ECT* (Trento), May 21-25 2012

Drell-Yan Scattering and the Structure of Hadrons

Theoretical Summary

Marco Radici

Disclaimer

the unusual combination of being an organizer
and at same time giving the (theory) summary talk
is <u>accidental</u>

summary = report on main message from each of previous talks
but unavoidably filtered through my perspective

apologies for every missing/wrong citation

Drell-Yan is important for several good reasons :

1. address the $\bar{q}$ distribution (in N, $\bar{N}$, ...)
2. explore the partonic structure of π
3. leading order (LO) is simple: $q\bar{q} \to \gamma^*$

   higher orders "easier" and under control
4. relevant for precision tests (W) and for Higgs searches

Physics motivation for DY studies at the LHC

At the LHC, both CC/NC DY reactions are of major importance for:

- extraction of PDFs in extended kinematics regions (high sensitivity to PDFs)
- best access to antiquark sea PDFs
- luminosity monitoring
- calibration of detectors (as "standard candles" for both Tevatron/LHC)
- the most precise ever definition of the W mass/width (CC from transverse mass)
- high precision SM tests (e.g. for Higgs physics)
- potential source of (or background for) many New Physics contributions
  (e.g. contact, 4-fermion interactions, extra W` and Z`, "unparticles" etc)
- we need unpolarised DY measurements from the LHC to use their results
  in later polarised DY experiments (e.g. will be useful for RHIC spin physics)

also use TMDs in low-x DY

see Pasechnik's talk

another good reason...

The DY cross section contains 48 structure functions [Arnold, Metz & Schlegel 2009]

$$
\begin{aligned}
\frac{\mathrm{d}^6\sigma}{\mathrm{d}^4 q\, \mathrm{d}\Omega} = \frac{\alpha_{\mathrm{em}}^2}{6 s Q^2} \Big\{ & \Big[(1+\cos^2\theta)\, W_{UU}^1 + \sin^2\theta\, W_{UU}^2 + \sin 2\theta\ \cos\phi\, W_{UU}^{\cos\phi} + \sin^2\theta\ \cos 2\phi\, W_{UU}^{\cos 2\phi} \Big] \\
& + S_{1T} \Big[\sin\phi_{S_1} \Big((1+\cos^2\theta)\, W_{TU}^1 + \sin^2\theta\, W_{TU}^2 + \sin 2\theta\ \cos\phi\, W_{TU}^{\cos\phi} + \sin^2\theta\ \cos 2\phi\, W_{TU}^{\cos 2\phi} \Big) \\
& + \cos\phi_{S_1} \left(\sin 2\theta\ \sin\phi\, W_{TU}^{\sin\phi} + \sin^2\theta\ \sin 2\phi\, W_{TU}^{\sin 2\phi} \right) \Big] + (1 \leftrightarrow 2, T \leftrightarrow U) \\
& + S_{1T}\, S_{2T} \Big[\cos(\phi_{S_1} + \phi_{S_2}) \ \Big((1+\cos^2\theta)\, W_{TT}^1 + \sin^2\theta\, W_{TT}^2 \\
& + \sin 2\theta\ \cos\phi\, W_{TT}^{\cos\phi} + \sin^2\theta\ \cos 2\phi\, W_{TT}^{\cos 2\phi} \Big) \\
& + \cos(\phi_{S_1} - \phi_{S_2}) \ \Big((1+\cos^2\theta)\, \overline{W}_{TT}^1 + \sin^2\theta\, \overline{W}_{TT}^2 + \sin 2\theta\ \cos\phi\, \overline{W}_{TT}^{\cos\phi} + \sin^2\theta\ \cos 2\phi\, \overline{W}_{TT}^{\cos 2\phi} \Big) \\
& + \sin(\phi_{S_1} + \phi_{S_2}) \left(\sin 2\theta \sin\phi\, W_{TT}^{\sin\phi} + \sin^2\theta\ \sin 2\phi\, W_{TT}^{\sin 2\phi} \right) \\
& + \sin(\phi_{S_1} - \phi_{S_2}) \left(\sin 2\theta \sin\phi\, \overline{W}_{TT}^{\sin\phi} + \sin^2\theta\ \sin 2\phi\, \overline{W}_{TT}^{\sin 2\phi} \right) \Big] + \ldots \Big\} \,.
\end{aligned}
$$

Integrating upon q_T only three structure functions survive:

W_{UU}^1, W_{LL}^1 and $W_{TT}^{\cos(2\phi - \phi_{S_1} - \phi_{S_2})} \equiv \frac{1}{2}(W_{TT}^{\sin 2\phi} + W_{TT}^{\cos 2\phi})$

24 structure functions appear at leading twist

from Barone's talk

a lot of interesting spin phenomena

most interesting "physics cases" in hadronic spin physics involve DY measurements :

1. double transversely polarized DY (DY$^{\uparrow\uparrow}$)

$$A_{TT}^{DY} = \frac{d\sigma^{\uparrow\uparrow} - d\sigma^{\uparrow\downarrow}}{d\sigma^{\uparrow\uparrow} + d\sigma^{\uparrow\downarrow}} \longrightarrow A_{TT}^{DY} = a_{TT} \frac{\sum_q e_q^2 \, h_{1q}(x_1, Q^2) \, \bar{h}_{1q}(x_2, Q^2) + [1 \leftrightarrow 2]}{\sum_q e_q^2 \, f_{1q}(x_1, Q^2) \, \bar{f}_{1q}(x_2, Q^2) + [1 \leftrightarrow 2]}$$

from Barone's talk

most interesting "physics cases" in hadronic spin physics involve DY measurements :

1. double transversely polarized DY ($DY^{\uparrow\uparrow}$)

$$A_{TT}^{DY} = \frac{d\sigma^{\uparrow\uparrow} - d\sigma^{\uparrow\downarrow}}{d\sigma^{\uparrow\uparrow} + d\sigma^{\uparrow\downarrow}} \longrightarrow A_{TT}^{DY} = a_{TT} \frac{\sum_q e_q^2 \, h_{1q}(x_1, Q^2) \, \bar{h}_{1q}(x_2, Q^2) + [1 \leftrightarrow 2]}{\sum_q e_q^2 \, f_{1q}(x_1, Q^2) \, \bar{f}_{1q}(x_2, Q^2) + [1 \leftrightarrow 2]}$$

collinear, LO, leading twist

cleanest access to transversity (Ralston-Soper '79)

Numerical predictions: ATT ~ 2-3% at RHIC (too low x)

~ 20-30% at PAX (but for $M < M_{J/\psi}$)

~ 10-20% at J-Parc

from Barone's talk

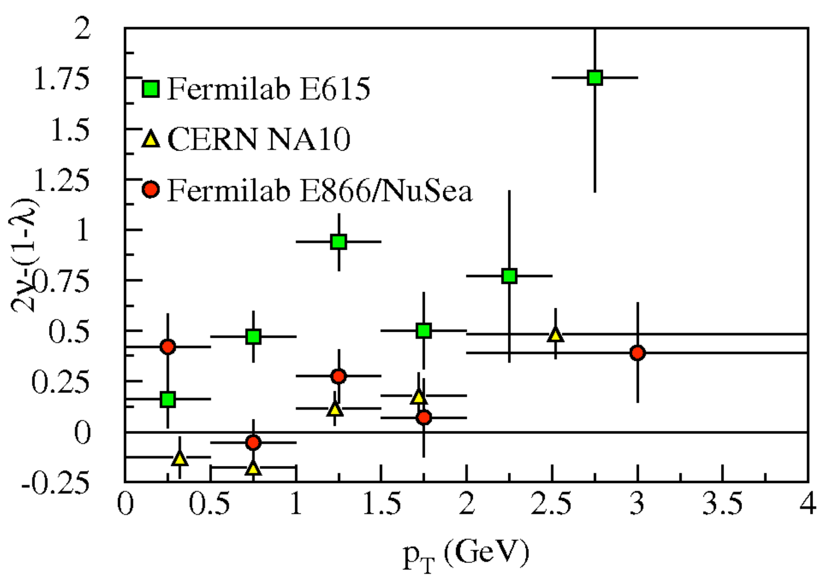
most interesting "physics cases" cont'ed

2. violation of Lam-Tung sum rule $1-\lambda = 2\nu$

$$\frac{\mathrm{d}^6\sigma_{UU}}{\mathrm{d}^4q\,\mathrm{d}\Omega} = \frac{\alpha_{\mathrm{em}}^2}{6sQ^2}\left\{(1+\cos^2\theta)\,W_{UU}^1 + \sin^2\theta\,W_{UU}^2 + \sin 2\theta\,\cos\phi\,W_{UU}^{\cos\phi} + \sin^2\theta\cos 2\phi\,W_{UU}^{\cos 2\phi}\right\}$$

unpol. DY cross section

$$\frac{1}{N_{\mathrm{tot}}}\frac{\mathrm{d}N}{\mathrm{d}\Omega} = \frac{3}{4\pi}\,\frac{1}{\lambda+3}\left(1+\lambda\,\cos^2\theta + \mu\,\sin 2\theta\,\cos\phi + \frac{\nu}{2}\,\sin^2\theta\cos 2\phi\right)$$



talk by P. Reimer at DY@BNL workshop

	pQCD
$2\nu - (1-\lambda) = 0$	LO
$\gtrsim 0$	NLO

from Bacchetta's talk

most interesting “physics cases” cont’ed

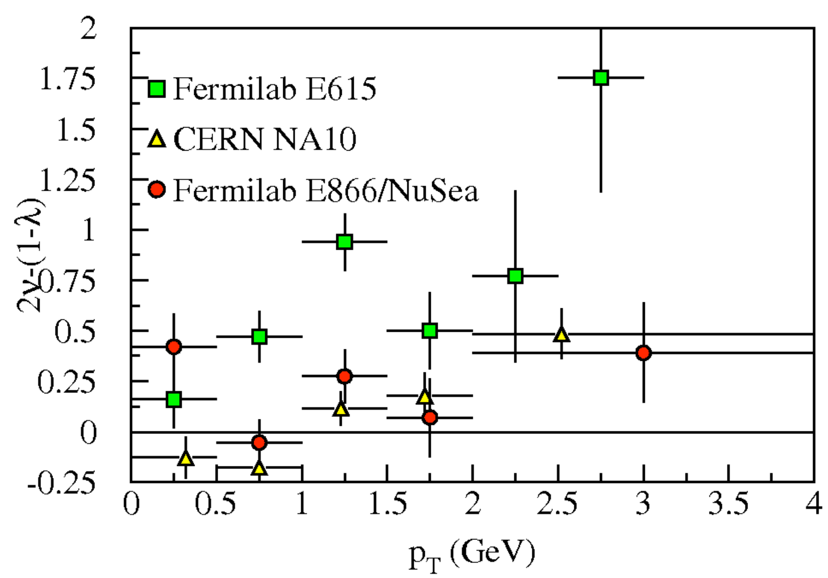
2. violation of Lam-Tung sum rule $1-\lambda = 2\nu$

unpol. DY cross section

$$\frac{d^6\sigma_{UU}}{d^4q\, d\Omega} = \frac{\alpha_{\rm em}^2}{6sQ^2}\Big\{(1+\cos^2\theta)\, W_{UU}^1 + \sin^2\theta\, W_{UU}^2 + \sin 2\theta\, \cos\phi\, W_{UU}^{\cos\phi} + \sin^2\theta\cos 2\phi\, W_{UU}^{\cos 2\phi}\Big\}$$

$$\frac{1}{N_{\rm tot}}\frac{dN}{d\Omega} = \frac{3}{4\pi}\frac{1}{\lambda+3}\left(1+\lambda\, \cos^2\theta + \mu\, \sin 2\theta\, \cos\phi + \frac{\nu}{2}\, \sin^2\theta\cos 2\phi\right)$$

new pT-dep. non-pert. effect



talk by P. Reimer at DY@BNL workshop

	pQCD
$2\nu - (1-\lambda) = 0$	LO
$\gtrsim 0$	NLO

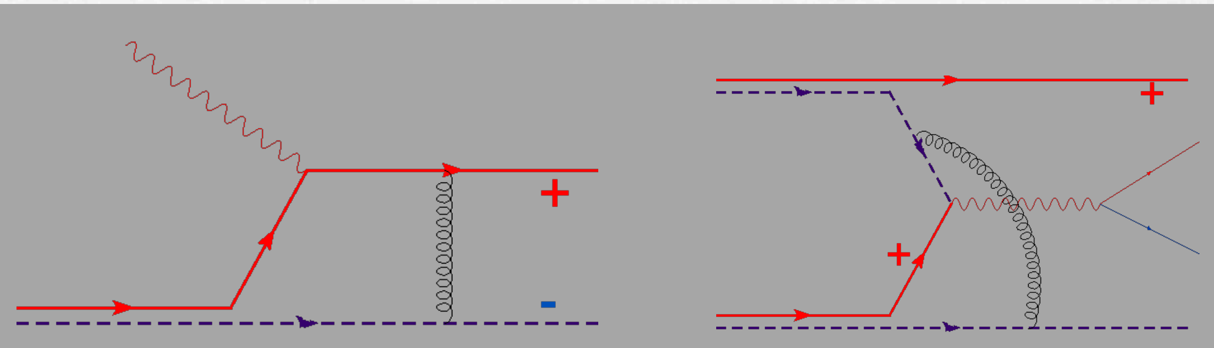
from Bacchetta’s talk

most interesting "physics cases" cont'ed

3. structure of interaction between colored objects dictated by gauge invariance (Wilson lines)

→ predict sign change of T-odd operators from SIDIS to DY

Ex. : the Sivers effect and process dependence of $f_{1T}^{\perp}$



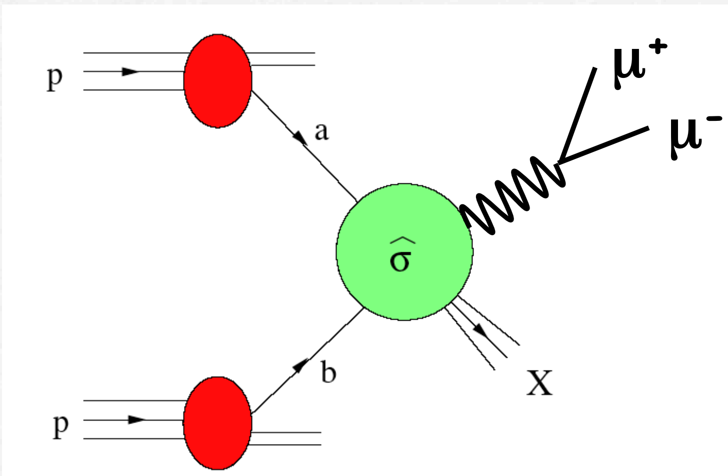
Brodsky,Hwang, Schmidt
Belitsky,Ji,Yuan
Collins
Boer,Mulders,Pijlman, etc

$$f_{1T}^{\perp SIDIS} = -f_{1T}^{\perp DY}$$

One of the main goals is to verify this relation.
It goes beyond "just" check of TMD factorization.
Motivates Drell-Yan experiments

from Prokudin's talk

DY and pert. QCD: 1. collinear factorization, scale/energy dependence of cross section, and all that..

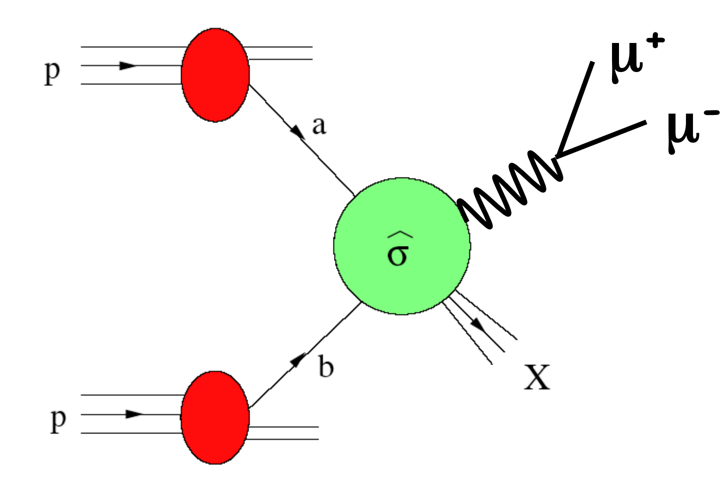


hard scale Q = inv. mass of $\mu^+\mu^-$

$$Q^4 \frac{d\sigma}{dQ^2} = \sum_{ab} \int dx_a dx_b \, f_a(x_a,\mu) \, f_b(x_b,\mu) \, \omega_{ab} + o(1/Q^2)$$

from Vogelsang's talk

DY and pert. QCD: 1. collinear factorization, scale/energy dependence of cross section, and all that..



hard scale Q = inv. mass of $\mu^+\mu^-$

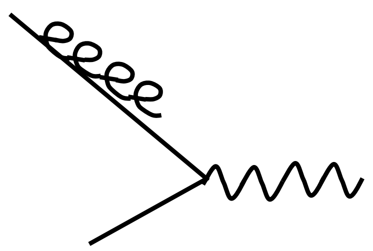
$$Q^4 \frac{d\sigma}{dQ^2} = \sum_{ab} \int dx_a dx_b \, f_a(x_a,\mu)\, f_b(x_b,\mu)\, \omega_{ab} + o(1/Q^2)$$

$$\omega_{ab} = \omega_{ab}^{(\mathrm{LO})} + \frac{\alpha_s}{2\pi}\,\omega_{ab}^{(\mathrm{NLO})} + \left(\frac{\alpha_s}{2\pi}\right)^2 \omega_{ab}^{(\mathrm{NNLO})} + \dots$$

	Unpol.	Long. pol.	Trans. pol.
NLO	Kubar et al. Altarelli, Ellis,Martinelli Harada et al.	Ratcliffe Weber Gehrmann Kamal de Florian, WV	Weber, WV WV Contogouris et al. Barone et al.
NNLO	Hamberg, van Neerven, Matsuura Harlander, Kilgore Anastasiou, Dixon, Melnikov, Petriello Catani, Cieri, Ferrera, de Florian, Grazzini	Smith, v.Neerven, Ravindran	

from Vogelsang's talk

DY and pert. QCD: 1. collinear factorization… cont'ed



Collinear singularity
→ factorization into PDFs
→ scheme dependence

- DGLAP evolution:

$$\mu^2 \frac{\mathrm{d}}{\mathrm{d}\mu^2} \begin{pmatrix} q(x,\mu^2) \\ g(x,\mu^2) \end{pmatrix} = \int_x^1 \frac{dz}{z} \begin{pmatrix} \mathcal{P}_{qq} & \mathcal{P}_{qg} \\ \mathcal{P}_{gq} & \mathcal{P}_{gg} \end{pmatrix} \begin{pmatrix} q \\ g \end{pmatrix} \left(\frac{x}{z},\mu^2\right)$$

$$\mathcal{P}_{ij} = \frac{\alpha_s}{2\pi} \mathcal{P}_{ij}^{\mathrm{LO}} + \left(\frac{\alpha_s}{2\pi}\right)^2 \mathcal{P}_{ij}^{\mathrm{NLO}} + \left(\frac{\alpha_s}{2\pi}\right)^3 \mathcal{P}_{ij}^{\mathrm{NNLO}} + \ldots$$

Ahmed,Ross
Altarelli,Parisi,…

Curci, Furmanski,
Petronzio
Antoniadis,Kounnas,
Lacaze
Mertig, van Neerven
WV
Kumano et al.
Koike et al.
WV

Moch,Vermaseren,
Vogt, Rogal

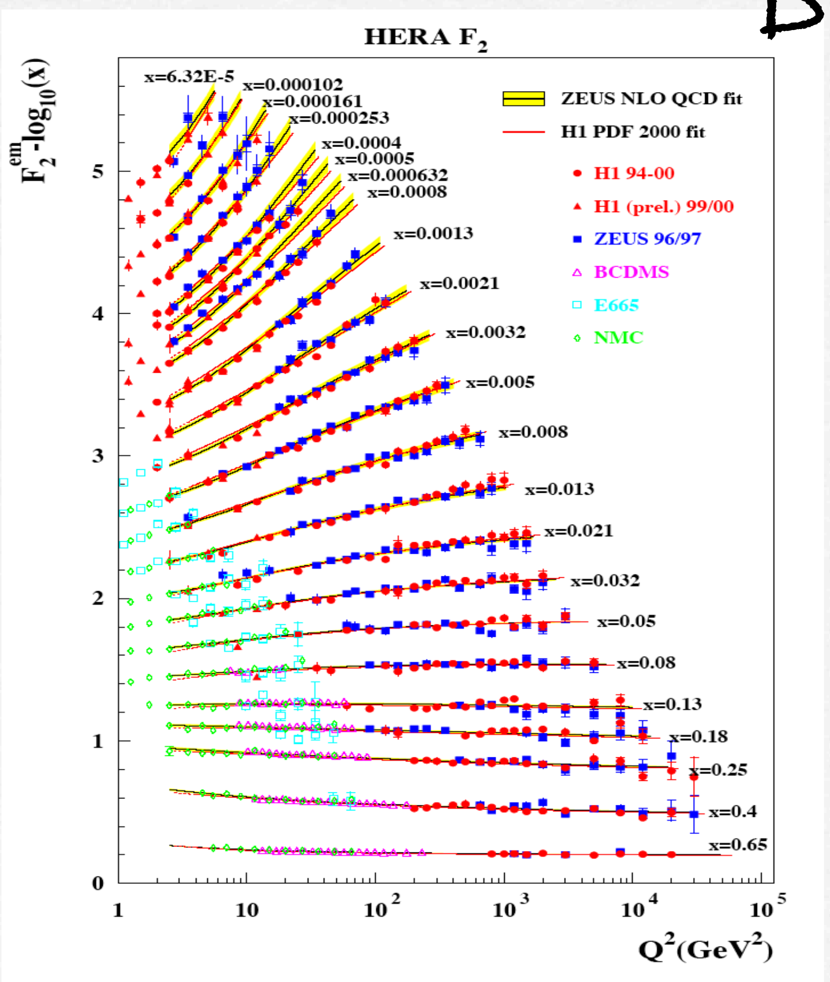
from Vogelsang's talk

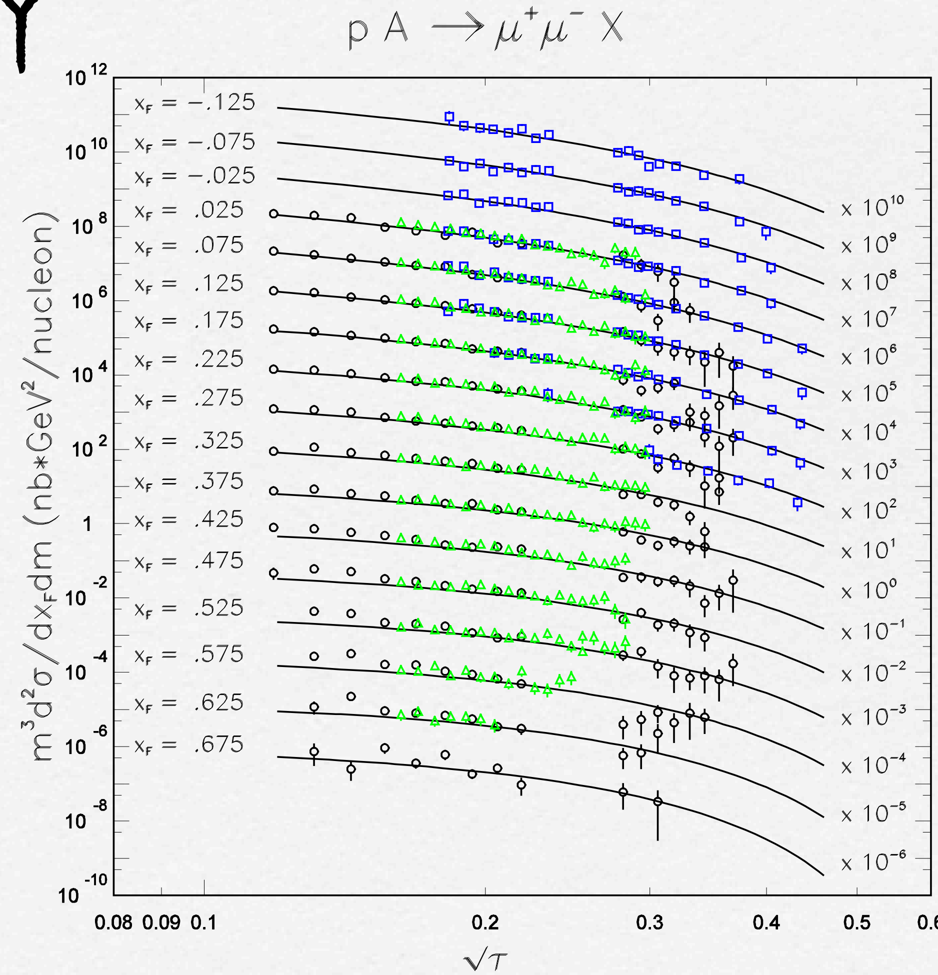
DY and pert. QCD: 1. collinear factorization... cont'ed

NLO and NNLO calc. reduce fact. scale uncertainty

NLO calc. already very successful

DIS ⟶ DY

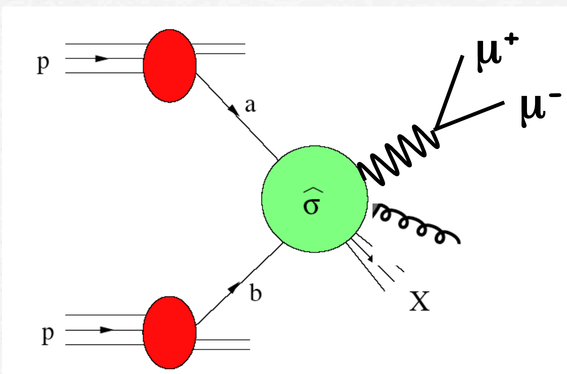




Ann.Rev.Nucl. Part. Sci. 49 (1999) 217

from Vogelsang's and Peng's talk

DY and pert. QCD: 1. collinear factorization... cont'ed



$$z = \frac{Q^2}{\hat{s}}$$

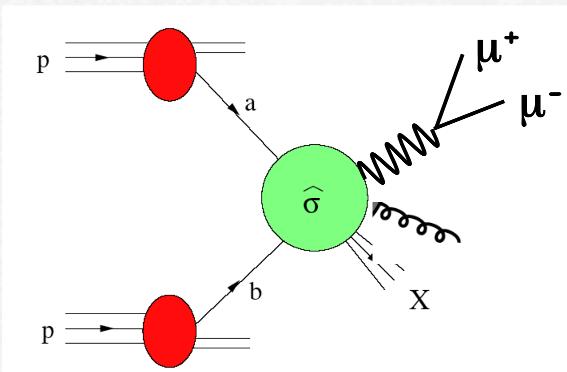
$z \to 1$:

$$\omega_{q\bar{q}}^{(\mathrm{NLO})} \propto \alpha_s \left(\frac{\log(1-z)}{1-z} \right)_+ + \ldots$$

threshold ($z \to 1$) log's
large, may spoil
the pert. series unless
resummed to all orders

from Vogelsang's talk

DY and pert. QCD: 1. collinear factorization… cont'ed



$$z = \frac{Q^2}{\hat{s}}$$

$$z \to 1 :$$

$$\omega_{q\bar{q}}^{(\mathrm{NLO})} \propto \alpha_s \left(\frac{\log(1-z)}{1-z} \right)_+ + \ldots$$

threshold (z→1) log's
large, may spoil
the pert. series unless
resummed to all orders

Fixed order

Resummation

LO	1				
NLO	$\alpha_s \mathbf{L}^2$	$\alpha_s \mathbf{L}$	α_s		$+\ldots$
NNLO	$\alpha_s^2 \mathbf{L}^4$	$\alpha_s^2 \mathbf{L}^3$	$\alpha_s^2 \mathbf{L}^2$	$\alpha_s^2 \mathbf{L}$	$+\ldots$
	$\alpha_s^3 \mathbf{L}^6$	$\alpha_s^3 \mathbf{L}^5$			
	$\alpha_s^4 \mathbf{L}^8$	$\alpha_s^4 \mathbf{L}^7$			
	$\vdots$	$\vdots$			
	$\alpha_s^k \mathbf{L}^{2k}$	$\alpha_s^k \mathbf{L}^{2k-1}$			
	LL	NLL			

from Vogelsang's talk

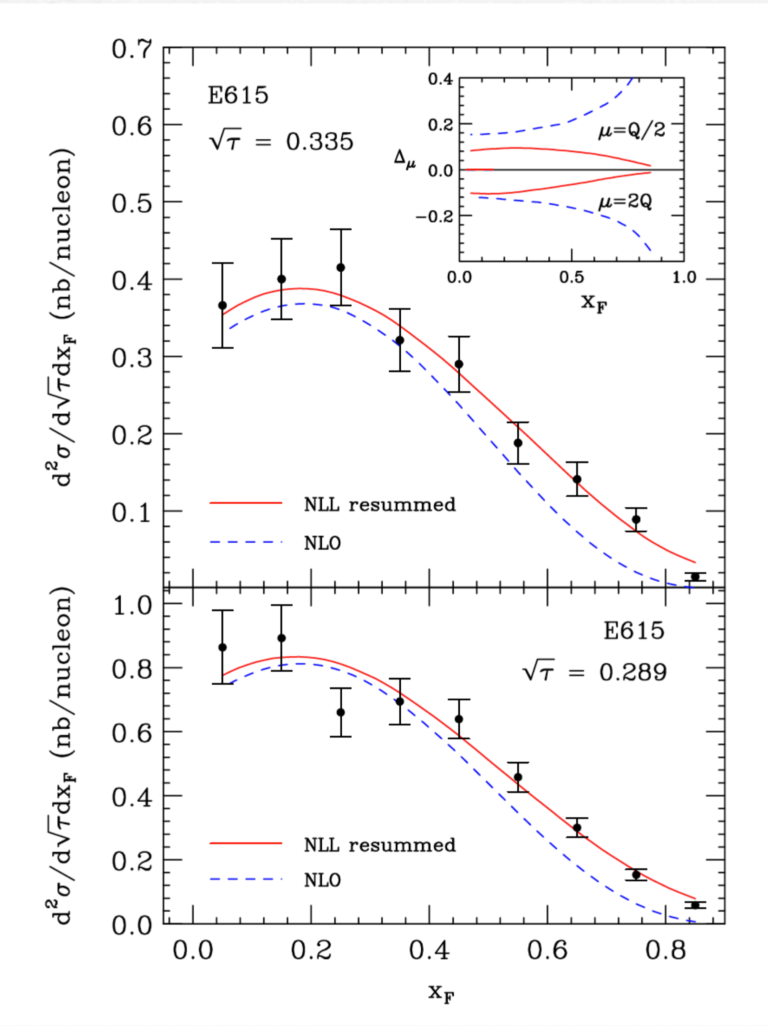
DY and pert. QCD: 1. collinear factorization... cont'ed

