

TMDS IN UNPOLARIZED DRELL-YAN

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- I am presenting mainly work done by others. In particular, Marc Schlegel, Zhun Lu and Stefano Melis among the workshop participants

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- I am presenting mainly work done by others. In particular, Marc Schlegel, Zhun Lu and Stefano Melis among the workshop participants
- I will repeat some things that have been said in the talks by Werner and Enzo

General formula

$$\frac{d\sigma}{d^4q d\Omega} \propto \frac{\alpha^2}{Q^2} \left[(1 + \cos^2 \theta) F_{UU}^1 + (1 - \cos^2 \theta) F_{UU}^2 + \sin 2\theta \cos \phi F_{UU}^{\cos \phi} + \sin^2 \theta \cos 2\phi F_{UU}^{\cos 2\phi} \right]$$

Boer, Vogelsang, PRD 74 (06)

Arnold, Metz, Schlegel, PRD 79 (09)

General formula

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“D-Y F_T ”

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“D-Y F_T ”

“D-Y F_L ”

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General formula

$$\frac{d\sigma}{d^4q d\Omega} \propto \frac{\alpha^2}{Q^2} \left[(1 + \cos^2 \theta) F_{UU}^1 + (1 - \cos^2 \theta) F_{UU}^2 + \sin 2\theta \cos \phi F_{UU}^{\cos \phi} + \sin^2 \theta \cos 2\phi F_{UU}^{\cos 2\phi} \right]$$

“D-Y F_T ”

“D-Y F_L ”

“Cahn”

Boer, Vogelsang, PRD 74 (06)

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General formula

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“D-Y F_T ”

“D-Y F_L ”

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“Boer-Mulders”

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“D-Y F_T ”

“D-Y F_L ”

“Cahn”

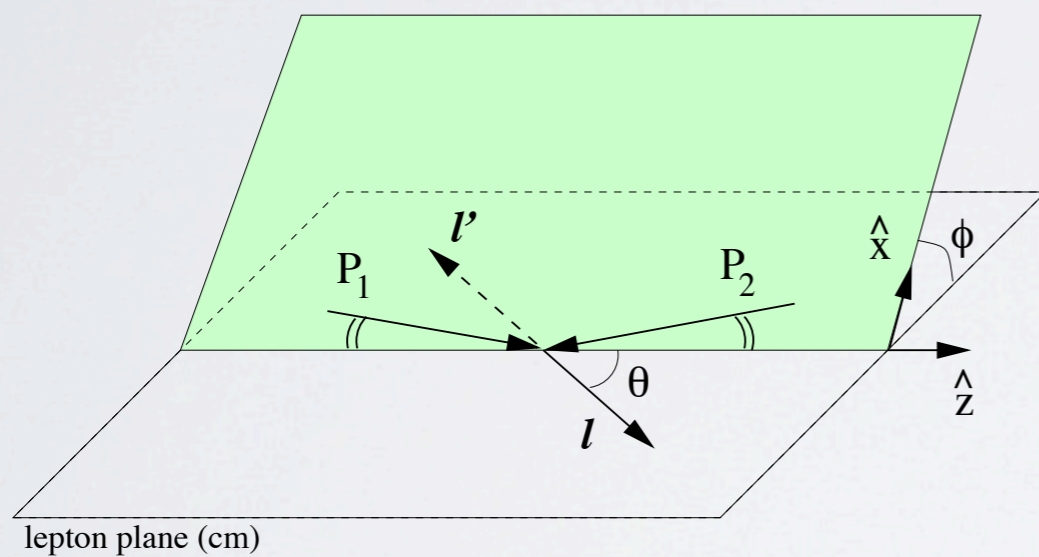
“Boer-Mulders”

$$\lambda = \frac{F_{UU}^1 - F_{UU}^2}{F_{UU}^1 + F_{UU}^2}, \quad \mu = \frac{F_{UU}^{\cos \phi}}{F_{UU}^1 + F_{UU}^2}, \quad \nu = \frac{2 F_{UU}^{\cos 2\phi}}{F_{UU}^1 + F_{UU}^2}$$

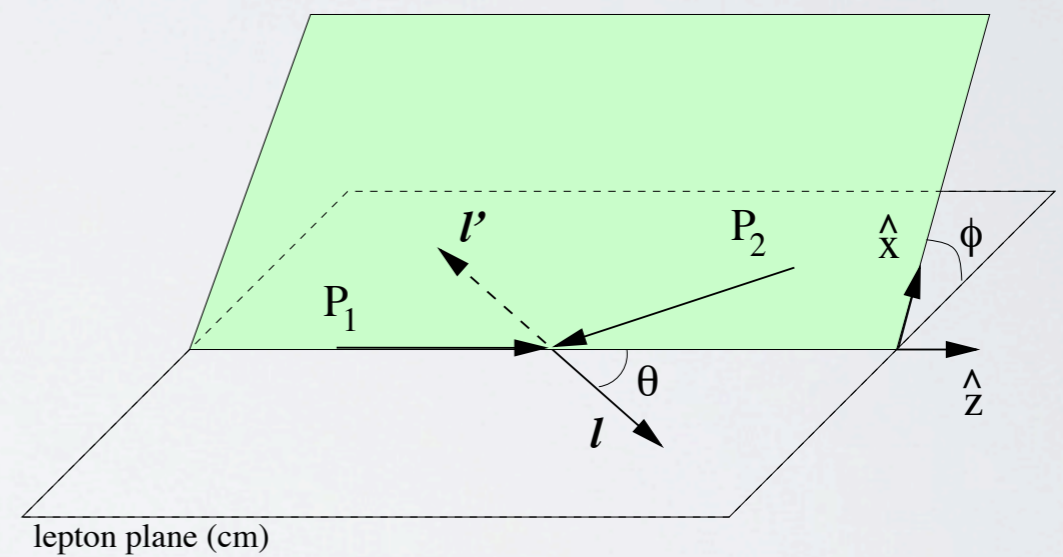
Boer, Vogelsang, PRD 74 (06)

Arnold, Metz, Schlegel, PRD 79 (09)

Different frames



Collins-Soper



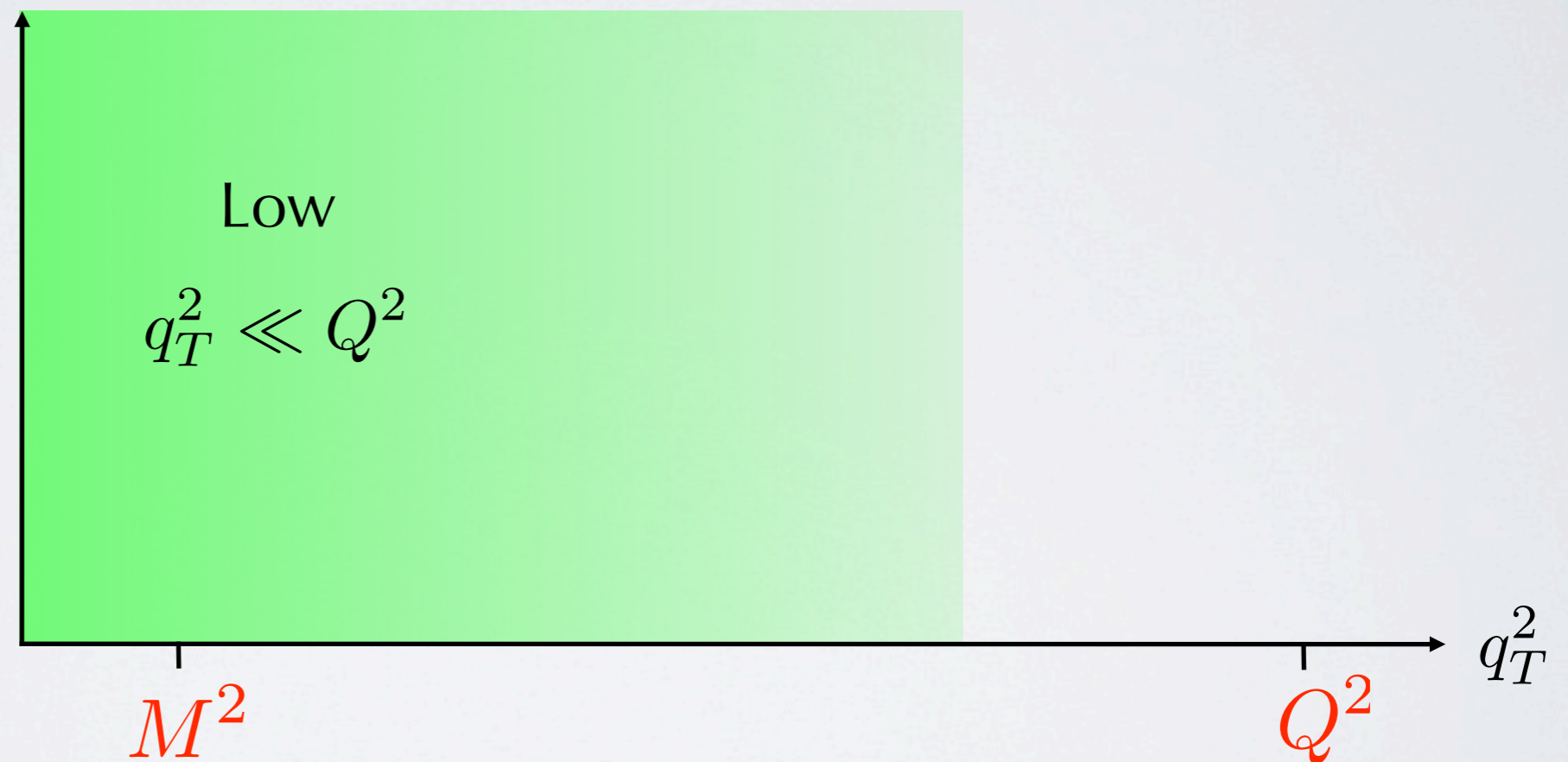
Gottfried-Jackson

Connecting the frames

$$\begin{pmatrix} F_{UU}^1 \\ F_{UU}^2 \\ F_{UU}^{\cos \phi} \\ F_{UU}^{\cos 2\phi} \end{pmatrix}_{\text{GJ}} = \frac{1}{1 + \rho^2} \begin{pmatrix} 1 + \frac{1}{2}\rho^2 & \frac{1}{2}\rho^2 & -\rho & \frac{1}{2}\rho^2 \\ \rho^2 & 1 & 2\rho & -\rho^2 \\ \rho & -\rho & 1 - \rho^2 & -\rho \\ \frac{1}{2}\rho^2 & -\frac{1}{2}\rho^2 & \rho & 1 + \frac{1}{2}\rho^2 \end{pmatrix} \begin{pmatrix} F_{UU}^1 \\ F_{UU}^2 \\ F_{UU}^{\cos \phi} \\ F_{UU}^{\cos 2\phi} \end{pmatrix}_{\text{CS}}$$

