chiral-odd TMDs at twist 2 (and beyond) from (single-/un)polarized Drell-Yan asymmetries

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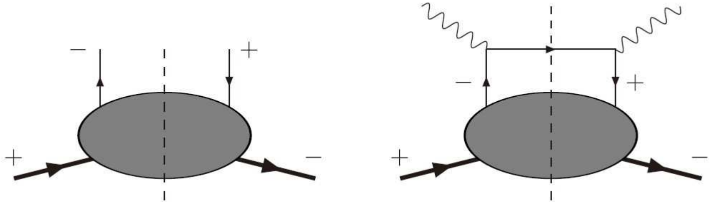
The TMD correlator for the nucleon (Goeke et.al, 05; Bacchetta et.al, 06):

$$\Phi(x,p_T) = \frac{1}{2}\Big\{ f_1\, \not{n}_+ - f_{1T}^{\perp}\, \frac{\epsilon_T^{\rho\sigma} p_{T\rho} S_{T\sigma}}{M}\, \not{n}_+ + g_{1s}\gamma_5\, \not{n}_+ + h_{1T}\, \frac{[\not{S}_T, \not{n}_+]\gamma_5}{2} + h_{1s}^{\perp}\, \frac{[\not{p}_T, \not{n}_+]\gamma_5}{2M} + i\, h_1^{\perp}\, \frac{[\not{p}_T, \not{n}_+]}{2M} \Big\}$$

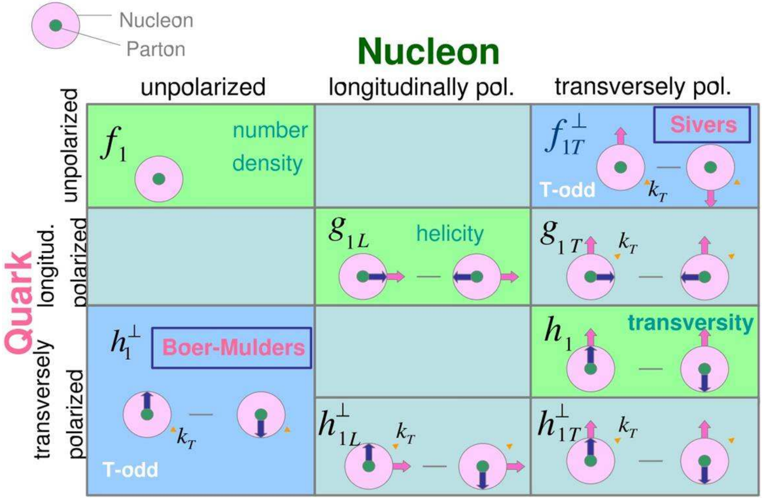
$$h_{1s}^{\perp} = S_L h_{1L}^{\perp} - \frac{p_T \cdot S_T}{M} h_{1T}^{\perp}$$

The TMD correlator for the pion (twist-2):

$$\Phi(x,p_T) = \frac{1}{2}\Big\{ f_1\, \not{n}_+ + i\, h_1^{\perp}\, \frac{[\not{p}_T, \not{n}_+]}{2M} \Big\}$$

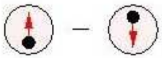


- describe helicity-flipped quark structure.
- handbag diagram forbidden by chirality conservation
- chiral-odd distributions participate in the processes involving at least two hadrons (SIDIS, Drell-Yan)
- chiral-odd TMDs appear as pair in high energy process


Nucleon
Parton
Nucleon
unpolarized
longitudinally pol.
transversely pol.
Quark
unpolarized
longitud. polarized
transversely polarized
f_1
number density
$f_{1T}^{\perp}$
Sivers
T-odd
k_T
g_{1L}
helicity
g_{1T}
k_T
$h_1^{\perp}$
Boer-Mulders
k_T
T-odd
h_1
transversity
$h_{1L}^{\perp}$
k_T
$h_{1T}^{\perp}$
k_T

Boer-Mulders functions: model calculations

$$i\,\frac{p_T^{\alpha}}{M}\,h_1^{\perp}(x,p_T^2) \quad = $$



- T-odd: $h_1^{\perp}|_{\mathrm{DY}} = -h_1^{\perp}|_{\mathrm{SIDIS}}$. (Collins 02)
- model calculations for the nucleon:
  - spectator model: Gamberg, Goldstein 02; Bacchetta, Schaefer, Yang 03; Gamberg, Goldstein, Schlegel 07; Bacchetta, Conti, Radici 08.
  - constituent (light-cone) quark model: Courtoy, Scopetta, Vento 09; Pasquini and Yuan 10.
  - bag model: Yuan 03; Courtoy, Scopetta, Vento 09
  - baryon-meson fluctuation model for the sea quarks: ZL, Ma, Schmidt 07.
- model calculations for the pion: ZL, Ma 04; Meissner, Metz, Schlegel, Goeke 08; Gamberg, Schlegel 10.

Boer-Mulders functions: model calculations

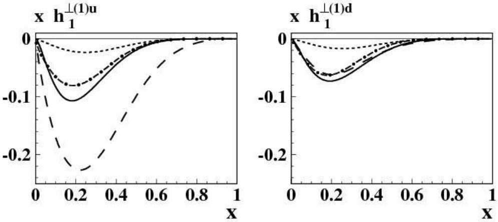


Figure: From arxiv:1108.1713

- solid curve: Pasquini and Yuan 10.
- dashed curve: Bacchetta, Conti, Radici 08.
- dotted curve: Courtoy, Scopetta, Vento 09.
- dashed-dotted curve: parametrization by Barone, Melis, Prokudin 10

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Theoretical status on the Boer-Mulders functions of the proton:

- Theoretical approaches and model calculations suggest $h_1^{\perp u}$ and $h_1^{\perp d}$ have the same sign (negative in SIDIS).
    - Bag model
    - constituent (light-cone) quark model
    - Axia-diquark spectator model
    - Large N_c limit (Pobylitsa 03)
    - Lattice calculation (QCDSF/UKQCD 06; Musch et.al 11)
    - GPD approach (Burkardt 05; Pasquini, Boffi 07; Burkardt, Hannafiou 08)
- $h_1^{\perp u}$ and $h_1^{\perp d}$ are expected to have the same order of magnitude
- The size of Boer-Mulders function is comparable to that of the other T-odd TMD, the Sivers function