- enhance cross section
  improve on NLO

- reduce fact. scale
  uncertainty

threshold log's (NLL)

- get expected
  $x\, v^{\pi}(x, Q_0^2) \xrightarrow[x \to 1]{} (1-x)^2$
  $Q_0^2 = 1\ \mathrm{GeV}^2$

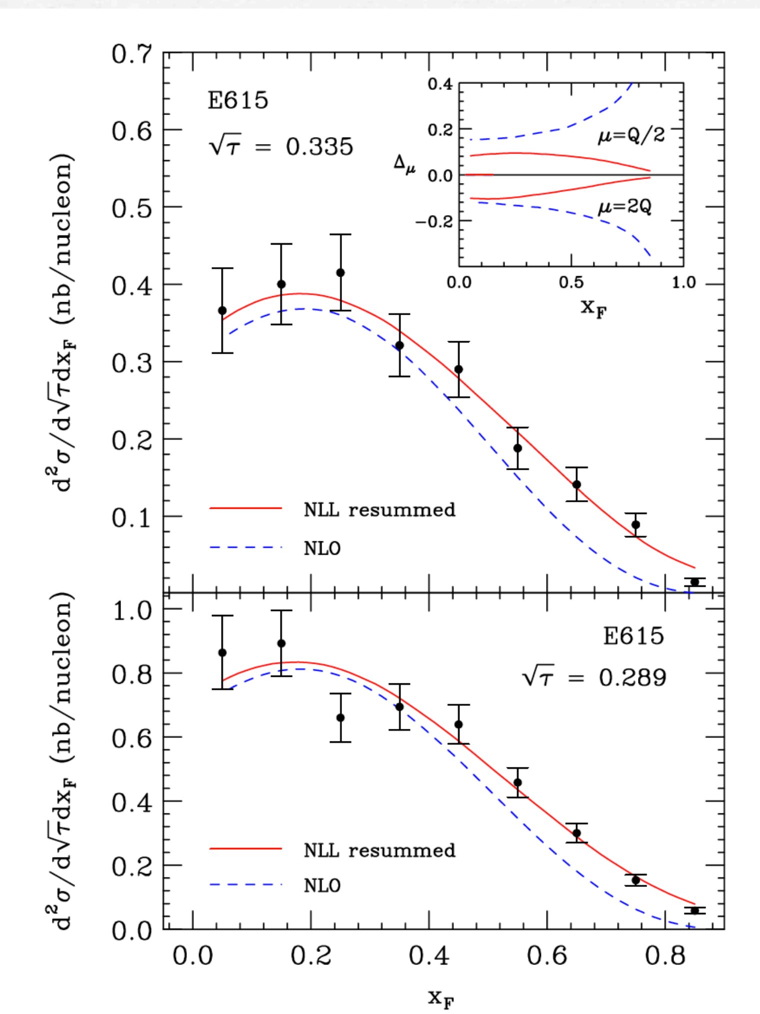


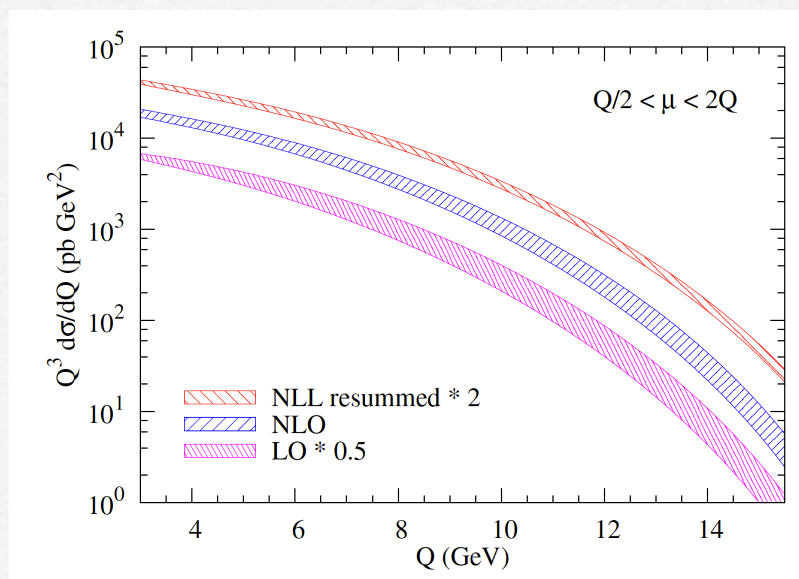
from Vogelsang's talk

DY and pert. QCD: 1. collinear factorization... cont'd

threshold log's (NLL)

- enhance cross section
  improve on NLO
- reduce fact. scale
  uncertainty
- get expected
  $x\, v^{\pi}(x, Q_0^2) \xrightarrow[x \to 1]{} (1-x)^2$
  $Q_0^2 = 1\ \mathrm{GeV}^2$



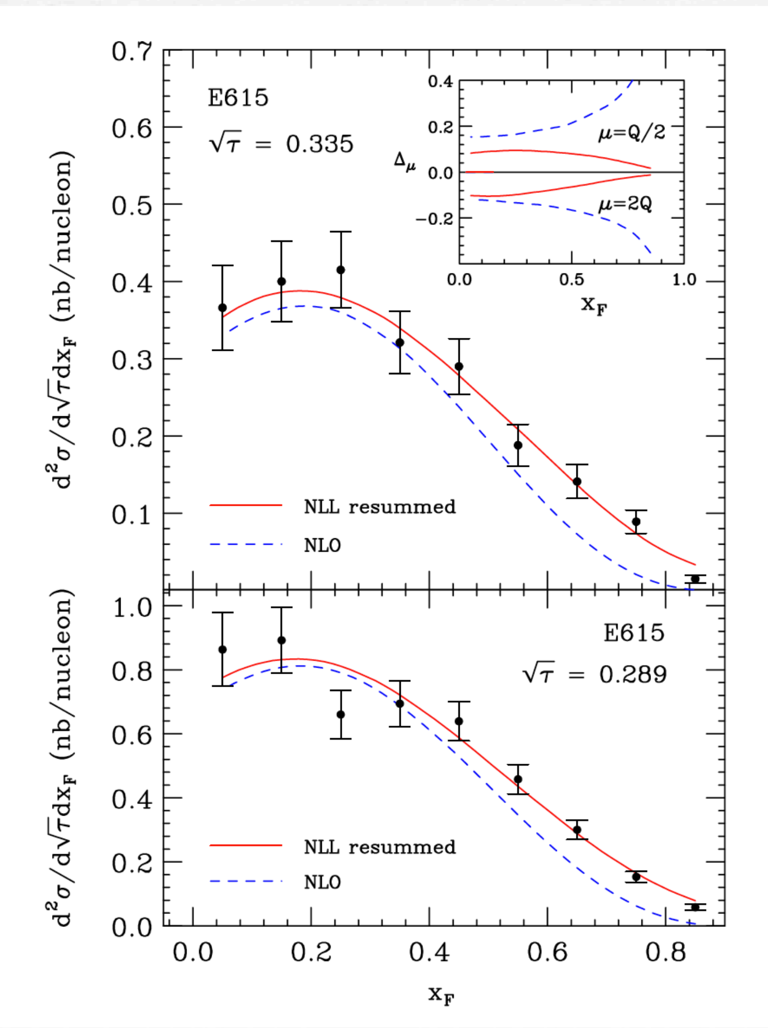


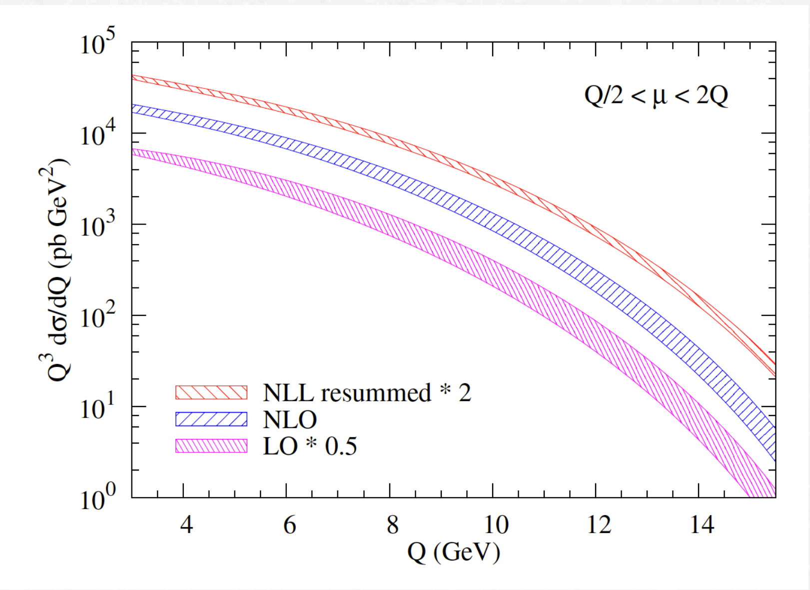
from Vogelsang's talk

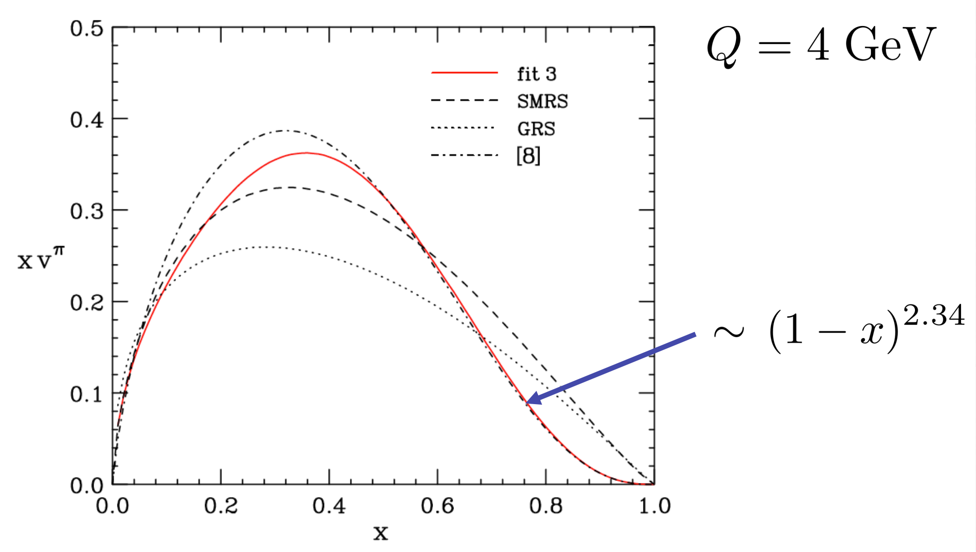
DY and pert. QCD: 1. collinear factorization... cont'ed

threshold log's (NLL)

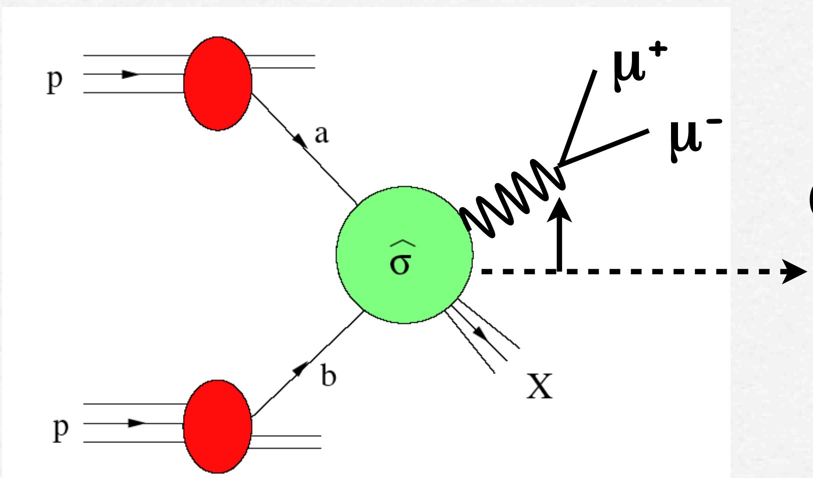
- enhance cross section
  improve on NLO
- reduce fact. scale
  uncertainty
- get expected
  $x v^{\pi}(x, Q_0^2) \xrightarrow[x \to 1]{} (1-x)^2$
  $Q_0^2 = 1\ \mathrm{GeV}^2$



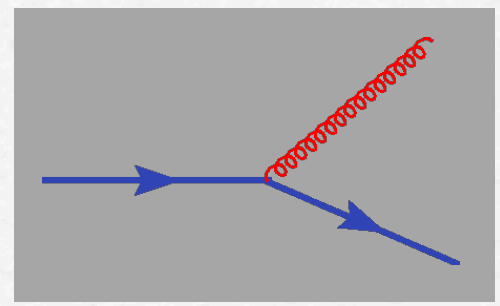




DY and pert. QCD: 2. "non-collinear" lepton pairs
($q_T \neq 0$ measured)



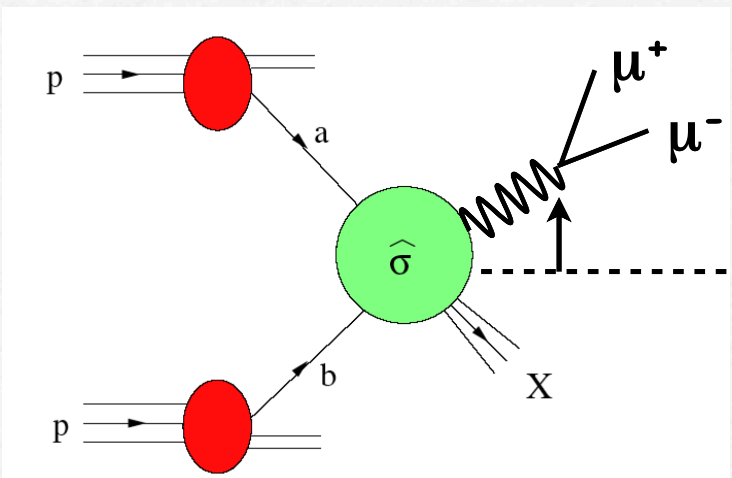
$q_T \neq 0$



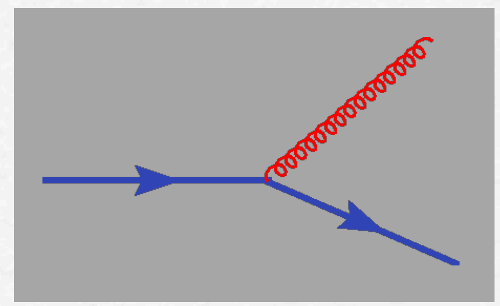
$$\alpha_s^k \, \frac{\log^{2k-1}\left(\frac{Q^2}{q_T^2}\right)}{q_T^2} + \ldots$$

gluon radiation creates transverse momenta

DY and pert. QCD: 2. "non-collinear" lepton pairs
($q_T \neq 0$ measured)



$q_T \neq 0$



$$\alpha_s^k \, \frac{\log^{2k-1}\left(\frac{Q^2}{q_T^2}\right)}{q_T^2} + \dots$$

gluon radiation creates
transverse momenta

emergence of Sudakov log's
at $q_T^2 << Q^2$
resummed at all orders for
unpol. cross section

Dokshitzer, Dyakonov, Troyan 1980
Parizi, Petronzio 1979
Collins, Soper 1982
Collins, Soper, Sterman 1985

$$\frac{d\sigma}{dq_T} \sim \int d^2 b_T e^{i q_T \cdot b_T} \hat{W}(x_1, x_2, b_T) e^{-S(b_T,Q)} + Y(q_T, Q)$$

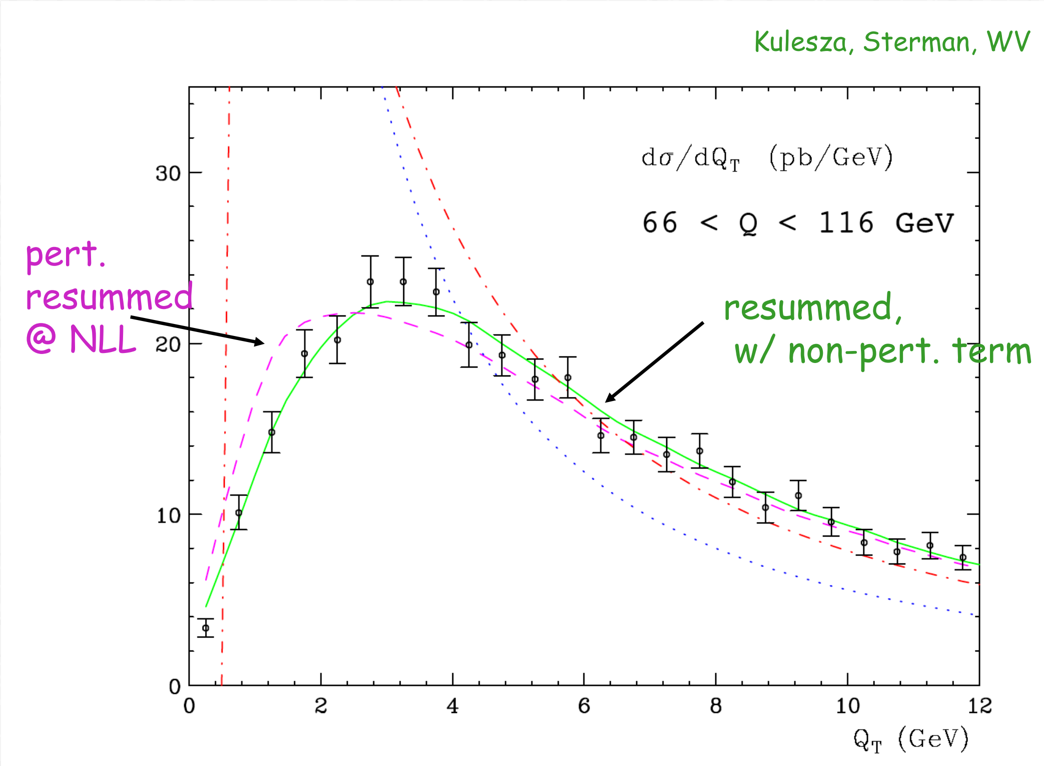
"hard" radiation
$q_T \approx Q$

DY and pert. QCD: 2. "non-collinear" lepton pairs ($q_T \neq 0$ measured)

Sudakov log's

- can be resummed with threshold log's
- reduce the cross section

better agreement with data

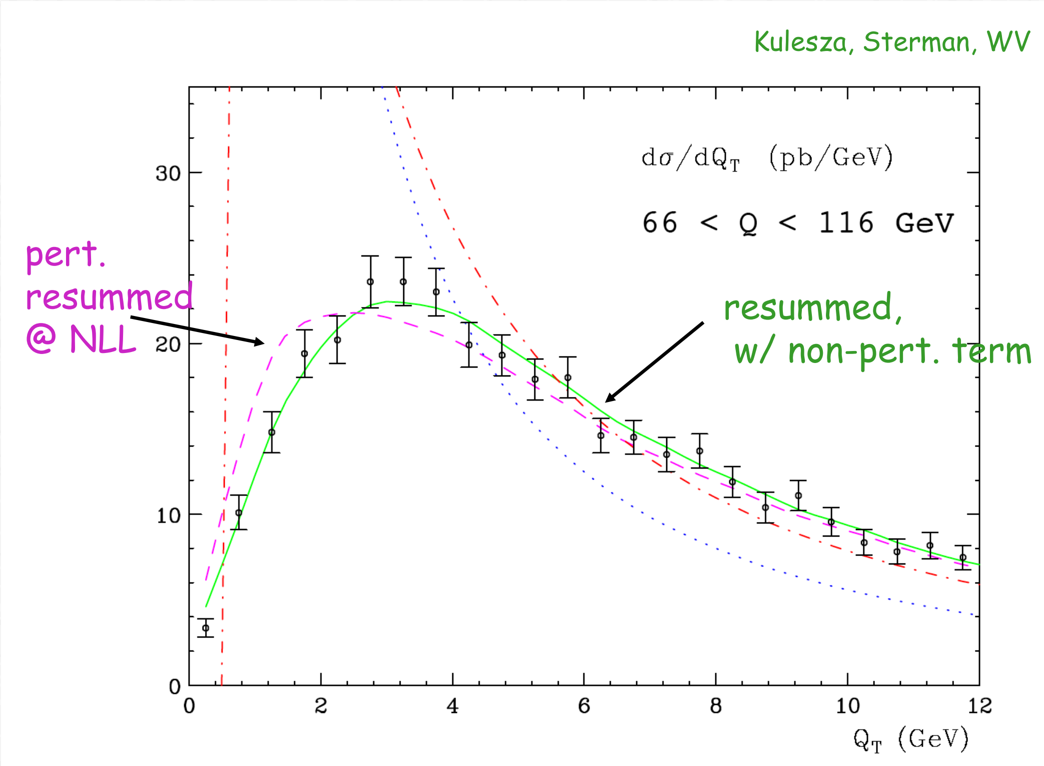


from Vogelsang's talk

DY and pert. QCD: 2. "non-collinear" lepton pairs
($q_T \neq 0$ measured)

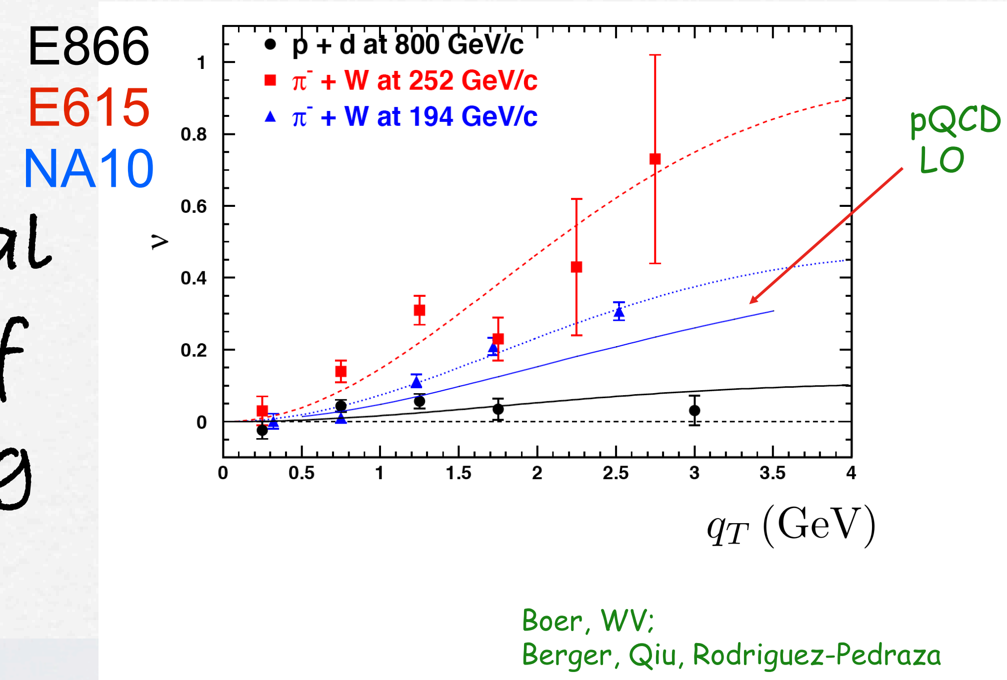
Sudakov log's

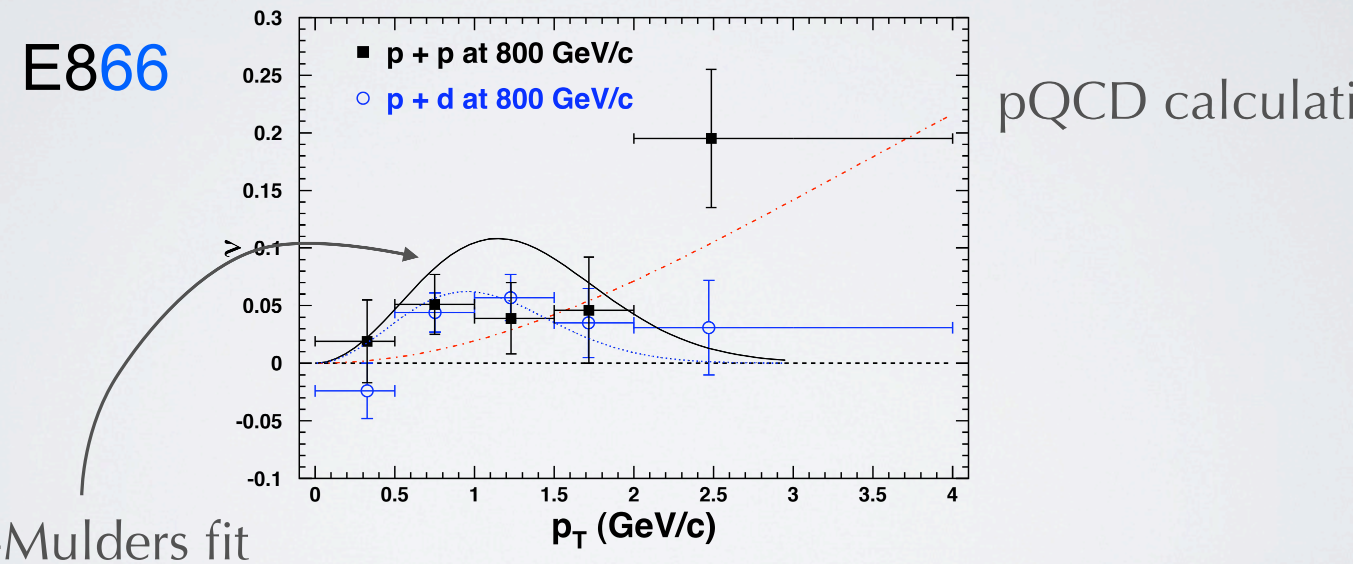
- can be resummed with threshold log's
- reduce the cross section better agreement with data



But

only partial account of Lam-Tung violation





Boer-Mulders fit

Zhang, Lu, Ma, Schmidt, PRD78 (08)

from Vogelsang's talk

Enzo Barone's talk

TMD fact.

pQCD collinear fact.

theory robust

but problems with

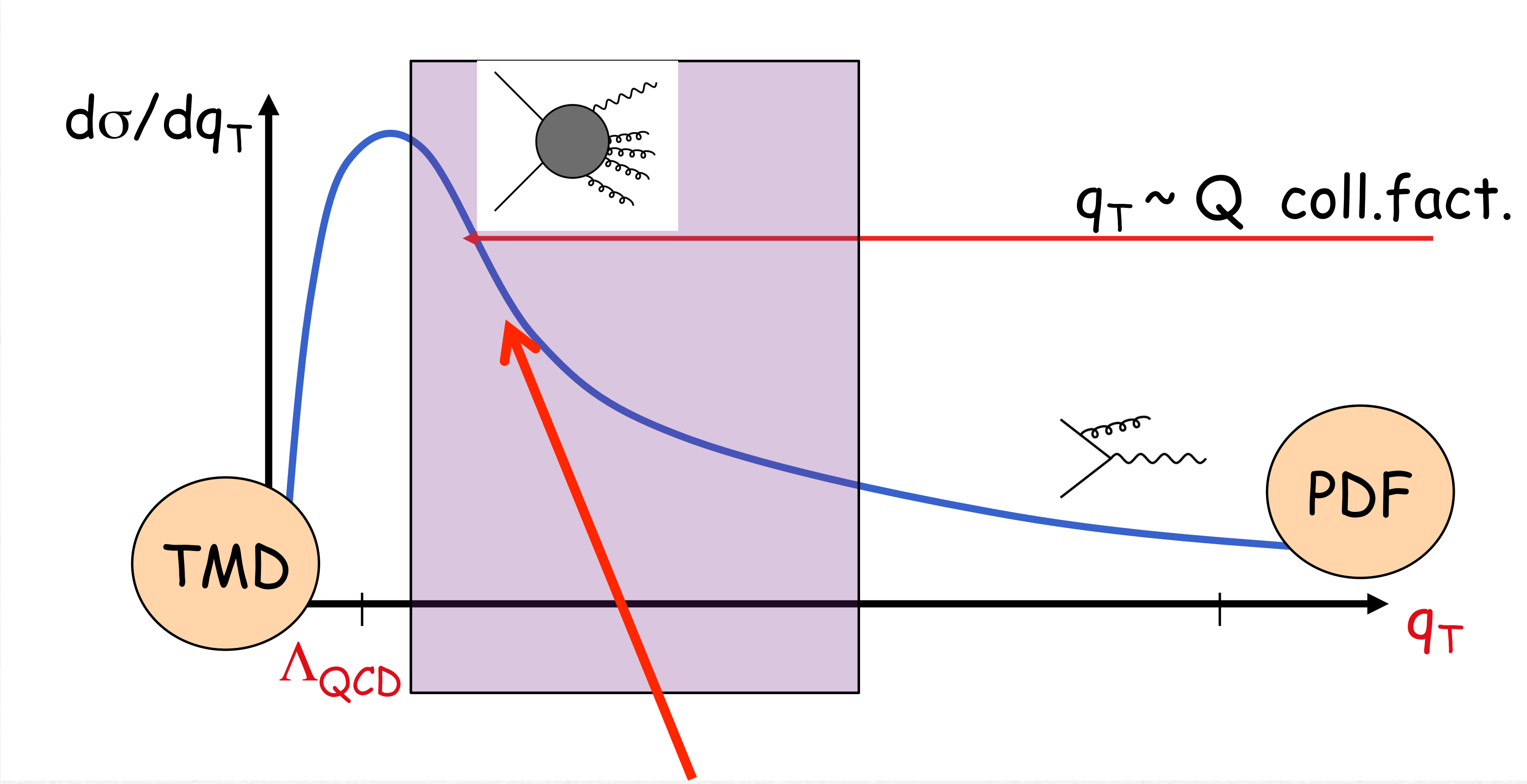
low q_T phenomenology

TMD fact.

rapidly
growing

pQCD collinear fact.

theory robust
but problems with
low q_T phenomenology



TMD fact.