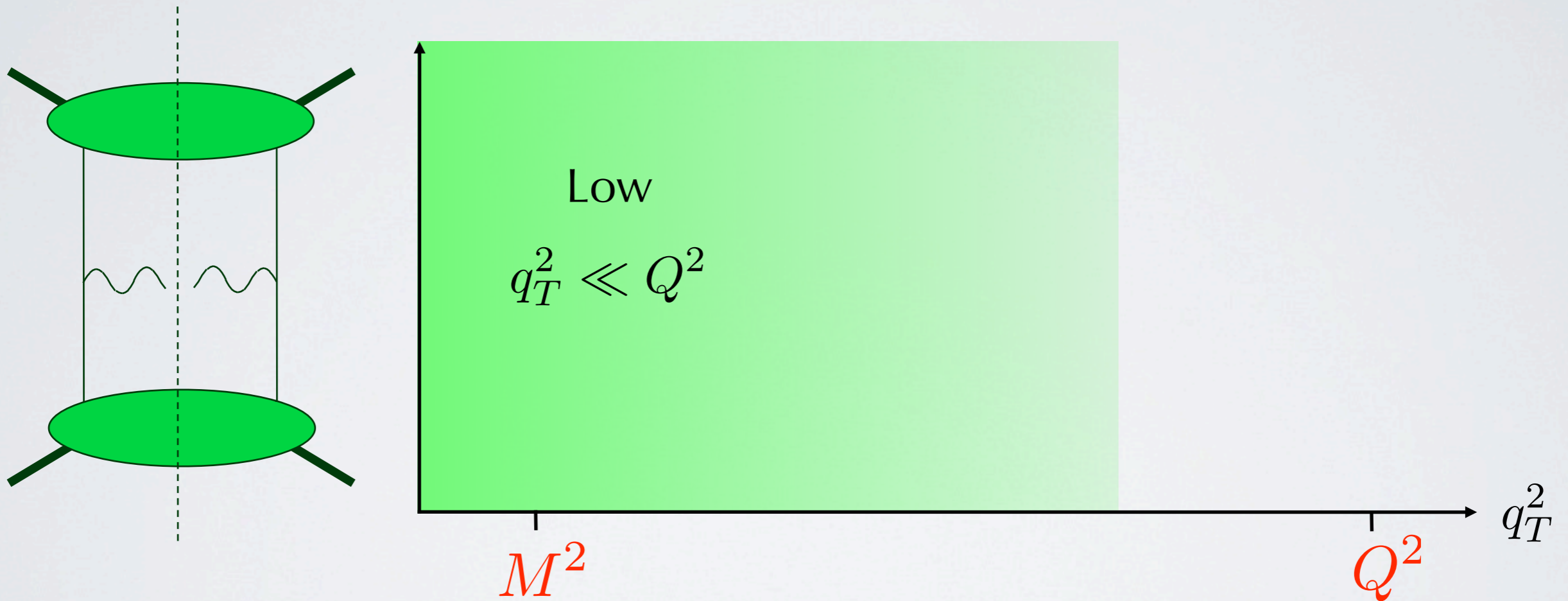
$$\rho \equiv \frac{q_T}{Q}$$

Low transverse momentum



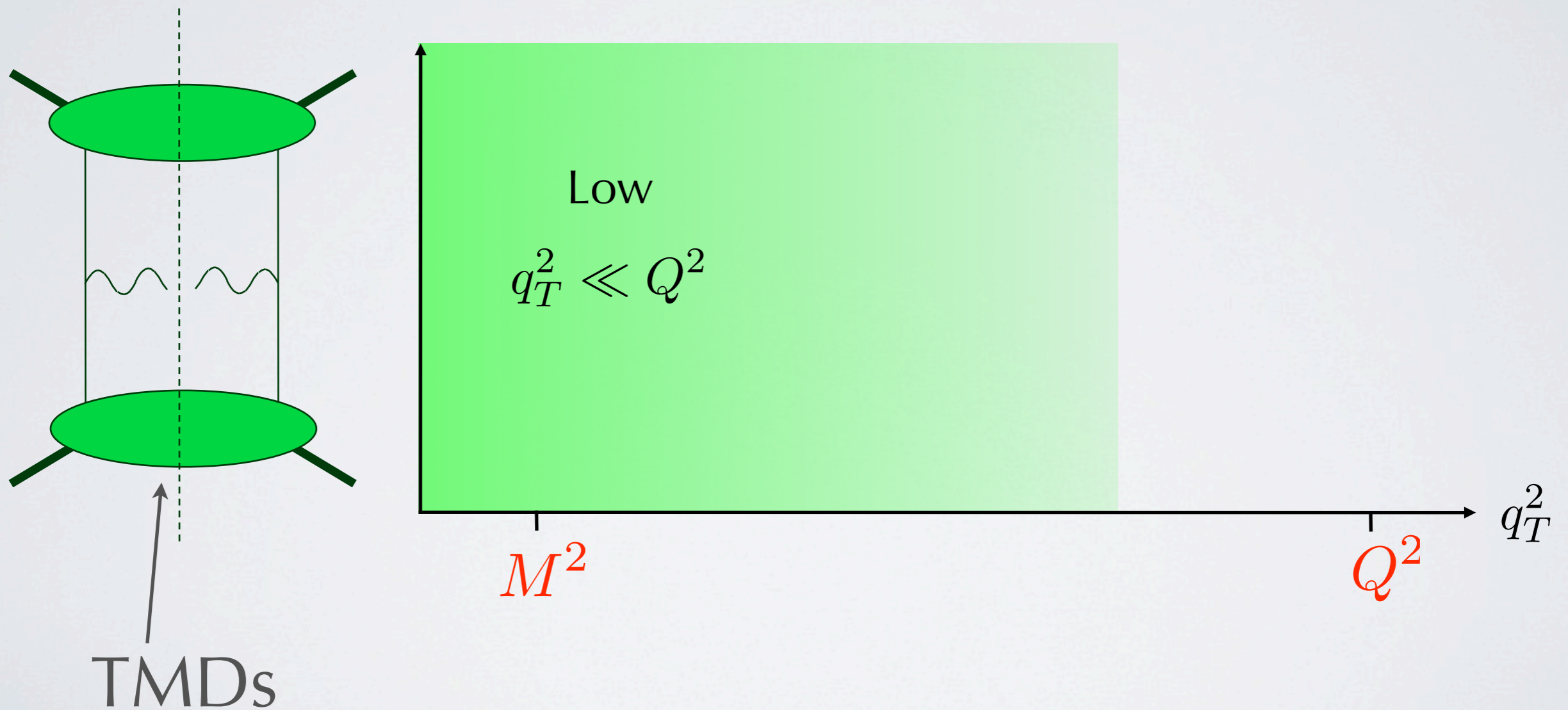
see Werner's talk

Low transverse momentum



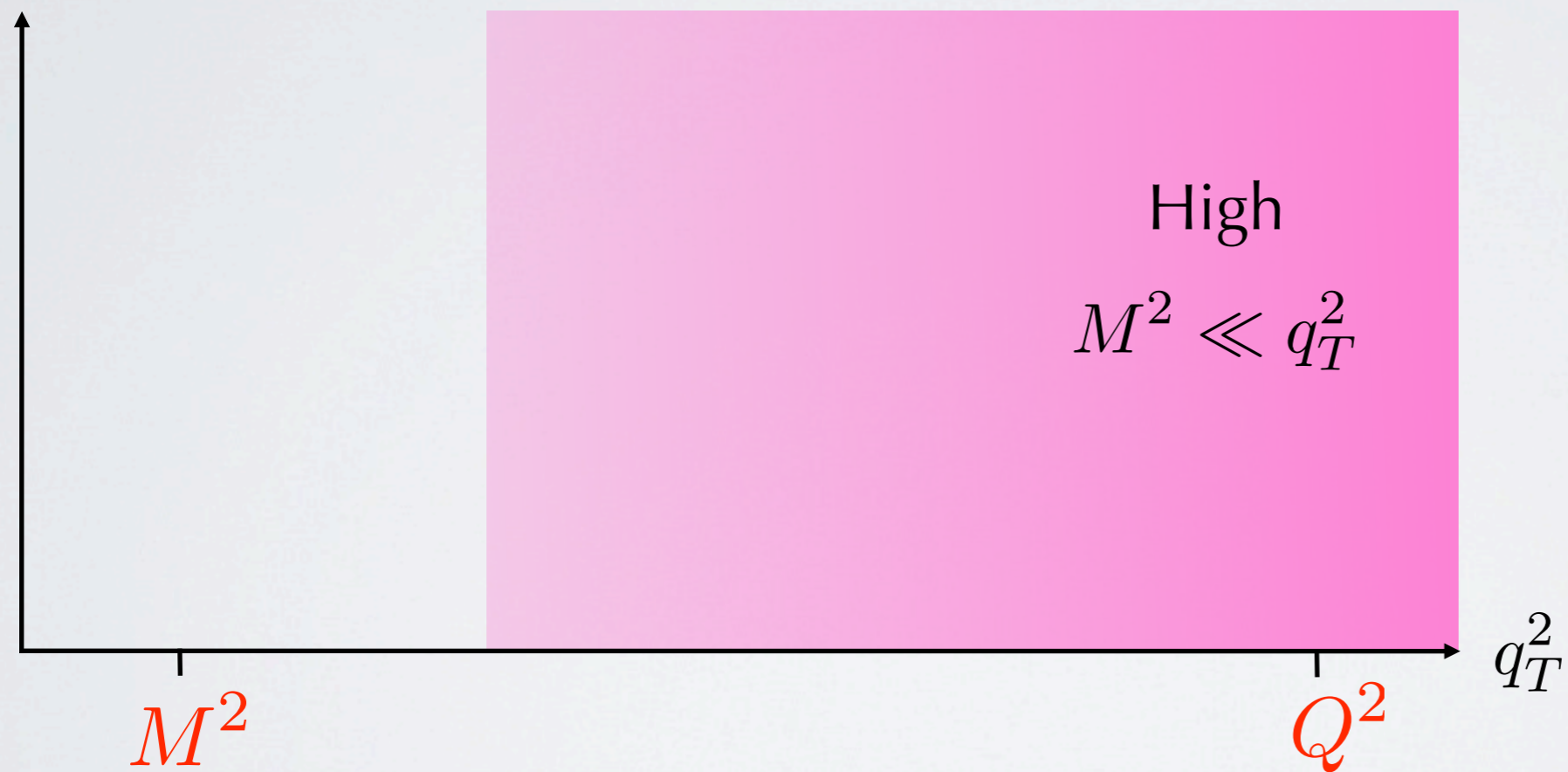
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Low transverse momentum

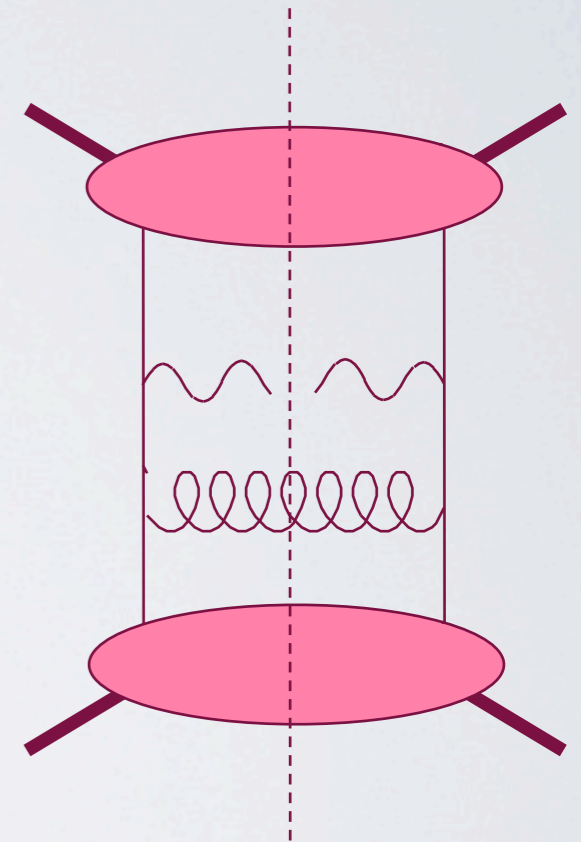
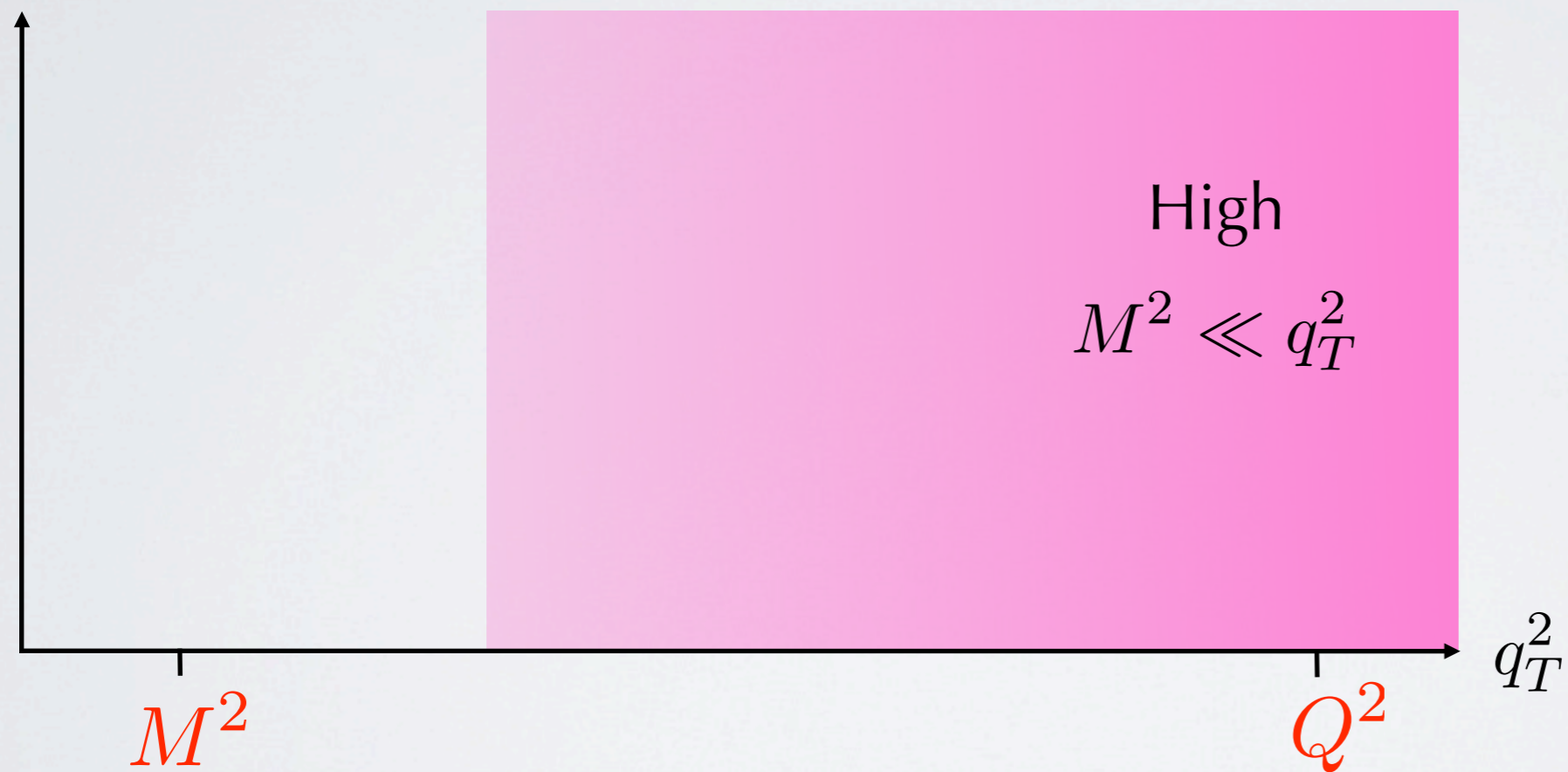