- Boer-Mulders function vs Sivers function:

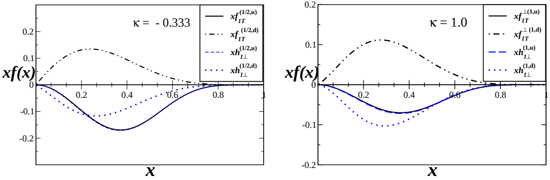


Figure: from Gamberg, Goldstein, Schlegel 2007

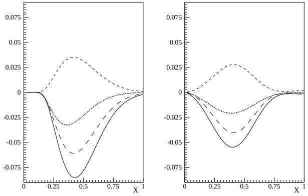
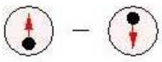


Figure: from Courtoy, Scopetta, Vento 2009

Boer-Mulders functions: extractions from data

$$i\,\frac{p_T^{\alpha}}{M}\,h_1^{\perp}(x,p_T^2) =$$



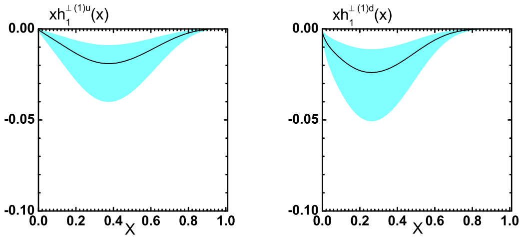
- parameterizations for the nucleon:
  - from anomalous tensor magnetic moment κ_T^q: Barone, Ma, prokudin 08;
  - from pp and pd Drell-Yan data (E866/Nusea): Zhang, ZL, Ma, Schmidt 08 (valence and sea quarks); ZL, Schmidt 09; Barone, Melis, and Prokudin 10 (sea quarks).
  - from SIDIS data (COMPASS, HERMES): Barone, Melis, and Prokudin 10.
- no existing parametrization for the pion.

Boer-Mulders functions: extraction from DY data

Parameterizations by Zhang, ZL, Ma, Schmidt 08; ZL, Schmidt 10:

$$h_1^{\perp q}(x,\boldsymbol{p}_T^2) = h_1^{\perp q}(x)\,\frac{1}{\pi p_{bm}^2}\,\exp\left(-\frac{\boldsymbol{p}_T^2}{p_{bm}^2}\right).$$

$$h_1^{\perp q}(x) = \omega H_q\,x^{c_q}\,(1-x)^b\,f_1^q(x);\quad h_1^{\perp \bar{q}}(x) = \frac{1}{\omega}H_{\bar{q}}\,x^{c_{\bar{q}}}\,(1-x)^b\,f_1^{\bar{q}}(x).$$

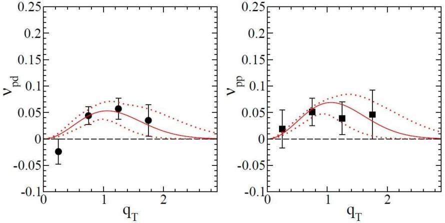


The shadow is the region allowed by the positivity bounds (corresponding to $0.48 < \omega < 2.1$).

- parameterizations confront data ($q_T \leq 2$ GeV).
- data from L.Y. Zhu, et,al. (E866/NuSea) 2006 (pd data), 2008 (pp data)

$$\frac{1}{\sigma}\frac{d\sigma}{d\Omega} = \frac{3}{4\pi}\frac{1}{\lambda+3}(1+\lambda\cos^2\theta+\mu\sin 2\theta\cos\phi+\frac{\nu}{2}\sin^2\theta\cos 2\phi).$$

$$\nu(x_1,x_2,q_T) = \frac{\mathcal{F}\left[((2\hat{\boldsymbol{h}}\cdot\boldsymbol{p}_{1T}\hat{\boldsymbol{h}}\cdot\boldsymbol{p}_{2T})-(\boldsymbol{p}_{1T}\cdot\boldsymbol{p}_{2T}))\frac{h_1^{\perp\,q}h_1^{\perp\,\bar{q}}}{M_1M_2}\right]}{\mathcal{F}[f_1^q f_1^{\bar{q}}]}$$



- TMD factorization is valid in low q_T region. At higher q_T, ν is subject to pQCD effects (Mirkes, Ohnemus 94; Boer, Vogelsang, 06; Berger, Qiu, Rodriguez-Pedraza 07).
- Product $h_1^{\perp\,q}h_1^{\perp\,\bar{q}}$ brings uncertainties on the absolute normalization for each flavor, which is constrained by the positivity bound (Bacchetta 99).

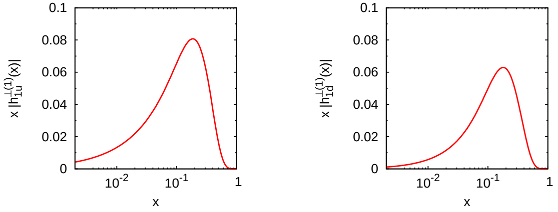
Boer-Mulders functions: extraction from SIDIS data

- Parameterizations by Barone, Melis, and Prokudin 10:

$$h_1^{\perp q}(x, k_T^2) = \lambda_q\, f_{1T}^{\perp q}(x, k_T^2) = \lambda_q\, \rho_q(x)\, \eta(k_T)\, f_1^q(x, \mathbf{k}_T^2)\,,$$

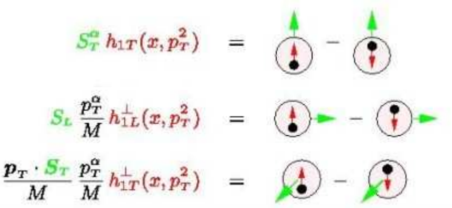
Extracted from $\cos 2\phi$ asymmetry in SIDIS (HERMES preliminary 09; COMPASS preliminary 08, 09)

$$\int \mathrm{d}\sigma \cos 2\phi = \frac{4\pi\alpha_{\rm em}^2 s}{Q^4} \int \sum_a e_a^2\, x(1-y)\, \{\mathcal{A}[f_1^a, D_1^a] + \frac{1}{2}\mathcal{B}[h_1^{\perp a}, H_1^{\perp a}]\}$$



- Knowledge on the Cahn effect $\mathcal{A}[f_1^a, D_1^a]$ (Boglinoe, Melis, Prokudin 11)

T-even chiral-odd TMDs: model calculations



- spectator model: Jakob, Mulders, Rodrigues, 97; Bacchetta, Conti, Radici 08
- constituent quark model: Pasquini and Yuan 10.
- bag model: Avakian, Efremov, Schweitzer, Yuan 10.
- light-cone diquark (spectator) model: She, Zhu, Ma 09; Zhu, Ma 11.