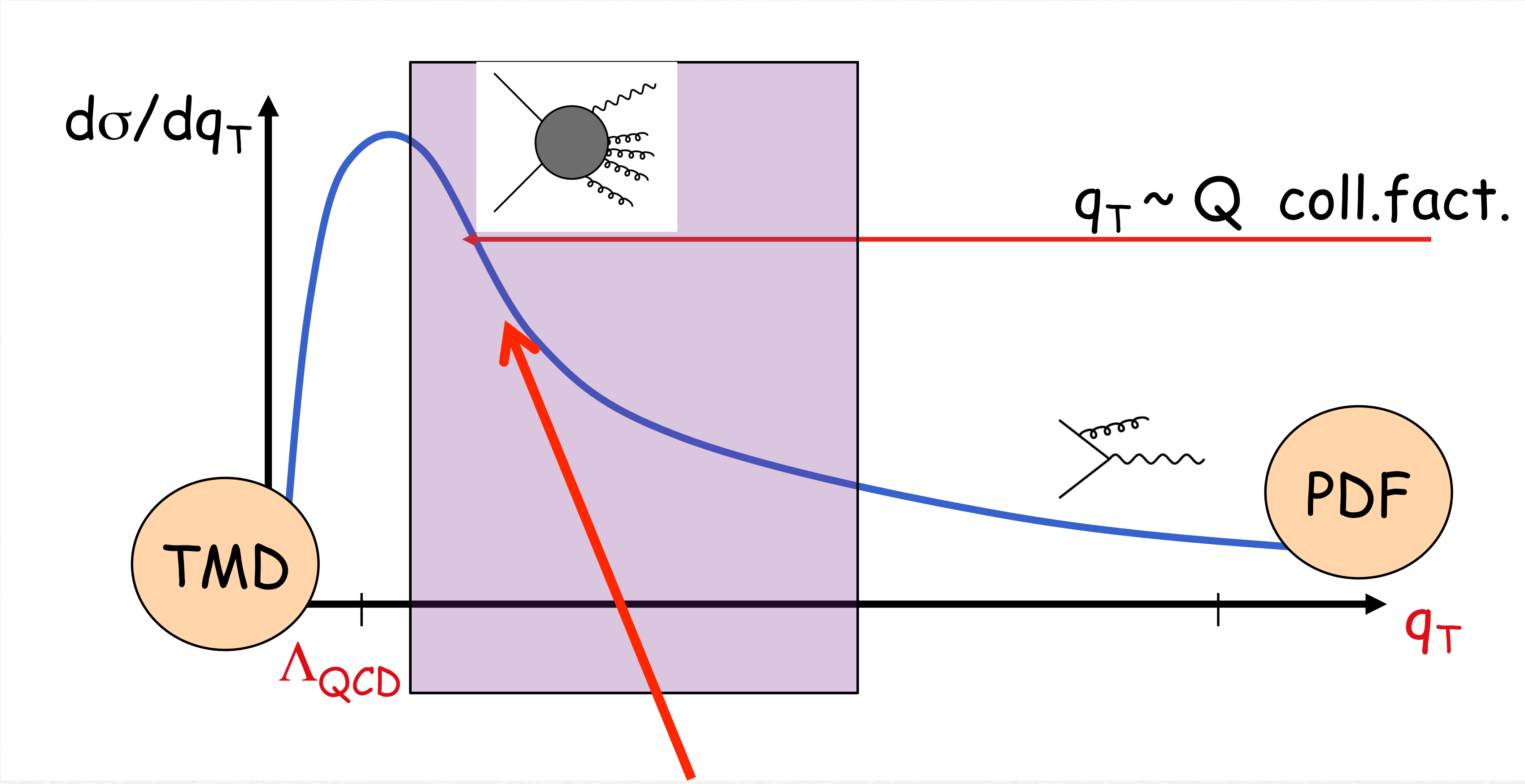
pQCD collinear fact.

rapidly growing

theory robust

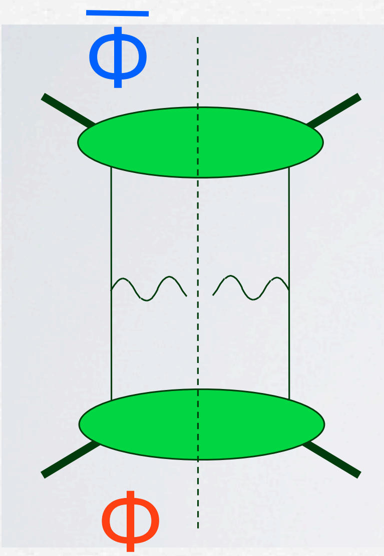
but problems with

low q_T phenomenology



overlapping region: do they match ?

TMD factorization approach
a rapidly growing field

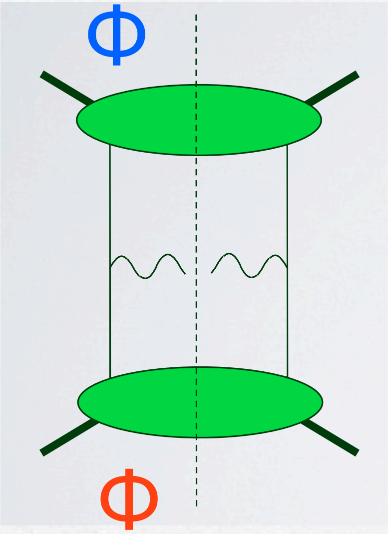


All-order TMD factorization theorem $\boxed{\text{at leading twist}}$

$$\boxed{W^{\mu\nu} \sim \int d^2k_{aT} d^2k_{bT}\, \delta^{(2)}(\vec{k}_{aT} + \vec{k}_{bT} - \vec{q}_T)\, \mathrm{Tr}[\hat{M}^\mu \Phi(x_a, \vec{k}_{aT}) (\hat{M}^\nu)^\dagger \bar{\Phi}(x_b, \vec{k}_{bT})] + Y^{\mu\nu}}$$

$q_T \ll Q$ $\qquad$ $q_T \simeq Q$

TMD factorization approach
a rapidly growing field



All-order TMD factorization theorem

at leading twist

$$W^{\mu\nu} \sim \int d^2k_{aT} d^2k_{bT}\, \delta^{(2)}(\vec{k}_{aT} + \vec{k}_{bT} - \vec{q}_T)\, \mathrm{Tr}[\hat{M}^{\mu} \Phi(x_a, \vec{k}_{aT}) (\hat{M}^{\nu})^{\dagger} \bar{\Phi}(x_b, \vec{k}_{bT})] + Y^{\mu\nu}$$

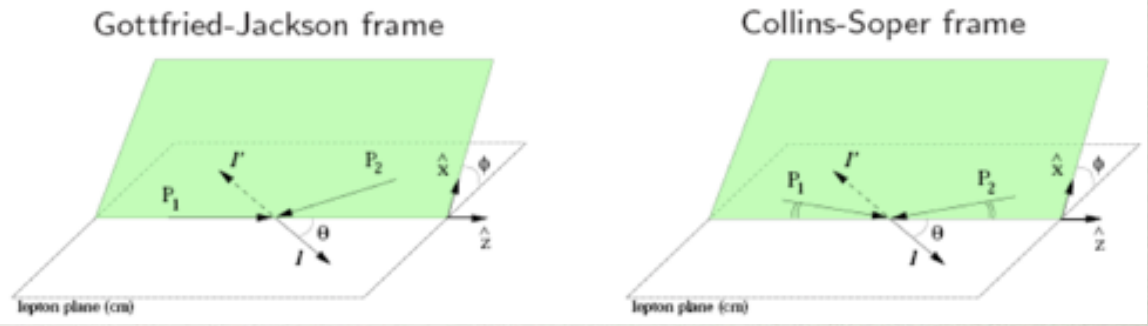
$q_T \ll Q$

$q_T \simeq Q$

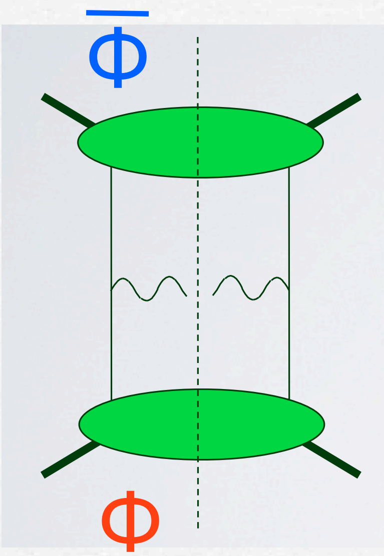
DY cross section

$$\frac{d\sigma}{d^4q\, d\Omega} \propto L_{\mu\nu} W^{\mu\nu}$$

Kinematics easy in dilepton rest frame



TMD factorization approach
a rapidly growing field



All-order TMD factorization theorem

at leading twist

$$W^{\mu\nu} \sim \int d^2k_{aT}\, d^2k_{bT}\, \delta^{(2)}(\vec{k}_{aT}+\vec{k}_{bT}-\vec{q}_T)\, \mathrm{Tr}[\hat{M}^{\mu}\Phi(x_a,\vec{k}_{aT})(\hat{M}^{\nu})^{\dagger}\bar{\Phi}(x_b,\vec{k}_{bT})] + Y^{\mu\nu}$$

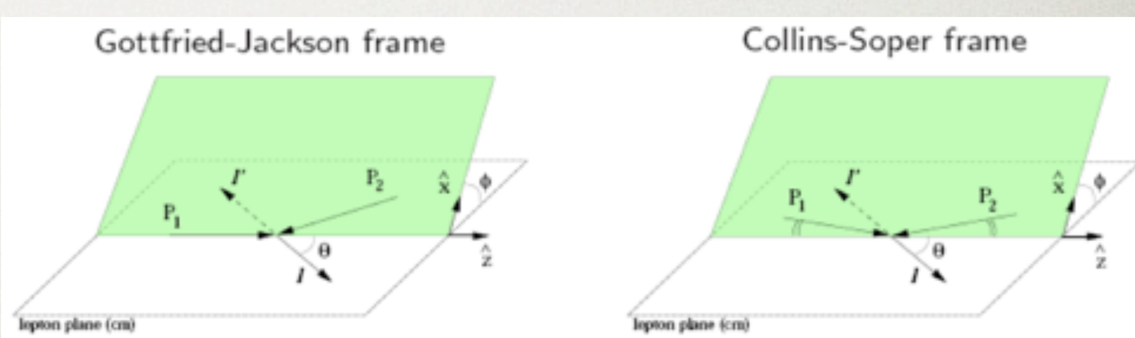
$q_T \ll Q$ $\qquad$ $q_T \simeq Q$

DY cross section

$$\frac{d\sigma}{d^4q\, d\Omega} \propto L_{\mu\nu} W^{\mu\nu}$$

but worry about q_T/Q differences
at twist 3 (see Bacchetta's talk)

Kinematics easy in dilepton rest frame



? new DY Trento conventions
on the definition of:
- azimuthal angles
- parametrization of $W^{\mu\nu}$ in terms of structure functions

TMD factorization approach cont'ed

Foundations of perturbative QCD
Collins 2011

hard dσ

$$W^{\mu\nu} = \sum_f |H_f(Q^2, \mu|^{\mu\nu}$$
$$\times \int d^2\mathbf{k}_{1T} d^2\mathbf{k}_{2T} F_{f/P_1}(x_1, \mathbf{k}_{1T}; \mu, \zeta_F) F_{\bar{f}/P_1}(x_2, \mathbf{k}_{2T}; \mu, \zeta_F)$$
$$\times \delta^{(2)}(\mathbf{k}_{1T} + \mathbf{k}_{2T} - \mathbf{q}_T) + Y(\mathbf{q}_T, Q)$$

μ = renorm./fact. scale

ζ_F = regulator for rapidity divergences (do not cancel as in coll. case but they cancel in $W^{\mu\nu}$)

TMD factorization approach cont'ed

Foundations of perturbative QCD
Collins 2011

hard dσ

$$W^{\mu\nu} = \sum_f |H_f(Q^2,\mu|^{\mu\nu}$$
$$\times \int d^2\mathbf{k}_{1T} d^2\mathbf{k}_{2T} F_{f/P_1}(x_1,\mathbf{k}_{1T};\mu,\zeta_F) \boxed{F_{\bar{f}/P_1}(x_2,\mathbf{k}_{2T};\mu,\zeta_F)}$$
$$\times \delta^{(2)}(\mathbf{k}_{1T}+\mathbf{k}_{2T}-\mathbf{q}_T) + Y(\mathbf{q}_T,Q)$$

unpol. TMD f_1

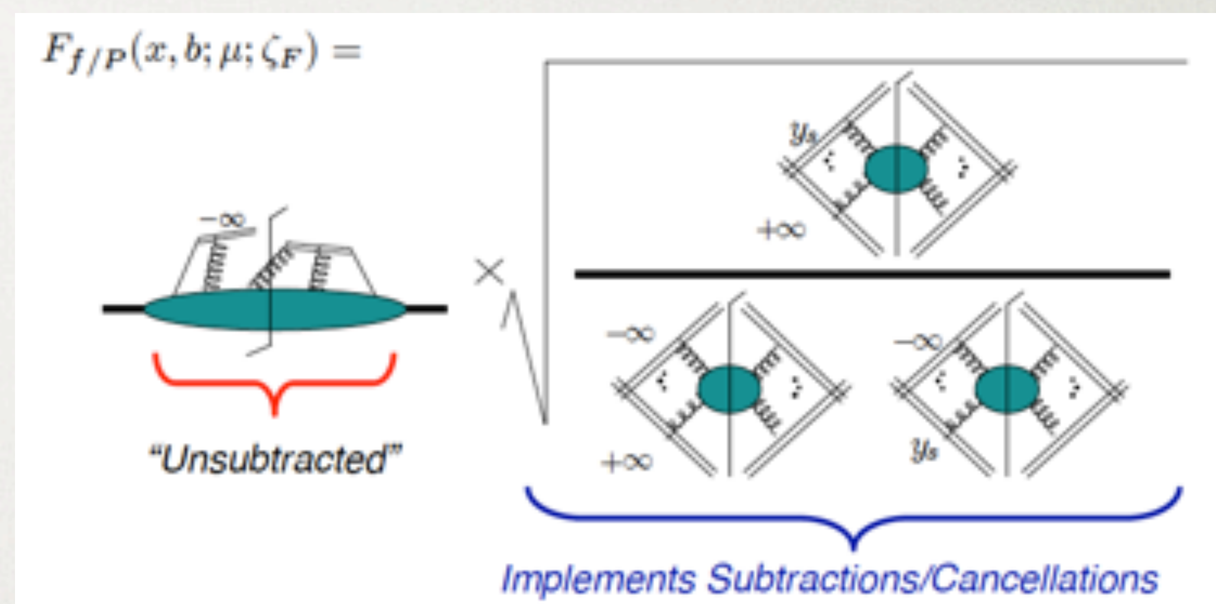
μ = renorm./fact. scale

ζ_F = regulator for rapidity divergences (do not cancel as in coll. case but they cancel in $W^{\mu\nu}$)

<u>Exact TMD definition beyond tree-level:</u>

1) Wilson lines are off the light cone
 ➙ ζ_F regulates light cone divergences
 ➙ "unsubtracted " TMD

2) "Soft factors" implemented



TMD factorization formalism cont'ed

evolution equations in ζ (in b space)

$$\frac{\partial \ln \tilde{F}(x, b_T, \mu, \zeta)}{\partial \ln \sqrt{\zeta}} = \tilde{K}(b_T, \mu)$$ ⟵ CSS kernel

anomalous dimensions

$$\frac{d\tilde{K}(b_T, \mu)}{d \ln \mu} = -\gamma_K(g(\mu))$$

$$\frac{d \ln \tilde{F}(x, b_T, \mu, \zeta)}{d \ln \mu} = \gamma_F(g(\mu), \zeta)$$

TMD:
Collins 2011
Rogers, Aybat 2011
Aybat, Collins, Qiu, Rogers 2011

final solution

$$f_1(x, k_T; Q) = \frac{1}{2\pi} \int d^2 b_T e^{-ik_T \cdot b_T} [C \otimes f_1](x, b_T)\, e^{-S'(b_T, Q)}\, e^{-S'_{\rm NP}(x, b_T, Q, \alpha_i)}$$

PDF

pQCD

non-pert. input

TMD factorization formalism cont'ed

evolution equations in ζ

(in b space)

$$\frac{\partial \ln \tilde{F}(x, b_T, \mu, \zeta)}{\partial \ln \sqrt{\zeta}} = \tilde{K}(b_T, \mu)$$ ← CSS kernel

anomalous dimensions

$$\frac{d\tilde{K}(b_T, \mu)}{d \ln \mu} = -\gamma_K(g(\mu))$$

$$\frac{d \ln \tilde{F}(x, b_T, \mu, \zeta)}{d \ln \mu} = \gamma_F(g(\mu), \zeta)$$

TMD:
Collins 2011
Rogers, Aybat 2011
Aybat, Collins, Qiu, Rogers 2011

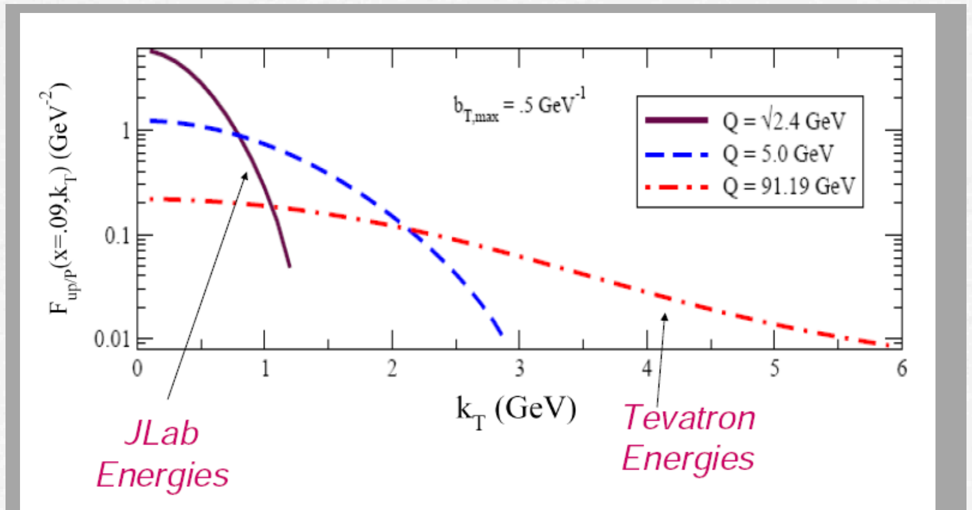
final solution

$$f_1(x, k_T; Q) = \frac{1}{2\pi} \int d^2 b_T e^{-i k_T \cdot b_T} [C \otimes f_1](x, b_T)\, e^{-S'(b_T, Q)}\, e^{-S'_{\rm NP}(x, b_T, Q, \alpha_i)}$$

PDF

pQCD

non-pert. input



Gaussian k_T tail appropriate only in restricted energy range

$<k_T^2>$ can depend on x

unpol. TMD f_1 from DY data

$$f_1(x, k_T; Q) = \frac{1}{2\pi} \int d^2 b_T e^{-i k_T \cdot b_T} [C \otimes f_1](x, b_T)\, e^{-S'(b_T, Q)}\, e^{-S'_{\rm NP}(x, b_T, Q, \alpha_i)}$$

small b_T (matching) large b_T

perturbative (prescription) non-perturbative

$$b_*(b_T) = \frac{b_T}{\sqrt{1 + b_T^2/b_{max}^2}}$$

unpol. TMD f_1 from DY data

$$f_1(x,k_T;Q) = \frac{1}{2\pi}\int d^2b_T e^{-ik_T\cdot b_T}[C\otimes f_1](x,b_T)\, e^{-S'(b_T,Q)}\, e^{-S'_{\rm NP}(x,b_T,Q,\alpha_i)}$$

small b_T (matching) large b_T
perturbative prescription non-perturbative

$$b_*(b_T) = \frac{b_T}{\sqrt{1+b_T^2/b_{max}^2}}$$

extracted from fits

BLNY fits

Landry, Brock, Nadolsky, Yuan, PRD67 (03)

Experiment	Reference	Reaction	$\sqrt{S}$ (GeV)	δN_{exp}
R209	[14]	$p+p\to\mu^+\mu^-+X$	62	10%
E605	[15]	$p+Cu\to\mu^+\mu^-+X$	38.8	15%
E288	[16]	$p+Cu\to\mu^+\mu^-+X$	27.4	25%
CDF-Z (Run-0)	[17]	$p+\bar{p}\to Z+X$	1800	–
DØ -Z (Run-1)	[18]	$p+\bar{p}\to Z+X$	1800	4.3%
CDF-Z (Run-1)	[19]	$p+\bar{p}\to Z+X$	1800	3.9%

COMPASS, E906, NICA

RHIC

from Bacchetta's talk

unpol. TMD f_1 from DY data

$$f_1(x,k_T;Q) = \frac{1}{2\pi}\int d^2b_T e^{-ik_T\cdot b_T}[C\otimes f_1](x,b_T)\, e^{-S'(b_T,Q)}\, e^{-S'_{\mathrm{NP}}(x,b_T,Q,\alpha_i)}$$

small b_T perturbative (matching prescription) large b_T non-perturbative

$$b_*(b_T) = \frac{b_T}{\sqrt{1+b_T^2/b_{max}^2}}$$

extracted from fits

BLNY fits

Landry, Brock, Nadolsky, Yuan, PRD67 (03)

DY (+ Z prod.) is major source of information for TMD

Experiment	Reference	Reaction	$\sqrt{S}$ (GeV)	δN_{exp}
R209	[14]	$p+p\to\mu^+\mu^-+X$	62	10%
E605	[15]	$p+Cu\to\mu^+\mu^-+X$	38.8	15%
E288	[16]	$p+Cu\to\mu^+\mu^-+X$	27.4	25%
CDF-Z (Run-0)	[17]	$p+\bar{p}\to Z+X$	1800	–
DØ -Z (Run-1)	[18]	$p+\bar{p}\to Z+X$	1800	4.3%
CDF-Z (Run-1)	[19]	$p+\bar{p}\to Z+X$	1800	3.9%

COMPASS, E906, NICA

RHIC

from Bacchetta's talk

TMD factorization formalism : another prescription

QCD current

$$J^\mu = \sum_q e_q \bar{\psi}\gamma^\mu\psi$$

SCET-q_T current

$$J^\mu = C(Q^2/\mu^2)\sum_q e_q \bar{\xi}_{\bar{n}} W_{\bar{n}}^T S_{\bar{n}}^{T\dagger}\gamma^\mu S_n^T W_n^{T\dagger}\xi_n$$

decouple collinear $W_n^{T\dagger}\xi_n$ fields from soft S_n^T fields

Collins:

regulate rapidity divergences going off LC and using proper soft factors depending on rapidity cut-offs

$$\tilde{F}_{f/P}(x,b;\zeta_A,\mu) = \boxed{\frac{\tilde{F}^{\mathrm{unsub}}_{f/P}(x,b;\mu)}{\sqrt{\tilde{S}(b;+\infty,-\infty)}}}\,\boxed{\sqrt{\frac{\tilde{S}(b;+\infty,y_n)}{\tilde{S}(b;y_n,-\infty)}}}$$

TMD depends on regulator ζ

$$\frac{\partial \ln \tilde{F}(x,b_T,\mu,\zeta)}{\partial \ln\sqrt{\zeta}} = \tilde{K}(b_T,\mu)$$

gives CSS evolution eq.

TMD factorization formalism : another prescription

QCD current

$$J^\mu = \sum_q e_q \bar{\psi} \gamma^\mu \psi$$

SCET-q_T current

$$J^\mu = C(Q^2/\mu^2) \sum_q e_q \bar{\xi}_{\bar{n}} W_{\bar{n}}^T S_{\bar{n}}^{T\dagger} \gamma^\mu S_n^T W_n^{T\dagger} \xi_n$$

decouple collinear $W_n^{T\dagger} \xi_n$ fields from soft S_n^T fields

Collins:
regulate rapidity divergences going off LC and using proper soft factors depending on rapidity cut-offs

$$\tilde{F}_{f/P}(x, b; \zeta_A, \mu) = \boxed{\frac{\tilde{F}^{\rm unsub}_{f/P}(x, b; \mu)}{\sqrt{\tilde{S}(b; +\infty, -\infty)}}} \cancel{\sqrt{\frac{\tilde{S}(b; +\infty, y_n)}{\tilde{S}(b; y_n, -\infty)}}}$$

TMD depends on regulator ζ

$$\frac{\partial \ln \tilde{F}(x, b_T, \mu, \zeta)}{\partial \ln \sqrt{\zeta}} = \tilde{K}(b_T, \mu)$$

gives CSS evolution eq.

new approach: reabsorb soft factor in definition of TMD → no dependence on ζ
regulate divergencies staying on LC with regulator Δ

$$\frac{d \ln \tilde{F}_{f/P}}{d \ln \Delta} = 0$$

simple evol. eq. at 2 loops

from Garcia's talk

TMD another prescription cont'ed

$\mathrm{TMD}_{SCET} = \mathrm{TMD}_{Collins}$ with $\zeta \leftrightarrow Q$

still a non-perturbative part dependent on b*, to be fitted to data

$\int \mathrm{TMD}_{SCET}\; dk_T = \mathrm{PDF}$ (but the bare one!)

from Garcia's talk

TMD another prescription cont'ed

$\mathrm{TMD}_{SCET} = \mathrm{TMD}_{Collins}$ with $\zeta \leftrightarrow Q$

still a non-perturbative part dependent on b*, to be fitted to data

$\int \mathrm{TMD}_{SCET}\, dk_T = \mathrm{PDF}$ (but the bare one!)

different soft S_n^T fields for different processes

→ unpol. TMD_{SCET} universal, pol. TMD_{SCET}?

from Garcia's talk

TMD another prescription cont'ed

$\mathrm{TMD}_{SCET} = \mathrm{TMD}_{Collins}$ with $\zeta \leftrightarrow Q$

still a non-perturbative part dependent on b*, to be fitted to data

$\int \mathrm{TMD}_{SCET}\, dk_T = \mathrm{PDF}$ (but the bare one!)

different soft S_n^T fields for different processes

$\rightarrow$ unpol. TMD_{SCET} universal, pol. TMD_{SCET} ?

in b space $W^{\mu\nu} = H(Q^2/\mu^2)\, \tilde{F}_{f/P}(x_1, b; Q^2, \mu)\, \tilde{F}_{\bar{f}/\bar{P}}(x_2, b; Q^2, \mu)$

no soft factor

RGE $\dfrac{d\ln W^{\mu\nu}}{d\ln\mu} = 0 = \gamma_H + \gamma_n + \gamma_{\bar{n}}$

known at NNLO $\gamma_H = A(\alpha_s)\ln\dfrac{Q^2}{\mu^2} + B(\alpha_s)$

anomalous dim. of TMD known at NNLO !

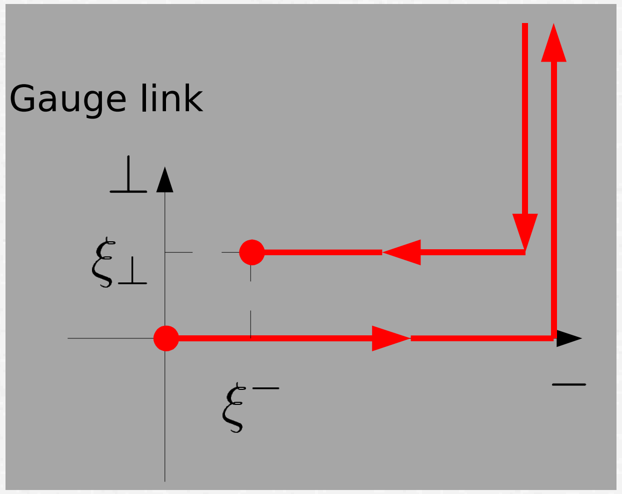
from Garcia's talk

TMD $\leftrightarrow$ PDF ?

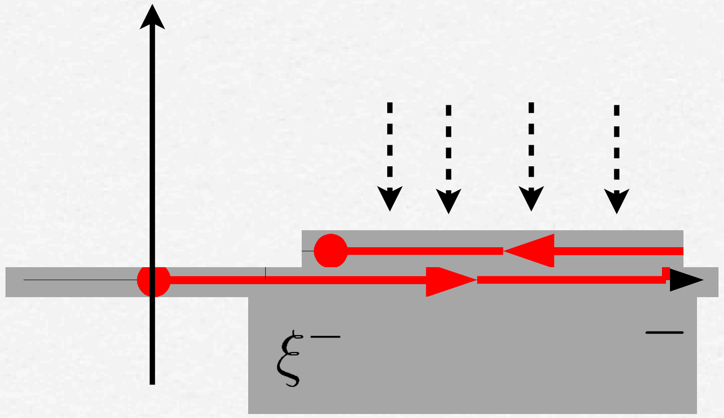
"TMD_{SCET}"

$\int \text{TMD}\; dk_T = \text{PDF}$ (bare)

TMD



PDF



from Prokudin's talk

TMD ↔ PDF ?

