Boer-Mulders and Sivers effects in the Drell-Yan process

Drell-Yan scattering and the structure of hadrons
Trento, May 21-25, 2012

Stefano Melis
European Centre for Theoretical Studies
in Nuclear Physics and Related Areas
ECT*, Trento

In collaboration with
M. Anselmino, E. Boglione, V. Barone, A. Prokudin

Outline

- Boer-Mulders extraction from SIDIS data (2010)
- Boer-Mulders in DY processes (2010)
- Open problems
- Sivers in DY processes (2009)
- Sivers from SIDIS data with TMD evolution
- Sivers TMD evolution & DY processes

Boer-Mulders function extraction from $A^{\cos 2\phi}$ in unpolarized SIDIS

V. Barone, S. Melis and A. Prokudin Phys. Rev. D81, 114026 (**2010**)

Extraction of the Boer-Mulders functions

- The angular distribution in the unpolarized SIDIS can be written as

$$d\sigma = A + B\cos\phi + C\cos 2\phi$$

- $A =\propto f_1 \otimes D_1$ is the usual ϕ-independent contribution
- $C \propto h_1^\perp \otimes H_1^\perp + \frac{1}{Q^2} f_1 \otimes D_1$ BM effect+Twist-4 Cahn effect

$$\boxed{A^{\cos 2\phi} = 2\frac{\int d\sigma \cos 2\phi}{\int d\sigma} = \frac{C}{A}}$$

Extraction of the Boer-Mulders functions

- The angular distribution in the unpolarized SIDIS can be written as

$$d\sigma = A + B\cos\phi + C\cos 2\phi$$

- $A = \propto f_1 \otimes D_1$ is the usual ϕ-independent contribution
- $C \propto h_1^{\perp} \otimes H_1^{\perp} + \frac{1}{Q^2} f_1 \otimes D_1$ BM effect+Twist-4 Cahn effect

Unpolarized PDF&FF gaussian as in Anselmino et al. [1]

[1] Anselmino et. Phys. ReV D71, 074006 (2005)

Extraction of the Boer-Mulders functions

- The angular distribution in the unpolarized SIDIS can be written as

$$d\sigma = A + B\cos\phi + C\cos 2\phi$$

- $A = \propto f_1 \otimes D_1$ is the usual ϕ-independent contribution
- $C \propto h_1^{\perp} \otimes H_1^{\perp} + \frac{1}{Q^2} f_1 \otimes D_1$ BM effect+Twist-4 Cahn effect

Collins function as in Anselmino et. al arXiv: 0812.4366v1

Extraction of the Boer-Mulders functions

➢ The angular distribution in the unpolarized SIDIS can be written as

$$d\sigma = A + B\cos\phi + C\cos 2\phi$$

- $A = \propto f_1 \otimes D_1$ is the usual ϕ-independent contribution
- $C \propto h_1^\perp \otimes H_1^\perp + \frac{1}{Q^2} f_1 \otimes D_1$ BM effect+Twist-4 Cahn effect

BM that we want to extract from the fit of $A^{\cos 2\phi}$ data

Extraction of the Boer-Mulders functions

- Simple parametrization of the Boer-Mulders functions:
  - $h_1^{\perp q}(x, k_\perp) = \lambda_q \, f_{1T}^{\perp q}(x, k_\perp)$ for valence quarks
  - $h_1^{\perp q}(x, k_\perp) = -|f_{1T}^{\perp q}(x, k_\perp)|$ for sea quarks

V. Barone, S. Melis and A. Prokudin Phys. Rev. D81, 114026 (2010)

Extraction of the Boer-Mulders functions

- Simple parametrization of the Boer-Mulders functions:
  - $h_1^{\perp q}(x, k_\perp) = \lambda_q \, f_{1T}^{\perp q}(x, k_\perp)$ for valence quarks
  - $h_1^{\perp q}(x, k_\perp) = -|f_{1T}^{\perp q}(x, k_\perp)|$ for sea quarks

- Inspired by models:

$$h_1^{\perp q}(x, k_\perp) = \frac{\mathcal{K}_T^q}{\mathcal{K}^q} f_{1T}^{\perp q}(x, k_\perp)$$

Tensor magnetic moment

Anomalous magnetic moment

Burkardt, Phys. Rev. D72, 094020 (2005)

Gockeler, Phys.Rev.Lett.98:222001,2007.

Extraction of the Boer-Mulders functions

- Simple parametrization of the Boer-Mulders functions:

  - $h_1^{\perp q}(x, k_\perp) = \lambda_q \, f_{1T}^{\perp q}(x, k_\perp)$ for valence quarks
  - $h_1^{\perp q}(x, k_\perp) = -|f_{1T}^{\perp q}(x, k_\perp)|$ for sea quarks

- Models inspired:

$$h_1^{\perp q}(x, k_\perp) = \frac{\mathcal{K}_T^q}{\mathcal{K}^q} f_{1T}^{\perp q}(x, k_\perp)$$

  - $h_1^{\perp u}(x, k_\perp) \simeq 1.80 f_{1T}^{\perp u}(x, k_\perp) < 0$
  - $h_1^{\perp d}(x, k_\perp) \simeq -0.94 f_{1T}^{\perp d}(x, k_\perp) < 0$

Extraction of the Boer-Mulders functions

FIT I

- HERMES proton and deuteron target (x,z,P_T) charged hadrons

HERMES, Giordano:arXiv:0901.2438

- COMPASS deuteron target (x,z) charged hadrons

COMPASS, Kafer: arXiv 0808.0114

- 2 free parameters:

$\lambda_u \quad \lambda_d$

✔GRV98 PDF

✔DSS FF

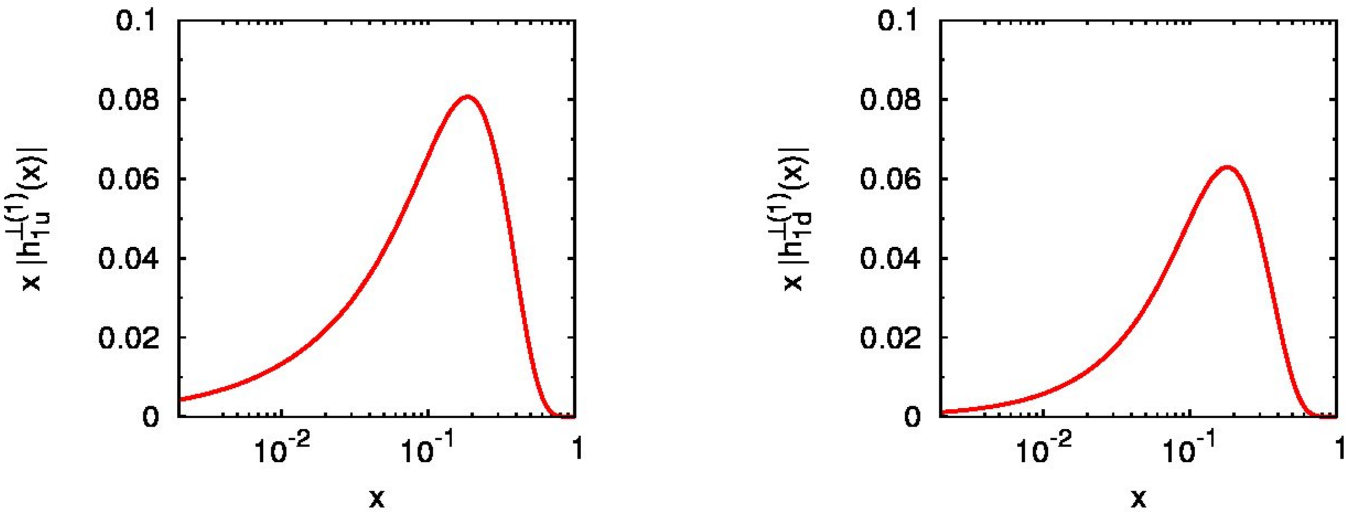
✔Gaussians: $\langle k_\perp^2\rangle$=0.25 $(GeV/c)^2$
$\langle p_\perp^2\rangle$=0.20 $(GeV/c)^2$
(from Cahn effect)

✔ $h_1^{\perp q}(x,k_\perp) = \lambda_q \, f_{1T}^{\perp q}(x,k_\perp)$

✔ $h_1^{\perp q}(x,k_\perp) = -|f_{1T}^{\perp q}(x,k_\perp)|$

Sivers functions from
Anselmino et al. Eur. Phys. J. A39,89

Extraction of the Boer-Mulders functions



$\diamond\chi^2/d.o.f. = 3.73$

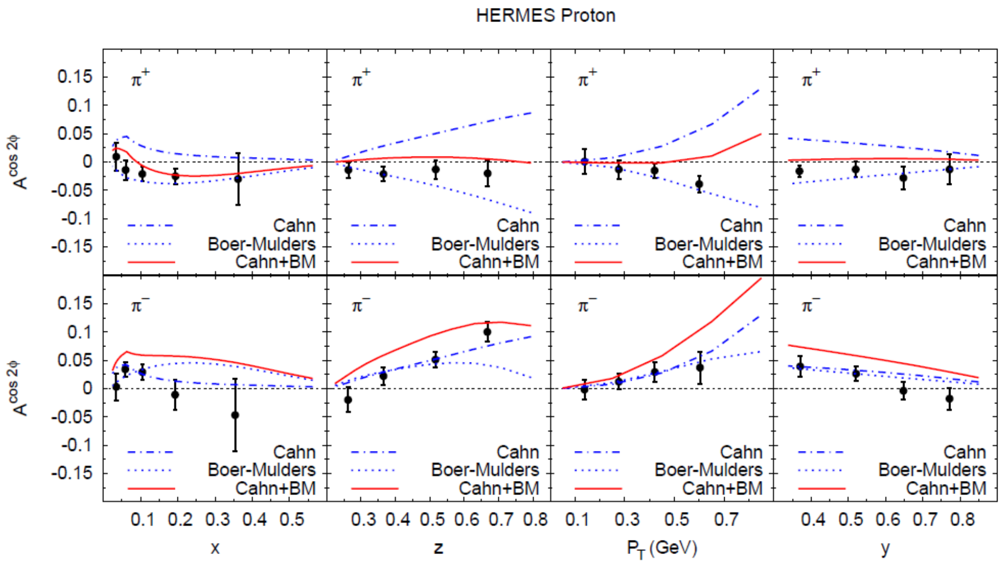
$\bullet\lambda_u = 2.0 \pm 0.1$

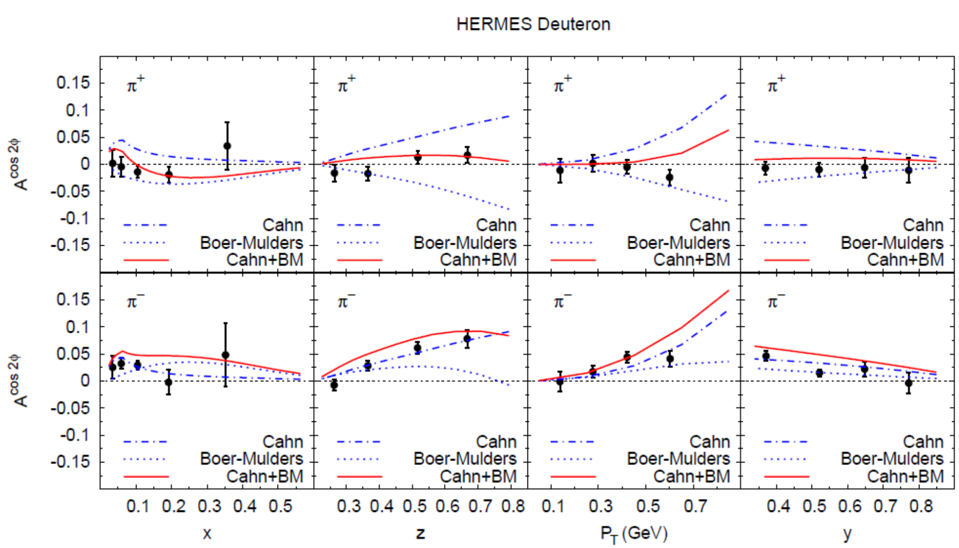
$\bullet\lambda_d = -1.11^{+0.00}_{-0.02}$

$\Rightarrow$ $h_1^{\perp d}$ and $h_1^{\perp u}$ both negative

Compatible with models predictions

Extraction of the Boer-Mulders functions

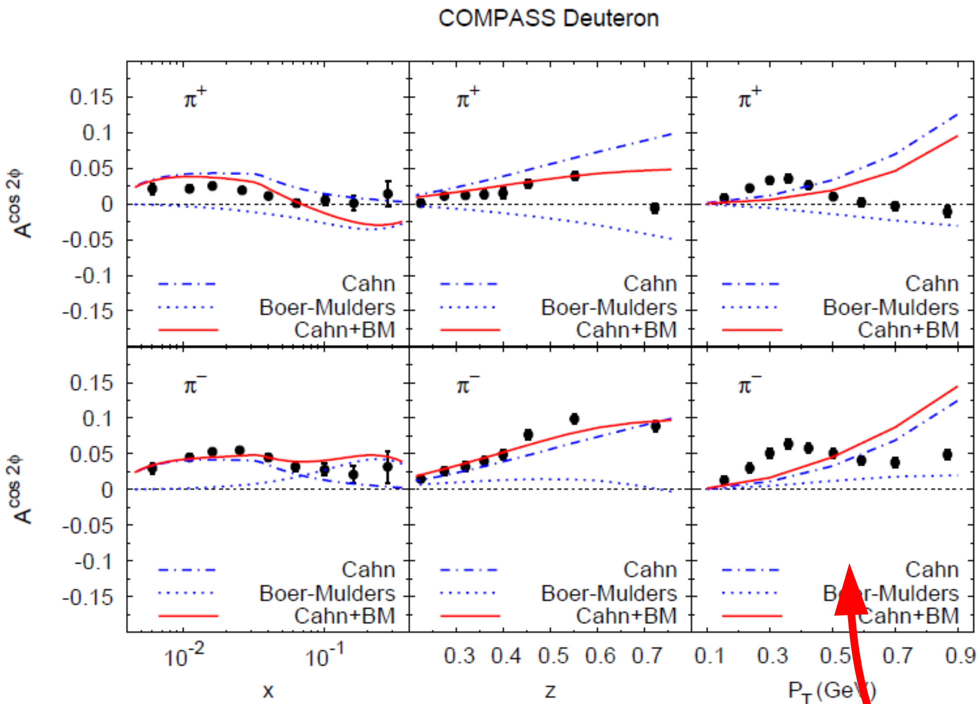




- ✔Cahn effect (Twist-4)comparable to BM effect
- ✔Same sign of Cahn contribution for positive and negative pions
- ✔BM contribution opposite in sign for positive and negative pions

HERMES, Giordano:arXiv:0901.2438

Extraction of the Boer-Mulders functions



- ✔Cahn effect (Twist-4)comparable to BM effect
- ✔Same sign of Cahn contribution for positive and negative pions
- ✔BM contribution opposite in sign for positive and negative pions

Data in p_T not included in the fit

COMPASS, Kafer: arXiv 0808.0114

Extraction of the Boer-Mulders Function

- The Cahn effect is a crucial ingredient

✔ Gaussians: $\langle k_\perp^2 \rangle = 0.25\ (\mathrm{GeV}/c)^2$
$\langle p_\perp^2 \rangle = 0.20\ (\mathrm{GeV}/c)^2$

From Ref.[*]: analysis of Cahn $\cos\phi$ effect from EMC data

COMPASS	HERMES
$\langle k_\perp^2 \rangle = 0.25\ (\mathrm{GeV}/c)^2$	$\langle k_\perp^2 \rangle = 0.18\ (\mathrm{GeV}/c)^2$
$\langle p_\perp^2 \rangle = 0.20\ (\mathrm{GeV}/c)^2$	$\langle p_\perp^2 \rangle = 0.20\ (\mathrm{GeV}/c)^2$
~EMC	~HERMES MC

[*] Anselmino et al. Phys. Rev. D71 074006 (2005)

Extraction of the Boer-Mulders Function

- FIT II

COMPASS

$\langle k_\perp^2 \rangle = 0.25\ (GeV/c)^2$

$\langle p_\perp^2 \rangle = 0.20\ (GeV/c)^2$

~EMC

HERMES

$\langle k_\perp^2 \rangle = 0.18\ (GeV/c)^2$

$\langle p_\perp^2 \rangle = 0.20\ (GeV/c)^2$

~HERMES MC

FIT II

$\diamond \chi^2/d.o.f. = 2.41$

$\bullet \lambda_u = 2.1 \pm 0.1$

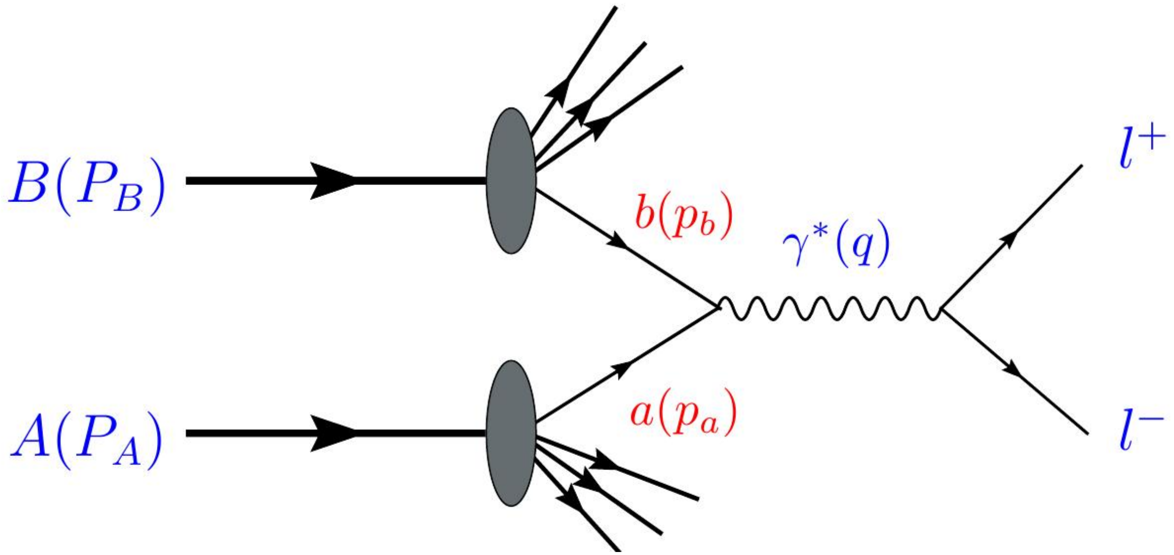
$\bullet \lambda_d = -1.11^{+0.00}_{-0.02}$

Better description of HERMES but the BM is unchanged

Conclusions I ...2010

- u and d BM functions have the same sign. They are compatible with models
- Twist-4 Cahn effect cannot be neglected at HERMES and COMPASS.
- Different average transverse momenta for different experiments or evidence of evolution?

