Nuclear shape and structure in neutron-rich ^{110,111}Tc

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The high-spin nuclear structure of Tc isotopes is extended to more neutron-rich regions based on the measurements of prompt γ rays from the spontaneous fission of ²⁵²Cf at the Gammasphere. The high-spin level scheme of N = 67 neutron-rich ¹¹⁰Tc (Z = 43) is established for the first time, and that of ¹¹¹Tc is extended and expanded. The ground band of ¹¹¹Tc reaches the band-crossing region, and the new observation of the weakly populated $\alpha = -1/2$ member of the band provides important information on signature splitting. The systematics of band crossings in the isotopic and isotonic chains and a CSM calculation suggest that the band crossing of the ground band of ¹¹¹Tc is due to alignment of a pair of $h_{11/2}$ neutrons. The best fit to signature splitting, branching ratios, and excitations of the ground band of ¹¹¹Tc by the rigid triaxial rotor plus particle model calculations result in a shape of $\varepsilon_2 = 0.32$ and $\gamma = -26^\circ$ for this nucleus. Its triaxiality is larger than that of ^{107,109}Tc, which indicates increasing triaxiality in Tc isotopes with increasing neutron number. The identification of the weakly populated K+2 satellite band provides strong evidence for the large triaxiality of ¹¹¹Tc. In ¹¹⁰Tc, the four lowest-lying levels observed are very similar to those in ¹⁰⁸Tc. At an excitation of 478.9 keV above the lowest state observed, ten states of a $\Delta I = 1$ band are observed. This band of ¹¹⁰Tc is very analogous to the $\Delta I = 1$ bands in ^{106,108}Tc, but it has greater and reversal signature splitting at higher spins.

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I. INTRODUCTION

The studies of shape coexistence and shape transitions in the neutron-rich $A \sim 100$ region have long been of major interest [1,2]. In this region, shape transitions from spherical to strongly deformed shapes are observed along the $Z \sim 40$ isotopic chains, and quadrupole deformations are found to decrease with increasing proton number between 38 and 42 [3–6]. Our systematic studies of neutron-rich odd-Z Y-Nb-Tc-Rh (Z = 39-41-43-45) isotopes identified a shape transition from an axially symmetric shape with very large quadrupole deformations in ^{99,101}Y to large triaxial deformations in ^{107,109}Tc and ^{111,113}Rh isotopes [7–9].

It is of interest to explore further their structure along isotopic chains to the more neutron-rich region. For Rh (Z = 45) isotopes, N = 67 isotope ¹¹²Rh and N = 68 isotope ¹¹³Rh have been reached [7]. For Tc (Z = 43) isotopes, however, although the heavy N = 72 isotope ¹¹⁵Tc was observed by using a projectile fragmentation technique [10], the high-spin structure of Tc isotopes was available only up to the N = 66 isotope ¹⁰⁹Tc [8,11]. In the previous studies of lighter Tc isotopes, high-spin level schemes were established and a shift from a weak-coupling scheme toward a strong-coupling scheme from ${}^{97}\text{Tc}$ to ${}^{105}\text{Tc}$ was reported [12]. Afterward, the strong-coupling scheme and large signature splitting were extended to ${}^{107,109}\text{Tc}$ [8,11]. This evolution of coupling schemes was interpreted as due to the location of the Fermi level changing with deformation as the neutron number increases [8,10]. The proton Fermi level, being close to $1/2^+$ and $3/2^+$ of the $\pi g_{9/2}$ subshell for ${}^{97,99}\text{Tc}$ with small deformations, approaches the $5/2^+$ of the same subshell for ${}^{103-109}\text{Tc}$ with larger deformations. A quadrupole deformation $\varepsilon_2 = 0.32$ and triaxiality $\gamma = -22.5^\circ$ were deduced in ${}^{107}\text{Tc}$ [8].

In the present paper, we report new experimental results obtained for the high-spin structure of the N = 67 isotope ¹¹⁰Tc and N = 68 isotope ¹¹¹Tc, which were achieved at almost the same time as we identified for the first time the high-spin level scheme of ¹³⁸Cs [13]. No low-lying yrast cascade had been reported in ^{110,111}Tc before the present work. Seven non-yrast levels up to a low excitation energy of 740.8 keV in ¹¹⁰Tc were reported in Nuclear Data Sheets [14] by measurements of ¹¹⁰Mo β^- decay. In the present paper, the high-spin level scheme of ¹¹⁰Tc is proposed for the first time,



FIG. 1(a). Double-gated triple-coincidence spectrum for ¹¹⁰Tc data analysis. Gates are set on the 53.5 and 126.3 keV transitions of ¹¹⁰Tc. All transitions identified in ¹¹⁰Tc coincident with the gates are simultaneously seen in the spectrum with those of the complementary fission partners ^{137,138,139}Cs.