T-even chiral-odd TMDs: model calculations

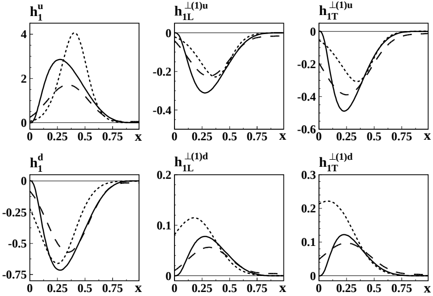


Figure: From arxiv:1108.1713

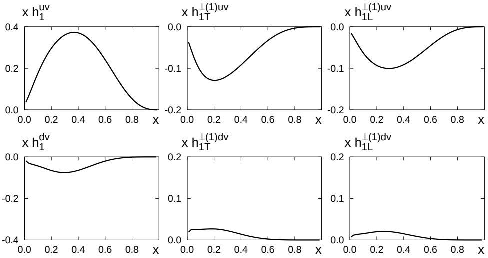
- solid curve: Pasquini and Yuan 10.
- dashed curve: Jakob, P. J. Mulders, and J. Rodrigues, 97
- dotted curve: Avakian, Efremov, Schweitzer, Yuan 10.
- Different model results qualitatively agree with each other.

T-even chiral-odd TMDs: model calculations

light-cone diquark model: She, Zhu, Ma 09; Zhu, Ma 11:

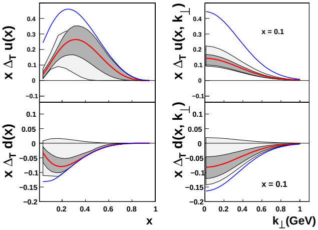
$$j^{u_v}(x,k_T^2) = \left[f_1^{u_v}(x,k_T^2) - \frac{1}{2}f_1^{d_v}(x,k_T^2)\right]W_S^j(x,k_T^2) - \frac{1}{6}f_1^{d_v}(x,k_T^2)W_V^j(x,k_T^2),$$

$$j^{d_v}(x,k_T^2) = -\frac{1}{3}f_1^{d_v}(x,k_T^2)W_V^j(x,k_T^2), \quad j = h_1,\ h_{1T}^{\perp},\ h_{1L}^{\perp}.$$



T-even chiral-odd TMDs: extraction from data

- most recent extraction of transversity: Anselmino et.al 08 (from HERMES and COMPASS SIDIS data on Collins asymmetry)



$$A_{UT}^{\sin(\phi_h+\phi_S)} = 2\,\frac{\int d\phi_S\, d\phi_h\, [d\sigma^{\uparrow} - d\sigma^{\downarrow}]\, \sin(\phi_h+\phi_S)}{\int d\phi_S\, d\phi_h\, [d\sigma^{\uparrow} + d\sigma^{\downarrow}]}$$

$$\propto \frac{\sum_q e_q^2\, \Delta_T q(x,k_\perp) \otimes \Delta^N D_{h/q^{\uparrow}}(z,p_\perp)}{\sum_q e_q^2\, f_{q/p}(x,k_\perp) \otimes D_{h/q}(z,p_\perp)}\,.$$

- no extractions of pretzelosity and $h_{1L}^{\perp}$.

$$h_1(P_1) + h_2(P_2) \to \gamma^*(q) + X \to \ell(l) + \bar{\ell}(l') + X.$$

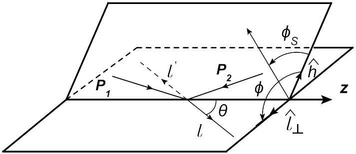
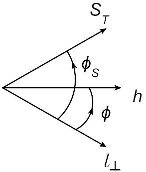
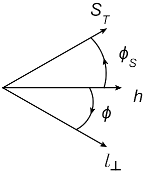


Figure: kinematics of Drell-Yan process in the Collins-Soper frame





- Left: Definition of ϕ and ϕ_S by Boer (99).
- right: Definition of ϕ and ϕ_S by Arnold, Metz, Schlegel (08).
- Two conventions are connected by $\phi \leftrightarrow -\phi$ and $\phi_S \leftrightarrow \phi_S - \phi$

- cross-section contributed by Chiral-odd TMDs: (Boer 99; Arnold, Metz, Schlegel 09)

$$\frac{d\sigma(h_1h_2\to l^+l^-X)}{dx_1\ dx_2\ d^2\boldsymbol{q}_T\ d\Omega}=\frac{\alpha_{em}^2}{12Q^2}\Big\{\sin^2\theta\cos2\phi F_{UU}^{\cos2\phi}+S_L\sin^2\theta\sin2\phi F_{LU}^{\sin2\phi}$$
$$+|\boldsymbol{S}_T|\sin^2\theta\left[\sin(2\phi+\phi_S)F_{TU}^{\sin(2\phi+\phi_S)}+\sin(2\phi-\phi_S)F_{TU}^{\sin(2\phi-\phi_S)}\right]+\cdots\Big\}.$$

$$F_{UU}^{\cos2\phi}=\mathcal{C}\Big[\frac{2(\boldsymbol{h}\cdot\boldsymbol{p}_{1T})(\boldsymbol{h}\cdot\boldsymbol{p}_{2T})-\boldsymbol{p}_{1T}\cdot\boldsymbol{p}_{2T}}{M_1M_2}h_1^\perp\bar{h}_1^\perp\Big],$$