Boer-Mulders function extraction from ν in unpolarized DY processes



Boer-Mulders function in DY from fits

- General expression for the dilepton angular distributions in the dilepton rest frame:

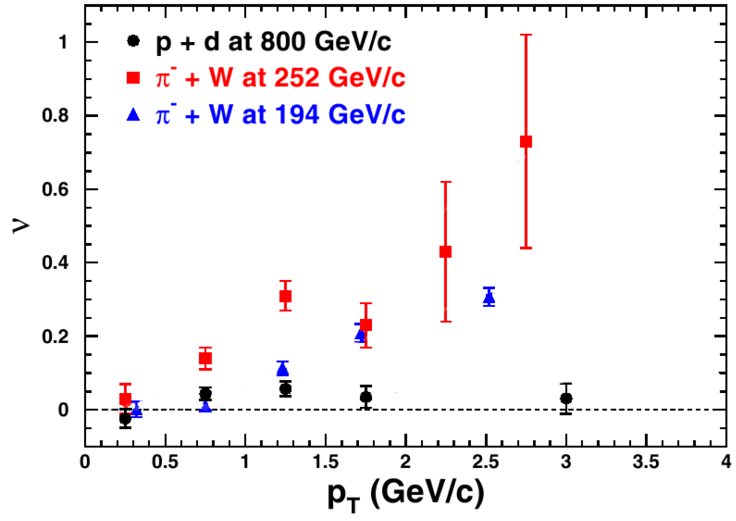
$$\frac{1}{\sigma}\frac{d\sigma}{d\Omega} = \frac{3}{4\pi(\lambda+3)}\left[1+\lambda\cos^2\theta+\mu\sin 2\theta\cos\phi+(\nu/2)\ \sin^2\theta\cos 2\phi\right]$$

Boer-Mulders function in DY from fits

➢ General expression for the dilepton angular distributions in the dilepton rest frame:

$$\frac{1}{\sigma}\frac{d\sigma}{d\Omega} = \frac{3}{4\pi(\lambda+3)}\left[1+\lambda\cos^2\theta+\mu\sin 2\theta\cos\phi+(\nu/2)\ \sin^2\theta\cos 2\phi\right]$$

ν



Boer-Mulders function in DY from fits

- General expression for the dilepton angular distributions in the dilepton rest frame:

$$\frac{1}{\sigma}\frac{d\sigma}{d\Omega} = \frac{3}{4\pi(\lambda+3)}\left[1+\lambda\cos^2\theta+\mu\sin 2\theta\cos\phi+(\nu/2)\,\sin^2\theta\cos 2\phi\right]$$

- TMDs approach

Boer-Mulders functions

$$\nu \propto \frac{h_1^{\perp a}\otimes h_1^{\perp b}}{f_1^a\otimes f_1^b}$$

Unpolarized PDFs

Boer-Mulders function in DY from fits

- We performed in 2010 an analysis of E866 data on pp and pD Drell-Yan

$$\nu \propto \frac{h_1^{\perp a} \otimes h_1^{\perp b}}{f_1^a \otimes f_1^b}$$

Boer-Mulders function in DY from fits

- We performed an analysis of E866 data on pp and pD Drell-Yan

$$\nu \propto \frac{h_1^{\perp a} \otimes h_1^{\perp b}}{f_1^a \otimes f_1^b}$$

Gaussian smearing for PDFs

- $f_{q/p}(x, k_\perp) = f_q(x) \frac{1}{\pi \langle k_\perp^2 \rangle} e^{-k_\perp^2/\langle k_\perp^2 \rangle}$

[*] $\langle k_\perp^2 \rangle = 0.25\ (\mathrm{GeV}/c)^2$

[*]Anselmino et. Phys. ReV D71, 074006 (2005)

Boer-Mulders function in DY from fits

- We performed an analysis of E866 data on pp and pD Drell-Yan

$$\nu \propto \frac{h_1^{\perp a} \otimes h_1^{\perp b}}{f_1^a \otimes f_1^b}$$

✎ u and d Boer-Mulders functions as extracted from SIDIS

- $h_1^{\perp q}(x, k_\perp) = \lambda_q \, f_{1T}^{\perp q}(x, k_\perp)$ [*]

$$\lambda_u = 2.0 \pm 0.1$$

$$\lambda_d = -1.11^{+0.00}_{-0.02}$$

[*]Sivers functions from Anselmino et al. Eur. Phys. J. A39,89

Boer-Mulders function in DY from fits

- We performed an analysis of E866 data on pp and pD Drell-Yan

$$\nu \propto \frac{h_1^{\perp a} \otimes h_1^{\perp b}}{f_1^a \otimes f_1^b}$$

✎ $\bar{u}$ and $\bar{d}$ Boer-Mulders parametrized similarly:

$$h_1^{\perp \bar{q}}(x, k_\perp) = \lambda_{\bar{q}} f_{1T}^{\perp q}(x, k_\perp)$$ [*]

[*]Sivers functions from Anselmino et al. Eur. Phys. J. A39,89

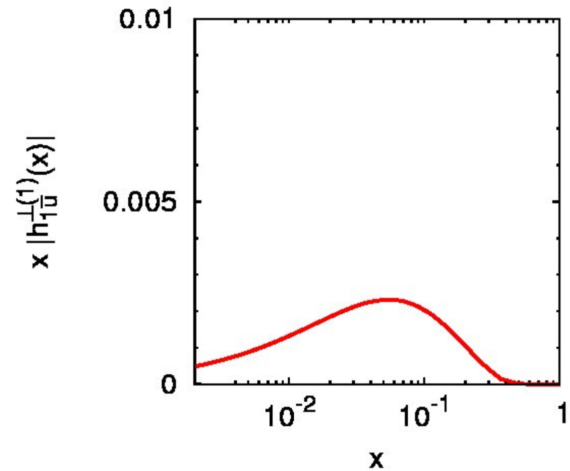
Boer-Mulders function in DY from fits

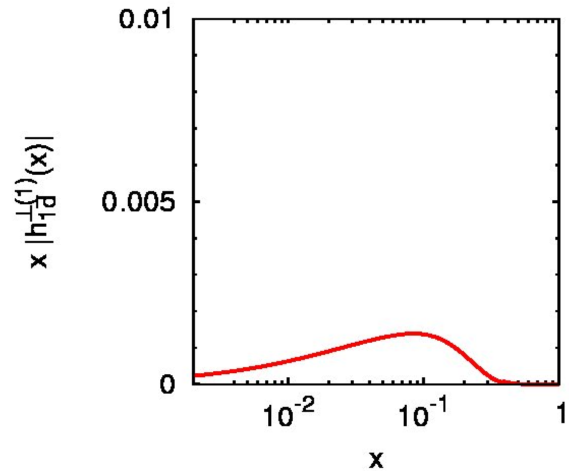
- Results of the analysis of E866 data on pp and pD Drell-Yan

$$h_1^{\perp\bar{q}}(x,k_\perp) = \lambda_{\bar{q}} f_{1T}^{\perp q}(x,k_\perp)$$ [*]

$$\boxed{\begin{array}{cc} \lambda_{\bar{u}} = 3.25 \pm 0.75 & \\ & \chi^2_{d.o.f} = 1.24 \\ \lambda_{\bar{d}} = -0.15 \pm 0.13 & \end{array}}$$

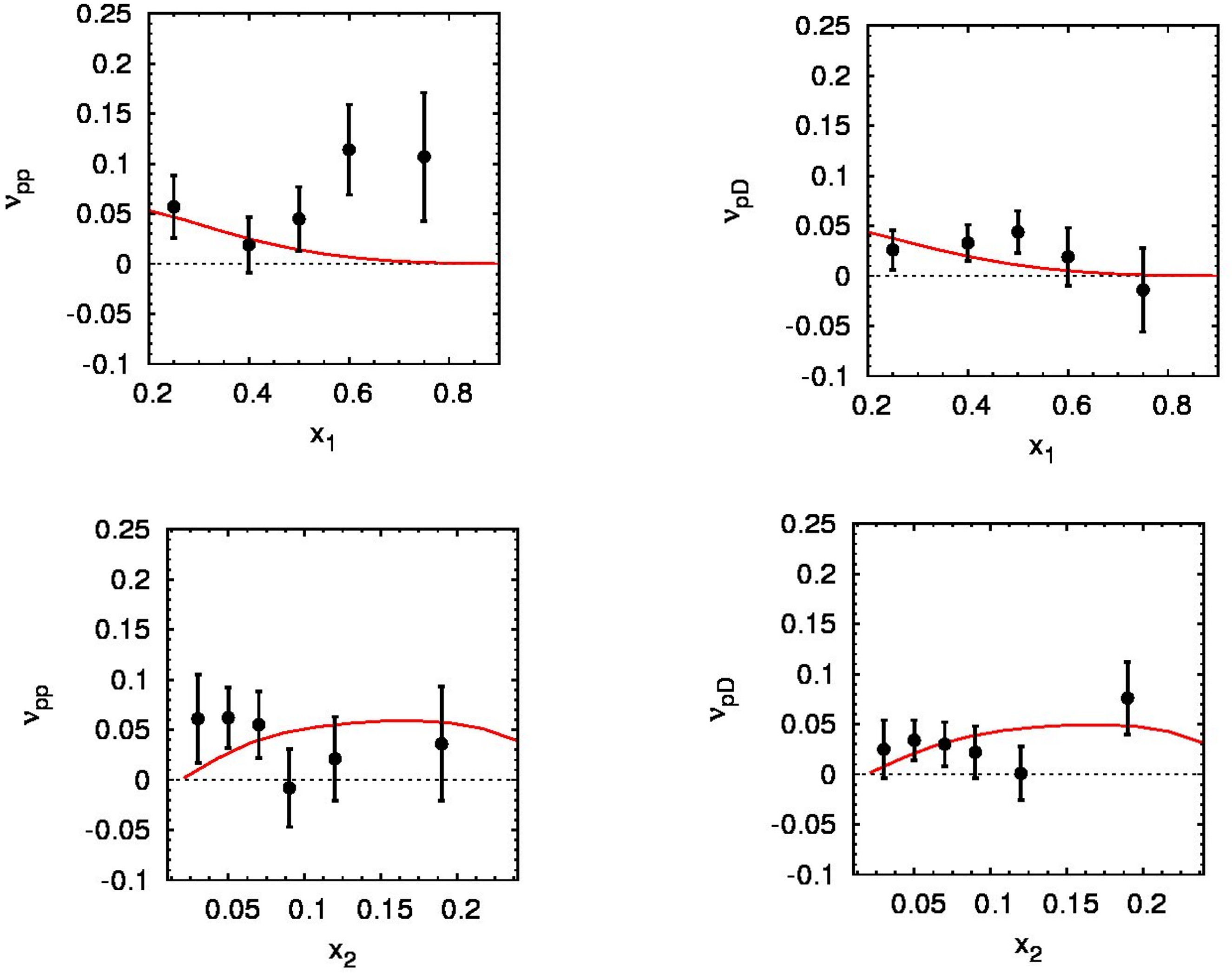
FIT I



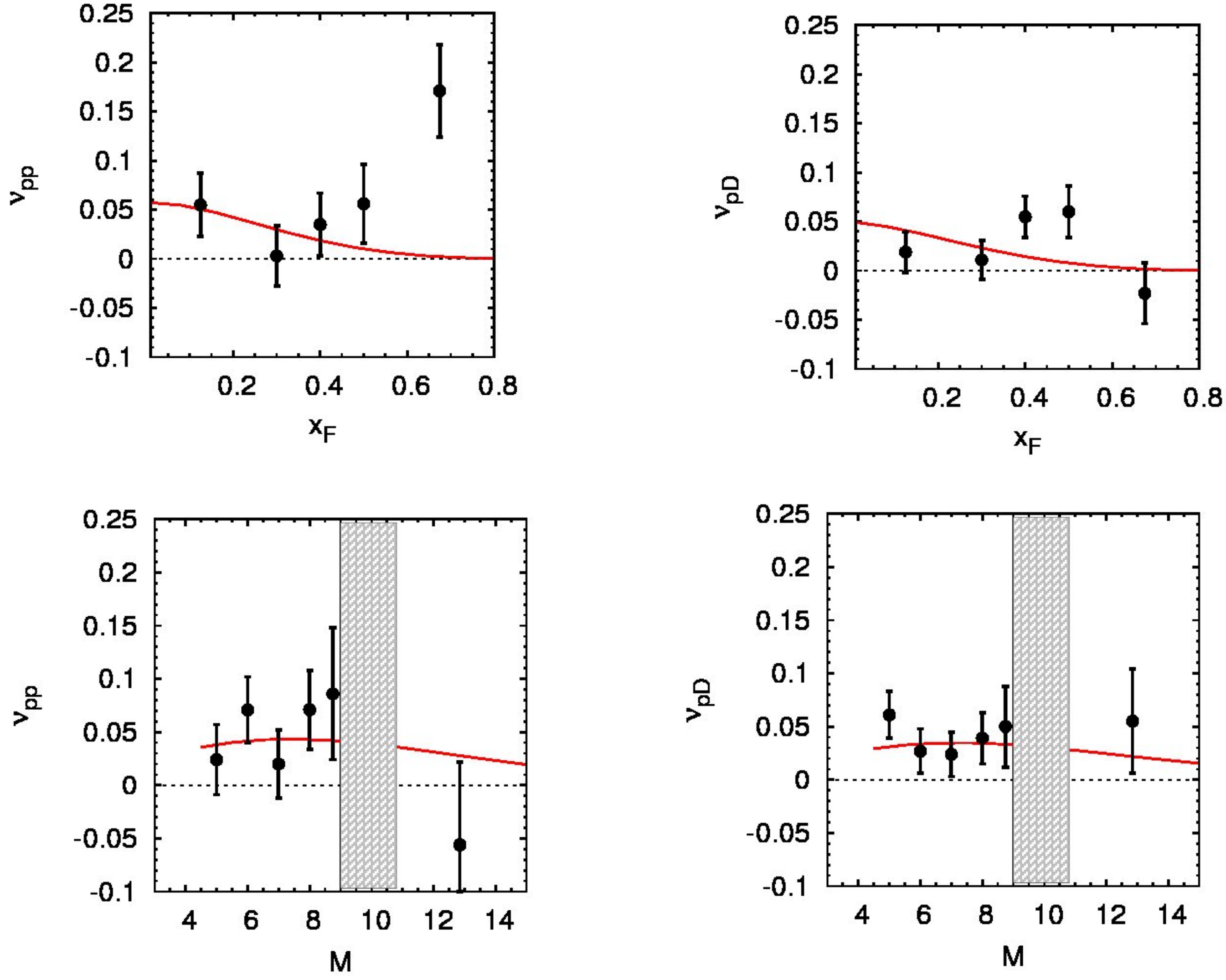


[*] Sivers functions from Anselmino et al. Eur. Phys. J. A39,89

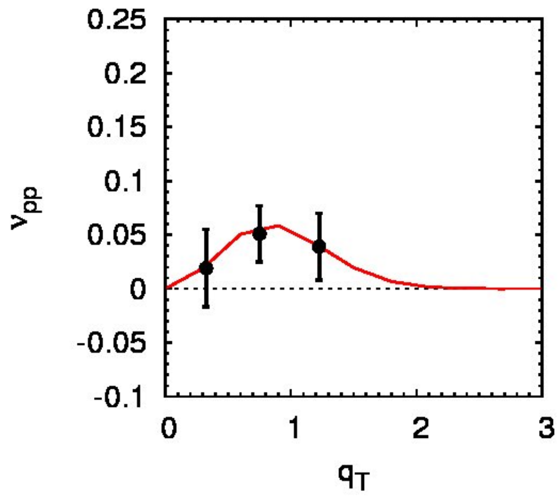
Boer-Mulders function in DY from fits

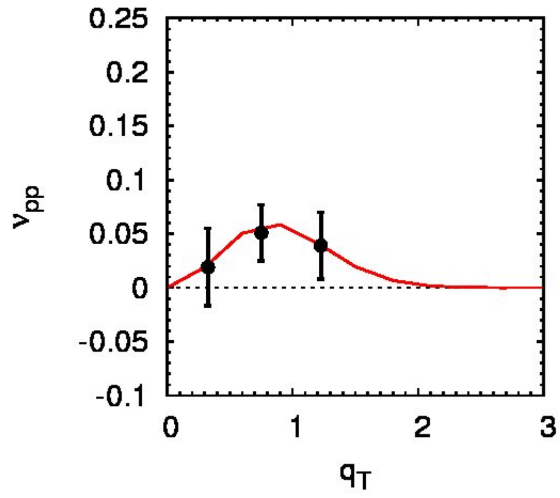


Boer-Mulders function in DY from fits



Boer-Mulders function in DY from fits





Boer-Mulders function in DY from fits

- Can we safely assume that the average transverse momentum is the same in SIDIS and in DY?