FIG. 1(b). Same as Fig. 1(a), but gates are set on the 178.3 and 178.9 keV transitions of ¹¹⁰Tc.



FIG. 1(c). Same as Fig. 1(a), but gates are set on the 126.3 and 178.3 keV transitions of ¹¹⁰Tc.



FIG. 1(d). Same as Fig. 1(a), but one of the gates is set on the 104.5 keV transition of ¹¹⁰Tc and the other on the 1184.7 keV transition of ¹³⁷Cs. Transitions of the 5n fission partner pair ¹¹⁰Tc-¹³⁷Cs are simultaneously seen.



FIG. 1(e). Same as Fig. 1(a), but one of the gates is set on the 222.0 keV transition of ¹¹⁰Tc and the other on the 1156.9 keV transition of ¹³⁸Cs. Transitions of the 4n fission partner pair ¹¹⁰Tc-¹³⁸Cs are simultaneously seen. This gated spectrum provides more evidence for the identifications of the 222.0 and 487.7 keV transition of ¹¹⁰Tc which are, respectively, overlapped by the 222.2 and 487.1 keV transition of the 5n fission partner ¹³⁷Cs.



FIG. 1(f). Same as Fig. 1(a), but gates are set on the 132.0 and 552.1 keV transitions of ¹¹¹Tc.



FIG. 1(g). Same as Fig. 1(a), but one of the gates is set on the 132.0 keV transition of ¹¹¹Tc and the other on the 1184.7 keV transition of ¹³⁷Cs. Transitions of the 4n fission partner pair ¹¹¹Tc-¹³⁷Cs are simultaneously seen. Some newly observed transitions in ¹¹¹Tc, which, however, are not coincident with the gates and thus do not show up in Fig. 1(f), can be seen in this gated spectrum.

which reports the newly observed yrast and near-yrast levels of the nucleus. After we completed the work of both 110,111 Tc, a paper on 111 Tc was published by Urban *et al.* [15], which reported a less extended level scheme of 111 Tc (see Sec. II for details).

Comparison and discussion are made in the present paper for the lowest levels and the $\Delta I = 1$ yrast band of ¹¹⁰Tc observed in the present work and those in ^{106,108}Tc [16]. The former shows an overall similarity to the latter for low-lying levels and a sharp increase and reversal in the signature splitting compared to ^{106,108}Tc [16] for higher spin levels, implying probably a significant increase in triaxiality in ¹¹⁰Tc. Model calculations performed for ¹¹¹Tc are presented. The extension of the ground band and the observation of the band crossing in this band of ¹¹¹Tc allow the study of systematics of band crossing of Tc isotopes. The systematics of the band crossing frequencies of the Tc isotopes and cranking shell model (CSM) calculations performed in the present work for ¹¹¹Tc suggest that this band crossing is caused by alignment of an $h_{11/2}$ neutron pair. The observation of the weakly populated $\alpha = -1/2$ member of the ground band of ¹¹¹Tc provides important information on signature splitting in this nucleus. The rigid triaxial rotor plus particle (RTRP) model is employed to calculate the signature splitting, excitation energies, and branching ratios of the ground band of ¹¹¹Tc. The best fits of the RTRP calculation to the experiments for ¹¹¹Tc result in a shape of $\varepsilon_2 = 0.32$ and $\gamma = -26^\circ$, a larger triaxiality than those in the lighter Tc isotopes, indicating an increase of triaxiality in Tc with increasing neutron number.

II. EXPERIMENTAL RESULTS

The populations and detections of the high-spin levels of ^{110,111}Tc were made by using spontaneous fission and measuring the prompt γ rays emitted in a multi- γ detection array [17]. The ^{110,111}Tc were produced as complementary fission fragments of Cs isotopes. A ²⁵²Cf source of 62 μ Ci, sandwiched between two 10 mg cm⁻² Fe foils, was placed in an 8 cm polyethylene ball centered in the Gammasphere, which consisted of 102 Compton-suppressed Ge detectors. Over

 5.7×10^{11} triple- and higher-fold events were accumulated. A RADWARE cube three-dimensional histogram was created . A less compressed RADWARE cube was also used to clarify



FIG. 2. High-spin level scheme of ¹¹⁰Tc proposed for the first time in the present work, assuming bandhead spin *I*, as for ^{106,108}Tc in [16]. The coincident 53.5–131.3 keV transitions are also reported by measurements of ¹¹⁰Mo β^- decay [14].