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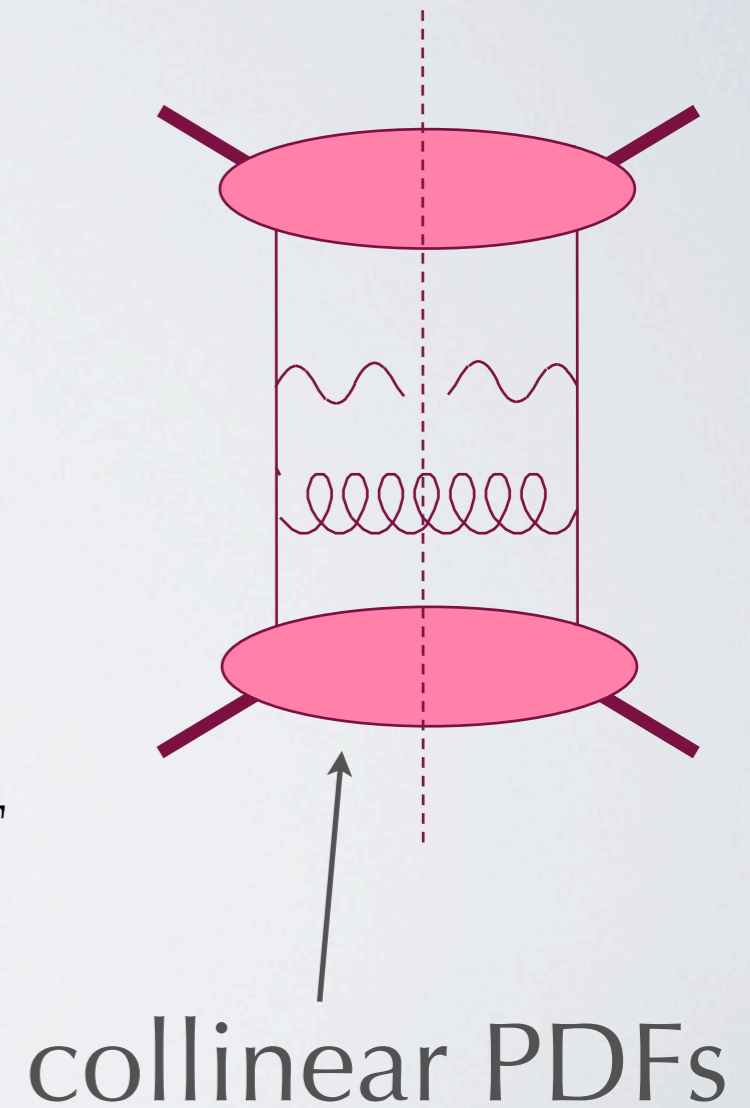
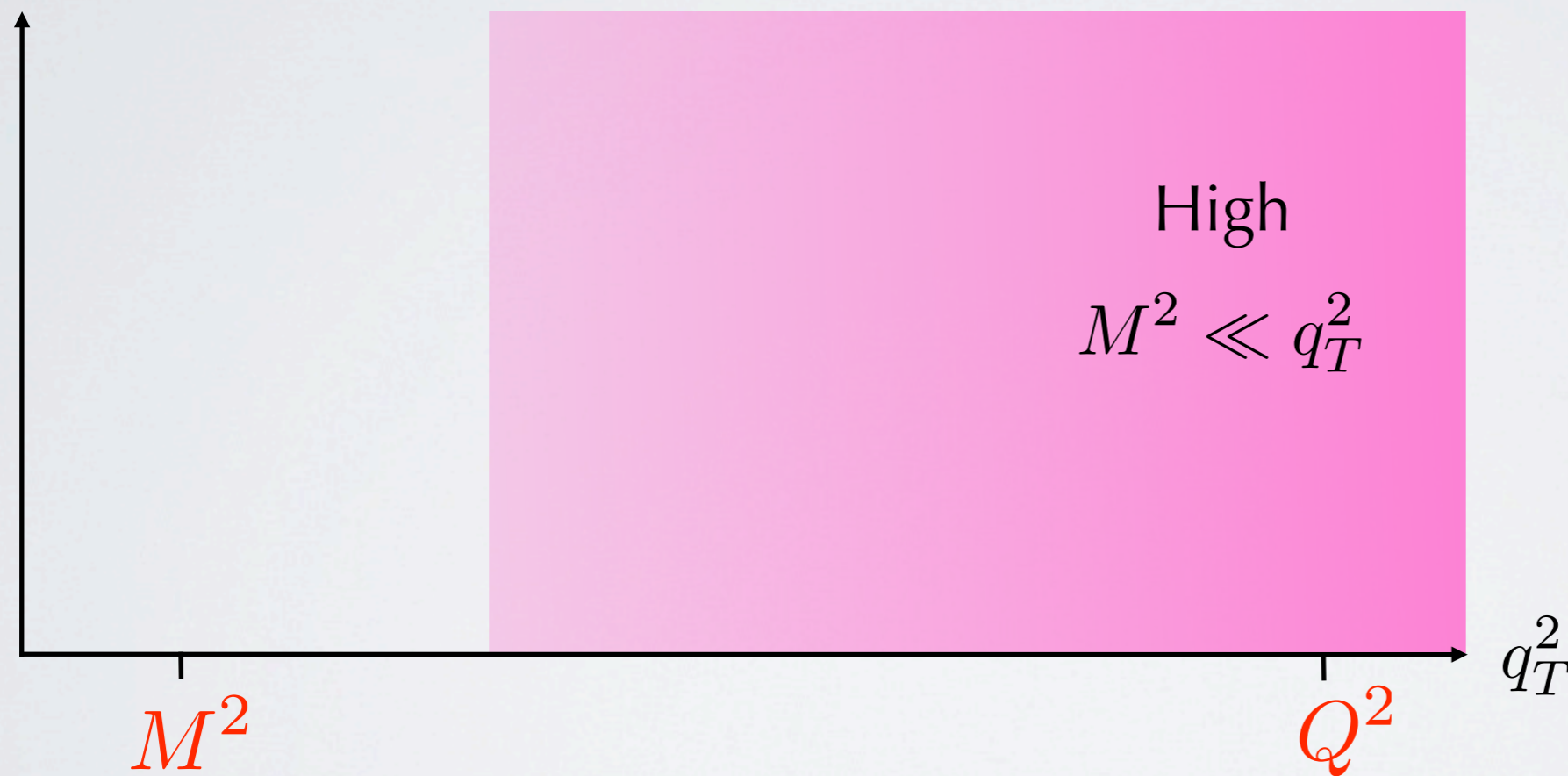
High transverse momentum



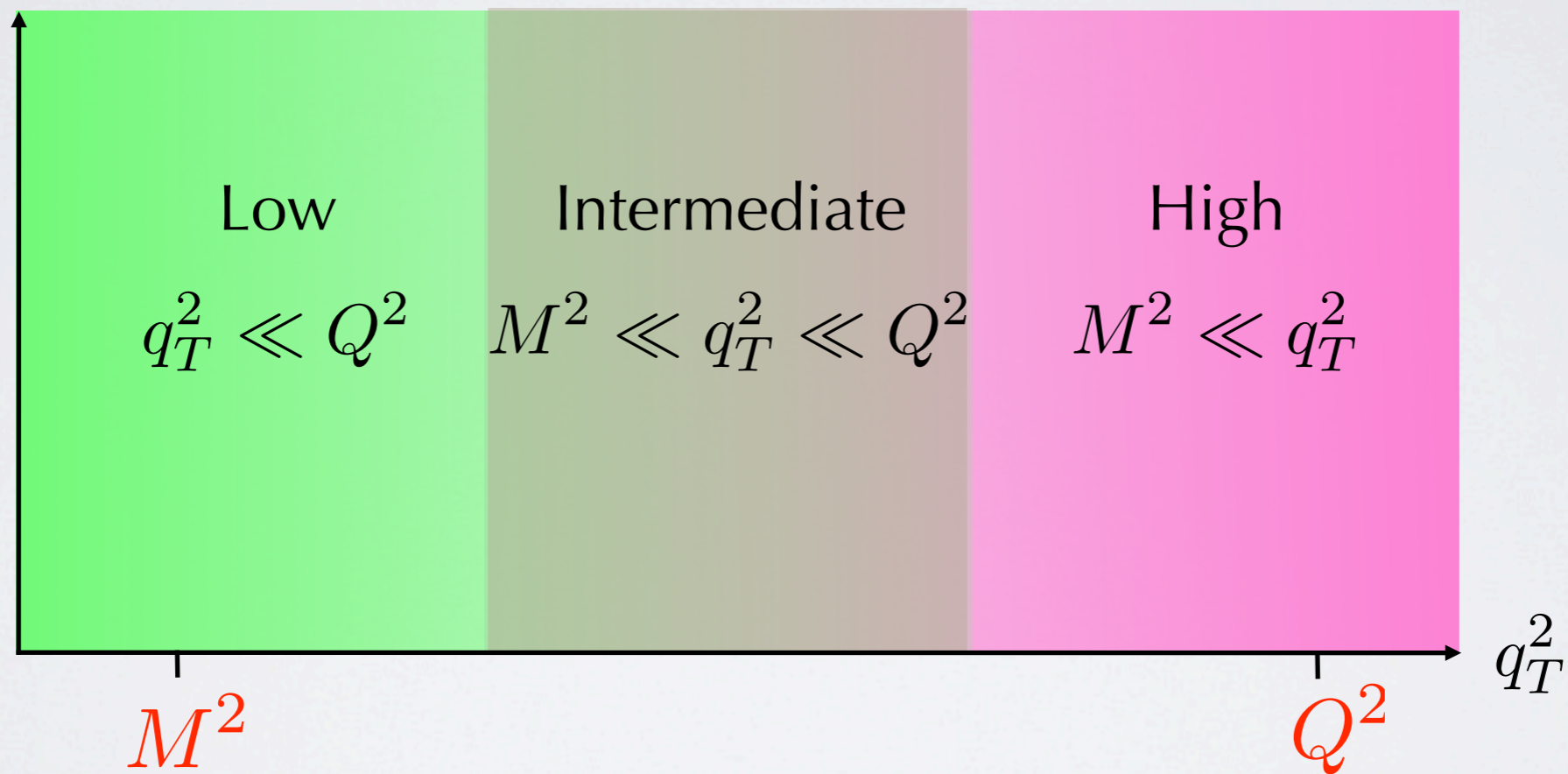
High transverse momentum



High transverse momentum



Matching?



Low transverse momentum

$$\frac{d\sigma}{d^4q d\Omega} \propto \frac{\alpha^2}{Q^2} \left[(1 + \cos^2 \theta) F_{UU}^1 + \sin^2 \theta \cos 2\phi F_{UU}^{\cos 2\phi} \right]$$

Low transverse momentum

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Twist 2

Twist 2

Low transverse momentum

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Twist 2

Twist 4

Twist 3

Twist 2

High transverse momentum

$$\frac{d\sigma}{d^4q d\Omega} \propto \frac{\alpha^2}{Q^2} \left[(1 + \cos^2 \theta) F_{UU}^1 + (1 - \cos^2 \theta) F_{UU}^2 + \sin 2\theta \cos \phi F_{UU}^{\cos \phi} + \sin^2 \theta \cos 2\phi F_{UU}^{\cos 2\phi} \right]$$

High transverse momentum

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All twist 2

Matching?

Low q_T

High q_T

$$F \sim \frac{M^2 q_T^2}{M^6 + q_T^6}$$

$$F \sim \frac{M^2}{q_T^4}$$

Matching?

Low q_T

$$F \sim \frac{1}{M^2 + q_T^2}$$

$$F \sim \frac{M^2 q_T^2}{M^6 + q_T^6}$$

High q_T

$$F \sim \frac{M^2}{q_T^4}$$

Matching?

Low q_T

$$F \sim \frac{1}{M^2}$$

High q_T

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$$F \sim \frac{M^2 q_T^2}{M^6 + q_T^6}$$

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Matching?

Low q_T

Tw 2 $F \sim \frac{1}{M^2}$

$$F \sim \frac{1}{M^2 + q_T^2}$$

High q_T

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Matching?

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Matching?

Low q_T

Tw 2 $F \sim \frac{1}{M^2}$

$$F \sim \frac{1}{M^2 + q_T^2}$$

High q_T

$F \sim \frac{1}{q_T^2}$ **Tw 2**

$$F \sim \frac{M^2 q_T^2}{M^6 + q_T^6}$$

$$F \sim \frac{M^2}{q_T^4}$$

Matching?

Low q_T

Tw 2 $F \sim \frac{1}{M^2}$

$$F \sim \frac{1}{M^2 + q_T^2}$$

$$F \sim \frac{q_T}{Q} \frac{1}{M^2 + q_T^2}$$

$$F \sim \frac{M^2 q_T^2}{M^6 + q_T^6}$$

High q_T

$F \sim \frac{1}{q_T^2}$ **Tw 2**

$$F \sim \frac{M^2}{q_T^4}$$

Matching?