  - Gaussian smearing for unpolarized PDFs

    - $f_{q/p}(x, k_\perp) = f_q(x) \, \frac{1}{\pi \langle k_\perp^2 \rangle} \, e^{-k_\perp^2/\langle k_\perp^2 \rangle}$

    From SIDIS: $\langle k_\perp^2 \rangle = 0.25 \; (\mathrm{GeV}/c)^2$

    Typical DY : $\langle k_\perp^2 \rangle \simeq 0.5 - 1 \; (\mathrm{GeV}/c)^2$

    ➔ Let us try to change this value

Boer-Mulders function in DY from fits

- Notice taht BM functions are proportional to the unpolarized pdf

✎ $$h_1^{\perp q}(x, k_T^2) = \lambda_q \, f_{1T}^{\perp q}(x, k_T^2) = \lambda_q \, \rho_q(x) \, \eta(k_T) \, f_1^q(x, \mathbf{k}_T^2)$$

Unpolarized PDF

[*]Sivers functions from Anselmino et al. Eur. Phys. J. A39,89

Boer-Mulders function in DY from fits

- As an exercise let us assume different average transverse momentum in the unpolarized PDF.

$\boxed{\text{FIT II}}$ as Fit I but with $\langle k_\perp^2 \rangle \simeq 0.64\ (\mathrm{GeV}/c)^2$ [*]

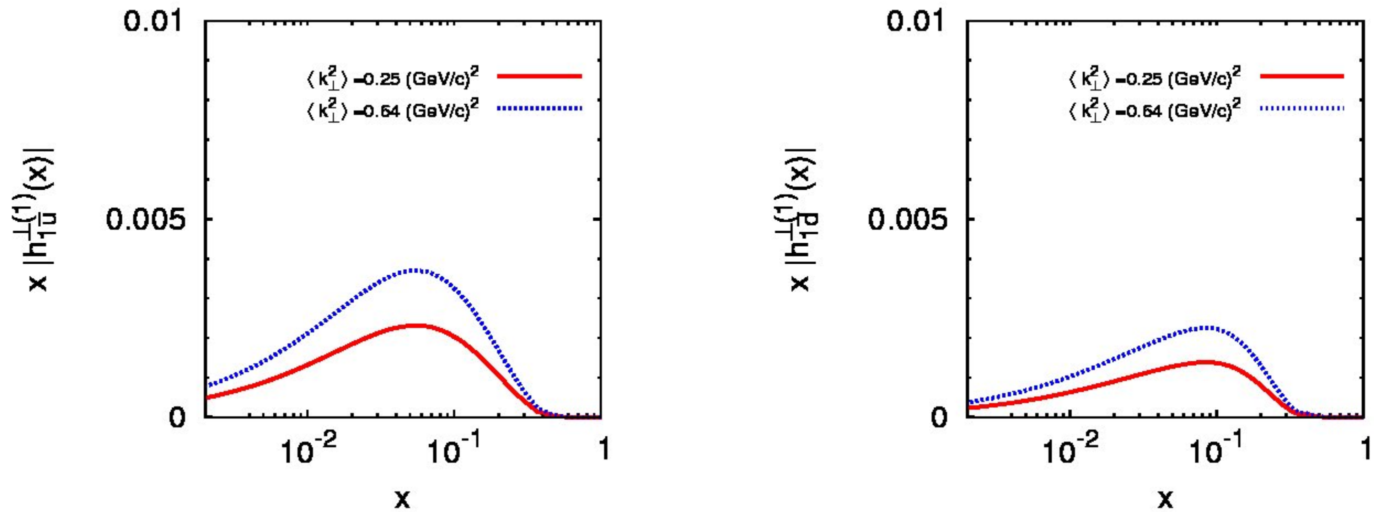
[*] U. D'Alesio and F. Murgia, Phys. Rev. D67,

Boer-Mulders function in DY from fits

$$\lambda_{\bar{u}} = 5.5 \pm 1.5$$
$$\lambda_{\bar{d}} = -0.25 \pm 0.20$$
$$\chi^2_{d.o.f} = 1.24$$

FIT II

Same description of the data!



Conclusions II ...2010

- $\bar{u}$ and $\bar{d}$ BM functions are different from zero but not well constrained from E866 data alone.
- Different average transverse momenta for different processes?

Conclusions??

Why such a large Cahn effect?

- The Cahn effect is suppressed by powers of Q:

$$d\sigma = A + B\cos\phi + C\cos 2\phi$$

- $A = \propto f_1 \otimes D_1$ is the usual ϕ-independent contribution
- $B \propto \frac{1}{Q}\left(f_1 \otimes D_1 + h_1^{\perp} \otimes H_1^{\perp}\right)$ subleading Cahn+Boer-Mulders effect
- $C \propto h_1^{\perp} \otimes H_1^{\perp} + \frac{1}{Q^2} f_1 \otimes D_1$ BM effect+Twist-4 Cahn effect

$$\boxed{\frac{k_\perp}{Q} \ll 1 \text{ ??}}$$

Why such a large Cahn effect?

- HERMES and COMPASS:

$$\langle Q^2 \rangle \simeq 2\ \mathrm{GeV}^2$$

$$Q^2 > 1\ \mathrm{GeV}^2$$

- Analytical integration of the transverse momenta

$$f_{q/p}(x, k_\perp) = f(x) \frac{e^{-k_\perp^2/\langle k_\perp^2 \rangle}}{\pi \langle k_\perp^2 \rangle}$$

$$\langle k_\perp^2 \rangle \simeq 0.25\ (\mathrm{GeV}/c)^2$$

$$\int d^2\boldsymbol{k}_\perp \Rightarrow \int_0^{2\pi} d\varphi \int_0^\infty dk_\perp\, k_\perp$$

Bounds on the intrinic transverse momenta

✔The integration from 0 to infinity can be a crude assumption

✔The parton model provides kinematical limits on the transverse momentum size

- By requiring the energy of the parton to be smaller than the energy of its parent hadron, we have

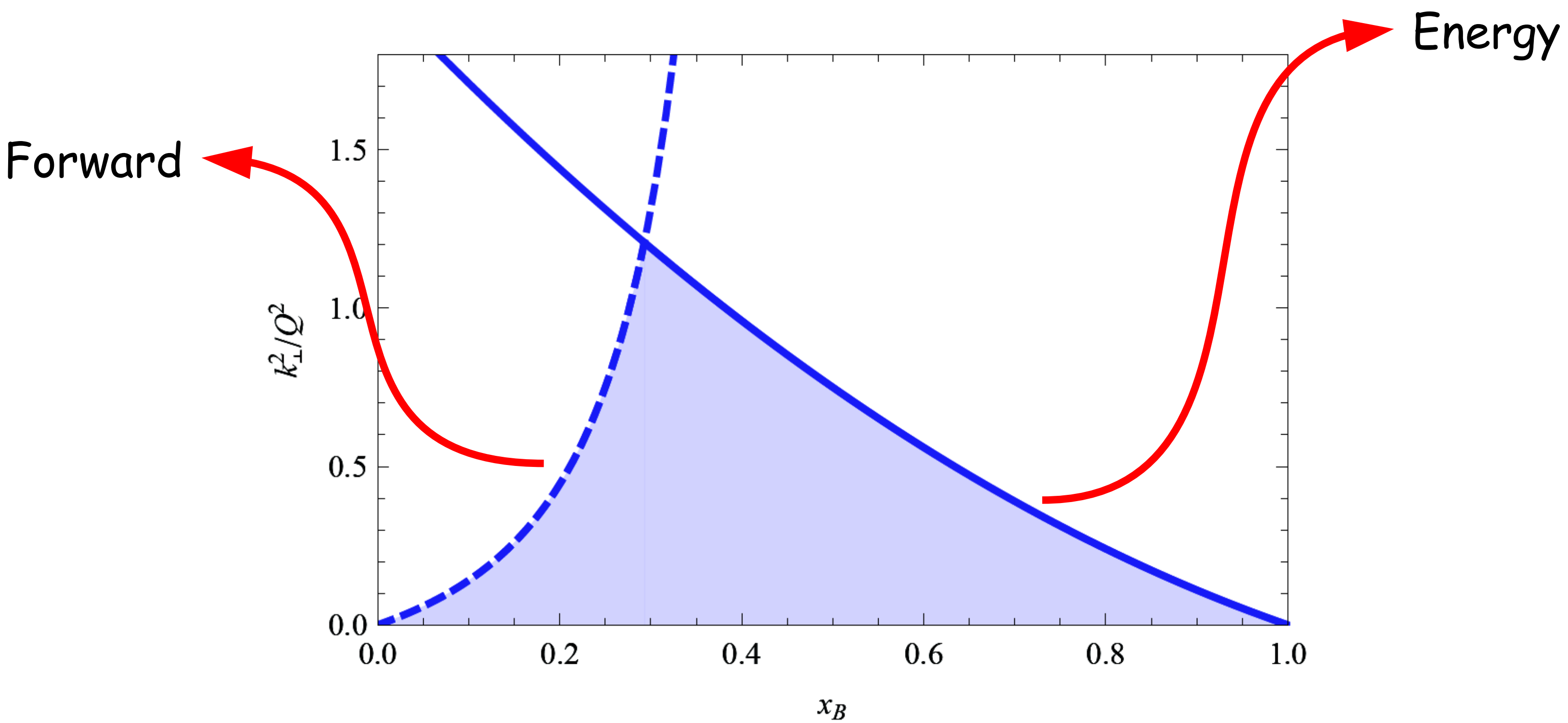
$$\boxed{k_\perp^2 \leq (2 - x_B)(1 - x_B)Q^2 \quad , \quad 0 < x_B < 1}$$

- By requiring the parton not to move backward with respect to its parent hadron, we find

$$\boxed{k_\perp^2 \leq \frac{x_B(1 - x_B)}{(1 - 2x_B)^2}Q^2 \; , \quad x_B < 0.5}$$

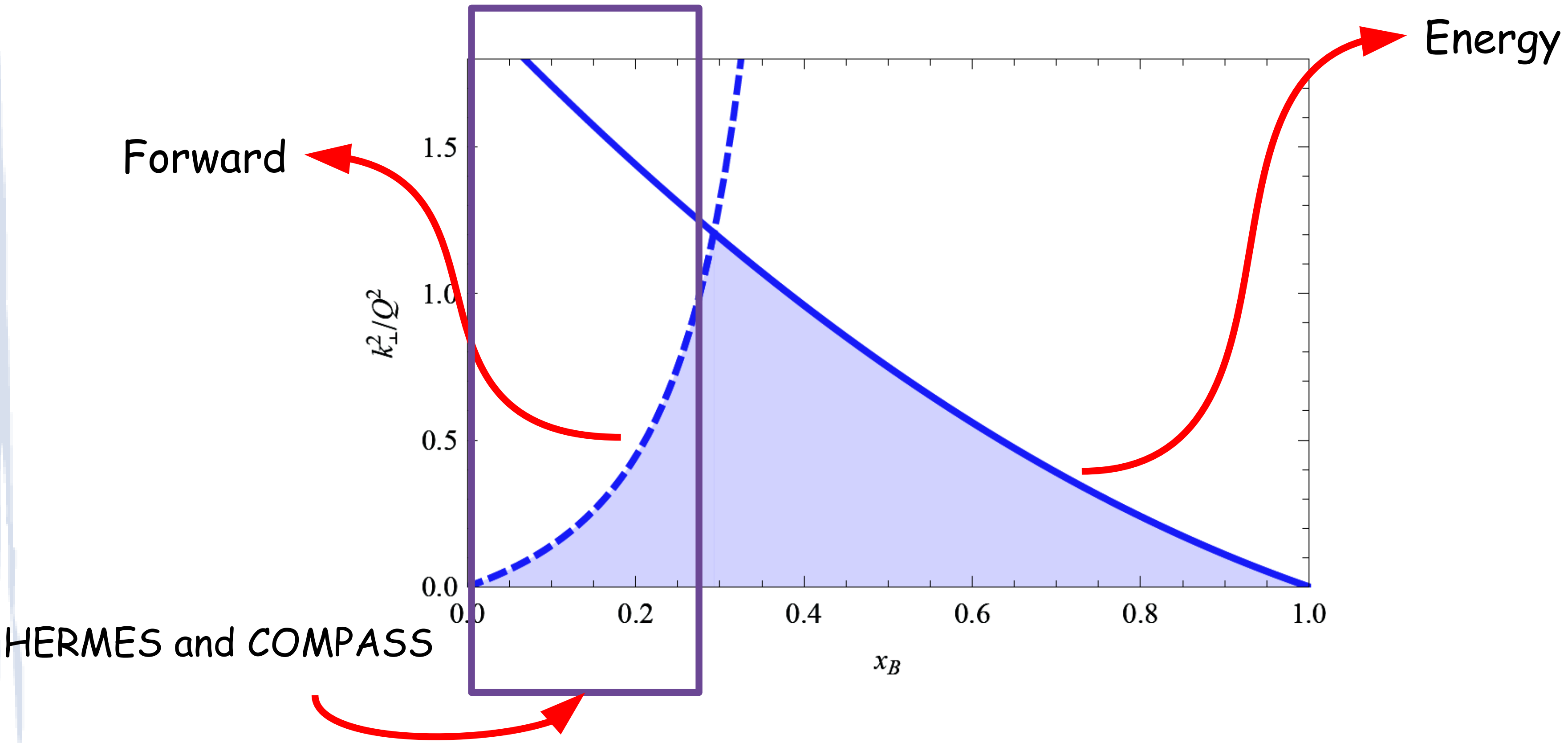
Bounds on the intrinic transverse momenta

- ✔The integration from 0 to infinity can be a crude assumption
- ✔The parton model provides kinematical limits on the transverse momentum size

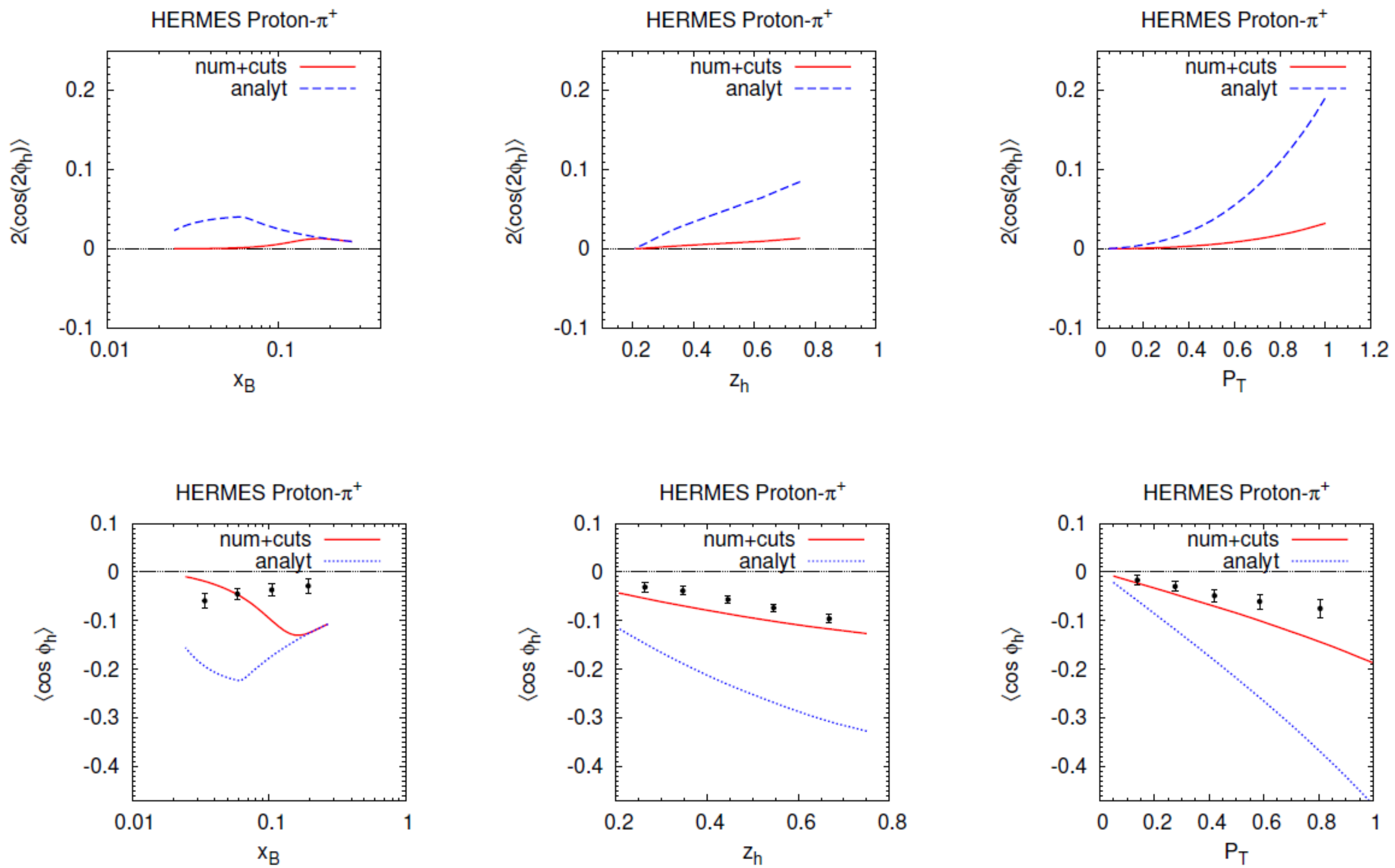


Bounds on the intrinic transverse momenta

- ✔The integration from 0 to infinity can be a crude assumption
- ✔The parton model provides kinematical limits on the transverse momentum size