$$F_{TU}^{\sin(2\phi-\phi_S)}=\mathcal{C}\Big[\frac{\boldsymbol{h}\cdot\boldsymbol{p}_{1T}}{M_1}h_1\bar{h}_1^\perp\Big],$$

$$F_{TU}^{\sin(2\phi+\phi_S)}=\mathcal{C}\Big[\frac{2(\boldsymbol{h}\cdot\boldsymbol{p}_{1T})[2(\boldsymbol{h}\cdot\boldsymbol{p}_{1T})(\boldsymbol{h}\cdot\boldsymbol{p}_{2T})-\boldsymbol{p}_{1T}\cdot\boldsymbol{p}_{2T}]-\boldsymbol{p}_{1T}^2(\boldsymbol{h}\cdot\boldsymbol{p}_{2T})}{2M_1M_2^2}\times h_{1T}^\perp\bar{h}_1^\perp\Big],$$

$$F_{LU}^{\sin2\phi}=\mathcal{C}\Big[\frac{2(\boldsymbol{h}\cdot\boldsymbol{p}_{1T})(\boldsymbol{h}\cdot\boldsymbol{p}_{2T})-\boldsymbol{p}_{1T}\cdot\boldsymbol{p}_{2T}}{M_1M_2}h_{1L}^\perp\bar{h}_1^\perp\Big].$$

Asymmetries in πN DY at COMPASS

The azimuthal asymmetries are defined as

$$A_{UT}^{\sin(2\phi\pm\phi_S)} = \frac{2\int_0^{2\pi} d\phi\, \sin(2\phi\pm\phi_S)\,[d\sigma^{\uparrow} - d\sigma^{\downarrow}]}{\int_0^{2\pi} d\phi [d\sigma^{\uparrow} + d\sigma^{\downarrow}]}$$

for transversely polarized target, and

$$A_{UL}^{\sin(2\phi)} = \frac{2\int_0^{2\pi} d\phi\, \sin 2\phi\,[d\sigma^{\Rightarrow} - d\sigma^{\Leftarrow}]}{\int_0^{2\pi} d\phi [d\sigma^{\Rightarrow} + d\sigma^{\Leftarrow}]}.$$

for longitudinally polarized target.

$$F_{UT}^{\sin(2\phi-\phi_S)}(h_1,h_2) = -F_{TU}^{\sin(2\phi-\phi_S)}(h_2,h_1)$$
$$F_{UT}^{\sin(2\phi+\phi_S)}(h_1,h_2) = -F_{TU}^{\sin(2\phi+\phi_S)}(h_2,h_1)$$
$$F_{UL}^{\sin 2\phi}(h_1,h_2) = -F_{LU}^{\sin 2\phi}(h_2,h_1)$$

Asymmetries in πN DY at COMPASS

- Kinematics:

$$\sqrt{s} = 18.9\ \mathrm{GeV},\ 0.1 < x_1 < 1,\ 0.05 < x_2 < 0.5,$$
$$4 \leqslant M \leqslant 8.5\ \mathrm{GeV},\ 0 \leqslant q_T \leqslant 2\ \mathrm{GeV}.$$

190 GeV pion beams collide on nucleon target

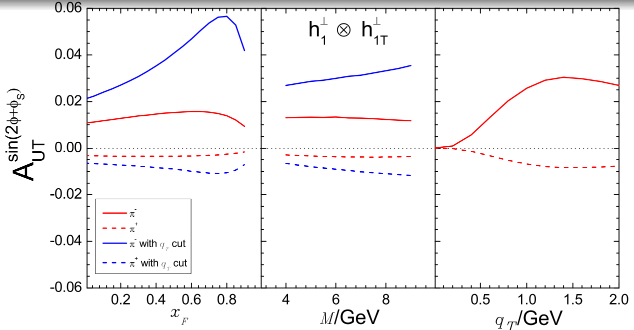
- isospin symmetry and charge conjugation ($\bar{\Phi}^{q\,[\Gamma]} = \Phi^{\bar{q}\,[\Gamma]}$for $\gamma^\mu,\ i\sigma^{\mu\nu}\gamma_5$) $\Rightarrow$ simple flavor structure of the pion:

$$f^{\bar{u}/\pi^-} = f^{d/\pi^-} = f^{\bar{d}/\pi^+} = f^{u/\pi^+}, \quad f = f_1 \text{ or } h_1^\perp$$

- ideal in unraveling the flavor content of nucleon TMDs:

$$f_1 \otimes f_{1T}^\perp; \quad h_1^\perp \otimes h_1^\perp; \quad h_1^\perp \otimes h_{1T}^\perp; \quad h_1^\perp \otimes h_1; \quad h_1^\perp \otimes h_{1L}^\perp$$

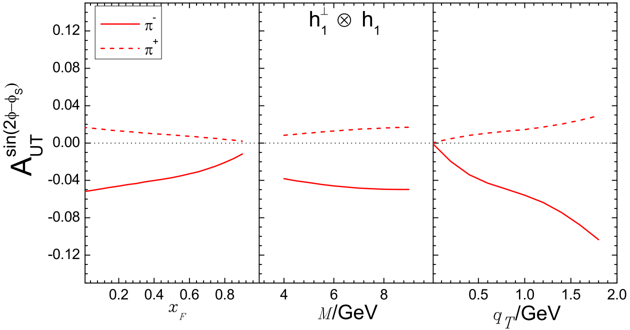
Asymmetries in $\pi p^{\uparrow}$ DY at COMPASS by $h_{1T}^{\perp}$



- $A_{UT}^{\sin(2\phi+\phi_S)}$ in $\pi^- p$ DY: positive. $(-h_1^{\perp\bar{u}/\pi^-} \otimes h_{1T}^{\perp u})$
- $A_{UT}^{\sin(2\phi+\phi_S)}$ in $\pi^+ p$ DY: negative, consistent with zero $(-h_1^{\perp\bar{d}/\pi^+} \otimes h_{1T}^{\perp d})$. Suppressed by the smaller size of $h_{1T}^{\perp d}$
- Blue curves: q_T is integrated from 1 GeV to 2 GeV, increase the asymmetries.