Low q_T

Tw 2 $F \sim \frac{1}{M^2}$

$$F \sim \frac{1}{Q} \frac{q_T}{M^2}$$

$$F \sim \frac{1}{M^2 + q_T^2}$$

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High q_T

$F \sim \frac{1}{q_T^2}$ **Tw 2**

$$F \sim \frac{M^2}{q_T^4}$$

Matching?

Low q_T

Tw 2 $F \sim \frac{1}{M^2}$

Tw 3 $F \sim \frac{1}{Q} \frac{q_T}{M^2}$

$$F \sim \frac{1}{M^2 + q_T^2}$$

$$F \sim \frac{q_T}{Q} \frac{1}{M^2 + q_T^2}$$

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High q_T

$F \sim \frac{1}{q_T^2}$ **Tw 2**

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High q_T

$F \sim \frac{1}{q_T^2}$ **Tw 2**

$$F \sim \frac{1}{Q q_T}$$

$$F \sim \frac{M^2}{q_T^4}$$

Matching?

Low q_T

Tw 2 $F \sim \frac{1}{M^2}$

Tw 3 $F \sim \frac{1}{Q} \frac{q_T}{M^2}$

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High q_T

$F \sim \frac{1}{q_T^2}$ **Tw 2**

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$$F \sim \frac{M^2}{q_T^4}$$

Matching?

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Tw 3 $F \sim \frac{1}{Q} \frac{q_T}{M^2}$

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High q_T

$F \sim \frac{1}{q_T^2}$ **Tw 2**

$F \sim \frac{1}{Q q_T}$ **Tw 2**

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Matching?

Low q_T

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High q_T

$F \sim \frac{1}{q_T^2}$ **Tw 2**

$F \sim \frac{1}{Q q_T}$ **Tw 2**

$$F \sim \frac{M^2}{q_T^4}$$

Matching?

Low q_T

Tw 2 $F \sim \frac{1}{M^2}$

Tw 3 $F \sim \frac{1}{Q} \frac{q_T}{M^2}$

Tw 4 $F \sim \frac{1}{Q^2} \frac{q_T^2}{M^2}$

$$F \sim \frac{1}{M^2 + q_T^2}$$

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High q_T

$F \sim \frac{1}{q_T^2}$ Tw 2

$F \sim \frac{1}{Q q_T}$ Tw 2

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Matching?

Low q_T

Tw 2 $F \sim \frac{1}{M^2}$

Tw 3 $F \sim \frac{1}{Q} \frac{q_T}{M^2}$

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High q_T

$F \sim \frac{1}{q_T^2}$ Tw 2

$F \sim \frac{1}{Q q_T}$ Tw 2

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Matching?

Low q_T

Tw 2 $F \sim \frac{1}{M^2}$

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High q_T

$F \sim \frac{1}{q_T^2}$ Tw 2

$F \sim \frac{1}{Q q_T}$ Tw 2

$F \sim \frac{1}{Q^2}$ Tw 2

$F \sim \frac{M^2}{q_T^4}$

Matching?

Low q_T

Tw 2 $F \sim \frac{1}{M^2}$

Tw 3 $F \sim \frac{1}{Q} \frac{q_T}{M^2}$

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$$F \sim \frac{M^2 q_T^2}{M^6 + q_T^6}$$

High q_T

$F \sim \frac{1}{q_T^2}$ Tw 2

$F \sim \frac{1}{Q q_T}$ Tw 2

$F \sim \frac{1}{Q^2}$ Tw 2

$$F \sim \frac{M^2}{q_T^4}$$

Matching?

Low q_T

Tw 2 $F \sim \frac{1}{M^2}$

Tw 3 $F \sim \frac{1}{Q} \frac{q_T}{M^2}$

Tw 4 $F \sim \frac{1}{Q^2} \frac{q_T^2}{M^2}$

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High q_T

$F \sim \frac{1}{q_T^2}$ **Tw 2**

$F \sim \frac{1}{Q q_T}$ **Tw 2**

$F \sim \frac{1}{Q^2}$ **Tw 2**

$F \sim \frac{M^2}{q_T^4}$

Matching?

Low q_T

High q_T

Tw 2 $F \sim \frac{1}{M^2}$

$$F \sim \frac{1}{M^2 + q_T^2}$$

$F \sim \frac{1}{q_T^2}$ **Tw 2**

Tw 3 $F \sim \frac{1}{Q} \frac{q_T}{M^2}$

$$F \sim \frac{q_T}{Q} \frac{1}{M^2 + q_T^2}$$

$F \sim \frac{1}{Q q_T}$ **Tw 2**

Tw 4 $F \sim \frac{1}{Q^2} \frac{q_T^2}{M^2}$

$$F \sim \frac{q_T^2}{Q^2} \frac{1}{M^2 + q_T^2}$$

$F \sim \frac{1}{Q^2}$ **Tw 2**

Tw 2 $F \sim \frac{q_T^2}{M^4}$

$$F \sim \frac{M^2 q_T^2}{M^6 + q_T^6}$$

$F \sim \frac{M^2}{q_T^4}$ Tw 4

Tw 2 $F \sim \frac{q_T^2}{M^4}$

$$F \sim \frac{M^2 q_T^4}{M^6 + q_T^6}$$

$$F \sim \frac{1}{q_T^4}$$

Tw 4

1

F^1
 UU

Matching?

