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Extending the region of triaxial superdeformation: candidate TSD bands in ^{174}Hf

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Abstract

Three, possibly four, regularly spaced rotational bands with large dynamic moments of inertia have been identified in ^{174}Hf. Their properties are consistent with known triaxial superdeformed bands of the Lu/Hf region. Calculations predict substantial triaxial deformation ($\gamma \approx \pm 17°$) for ^{174}Hf structures with deformation $\epsilon_2 \approx 0.45$, despite the fact that ^{174}Hf is eight neutrons away from the previously established $N = 94$ triaxial superdeformed gap. Shell gaps at $N = 100$ and 106 with $\gamma \geqslant 15°$ are predicted for $\epsilon_2 \approx 0.45$, and are most likely responsible for the calculated TSD minima in ^{174}Hf.

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Past nuclear spectroscopic studies have revealed that the nucleus can exhibit a variety of shapes associated with deformations of different magnitudes. These encompass spheres, oblate and prolate spheroids that include prolate deformations which range from normal ($\epsilon_2 \approx 0.2$) to superformed ($\epsilon_2 \approx 0.5$). Most established cases, however, are axially symmetric, that is, there is an axis about which the mass distribution is symmetric. The identification of stable triaxial nuclei, where the mass distribution (and therefore the moments of inertia) are different along all three principal body-fixed axes, has been more elusive, partly because direct experimental characteristics of triaxiality are scarce. Nevertheless triaxiality has been invoked to describe various (relatively low-spin) phenomena, such as anomalous signature splitting [1], signature inversion [2], and chiral-twin bands [3,4].

Recent calculations [5,6] suggested that stable triaxial deformation would occur in the superdeformed well, with a well-defined energy minimum present up to large rotational frequencies in nuclei with $Z \approx 72$ and $N \approx 92, 94$. Therefore, structures that are thought to reside in these minima are often referred to as triaxial superdeformed (TSD) bands. Schnack-Petersen et al. [5] demonstrated that single-particle shell gaps exist at high deformation ($\epsilon_2 \approx 0.39$) for $Z = 72$ and $N = 94$ within the framework of the ultimate cranker (UC) model [7]. The neutron shell gap was found to be associated with a substantial triaxial deformation of $\gamma \approx 20°$. This led the authors of Ref. [5] to suggest that the presence of these gaps (which stabilize the nuclear shape near the deformations given) virtually guarantees the appearance of TSD minima in the total energy surfaces (TES) of nuclei near $N = 94$. Bengtsson [6] performed systematic UC calculations for $N \approx 94$, Yb–Hf–W nuclei and found that the $N = 92$ isotopes may have the energetically lowest TSD bands with respect to the ground-state sequences. Thus, most of the experimental searches for triaxial superdeformed bands have concentrated on the $N \approx 92$, 94 region.

Superdeformed structures have been experimentally verified in $^{163\text{–}165}_{71}$Lu [8–10] and $^{168}_{72}$Hf [11] by the measurement of transitional quadrupole moments. Good candidates for superdeformed bands also exist in ^{161,162,167}Lu [12,13] and ^{170}Hf [14]. It is likely that all these sequences are based on the excitation of at least one $i_{13/2}$ proton. The assertion of triaxiality for these bands has largely been based on UC calculations, as mentioned above. However, the observation of excited TSD bands, and the properties of linking transitions between these sequences in ^{163}Lu [15–17], ^{165}Lu [18], and ^{167}Lu [13] have recently been shown to be consistent with the behavior of “wobbling” excitations [19,20] resulting from the rotation of triaxial nuclei as predicted by Bohr and Mottelson [21]. Therefore, these odd-A Lu nuclei are the best examples of stable triaxiality currently known. Surprisingly, no TSD bands have been found in the $N = 92$, 94 ^{164,166}Hf nuclei to date. In this context, the present observation of three, and possibly four, candidate TSD bands in ^{174}Hf$_{102}$, eight neutrons away from the previously established TSD gap at $N = 94$, is particularly surprising.