TABLE I. Transition energies and relative intensities of the transitions in $^{110}\mathrm{Tc}.$

E_{γ} (keV)	Relative intensities	E_{γ}^{a} (keV)	Band	Initial level (keV)
53.5		53.3	1	53.5
81.9			2	266.7
88.1			1	240.5
98.9			1	152.4
99.3			2	366.0
104.5			1	256.9
126.3	100		1	605.2
128.8	2.4		1	2046.5
131.3		131.2	2-1	184.8
146.2			2	366.0
159.3	5.1		1	1430.4
178.3	57.8		1	783.5
178.9	25.5		1	962.3
222.0	89.6		1	478.9
238.4	32.1		1	478.9
304.5	19.9		1	783.5
309.0	11.6		1	1271.2
357.1	32.2		1	962.3
468.1	24.9		1	1430.4
487.3	3.3		1	1917.8
487.7	15.0		1	1271.2
616.1	14.5		1	2046.5
646.6	7.1		1	1917.8
(650.3)			1	2699.5
745.8	3.7		1	2792.3
781.7	1.9		1	2699.5
884.9	0.5		1	3677.3

TABLE II. Transition energies and relative intensities of our measured transitions in ¹¹¹Tc and comparison energies E_{γ}^{*} from the recent work of Urban *et al.* [15].

$\frac{E_{\gamma}}{(\text{keV})}$	Relative intensities	E_{γ}^{*} (keV)	Band	Initial level (keV)
67.5		67.0	1	67.5
(124.0)			1	1830.7
126.5	14.3	126.5	1	610.1
132.0	100	131.6	1	199.5
132.6	3.6		1	1162.2
284.2	26.1	284.1	1	483.7
(293.6)			6	1182.0
313.1	4.2	312.2	6	888.4
375.8	13.6	375.8	6-1	575.3
410.6	47.7	410.6	1	610.1
416.3	12.9	415.5	1	483.7
419.5	7.0		1	1029.5
(507.9)			6-1	575.3
544.5	2.6		1	1706.8
545.8	6.9		1	1029.5
552.1	28.7	552.1	1	1162.2
606.7	3.2		6	1182.0
660.2	4.3		1	3214.5
668.5	19.2	668.4	1	1271.2
677.3	2.1		1	1706.8
723.6	9.5	723.4	1	2554.3
737.8	0.7		1	3952.3

^aTwo low-lying transitions reported in [14].

ambiguities caused by peaks overlapping, which was discussed in detail in [8].

As described in detail in our previous papers (e.g., Ref. [7]), the identifications of the transitions of ^{110,111}Tc were based on cross-checking the coincident relationships and relative transition intensities with those of the complementary fission partner Cs isotopes ^{137,138,139}Cs, and with the relevant transitions in ^{110,111}Tc as well. Careful background subtractions were always performed to eliminate possible accidental coincidences. Figures 1(a)-1(c) show typical examples of double-gated triple-coincidence spectra with both gates set on transitions in ¹¹⁰Tc for data analysis of ¹¹⁰Tc. Figures 1(d) and 1(e) are double-gated triple-coincidence spectra with one gate set on a transition of ¹¹⁰Tc and the other on the ground transition of its fission partner ¹³⁷Cs and 138 Cs, respectively. In Figs. 1(a)–1(c), transitions of ¹¹⁰Tc coincident with the gates are simultaneously seen with the transitions of all the complementary fission partner Cs isotopes. In Fig. 1(d), transitions of the 5n fission partner pair 110 Tc- 137 Cs [and the transitions of the 4*n* fission partner pair ¹¹⁰Tc-¹³⁸Cs in Fig. 1(e)] coincident with the gates are observed simultaneously. All the cross-checks showing the coincident relationship between the transitions of ¹¹⁰Tc and the transitions of all the fission partners ^{137,138,139}Cs provide unambiguous evidence of the identification of the level scheme of ¹¹⁰Tc. It is

worth mentioning that the gated spectrum in Fig. 1(e) with one gate on the 222.0 keV transition of ¹¹⁰Tc and the other on the 1156.9 keV transition of the 4*n* fission partner ¹³⁸Cs shows more evidence supporting the identifications of the 222.0 and 487.7 keV transitions identified in ¹¹⁰Tc, which are overlapped by the 222.2 and 487.1 keV transitions of the 5*n* fission partner ¹³⁷Cs, respectively. Similarly, Figs. 1(f) and Fig. 1(g) show evidence of the extension of the level scheme of ¹¹¹Tc.