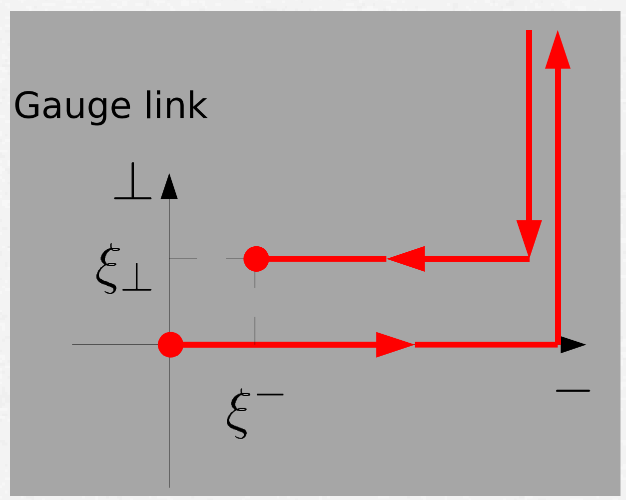
"TMD_{SCET}"

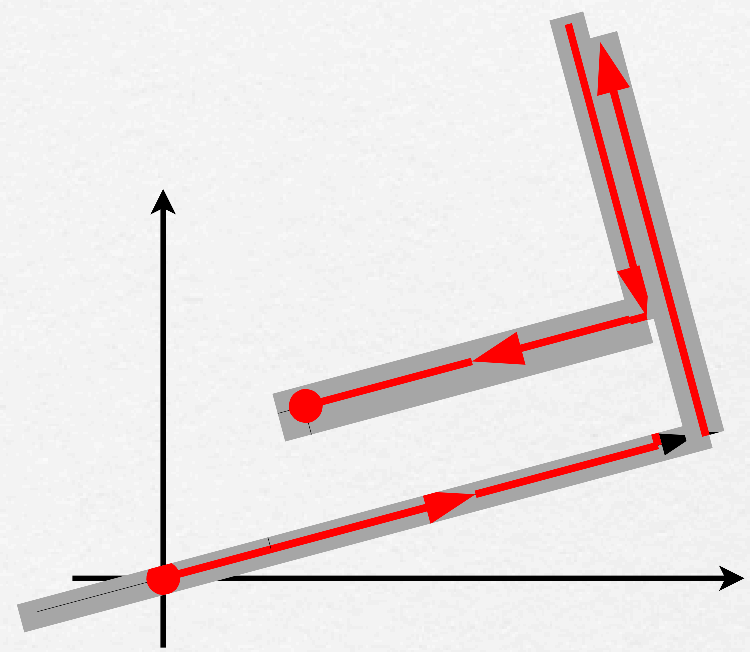
$\int \text{TMD}\ dk_T = \text{PDF}$ (bare)

"$\text{TMD}_{Collins}$"

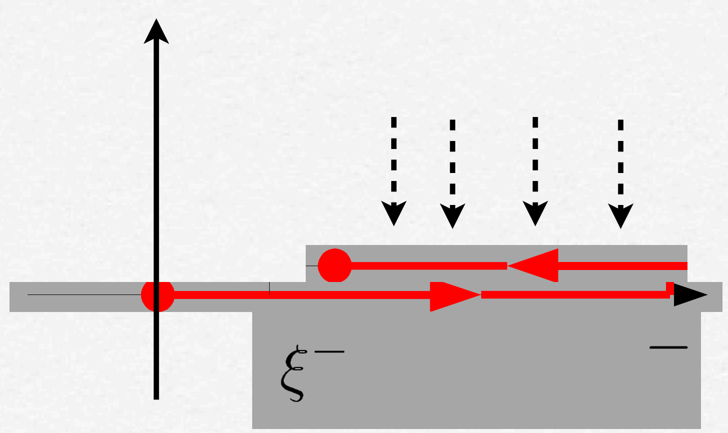
$\int \text{TMD}\ dk_T \neq \text{PDF}$

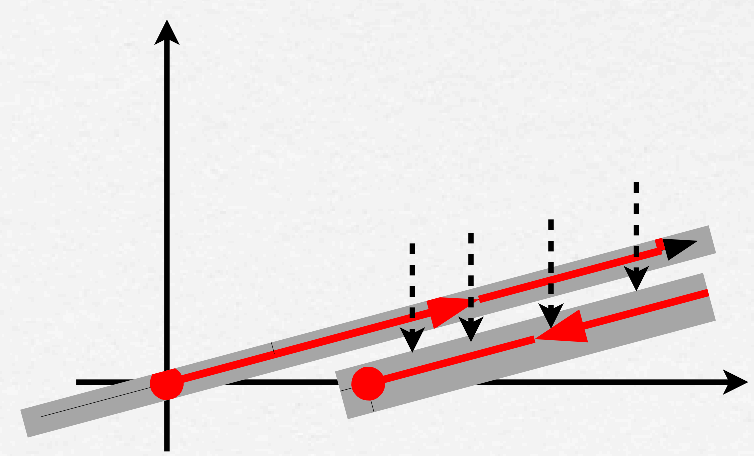
TMD





PDF



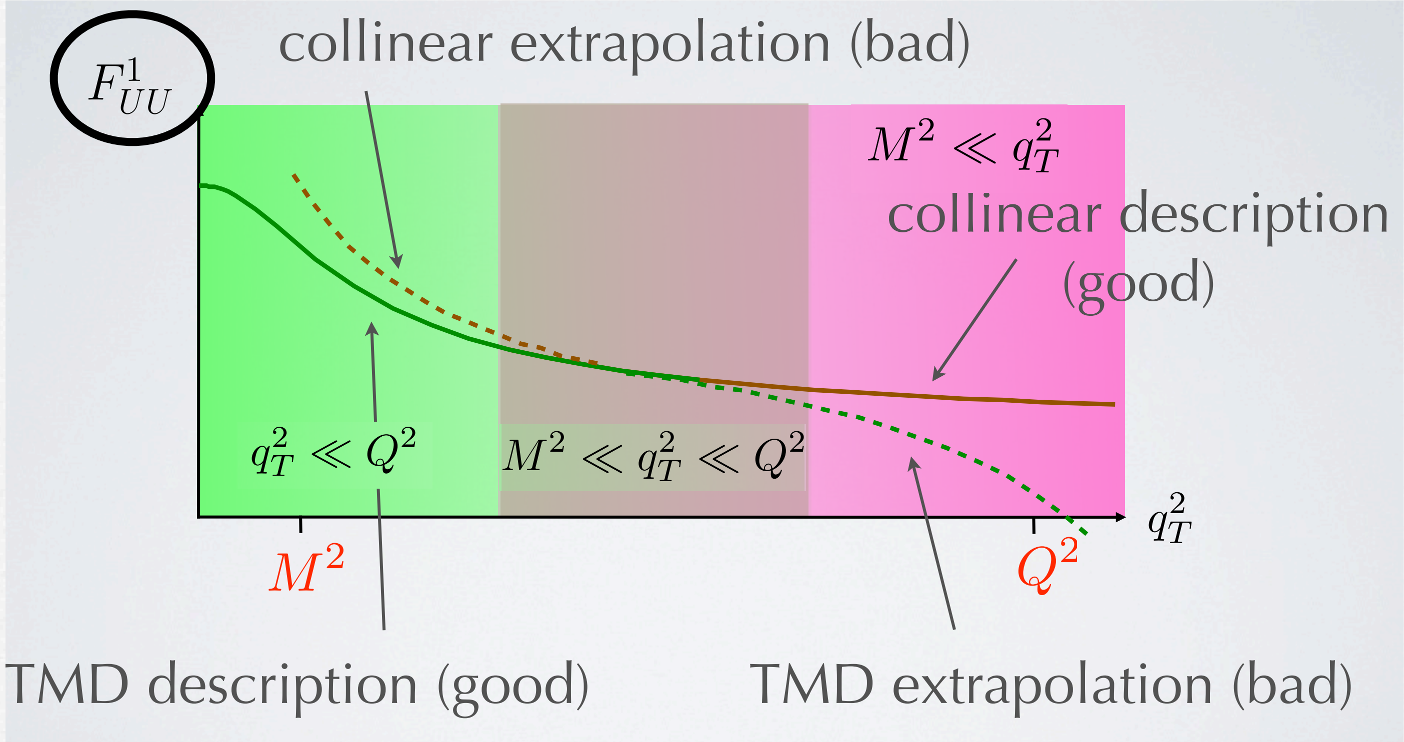


from Prokudin's talk

TMD $\leftrightarrow$ PDF cont'ed

$$
\begin{aligned}
\frac{\mathrm{d}^6\sigma}{\mathrm{d}^4q\,\mathrm{d}\Omega} = \frac{\alpha_{\mathrm{em}}^2}{6sQ^2}\Big\{ &\Big[(1+\cos^2\theta)\,W_{UU}^1 + \sin^2\theta\,W_{UU}^2 + \sin 2\theta\,\cos\phi\,W_{UU}^{\cos\phi} + \sin^2\theta\,\cos 2\phi\,W_{UU}^{\cos 2\phi}\Big] \\
&+S_{1T}\,\Big[\sin\phi_{S_1}\,\Big((1+\cos^2\theta)\,W_{TU}^1 + \sin^2\theta\,W_{TU}^2 + \sin 2\theta\,\cos\phi\,W_{TU}^{\cos\phi} + \sin^2\theta\,\cos 2\phi\,W_{TU}^{\cos 2\phi}\Big) \\
&+\cos\phi_{S_1}\,(\sin 2\theta\,\sin\phi\,W_{TU}^{\sin\phi} + \sin^2\theta\,\sin 2\phi\,W_{TU}^{\sin 2\phi})\Big] + (1\leftrightarrow 2, T\leftrightarrow U) \\
&+S_{1T}\,S_{2T}\,\Big[\cos(\phi_{S_1}+\phi_{S_2})\,\Big((1+\cos^2\theta)\,W_{TT}^1 + \sin^2\theta\,W_{TT}^2 \\
&+\sin 2\theta\,\cos\phi\,W_{TT}^{\cos\phi} + \sin^2\theta\,\cos 2\phi\,W_{TT}^{\cos 2\phi}\Big) \\
&+\cos(\phi_{S_1}-\phi_{S_2})\,\Big((1+\cos^2\theta)\,\overline{W}_{TT}^1 + \sin^2\theta\,\overline{W}_{TT}^2 + \sin 2\theta\,\cos\phi\,\overline{W}_{TT}^{\cos\phi} + \sin^2\theta\,\cos 2\phi\,\overline{W}_{TT}^{\cos 2\phi}\Big) \\
&+\sin(\phi_{S_1}+\phi_{S_2})\,(\sin 2\theta\,\sin\phi\,W_{TT}^{\sin\phi} + \sin^2\theta\,\sin 2\phi\,W_{TT}^{\sin 2\phi}) \\
&+\sin(\phi_{S_1}-\phi_{S_2})\,(\sin 2\theta\,\sin\phi\,\overline{W}_{TT}^{\sin\phi} + \sin^2\theta\,\sin 2\phi\,\overline{W}_{TT}^{\sin 2\phi})\Big] + \ldots\Big\}\,.
\end{aligned}
$$

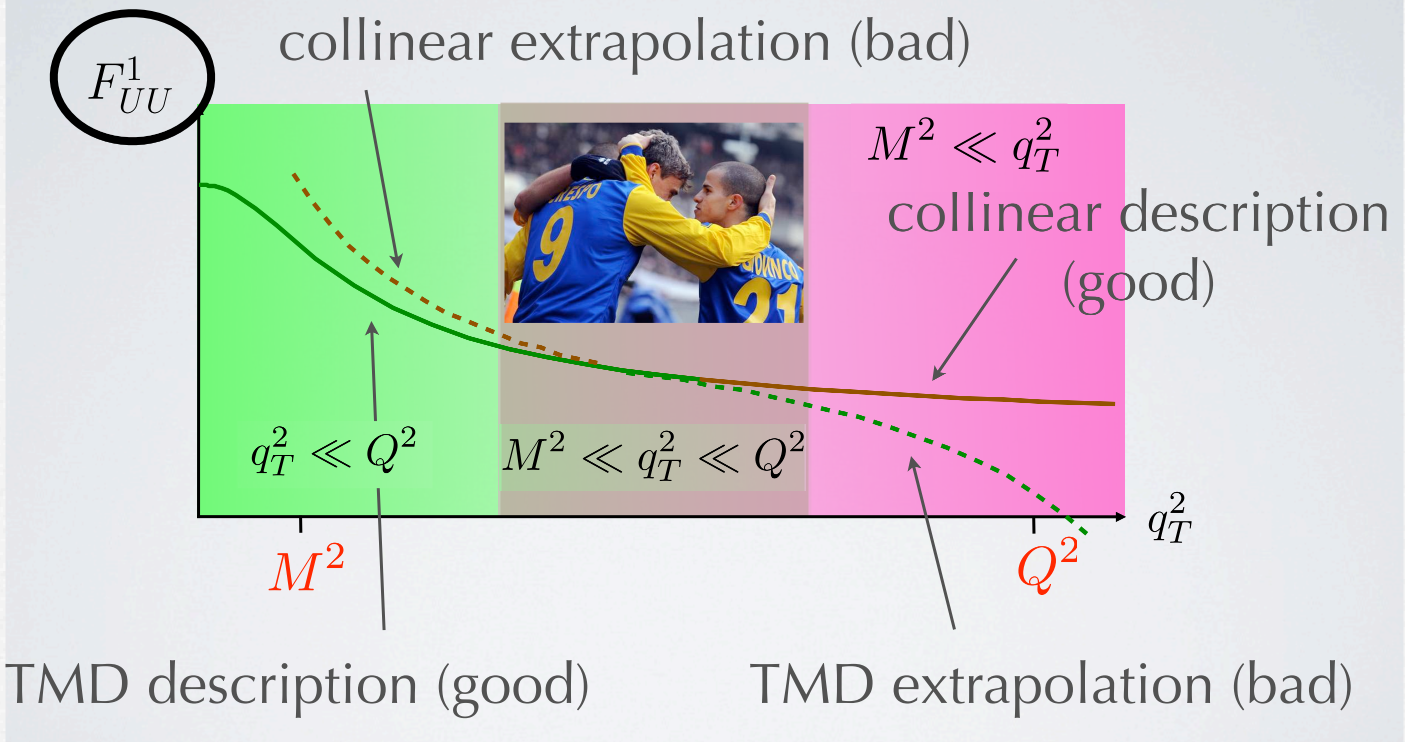
TMD $\leftrightarrow$ PDF cont'ed



from Bacchetta's talk

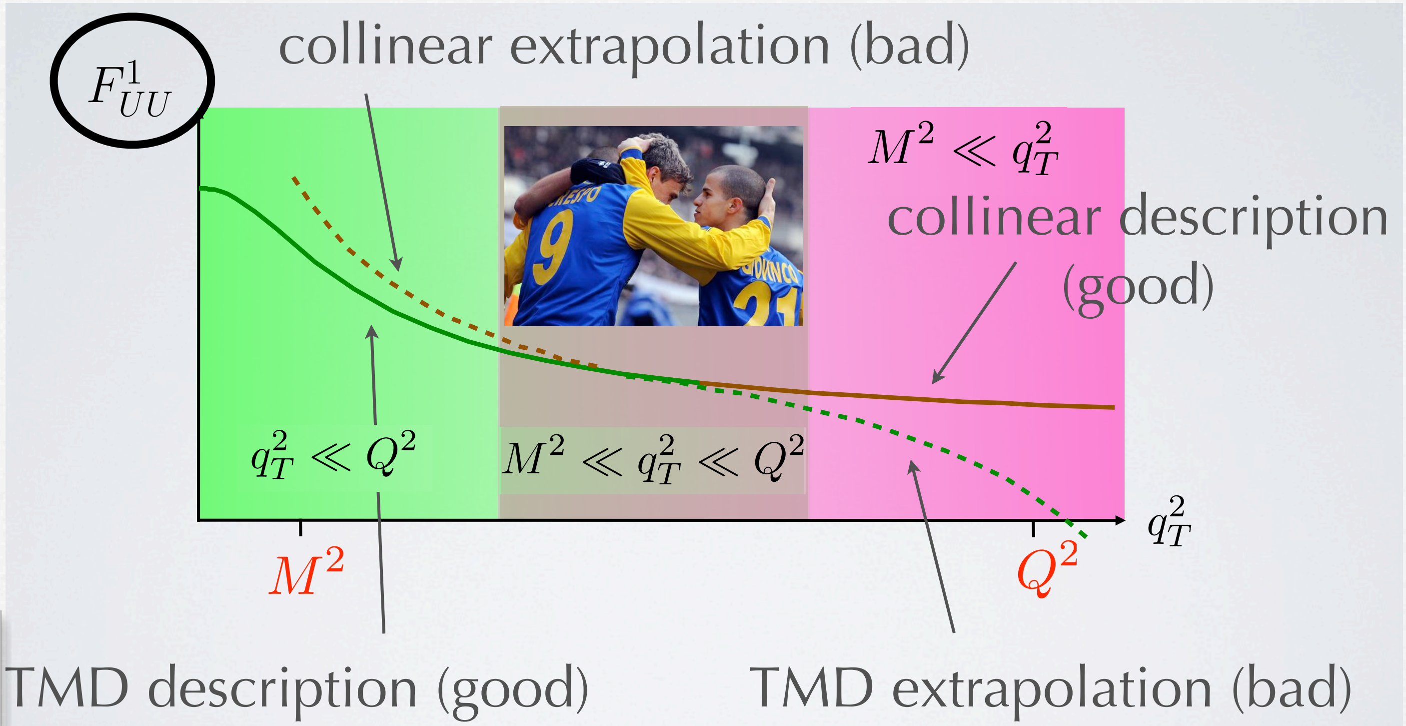
TMD ↔ PDF cont'ed

matching
is ok

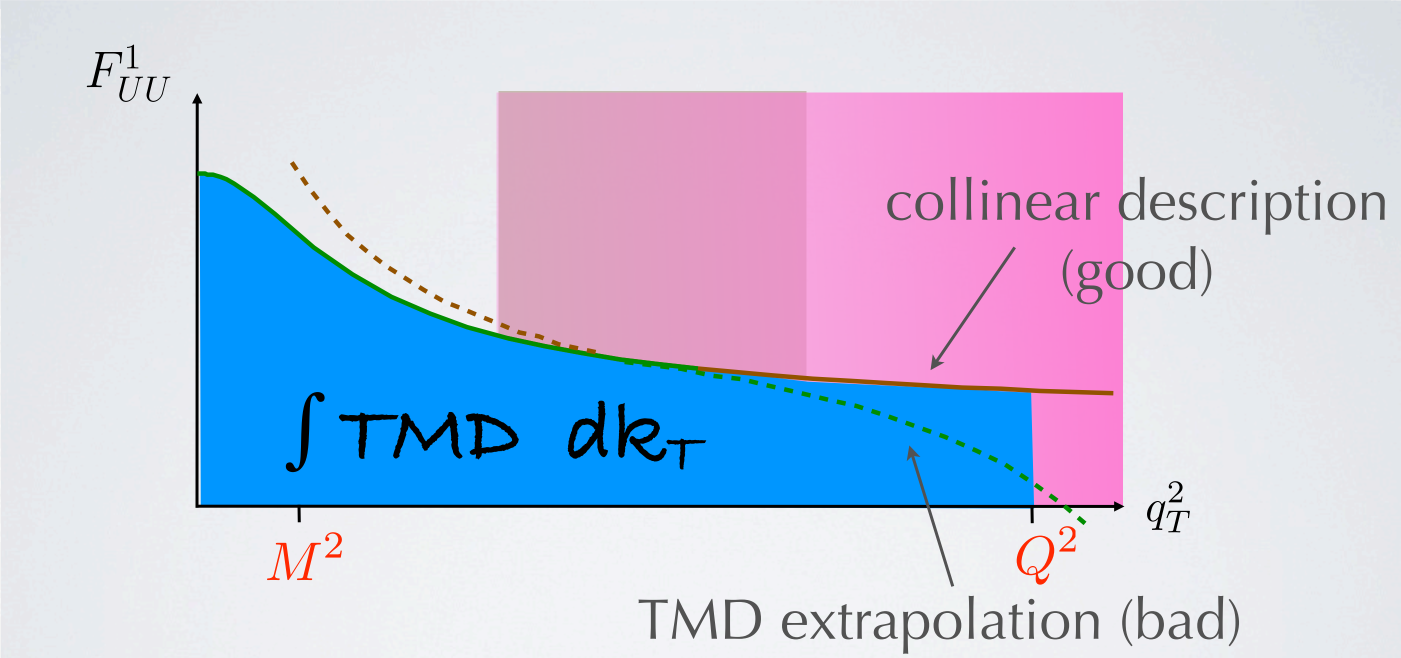


from Bacchetta's talk

TMD ↔ PDF cont'ed



matching
is ok



but
integrate TMD up to high q_T where TMD extrapolation is bad

$\Downarrow$

expect

$\int$ TMD $dk_T \neq$ PDF

from Bacchetta's talk

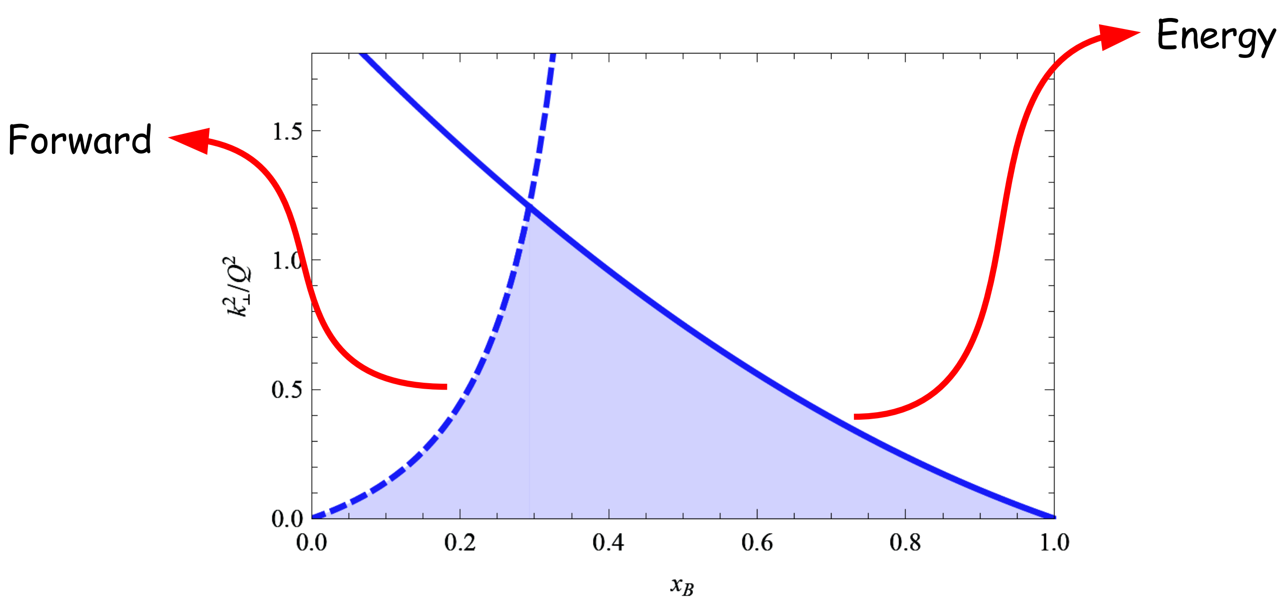
TMD $\leftrightarrow$ PDF cont'd

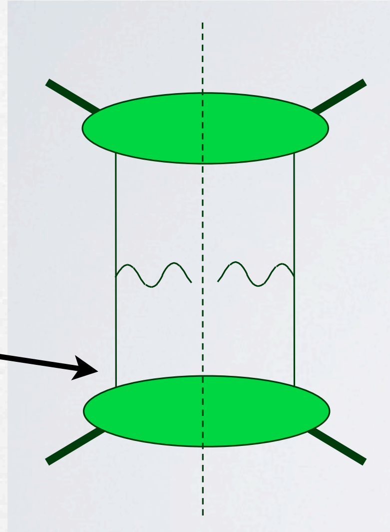
- By requiring the energy of the parton to be smaller than the energy of its parent hadron, we have

$$\boxed{k_\perp^2 \le (2 - x_B)(1 - x_B)Q^2 \quad , \quad 0 < x_B < 1}$$

- By requiring the parton not to move backward with respect to its parent hadron, we find

$$\boxed{k_\perp^2 \le \frac{x_B(1 - x_B)}{(1 - 2x_B)^2} Q^2 \ , \ x_B < 0.5}$$





from Melis' talk
(see also Zavada)

The unpolarized DY : pQCD

$$\frac{\mathrm{d}^6\sigma_{UU}}{\mathrm{d}^4q\,\mathrm{d}\Omega} = \frac{\alpha_{\mathrm{em}}^2}{6sQ^2}\,\Big\{(1+\cos^2\theta)\,W_{UU}^1 + \sin^2\theta\,W_{UU}^2$$

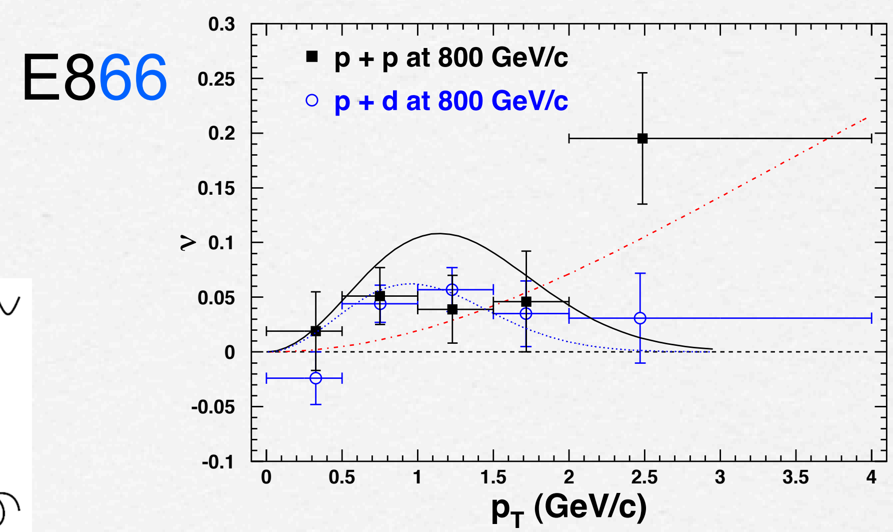
$$+\sin 2\theta\,\cos\phi\,W_{UU}^{\cos\phi} + \sin^2\theta\cos 2\phi\,W_{UU}^{\cos 2\phi}\Big\}$$

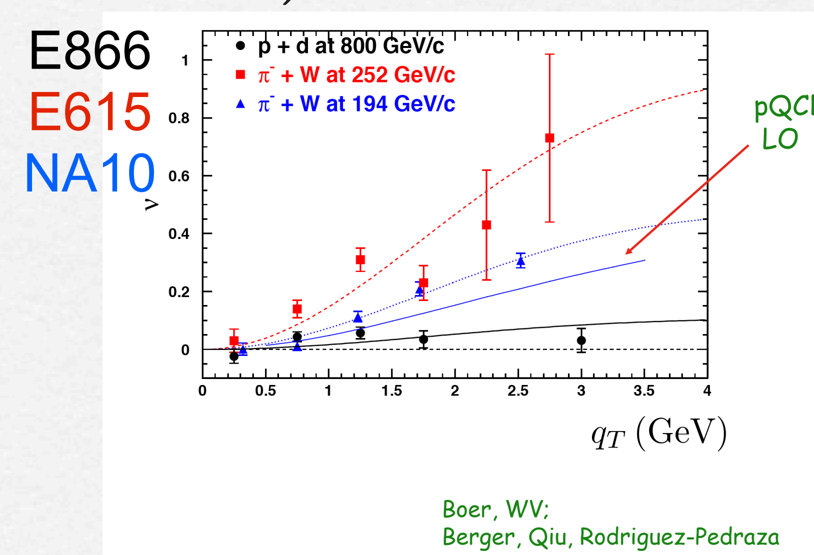
$$\frac{1}{N_{\mathrm{tot}}}\frac{\mathrm{d}N}{\mathrm{d}\Omega} = \frac{3}{4\pi}\,\frac{1}{\lambda+3}\left(1+\lambda\,\cos^2\theta + \mu\,\sin 2\theta\,\cos\phi + \frac{\nu}{2}\,\sin^2\theta\cos 2\phi\right)$$

The unpolarized DY : pQCD

$$\frac{d^6\sigma_{UU}}{d^4q\,d\Omega} = \frac{\alpha_{\rm em}^2}{6sQ^2}\Big\{(1+\cos^2\theta)\,W_{UU}^1 + \sin^2\theta\,W_{UU}^2 + \sin 2\theta\,\cos\phi\,W_{UU}^{\cos\phi} + \sin^2\theta\cos 2\phi\,W_{UU}^{\cos 2\phi}\Big\}$$

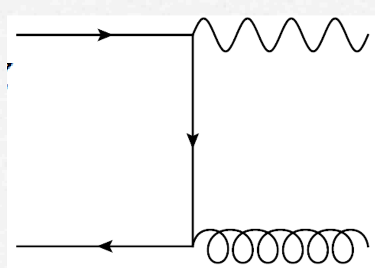
$$\frac{1}{N_{\rm tot}}\frac{dN}{d\Omega} = \frac{3}{4\pi}\frac{1}{\lambda+3}\left(1+\lambda\,\cos^2\theta + \mu\,\sin 2\theta\,\cos\phi + \frac{\nu}{2}\,\sin^2\theta\cos 2\phi\right)$$



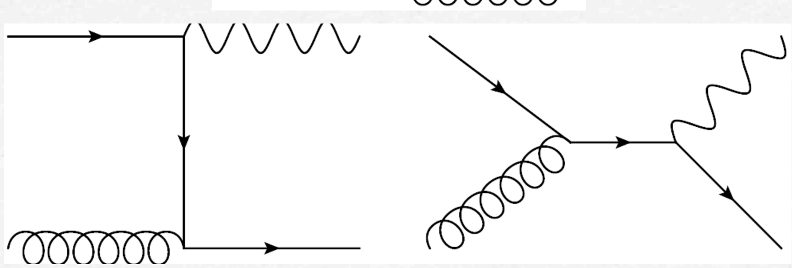


pQCD

gluon bremsstrahlung



QCD Compton



\+ resummation of Sudakov log's

$\log^k(Q^2/q_T^2)$

from Vogelsang's and Barone's talk

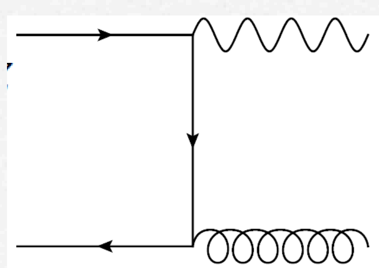
The unpolarized DY : pQCD

$$\frac{d^6\sigma_{UU}}{d^4q\, d\Omega} = \frac{\alpha_{em}^2}{6sQ^2}\left\{(1+\cos^2\theta)\, W_{UU}^1 + \sin^2\theta\, W_{UU}^2 + \sin 2\theta\, \cos\phi\, W_{UU}^{\cos\phi} + \sin^2\theta \cos 2\phi\, W_{UU}^{\cos 2\phi}\right\}$$