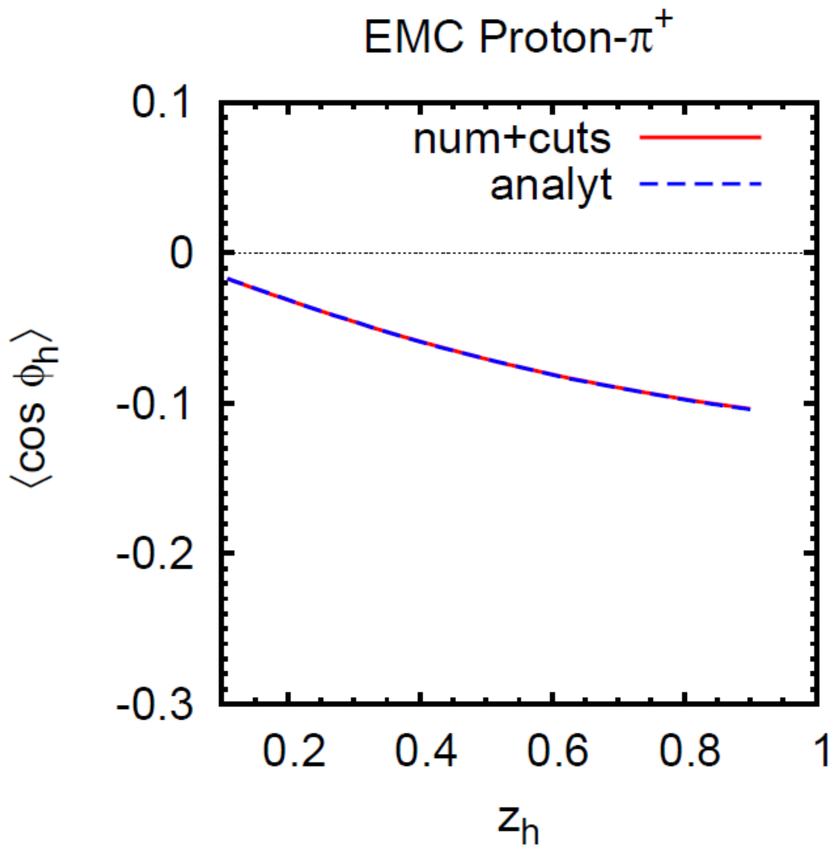


Smaller Cahn effect...



Boglione, Melis, Prokudin
Phys. Rev. D 84, 034033 (2011)

No effects in “true” DIS regime...



EMC like kinematics:

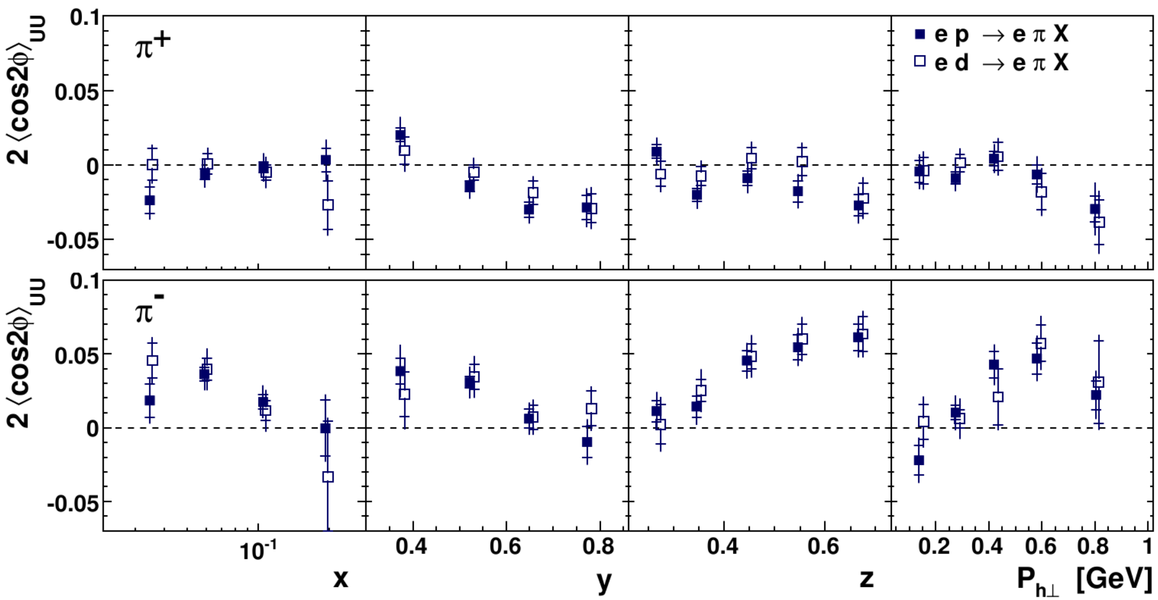
$$Q^2 \geq 5\ \mathrm{GeV}^2$$

Conclusions??

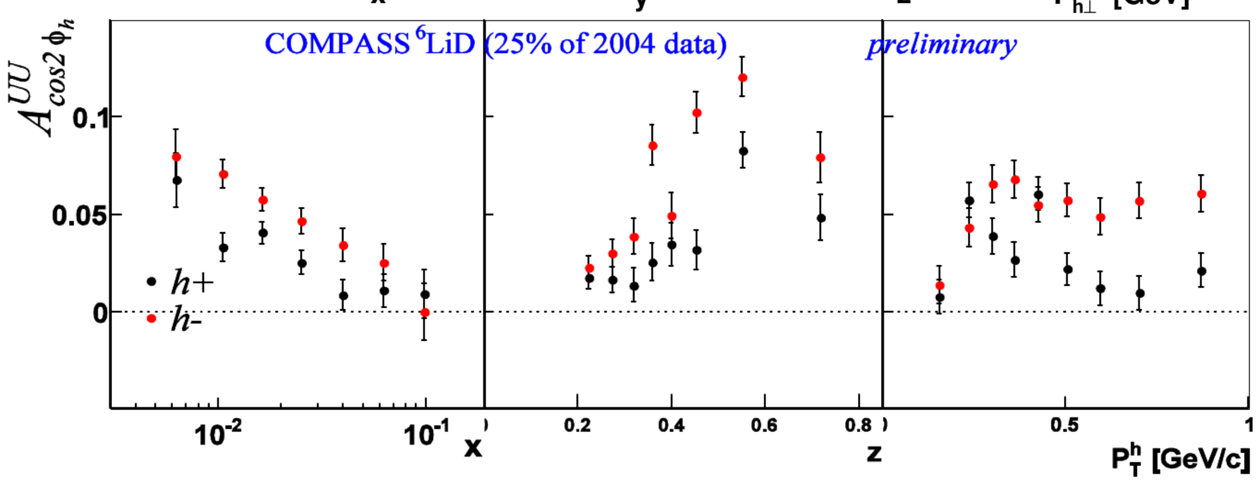
- New data on cos2Φ (and cosΦ) from SIDIS
- Bounds on transverse momenta?&/or
- Different average transverse momenta &/or
- Evolution Equation?

Conclusions??

- New data from HERMES & COMPASS! Re-analysis needed!



Arxiv:1204.4161



Sbrizzai, Transversity 2011

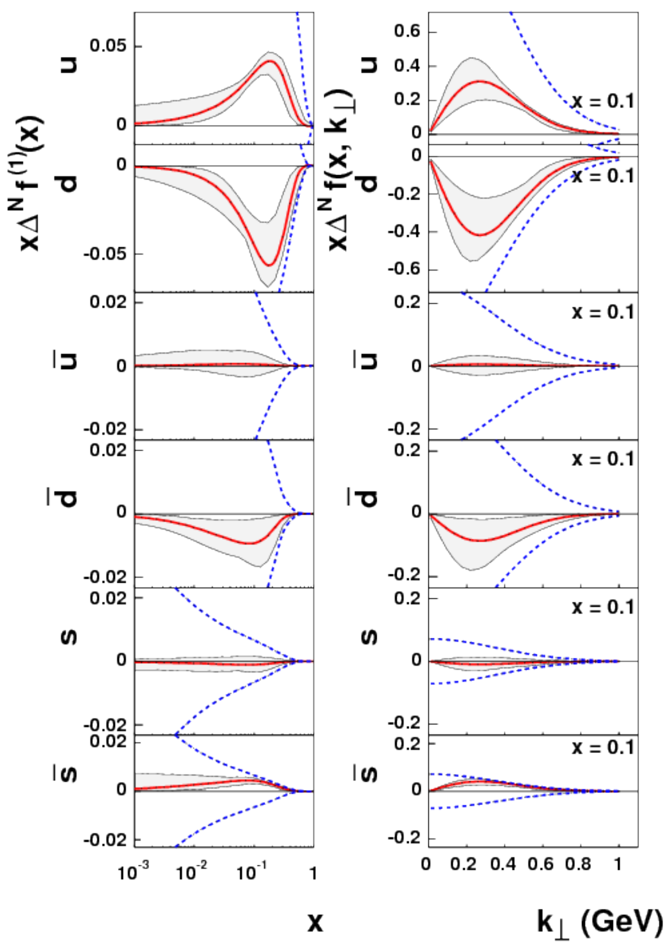
Conclusions??

- New data on cos2Φ (and cosΦ) from SIDIS
- Bounds on transverse momenta?&/or
- Different average transverse momenta &/or
- Evolution Equation?

Sivers function in SIDIS from fits

Sivers function in SIDIS from fits

- In 2009 we performed a fit of HERMES (2002-5) and COMPASS (Deuteron 2003-4) data on π and K production



✔Valence quark

- $\Delta^N f_{u/p^\uparrow} > 0 \quad \Rightarrow f_{1T}^{\perp u} < 0$
- $\Delta^N f_{d/p^\uparrow} < 0 \quad \Rightarrow f_{1T}^{\perp d} > 0$

✔Sea quarks

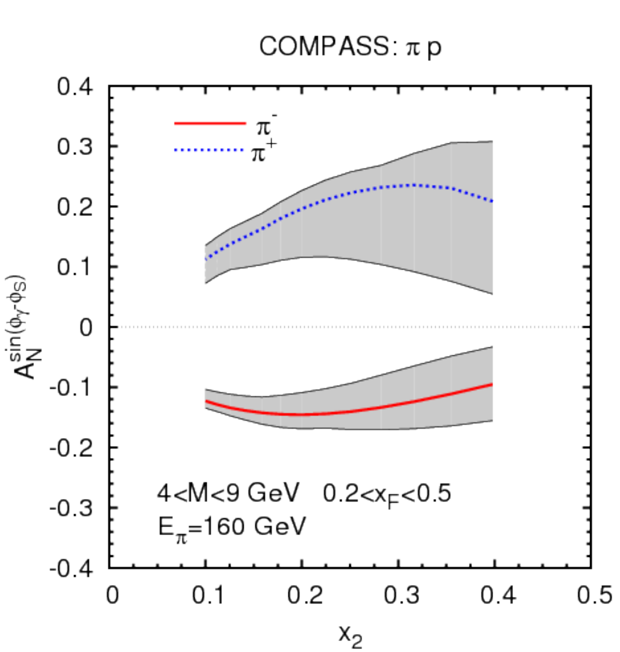
- $\Delta^N f_{\bar{s}/p^\uparrow} > 0 \quad \Rightarrow f_{1T}^{\perp \bar{s}} < 0$

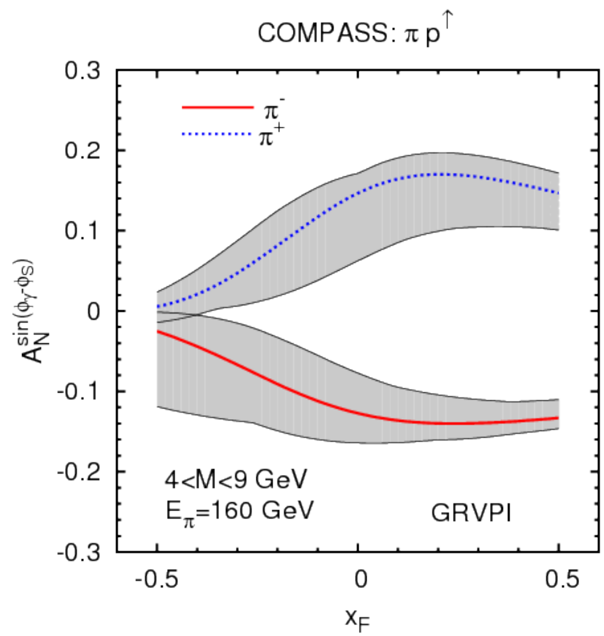
→ $\Delta^N f_q^{(1)}(x) \equiv \int d^2 \boldsymbol{k}_\perp \frac{k_\perp}{4m_p} \Delta^N f_{q/p^\uparrow}(x, k_\perp) = -f_{1T}^{\perp(1)q}(x)$

Anselmino et al. , Eur. Phys. J. A39, 89-100 (2009)

Predictions for COMPASS DY

- Polarized NH_3
- Pion beam
- Valence region for the Sivers function



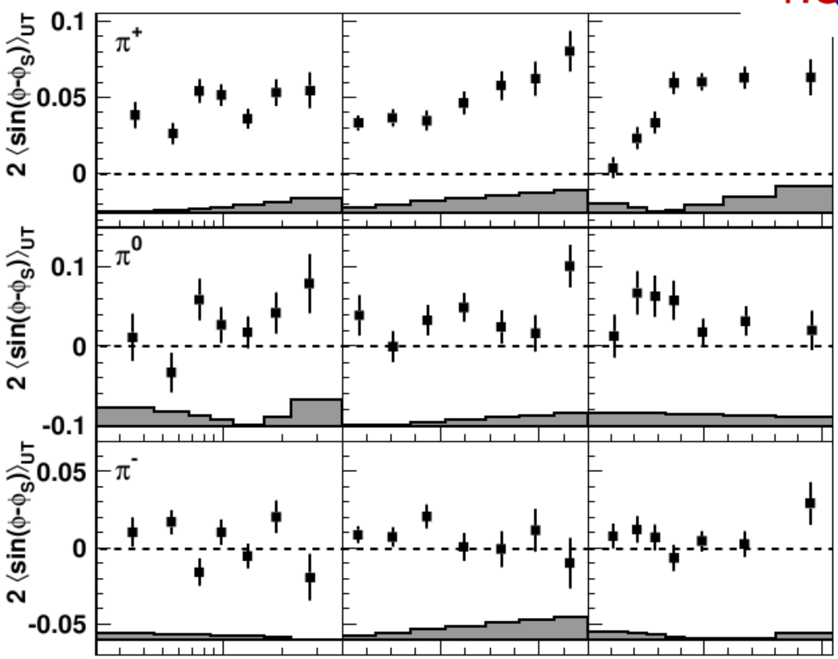


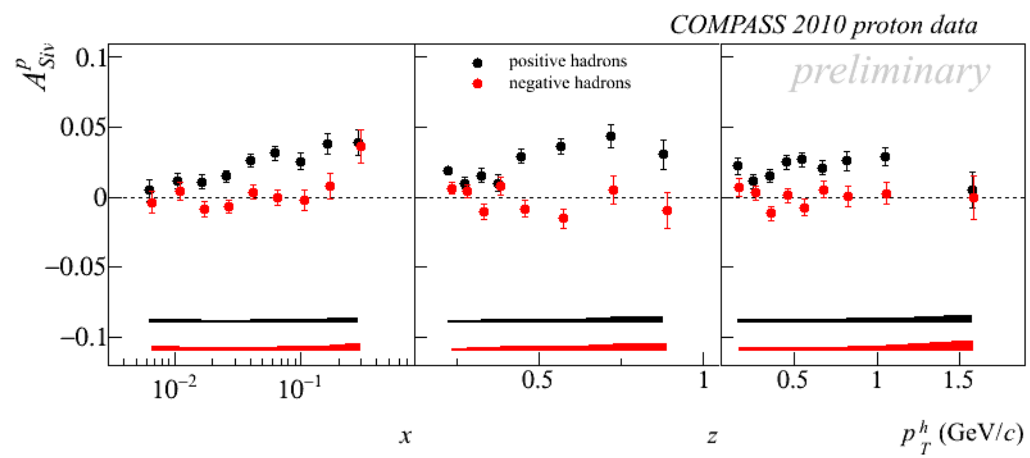
Large measurable asymmetry

- Anselmino et al. Phys. Rev. D79,054010

Sivers function in SIDIS from fits

- New SIDIS data from HERMES and COMPASS





Phys.Rev.Lett.103:152002,2009

Bradamante, Transversity 2011

Sivers function in SIDIS from fits

- New theoretical tools: TMD evolution!

