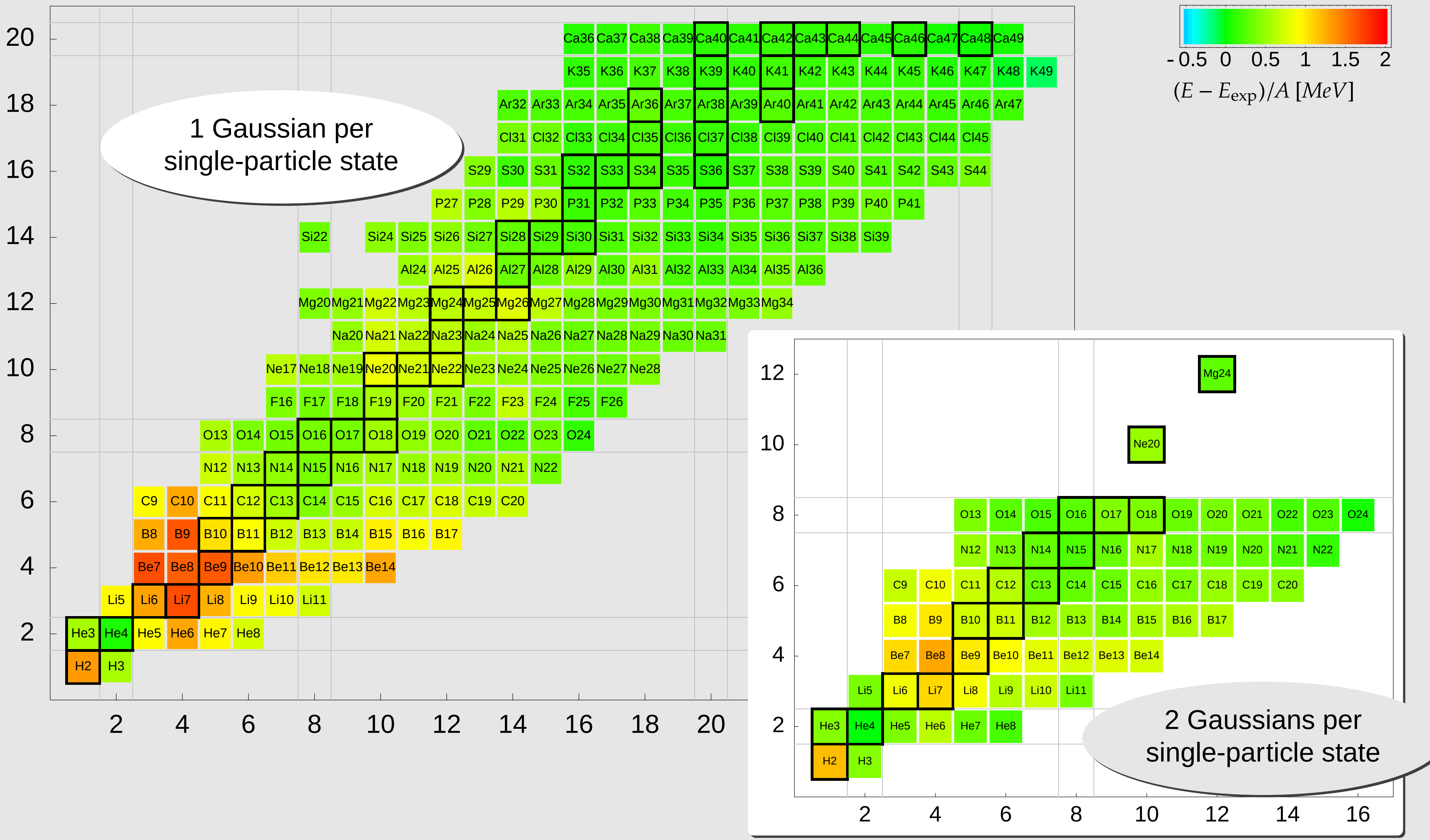
^{16}O and ^{40}Ca are not “closed shell” nuclei !

Nuclear Chart



Nuclear Chart

Variation



How to improve ?

Projection After Variation (PAV)

- mean-field may break symmetries of Hamiltonian
- restore inversion and rotational symmetry by parity and angular-momentum projection $\underset{\sim}{P}^{J^\pi}_{MK}$

$$\underset{\sim}{P}^{J}_{MK} = \frac{2J+1}{8\pi^2} \int d^3\Omega \, D^{J}_{MK}{}^{\star}(\Omega) \, \underset{\sim}{R}(\Omega)$$

$$\sum_{K'} \left\langle Q \middle| \underset{\sim}{H} \underset{\sim}{P}^{J^\pi}_{KK'} \middle| Q \right\rangle \cdot c_{K'} = E^{J^\pi}_{K} \sum_{K'} \left\langle Q \middle| \underset{\sim}{P}^{J^\pi}_{KK'} \middle| Q \right\rangle \cdot c_{K'}$$

Variation After Projection (VAP)

- effect of projection can be large
- perform Variation after Parity Projection V^π
- perform VAP by applying **constraints** on radius, dipole moment, quadrupole moment or octupole moment and minimize the energy in the projected energy surface