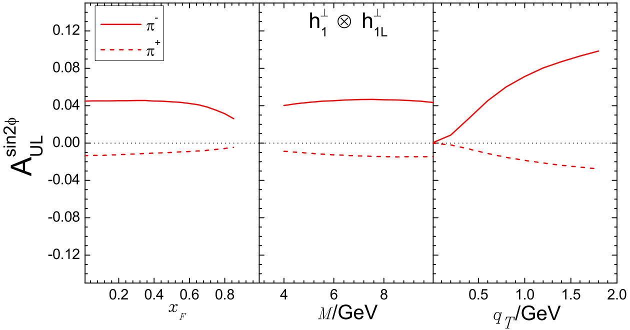
ZL, Ma and She, 11

Asymmetries in $\pi p^{\uparrow}$ DY at COMPASS by h_1



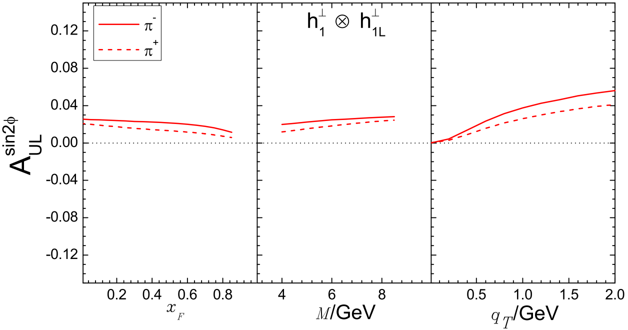
- $A_{UT}^{\sin(2\phi-\phi_S)}$ in $\pi^- p$ DY: negative $(-\, h_1^{\perp\bar{u}/\pi^-} \otimes h_1^u)$.
- $A_{UT}^{\sin(2\phi-\phi_S)}$ in $\pi^+ p$ DY: positive $(-\, h_1^{\perp\bar{d}/\pi^+} \otimes h_1^d)$.

Asymmetries in $\pi p^{\Rightarrow}$ DY at COMPASS by $h_{1L}^{\perp}$



- $A_{UL}^{\sin 2\phi}$ in $\pi^- p$ DY: positive $(-h_1^{\perp \bar{u}/\pi^-} \otimes h_{1L}^{\perp u})$.
- $A_{UL}^{\sin 2\phi}$ in $\pi^+ p$ DY: negative $(-h_1^{\perp \bar{d}/\pi^+} \otimes h_{1L}^{\perp d})$.

Asymmetries in $\pi D^{\Rightarrow}$ DY at COMPASS by $h_{1L}^{\perp}$



- $A_{UL}^{\sin(2\phi)}$ in $\pi^- D$ DY: positive $(-h_1^{\perp\bar{u}/\pi^-} \otimes (h_{1L}^{\perp u} + h_{1L}^{\perp d}))$.
- $A_{UL}^{\sin(2\phi)}$ in $\pi^+ D$ DY: positive $(-h_1^{\perp\bar{d}/\pi^+} \otimes (h_{1L}^{\perp u} + h_{1L}^{\perp d}))$.
- Asymmetries for the π^- and π^+ beams are similar.

q_T weighted asymmetries

- Bacchetta, Conti, Radici, Guagnelli 10 :

$$A_{UT}^{q_T \sin(2\phi-\phi_{S_2})} = 2\,\frac{\left\langle \frac{q_T}{M_1}\,\sin(2\phi-\phi_{S_2})\right\rangle_{UT}}{\langle 1\rangle_{UU}} = -2\,\frac{B(y)}{A(y)}\,\frac{\sum_a e_a^2\, x_1 h_1^{\perp(1)\bar{a}}(x_1)\, x_2 h_1^a(x_2)}{\sum_a e_a^2\, x_1 f_1^{\bar{a}}(x_1)\, x_2 f_1^a(x_2)}\,,$$

$$A_{UT}^{q_T^3 \sin(2\phi+\phi_{S_2})} = 2\,\frac{\left\langle \frac{q_T^3}{6M_1M_2^2}\,\sin(2\phi+\phi_{S_2})\right\rangle_{UT}}{\langle 1\rangle_{UU}}$$

$$= -\,2\,\frac{B(y)}{A(y)}\,\frac{\sum_a e_a^2\, x_1 h_1^{\perp(1)\bar{a}}(x_1)\, x_2 h_{1T}^{\perp(2)a}(x_2)}{\sum_a e_a^2\, x_1 f_1^{\bar{a}}(x_1)\, x_2 f_1^a(x_2)}\,,$$

$$A_{UL}^{q_T \sin 2\phi} = 2\,\frac{\left\langle \frac{q_T^2}{4M_1M_2}\,\sin 2\phi\right\rangle_{UL}}{\langle 1\rangle_{UU}} = -2\,\frac{B(y)}{A(y)}\,\frac{\sum_a e_a^2\, x_1 h_1^{\perp(1)\bar{a}}(x_1)\, x_2 h_{1L}^{\perp(1)a}(x_2)}{\sum_a e_a^2\, x_1 f_1^{\bar{a}}(x_1)\, x_2 f_1^a(x_2)}\,,$$

- ZL, Ma, Schmidt 06:

$$A_{UU}^{q_T^2 \cos 2\phi} = 2\,\frac{\left\langle \frac{q_T^2}{4M_1M_2}\,\cos(2\phi)\right\rangle_{UU}}{\langle 1\rangle_{UU}} = 2\,\frac{B(y)}{A(y)}\,\frac{\sum_a e_a^2\, x_1 h_1^{\perp(1)\bar{a}}(x_1)\, x_2 h_1^{\perp(1)a}(x_2)}{\sum_a e_a^2\, x_1 f_1^{\bar{a}}(x_1)\, x_2 f_1^a(x_2)}\,,$$