•*J.C. Collins, Foundation of Perturbative QCD, Cambridge Monographs on Particle Physics, Nuclear Physics and Cosmology, No. 32, Cambridge University Press, 2011.*
•*S. M. Aybat and T. C. Rogers, Phys. Rev. D83, 114042 (2011), arXiv:1101.5057 [hep-ph]*
•*S. M. Aybat, J. C. Collins, J.-W. Qiu and T.C. Rogers, arXiv:1110.6428 [hep-ph]*

- What are the consequences from the phenomenological point of view??

Turin standard approach (DGLAP)

Turin standard approach (DGLAP)

- Unpolarized TMDs are factorized in x and $k_\perp$. Only the collinear part evolves with DGLAP evolution equation. No evolution in the transverse momenta:

$$\boxed{\widehat{f}_{q/p}(x, k_\perp; Q) = f_{q/p}(x; Q)\,\frac{e^{-k_\perp^2/\langle k_\perp^2\rangle}}{\pi\langle k_\perp^2\rangle}}$$

Collinear PDF (DGLAP evolution)

Normalized Gaussian: no evolution

Turin standard approach (DGLAP)

- The Sivers function is factorized in x and $k_\perp$ and proportional to the unpolarized PDF.

$$\boxed{\begin{aligned}\Delta^N \widehat{f}_{q/p^\uparrow}(x,k_\perp;Q) &= 2\mathcal{N}_q(x)h(k_\perp)\widehat{f}_{q/p}(x,k_\perp;Q)\\ &= 2\mathcal{N}_q(x)f_{q/p}(x;Q)\sqrt{2e}\frac{k_\perp}{M_1}\frac{e^{-k_\perp^2/\langle k_\perp^2\rangle_S}}{\pi\langle k_\perp^2\rangle}\end{aligned}}$$

Collinear PDF (DGLAP)

$$\mathcal{N}_q(x) = N_q\, x^{\alpha_q}(1-x)^{\beta_q}\,\frac{(\alpha_q+\beta_q)^{(\alpha_q+\beta_q)}}{\alpha_q^{\alpha_q}\,\beta_q^{\beta_q}}$$

$$\langle k_\perp^2\rangle_S = \frac{M_1^2\,\langle k_\perp^2\rangle}{M_1^2+\langle k_\perp^2\rangle}$$

$$\Delta^N \widehat{f}_{q/p^\uparrow}(x,k_\perp) = -\frac{2k_\perp}{m_p}f_{1T}^\perp(x,k_\perp)$$

TMD evolution formalism*

*•*J.C. Collins, Foundation of Perturbative QCD, Cambridge Monographs on Particle Physics, Nuclear Physics and Cosmology, No. 32, Cambridge University Press, 2011.*
•*S. M. Aybat and T. C. Rogers, Phys. Rev. D83, 114042 (2011), arXiv:1101.5057 [hep-ph]*
•*S. M. Aybat, J. C. Collins, J.-W. Qiu and T.C. Rogers, arXiv:1110.6428 [hep-ph]*

TMD evolution formalism

- Let us denote with $\tilde{F}$ either a PDF (or a FF) or the first derivative of the Sivers function in the impact parameter space:

$$\widetilde{F}(x, \boldsymbol{b}_T; Q) \to \widetilde{f}_{q/p}(x, \boldsymbol{b}_T; Q)$$ Unpolarized PDF

$$\widetilde{F}(x, \boldsymbol{b}_T; Q) \to \widetilde{D}_{h/q}(z, \boldsymbol{b}_T; Q)$$ Unpolarized FF

$$\widetilde{F}(x, \boldsymbol{b}_T; Q) \to \widetilde{f}'^{\perp f}_{1T}(x, \boldsymbol{b}_T; Q)$$ First derivative of the Sivers function

TMD evolution formalism

- At LO the evolution equation can be summarized by the following expression:

$$\widetilde{F}(x, \boldsymbol{b}_T; Q) = \widetilde{F}(x, \boldsymbol{b}_T; Q_0)\, \widetilde{R}(Q, Q_0, b_T)\ \exp\left\{-g_K(b_T)\ln\frac{Q}{Q_0}\right\}$$

Corresponding to Eq. 44 of Ref [*] with $\tilde{K}$=0 and : $\boxed{\mu^2 = \zeta_F = \zeta_D = Q^2}$

- *[*]S. M. Aybat, J. C. Collins, J.-W. Qiu and T.C. Rogers, arXiv:1110.6428 [hep-ph]*

TMD evolution formalism

- At LO the evolution equation can be summarized by the following expression:

$$\boxed{\widetilde{F}(x,\boldsymbol{b}_T;Q) = \widetilde{F}(x,\boldsymbol{b}_T;Q_0)\,\widetilde{R}(Q,Q_0,b_T)\;\exp\left\{-g_K(b_T)\ln\frac{Q}{Q_0}\right\}}$$

Output function at the scale **Q**
in the impact parameter space

Input function at the scale $\mathbf{Q_0}$
in the impact parameter space

Evolution kernel

TMD evolution formalism

- At LO the evolution equation can be summarized by the following expression:

$$\widetilde{F}(x, \boldsymbol{b}_T; Q) = \widetilde{F}(x, \boldsymbol{b}_T; Q_0)\, \widetilde{R}(Q, Q_0, b_T)\, \exp\left\{-g_K(b_T) \ln \frac{Q}{Q_0}\right\}$$

- **Perturbative** part of the evolution kernel

TMD evolution formalism

➢ At LO the evolution equation can be summarized by the following expression:

$$\widetilde{F}(x, \boldsymbol{b}_T; Q) = \widetilde{F}(x, \boldsymbol{b}_T; Q_0) \boxed{\widetilde{R}(Q, Q_0, b_T)} \exp\left\{-g_K(b_T) \ln \frac{Q}{Q_0}\right\}$$

➢ **Perturbative** part of the evolution kernel

$$\widetilde{R}(Q, Q_0, b_T) \equiv \exp\left\{\ln \frac{Q}{Q_0} \int_{Q_0}^{\mu_b} \frac{d\mu'}{\mu'} \gamma_K(\mu') + \int_{Q_0}^{Q} \frac{d\mu}{\mu} \gamma_F\left(\mu, \frac{Q^2}{\mu^2}\right)\right\}$$

TMD evolution formalism

- At LO the evolution equation can be summarized by the following expression:

$$\widetilde{F}(x, \boldsymbol{b}_T; Q) = \widetilde{F}(x, \boldsymbol{b}_T; Q_0)\, \widetilde{R}(Q, Q_0, b_T)\, \exp\left\{-g_K(b_T)\ln\frac{Q}{Q_0}\right\}$$

- **Perturbative** part of the evolution kernel

$$\widetilde{R}(Q, Q_0, b_T) \equiv \exp\left\{\ln\frac{Q}{Q_0}\int_{Q_0}^{\mu_b}\frac{d\mu'}{\mu'}\gamma_K(\mu') + \int_{Q_0}^{Q}\frac{d\mu}{\mu}\gamma_F\left(\mu, \frac{Q^2}{\mu^2}\right)\right\}$$

$$\gamma_K(\mu) = \alpha_s(\mu)\frac{2\,C_F}{\pi}$$

$$\gamma_F(\mu; \frac{Q^2}{\mu^2}) = \alpha_s(\mu)\frac{C_F}{\pi}\left(\frac{3}{2} - \ln\frac{Q^2}{\mu^2}\right)$$

TMD evolution formalism

- At LO the evolution equation can be summarized by the following expression:

$$\widetilde{F}(x,\boldsymbol{b}_T;Q) = \widetilde{F}(x,\boldsymbol{b}_T;Q_0)\,\boxed{\widetilde{R}(Q,Q_0,b_T)}\,\exp\left\{-g_K(b_T)\ln\frac{Q}{Q_0}\right\}$$

- **Perturbative** part of the evolution kernel

$$\widetilde{R}(Q,Q_0,b_T) \equiv \exp\left\{\ln\frac{Q}{Q_0}\int_{Q_0}^{\mu_b}\frac{d\mu'}{\mu'}\gamma_K(\mu') + \int_{Q_0}^{Q}\frac{d\mu}{\mu}\gamma_F\left(\mu,\frac{Q^2}{\mu^2}\right)\right\}$$

Scale that separates the perturbative region
from the non perturbative one

TMD evolution formalism

- At LO the evolution equation can be summarized by the following expression:

$$\widetilde{F}(x, \boldsymbol{b}_T; Q) = \widetilde{F}(x, \boldsymbol{b}_T; Q_0) \boxed{\widetilde{R}(Q, Q_0, b_T)} \exp\left\{ -g_K(b_T) \ln \frac{Q}{Q_0} \right\}$$

- **Perturbative** part of the evolution kernel

$$\widetilde{R}(Q, Q_0, b_T) \equiv \exp\left\{ \ln \frac{Q}{Q_0} \int_{Q_0}^{\mu_b} \frac{\mathrm{d}\mu'}{\mu'} \gamma_K(\mu') + \int_{Q_0}^{Q} \frac{\mathrm{d}\mu}{\mu} \gamma_F\left(\mu, \frac{Q^2}{\mu^2}\right) \right\}$$

$$\boxed{\mu_b = \frac{C_1}{b_*(b_T)} \qquad b_*(b_T) \equiv \frac{b_T}{\sqrt{1 + b_T^2/b_{\max}^2}} \qquad C_1 = 2e^{-\gamma_E}}$$

One of the possible prescription to separate the perturbative region from the non perturbative one

TMD evolution formalism

- At LO the evolution equation can be summarized by the following expression:

$$\widetilde{F}(x, \boldsymbol{b}_T; Q) = \widetilde{F}(x, \boldsymbol{b}_T; Q_0)\, \widetilde{R}(Q, Q_0, b_T)\, \exp\left\{ -\boxed{g_K(b_T)} \ln \frac{Q}{Q_0} \right\}$$

- **Non Perturbative** (scale independent) part of the evolution kernel that needs to be empirically modeled

$$\boxed{g_K(b_T) = \frac{1}{2}\, g_2\, b_T^2}$$

Common choice used in the unpolarized DY data analyses in the CSS formalism

$$g_2 = 0.68\ \mathrm{GeV}^2$$

Landry et al. Phys Rev D67, 073016

TMD evolution formalism

- One can get the TMD in the momentum space by Fourier trasforming:

$$\widehat{f}_{q/p}(x,k_\perp;Q) = \frac{1}{2\pi}\int_0^\infty \mathrm{d}b_T\, b_T\, J_0(k_\perp b_T)\, \widetilde{f}_{q/p}(x,b_T;Q)$$

$$\widehat{D}_{h/q}(z,p_\perp;Q) = \frac{1}{2\pi}\int_0^\infty \mathrm{d}b_T\, b_T\, J_0(\mathrm{k}_T b_T)\, \widetilde{D}_{h/q}(z,b_T;Q)$$

$$\widehat{f}_{1T}^{\perp f}(x,k_\perp;Q) = \frac{-1}{2\pi k_\perp}\int_0^\infty \mathrm{d}b_T\, b_T\, J_1(k_\perp b_T)\, \widetilde{f}'^{\perp q}_{1T}(x,b_T;Q)$$

$$\begin{aligned} f_{q/p^\uparrow}(x,\boldsymbol{k}_\perp,\boldsymbol{S};Q) &= f_{q/p}(x,k_\perp;Q) - f_{1T}^{\perp q}(x,k_\perp;Q)\frac{\epsilon_{ij}\,k_\perp^i\,S^j}{M_p} \\ &= f_{q/p}(x,k_\perp;Q) + \frac{1}{2}\Delta^N f_{q/p^\uparrow}(x,k_\perp;Q)\frac{\epsilon_{ij}\,k_\perp^i\,S^j}{k_\perp} \end{aligned}$$

Parametrization ot the input functions

$$\widetilde{F}(x, \boldsymbol{b}_T; Q) = \boxed{\widetilde{F}(x, \boldsymbol{b}_T; Q_0)}\, \widetilde{R}(Q, Q_0, b_T)\ \exp\left\{-g_K(b_T) \ln \frac{Q}{Q_0}\right\}$$

- We want to compare the effect of TMD evolution vs our traditional approach (DGLAP)

$$\downarrow$$

- Same parametrization of the input function at the initial scale in the trasverse momentum space.

Parametrization ot the input functions

$$\boxed{\widetilde{F}(x, \boldsymbol{b}_T; Q) = \widetilde{F}(x, \boldsymbol{b}_T; Q_0)\, \widetilde{R}(Q, Q_0, b_T)\ \exp\left\{-g_K(b_T) \ln \frac{Q}{Q_0}\right\}}$$

Example: unpolarized pdf

$$\tilde{F}(x, b_T; Q_0) = \tilde{f}_{q/p}(x, b_T; Q_0) \xrightarrow{\text{Fourier transf.}} \widehat{f}_{q/p}(x, k_\perp; Q_0)$$

$$\widehat{f}_{q/p}(x, k_\perp; Q_0) = f_{q/p}(x, Q_0)\, \frac{1}{\pi \langle k_\perp^2 \rangle}\, e^{-k_\perp^2/\langle k_\perp^2 \rangle}$$

Parametrization ot the input functions

$$\widetilde{F}(x, \boldsymbol{b}_T; Q) = \widetilde{F}(x, \boldsymbol{b}_T; Q_0)\, \widetilde{R}(Q, Q_0, b_T)\ \exp\left\{ -g_K(b_T) \ln \frac{Q}{Q_0} \right\}$$

$$\widetilde{f}_{q/p}(x, b_T; Q_0) = f_{q/p}(x, Q_0) \exp\left\{ -\alpha^2\, b_T^2 \right\}$$

$$\widehat{f}_{q/p}(x, k_\perp; Q_0) = f_{q/p}(x, Q_0)\, \frac{1}{\pi \langle k_\perp^2 \rangle}\, e^{-k_\perp^2/\langle k_\perp^2 \rangle}$$

$$\alpha^2 = \langle k_\perp^2 \rangle / 4$$

Parametrization of the input functions

$$\widetilde{F}(x, \boldsymbol{b}_T; Q) = \widetilde{F}(x, \boldsymbol{b}_T; Q_0)\, \widetilde{R}(Q, Q_0, b_T)\ \exp\left\{-g_K(b_T) \ln \frac{Q}{Q_0}\right\}$$

$$\widetilde{D}_{h/q}(z, b_T; Q_0) = \frac{1}{z^2}\, D_{h/q}(z, Q_0)\ \exp\left\{-\beta^2\, b_T^2\right\}$$

$$\widehat{D}_{h/q}(z, p_\perp; Q_0) = D_{h/q}(z, Q_0)\, \frac{1}{\pi \langle p_\perp^2 \rangle}\, e^{-p_\perp^2/\langle p_\perp^2 \rangle}$$

$$\beta^2 = \langle p_\perp^2 \rangle / 4z^2$$

Parametrization of the input functions

$$\widetilde{F}(x,\boldsymbol{b}_T;Q) = \boxed{\widetilde{F}(x,\boldsymbol{b}_T;Q_0)}\,\widetilde{R}(Q,Q_0,b_T)\ \exp\left\{-g_K(b_T)\ln\frac{Q}{Q_0}\right\}$$

$$\widetilde{f}'^{\perp}_{1T}(x,b_T;Q_0) = -2\,\gamma^2\, f^{\perp}_{1T}(x;Q_0)\, b_T\, e^{-\gamma^2\, b_T^2}$$

$$\widehat{f}^{\perp}_{1T}(x,k_\perp;Q_0) = f^{\perp}_{1T}(x;Q_0)\,\frac{1}{4\,\pi\,\gamma^2}\, e^{-k_\perp^2/4\gamma^2}$$

$$4\,\gamma^2 \equiv \langle k_\perp^2\rangle_S = \frac{M_1^2\,\langle k_\perp^2\rangle}{M_1^2+\langle k_\perp^2\rangle}$$

Parametrization of the input functions

➢Then the evolution equations for unpolarized TMDs become simply:

$$\boxed{\widetilde{f}_{q/p}(x, b_T; Q) = f_{q/p}(x, Q_0)\, \widetilde{R}(Q, Q_0, b_T)\; \exp\left\{-b_T^2 \left(\alpha^2 + \frac{g_2}{2} \ln \frac{Q}{Q_0}\right)\right\}}$$

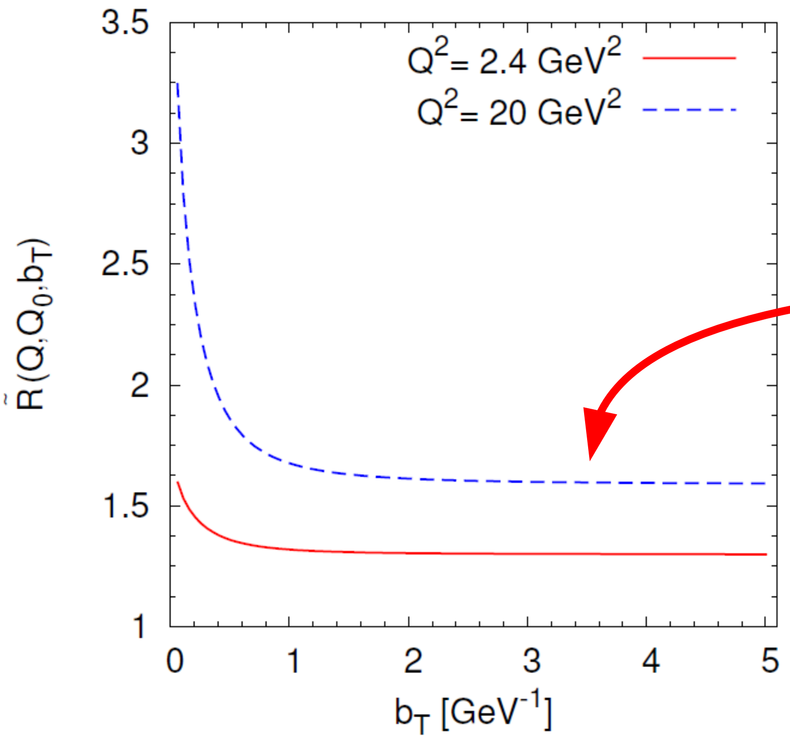
$$\boxed{\widetilde{D}_{h/q}(z, b_T; Q) = \frac{1}{z^2} D_{h/q}(z, Q_0)\, \widetilde{R}(Q, Q_0, b_T)\; \exp\left\{-b_T^2 \left(\beta^2 + \frac{g_2}{2} \ln \frac{Q}{Q_0}\right)\right\}}$$