Multiconfiguration Calculations

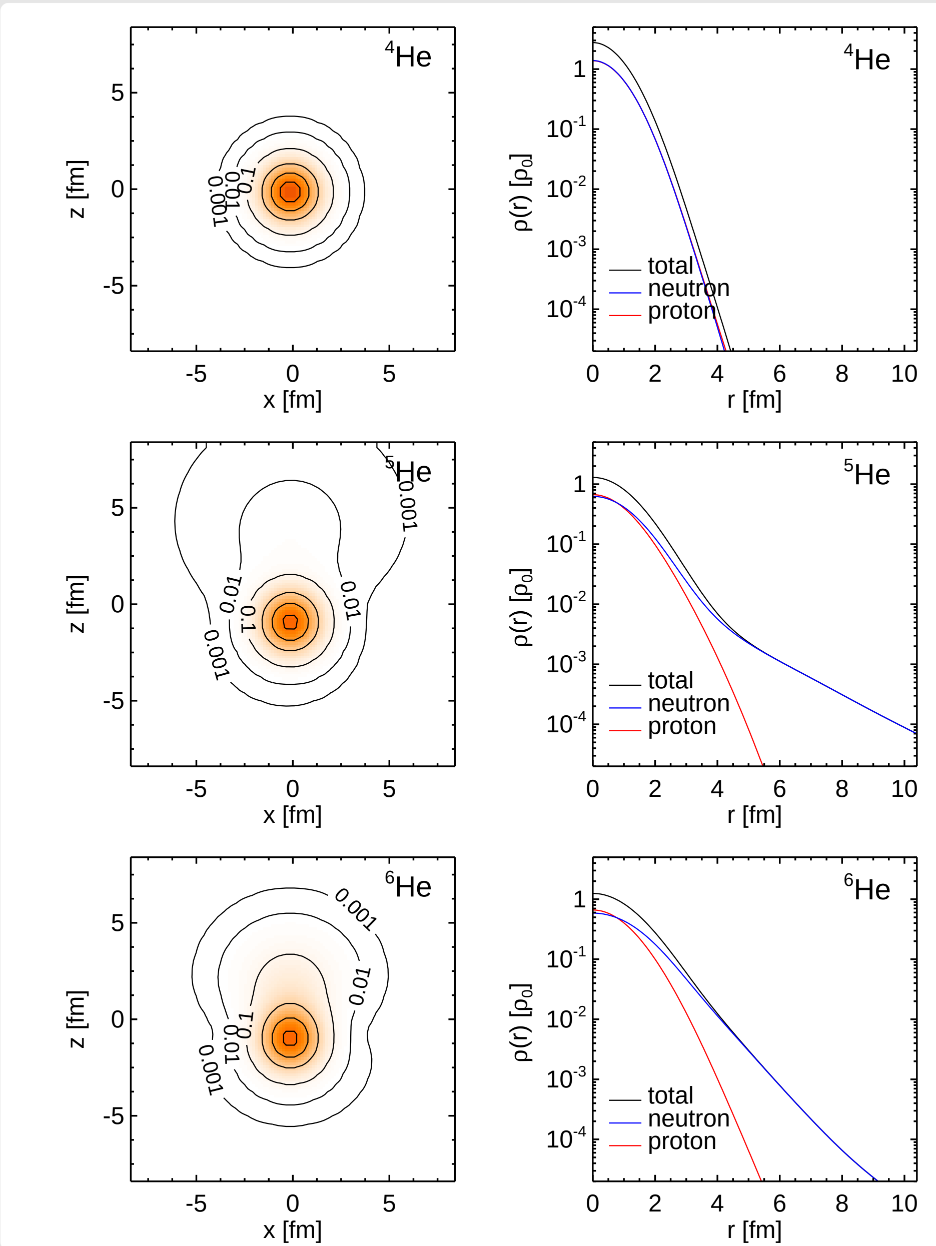
- **diagonalize** Hamiltonian in a set of projected intrinsic states

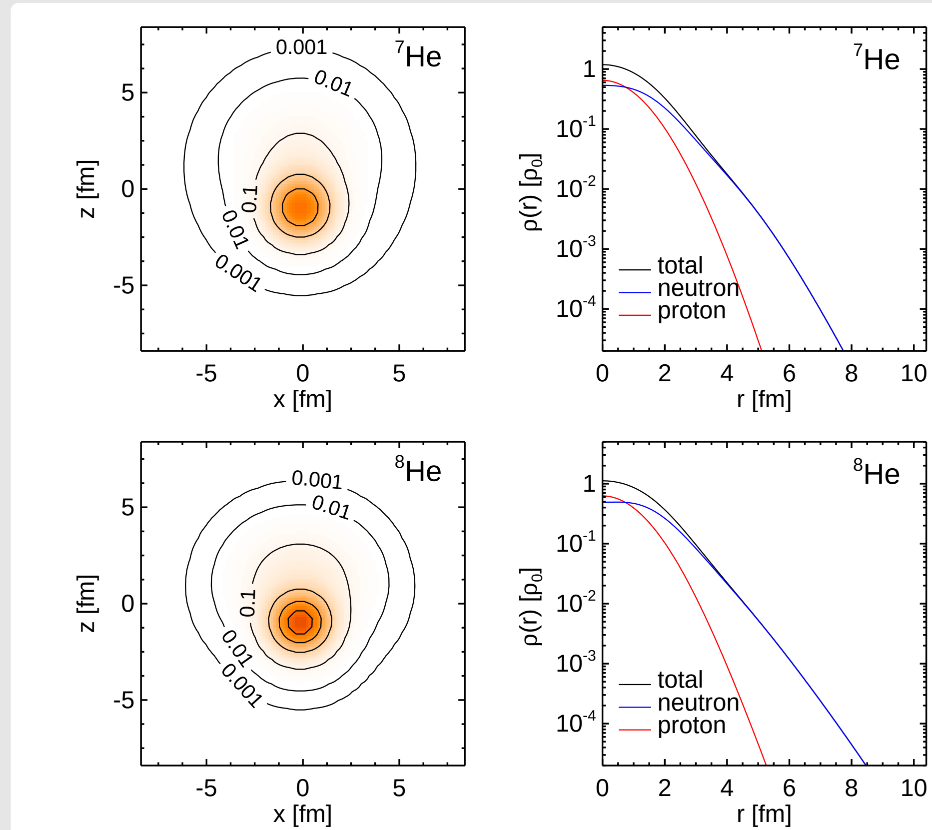
$$\left\{ \left| Q^{(a)} \right\rangle, \quad a = 1, \ldots, N \right\}$$

(Hill-Wheeler equation)