- $A(y)$ and $B(y)$ are defined as:

$$A(y) = \frac{1}{2} - y + y^2 \overset{\text{cm}}{=} \frac{1}{4}(1+\cos^2\theta), \qquad B(y) = y(1-y) \overset{\text{cm}}{=} \frac{1}{4}\sin^2\theta.$$

- In the region where x_1 and x_2 are not small (ZL, Ma, Schmidt 06):

$$\frac{A_{UU,\pi^- D}^{q_T^2\ \cos 2\phi}}{A_{UU,\pi^- p}^{q_T^2\ \cos 2\phi}} \approx \frac{1+\frac{h_1^{\perp(1)d}(x_2)}{h_1^{\perp(1)u}(x_2)}}{1+\frac{f_1^d(x_2)}{f_1^u(x_2)}}; \quad \frac{A_{UU,\pi^+ p}^{q_T^2\ \cos 2\phi}}{A_{UU,\pi^- p}^{q_T^2\ \cos 2\phi}} \approx \frac{\frac{h_1^{\perp(1)d}(x_2)}{h_1^{\perp(1)u}(x_2)}}{\frac{f_1^d(x_2)}{f_1^u(x_2)}}.$$

similarly:

$$\frac{A_{UT,\pi^- D}^{q_T\ \sin(2\phi-\phi_{S_2})}}{A_{UT,\pi^- p}^{q_T\ \sin(2\phi-\phi_{S_2})}} \approx \frac{1+\frac{h_1^d(x_2)}{h_1^u(x_2)}}{1+\frac{f_1^d(x_2)}{f_1^u(x_2)}}; \quad \frac{A_{UT,\pi^+ p}^{q_T\ \sin(2\phi-\phi_{S_2})}}{A_{UT,\pi^- p}^{q_T\ \sin(2\phi-\phi_{S_2})}} \approx \frac{\frac{h_1^d(x_2)}{h_1^u(x_2)}}{\frac{f_1^d(x_2)}{f_1^u(x_2)}}.$$

- Apply different beams/targets to explore the flavor contents of TMDs

$$p^{\uparrow/\rightarrow}(P_1) + p(P_2) \to \gamma^*(q) + X \to \ell(l) + \bar{\ell}(l') + X.$$

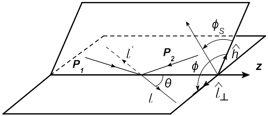


Figure: kinematics of Drell-Yan process in the Collins-Soper frame

- current/planned polarized pp program

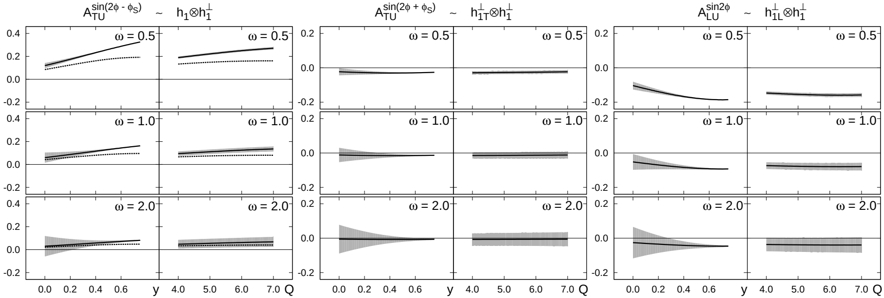
	c.m. energy (GeV)	mode
RHIC	200, 510	collider
RHIC	22	fixed-target
J-PARC	10	fixed-target
E906	15	fixed-target
NICA	$12 \div 27$	collider
SPASCHARM	10.7	fixed-target

- In PRD84 074036 (ZL, Ma, Zhu 11), the asymmetries is defined as

$$\begin{aligned}
\tilde{A}_{TU}^{\sin(2\phi-\phi_S)}(x_1,\,x_2,\,q_T) &= \frac{F_{TU}^{\sin(2\phi-\phi_S)}}{F_{UU}^{1}},\\
\tilde{A}_{TU}^{\sin(2\phi+\phi_S)}(x_1,\,x_2,\,q_T) &= \frac{F_{TU}^{\sin(2\phi+\phi_S)}}{F_{UU}^{1}},\\
\tilde{A}_{LU}^{\sin 2\phi}(x_1,\,x_2,\,q_T) &= \frac{F_{LU}^{\sin 2\phi}}{F_{UU}^{1}}.
\end{aligned}$$

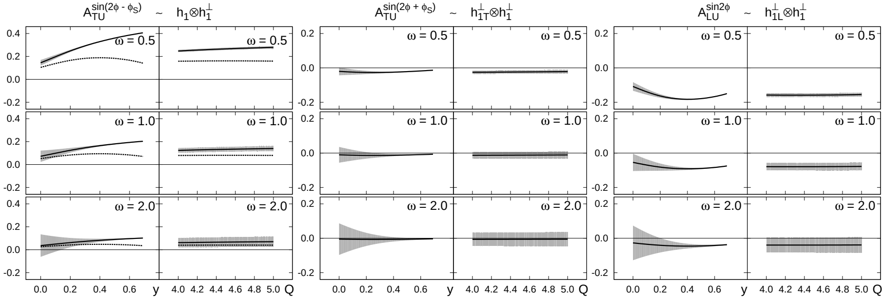
- In an analogy with the definition of the $\cos 2\phi$: $\nu = 2F_{UU}^{\cos 2\phi}/F_{UU}^{1}$

$$A_{PU}^{W(\phi,\phi_S)} = \frac{2\,A(y)\,\int_0^{2\pi} d\phi W(\phi,\phi_S)[d\sigma^{\uparrow/\Rightarrow} - d\sigma^{\uparrow/\Leftarrow}]}{B(y)\,\int_0^{2\pi} d\phi[d\sigma^{\uparrow/\Rightarrow} + d\sigma^{\uparrow/\Leftarrow}]}, \quad P = L, T$$