High-spin states in ^{174}Hf were produced with the ^{130}Te(^{48}Ca, 4n) reaction at a beam energy of 194 MeV. The target consisted of ~ 0.5 mg/cm^2 of enriched ^{130}Te covered with a thin (~ 0.2 mg/cm^2) Au flashing, and the ^{48}Ca beam was provided by the ATLAS facility at Argonne National Laboratory (ANL). Decay γ radiation was detected with the Gammasphere array [22], which contained 100 Compton-suppressed Ge detectors. A total of $\sim 5.5 \times 10^8$ three or higher fold coincidence events were recorded in approximately *one day of beam time*. It should be noted that the primary objective of the experiment was to observe rotational levels built on the high-K isomers in ^{174,175}Hf [23]. As these sequences may be low in multiplicity, a relatively low trigger condition of only three or more Ge detectors in coincidence was used. Thus, this experiment was not optimized for a search of superdeformed bands, as higher multiplicity triggers and longer beam times are normally required to observe these weak structures. The beam wobbling device at ANL was utilized so that a higher beam intensity (~ 1.7 pnA) could be deposited on the Te target, which has a relatively low melting point. The transitions were Doppler corrected and then sorted into a

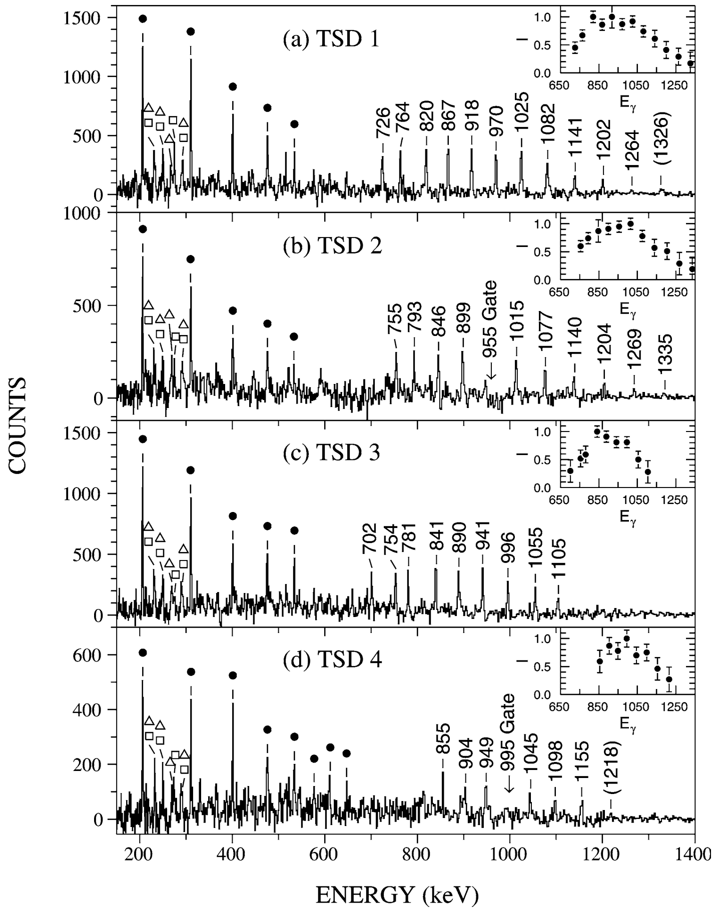

Fig. 1. Summed coincidence spectra of the candidate TSD bands of ^{174}Hf. All clean combinations of double coincidence gates with the five lowest inband transitions produce the spectra for TSD bands 1 and 3. For TSD bands 2 and 4, double gates of the lowest transitions with the 955 and 995 keV transitions, respectively, were used. Transitions denoted with γ-ray energies are assigned to the TSD bands, while peaks marked with a filled circle are ground-band transitions in ^{174}Hf [25]. Peaks denoted with an open square and a triangle indicate transitions from the $K^\pi = 8^-$ and 14^+ bands, respectively. The relative intensity profile for each band is shown in the top right-hand corner of each panel.