Helium Isotopes

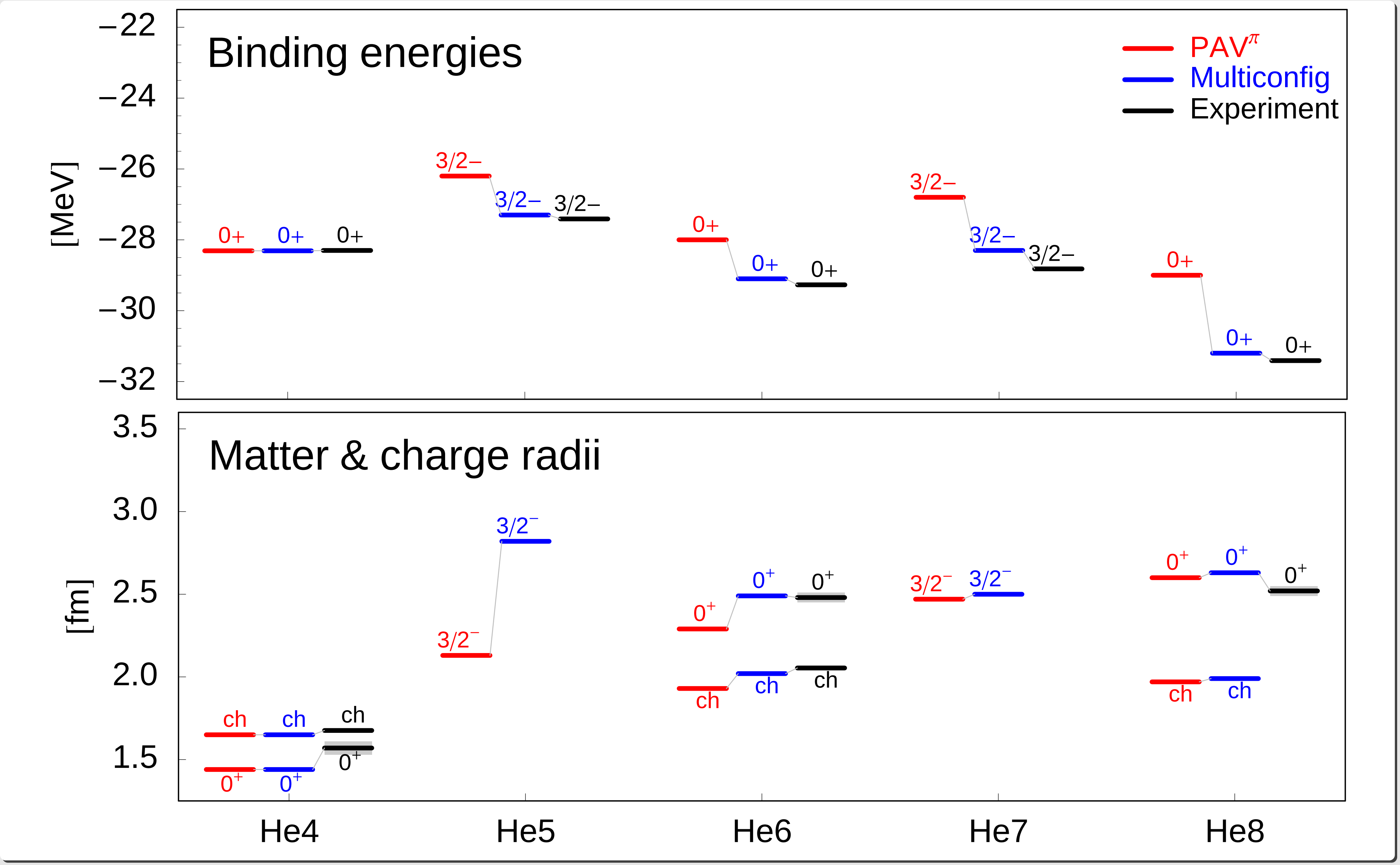
dipole and quadrupole constraints



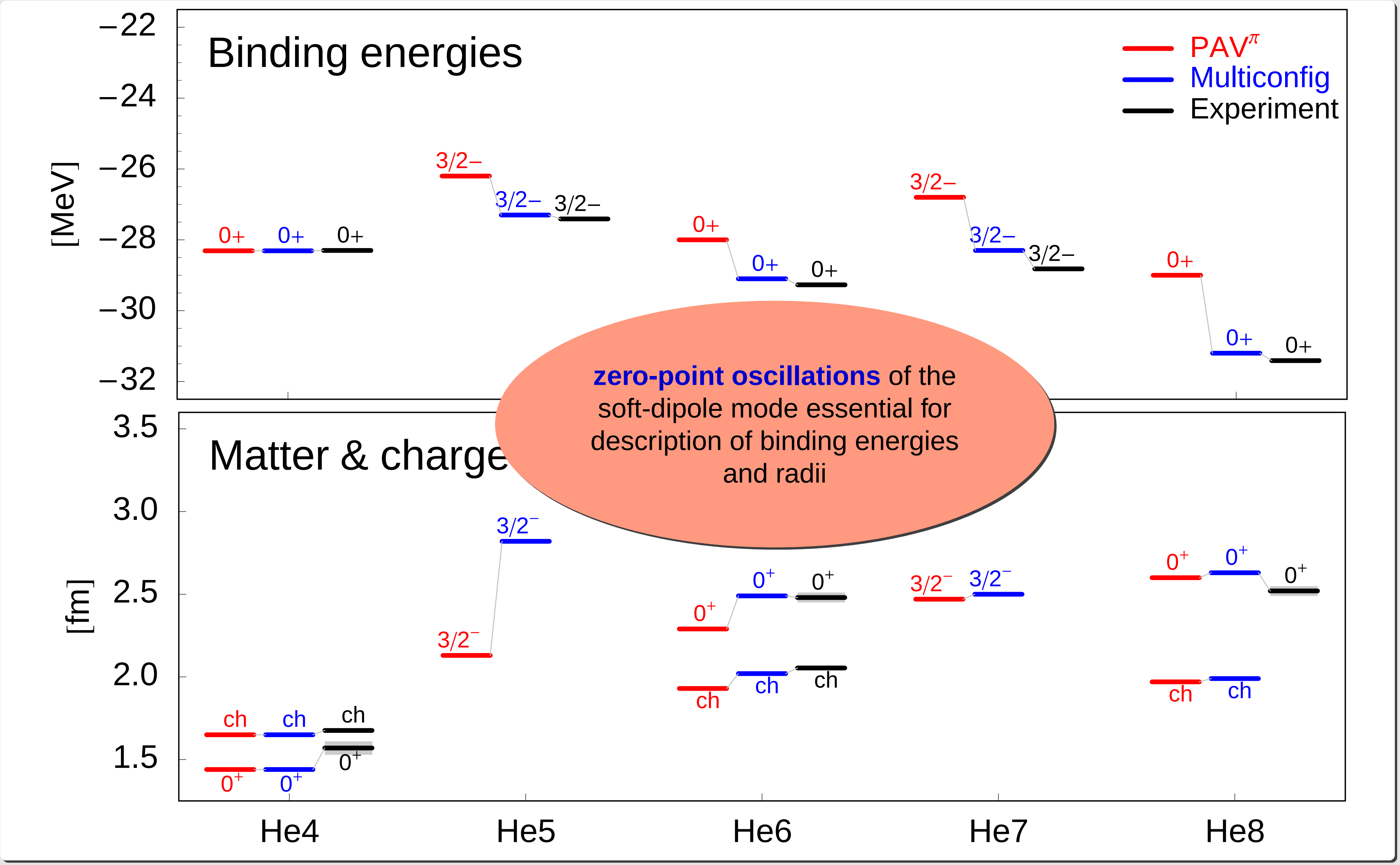


- intrinsic nucleon densities of VAP states
- radial densities from multiconfiguration calculations