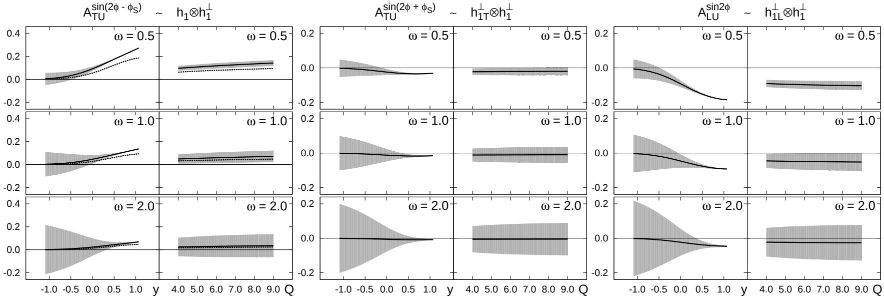
- $h_1^\perp$ from the parametrization by ZL, Schmidt 09; T-even chiral-odd TMDs from light-cone diquark model (She, Zhu, Ma 09; Zhu, Ma 11)
- Dashed line in the left panels: calculated from the parameterization for h_1 by Anselmino et.al., 08
- Shaded regions: ranges of the asymmetries by considering the additional contribution from $h_1^{\bar{q}}$, $h_{1T}^{\perp\bar{q}}$ and $h_{1L}^{\perp\bar{q}}$

$$\sqrt{s} = 15 \text{ GeV}, \quad 0.3 < x_1 < 0.7, \quad 0.1 < x_2 < 0.3,$$
$$0 < q_T < 1 \text{ GeV}, \quad 4 \text{ GeV} < Q < 7 \text{ GeV},$$



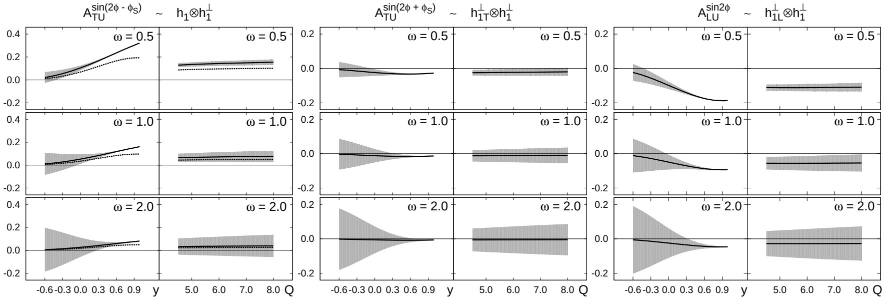
kinematical cuts at J-PARC:

$$4 \text{ GeV} < Q < 5 \text{ GeV}, \quad 0 < q_T < 1 \text{ GeV},$$
$$0.5 < x_1 < 0.9,$$



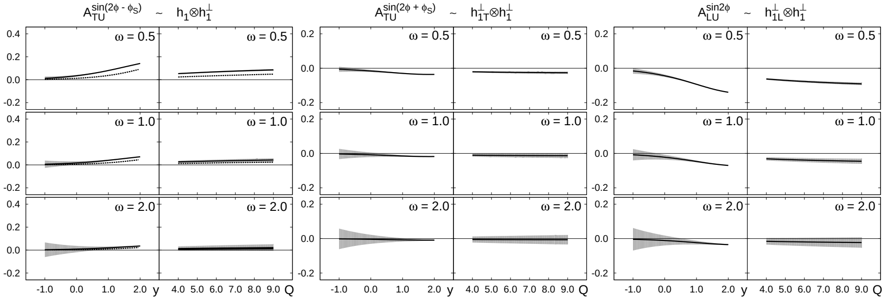
The kinematics cuts at NICA:

$$\sqrt{s} = 27 \text{ GeV}, \quad 4 \text{ GeV} < Q < 9 \text{ GeV},$$
$$0 < q_T < 1 \text{ GeV}, \quad 0.1 < x_1 < 0.8,$$



RHIC kinematics for the fixed-target experiment:

$$\sqrt{s} = 22\ \text{GeV}, \quad 4.5\ \text{GeV} < Q < 8\ \text{GeV},$$
$$0 < q_T < 1\ \text{GeV}, \quad 0.2 < x_1 < 0.6,$$



kinematics for collider experiment at RHIC-STAR:

$$\sqrt{s} = 200 \text{ GeV}, \quad 4 \text{ GeV} < Q < 9 \text{ GeV},$$
$$0 < q_T < 1 \text{ GeV}, \quad -1 < y < 2.$$

- angular dependence of DY at $\mathcal{O}(1/Q)$ (Arnold, Metz, Schlegel 09, ZL, Schmidt 11):

$$\frac{d\sigma^{\text{twist-3}}}{dx_1 dx_2 d^2\boldsymbol{q}_T\, d\Omega} = \frac{\alpha_{em}^2}{3Q^2}\sin 2\theta \Big\{ \cos\phi\, F_{UU}^{\cos\phi} + S_{1L}\sin\phi\, F_{LU}^{\sin\phi} + S_{2L}\sin\phi\, F_{UL}^{\sin\phi} + |\vec{S}_{1T}|\Big[\sin(\phi_1+\phi)F_{TU}^{\sin(\phi_{S_1}+\phi)} + \sin(\phi_{S_1}-\phi)F_{TU}^{\sin(\phi_{S_1}-\phi)}\Big] + |\vec{S}_{2T}|\Big[\sin(\phi_{S_2}+\phi)F_{UT}^{\sin(\phi_{S_2}+\phi)} + \sin(\phi_{S_2}-\phi)F_{UT}^{\sin(\phi_{S_2}-\phi)}\Big]$$

- TMD correlator at twist-3 (Goeke et.al, 05; Bacchetta et.al, 06):

$$\Phi(x,p_T) = \ldots + \frac{M}{2P^+}\Big\{ e - i\, e_s\, \gamma_5 - e_T^\perp \frac{\epsilon_T^{\rho\sigma} p_{T\rho} S_{T\sigma}}{M} + f^\perp \frac{\not{p}_T}{M} - f_T'\, \epsilon_T^{\rho\sigma}\gamma_\rho S_{T\sigma} - f_s^\perp \frac{\epsilon_T^{\rho\sigma}\gamma_\rho p_{T\sigma}}{M} + g_T'\,\gamma_5\, \not{S}_T + g_s^\perp \gamma_5 \frac{\not{p}_T}{M} - g^\perp \gamma_5 \frac{\epsilon_T^{\rho\sigma}\gamma_\rho p_{T\sigma}}{M} + h_s \frac{[\not{n}_+, \not{n}_-]\gamma_5}{2} + h_T^\perp \frac{[\not{S}_T, \not{p}_T]\gamma_5}{2M} + i\, h \frac{[\not{n}_+, \not{n}_-]}{2} \Big\}.$$