$E_\gamma \times E_\gamma \times E_\gamma$ coincidence cube. Subsequent analysis of this cube was accomplished using the Radware package [24].

Representative spectra of the four candidate TSD sequences are displayed in Fig. 1. Unfortunately, none of the bands could be linked into the normal deformed structures of ^{174}Hf [25] despite the strong coincidence relationships observed with γ rays in the ground-state structure (see Fig. 1). Band 1, shown in Fig. 1(a), is the strongest of the four sequences with a relative intensity of 1.1(3)% of the total population of ^{174}Hf. Ground-state transitions up to the $I^\pi = 12^+\ \hbar$ state are found in coincidence with this structure. Bands 2 and 3 have intensities of 0.9(4)%, and they also feed the ground-state band as high as $I = 12\ \hbar$. TSD band 4 is substantially weaker than the other three with an intensity of 0.3(2)%. Due to the low intensity and contamination in the coincidence gates, we only tentatively assign this sequence to ^{174}Hf. Higher statistics are needed to confirm its placement, but the indications from the present data are that it feeds the yrast sequence as high as $I = 18\ \hbar$, which is higher than the other three bands.

The large deformation for these bands has been inferred from their dynamic moments of inertia, which are plotted in Fig. 2(a) assuming that the inband tran-

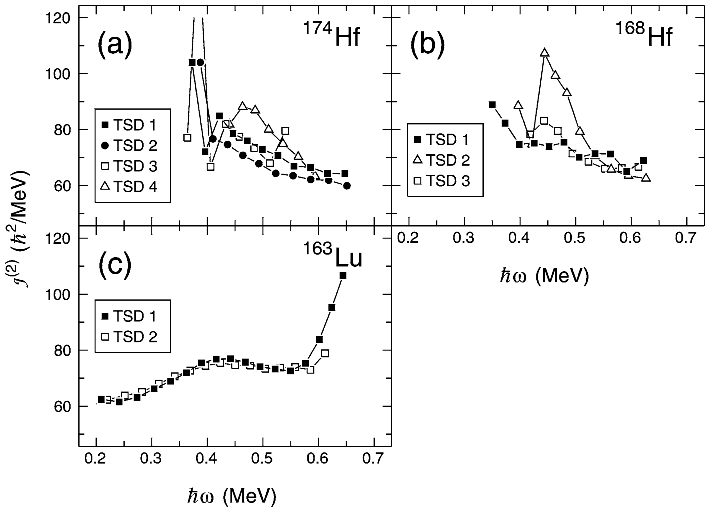

Fig. 2. Dynamic moments of inertia of the TSD structures in (a) ^{174}Hf, (b) ^{168}Hf, and (c) ^{163}Lu as a function of rotational frequency.

sitions are of stretched E2 character. One may note that the moments of inertia for the TSD bands in ^{174}Hf have similar values to those in ^{168}Hf and ^{163}Lu shown in Fig. 2(b) and (c), respectively. Large transition quadrupole moments (Q_t) have been confirmed for the yrast TSD bands in the latter nuclei with $Q_t = 11.4^{+1.1}_{-1.2}\,e\,\mathrm{b}$ [11] and $7.4^{+0.7}_{-0.4}\,e\,\mathrm{b}$ [10], respectively. (Normal deformed states in ^{168}Hf and ^{163}Lu have $Q_t \approx 6\,e\,\mathrm{b}$ and $5\,e\,\mathrm{b}$ [9], respectively.) The similarity between the values of the moments of inertia (which is often associated with deformation) for the three nuclei indicate that the bands in ^{174}Hf are likely superdeformed as well. However, this assertion must be verified through lifetime measurements that will determine the quadrupole moments of these structures.