Tables I and II summarize the transition energies and relative intensities determined in the present work for all the transitions identified in ^{110,111}Tc, respectively. Those reported in [15] for ¹¹¹Tc and the low-lying 53.3 and 131.2 keV transitions in ¹¹⁰Tc reported in [14] are also included in the tables.

Figures 2 and 3 show the high-spin level schemes of ¹¹⁰Tc and ¹¹¹Tc proposed in the present work, respectively. Figure 2 represents the first observation of the high-spin level scheme of ¹¹⁰Tc. The lowest 53.5 keV transition observed in ¹¹⁰Tc is most likely the low-lying *M*1 transition of 53.3 keV reported in [14] by measurement of ¹¹⁰Mo β^- decay. The 131.2 keV transition in coincidence with the 53.3 keV transition reported in [14] and a new weak cascade 81.9–99.3–146.2 keV above it are also identified in the present work. The observation of the low-lying 53.5–131.3 keV cascade in ¹¹⁰Tc by both β decay [14] and by fission of ²⁵²Cf in the present work thus supports the identification of the high-spin level scheme of ¹¹⁰Tc. For ¹¹⁰Tc, we have so far not been able to assign spins-parities to the levels observed. The levels of band 1 observed in ¹¹⁰Tc. Fig. 2, however, are quite similar to those in ^{106,108}Tc. The

(1)

(6)



FIG. 3. High-spin level scheme of ¹¹¹Tc proposed in the present work. The $\alpha = +1/2$ member of the ground band reaches the band-crossing region; the weakly populated $\alpha = -1/2$ member of the ground band is identified for the first time. A K + 2 satellite band, band 6, similar to those in ^{105,107,109}Tc [8] is also identified in ¹¹¹Tc.

lowest energy levels in ¹¹⁰Tc are rather similar to those in ¹⁰⁸Tc [16]. Then at 478.9 keV above the lowest state observed, a $\Delta I = 1$ band with cascade and cross-over transitions begins that is very similar to those in ^{106,108}Tc, as shown in Fig. 4, where the level energies of these high-spin bands are compared. In Fig. 4, one also sees a change in the signature splitting of this band in ¹¹⁰Tc compared to ^{106,108}Tc. Assuming the same bandhead spin of *I* for each of the three nuclei, one sees by the I + 4 level a sharp increase and reversal in the splitting with the I + 4 member somewhat closer to the I + 3 level in ¹⁰⁸Tc, but the I + 4 level is much closer to the I + 5 level in ¹¹⁰Tc. The splitting is also much greater in ¹¹⁰Tc. The sharp increase in signature splitting in ¹¹⁰Tc compared to ^{106,108}Tc may also be interpreted as a significant increase in triaxiality in ¹¹⁰Tc.

The numbering of the bands identified in ¹¹¹Tc follows those for ^{105,107,109}Tc in [8]. The level scheme of ¹¹¹Tc is considerably more extended than that reported in [15]. The $\alpha = +1/2$ branch of the ground band (band 1) of ¹¹¹Tc reported in [15] reaches 2553.1 keV, (25/2⁺) with no band-crossing observed; for the $\alpha = -1/2$ branch of band 1, only two levels at 67 and at 482.7 keV were reported in [15]. In Fig. 3, the $\alpha = +1/2$ branch of band 1 of ¹¹¹Tc identified in the present work reaches 3952.3 keV, (33/2⁺) level and shows clearly a band crossing. The $\alpha = -1/2$ branch of band 1 reaches 1706.8 keV, (19/2⁺) level, which provides important information about signature splitting of band 1. As seen in Fig. 3, the band 6 observed in ^{105,107,109}Tc [8] is also observed in ¹¹¹Tc.