- h_L, h_T, $h_T^\perp$, and h might be probed in Drell-Yan process

un/single-polarized structure functions at twist-3

$$
\begin{aligned}
F_{UU}^{\cos\phi} &= \frac{2}{Q}\,\mathcal{C}\left[(\boldsymbol{h}\cdot\boldsymbol{k}_{1T})\left(\widehat{f}^{\perp}\overline{f}_1 - \frac{M_2}{M_1}h_1^{\perp}\widehat{\overline{h}}\right) - (\boldsymbol{h}\cdot\boldsymbol{k}_{2T})\left(f_1\widehat{\overline{f}}^{\perp} - \frac{M_1}{M_2}\widehat{h}\,\overline{h}_1^{\perp}\right)\right] \\
F_{LU}^{\sin\phi} &= \frac{2}{Q}\,\mathcal{C}\left[(\boldsymbol{h}\cdot\boldsymbol{k}_{1T})\left(\widehat{f}_L^{\perp}\overline{f}_1 + \frac{M_2}{M_1}\,h_{1L}^{\perp}\,\widehat{\overline{h}}\right) - (\boldsymbol{h}\cdot\boldsymbol{k}_{2T})\left(g_{1L}\widehat{\overline{g}}^{\perp} + \frac{M_1}{M_2}\widehat{h}_L\,\overline{h}_1^{\perp}\right)\right] \\
F_{TU}^{\sin(\phi_{S_1}-\phi)} &= \frac{1}{Q}\,\mathcal{C}\Big[2M_1\,\widehat{f}_T\overline{f}_1 + 2M_2\;h_1\widehat{\overline{h}} \\
&\quad + (\boldsymbol{k}_{1T}\cdot\boldsymbol{k}_{2T})\left(\frac{f_{1T}^{\perp}\widehat{\overline{f}}^{\perp}}{M_1} - \frac{g_{1T}\widehat{\overline{g}}^{\perp}}{M_1} - \frac{\widehat{h}_T\overline{h}_1^{\perp}}{M_2} + \frac{\widehat{h}_T^{\perp}\overline{h}_1^{\perp}}{M_2}\right)\Big] \\
F_{TU}^{\sin(\phi_{S_1}+\phi)} &= \frac{1}{Q}\,\mathcal{C}\Big[-\left(2\,(\boldsymbol{h}\cdot\boldsymbol{k}_{1T})^2 - \boldsymbol{k}_{1T}^2\right)\left(\frac{\widehat{f}_T^{\perp}\overline{f}_1}{M_1} + \frac{M_2\,h_{1T}^{\perp}\widehat{\overline{h}}}{M_1^2}\right) \\
&\quad + (2\boldsymbol{h}\cdot\boldsymbol{k}_{1T}\boldsymbol{h}\cdot\boldsymbol{k}_{2T} - \boldsymbol{k}_{1T}\cdot\boldsymbol{k}_{2T})\left(\frac{f_{1T}^{\perp}\widehat{\overline{f}}^{\perp}}{M_1} + \frac{g_{1T}\widehat{\overline{g}}^{\perp}}{M_1} + \frac{\widehat{h}_T\overline{h}_1^{\perp}}{M_2} + \frac{\widehat{h}_T^{\perp}\overline{h}_1^{\perp}}{M_2}\right)\Big]
\end{aligned}
$$

$$
\widehat{f} = x_1\left((1-c)\,f + c\,\tilde{f}\right),\quad \widehat{\overline{f}} = x_2\left(c\,\overline{f} + (1-c)\,\tilde{\overline{f}}\right).\quad c = \frac{1}{2}\;:\;\text{CS frame}
$$

Accessible in DY?

- lesson from SIDIS

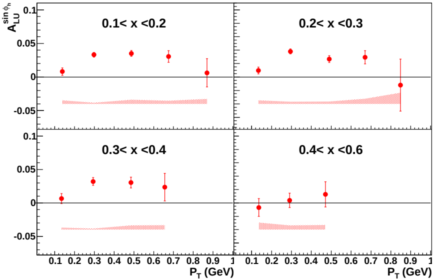


Figure: $\sin\phi$ asymmetry measured by CLAS (arxiv:1106.2293) for $Q > 1$ GeV, $W^2 > 4$ GeV^2 and $0.4 < z < 0.7$

- SIDIS at twist-3 (Bacchetta et.al,06)

$$F_{LU}^{\sin\phi_h} = \frac{2M}{Q}\mathcal{C}\left\{\frac{\hat{\boldsymbol{P}}_{\boldsymbol{T}}\cdot\boldsymbol{p_T}}{M}\left[\frac{M_h}{M}\,h_1^{\perp}\,\frac{\tilde{E}}{z}+x\,g^{\perp}D_1\right]-\frac{\hat{\boldsymbol{P}}_{\boldsymbol{T}}\cdot\boldsymbol{k_T}}{M_h}\left[\frac{M_h}{M}\,f_1\,\frac{\tilde{G^{\perp}}}{z}+x\,eH_1^{\perp}\right]\right\}.$$

- Drell-Yan at low mass ($2.0 < Q < 2.5$GeV) region (COMPASS, SPASCHARM)?

Summary

- The πN Drell-Yan program at COMPASS provides great opportunity to explore the flavor contents of chiral-odd TMDs
- Single polarized pp Drell-Yan programme at E906, RHIC, J-PARC, NICA is vital to access h_1, h_{1L} in the valence and sea region, through asymmetries $A_{TU}^{\sin(2\phi-\phi_S)}$ and $A_{LU}^{\sin 2\phi}$.
- Possibility for accessing twist-3 TMDs by Drell-Yan is explored.