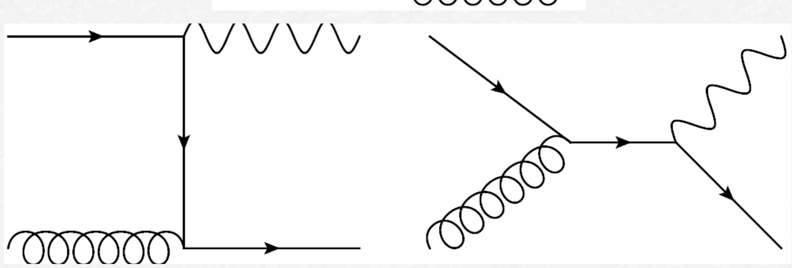
$$\frac{1}{N_{tot}}\frac{dN}{d\Omega} = \frac{3}{4\pi}\frac{1}{\lambda+3}\left(1 + \lambda\cos^2\theta + \mu\sin 2\theta\cos\phi + \frac{\nu}{2}\sin^2\theta\cos 2\phi\right)$$

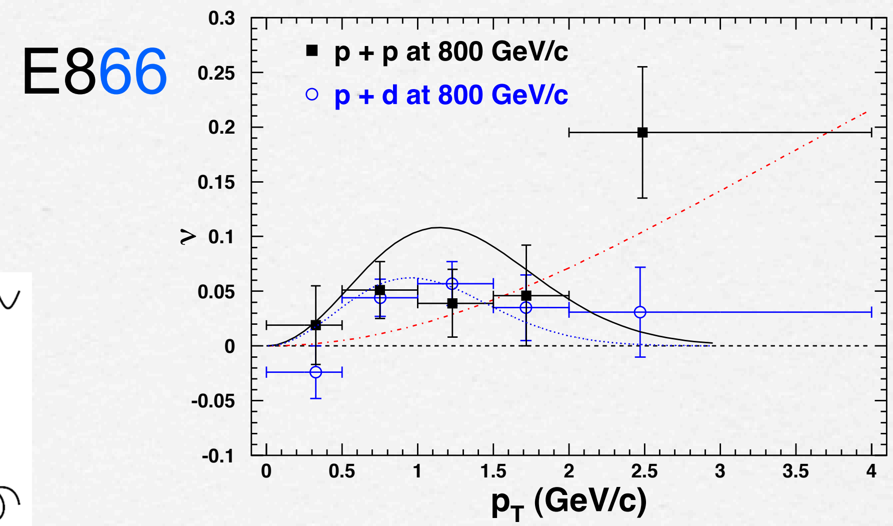
pQCD

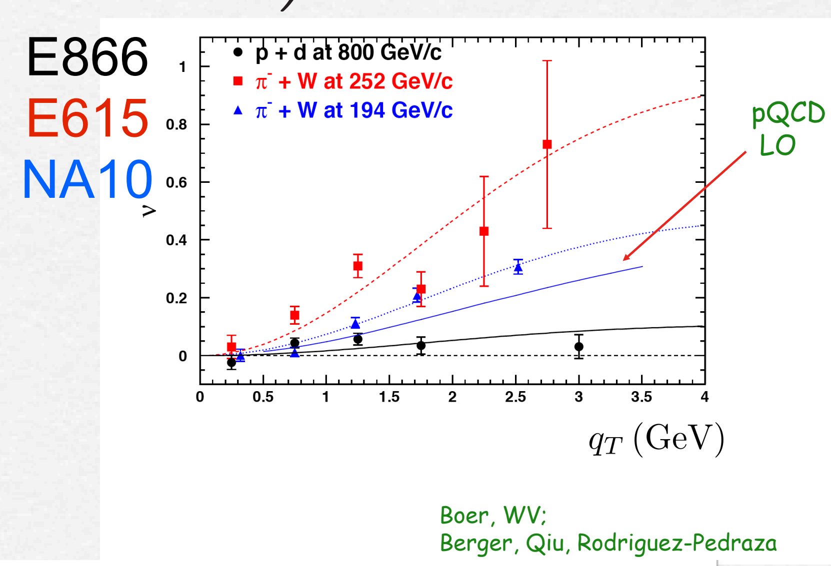
gluon bremsstrahlung



QCD Compton







partly accounts for ν_{pp}
but not for ν_{pD} and $\nu_{\pi N}$

\+ resummation of Sudakov log's
$\log^k(Q^2/q_T^2)$

from Vogelsang's and Barone's talk

The unpolarized DY : TMD

$\lambda \rightarrow \quad W_{UU}^{1} = \mathcal{C}\,[f_1\,\bar{f}_1]$

$$\mu \rightarrow \quad W_{UU}^{\cos\phi} = \frac{1}{Q}\,\mathcal{C}\left[[(\hat{\mathbf{q}}_T\cdot\mathbf{k}_{1T}) - (\hat{\mathbf{q}}_T\cdot\mathbf{k}_{2T})]\,f_1\,\bar{f}_1\right]$$ Cahn

$$+\frac{1}{Q}\,\mathcal{C}\left[\frac{(\hat{\mathbf{q}}_T\cdot\mathbf{k}_{1T})\mathbf{k}_{2T}^2 - (\hat{\mathbf{q}}_T\cdot\mathbf{k}_{2T})\mathbf{k}_{1T}^2}{2M_1M_2}\,h_1^{\perp}\,\bar{h}_1^{\perp}\right]$$ B.M.

$$\nu \rightarrow \quad W_{UU}^{\cos 2\phi} = \mathcal{C}\left[\frac{2(\hat{\mathbf{q}}_T\cdot\mathbf{k}_{1T})(\hat{\mathbf{q}}_T\cdot\mathbf{k}_{2T}) - \mathbf{k}_{1T}\cdot\mathbf{k}_{2T}}{M_1M_2}\,h_1^{\perp}\bar{h}_1^{\perp}\right]$$ B.M.

$$+\ \frac{1}{Q^2}\,\mathcal{C}\left[\left\{\frac{1}{2}[(\hat{\mathbf{q}}_T\cdot\mathbf{k}_{1T}) - (\hat{\mathbf{q}}_T\cdot\mathbf{k}_{2T})]^2 + 2\mathbf{k}_{1T}^2\mathbf{k}_{2T}^2\right\}\,f_1\,\bar{f}_1\right]$$ Cahn

+ unknown terms ....

from Barone's talk

The unpolarized DY : TMD

$\lambda \to \quad W_{UU}^{1} = \mathcal{C}\,[f_1 \bar{f}_1]$

$\mu \to \quad W_{UU}^{\cos\phi} = \frac{1}{Q}\,\mathcal{C}\left[[(\hat{\mathbf{q}}_T\cdot\mathbf{k}_{1T}) - (\hat{\mathbf{q}}_T\cdot\mathbf{k}_{2T})]\, f_1\,\bar{f}_1\right]$ Cahn

$+\frac{1}{Q}\,\mathcal{C}\left[\frac{(\hat{\mathbf{q}}_T\cdot\mathbf{k}_{1T})\mathbf{k}_{2T}^2 - (\hat{\mathbf{q}}_T\cdot\mathbf{k}_{2T})\mathbf{k}_{1T}^2}{2M_1M_2}\, h_1^\perp\,\bar{h}_1^\perp\right]$ B.M.

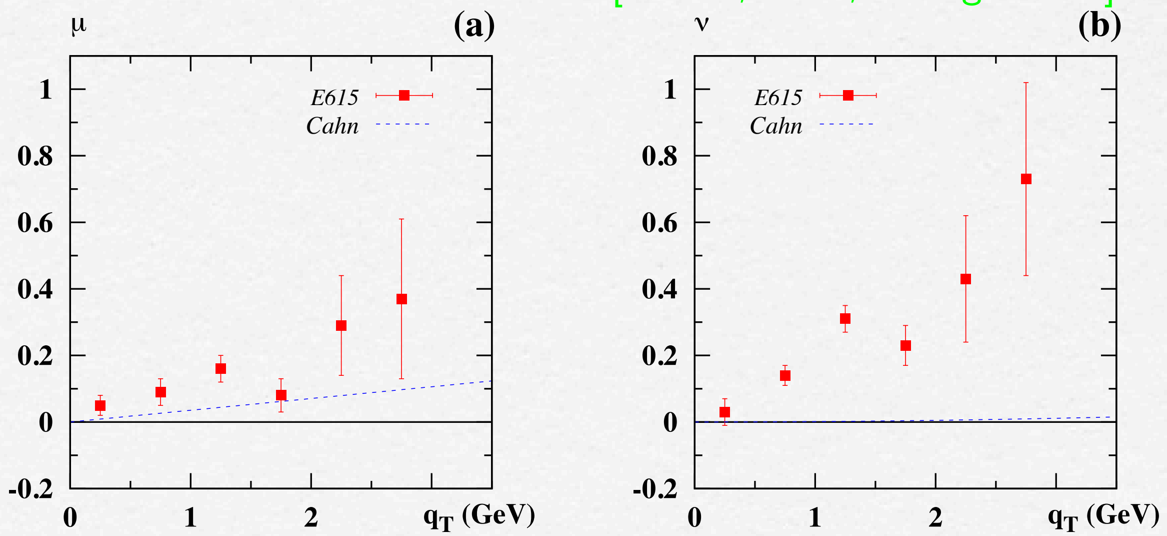
$\nu \to \quad W_{UU}^{\cos 2\phi} = \mathcal{C}\left[\frac{2(\hat{\mathbf{q}}_T\cdot\mathbf{k}_{1T})(\hat{\mathbf{q}}_T\cdot\mathbf{k}_{2T}) - \mathbf{k}_{1T}\cdot\mathbf{k}_{2T}}{M_1M_2}\, h_1^\perp \bar{h}_1^\perp\right]$ B.M.

$+\ \frac{1}{Q^2}\,\mathcal{C}\left[\left\{\frac{1}{2}[(\hat{\mathbf{q}}_T\cdot\mathbf{k}_{1T}) - (\hat{\mathbf{q}}_T\cdot\mathbf{k}_{2T})]^2 + 2\mathbf{k}_{1T}^2\mathbf{k}_{2T}^2\right\}\, f_1\,\bar{f}_1\right]$ Cahn

+ unknown terms ....

[Arnold, Metz, Schlegel 2009]

$\mu_{\text{Cahn}} \sim \frac{Q_T}{Q}\,\frac{\langle\mathbf{k}_{1T}^2\rangle - \langle\mathbf{k}_{2T}^2\rangle}{\langle Q_T^2\rangle}$



$\nu_{\text{Cahn}} \sim \frac{Q_T^2}{Q^2}\left(\frac{\langle\mathbf{k}_{1T}^2\rangle - \langle\mathbf{k}_{2T}^2\rangle}{\langle Q_T^2\rangle}\right)^2$

Cahn effect expected small

from Barone's talk

The unpolarized DY : parametrization of Boer-Mulders function

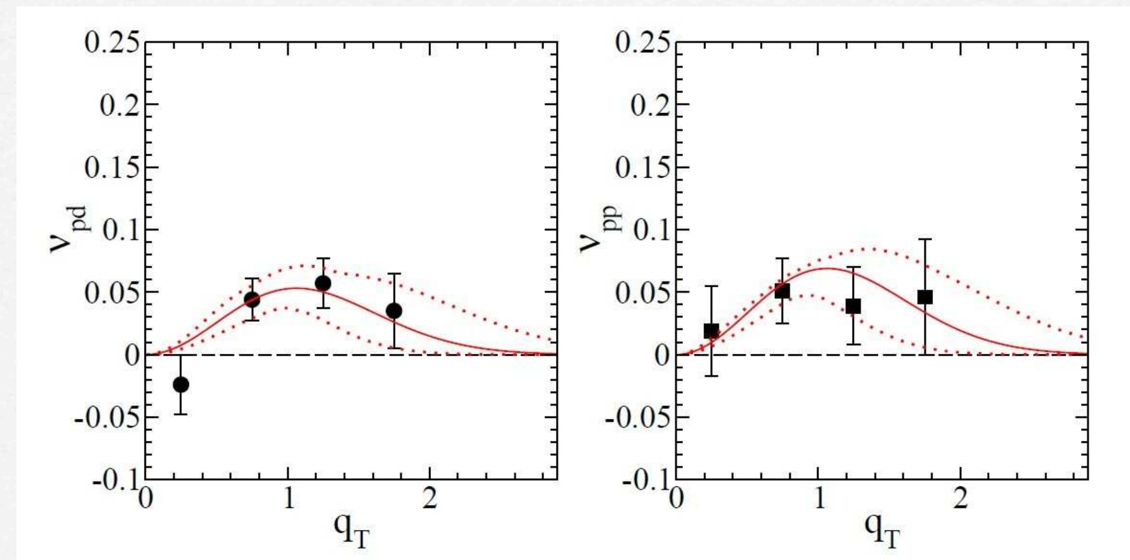
Zhang, ZL, Ma, Schmidt 08; ZL, Schmidt 10

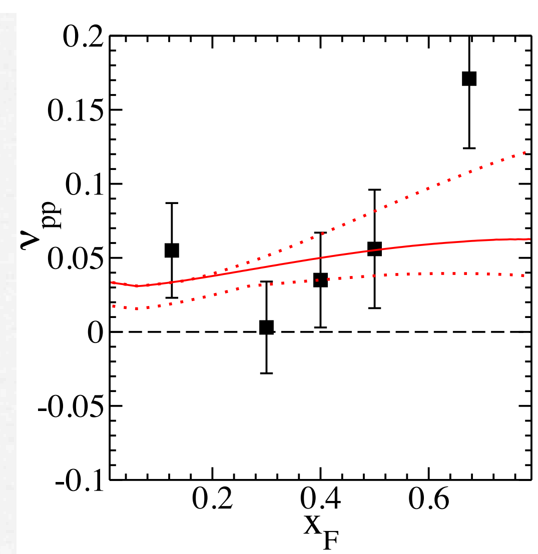
$$h_1^{\perp\, q}(x, \boldsymbol{p}_T^2) \;=\; h_1^{\perp\, q}(x)\,\frac{1}{\pi p_{bm}^2}\,\exp\left(-\frac{\boldsymbol{p}_T^2}{p_{bm}^2}\right)$$

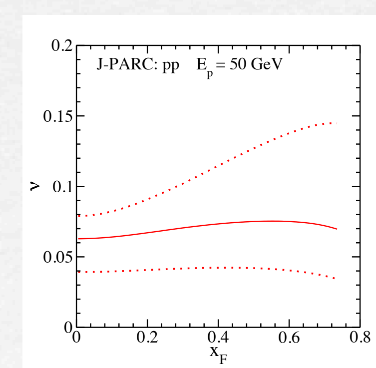
$$h_1^{\perp\, q}(x) \;=\; \omega H_q\, x^{c_q}\,(1-x)^b\, f_1^{\,q}(x)$$

$$h_1^{\perp\, \bar{q}}(x) = \frac{1}{\omega} H_{\bar{q}}\, x^{c_{\bar{q}}}\,(1-x)^b\, f_1^{\,\bar{q}}(x)$$

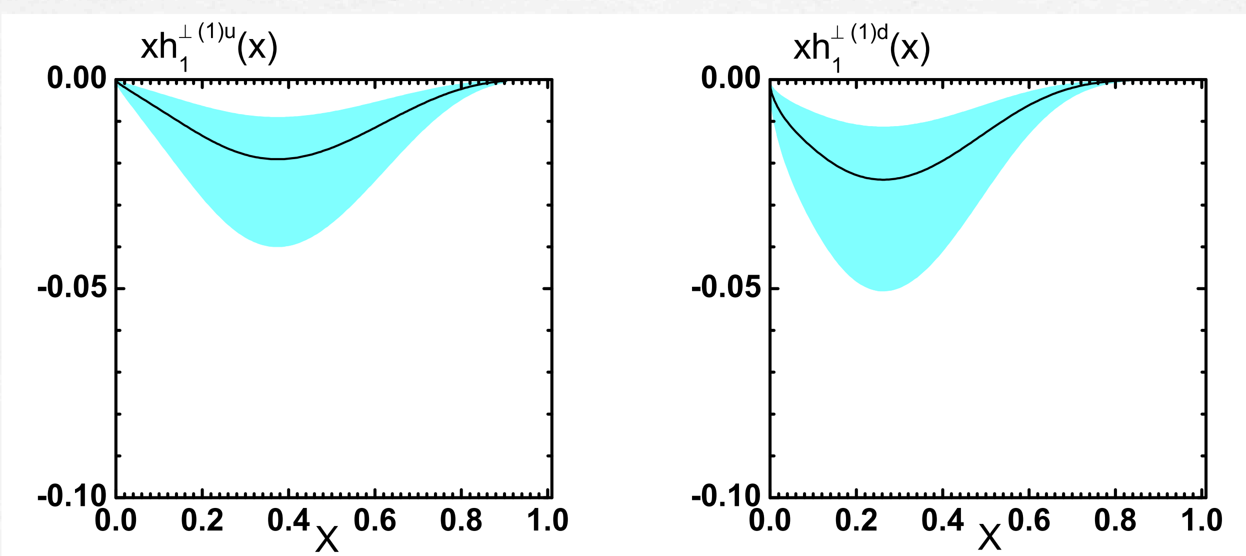
fitting the E866 data for ν







result



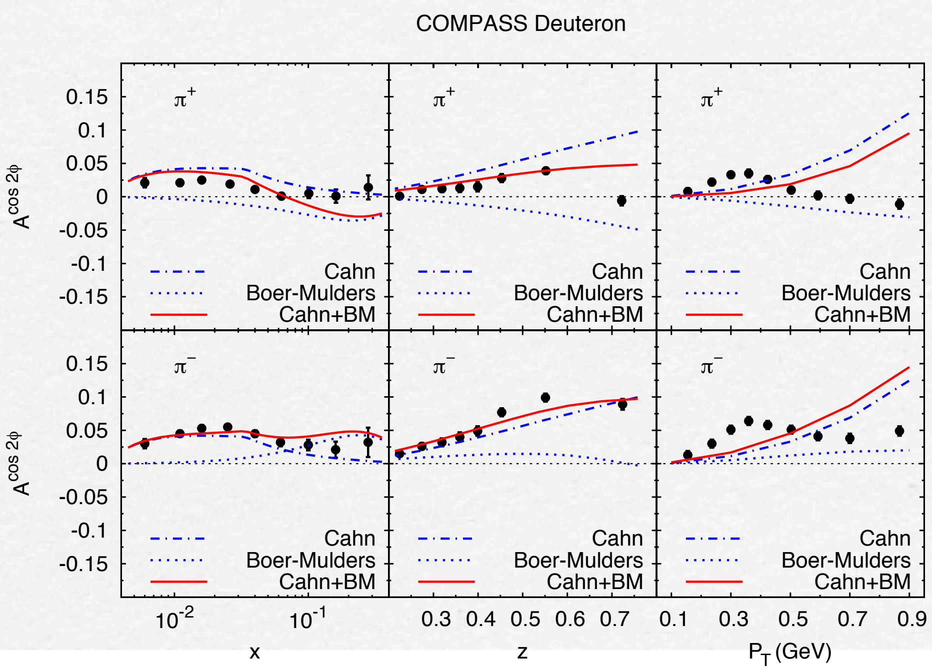
0.48<ω<2.1 normalization uncertainty

from Lu's talk

combined unpol. SIDIS - DY : parametrization of Boer-Mulders function

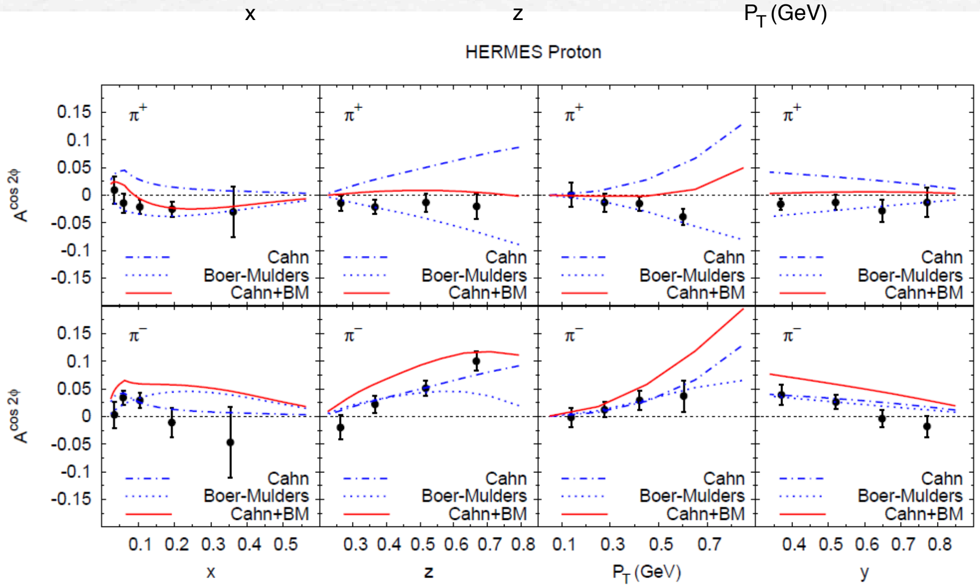
1st step : extract $h_1^{\perp u,d}$ from unpol. SIDIS $A_{UU}^{\cos 2\varphi}$

Anselmino et al., 09



COMPASS

$\langle k_\perp^2 \rangle$=0.25 (GeV/c)^2
$\langle p_\perp^2 \rangle$=0.20 (GeV/c)^2



HERMES

$\langle k_\perp^2 \rangle$=0.18 (GeV/c)^2
$\langle p_\perp^2 \rangle$=0.20 (GeV/c)^2

$$h_1^{\perp q}(x, k_\perp) = \lambda_q \, f_{1T}^{\perp q}(x, k_\perp)$$

$$\lambda_u = 2.0 \pm 0.1$$
$$\lambda_d = -1.11^{+0.00}_{-0.02}$$

$$\chi^2/d.o.f. = 2.41$$

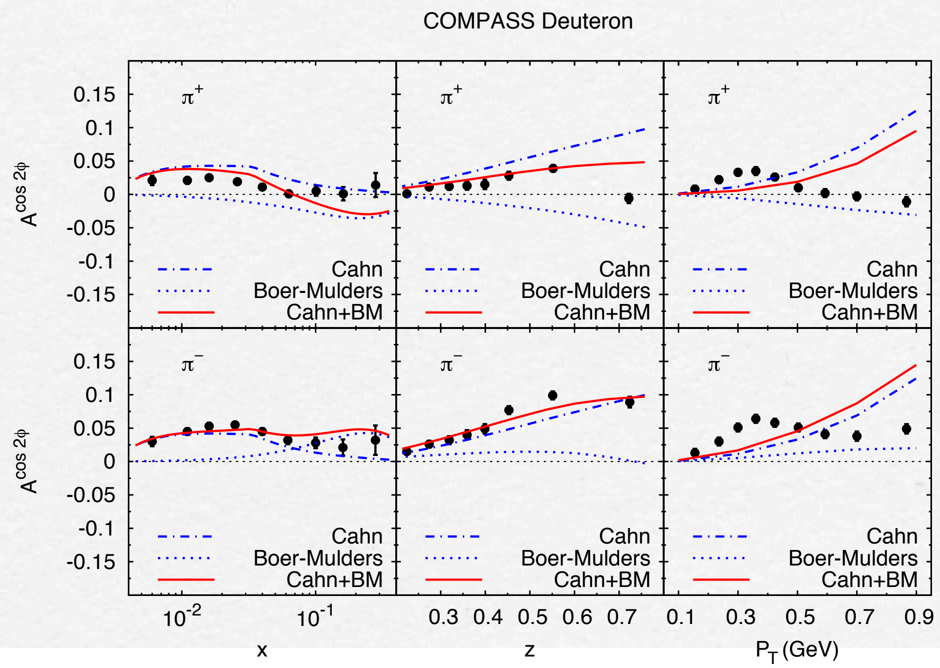
[VB, Ma, Melis, Prokudin (2008, 2010)]

from Melis' talk

combined unpol. SIDIS - DY : parametrization of Boer-Mulders function

1st step : extract $h_1^{\perp u,d}$ from unpol. SIDIS $A_{UU}^{\cos 2\varphi}$

Anselmino et al., 09



COMPASS

$\langle k_\perp^2\rangle = 0.25\ (GeV/c)^2$

$\langle p_\perp^2\rangle = 0.20\ (GeV/c)^2$

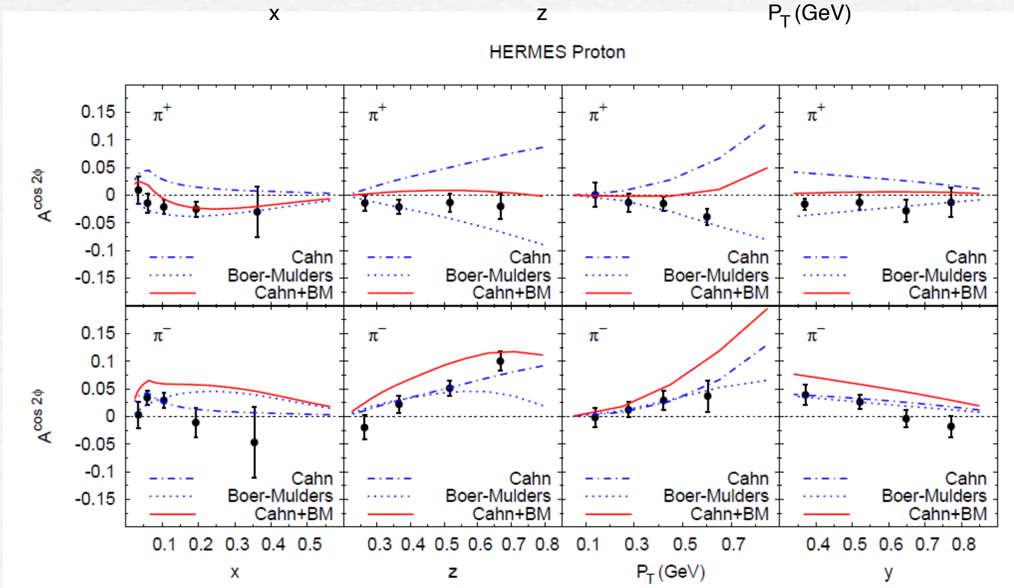
$$h_1^{\perp q}(x,k_\perp) = \lambda_q\, f_{1T}^{\perp q}(x,k_\perp)$$

$$\lambda_u = 2.0 \pm 0.1$$

$$\lambda_d = -1.11^{+0.00}_{-0.02}$$

$$\chi^2/d.o.f. = 2.41$$

[VB, Ma, Melis, Prokudin (2008, 2010)]



HERMES

$\langle k_\perp^2\rangle = 0.18\ (GeV/c)^2$

$\langle p_\perp^2\rangle = 0.20\ (GeV/c)^2$

sign and size as lattice and models

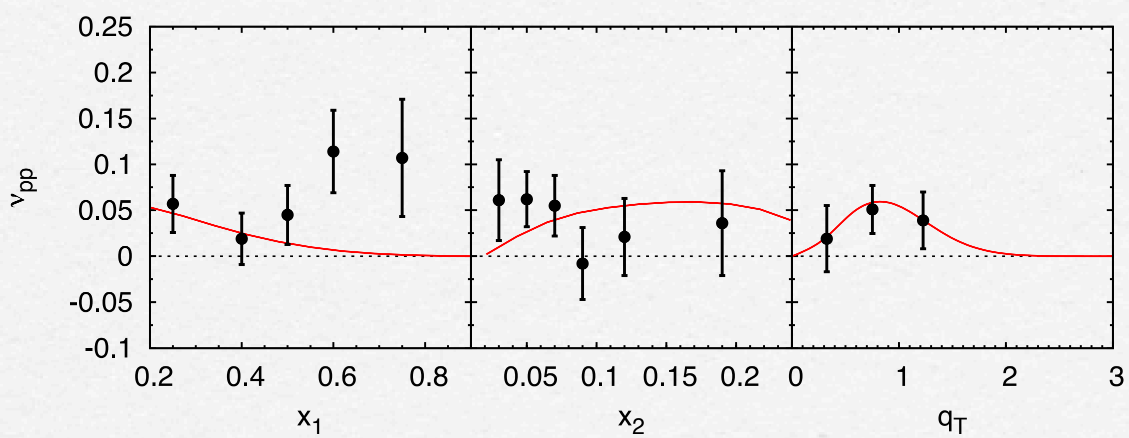
<u>Cahn effect very large</u>

from Melis' talk

combined unpol. SIDIS - DY : parametrization of Boer-Mulders function

2nd step : extract $h_1^{\perp \bar{u}, \bar{d}}$ from E866 pp and pD

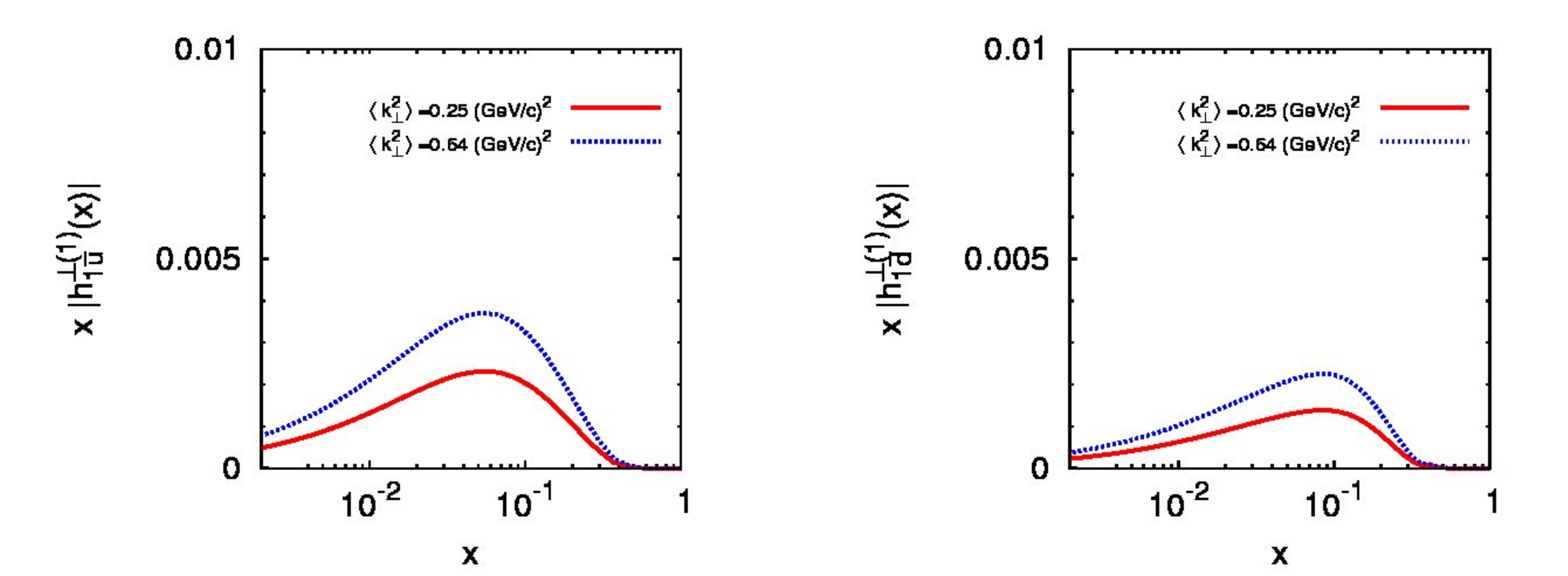
Anselmino et al., 09



$$h_1^{\perp \bar{q}}(x, k_\perp) = \lambda_{\bar{q}} f_{1T}^{\perp q}(x, k_\perp)$$