➢While for the Sivers function we have:

$$\boxed{\widetilde{f}'^{\perp}_{1T}(x, b_T; Q) = -2\, \gamma^2\, f^{\perp}_{1T}(x; Q_0)\, \widetilde{R}(Q, Q_0, b_T)\, b_T\; \exp\left\{-b_T^2 \left(\gamma^2 + \frac{g_2}{2} \ln \frac{Q}{Q_0}\right)\right\}}$$

Analytical (approximated) solution of the TMD evolution equation

- $\tilde{R}(Q,Q0,b_T)$ exhibits a non trivial dependence on b_T that prevents any analytical integration



$\tilde{R}(Q,Q0,b_T)$ becomes **constant** for b_T > 1 GeV $^{-1}$

We can therefore neglect the $\tilde{R}$ dependence on b_T and define:

$$\boxed{R(Q,Q_0) \equiv \tilde{R}(Q,Q_0,b_T \to \infty)}$$

Good approximation for large b_T i.e. small $k_\perp$

Analytical (approximated) solution of the TMD evolution equation

- For instance, replacing $\tilde{R}$ with R in the unpolarized, we get:

$$\widetilde{f}_{q/p}(x, \boldsymbol{b}_T; Q) = f_{q/p}(x, Q_0)\; R(Q, Q_0)\; \exp\left\{ -b_T^2 \left(\alpha^2 + \frac{g_2}{2} \ln \frac{Q}{Q_0} \right) \right\}$$

Which is Gaussian in b_T, and will then Fourier-transform into a Gaussian in $k_\perp$

$$\boxed{\widehat{f}_{q/p}(x, k_\perp; Q) = f_{q/p}(x, Q_0)\; R(Q, Q_0)\; \frac{e^{-k_\perp^2/w^2}}{\pi\, w^2}}$$

$$\boxed{w^2(Q, Q_0) = \langle k_\perp^2 \rangle + 2\, g_2 \ln \frac{Q}{Q_0}}$$

Analytical (approximated) solution of the TMD evolution equation

- Similarly, for the unpolarized TMD fragmentation function, we have

$$\boxed{\widehat{D}_{h/q}(z, p_\perp; Q) = D_{h/q}(z, Q_0)\, R(Q, Q_0)\, \frac{e^{-p_\perp^2/w_F^2}}{\pi w_F^2}}$$

$$\boxed{w_F^2 \equiv w_F^2(Q, Q_0) = \langle p_\perp^2 \rangle + 2z^2 g_2 \ln \frac{Q}{Q_0}}$$

Analytical (approximated) solution of the TMD evolution equation

- For the Sivers distribution function, we find:

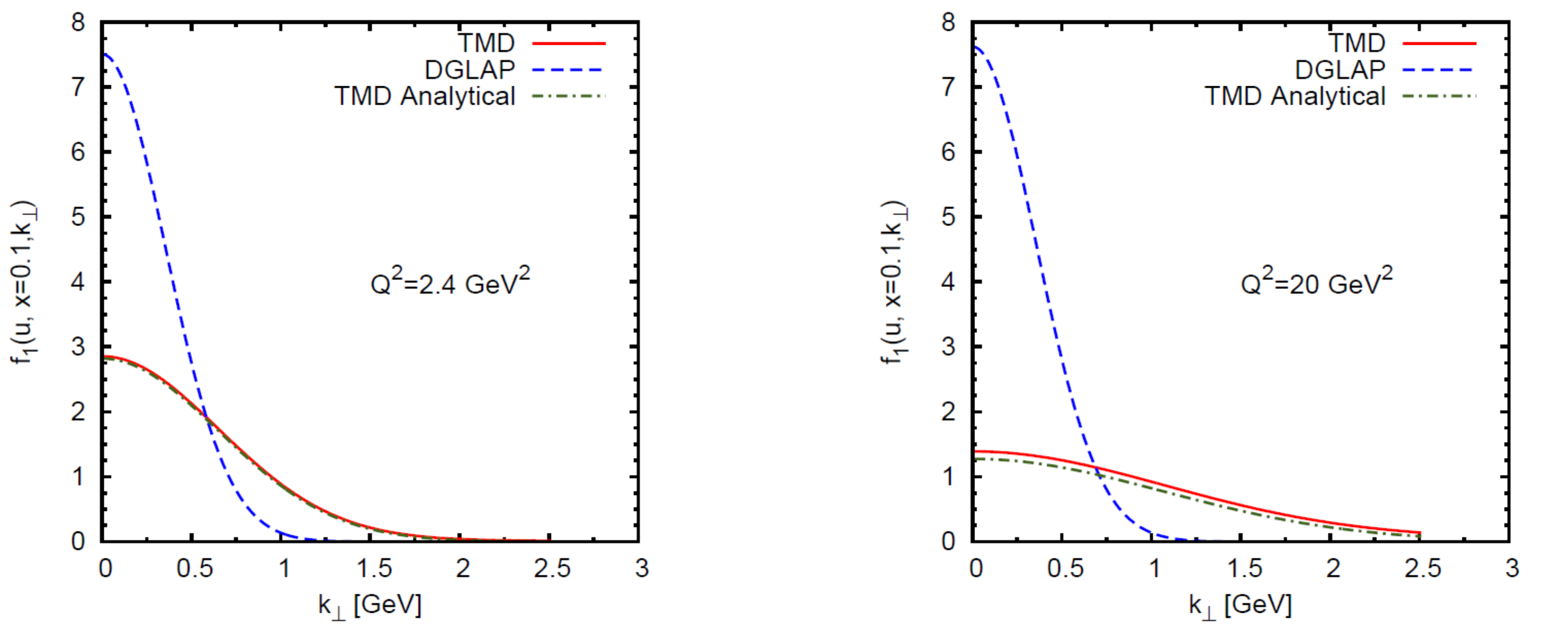
$$\Delta^N \widehat{f}_{q/p^\uparrow}(x, k_\perp; Q) = \frac{k_\perp}{M_1}\sqrt{2e}\,\frac{\langle k_\perp^2\rangle_S^2}{\langle k_\perp^2\rangle}\,\Delta^N f_{q/p^\uparrow}(x, Q_0)\, R(Q, Q_0)\,\frac{e^{-k_\perp^2/w_S^2}}{\pi w_S^4}$$

$$w_S^2(Q, Q_0) = \langle k_\perp^2\rangle_S + 2g_2 \ln\frac{Q}{Q_0}$$

$$\Delta^N \widehat{f}_{q/p^\uparrow}(x, k_\perp) = -\frac{2k_\perp}{m_p} f_{1T}^\perp(x, k_\perp)$$

$$\langle k_\perp^2\rangle_S = \frac{M_1^2\,\langle k_\perp^2\rangle}{M_1^2 + \langle k_\perp^2\rangle}$$

Comparative analysis of TMD evoultion equations

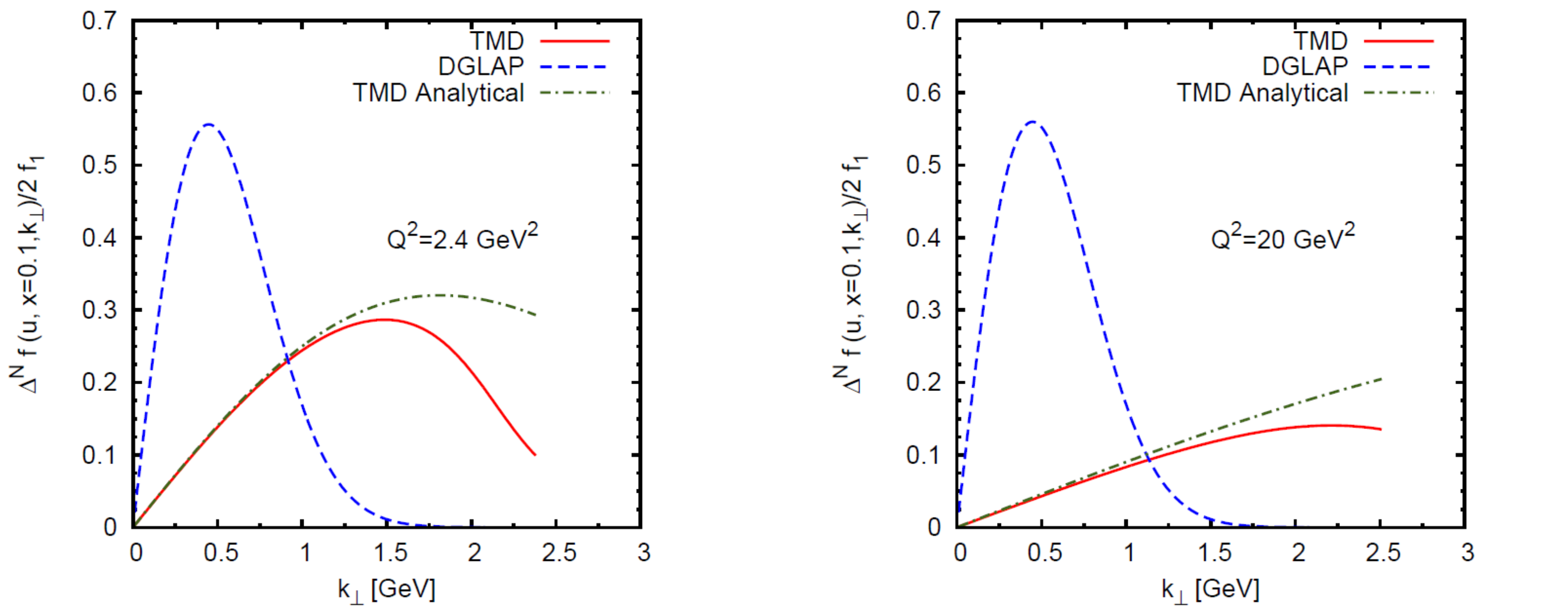


Starting scale Q_0=1 GeV
Same function at Q_0

DGLAP evolution is slow at moderate x and in this range of Q^2

For the unpolarized PDF, the analytical approximation holds up to large $k_\perp$

Comparative analysis of TMD evoultion equations



Starting scale Q_0=1 GeV
Same function at Q_0

For the Sivers function, the analytical approximation breaks down at large $k_\perp$ values

Fit of HERMES and COMPASS SIDIS data

$$A_{UT}^{\sin(\phi_h-\phi_S)} = \frac{\sum_q \int d\phi_S\, d\phi_h\, d^2\boldsymbol{k}_\perp\, \Delta^N f_{q/p^\uparrow}(x,k_\perp,Q)\sin(\varphi-\phi_S)\,\frac{d\hat{\sigma}^{\ell q\to\ell q}}{dQ^2}\, D_q^h(z,p_\perp,Q)\sin(\phi_h-\phi_S)}{\sum_q \int d\phi_S\, d\phi_h\, d^2\boldsymbol{k}_\perp\, f_{q/p}(x,k_\perp,Q)\,\frac{d\hat{\sigma}^{\ell q\to\ell q}}{dQ^2}\, D_q^h(z,p_\perp,Q)}$$

$$\Delta^N \widehat{f}_{q/p^\uparrow}(x,k_\perp;Q_0) = 2\mathcal{N}_q(x)h(k_\perp)\widehat{f}_{q/p}(x,k_\perp;Q_0)$$

$$\mathcal{N}_q(x) = N_q\, x^{\alpha_q}(1-x)^{\beta_q}\,\frac{(\alpha_q+\beta_q)^{(\alpha_q+\beta_q)}}{\alpha_q^{\alpha_q}\,\beta_q^{\beta_q}}$$

$$h(k_\perp) = \sqrt{2e}\,\frac{k_\perp}{M_1}\,e^{-k_\perp^2/M_1^2}$$

$$\widehat{f}_{q/p}(x,k_\perp;Q_0) = f_{q/p}(x,Q_0)\,\frac{1}{\pi\langle k_\perp^2\rangle}\,e^{-k_\perp^2/\langle k_\perp^2\rangle}$$

$$\widehat{D}_{h/q}(z,p_\perp;Q_0) = D_{h/q}(z,Q_0)\,\frac{1}{\pi\langle p_\perp^2\rangle}\,e^{-p_\perp^2/\langle p_\perp^2\rangle}$$

11 free parameters

$$\begin{array}{lll} N_{u_v} & N_{d_v} & N_s \\ N_{\bar{u}} & N_{\bar{d}} & N_{\bar{s}} \\ \alpha_{u_v} & \alpha_{d_v} & \alpha_{sea} \\ \beta & M_1\ (\mathrm{GeV}/c)\,. & \end{array}$$

Fixed parameters

$$\langle k_\perp^2\rangle = 0.25\ \mathrm{GeV}^2$$
$$\langle p_\perp^2\rangle = 0.20\ \mathrm{GeV}^2$$
$$g_2 = 0.68\ \mathrm{GeV}^2$$

Fit of HERMES and COMPASS SIDIS data

- We perform 3 different fits:
  - TMD-fit (computing TMD evolution equations numerically)
  - TMD-analytical fit (solving TMD evolution equations in the analytical approx.)
  - DGLAP fit (using DGLAP evolution equation for the collinear part of the TMD)

- Data sets:
  - HERMES (2009) π+ π- π^0 K+ K-
  - COMPASS Deuteron (2004) π+ π- K+ K-
  - COMPASS Proton (2011) h+ h-

Fit of HERMES and COMPASS SIDIS data

χ^2 tables

11 free parameters, 261 points

TMD Evolution (Exact)	TMD Evolution (Analytical)	DGLAP Evolution
$\boxed{\chi^2_{tot} = 255.8}$	$\boxed{\chi^2_{tot} = 275.7}$	$\boxed{\chi^2_{tot} = 315.6}$
$\chi^2_{d.o.f} = 1.02$	$\chi^2_{d.o.f} = 1.10$	$\chi^2_{d.o.f} = 1.26$

Fit of HERMES and COMPASS SIDIS data

χ^2 tables

11 free parameters, 261 points

TMD Evolution (Exact)	TMD Evolution (Analytical)	DGLAP Evolution
$\chi^2_{tot} = 255.8$	$\chi^2_{tot} = 275.7$	$\chi^2_{tot} = 315.6$
$\boxed{\chi^2_{d.o.f} = 1.02}$	$\boxed{\chi^2_{d.o.f} = 1.10}$	$\boxed{\chi^2_{d.o.f} = 1.26}$