The spin-parity assignments of the observed levels of ¹¹¹Tc are based on the level systematics observed in the Tc isotopic chains and on the observation of both cascade and linking transitions in the lower part of the bands. Shown in Fig. 5 are excitations of the levels of the ground $\pi g_{9/2}$ bands of ^{105,107,109,111}Tc. The smooth change of the level patterns with increasing neutron number supports the spin-parity assignments of the levels observed in ¹¹¹Tc (Fig. 3). It is reasonable to interpret the ground band of ¹¹¹Tc also as $\pi g_{9/2}$.

III. DISCUSSION AND CALCULATIONS

The extending of ground band 1 of ¹¹¹Tc allows a study of the systematics of band crossings in this band for the



FIG. 4. Level systematics of the high-spin bands in even-A 106,108,110 Tc. The $\Delta I = 1$ yrast band observed in 110 Tc is very analogous to those recently observed in 106,108 Tc [16], but it has greater and reversal signature splitting at higher spins.

odd-A Tc isotopes. The kinematic moment of inertias of band 1 of 105,107,109,110,111 Tc are given in Fig. 6(a). In the figure, a band crossing is observed in 111 Tc at a rotational



FIG. 5. Systematics of level patterns of ground bands of odd-A ¹⁰³⁻¹¹¹Tc isotopes. Data are from the present work and [8,12]. A smooth trend can be seen.





FIG. 6(a). (Color online) Kinematic moment of inertia $J^{(1)}$ of the ground bands of odd- $A^{105-111}$ Tc and 110 Tc. Crossing frequency decreases with increasing neutron number. However, no crossing is seen in 110 Tc in the frequency region. Data are from the present work and [8,12].

frequency of ~0.35 MeV, with crossing frequency decreasing with increasing neutron number for the Tc isotopes; for the odd-neutron neighbor ¹¹⁰Tc, no band crossing is observed in the frequency region. Figure 6(b) shows J(1) of the ground bands in N = 68 isotones ¹¹¹Tc and ¹¹³Rh [7]. The band crossing of the ground band of ¹¹¹Tc is observed at almost the same rotational frequency as that of ¹¹³Rh, also with no band crossing observed in the odd-neutron neighbor ¹¹²Rh [7], where there is odd-neutron blocking. All the observations imply that the band crossing of the ground band of ¹¹¹Tc can be interpreted to have the same origin as that of ¹¹³Rh, that is, the breaking of a pair of $h_{11/2}$ neutrons [7].

Cranking shell model (CSM) calculations described by Bengtsson and Frauendorf [19–21] were performed in the present work for ¹¹¹Tc to give total Routhian surface (TRS) and Routhian. The TRS calculations gave deformation parameters of $\beta_2 = 0.237$, $\beta_4 = -0.046$, $\gamma = 60^\circ$ at $\hbar\omega = 0.0$ MeV. It can



FIG. 6(b). (Color online) Kinematic moment of inertia $J^{(1)}$ of ground bands of the N = 68 isotones ¹¹¹Tc and ¹¹³Rh. $J^{(1)}$ of ¹¹²Rh is also shown. Almost the same crossing frequency is observed in these isotones. However, no crossing is seen in ¹¹²Rh in the frequency region. Data of ^{112,113}Rh are from [7].



FIG. 7. Polar coordinate plots of total Routhian surface (TRS) calculated at (a) $\hbar \omega = 0.2$ and (b) 0.4 MeV for ¹¹¹Tc.

be seen in Fig. 7 that a minimum of TRS of ¹¹¹Tc is observed around deformation parameters $\beta_2 = 0.295$, $\beta_4 = -0.004$, $\gamma = -28.9^{\circ}$ at $\hbar \omega = 0.2$ MeV and $\beta_2 = 0.262$, $\beta_4 = -0.024$, $\gamma = -38.7^{\circ}$ at $\hbar \omega = 0.4$ MeV, respectively. Inputting the β_2 and γ parameters obtained in the TRS calculations, the Routhian for ¹¹¹Tc is calculated by using CSM. An example of the Routhian calculations for ¹¹¹Tc is presented in Fig. 8 for quasiprotons (a) and quasineutrons (b). One can see that the calculations for ¹¹¹Tc predict an alignment caused by two $h_{11/2}$ neutrons at a rotational frequency of ~0.36 MeV, which is in good agreement with the observation of the band crossing of ¹¹¹Tc at ~0.35 MeV, supporting the interpretation of the band crossing as alignment of a pair of $h_{11/2}$ neutrons.