Fit I	Fit II
$\lambda_{\bar{u}} = 3.25 \pm 0.75$	$\lambda_{\bar{u}} = 5.5 \pm 1.5$
$\lambda_{\bar{d}} = -0.15 \pm 0.13$	$\lambda_{\bar{d}} = -0.25 \pm 0.20$
$\chi^2_{d.o.f} = 1.24$	$\chi^2_{d.o.f} = 1.24$
$\boxed{\langle k_\perp^2 \rangle = 0.25\ (\mathrm{GeV}/c)^2}$	$\boxed{\langle k_\perp^2 \rangle \simeq 0.64\ (\mathrm{GeV}/c)^2}$

result

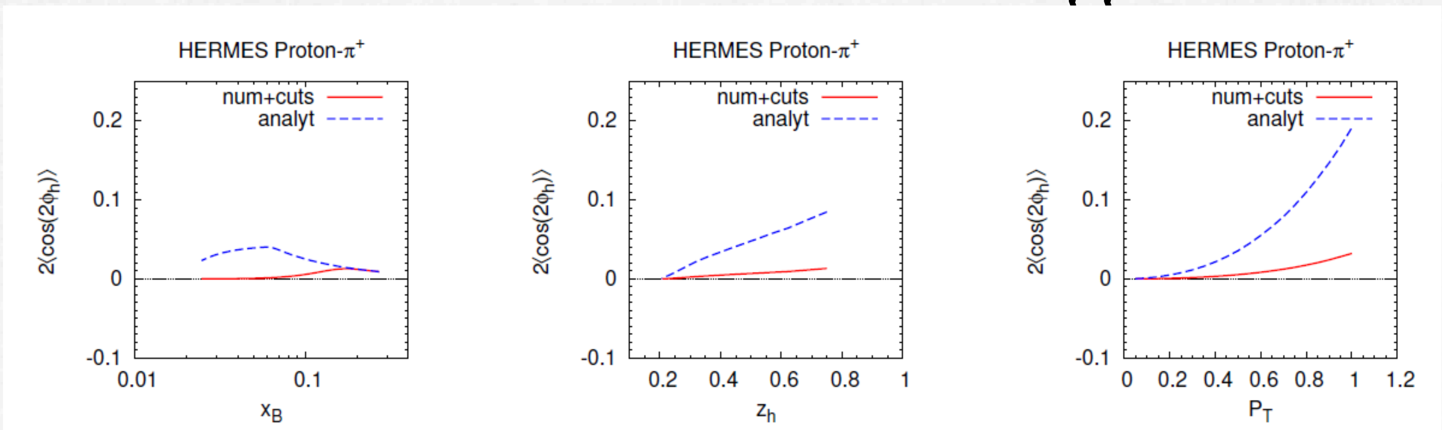


[VB, Melis, Prokudin 2010]

from Melis' talk

combined unpol. SIDIS - DY : parametrization of Boer-Mulders function

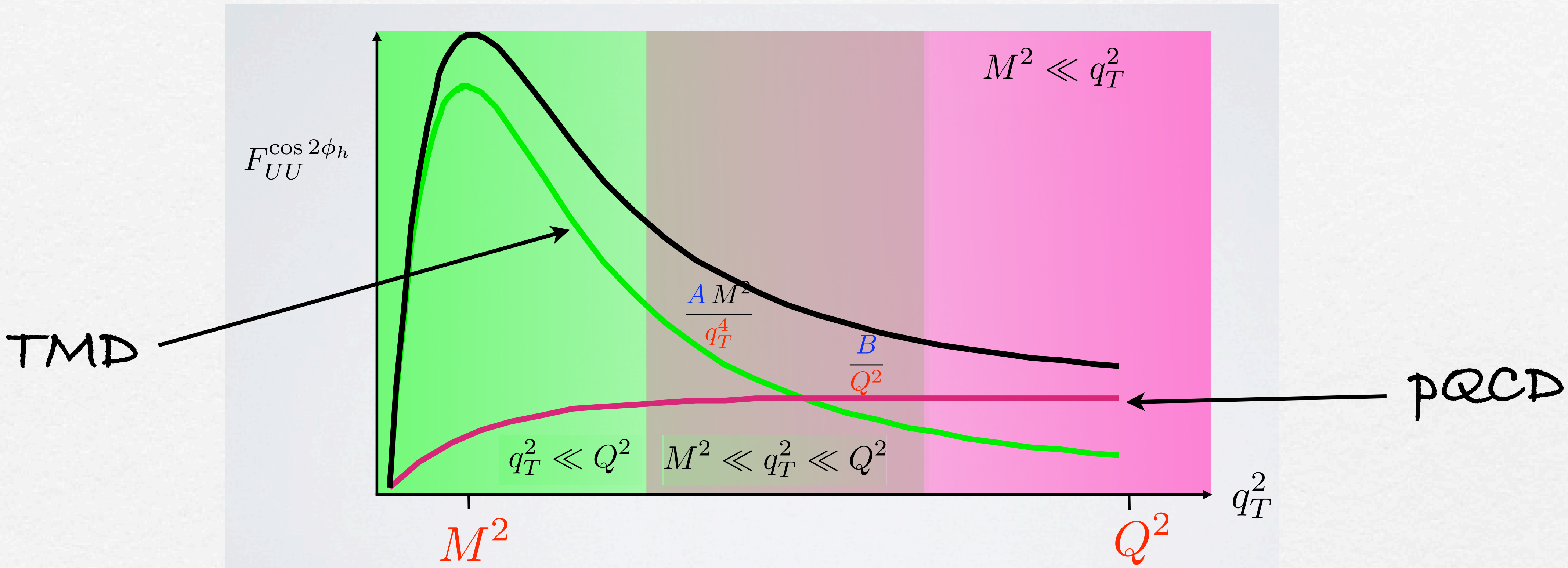
- new SIDIS data on <cos2φ> from
  - arXiv:1204.4161 $\Rightarrow$ redo the analysis
  - Sbrizzai, Transversity 2011
- results contaminated from huge Cahn effect in SIDIS
  if intrinsic $k_\perp^2$ is limited, then effect reduced



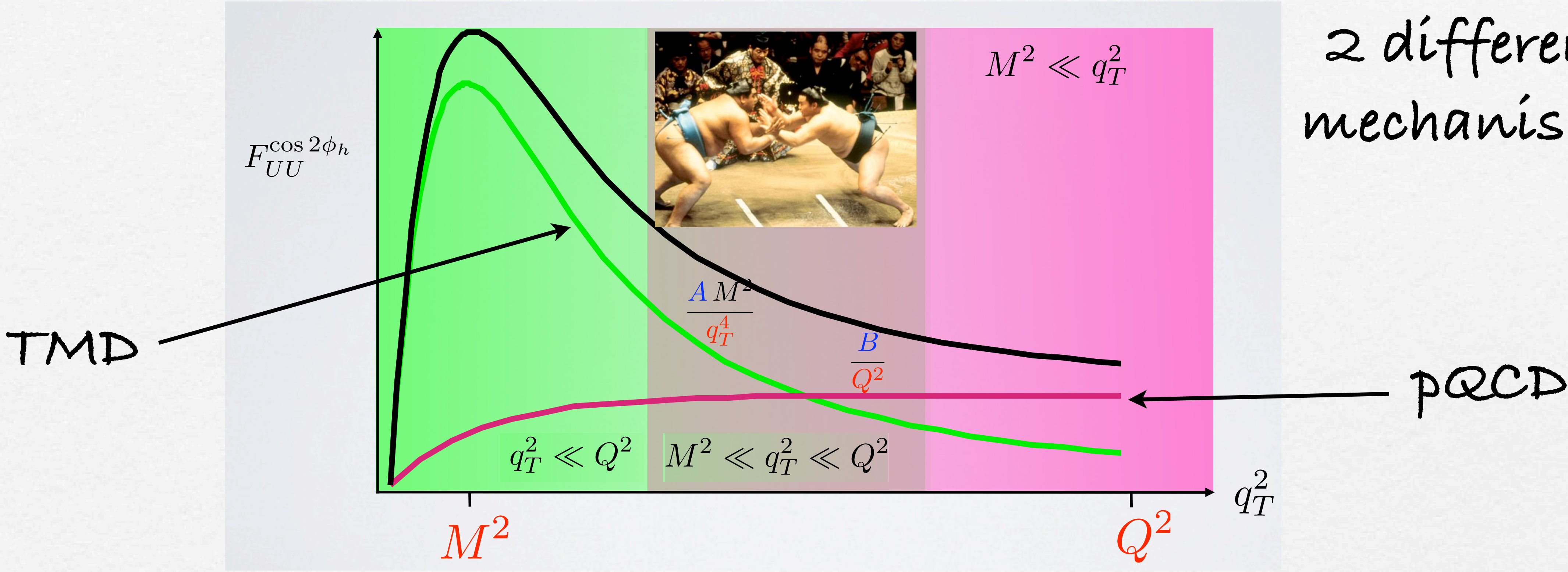
Boglione, Melis, Prokudin
Phys. Rev. D 84, 034033 (2011)

- TMD evolution missing
- further kinematic $1/Q^2$ and dynamical twist-4 terms
  in fact...

TMD → $A_{UU}^{\cos 2\phi}$ ← pQCD
expected mismatch

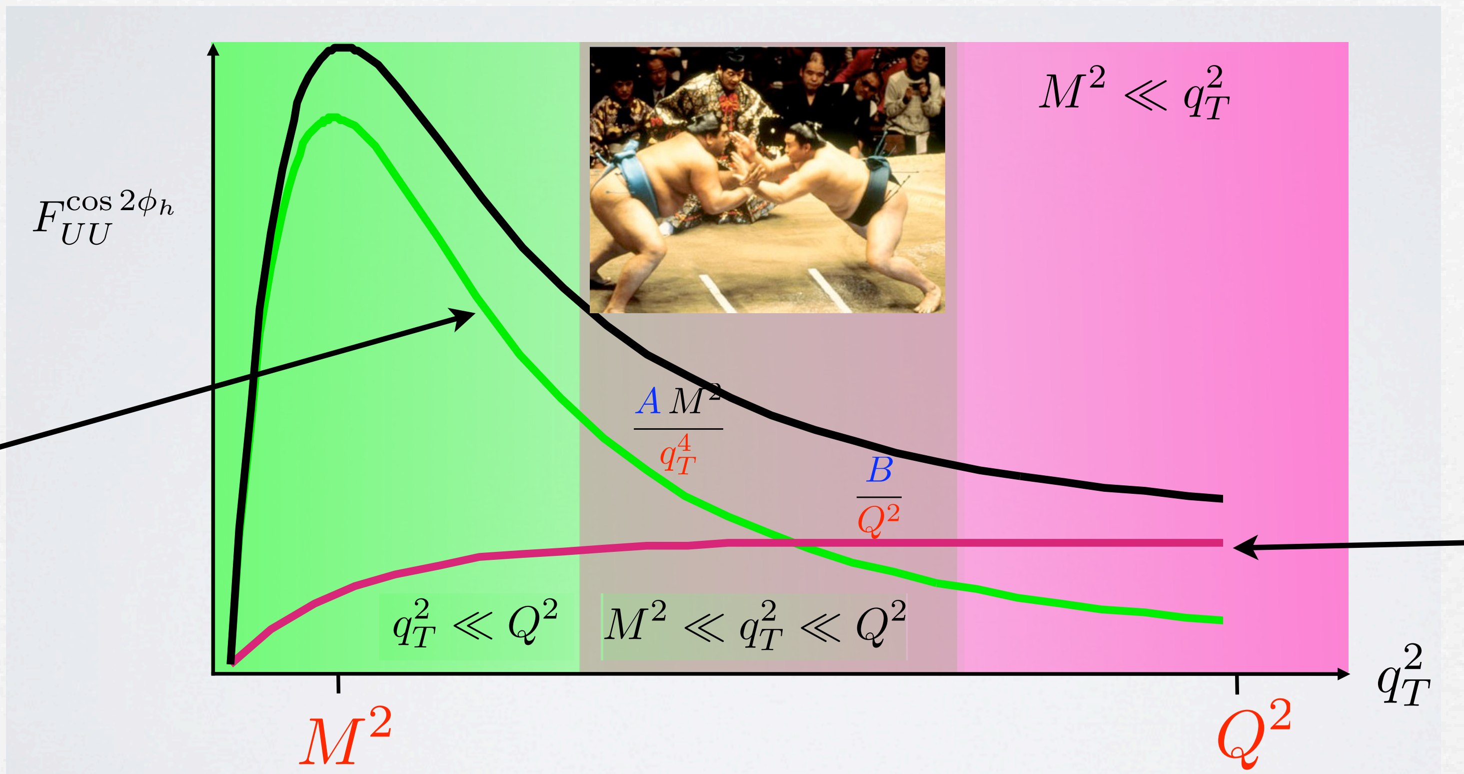


TMD → $A_{UU}^{\cos 2\phi}$ ← pQCD
expected mismatch



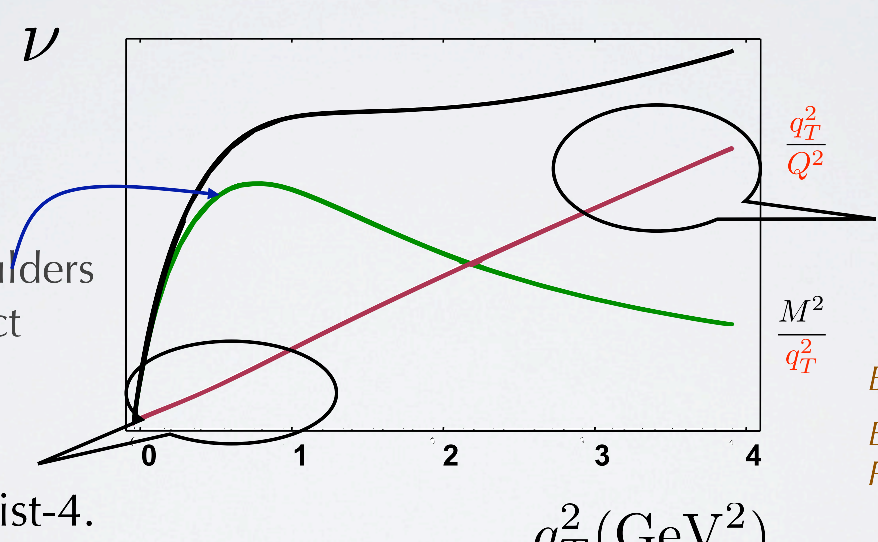
2 different mechanisms

TMD → $A_{UU}^{\cos 2\phi}$ ← pQCD
expected mismatch



2 different mechanisms

situation very complicated



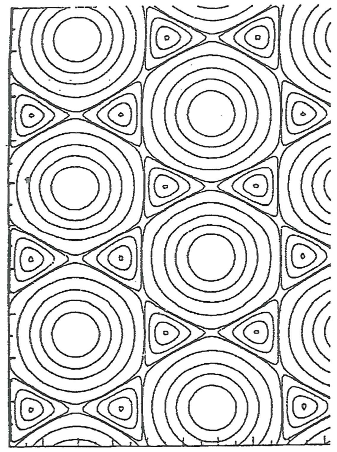
Can be calculated with pQCD. Resummation important.

Boer, Vogelsang, PRD74 (06)

Berger, Qiu, Rodrigues-Pedraza, PRD76 (07)

from Bacchetta's talk

The DY $A_{UU}^{\cos 2\phi}$: nonpert. QCD



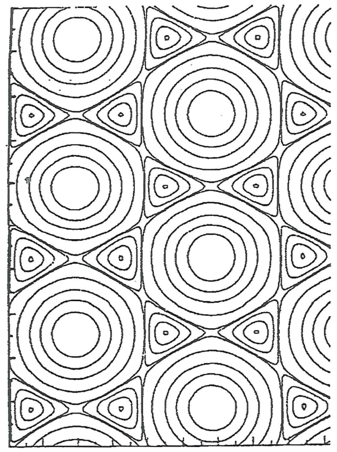
QCD vacuum

fluctuating vacuum
chromomagnetic field

- Deflection due to chromomagnetic Lorentz force
- Synchrotron emission of gluons and photons
- Spin-flip gluon synchrotron emission leading to a correlated polarisation of q and $\bar{q}$ (Chromomagnetic Sokolov-Ternov effect).

from Nachtmann's talk

The DY $A_{UU}^{\cos 2\phi}$: nonpert. QCD



QCD vacuum

fluctuating vacuum
chromomagnetic field

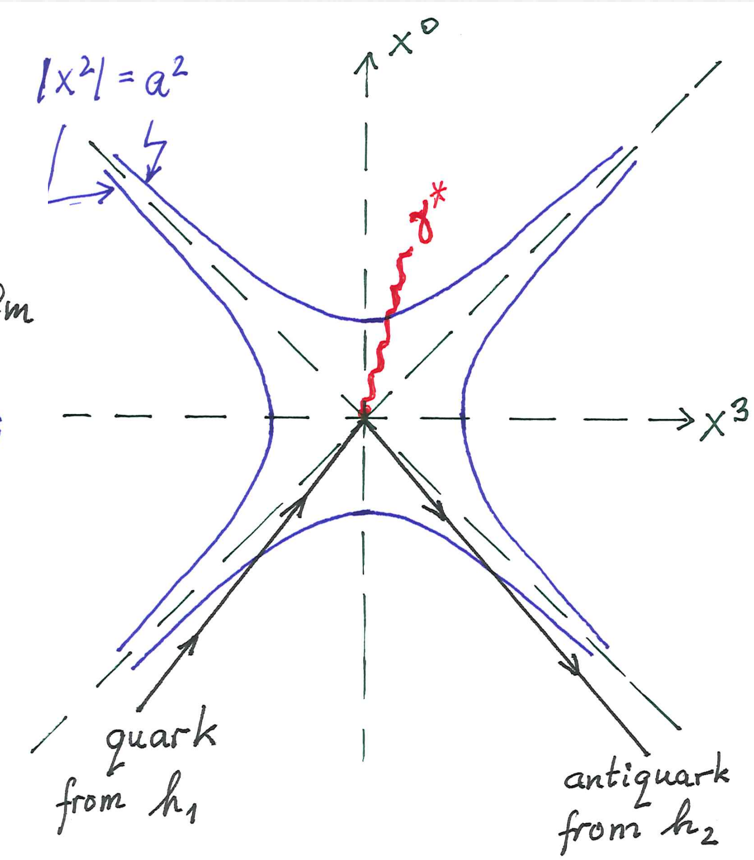
- Deflection due to chromomagnetic Lorentz force
- Synchrotron emission of gluons and photons
- Spin-flip gluon synchrotron emission leading to a correlated polarisation of q and $\bar{q}$ (Chromomagnetic Sokolov-Ternov effect).

In the Drell-Yan process the annihilating quark and antiquark can spend a long time in the same correlation region.

$|x^2| \lesssim a^2$

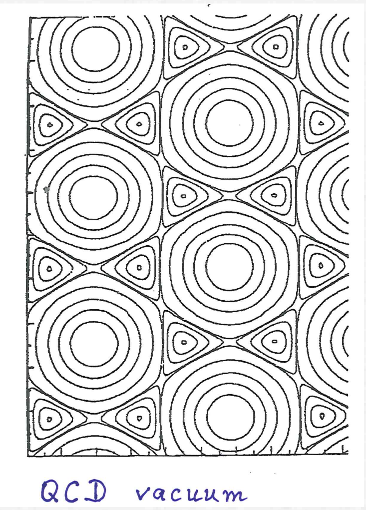
$a = 0.2$ to 0.3 fm

from lattic QCD;



from Nachtmann's talk

The DY $A_{UU}^{\cos 2\phi}$: nonpert. QCD



fluctuating vacuum
chromomagnetic field

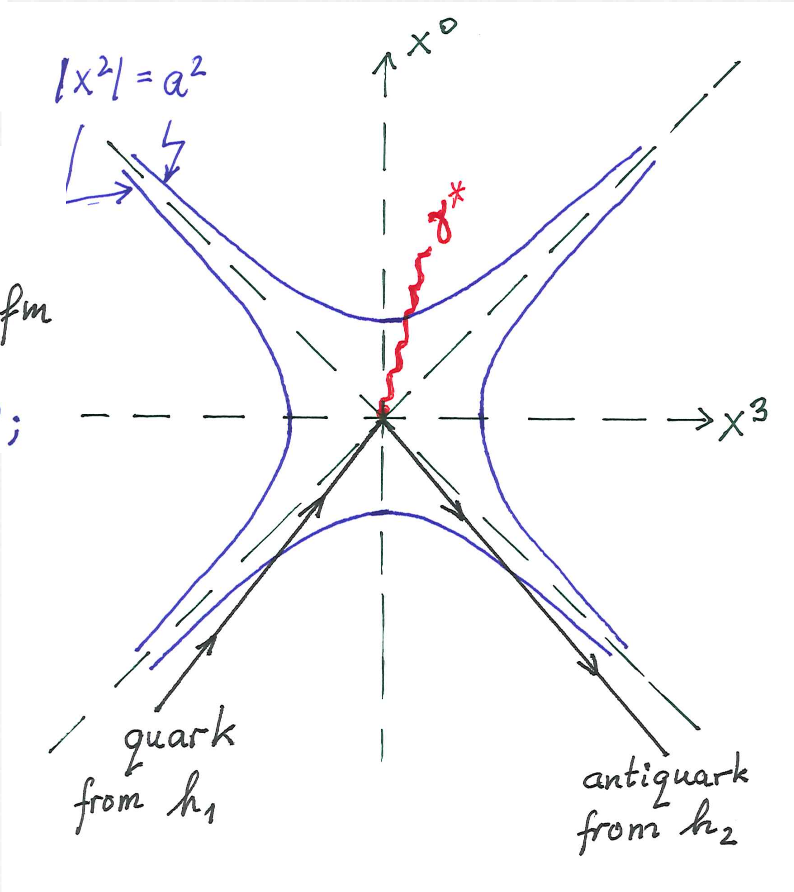
- Deflection due to chromomagnetic Lorentz force
- Synchrotron emission of gluons and photons
- Spin-flip gluon synchrotron emission leading to a correlated polarisation of q and $\bar{q}$ (Chromomagnetic Sokolov-Ternov effect).

In the Drell-Yan process the annihilating quark and antiquark can spend a long time in the <u>same</u> correlation region.

$|x^2| \lesssim a^2$

$a = 0.2$ to 0.3 fm

from lattic QCD;



q-q̄ density matrix

$$\rho^{(q\bar{q})}(\vec{k}_1^*, \vec{p}_1^*, \vec{p}_2^*) = \frac{1}{4}\{ \mathbb{1}\otimes\mathbb{1} + (\vec{F}\cdot\vec{\sigma})\otimes\mathbb{1} + \mathbb{1}\otimes(\vec{G}\cdot\vec{\sigma}) + H_{ij}(\vec{e}_i^{\,*}\cdot\vec{\sigma})\otimes(\vec{e}_j^{\,*}\cdot\vec{\sigma})\}$$

does it factorize, or is q-q̄ entanglement ?

from Nachtmann's talk

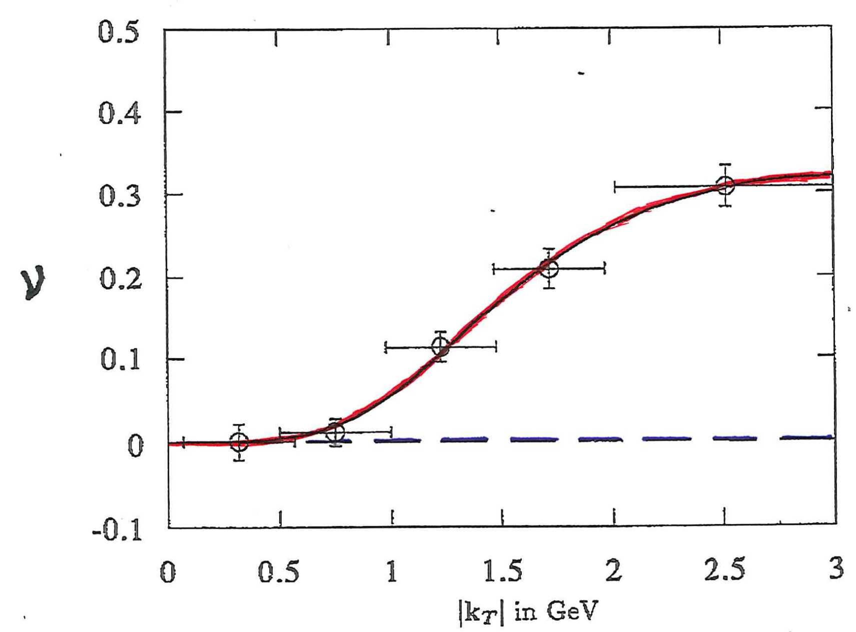
The DY $A_{UU}^{\cos 2\phi}$: nonpert. QCD

$$\varkappa = \frac{H_{22} - H_{11}}{1 + H_{33}}$$

Lam-Tung violation

$$1 - \lambda - 2\nu \approx -4\varkappa$$

Data: NA10, Theory: Brandenburg, O.N., Mirkes, 1993



$\varkappa = 0$ (dashed) $\qquad \varkappa = \varkappa_0 \dfrac{|\vec{k}_T|^4}{|\vec{k}_T|^4 + m_T^4}$ (solid red)

$\varkappa_0 = 0.17$, $m_T = 1.5$ GeV

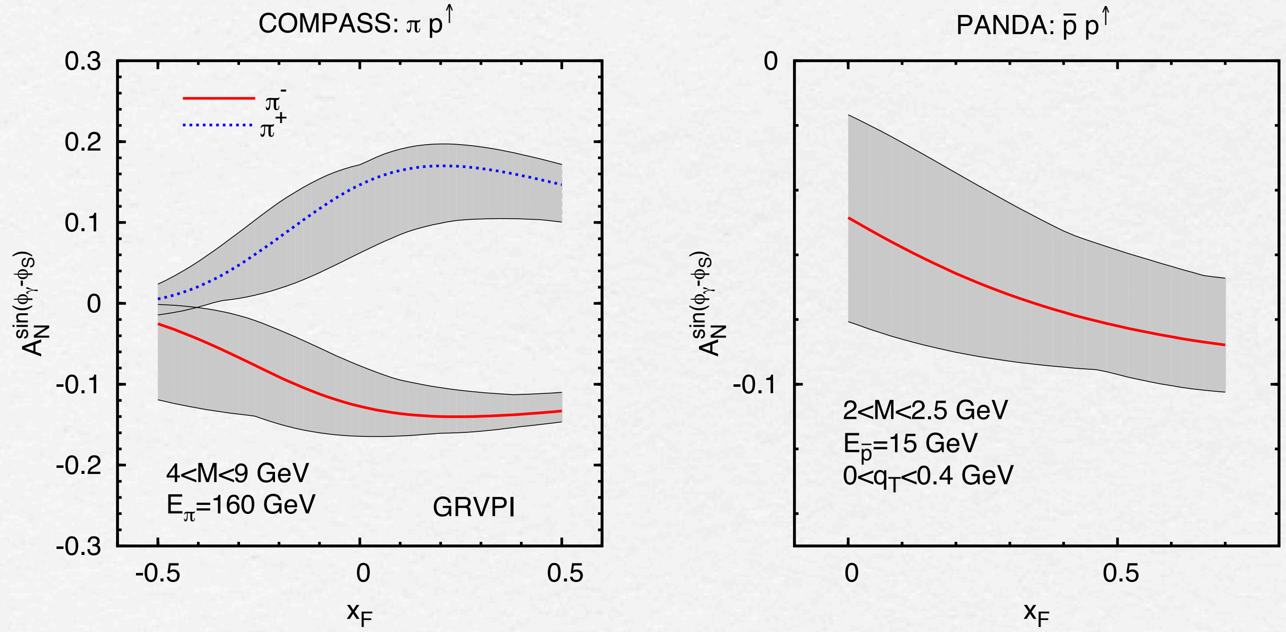
from Nachtmann's talk

The single-polarized DY : the Sivers effect

$$W_{UT}^{\sin(\phi-\phi_{S_2})} = \mathcal{C}\left[\frac{\hat{\mathbf{q}}_T\cdot\mathbf{k}_{2T}}{M_2}\, f_1\, \bar{f}_{1T}^{\perp}\right] \qquad \mathcal{A}^{\sin(\phi-\phi_S)} \equiv \frac{2\int d\phi\, \sin(\phi-\phi_S)\,(d\sigma^{\uparrow}-d\sigma^{\downarrow})}{\int d\phi\,(d\sigma^{\uparrow}+d\sigma^{\downarrow})}$$

large asymmetries predicted using “old style” analysis with DGLAP evolution of collinear part