Low q_T

High q_T

Tw 2

$$F \sim \frac{1}{M^2}$$

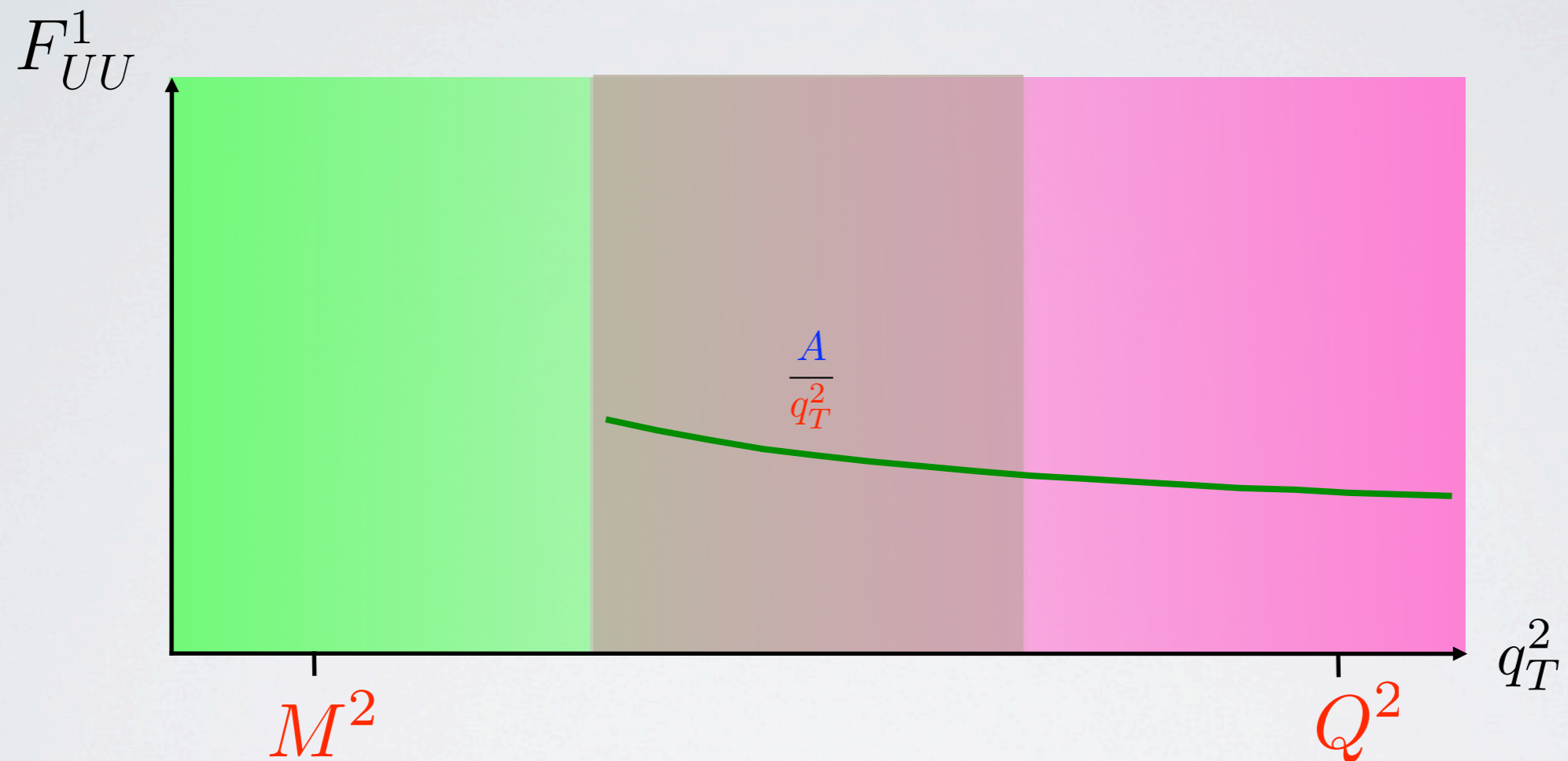
$$F \sim \frac{1}{M^2 + q_T^2}$$

$$F \sim \frac{1}{q_T^2}$$

Tw 2

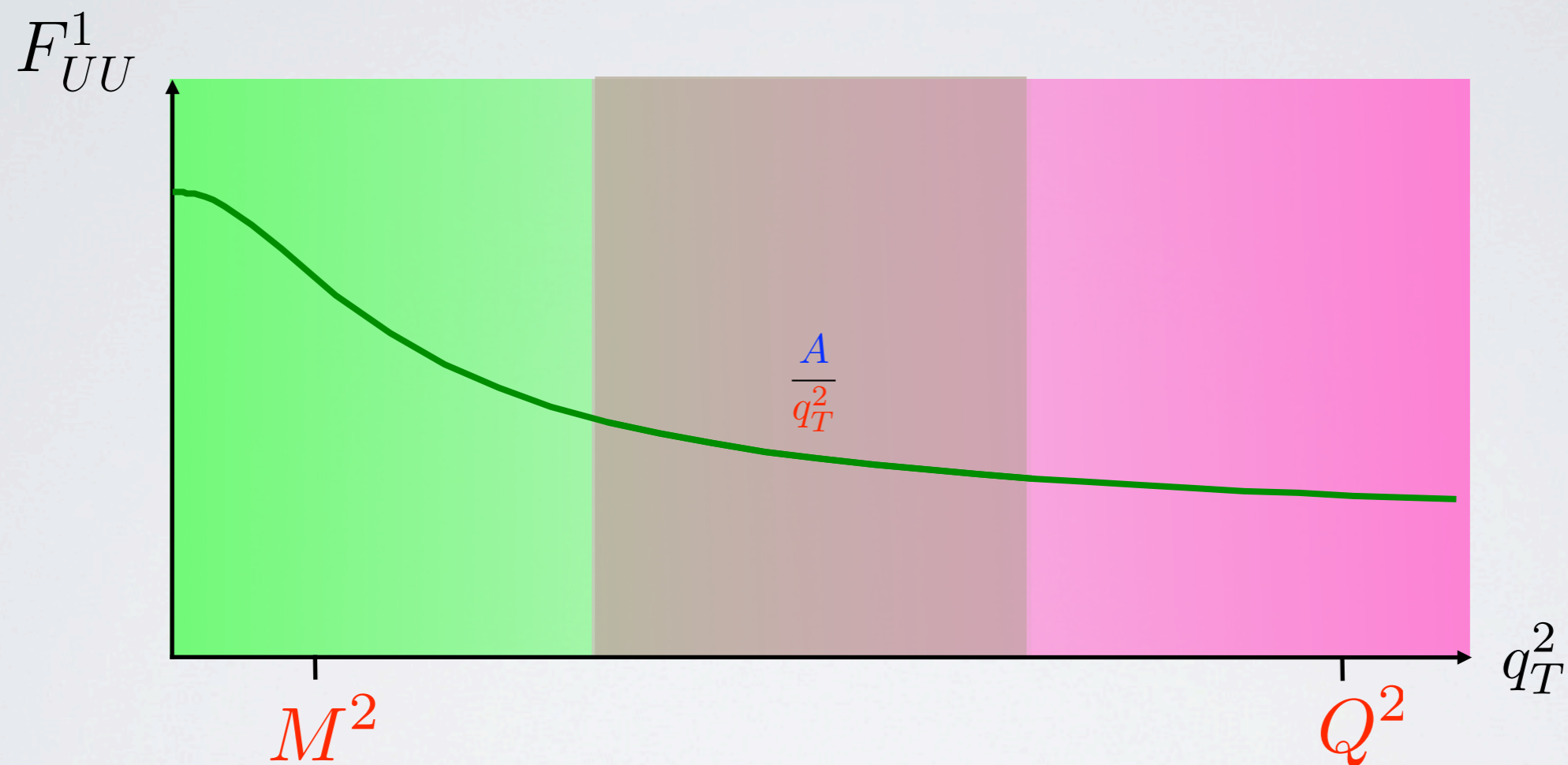
Yes

Matching behavior



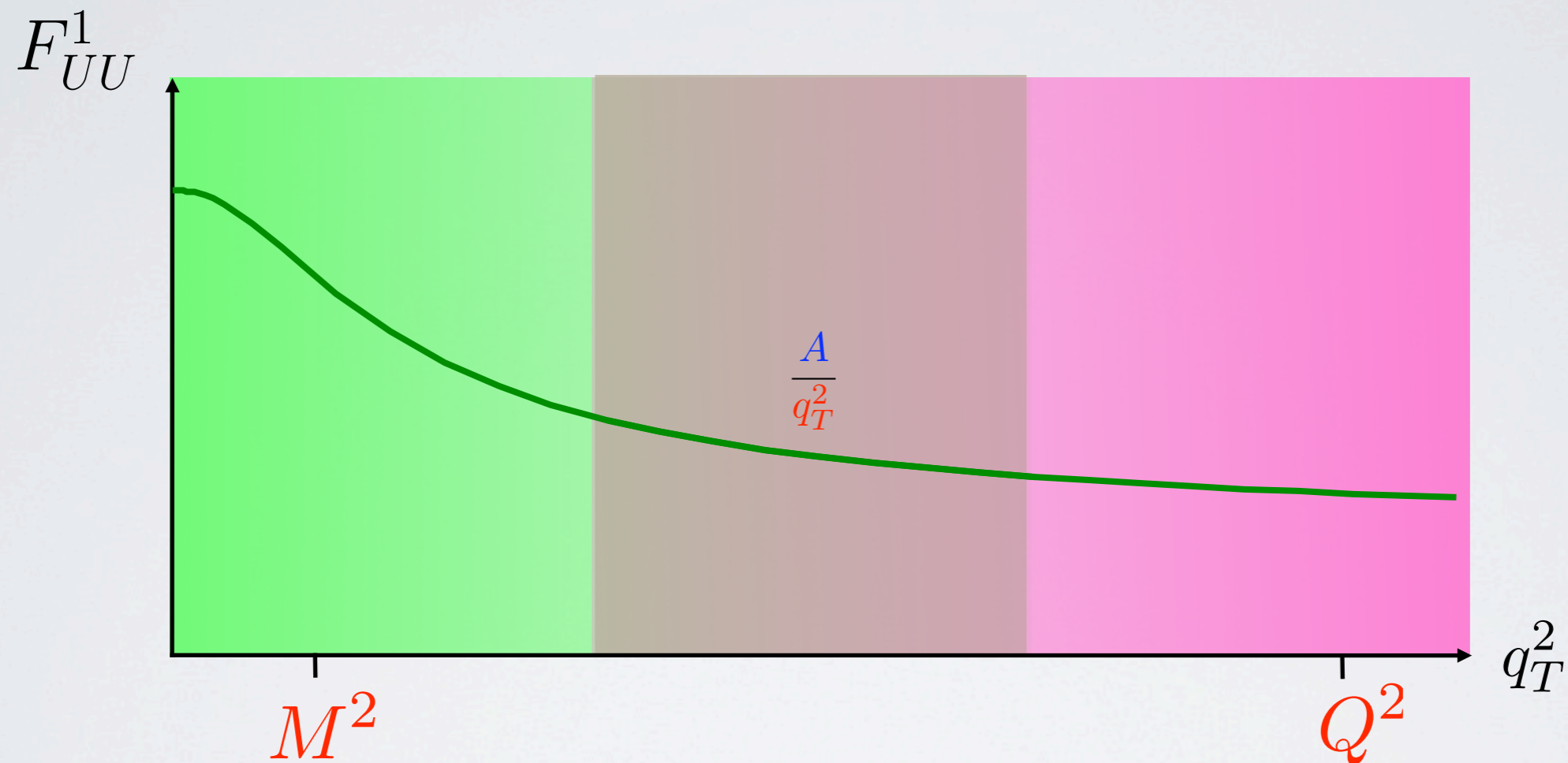
Collins, Soper, Sterman, NPB250 (85)

Matching behavior



Collins, Soper, Sterman, NPB250 (85)

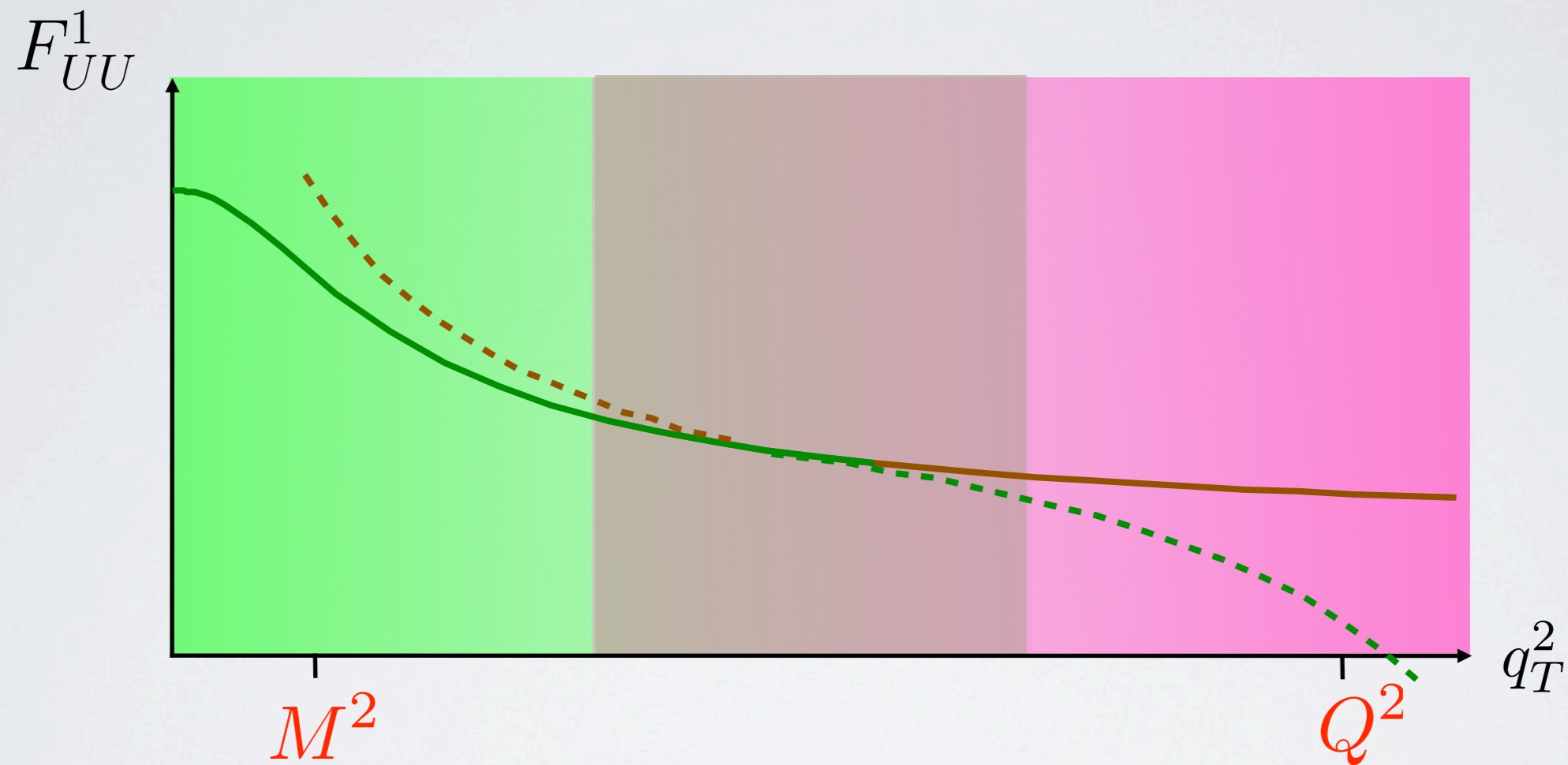
Matching behavior



The leading high- q_T part is just the “tail” of the leading low- q_T part. It is in principle possible to write a single expression from low to high q_T .

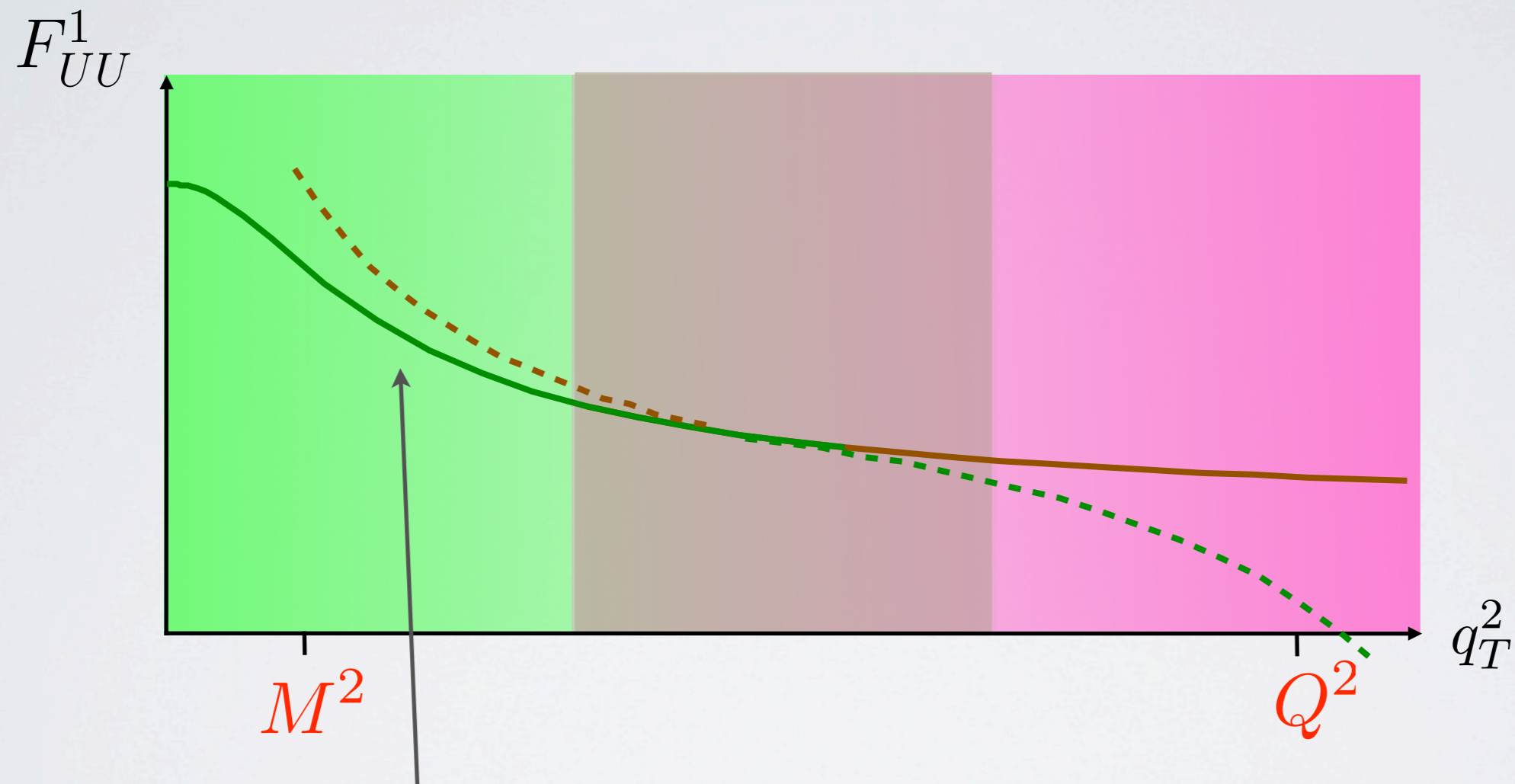
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Matching behavior



Collins, Soper, Sterman, NPB250 (85)