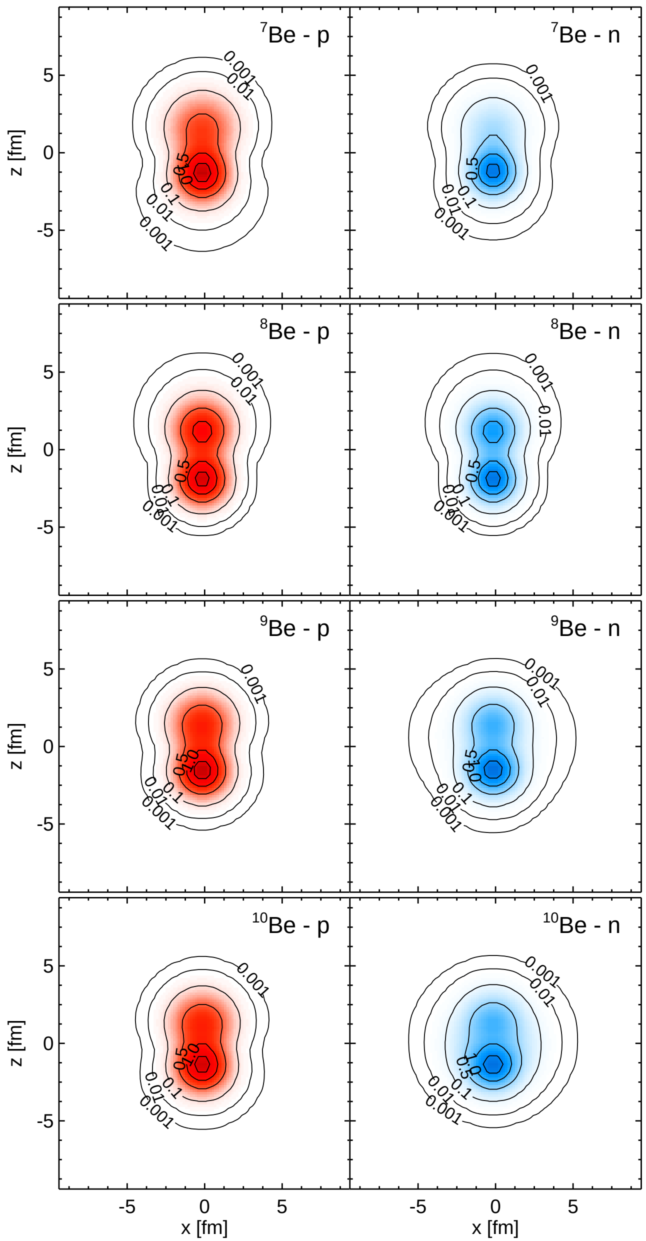
Helium Isotopes

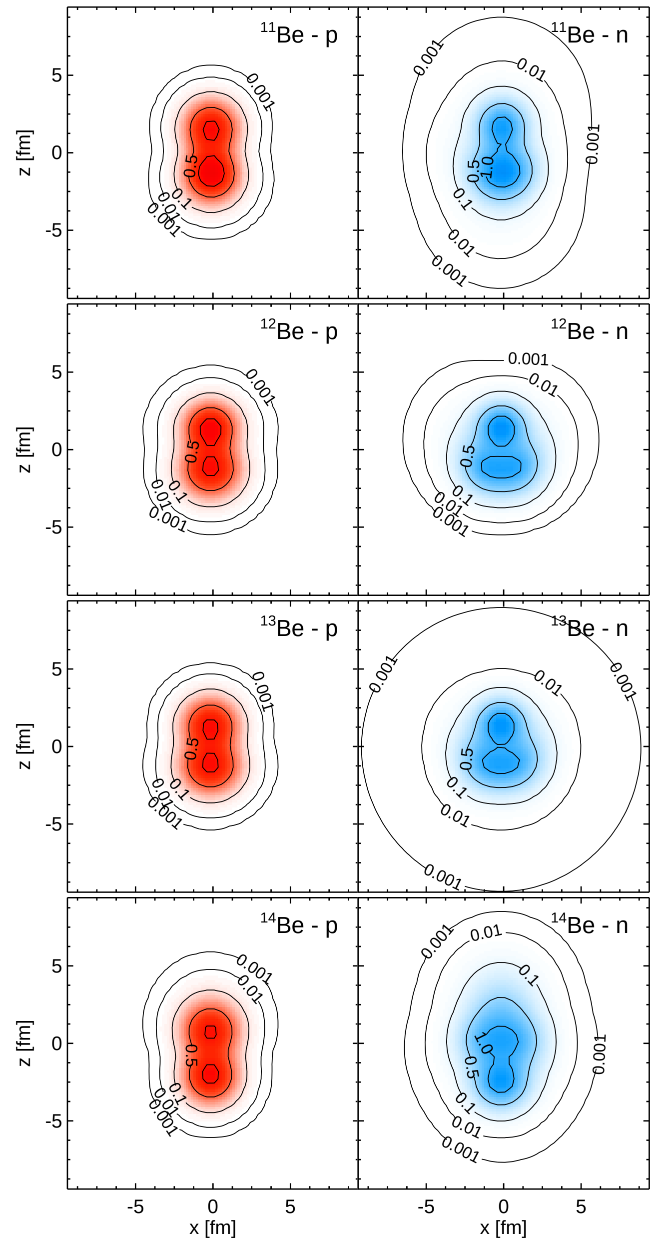


Helium Isotopes



Beryllium Isotopes

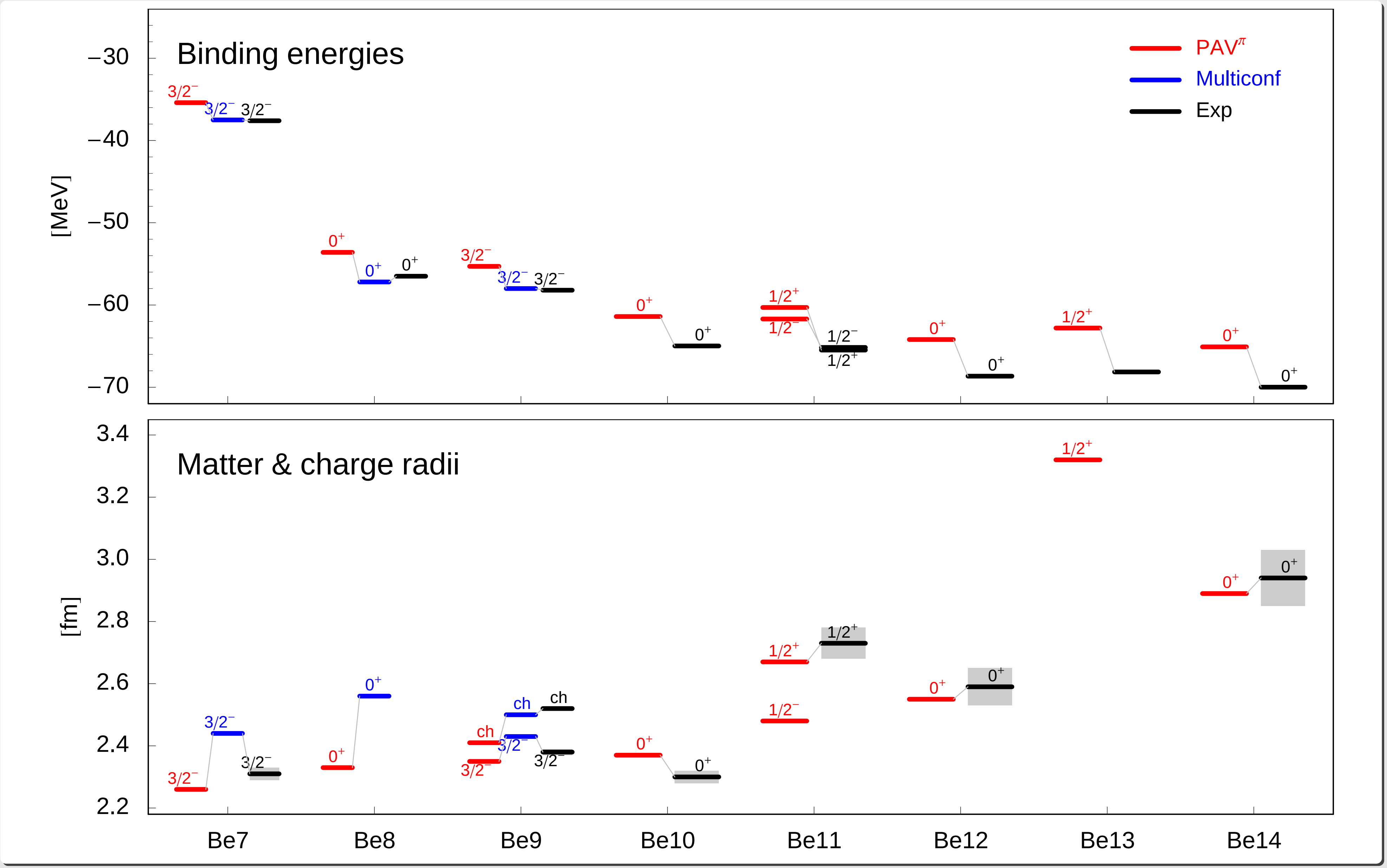




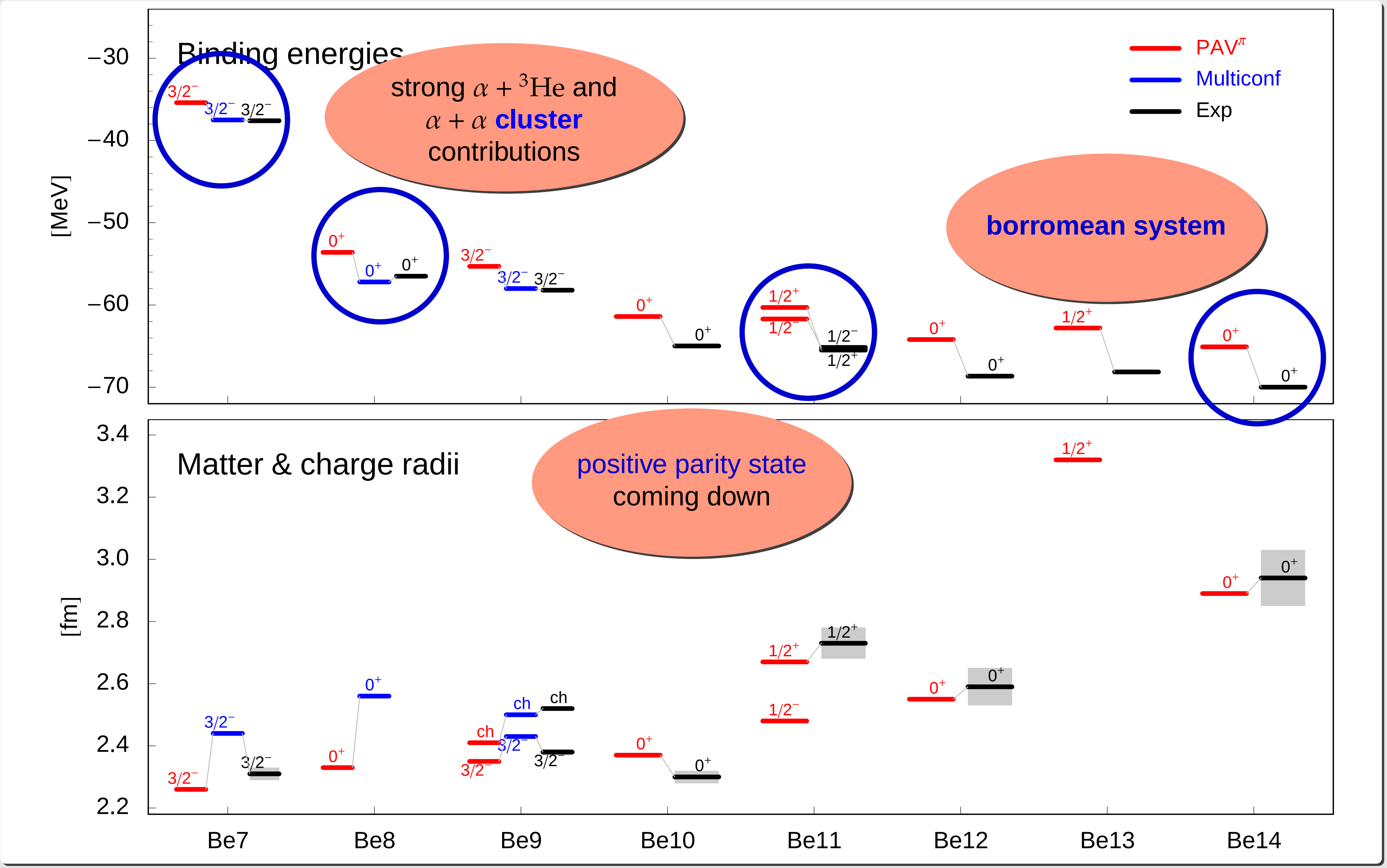
- intrinsic densities of V^{π} states

cluster structure evolves with addition of neutrons

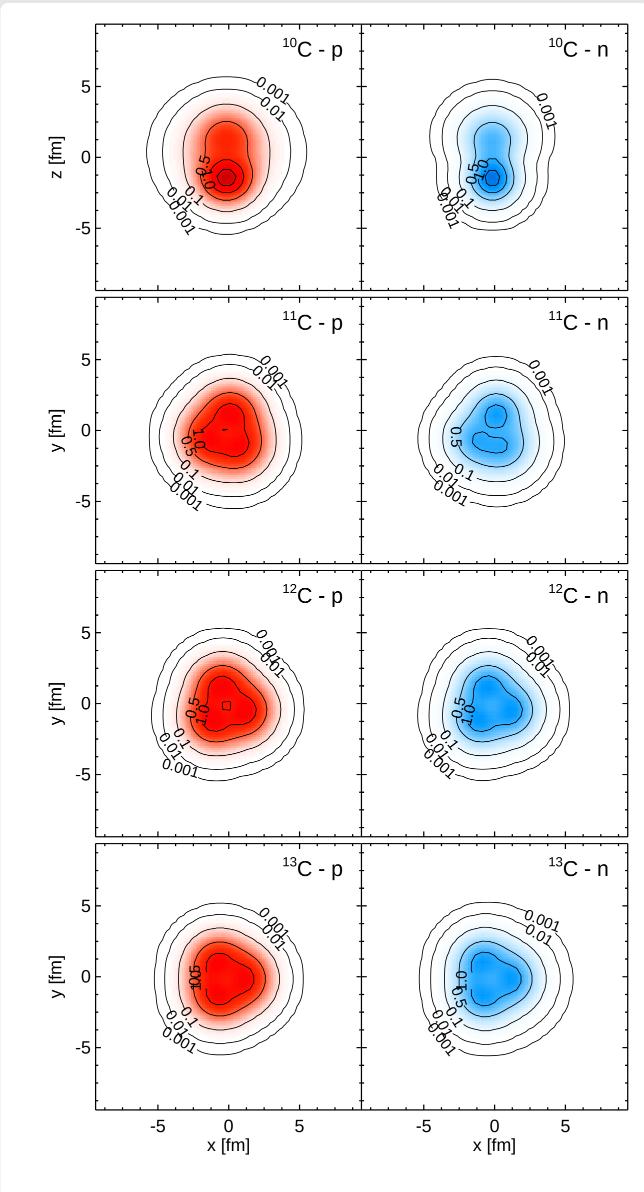
Beryllium Isotopes

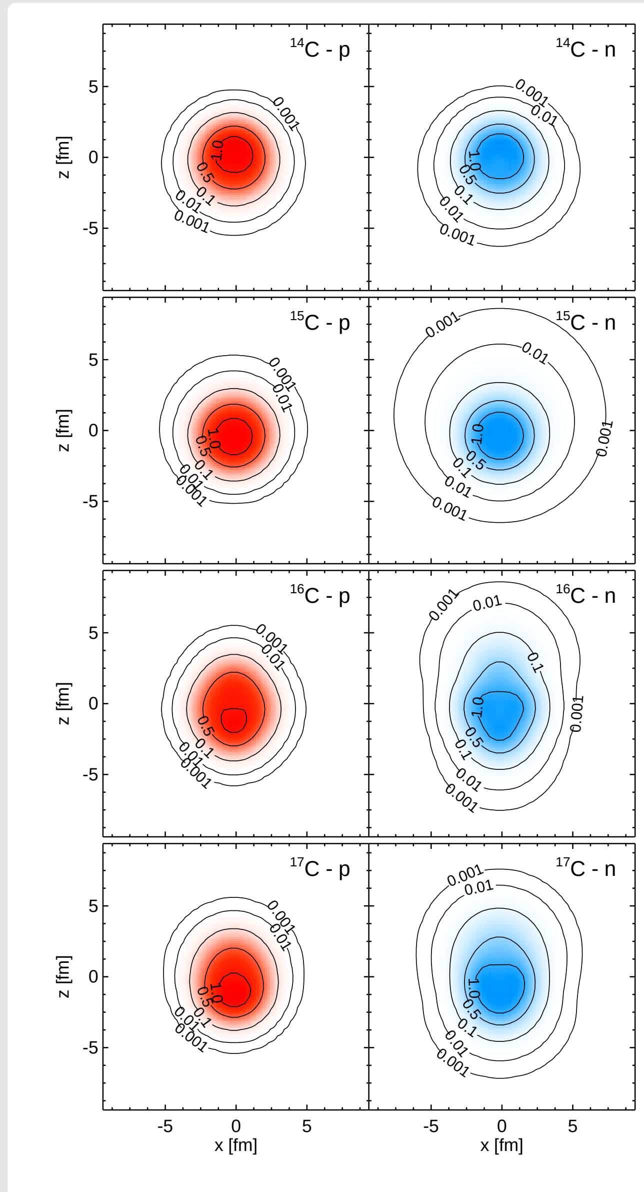


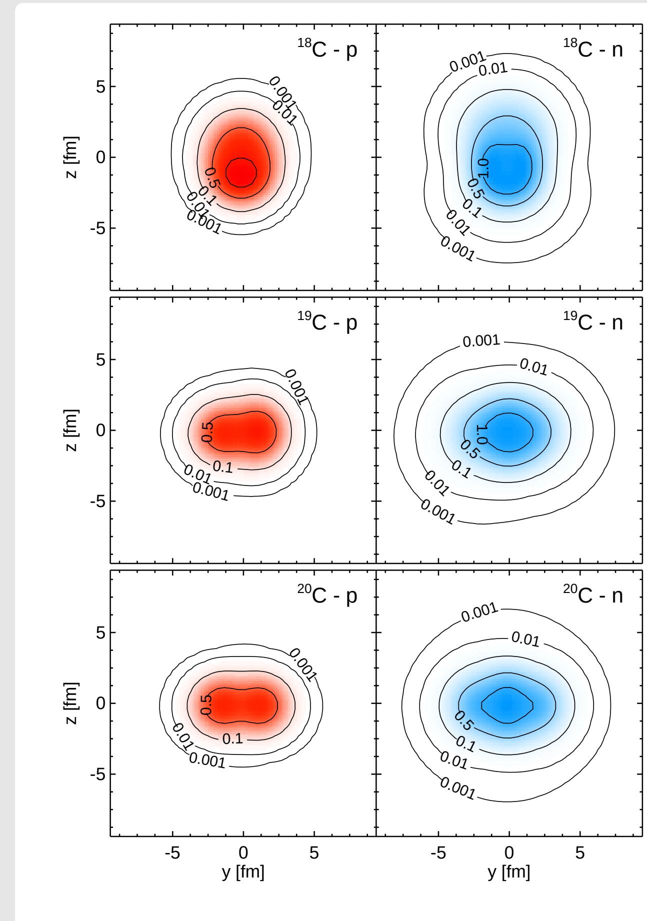
Beryllium Isotopes



Carbon Isotopes

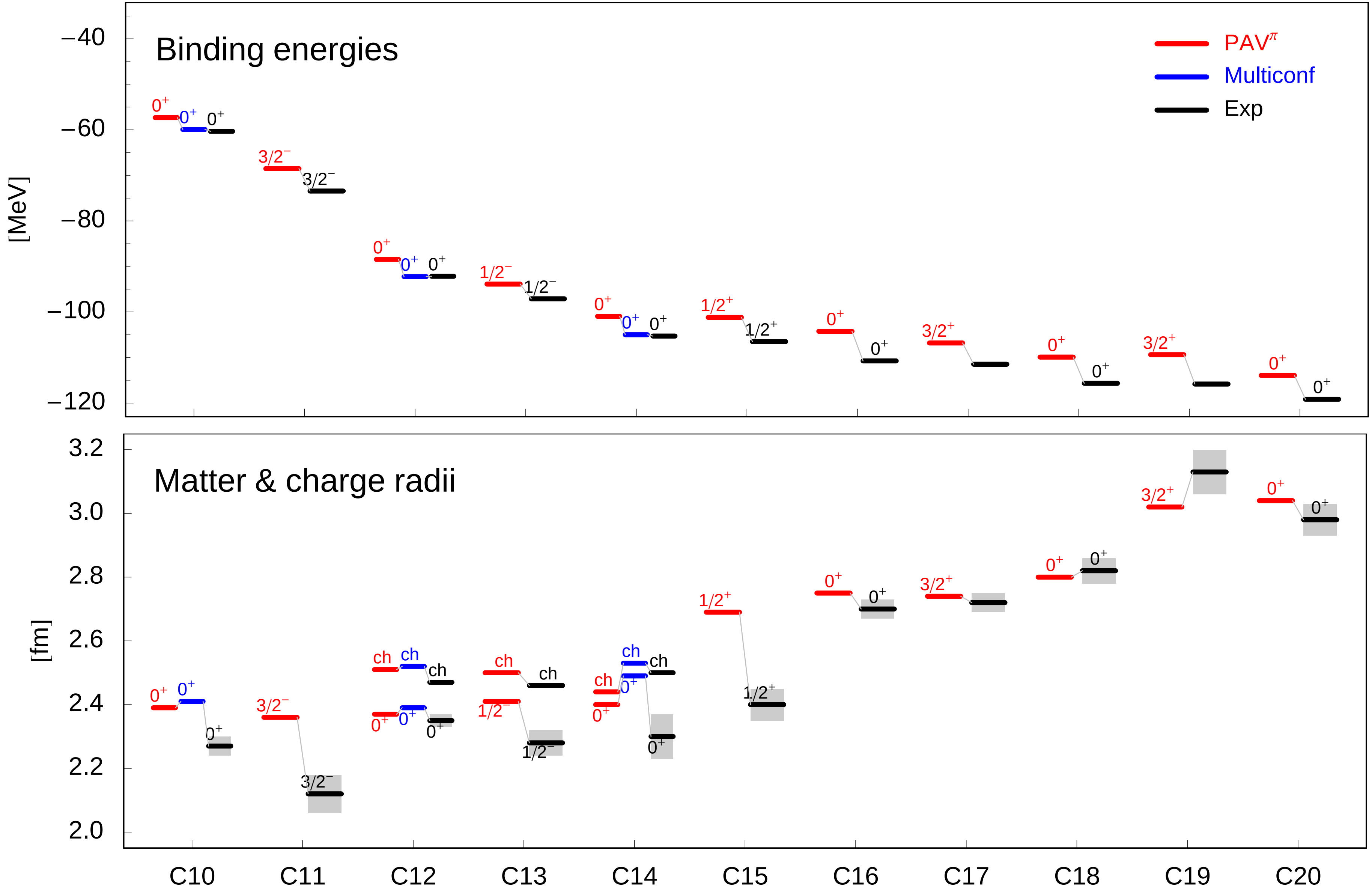