[Anselmino et al. 2009]



from Barone's talk

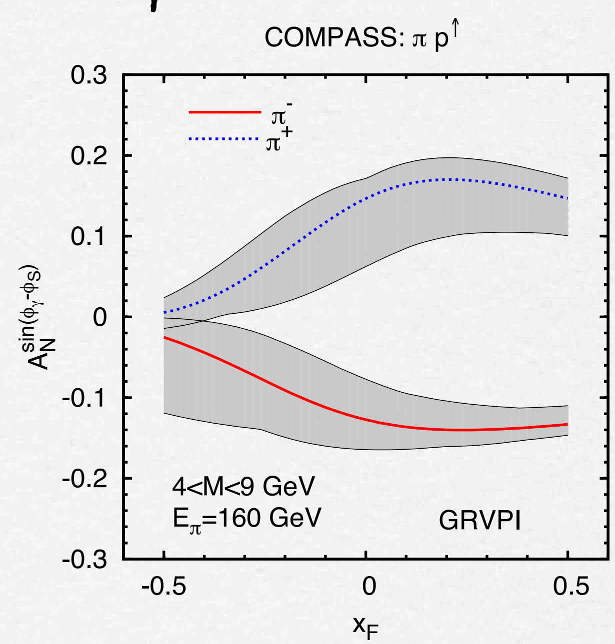
The single-polarized DY : the Sivers effect

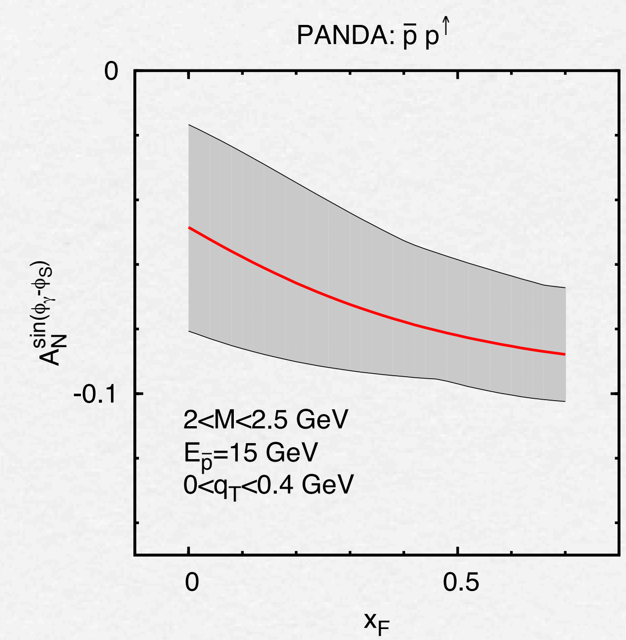
$$W_{UT}^{\sin(\phi-\phi_{S_2})} = \mathcal{C}\left[\frac{\hat{\mathbf{q}}_T\cdot\mathbf{k}_{2T}}{M_2}\, f_1\, \bar{f}_{1T}^{\perp}\right]$$

$$\mathcal{A}^{\sin(\phi-\phi_S)} \equiv \frac{2\int d\phi\, \sin(\phi-\phi_S)\,(d\sigma^{\uparrow}-d\sigma^{\downarrow})}{\int d\phi\,(d\sigma^{\uparrow}+d\sigma^{\downarrow})}$$

large asymmetries predicted using "old style" analysis
with DGLAP evolution of collinear part

[Anselmino et al. 2009]

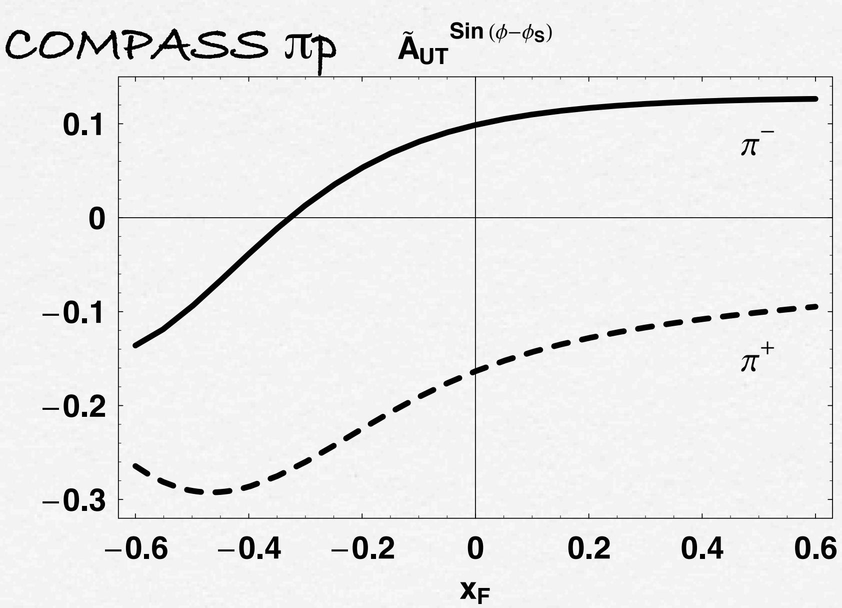




or with weighted asymmetries

$$\tilde{A}^{\sin(\phi-\phi_S)} \equiv \frac{2\int d\phi\, d\mathbf{q}_T^2\, \frac{Q_T}{M}\,\sin(\phi-\phi_S)\,(d\sigma^{\uparrow}-d\sigma^{\downarrow})}{\int d\phi\, d\mathbf{q}_T^2\,(d\sigma^{\uparrow}+d\sigma^{\downarrow})}$$

COMPASS πp



[Bacchetta et al. 2010]

Sivers funct. in spectator
diquark model with
(x,kT) unfactorized dep.

from Barone's talk

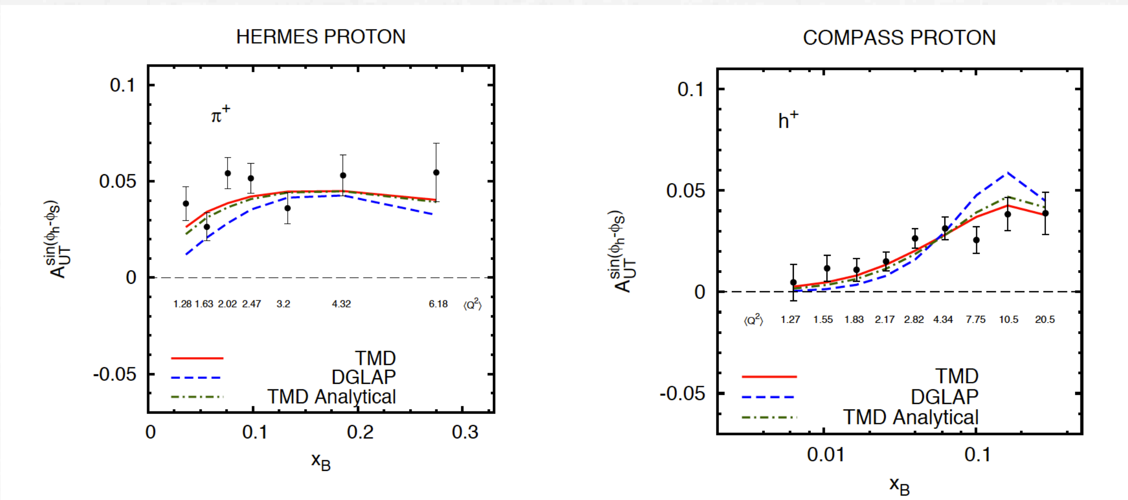
The single-polarized DY : the Sivers effect

revisit the analysis using TMD evolution

$$\widetilde{F}(x,\boldsymbol{b}_T;Q) = \widetilde{F}(x,\boldsymbol{b}_T;Q_0)\,\widetilde{R}(Q,Q_0,b_T)\,\exp\left\{-g_K(b_T)\ln\frac{Q}{Q_0}\right\}$$

Aybat, Collins, Qiu, Rogers 2012

pert. kernel nonpert.



Anselmino, Boglione, Melis

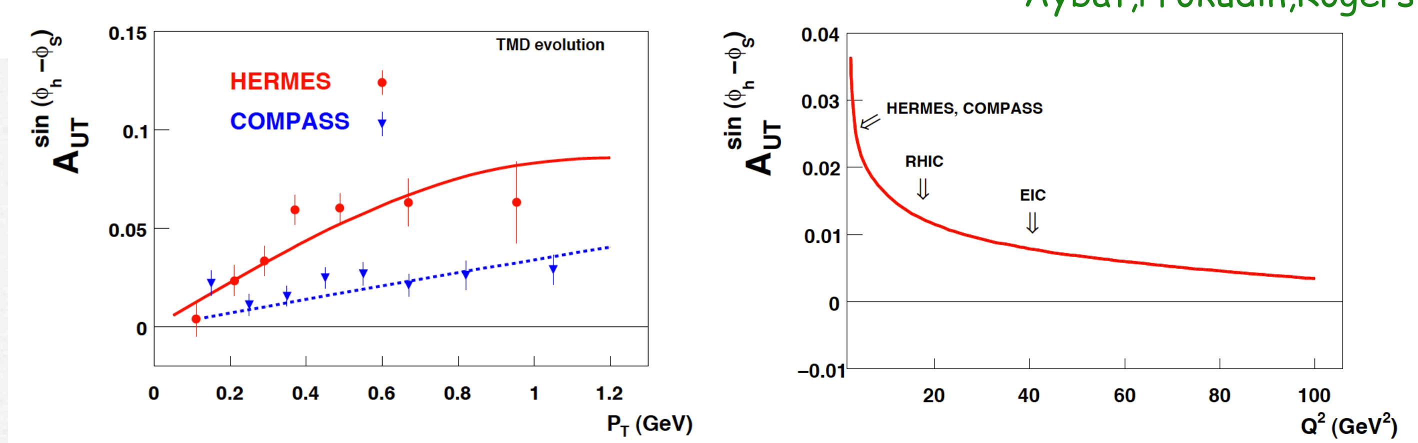
COMPASS $\langle Q^2\rangle=3.6$ GeV^2

HERMES $\langle Q^2\rangle=2.4$ GeV^2

first: SIDIS

$$\langle k_\perp^2\rangle = 0.25\ \mathrm{GeV}^2$$
$$\langle p_\perp^2\rangle = 0.20\ \mathrm{GeV}^2$$
$$g_2 = 0.68\ \mathrm{GeV}^2$$

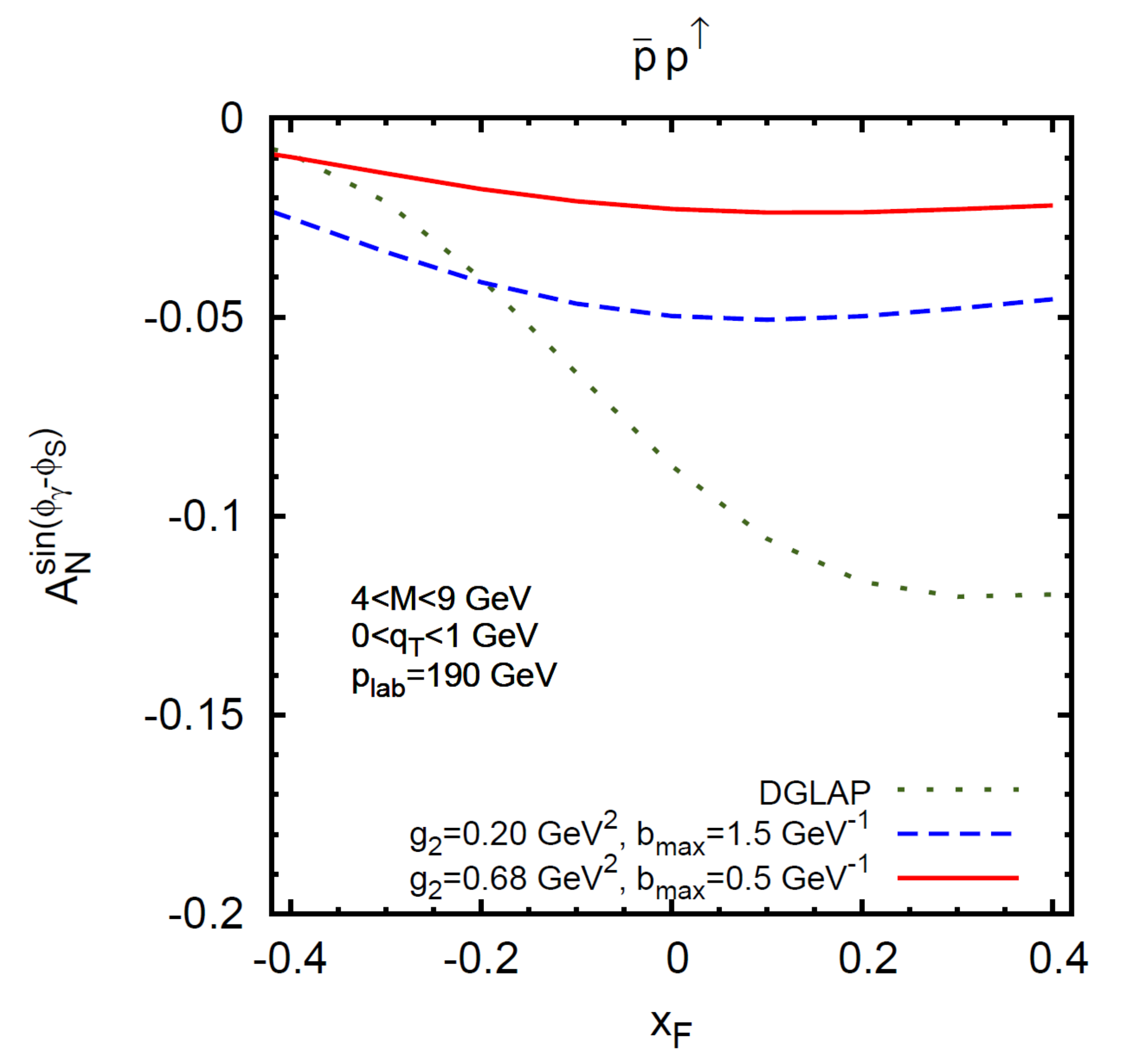
Aybat,Prokudin,Rogers



The single-polarized DY : the Sivers effect

next: DY

marked sensitivity to parameter of nonpert. kernel



from Melis' talk

The puzzle of the Sivers function

$$g_s T_F(x,x) = -2M f_{1T}^{\perp(1)}(x)$$

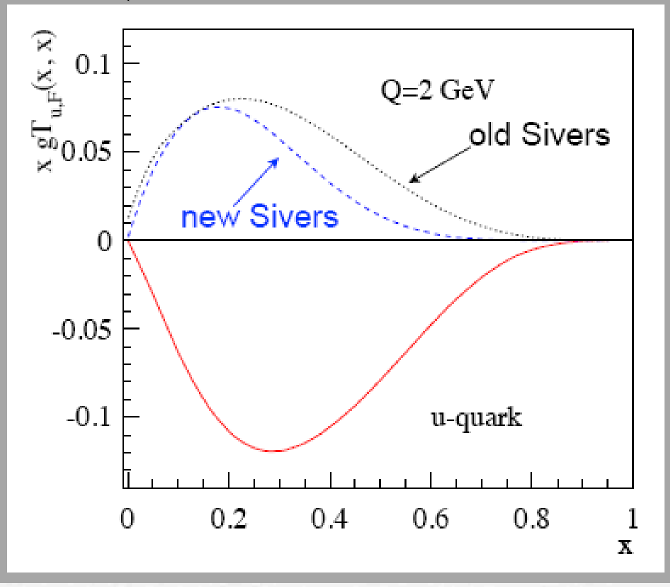
$p\, p^{\uparrow} \to \pi(p_T)\, X$ at RHIC

Collinear analysis: Kouvaris, Qiu, Vogelsang, Yuan (2006)

SIDIS

TMD analysis:

Anselmino et al (2008)



Kang, Qiu, Vogelsang, Yuan (2011)

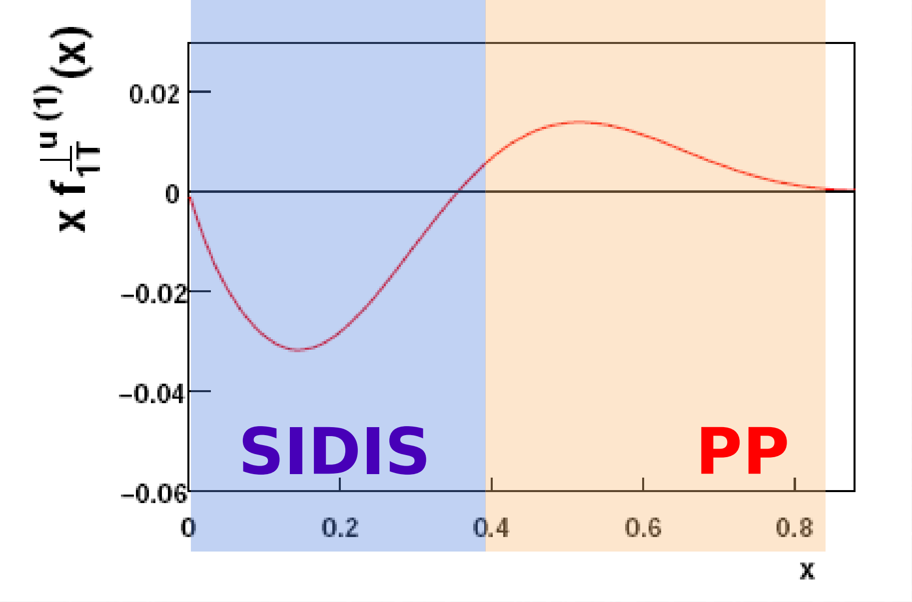
- Magnitudes are similar
- Sign is opposite

??

The puzzle of the Sivers function

Sivers function can have nodes in x.
Boer (2011)
Bacchetta et al, model calculation (2010)

Kang, AP (2012)

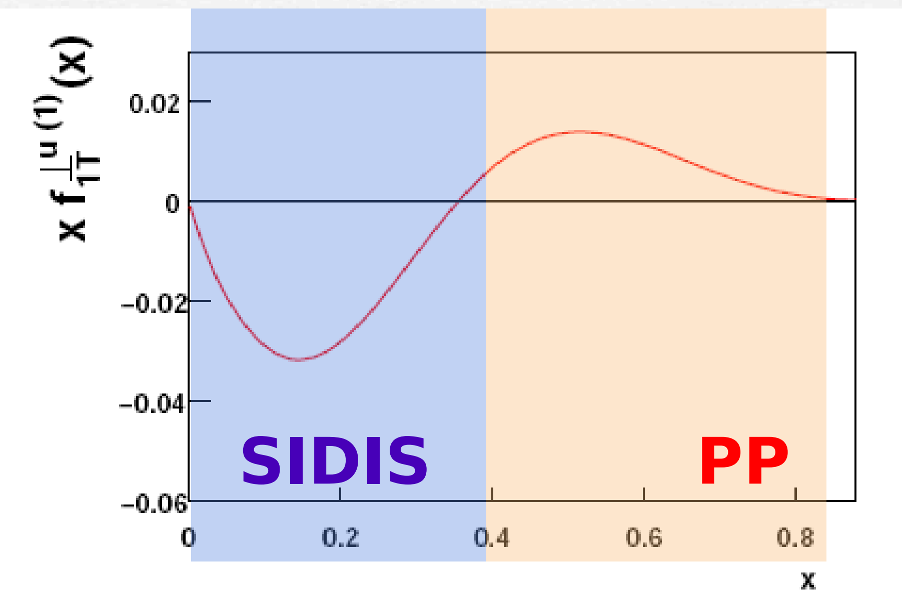


from Prokudin's talk

The puzzle of the Sivers function

Sivers function can have nodes in x.
Boer (2011)
Bacchetta et al, model calculation (2010)

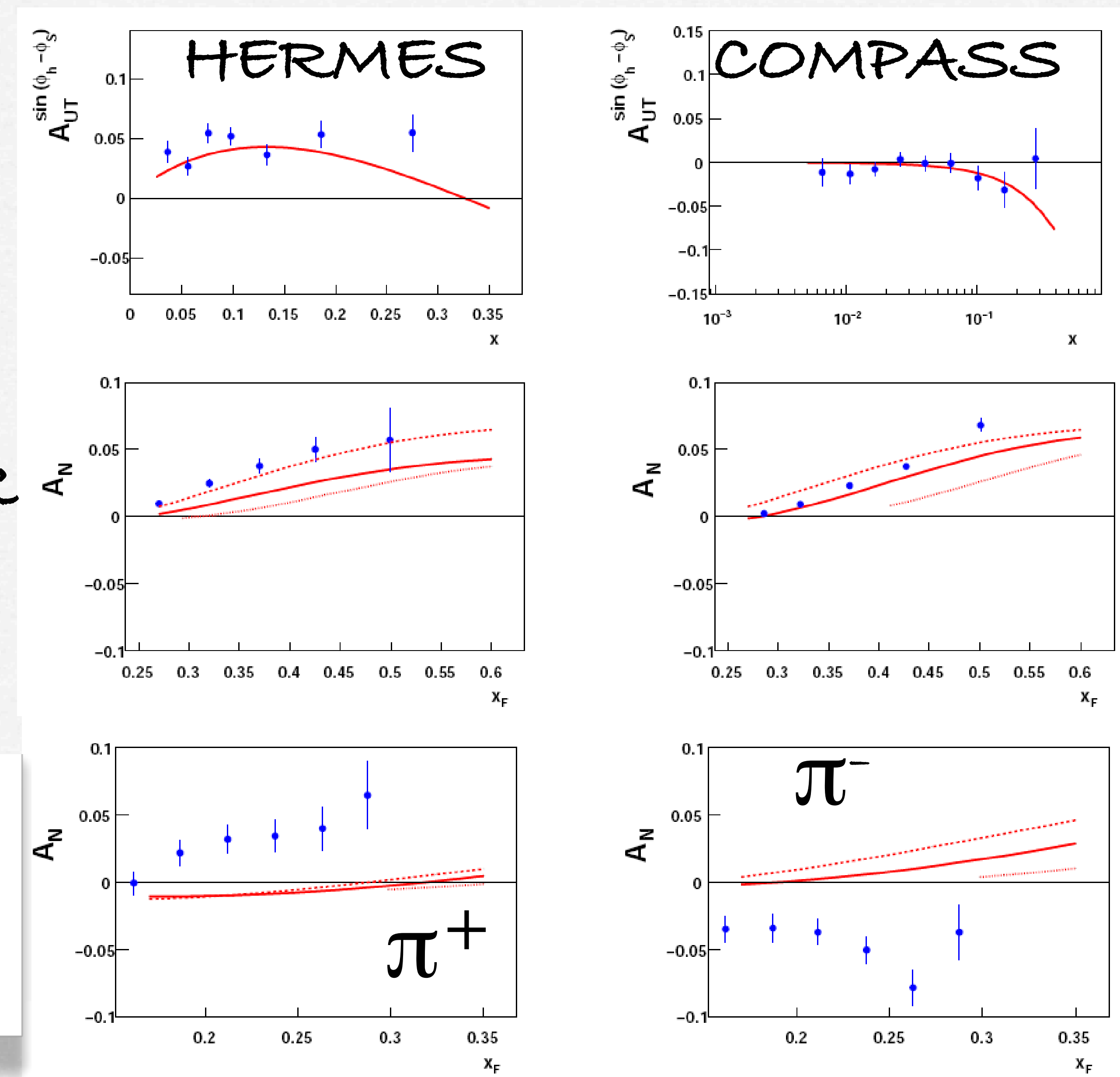
Kang, AP (2012)



OK

STAR
OK

BRAHMS
not OK !



from Prokudin's talk

The single-polarized DY : the Boer-Mulders

$$W_{UT}^{\sin(\phi+\phi_{S_2})} = \mathcal{C}\left[\frac{\hat{\mathbf{q}}_T \cdot \mathbf{k}_{1T}}{M_1}\, h_1^\perp \bar{h}_1\right]$$

$$W_{UT}^{\sin(3\phi-\phi_{S_2})} = \mathcal{C}\left[\frac{2(\hat{\mathbf{q}}_T \cdot \mathbf{k}_{2T})[2(\hat{\mathbf{q}}_T \cdot \mathbf{k}_{1T})(\hat{\mathbf{q}}_T \cdot \mathbf{k}_{2T}) - \mathbf{k}_{1T} \cdot \mathbf{k}_{2T}] - \mathbf{k}_{2T}^2(\hat{\mathbf{q}}_T \cdot \mathbf{k}_{1T})}{2M_1 M_2^2}\, h_1^\perp \bar{h}_{1T}^\perp\right]$$

$\pi\, p^\uparrow$ at COMPASS: explore B.M. of pion

(*)

The pion pdf from Dynamical χ Symmetry Breaking

double nature of pion as

q-$\bar{q}$ bound state and

Goldstone boson of DχSB

implementation of confinement from DχSB

solving Dyson-Schwinger eq.

$\Downarrow$

valence distribution of

confined partons in pion

see Roberts' talk

The single-polarized DY : the Boer-Mulders

$$W_{UT}^{\sin(\phi+\phi_{S_2})} = \mathcal{C}\left[\frac{\hat{\mathbf{q}}_T \cdot \mathbf{k}_{1T}}{M_1}\, h_1^\perp \bar{h}_1\right]$$

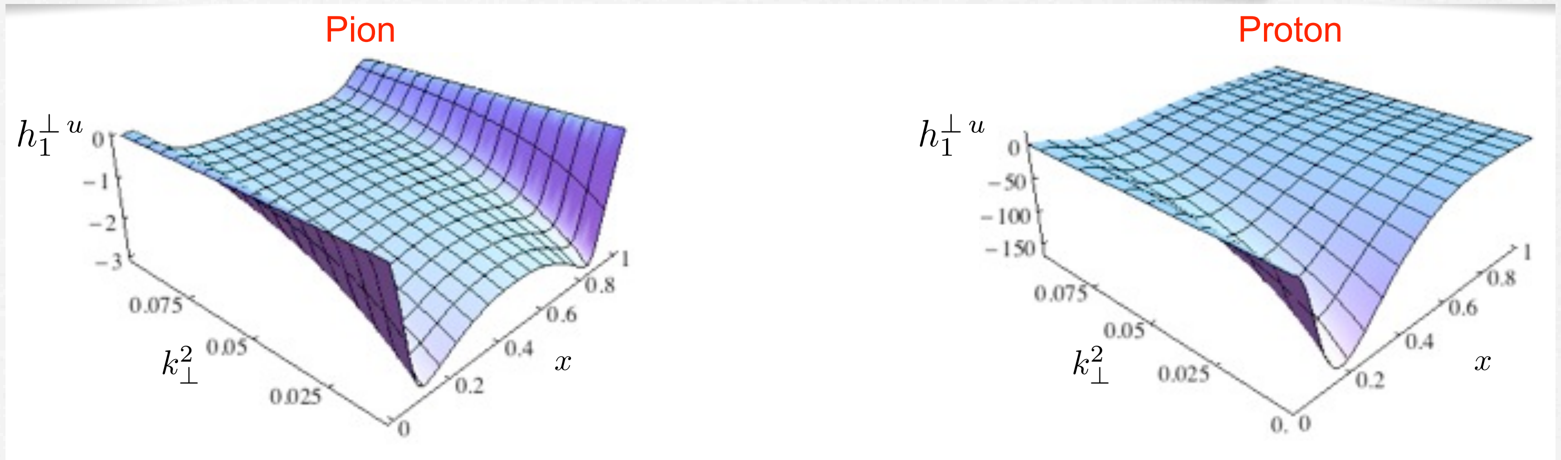
$$W_{UT}^{\sin(3\phi-\phi_{S_2})} = \mathcal{C}\left[\frac{2(\hat{\mathbf{q}}_T \cdot \mathbf{k}_{2T})[2(\hat{\mathbf{q}}_T \cdot \mathbf{k}_{1T})(\hat{\mathbf{q}}_T \cdot \mathbf{k}_{2T}) - \mathbf{k}_{1T} \cdot \mathbf{k}_{2T}] - \mathbf{k}_{2T}^2(\hat{\mathbf{q}}_T \cdot \mathbf{k}_{1T})}{2M_1 M_2^2}\, h_1^\perp \bar{h}_{1T}^\perp\right]$$

$\pi\, p^\uparrow$ at COMPASS: explore B.M. of pion

(*)

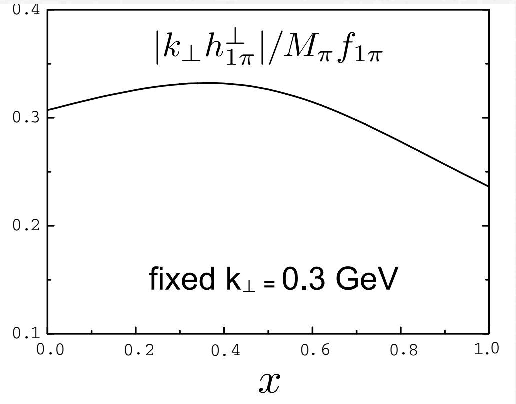
LCCQM

from Pasquini's talk



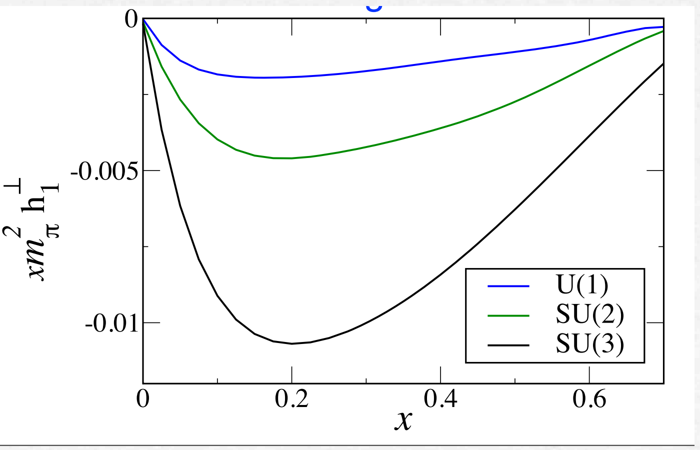
Spec. quark model

Zhun Lu, B.Q. Ma
PRD70 (2011)



eikonal method

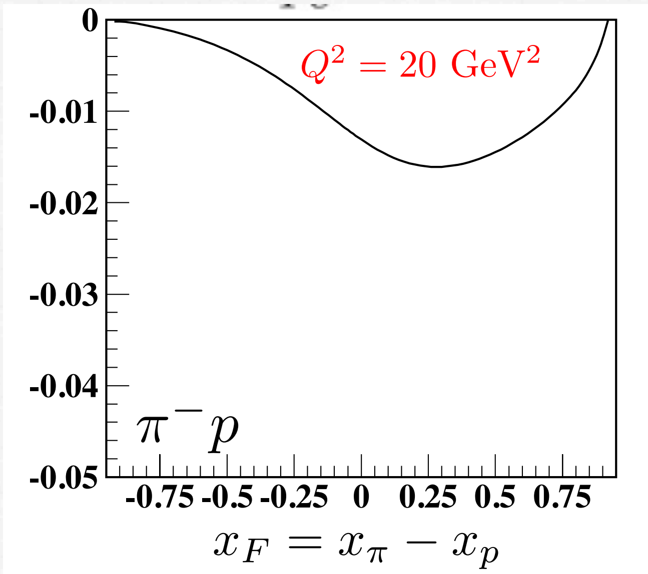
Gamberg, Schlegel
PLB685 (2010)