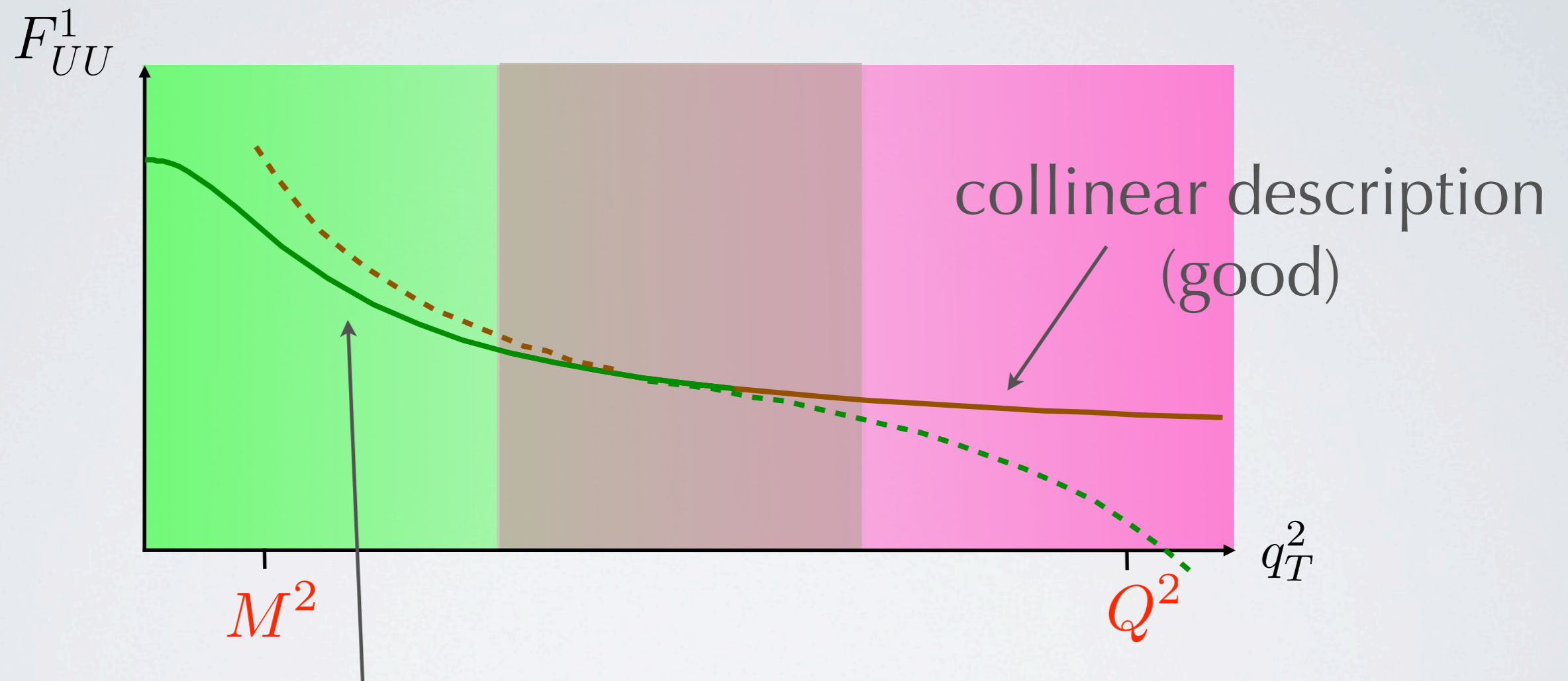
Matching behavior



TMD description (good)

Collins, Soper, Sterman, NPB250 (85)

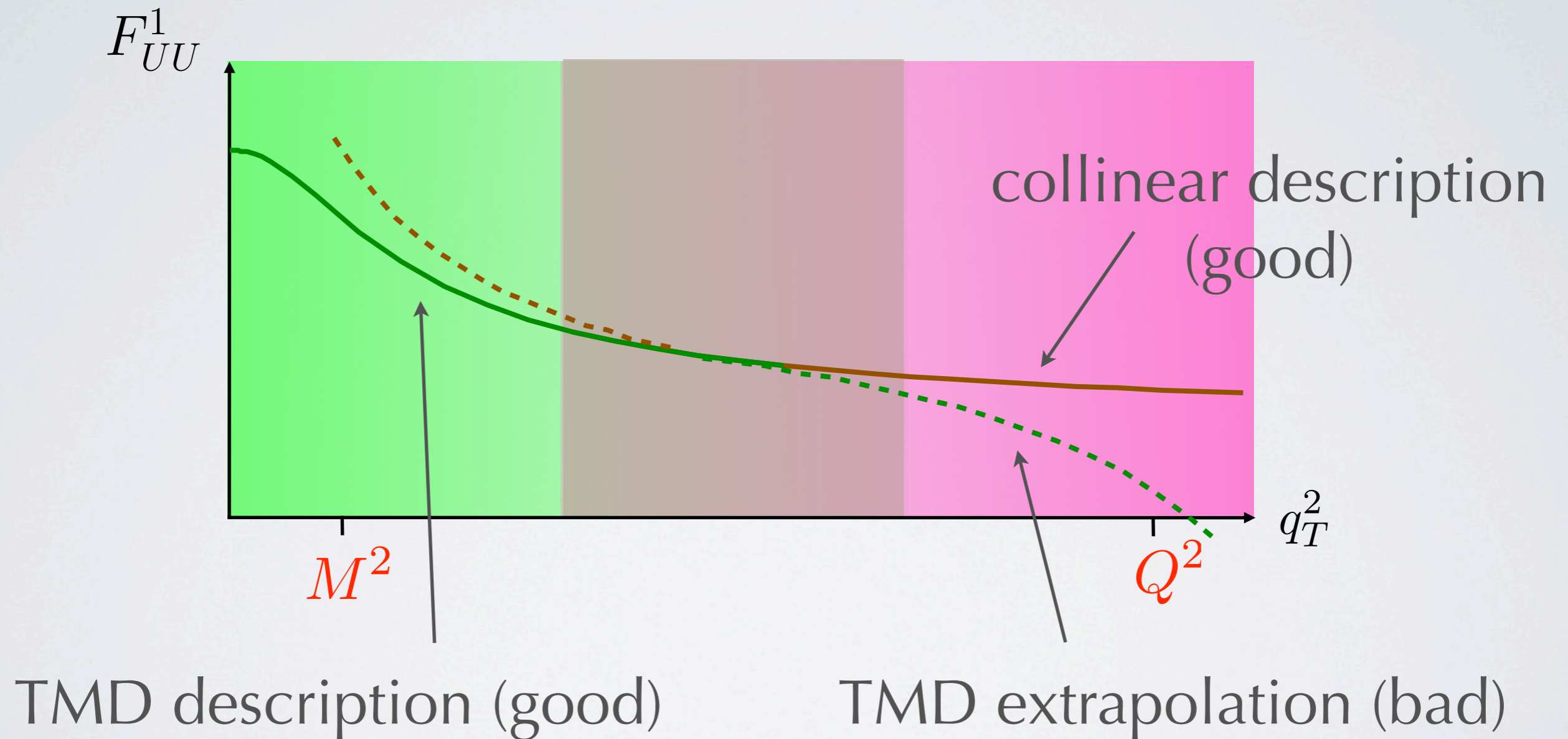
Matching behavior



TMD description (good)

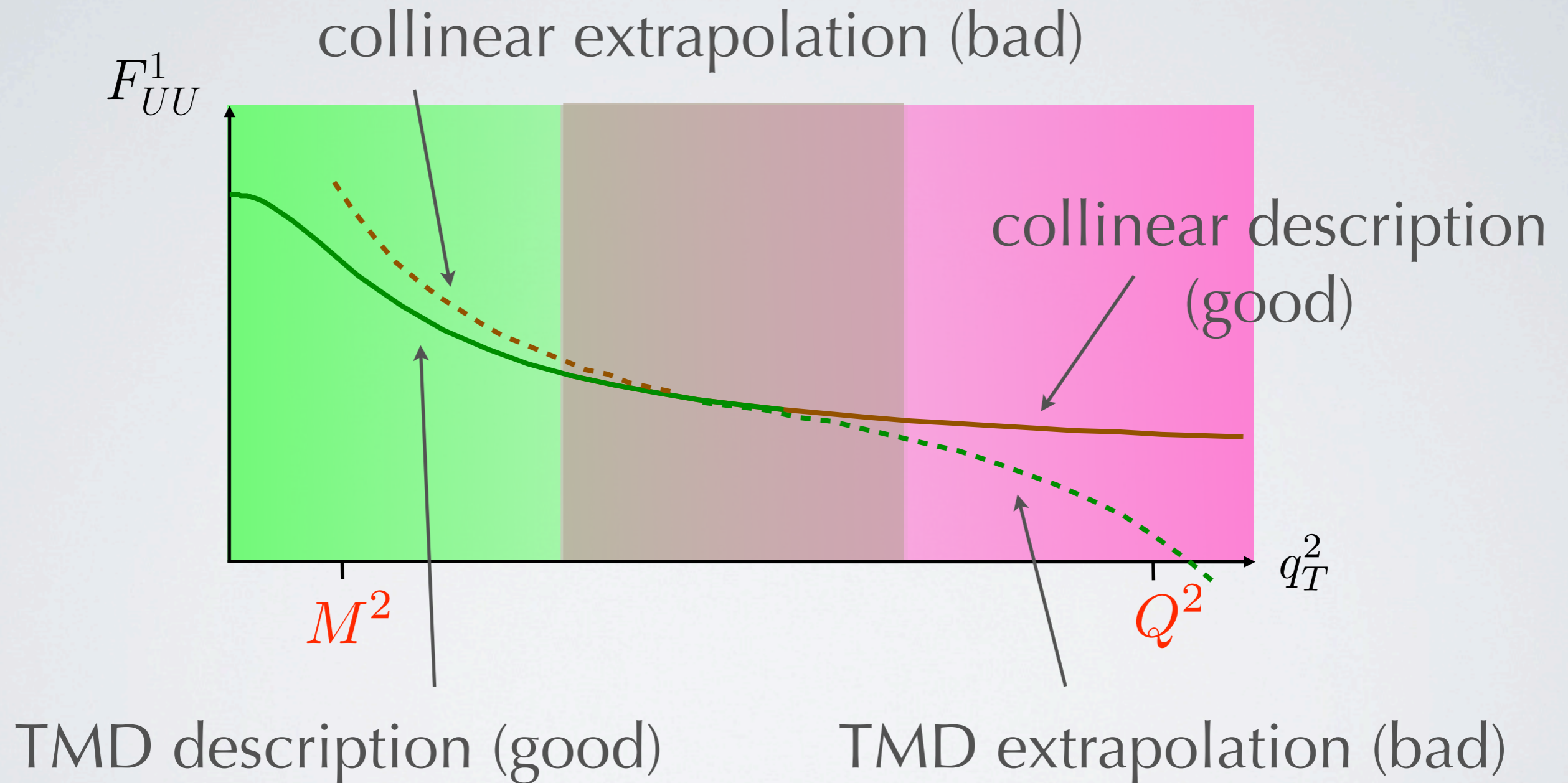
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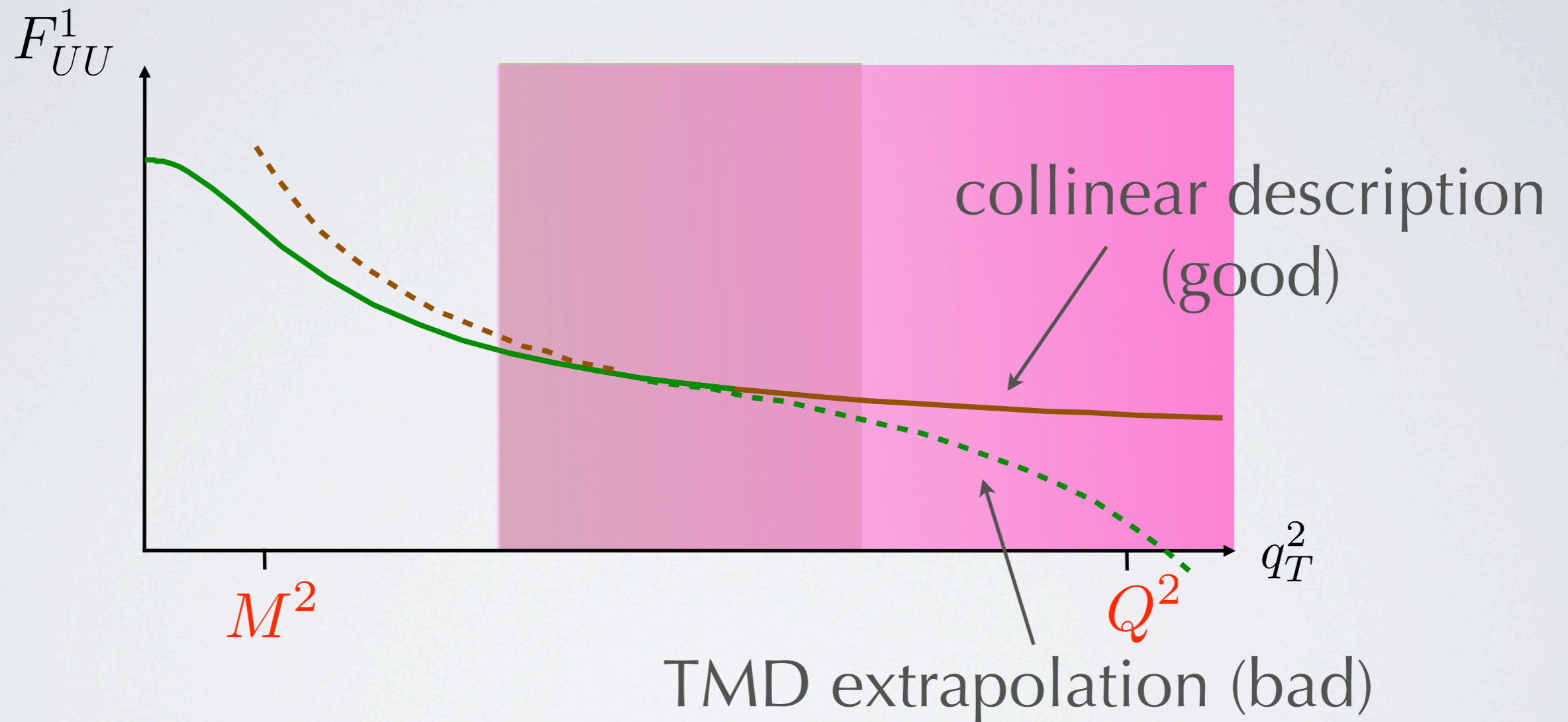
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Matching behavior