Bands 1–3 in ^{174}Hf display irregularities in $\mathcal{J}^{(2)}$ (see Fig. 2(a)) at their lowest frequencies. These are likely due to an interaction as the structures decay out of the superdeformed well. A smooth decrease in the $\mathcal{J}^{(2)}$ moment is observed above 0.4 MeV for bands 1 and 2, which is consistent with the general trend of TSD bands 1 and 3 in ^{168}Hf (see Fig. 2(b)). TSD bands 1 and 2 in ^{174}Hf have rather similar slopes in $\mathcal{J}^{(2)}$ throughout the observed frequency range, although TSD 2 has values consistently ∼10% lower than TSD 1. Band 4 has a somewhat different profile than any of the other sequences in Fig. 2(a) as an interaction is observed near 0.48 MeV. As seen in Fig. 2(b), TSD 2 in ^{168}Hf exhibits a similar bump in the $\mathcal{J}^{(2)}$ moment at approximately the same frequency.

The lack of discrete linking transitions for the TSD bands in ^{174}Hf makes definitive spin and parity assignments impossible. However, following the procedure suggested by Amro et al. [11], spins may be estimated by comparing the relative alignments for the sequences with respect to the normal deformed structures in ^{174}Hf, the TSD bands in the Lu isotopes, as well as the TSD bands in ^{168}Hf. It is likely that the TSD bands in ^{174}Hf are characterized by the presence of at least one $i_{13/2}$ proton in their configuration (since the TSD bands in the Lu nuclei are based on this orbital), therefore, their alignments should be at least equal to that of the $\pi i_{13/2}$ band in ^{163}Lu. The spins were adjusted until the alignments of the ^{174}Hf bands were approximately equal to those of the ^{163}Lu bands. Using this line of reasoning, spins of 23, 24, 22, and 28 $\hbar$ are suggested as the lower limits for bands 1, 2, 3, and 4, respectively.

It is also interesting to note that the TSD bands may decay into the $K^\pi = 8^-$ and 14^+ bands [26] as transitions from these sequences appear in Fig. 1. The band heads of these high-K structures have microsecond half lives resulting from ΔK selection rules [23], operational when the system is axially symmetric. Perhaps this decay between the TSD bands (with presumably low-Ω orbitals involved in their configuration), and high-K states is a consequence of

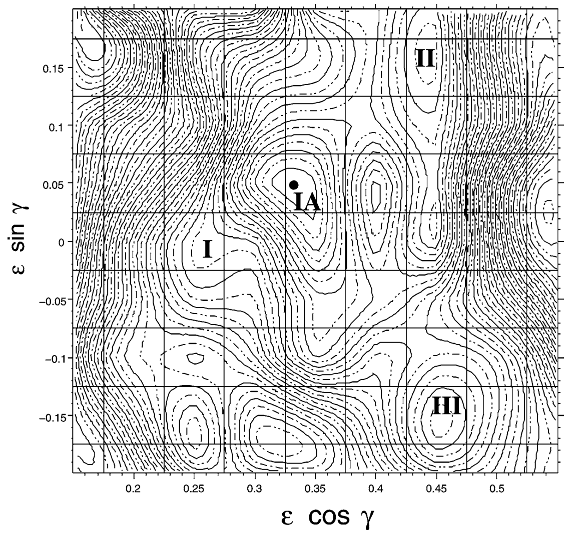

Fig. 3. Total Routhian surface from ultimate cranker calculations for ^{174}Hf at a frequency of 0.55 MeV. A positive-parity, $\alpha = 0$ configuration was considered. Minima are labeled using the convention defined by Bengtsson [6]. The energy separation between lines is 200 keV. It should be noted the concentric lines to the right of minimum IA (near $\epsilon \cos\gamma \approx 0.4$) and near $\epsilon \cos\gamma \approx 0.53$ define maxima, not minima.

the proposed triaxiality (as K is not a good quantum number for an axially asymmetric configuration), or the decay path may occur through many levels, with increasing K. Clearly, a higher statistics experiment is required to confirm and better understand the complex decay mechanisms of these TSD bands.