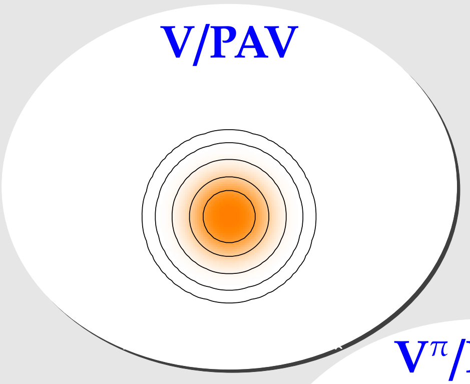
- intrinsic densities of V^{π} states

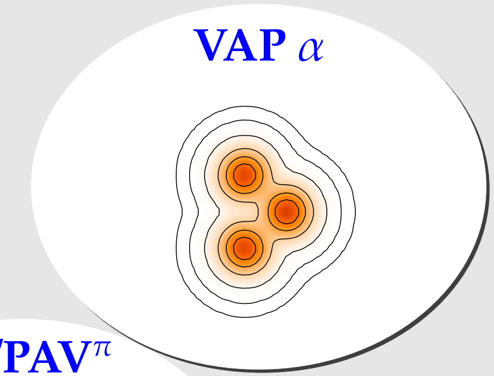
Carbon Isotopes

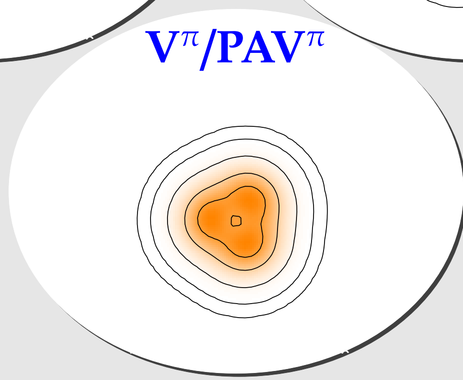


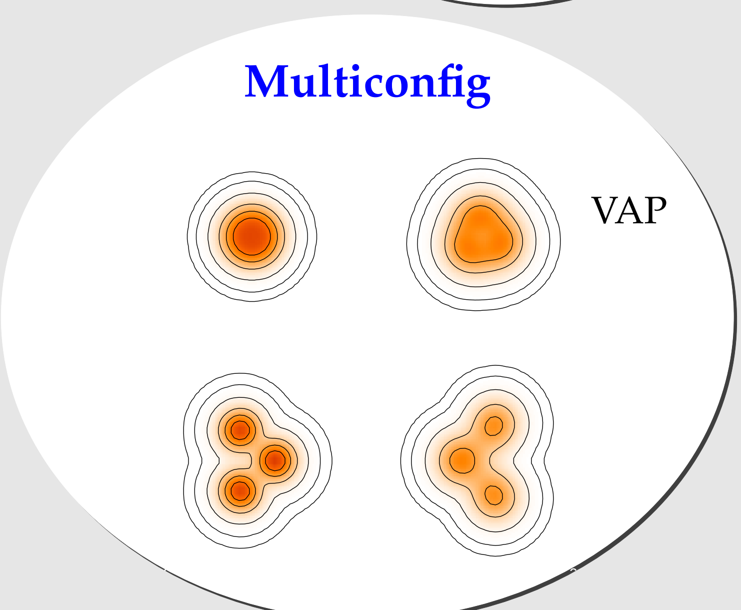
^{12}C

radius and octupole constraints

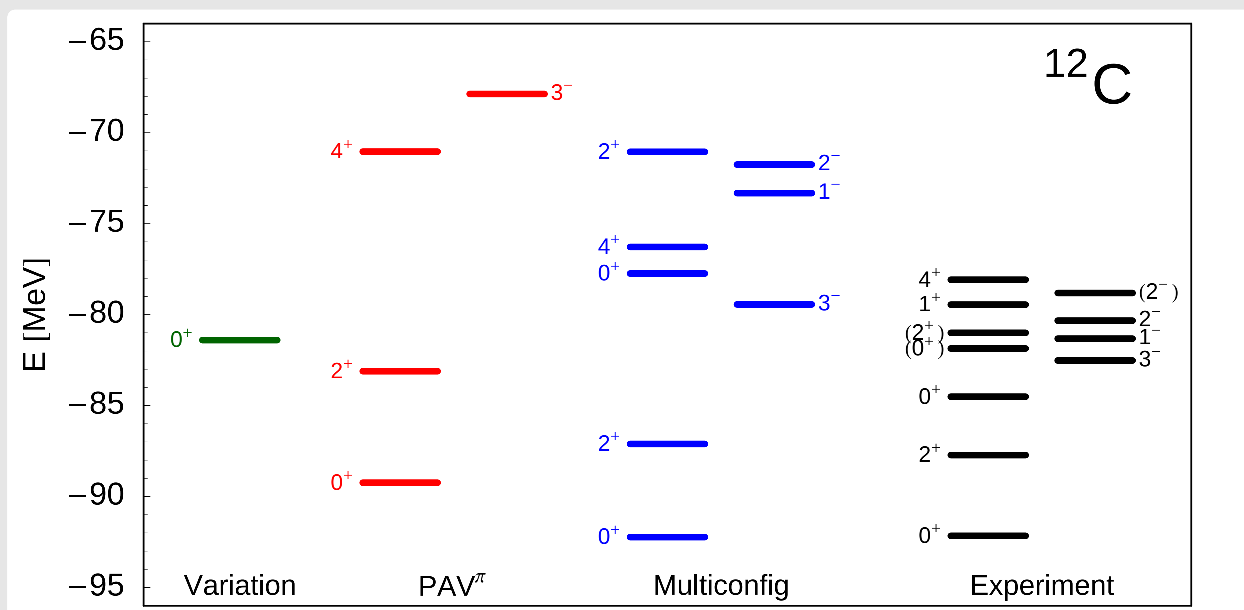








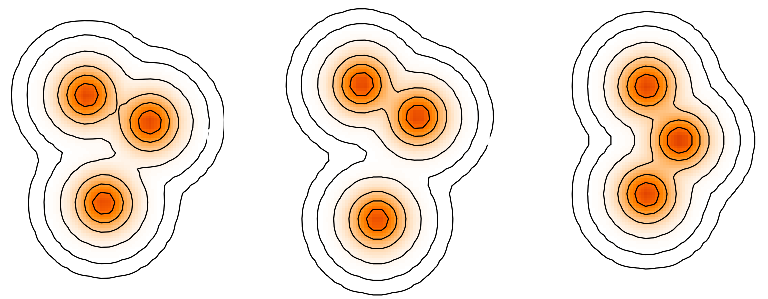
	E_b [MeV]	r_{charge} [fm]	$B(E2)$ [e^2fm^4]
V/PAV	81.4	2.36	-
VAP α-cluster	79.1	2.70	76.9
PAV$^\pi$	88.5	2.51	36.3
VAP	89.2	2.42	26.8
Multiconfig	92.2	2.52	42.8
Experiment	92.2	2.47	39.7 ± 3.3



^{12}C – excited 0^+ and 2^+ states