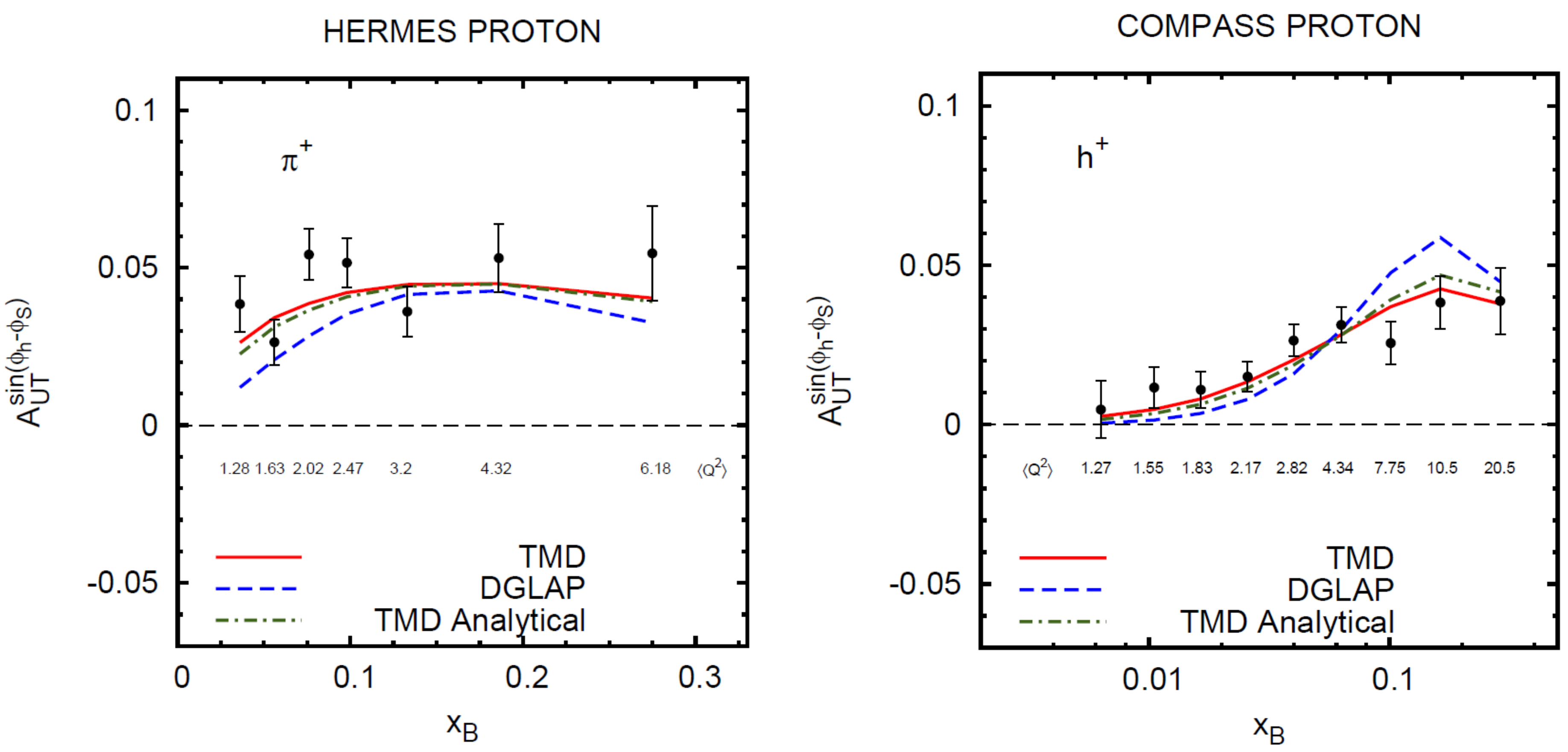
Fit of HERMES and COMPASS SIDIS data

χ^2 tables

11 free parameters, 261 points

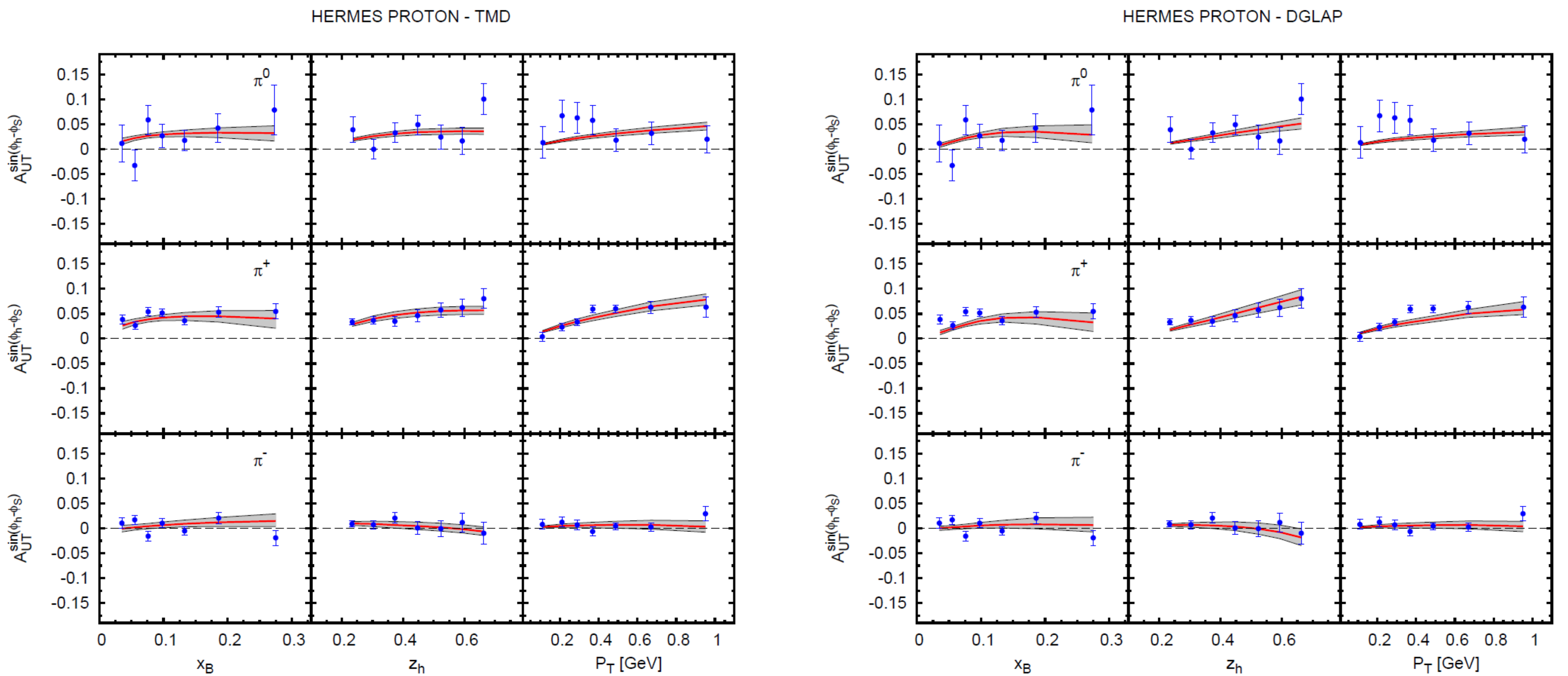
		TMD Evolution (Exact)		TMD Evolution (Analytical)	DGLAP Evolution
		$\chi^2_{tot} = 255.8$ $\chi^2_{d.o.f} = 1.02$		$\chi^2_{tot} = 275.7$ $\chi^2_{d.o.f} = 1.10$	$\chi^2_{tot} = 315.6$ $\chi^2_{d.o.f} = 1.26$
HERMES π^+		$\boxed{\chi^2_x = 10.7}$ $\chi^2_z = 4.3$ $\chi^2_{P_T} = 9.1$	7 points	$\boxed{\chi^2_x = 12.9}$ $\chi^2_z = 4.3$ $\chi^2_{P_T} = 10.5$	$\boxed{\chi^2_x = 27.5}$ $\chi^2_z = 8.6$ $\chi^2_{P_T} = 22.5$
COMPASS h^+		$\boxed{\chi^2_x = 6.7}$ $\chi^2_z = 17.8$ $\chi^2_{P_T} = 12.4$	9 points	$\boxed{\chi^2_x = 11.2}$ $\chi^2_z = 18.5$ $\chi^2_{P_T} = 24.2$	$\boxed{\chi^2_x = 29.2}$ $\chi^2_z = 16.6$ $\chi^2_{P_T} = 11.8$

Fit of HERMES and COMPASS SIDIS data



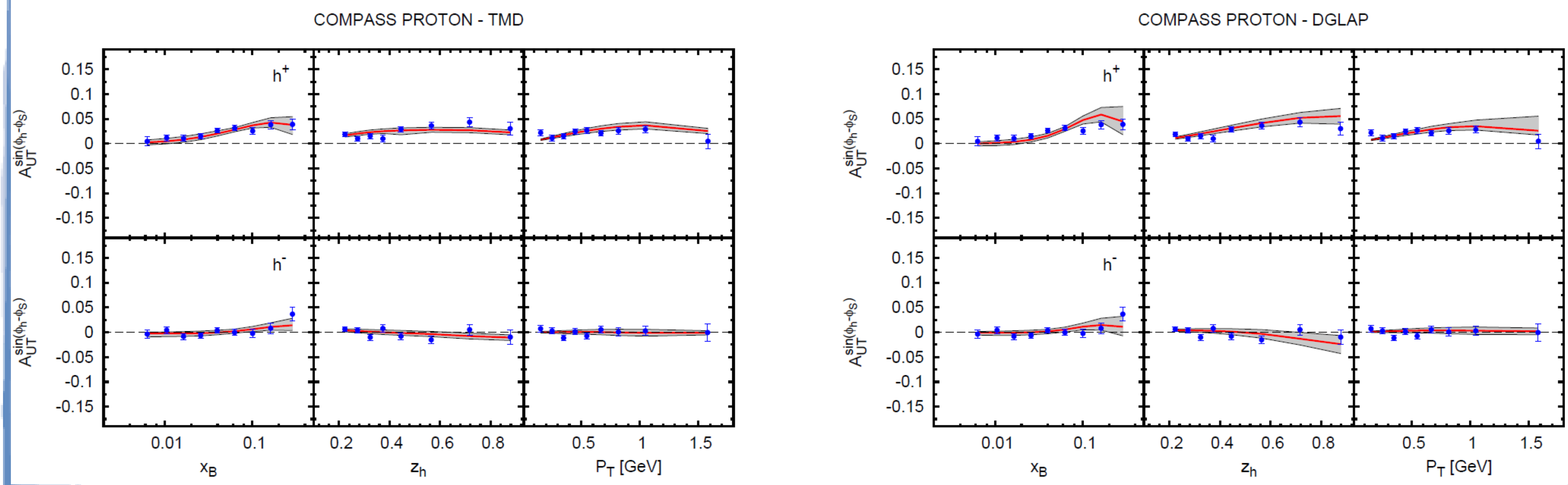
Fit of HERMES and COMPASS SIDIS data

A. Airapetian et al., Phys. Rev. Lett. 103, 152002 (2009), arXiv:0906.3918 [hep-ex]



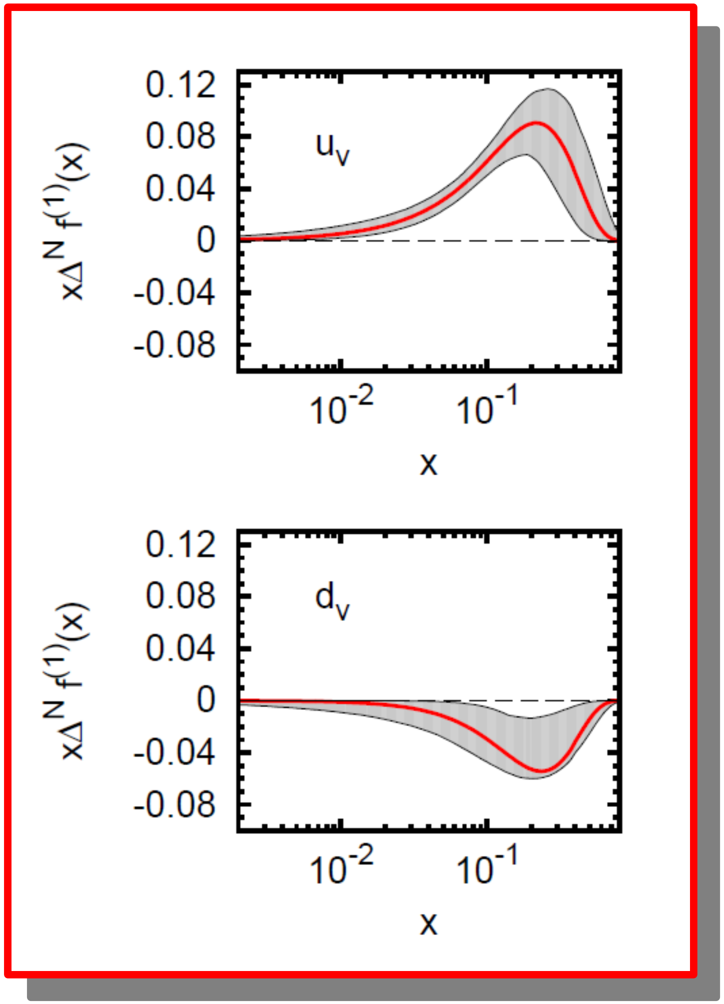
Fit of HERMES and COMPASS SIDIS data

F. Bradamante, arXiv:1111.0869 [hep-ex]



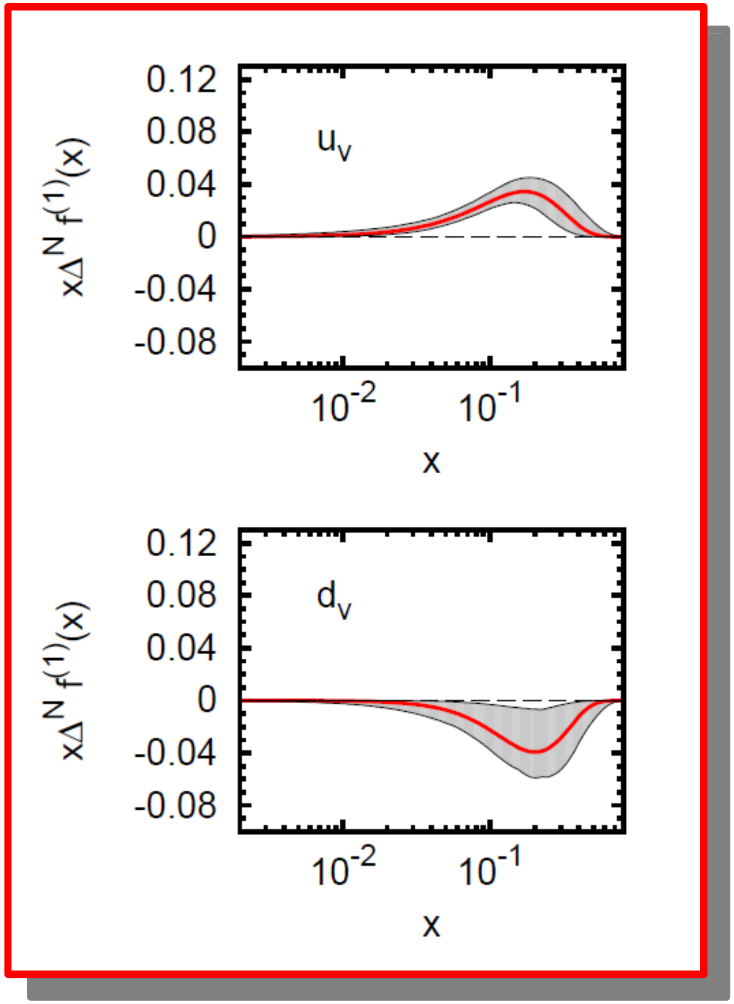
Fit of HERMES and COMPASS SIDIS data

TMD Evolution



Q_0=1 GeV

DGLAP Evolution



Consequences on DY data and warnings

- A rigorous fit need a 'fresh restart' i.e. the analysis of the SIDIS and DY unpolarized data

Fixed parameters in the fit

$$\boxed{\begin{aligned}\langle k_\perp^2\rangle &= 0.25\ \mathrm{GeV}^2\\ \langle p_\perp^2\rangle &= 0.20\ \mathrm{GeV}^2\\ g_2 &= 0.68\ \mathrm{GeV}^2\end{aligned}}$$

- In SIDIS, the Sivers asymmetry is not so strongly sensitive to these values.

- … however in DY they are crucial, in particular g_2

Consequences on DY data and warnings

- Numerator of the asymmetry in analytical approximation for a SIDIS process

$$N_{SIDIS} \propto \Delta^N f(x,Q_0) D(z,Q_0) \sqrt{2e}\frac{P_T}{M_1}\frac{z\,\langle k_\perp^2\rangle_{Siv}^2}{\langle k_\perp^2\rangle \langle P_T^2\rangle_{Siv}^2} e^{-P_T^2/\langle P_T^2\rangle_{Siv}}$$

$$\boxed{\langle P_T^2\rangle_{Siv}^{SIDIS} = z^2\omega_{Siv}^2 + \omega_{FF}^2}$$

$$\boxed{w_S^2(Q,Q_0) = \langle k_\perp^2\rangle_S + 2g_2 \ln\frac{Q}{Q_0}}$$

$$\boxed{w_F^2 \equiv w_F^2(Q,Q_0) = \langle p_\perp^2\rangle + 2z^2 g_2 \ln\frac{Q}{Q_0}}$$

- Here it is squared, strongly suppresses the asymmetry as it becomes larger and larger

- 0.2 <z<0.8

Consequences on DY data and warnings

- Numerator of the asymmetry in analytical approximation for a DY process

$$N_{DY} \propto \Delta^N f(x_1, Q_0) f(x_2, Q_0) \sqrt{2e} \frac{P_T}{M_1} \frac{\langle k_\perp^2 \rangle_{Siv}^2}{\langle k_{\perp 1}^2 \rangle \langle P_T^2 \rangle_{Siv}^2} e^{-P_T^2/\langle P_T^2 \rangle_{Siv}}$$

$$\boxed{\langle P_T^2 \rangle_{Siv}^{DY} = \omega_{Siv}^2 + \omega_2^2}$$

$$\boxed{w_S^2(Q, Q_0) = \langle k_\perp^2 \rangle_S + 2 g_2 \ln \frac{Q}{Q_0}}$$