Shown in Fig. 9 are the level systematics of band 6 in 105,107,109,111 Tc observed in [8] and in the present work. Those of the Rh isotopes [7] are also given in the figure. A clear tendency is seen in the figure that with increasing neutron number, the excitation energies of the bandhead of band 6 of the Tc isotopes are decreasing, even more rapidly than those in the Rh isotopes. Like the Rh and ^{105,107,109}Tc cases, band 6 of ¹¹¹Tc, built on the excited $11/2^+$ state, deexcites to the $g_{9/2}$ ground band with predominant feeding to the $9/2^+$ level and very weak decay to the $7/2^+$ level. In view of the low excitation energy of the bandhead and the near vanishing of the E2 decay-out transition from the $(11/2^+)$ bandhead to the $7/2^+$ level of the ground band, the γ phonon interpretation for band 6 given in [15] and [22] is unlikely. Instead, as for ^{111,113}Rh [7] and ^{105,107,109}Tc [8], we believe that the level energies and decay pattern of band 6 of $^{111}\mathrm{Tc}$ provide strong evidence of triaxiality. The quenching of the $(11/2^+)_2 \rightarrow 7/2^+_1$ transition was explained by examining the wave functions [7]. The main core component in the wave functions of both the initial and final states is the first 2^+ core state; thus the E2 transition strength is mainly dictated by the diagonal E2 reduced matrix element, which vanishes for $\gamma = -30^{\circ}$. However, the main core component of the $9/2_1^+$ state is the 0⁺ state of the core, resulting in a large $B(E2, 11/2^+_2 \rightarrow 9/2^+_1)$. Band 6 is thus considered to be in the collective family of the ground band, and the term "K+2satellite band" is suggested for it (see discussion in more detail in [8] and in the following paragraphs).

In our previous theoretical calculations dedicated to the investigation of the neutron-rich Y and Nb isotopes [9] and lighter odd-even Tc and Rh isotopes [7,8], we employed the rigid triaxial rotor plus particle (RTRP) model to calculate the energy levels and several E2 and M1 strengths of the ground bands and some yrare levels. The model described very well the basic properties of these nuclei, such as signature splitting, excitation energies, and branching ratios. In the present work, we describe the level structure of ¹¹¹Tc in the framework of the same model.

The model was introduced in detail in the paper by Larsson *et al.* [23], and calculations based on this model for several neutron-rich nuclei in the considered mass region can be found in [7–9]. In the RTRP model, the odd particle occupying a deformed single-particle orbital is coupled to a rigid triaxial core. The nuclear field is described by a deformed modified oscillator potential characterized by the deformation parameters ε_2 , ε_4 , and γ , which are kept constant throughout the calculations (so-called rigid shape). In the present work, a value $\varepsilon_4 = 0$ was assumed. The asymmetry parameter γ was fitted from the splitting of the levels with opposite signature by using the signature-splitting function S(I) suggested by



FIG. 8. Calculated Routhian in ¹¹¹Tc for (a) quasiprotons and (b) quasineutrons plotted vs rotational frequency. The parity and signature (π, α) of the state are solid lines (+,+); dotted lines (+,-); dot-dashed lines (-,+); dashed lines (-,-).

Zamfir et al. in Ref. [24], where

$$S(I) = \frac{E(I) - E(I-1)}{E(I) - E(I-2)} \cdot \frac{I(I+1) - (I-2)(I-1)}{I(I+1) - (I-1)I} - 1.$$
(1)

We use the Lund convention for γ [25], for which the interval $0 \ge \gamma \ge -60^{\circ}$ describes the collective rotation, with $\gamma = 0$ corresponding to a prolate shape and $\gamma = -60^{\circ}$ corresponding to an oblate shape. The model uses the hydrodynamical irrotational flow formula for the ratios of the moments of inertia along the principal axes, which depend only on the deformation parameter γ . They can be normalized by using the effective value $E(2^+)$ of the core, which represents a scaling factor of the rotational energy. Pairing is included in the model by a standard BCS calculation. It is also possible to reduce the strength of the Coriolis force by an attenuation factor ξ [26]. Because of

the triaxiality, the projection K of the total angular momentum I on the intrinsic axis 3 (quantization axis) is no longer equal to the projection Ω of the particle angular momentum J on the same axis. However, in triaxial nuclei, a rotational band can, in principle, be characterized by the K quantum number which has the dominant contribution to the total wave function.