The single-polarized DY : the Boer-Mulders

$h_1^\perp \otimes h_{1T}^\perp$

LCCQM

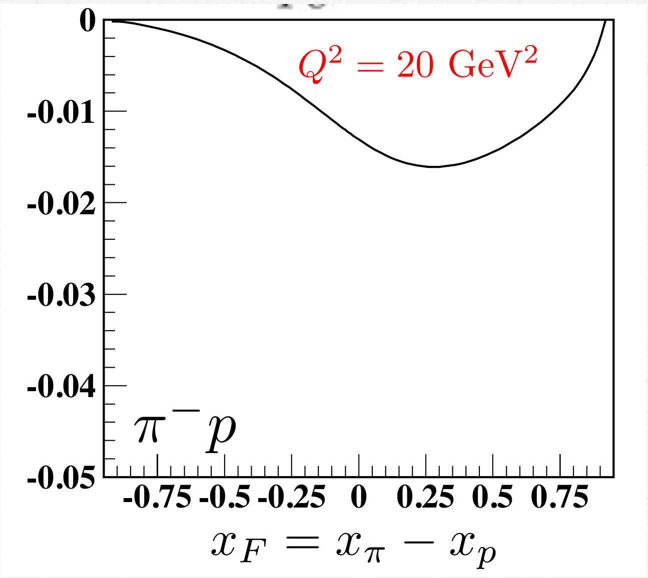


from Pasquini's talk

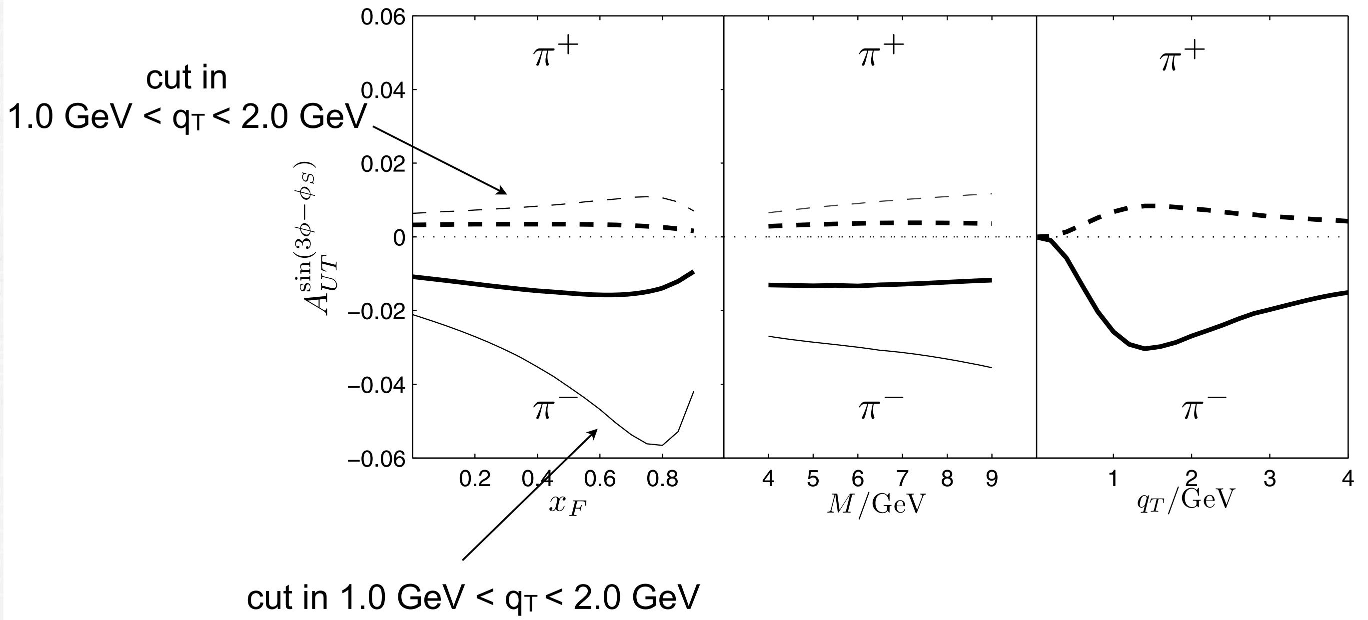
The single-polarized DY : the Boer-Mulders

$h_1^{\perp} \otimes h_{1T}^{\perp}$

LCCQM



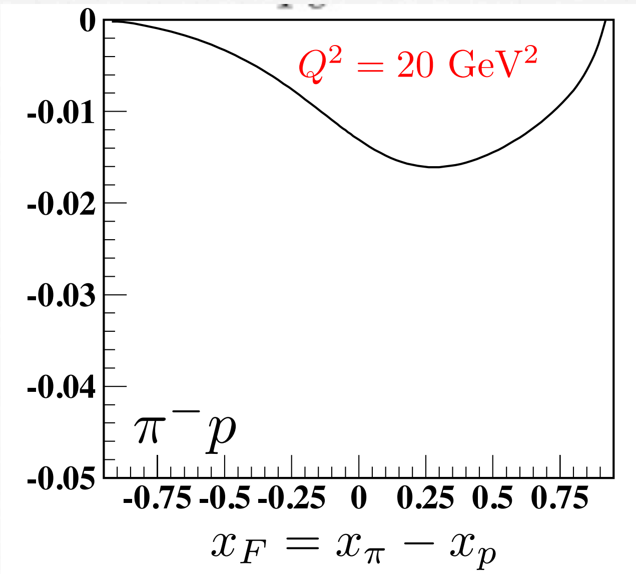
from Pasquini's talk



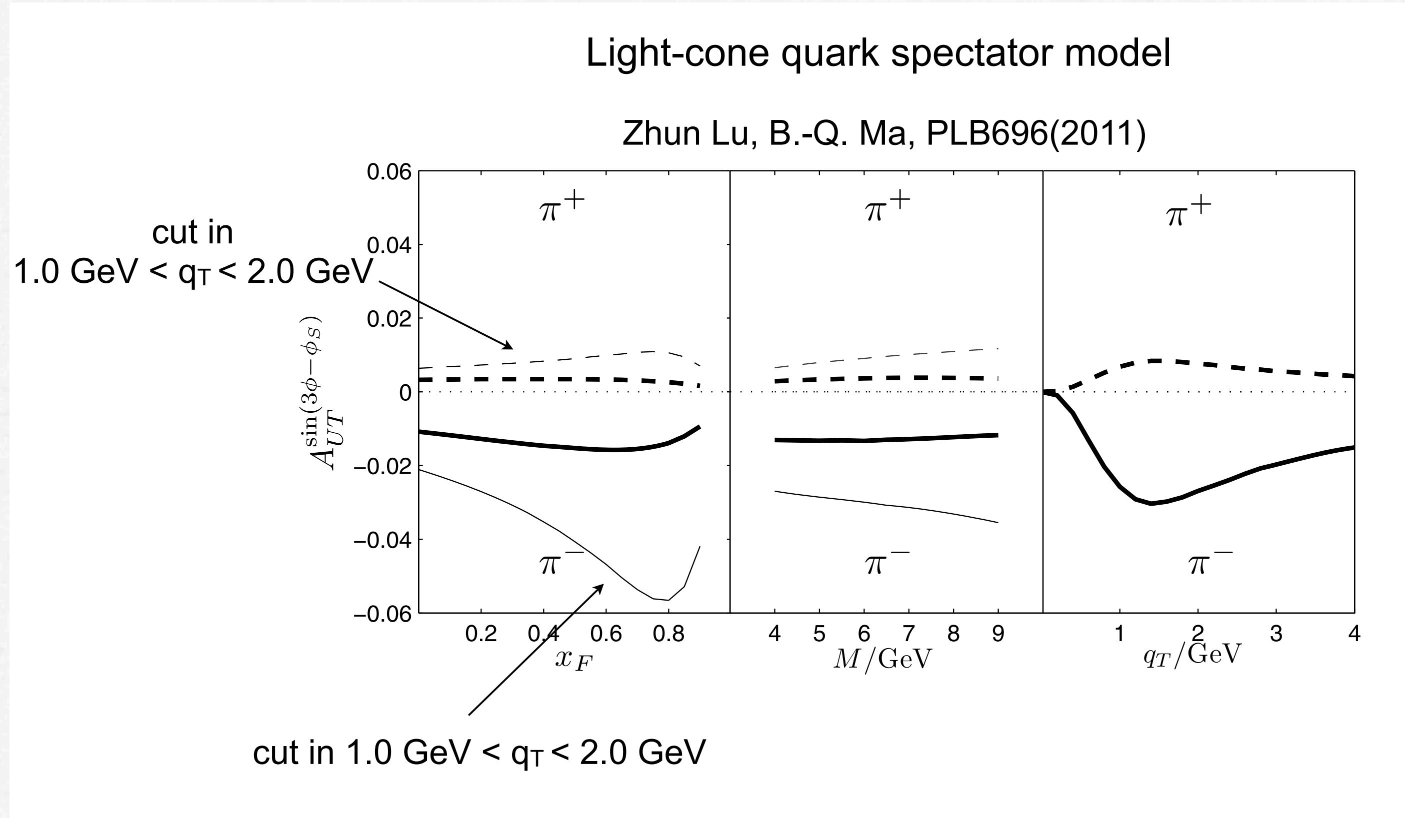
The single-polarized DY : the Boer-Mulders

$h_1^{\perp} \otimes h_{1T}^{\perp}$

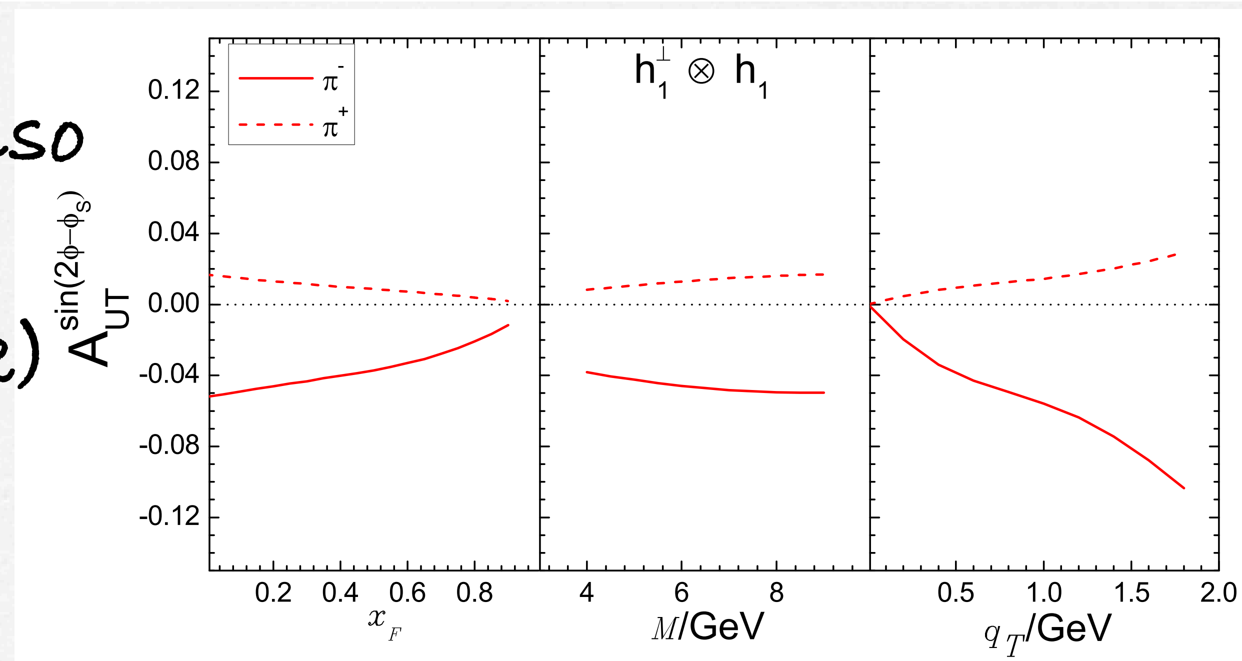
LCCQM



from Pasquini's talk



calculations also
for pp DY
(see Lu's talk)



all TMDs calculable also
in the covariant QPM
(see Zavada's talk)

DY at twist 3

- angular dependence of DY at $\mathcal{O}(1/Q)$ (Arnold, Metz, Schlegel 09, ZL, Schmidt 11):

$$
\begin{aligned}
\frac{d\sigma^{\text{twist-3}}}{dx_1 dx_2 d^2\boldsymbol{q}_T\, d\Omega} = & \frac{\alpha_{em}^2}{3Q^2}\sin 2\theta \Big\{ \cos\phi\, F_{UU}^{\cos\phi} \;+\; S_{1L}\sin\phi\, F_{LU}^{\sin\phi} \;+\; S_{2L}\sin\phi\, F_{UL}^{\sin\phi} \\
& + |\vec{S}_{1T}|\Big[\sin(\phi_1+\phi) F_{TU}^{\sin(\phi_{S_1}+\phi)} + \sin(\phi_{S_1}-\phi) F_{TU}^{\sin(\phi_{S_1}-\phi)}\Big] \\
& + |\vec{S}_{2T}|\Big[\sin(\phi_{S_2}+\phi) F_{UT}^{\sin(\phi_{S_2}+\phi)} + \sin(\phi_{S_2}-\phi) F_{UT}^{\sin(\phi_{S_2}-\phi)}\Big]
\end{aligned}
$$

$$
\begin{aligned}
F_{UU}^{\cos\phi} =& \frac{2}{Q}\,\mathcal{C}\left[(\boldsymbol{h}\cdot\boldsymbol{k}_{1T})\left(\widehat{f}^{\perp}\overline{f}_1 - \frac{M_2}{M_1}h_1^{\perp}\widehat{\overline{h}}\right) - (\boldsymbol{h}\cdot\boldsymbol{k}_{2T})\left(f_1\,\widehat{\overline{f}}^{\perp} - \frac{M_1}{M_2}\widehat{h}\,\overline{h}_1^{\perp}\right)\right] \\
F_{LU}^{\sin\phi} =& \frac{2}{Q}\,\mathcal{C}\left[(\boldsymbol{h}\cdot\boldsymbol{k}_{1T})\left(\widehat{f}_L^{\perp}\overline{f}_1 + \frac{M_2}{M_1}\,h_{1L}^{\perp}\,\widehat{\overline{h}}\right) - (\boldsymbol{h}\cdot\boldsymbol{k}_{2T})\left(g_{1L}\widehat{\overline{g}}^{\perp} + \frac{M_1}{M_2}\widehat{h}_L\,\overline{h}_1^{\perp}\right)\right] \\
F_{TU}^{\sin(\phi_{S_1}-\phi)} =& \frac{1}{Q}\,\mathcal{C}\Big[2M_1\,\widehat{f}_T\overline{f}_1 + 2M_2\;h_1\widehat{\overline{h}} \\
& + (\boldsymbol{k}_{1T}\cdot\boldsymbol{k}_{2T})\left(\frac{f_{1T}^{\perp}\widehat{\overline{f}}^{\perp}}{M_1} - \frac{g_{1T}\widehat{\overline{g}}^{\perp}}{M_1} - \frac{\widehat{h}_T\overline{h}_1^{\perp}}{M_2} + \frac{\widehat{h}_T^{\perp}\overline{h}_1^{\perp}}{M_2}\right)\Big] \\
F_{TU}^{\sin(\phi_{S_1}+\phi)} =& \frac{1}{Q}\,\mathcal{C}\Big[-\left(2\,(\boldsymbol{h}\cdot\boldsymbol{k}_{1T})^2 - \boldsymbol{k}_{1T}^2\right)\left(\frac{\widehat{f}_T^{\perp}\overline{f}_1}{M_1} + \frac{M_2\,h_{1T}^{\perp}\widehat{\overline{h}}}{M_1^2}\right) \\
& + (2\boldsymbol{h}\cdot\boldsymbol{k}_{1T}\boldsymbol{h}\cdot\boldsymbol{k}_{2T} - \boldsymbol{k}_{1T}\cdot\boldsymbol{k}_{2T})\left(\frac{f_{1T}^{\perp}\widehat{\overline{f}}^{\perp}}{M_1} + \frac{g_{1T}\widehat{\overline{g}}^{\perp}}{M_1} + \frac{\widehat{h}_T\overline{h}_1^{\perp}}{M_2} + \frac{\widehat{h}_T^{\perp}\overline{h}_1^{\perp}}{M_2}\right)\Big]
\end{aligned}
$$

$$
\widehat{f} = x_1\left((1-c)\,f + c\,\tilde{f}\right),\; \widehat{\overline{f}} = x_2\left(c\,\overline{f} + (1-c)\,\tilde{\overline{f}}\right).\;\; c = \frac{1}{2}:\; \text{CS frame}
$$

see Lu's talk

DY at twist 3

- angular dependence of DY at $\mathcal{O}(1/Q)$ (Arnold, Metz, Schlegel 09, ZL, Schmidt 11):

$$\frac{d\sigma^{\text{twist-3}}}{dx_1 dx_2 d^2\boldsymbol{q}_T\, d\Omega} = \frac{\alpha_{em}^2}{3Q^2}\sin 2\theta \Big\{ \cos\phi\, F_{UU}^{\cos\phi} + S_{1L}\sin\phi\, F_{LU}^{\sin\phi} + S_{2L}\sin\phi\, F_{UL}^{\sin\phi}$$
$$+ |\vec{S}_{1T}|\Big[\sin(\phi_1+\phi)F_{TU}^{\sin(\phi_{S_1}+\phi)} + \sin(\phi_{S_1}-\phi)F_{TU}^{\sin(\phi_{S_1}-\phi)}\Big]$$
$$+ |\vec{S}_{2T}|\Big[\sin(\phi_{S_2}+\phi)F_{UT}^{\sin(\phi_{S_2}+\phi)} + \sin(\phi_{S_2}-\phi)F_{UT}^{\sin(\phi_{S_2}-\phi)}\Big]$$

$$F_{UU}^{\cos\phi} = \frac{2}{Q}\,\mathcal{C}\left[(\boldsymbol{h}\cdot\boldsymbol{k}_{1T})\left(\widehat{f}^{\perp}\overline{f}_1 - \frac{M_2}{M_1}h_1^{\perp}\widehat{\overline{h}}\right) - (\boldsymbol{h}\cdot\boldsymbol{k}_{2T})\left(f_1\widehat{\overline{f}}^{\perp} - \frac{M_1}{M_2}\widehat{h}\,\overline{h}_1^{\perp}\right)\right]$$

$$F_{LU}^{\sin\phi} = \frac{2}{Q}\,\mathcal{C}\left[(\boldsymbol{h}\cdot\boldsymbol{k}_{1T})\left(\widehat{f}_L^{\perp}\overline{f}_1 + \frac{M_2}{M_1}h_{1L}^{\perp}\widehat{\overline{h}}\right) - (\boldsymbol{h}\cdot\boldsymbol{k}_{2T})\left(g_{1L}\widehat{\overline{g}}^{\perp} + \frac{M_1}{M_2}\widehat{h}_L\,\overline{h}_1^{\perp}\right)\right]$$

$$F_{TU}^{\sin(\phi_{S_1}-\phi)} = \frac{1}{Q}\,\mathcal{C}\Big[2M_1\,\widehat{f}_T\overline{f}_1 + 2M_2\,h_1\widehat{\overline{h}}$$
$$+ (\boldsymbol{k}_{1T}\cdot\boldsymbol{k}_{2T})\left(\frac{f_{1T}^{\perp}\widehat{\overline{f}}^{\perp}}{M_1} - \frac{g_{1T}\widehat{\overline{g}}^{\perp}}{M_1} - \frac{\widehat{h}_T\overline{h}_1^{\perp}}{M_2} + \frac{\widehat{h}_T^{\perp}\overline{h}_1^{\perp}}{M_2}\right)\Big]$$

$$F_{TU}^{\sin(\phi_{S_1}+\phi)} = \frac{1}{Q}\,\mathcal{C}\Big[-\left(2\,(\boldsymbol{h}\cdot\boldsymbol{k}_{1T})^2 - \boldsymbol{k}_{1T}^2\right)\left(\frac{\widehat{f}_T^{\perp}\overline{f}_1}{M_1} + \frac{M_2\,h_{1T}^{\perp}\widehat{\overline{h}}}{M_1^2}\right)$$
$$+ (2\boldsymbol{h}\cdot\boldsymbol{k}_{1T}\boldsymbol{h}\cdot\boldsymbol{k}_{2T} - \boldsymbol{k}_{1T}\cdot\boldsymbol{k}_{2T})\left(\frac{f_{1T}^{\perp}\widehat{\overline{f}}^{\perp}}{M_1} + \frac{g_{1T}\widehat{\overline{g}}^{\perp}}{M_1} + \frac{\widehat{h}_T\overline{h}_1^{\perp}}{M_2} + \frac{\widehat{h}_T^{\perp}\overline{h}_1^{\perp}}{M_2}\right)\Big]$$

$$\widehat{f} = x_1\left((1-c)\,f + c\,\tilde{f}\right),\ \widehat{\overline{f}} = x_2\left(c\,\overline{f} + (1-c)\,\tilde{\overline{f}}\right).\quad c = \frac{1}{2}:\ \text{CS frame}$$

see Lu's talk

but warning on mismatch and WW approx.

Ex: $$h = \frac{k_T^2}{M^2}\frac{h_1^{\perp}}{x} + \tilde{h}$$

see Bacchetta's talk

The TMDs-ology for gluons

$$\Gamma^{ij}(x,\vec{k}_T) = \frac{1}{xP^+}\int \frac{dz^-\, d^2z_T}{(2\pi)^3}\, \mathrm{e}^{ik\cdot z} \langle P,S|\, F^{+i}(0)\, \mathcal{W}[0\,;\, z]\, F^{+j}(z)\, |P,S\rangle\Big|_{z^+=0}$$

$\Gamma^{[T-even]}(x,\vec{k}_T)$			$\Gamma^{[T-odd]}(x,\vec{k}_T)$	
		flip		flip
U	f_1^g	$h_1^{\perp g}$		
L	$g_{1L}^{\perp g}$			$h_{1L}^{\perp g}$
T	$g_{1T}^{\perp g}$		$f_{1T}^{\perp g}$	h_1^g $h_{1T}^{\perp g}$

[Mulders, Rodriues, PRD 63,094021]

from Schlegel's talk

TMD → matching ← CSS resummation

Unpolarized pp→γγX Cross-Section at $q_T << Q$

$$\frac{d\sigma_{UU}}{d^4q\, d\Omega} \sim \left(\frac{2}{\sin^2\theta}\right)\left((1+\cos^2\theta)[f_1^q \otimes f_1^{\bar{q}}] + \cos(2\phi)\sin(2\theta)[h_1^{\perp q} \otimes h_1^{\perp \bar{q}}]\right)$$

quark contributions → almost identical to DY

$$+\left(\frac{\alpha_s}{2\pi}\right)^2\left(\mathcal{F}_1[f_1^g \otimes f_1^g] + \mathcal{F}_2[h_1^{\perp g} \otimes h_1^{\perp g}] + \cos(2\phi)\mathcal{F}_3[h_1^{\perp g} \otimes f_1^g + f_1^g \otimes h_1^{\perp g}] + \cos(4\phi)\mathcal{F}_4[h_1^{\perp g} \otimes h_1^{\perp g}]\right)$$

gluon contributions → absent in DY

from Schlegel's talk

TMD → matching ← CSS resummation

Unpolarized pp→γγX Cross-Section at $q_T << Q$

$$\frac{d\sigma_{UU}}{d^4q\,d\Omega} \sim \left(\frac{2}{\sin^2\theta}\right)\Big((1+\cos^2\theta)[f_1^q \otimes f_1^{\bar{q}}] + \cos(2\phi)\sin(2\theta)[h_1^{\perp q} \otimes h_1^{\perp \bar{q}}]\Big)$$

quark contributions → almost identical to DY

$$+\left(\frac{\alpha_s}{2\pi}\right)^2\Big(\mathcal{F}_1[f_1^g \otimes f_1^g] + \mathcal{F}_2[h_1^{\perp g} \otimes h_1^{\perp g}] + \cos(2\phi)\mathcal{F}_3[h_1^{\perp g} \otimes f_1^g + f_1^g \otimes h_1^{\perp g}] + \cos(4\phi)\mathcal{F}_4[h_1^{\perp g} \otimes h_1^{\perp g}]\Big)$$

gluon contributions → absent in DY

same structure as in CSS resummation in collinear factorization

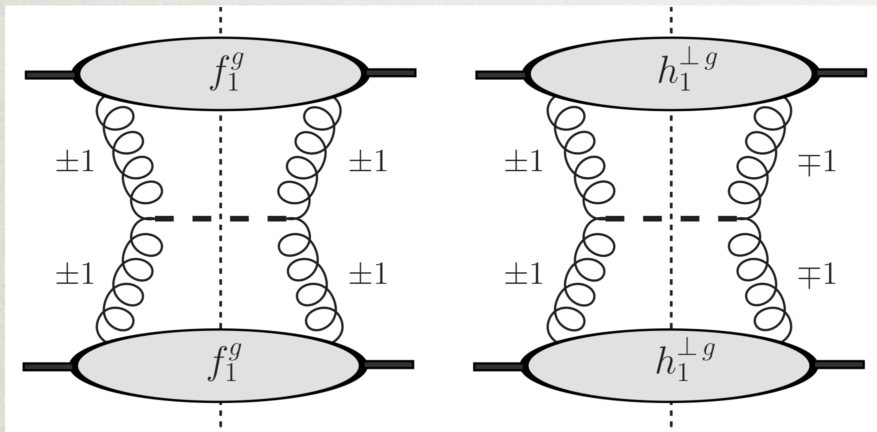
[Nadolsky, Balazs, Berger, Yuan; Catani, Grazzini, de Florian]

feasible at RHIC $\sqrt{s}=500$

from Schlegel's talk

using gluon TMD to guess Higgs parity

pure Higgs production via top-quark loop



again matches CSS resummed structure

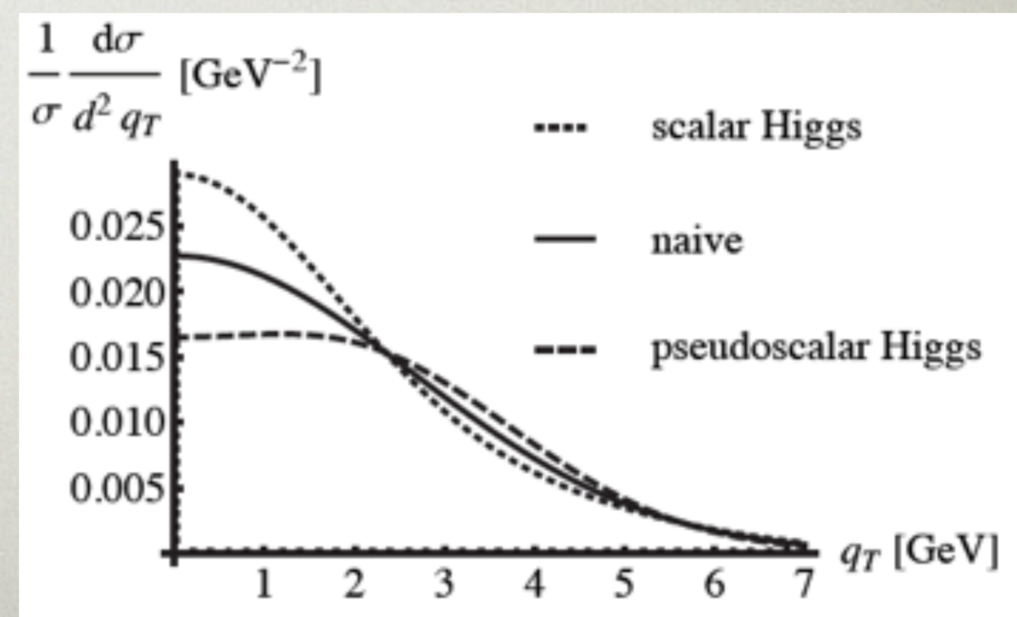
linearly polarized gluons sensitive to Higgs parity

$$[f_1^g \otimes f_1^g] \pm [h_1^{\perp g} \otimes h_1^{\perp g}]$$

+: scalar Higgs   -: pseudoscalar Higgs

$$\boxed{R = \frac{[h_1^{\perp g} \otimes h_1^{\perp g}]}{[f_1^g \otimes f_1^g]}}$$

$$\boxed{\frac{1}{\sigma}\frac{d\sigma}{d^2\vec{q}_T} = [1 \pm R(q_T)]\,\frac{1}{2\pi\langle p_T^2\rangle}e^{-q_T^2/2\langle p_T^2\rangle}}$$



from Schlegel's talk

not a summary of the summary... just recommend first of all

unpol. DY at several x, Q^2, s with different probes/targets

goal: constrain unpol. TMD as much as possible

then go to polarized case

not a summary of the summary... just recommend first of all
unpol. DY at several x, Q^2, s with different probes/targets
goal: constrain unpol. TMD as much as possible
then go to polarized case

Drell-Yan is important for several good reasons

that's why we very welcome the first DY measurement
after 15 years (E906, see Nakahara's talk)

and we're looking forward for upcoming COMPASS data

(and we are sad for A_NDY calcellation)