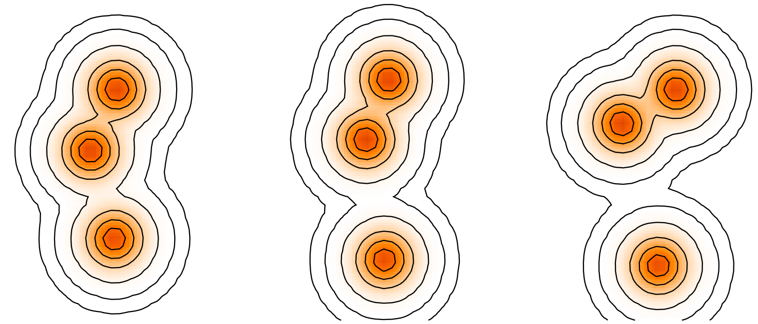
quadrupole and octupole constraints

0_2^+ state



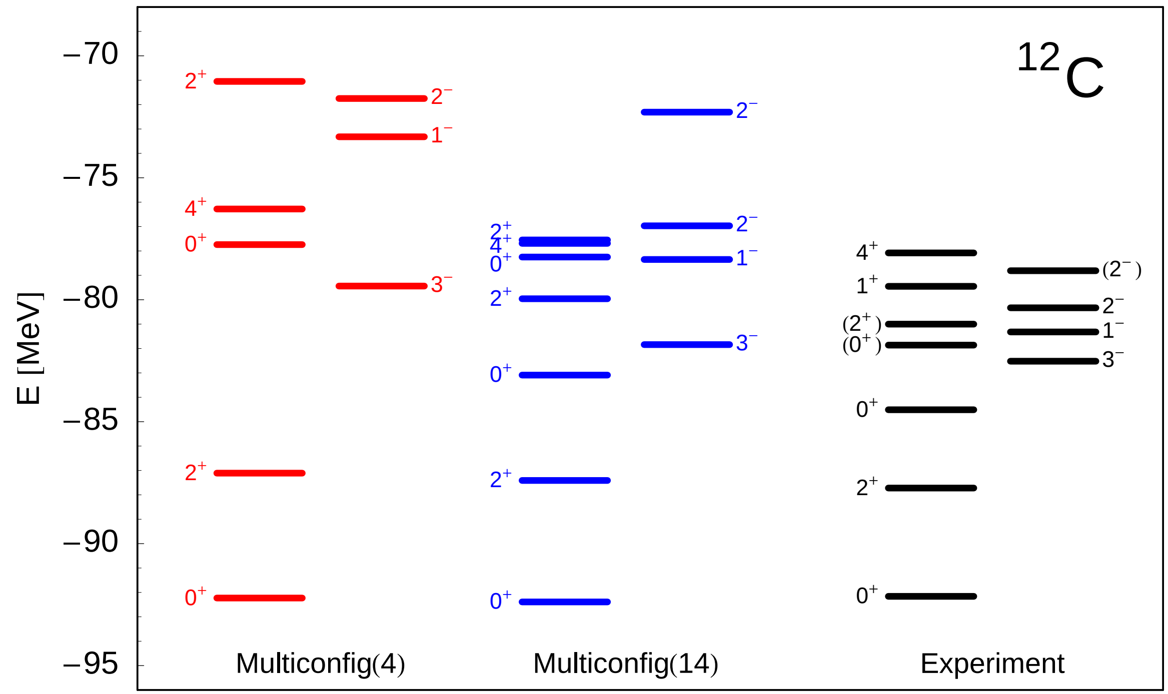
$|\langle \cdot | 0_2^+ \rangle| = 0.76$ $|\langle \cdot | 0_2^+ \rangle| = 0.71$ $|\langle \cdot | 0_2^+ \rangle| = 0.50$

0_3^+ state



$|\langle \cdot | 0_3^+ \rangle| = 0.69$ $|\langle \cdot | 0_3^+ \rangle| = 0.65$ $|\langle \cdot | 0_3^+ \rangle| = 0.44$

	Multiconfig	Experiment
E_b [MeV]	92.4	92.2
r_{charge}[fm]	2.52	2.47
$B(E2)(0_1^+ \to 2_1^+)$ [e^2fm^4]	42.9	39.7 ± 3.3
$M(E0)(0_1^+ \to 0_2^+)$[fm^2]	5.67	5.5 ± 0.2