In the case of ¹¹¹Tc, the fitted parameters are $\varepsilon_2 = 0.32$, $\gamma = -26^\circ$, $E(2^+) = 0.25$ MeV, and $\xi = 0.8$. The RTRP model reproduces very well the large splitting S(I) experimentally found in ¹¹¹Tc as seen in Fig. 10. In [8], the model parameters found to best fit the shape of ¹⁰⁷Tc were $\varepsilon_2 = 0.32$ and $\gamma = -22.5^\circ$. In the present work, RTRP calculations were also performed for ¹⁰⁹Tc, and the model parameters found to best fit the shape of ¹⁰⁹Tc were $\varepsilon_2 = 0.32$ and $\gamma = -25^\circ$. The RTRP calculations for ^{107,109,111}Tc show that for neutron-rich nuclei with Z = 43 in the $A \sim 110$ region,



FIG. 9. Level systematics of band 6 of $^{105-111}$ Tc and of the Rh isotopes. A rapid lowering of the low-lying bandhead of band 6 with increasing neutron number is observed for both Tc and Rh isotopes. Data are from the present work and [7,8,12,28].

with increasing neutron number, the quadrupole deformation remains practically unchanged while the asymmetry parameter γ becomes larger. The increase in triaxiality with increasing neutron number is in fact suggested by the increase in the signature splitting function when going from ¹⁰⁵Tc to ¹¹¹Tc, as seen in the plot of Fig. 11.

The model also describes quite well the excitation energies of the ground band and yrare band (see Fig. 12). The intrinsic wave functions for the states in both bands were found to be dominated by the $7/2^+$ [413] single-particle orbital. However, as in the case of ¹⁰⁷Tc [8], the model does not reproduce the



FIG. 11. (Color online) Experimental signature splittings of the ground bands of $^{105-111}$ Tc. \diamond 105 Tc; \Box 107 Tc; \bullet 109 Tc; \blacktriangle 111 Tc. Signature splitting of 111 Tc is the largest among all the Tc isotopes, implying a larger triaxiality. Data are from the present work and [8].

experimental $5/2^+ \cdot 7/2^+$ level ordering, with $5/2^+$ being the ground state. A Harris plot performed for the favored-signature band in ¹⁰⁷Tc showed that the $5/2^+$ state clearly deviates from the extrapolated plot, suggesting that the intrinsic structure of this state may be different from that of the rest of the band [8]. In our calculation, the $5/2^+$ level is dominated by a component with K = 5/2, while the higher-spin states were found to have a dominant K = 7/2 component. In the yrare band, however, the main contribution to the total wave function has K = 11/2. The different values of K in bands based on the same Nilsson state are caused by different orientations of the core angular momentum.

The comparison of theoretical and experimental branching ratios $I(I \rightarrow I - 1) / I(I \rightarrow I - 2)$ for the transitions within the ground band and the ratio of the decays of the $11/2_2^+$ state $I(11/2_2^+ \rightarrow 9/2_1^+) / I(11/2_2^+ \rightarrow 7/2_1^+)$ are given in Table III.



FIG. 10. Comparison of theoretical and experimental signature splitting of the ground band of ¹¹¹Tc. Calculations were performed using the RTRP model. Good agreement is obtained by using parameters $\varepsilon_2 = 0.32$, $\gamma = -26^\circ$, $E(2^+) = 0.25$ MeV, and $\xi = 0.8$.



FIG. 12. Comparison of theoretical and experimental excitation energies of the ground band (band 1) and band 6 of ¹¹¹Tc. Calculations were performed using the RTRP. Good agreement is obtained by using the same parameters as in Fig. 10.

TABLE III. Comparison of theoretical and experimental branching ratios of the ground band and of the decays of band 6 of ¹¹¹Tc.