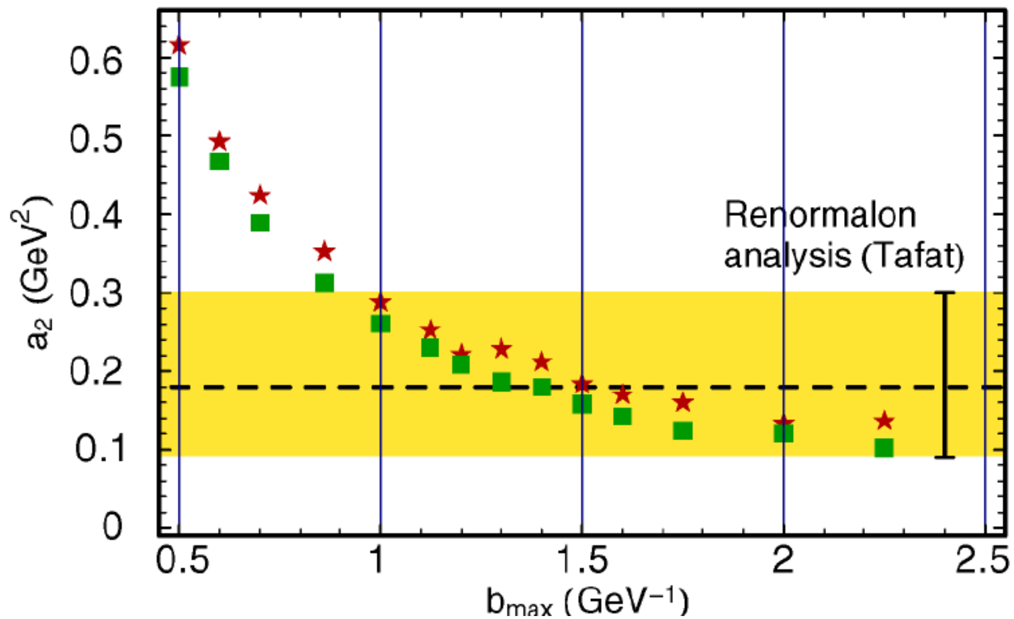
$$\boxed{w^2(Q, Q_0) = \langle k_\perp^2 \rangle + 2\, g_2 \ln \frac{Q}{Q_0}}$$

- Here it is squared, strongly suppresses the asymmetry as it becomes larger and larger

- g_2 is more crucial for DY processes than for the present SIDIS data (because of a wider kinematical range in Q^2)

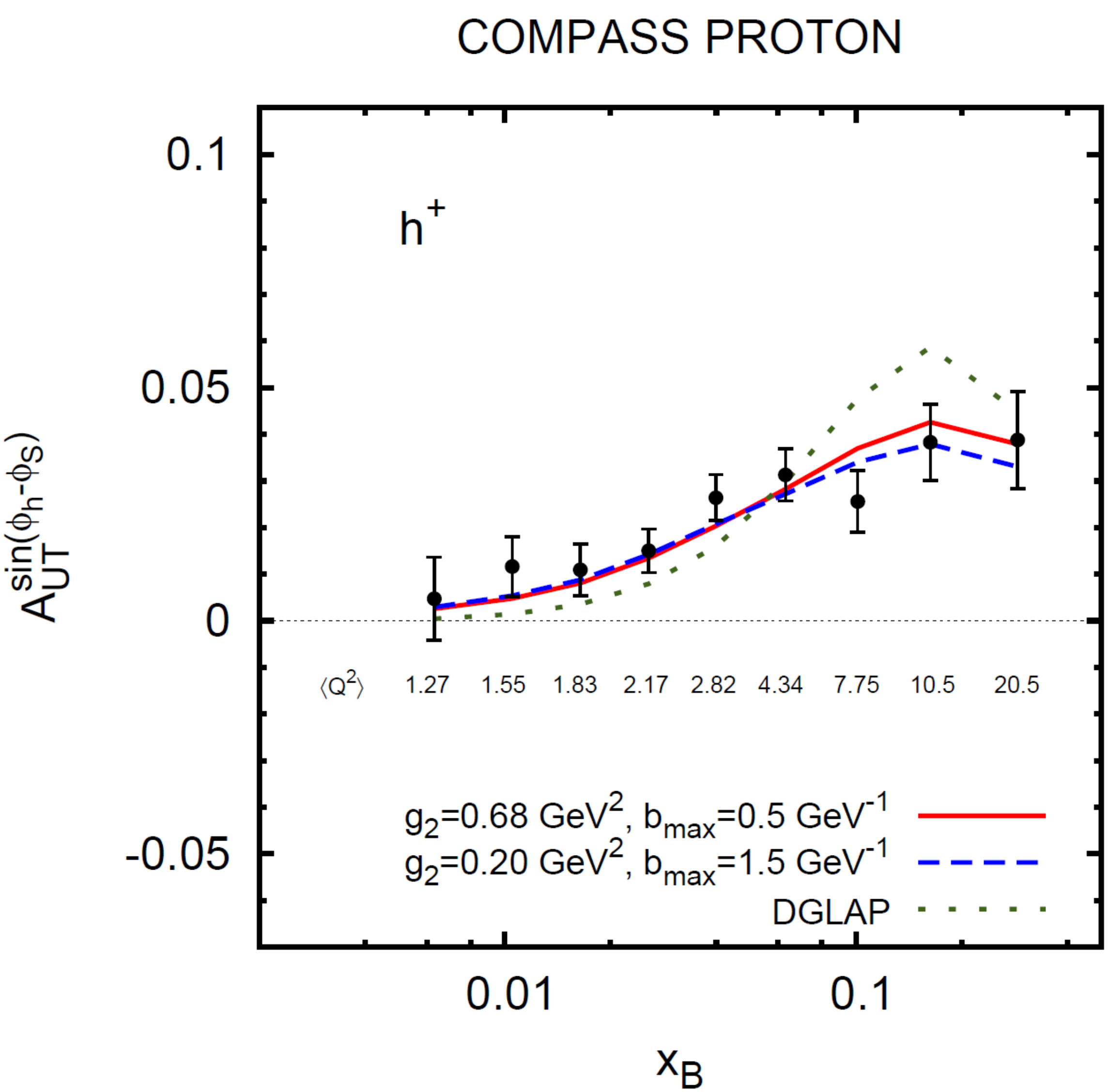
Consequences on DY data and warnings

➢ g_2 depends on the prescription for the separation of the perturbative region from the non -perturbative one. Depends also on the "order" at which you stop in the perturbative expansion.

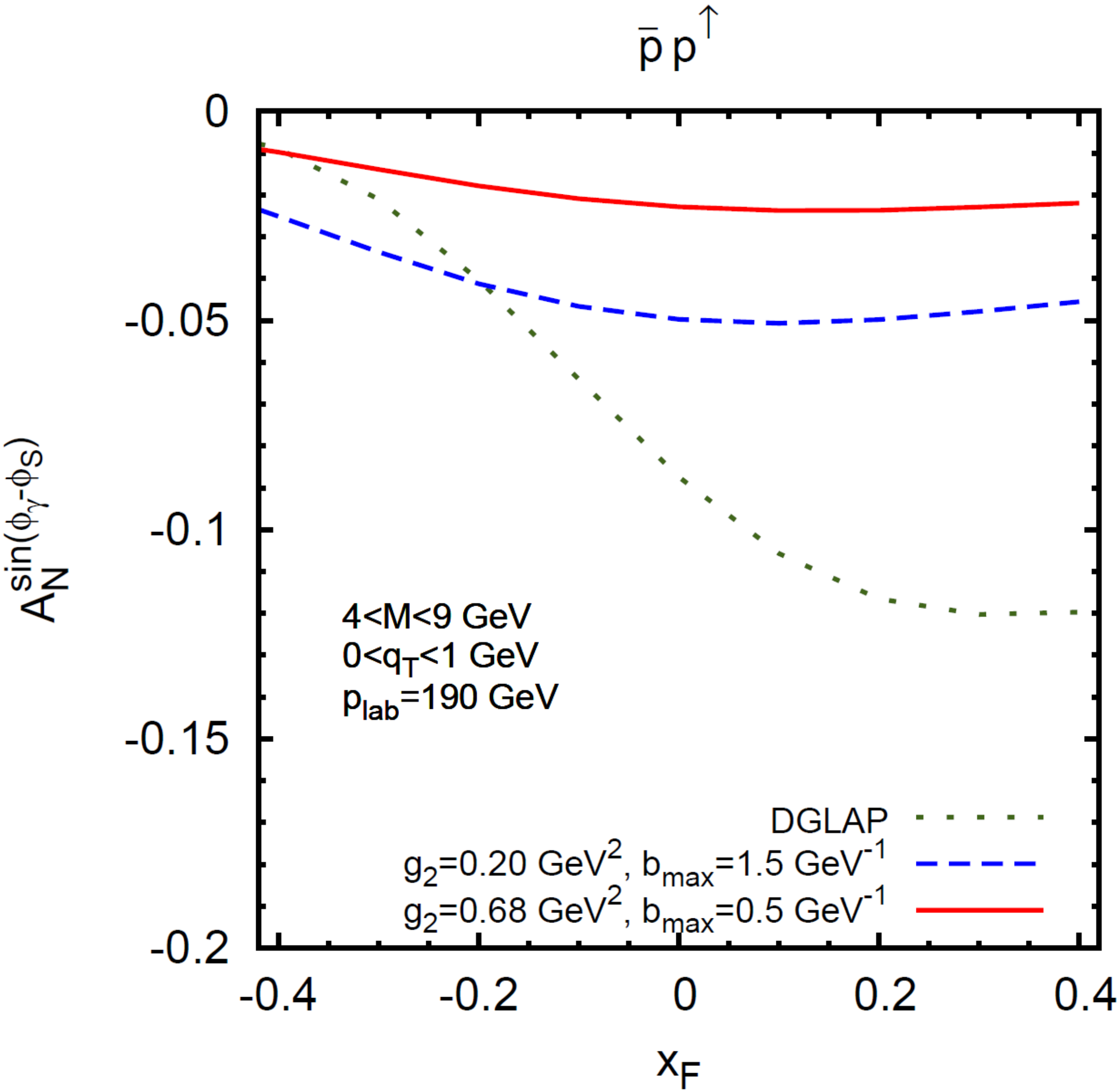


$a_2=g_2$, stars correspond to the choice $C1=2\exp(-\gamma_e)$, squares to $C1=4\exp(-\gamma_e)$

Konychev and Nadolsky, Phys. Lett. B633 (2006)



COMPASS PROTON
h+
0.1
0.05
0
-0.05
⟨Q2⟩ 1.27 1.55 1.83 2.17 2.82 4.34 7.75 10.5 20.5
g2=0.68 GeV2, bmax=0.5 GeV-1
g2=0.20 GeV2, bmax=1.5 GeV-1
DGLAP
0.01
0.1
xB



$\bar{p}\,p^{\uparrow}$
$A_N^{\sin(\phi_\gamma-\phi_S)}$
x_F
0
-0.05
-0.1
-0.15
-0.2
-0.4
-0.2
0
0.2
0.4
4<M<9 GeV
$0<q_T<1$ GeV
p_{lab}=190 GeV
DGLAP
g_2=0.20 GeV^2, b_{max}=1.5 GeV^{-1}
g_2=0.68 GeV^2, b_{max}=0.5 GeV^{-1}

Conclusions

- A first (very preliminary) analysis of evolution shows that it suppresses the Sivers effect.
  Evolutions is fast but not so fast to make the asymmetry negligible (at least in SIDIS) and helps to understand data

- Sivers asymmetry in SIDIS are not sufficient to extract crucial parameters for the evolution

- DY data are more sensitive to the evolution

- A combined analysis of DY&SIDIS (un)polarized data is needed

- Open phenomenological (different g's?) &theoretical problems (other TMD definitions, other prescriptions)

➢ Parametrization of the Collins function:

$$\Delta^N D_{\pi/q^\uparrow}(z, p_\perp) = 2\mathcal{N}_q^C(z)\, h(p_\perp)\, D_{\pi/q}(z, k_\perp)$$

Unpolarized FF

- $\mathcal{N}_q^C(z) = N_q^C\, z^\gamma (1-z)^\delta \dfrac{(\gamma+\delta)^{(\gamma+\delta)}}{\gamma^\gamma \delta^\delta}$

- $h(p_\perp) = \sqrt{2e}\,\dfrac{p_\perp}{M_h} e^{-p_\perp^2/M_h^2}$

N_q^C, γ, δ, M_h free parameters

✔ Bound:

$$\Delta^N D_{\pi/q^\uparrow}(z, p_\perp) \leq 2\, D_{\pi/q}(z, k_\perp)$$

✔ Torino vs Amsterdam notation

$$\Delta^N D_{\pi/q^\uparrow}(z, p_\perp) = \frac{2p_\perp}{zM} H_1^\perp(z, p_\perp)$$

➢Gaussian smearing for both unpolarized PDF and FF

$$f_{q/p}(x, k_\perp) = f_q(x) \, \frac{1}{\pi \langle k_\perp^2 \rangle} \, e^{-k_\perp^2/\langle k_\perp^2 \rangle}$$

GRV98 set

[*] $\langle k_\perp^2 \rangle = 0.25 \, (\mathrm{GeV}/c)^2$

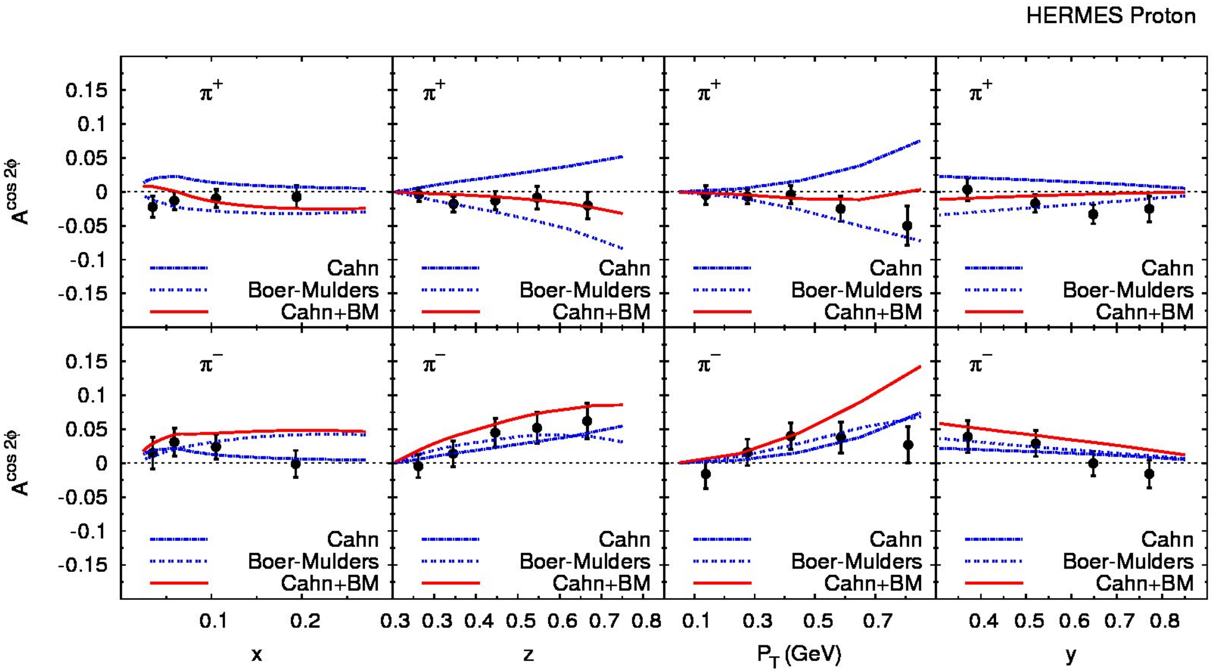
$$D_q^h(z, p_\perp) = D_q^h(z) \, \frac{1}{\pi \langle p_\perp^2 \rangle} \, e^{-p_\perp^2/\langle p_\perp^2 \rangle}$$

DSS set

[*] $\langle p_\perp^2 \rangle = 0.20 \, (\mathrm{GeV}/c)^2$

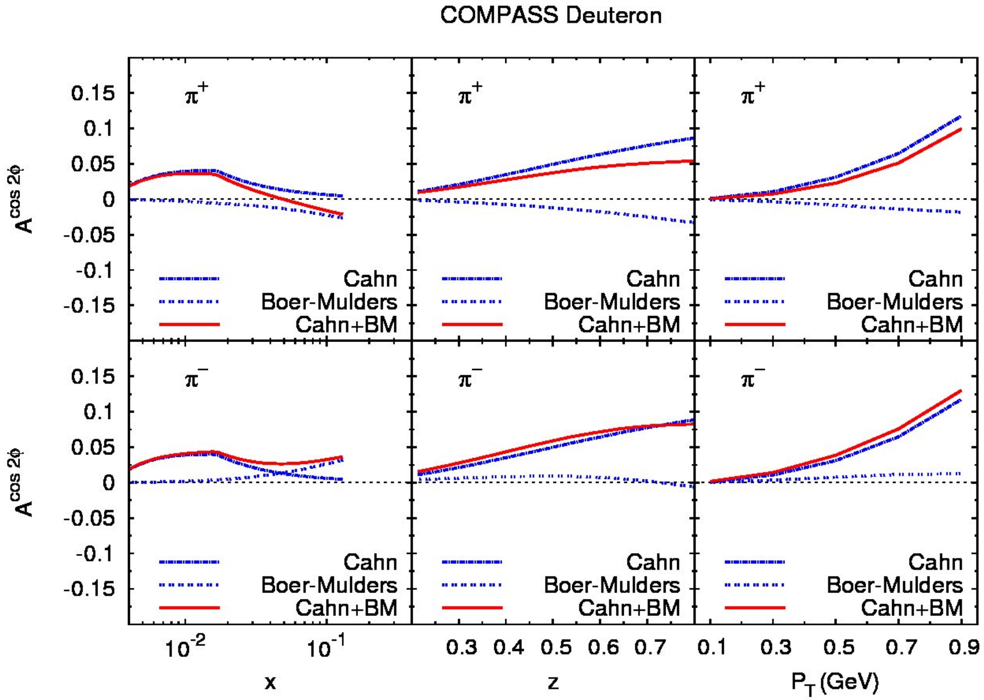
[*]Anselmino et. Phys. ReV D71, 074006 (2005)

Extraction of the Boer-Mulders Function



SPIN2010 (Francesca Giordano)

Extraction of the Boer-Mulders Function



New COMPASS data.
SPIN2010 Sbrizzai