Total Routhian surfaces (TRS) for ^{174}Hf were calculated using the ultimate cranker with standard parameters [27] to investigate the possible presence of triaxiality in these sequences. Since the only good quantum numbers in these calculations are parity and signature (π, α), the quasiparticle configuration associated with a given TRS cannot be identified straightforwardly. However, the UC does allow for configurations to be traced diabatically through crossings with small interaction strengths. The lowest energy configuration with $(\pi, \alpha) = (+, 0)$ at $\hbar\omega = 0.55$ MeV is shown in Fig. 3. Several minima appear in the TRS, and the labeling convention of Bengtsson [6] will be used to describe each of them. The remnants of minimum I, which is the lowest at frequencies less then 0.4 MeV, can still be seen in Fig. 3. It corresponds to the normal deformed well with $\epsilon_2 \approx 0.25$ and $\gamma \approx 0°$. Minimum IA is lowest for the given frequency of 0.55 MeV and has intermediate deformations of $\epsilon_2 \approx 0.34$ and $\gamma \approx +7°$. Another difference between minima I and IA is that the proton pairing is found to be $\sim 25\%$ lower in IA. Bengtsson observed this feature in his calculations for lighter Yb, Hf, and W nuclei [6]. In addition, the proton spin is found to contribute more to the total spin in minimum IA. Bengtsson concluded that this corresponds to an aligned pair of deformation driving protons present in minimum IA, but not in minimum I. Indeed, a crossing is observed [28] in the ground-state sequence of ^{174}Hf at $\hbar\omega_c = 0.49$ MeV, which we suggest involves the alignment of at least one deformation driving $h_{9/2}$ proton. Thus there appears good agreement between experimental results and these calculations.

Two superdeformed minima are observed in Fig. 3 and are marked as regions II and III. Minimum II is close in energy with respect to IA as it is only ~ 300 keV higher at this frequency (0.55 MeV) while minimum III is located $\sim$1 MeV above minimum II. This is consistent with the calculations performed for nuclei with $N \approx 94$ [5,6]. The proton pairing is observed to be as low in minimum II as IA, which once again suggests the presence of unpaired protons that likely includes at least one from the $i_{13/2}$ orbital. The deformations in the two TSD minima are $\epsilon_2 = 0.453$ and $\gamma = +16°$ for II, and $\epsilon_2 = 0.475$ and $\gamma = -19°$ for III. Note that a shallow superdeformed minimum also exists near $\epsilon_2 = 0.45$ and $\gamma \approx 0°$, but with an energy higher than that of minimum II. Thus, the UC predicts that superdeformed structures in ^{174}Hf will likely be triaxial. However, one may question why these triaxial minima appear when ^{174}Hf is eight neutrons away from the $N = 94$ TSD shell gap. To answer this question, the single-particle spectra were investigated in a manner similar to that used in Ref. [5].

One should first note the difference in predicted quadrupole deformations between the light $N = 92, 94$ Lu nuclei ($\epsilon_2 = 0.389$ [5]) and ^{174}Hf ($\epsilon_2 = 0.453$).[6] In fact, a larger deformation is also suggested for ^{168}Hf at $\epsilon_2 = 0.43$ [11]. The most recent Q_t value of the yrast TSD band in ^{163}Lu (7.4 e b) appears to confirm this

[6] Since the $\gamma > 0°$ well is favored over the $\gamma < 0°$ minimum, we will restrict the discussion of the ^{174}Hf deformation to the former.

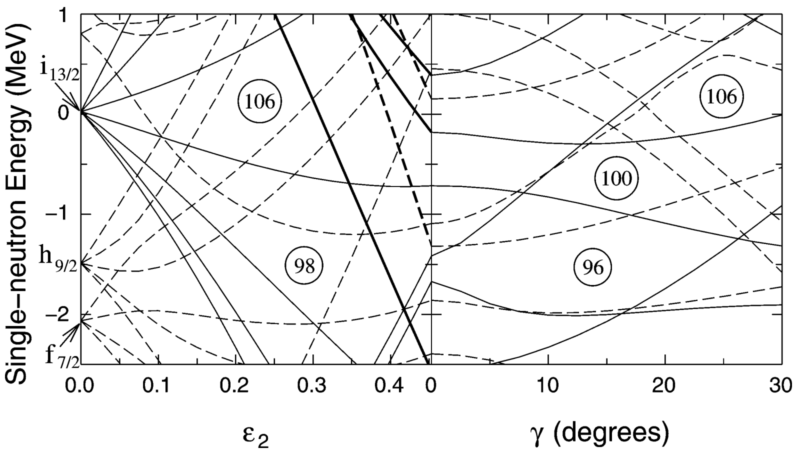

Fig. 4. Left panel: single-neutron energy as a function of ϵ_2 from ultimate cranker calculations. Right panel: single-neutron energy as a function of γ, where $\epsilon_2 = 0.453$ and $\epsilon_4 = 0$.