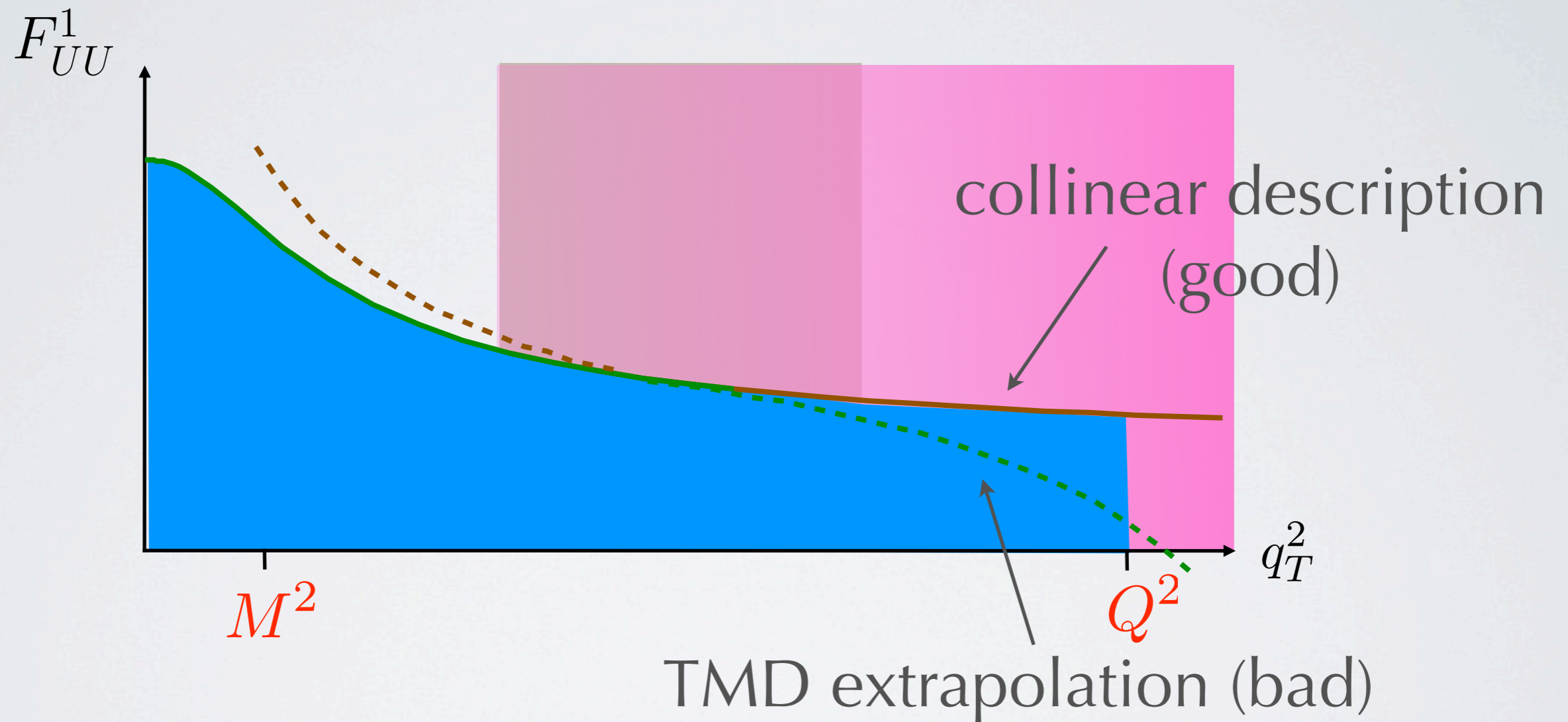


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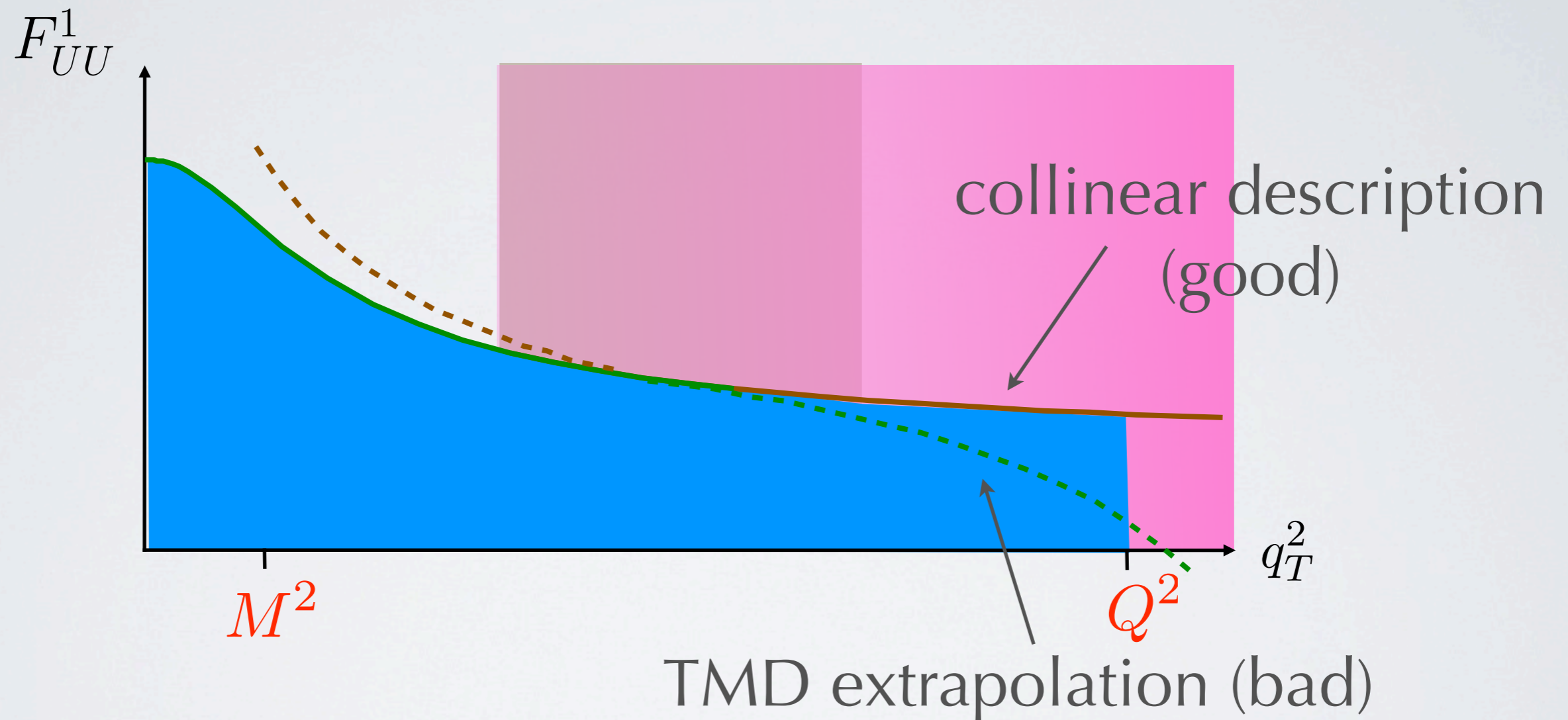
From TMD to collinear PDFs?



From TMD to collinear PDFs?



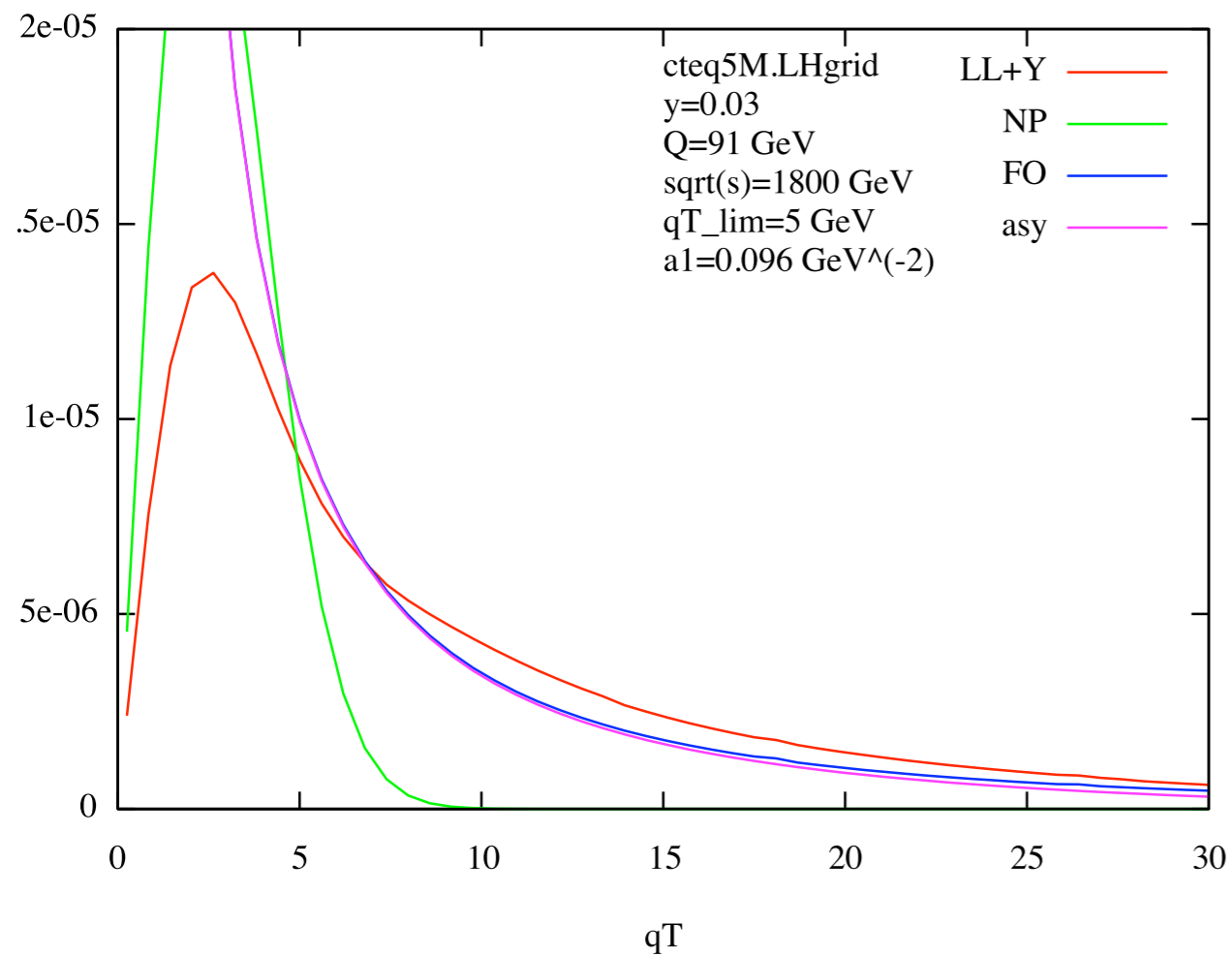
From TMD to collinear PDFs?



To obtain PDFs, you need to integrate into the high q_T region, where TMDs cannot be used.

You should NOT expect to recover PDFs from TMDs.