Branching ratios	Theory	Expt.
$\overline{I(11/2^+ \to 9/2^+)/I(11/2^+ \to 7/2^+)}$	1.74	2.02
$I(13/2^+ \rightarrow 11/2^+)/I(13/2^+ \rightarrow 9/2^+)$	0.2	0.3
$I(15/2^+ \rightarrow 13/2^+)/I(15/2^+ \rightarrow 11/2^+)$	0.85	1.0
$I(17/2^+ \rightarrow 15/2^+)/I(17/2^+ \rightarrow 13/2^+)$	0.07	0.125
$I(19/2^+ \rightarrow 17/2^+)/I(19/2^+ \rightarrow 15/2^+)$	0.55	1.24
$I(11/2_2^+ \rightarrow 9/2_1^+)/I(11/2_2^+ \rightarrow 7/2_1^+)$	10.8	>13.6

The agreement between experiment and theory is very good. The interesting feature observed in the lighter Tc isotopes, namely, the predominant decay of the $11/2^+_2$ state to the $9/2^+$ state of the ground band, is present also in ¹¹¹Tc. As a matter of fact, in ¹¹¹Tc, the $11/2^+_2 \rightarrow 7/2^+$ transition is so weak that its relative intensity could not be extracted in the present experiment. As mentioned above, the preference for the decay of the $11/2^+_2$ to the $9/2^+_1$ state was explained by invoking the composition of the core wave functions of the states involved. In ¹¹¹Tc, for instance, the model predicts that the states $11/2_2^+$, $13/2_2^+$, and $15/2_2^+$ have a dominant K = 11/2 component, and the dominant core angular momentum for the $11/2^+_2$ state is R = 2. The same core state has the maximum amplitude in the wave function describing the $7/2^+$ state of the ground band, which means that the E2 strength of the $11/2^+_2 \rightarrow 7/2^+$ transition is mainly determined by the diagonal matrix element of the quadrupole operator. For nuclei with $\gamma = -30^{\circ}$, this matrix element vanishes. For the $9/2^+$ state, the calculations reveal that the main contribution to the total wave function is determined by the R = 0 core state, resulting in a large matrix element of the quadrupole operator. This property is directly related to the triaxial deformation [27]. If we assume an upper limit of 0.01 relative intensity for the $11/2_2^+ \rightarrow 7/2_1^+$ transition, then the lower limit for the intensity ratio $I(11/2_2^+ \rightarrow 9/2_1^+) / 1/2_2^+$ $I(11/2+2 \rightarrow 7/2^+)$ is 13.6. Theoretically, we obtained 10.8 for the same ratio, which describes rather well the enhancement of the $11/2^+_2 \rightarrow 9/2^+_1$ transition with respect to the $11/2^+_2 \rightarrow$ $7/2^+$ transition.

Finally, it is necessary to compare the shape parameters used in the CSM and RTRP calculations performed in this work. The CSM calculations reproduced quite well the band crossing of band 1 in ¹¹¹Tc, and the RTRP calculations gave best fits to the signature splitting, excitations, and branching ratios of the band in the nucleus. Despite the differences in the absolute values of β_2 and γ parameters between CSM and RTRP calculations, the two calculations are in essential agreement, with very large β_2 for the ground band with slowly decreasing deformation with rotational frequency and triaxiality parameter γ that approaches -30° with rotational frequency. We need to keep in mind that the RTRP calculations deal with one-quasiparticle states and are thus only valid within a rotational frequency region below the band crossing. Since in our previous papers, RTRP calculations were performed for the Tc and Rh isotopes [7,8], we prefer to use the shape parameters for ¹¹¹Tc also obtained via RTRP calculations to study the shape evolutions in the isotopes.

In summary, a high-spin level scheme of ¹¹⁰Tc is established for the first time. Our extension of the level scheme of ¹¹¹Tc allows the study of band crossing and signature splitting in the more neutron-rich Tc isotopes. The systematics of band-crossing frequencies of the $^{105-111}$ Tc isotopes and N =68 Tc/Rh isotones and cranking shell model calculations suggest that the alignment of a pair of $h_{11/2}$ neutrons accounts for the band crossing of the ground band of ¹¹¹Tc. The very large signature splitting observed in ¹¹¹Tc (even larger than those observed in $^{105-109}$ Tc) is accounted for by large triaxial deformations in the nucleus. A shape of $\varepsilon_2 = 0.32$ and $\gamma = -26^{\circ}$ is deduced with the best fits to signature splitting, branching ratios, and excitations of the ground band of ¹¹¹Tc by the RTRP model calculations. This shows an increasing triaxiality with increasing neutron number for Tc isotopes. For ¹¹⁰Tc, a $\Delta I = 1$ band is observed that is similar overall to those in ^{106,108}Tc but shows larger signature splitting and a different sign of signature splitting, with the I + 4 level much close to the I + 5 level, and similarly for higher spins in ¹¹⁰Tc; while the I + 4 level is closer to the I + 3 level in ^{106,108}Tc.

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