difference as a larger moment was measured for ^{168}Hf (11.4 e b). A possible source for the larger deformation in the heavier nuclei may be found in the left-hand portion of Fig. 4, where the single-neutron energy is given as a function of ϵ_2. (The hexadecapole deformation was set to zero for this and all following calculations.) Orbitals that originate above the $N = 126$ spherical shell gap are strongly down-sloping in energy at higher deformations ($\epsilon_2 > 0.25$) and are highlighted with bold lines in Fig. 4. These "intruder" orbitals are based on $i_{11/2}$ (solid lines) and $j_{15/2}$ (dashed lines) states, and their occupation by neutrons results in significant deformation enhancement. The light ($N = 92, 94$) Lu and Hf nuclei are less likely to involve these intruder states as their Fermi surfaces are further below the orbitals than $N = 102$ for ^{174}Hf. Thus, the larger predicted deformation in ^{174}Hf may result from a higher occupancy of these $i_{11/2}$ and $j_{15/2}$ neutrons. In addition, a shell gap at $Z = 72$ is observed in the single-proton spectrum with $\epsilon_2 \approx 0.45$, which helps stabilize the superdeformed shape.

The right-hand panel of Fig. 4 displays the single-neutron orbitals as a function of γ deformation for ϵ_2 and ϵ_4 parameters fixed as $\epsilon_2 = 0.453$ and $\epsilon_4 = 0$. The $\sim 15\%$ increase in deformation from ^{163,165}Lu to ^{174}Hf shifts the relative placement of the neutron orbitals, which alters the single-neutron spectrum. Some notable differences occur in comparison with the calculations of Schnack-Petersen et al. [5], such as the evolution of the $N = 94$ gap to $N = 96$. The gap observed in Fig. 4 is nearly as large as the one previously calculated and spans a large range of γ values. More importantly, two other gaps are found at $N = 100$ and 106 (see Fig. 4) with well defined triaxiality, $\gamma \approx 15°$ and $\approx 25°$, respectively. The latter gaps are likely responsible for the TSD minima in Fig. 3. Therefore, the location of the neutron gaps is dependent on the quadrupole deformation. Most significantly, the observation of these gaps, and the candidate TSD bands in ^{174}Hf, extends the region to search for these triaxial bands toward heavier $Z \approx 72$ nuclei.

With the observation of several candidate TSD bands in ^{174}Hf, it is natural to consider whether any of the weaker sequences correspond to a wobbling excitation. However, it is critical to inspect the electromagnetic properties of linking transitions between TSD bands to determine if this mode is observed. As no such transitions are found at this time, it is impossible to draw any conclusions whether wobbling bands exist in ^{174}Hf.

In summary, three, tentatively four, TSD bands were identified in ^{174}Hf for the first time. The bands have moments of inertia similar to previously established TSD sequences in $A \approx 165$ nuclei. Ultimate cranker calculations suggest that these superdeformed bands are higher in deformation than those found in ^{163}Lu. This may be due to an increased occupation of intruder neutron orbitals in ^{174}Hf. Although TSD bands have previously been assumed to result from a single-neutron shell gap at $N = 94$, new calculations reveal that at deformations closer to those predicted for ^{174}Hf, gaps with significant triaxial deformation open at $N = 100$ and 106. Thus, these new calculations may explain the existence of TSD sequences in the $N = 102$ ^{174}Hf nucleus, and they suggest the presence of an extended region of triaxial superdeformation.

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