$r_{rms}(0_1^+)$[fm]	2.38	
$r_{rms}(0_2^+)$[fm]	3.42	
$r_{rms}(0_3^+)$[fm]	3.85	
$r_{rms}(2_1^+)$[fm]	2.44	
$r_{rms}(2_2^+)$[fm]	3.64	
$r_{rms}(2_3^+)$[fm]	3.63	
$Q(2_1^+)$[$e$fm^2]	5.85	
$Q(2_2^+)$[$e$fm^2]	-23.65	
$Q(2_3^+)$[$e$fm^2]	5.89	



Summary & Outlook

Summary

- consistent many-body approach for stable and exotic nuclei
- FMD basis is flexible enough to describe clustering, shell effects and halos, Neff, Feldmeier, Nuc. Phys. **A738** (2004) 357
- same effective NN interaction based on realistic interaction used for all nuclei, Neff, Feldmeier, Nuc. Phys. **A713** (2003) 311, Roth, Neff, Hergert, Feldmeier, nucl-th/0406021
- importance of VAP and multiconfiguration calculations
- binding energies and radii well described

Experimental data taken from:

- Audi, Wapstra, Nucl. Phys. **A595** (1995) 409
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Outlook

- systematic study of light nuclei
- other observables: momentum distributions, spectroscopic factors, two-body densities, electromagnetic and weak transitions
- “real” VAP ?
- investigate effects of long range part of tensor force