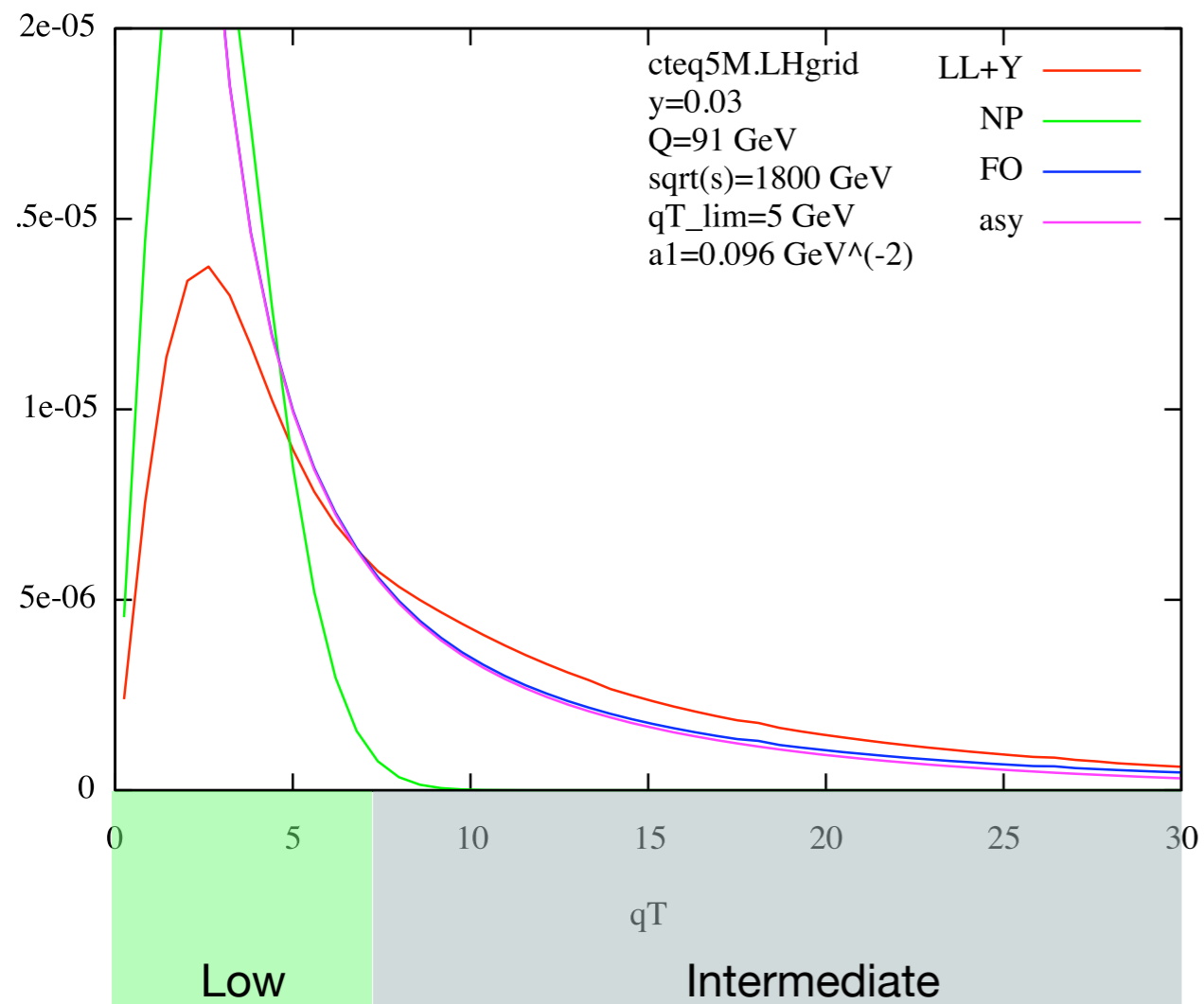
Low and high transverse momentum



Tevatron

see Werner's talk

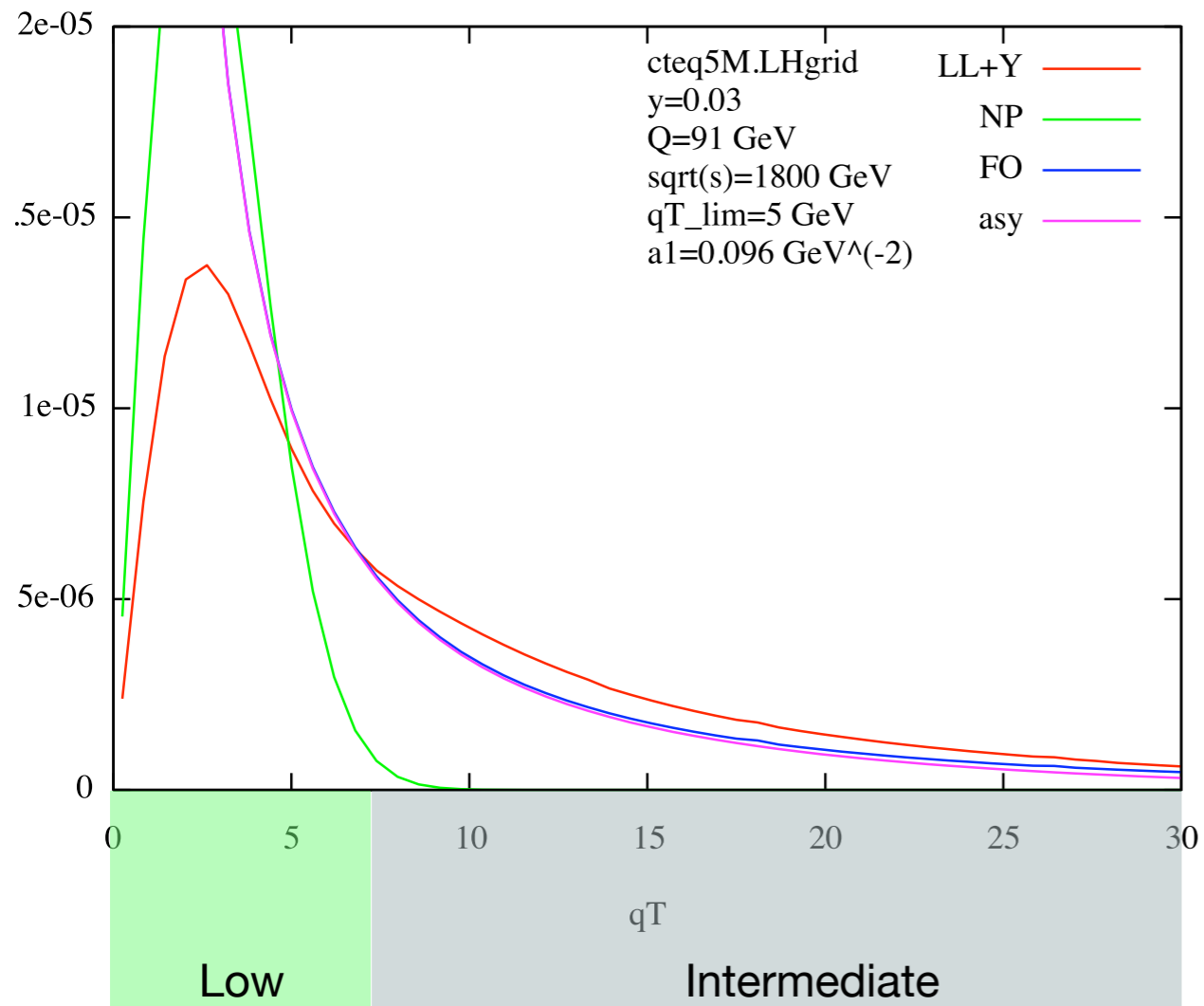
Low and high transverse momentum



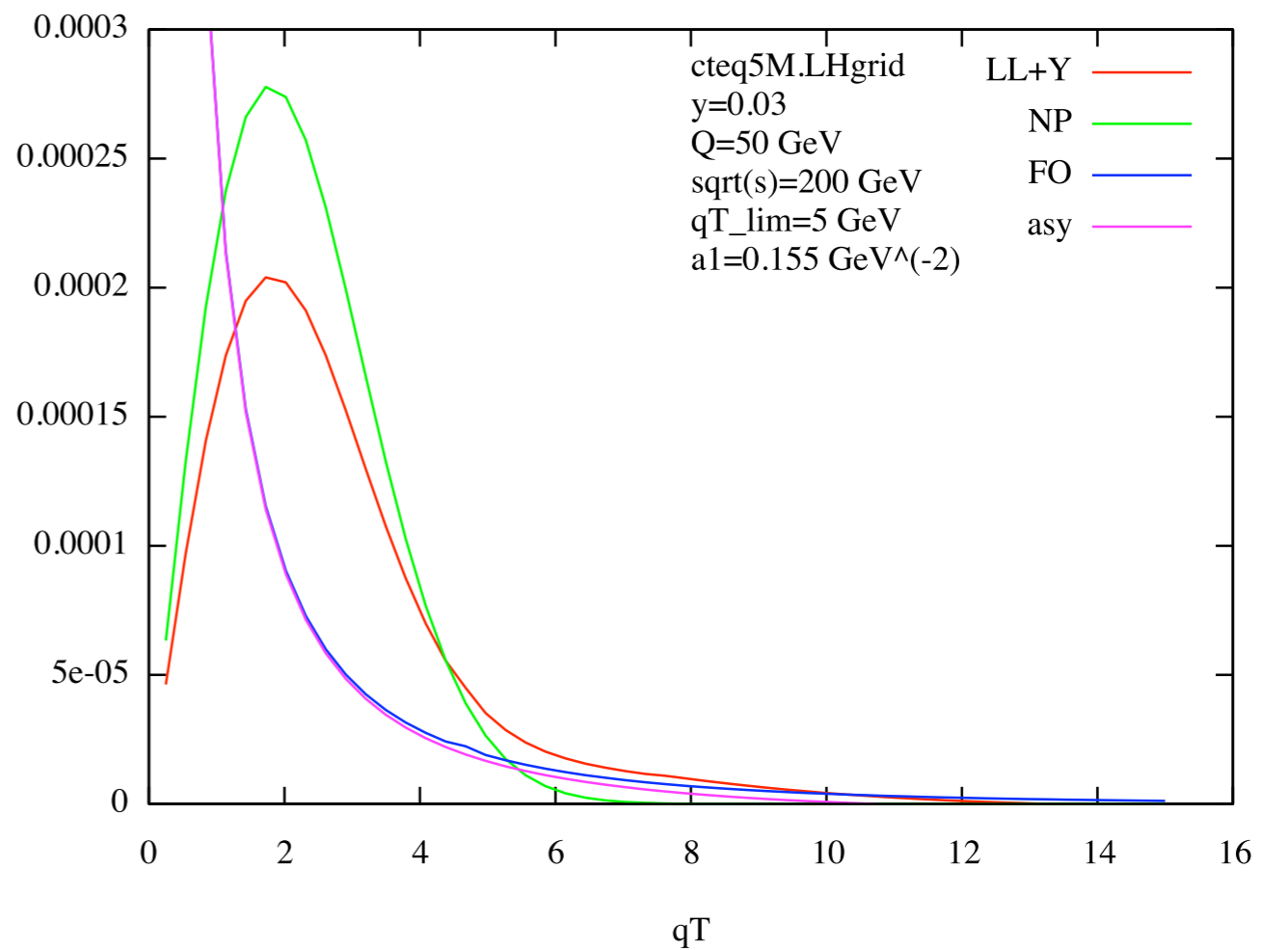
Tevatron

see Werner's talk

Low and high transverse momentum



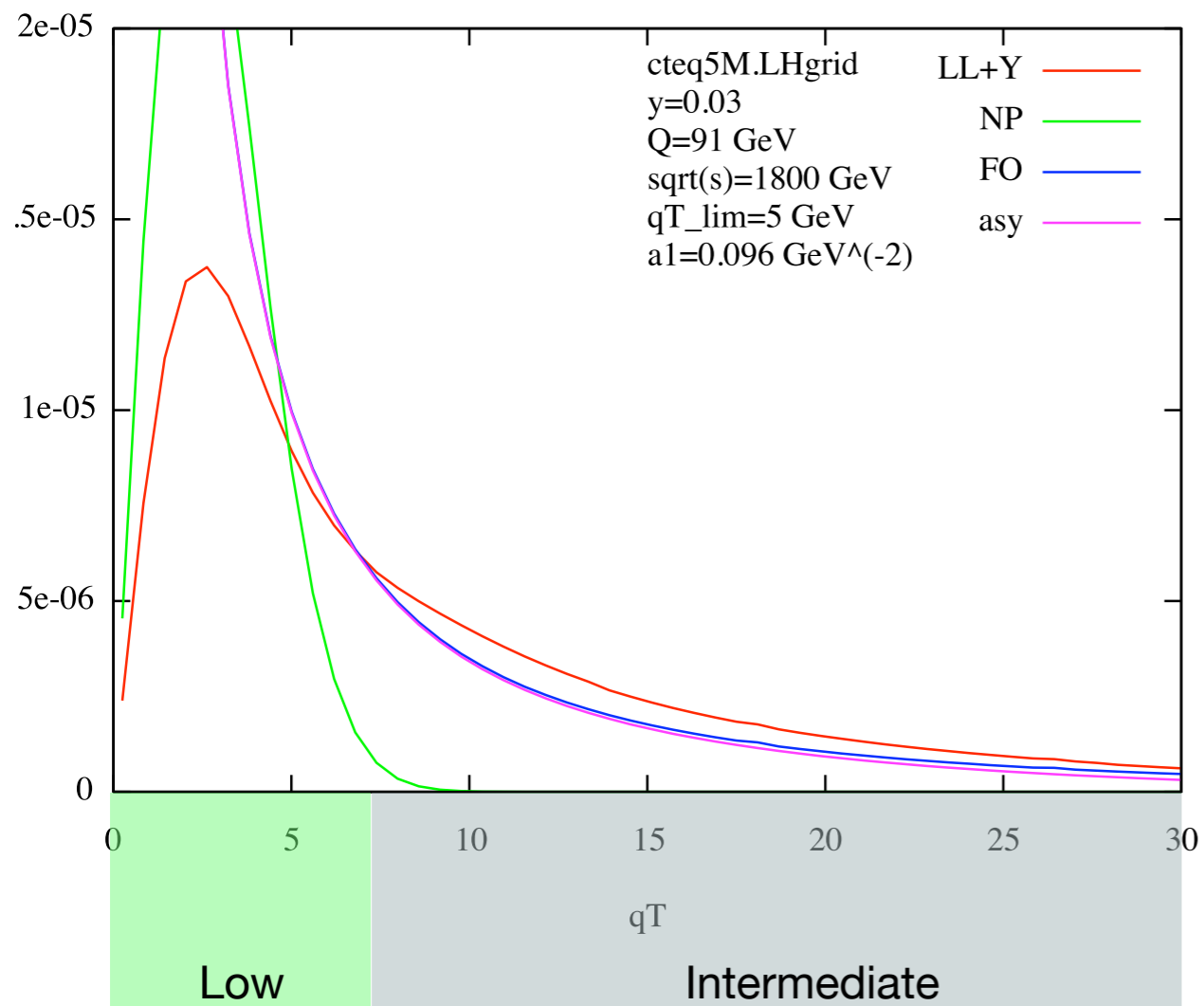
Tevatron



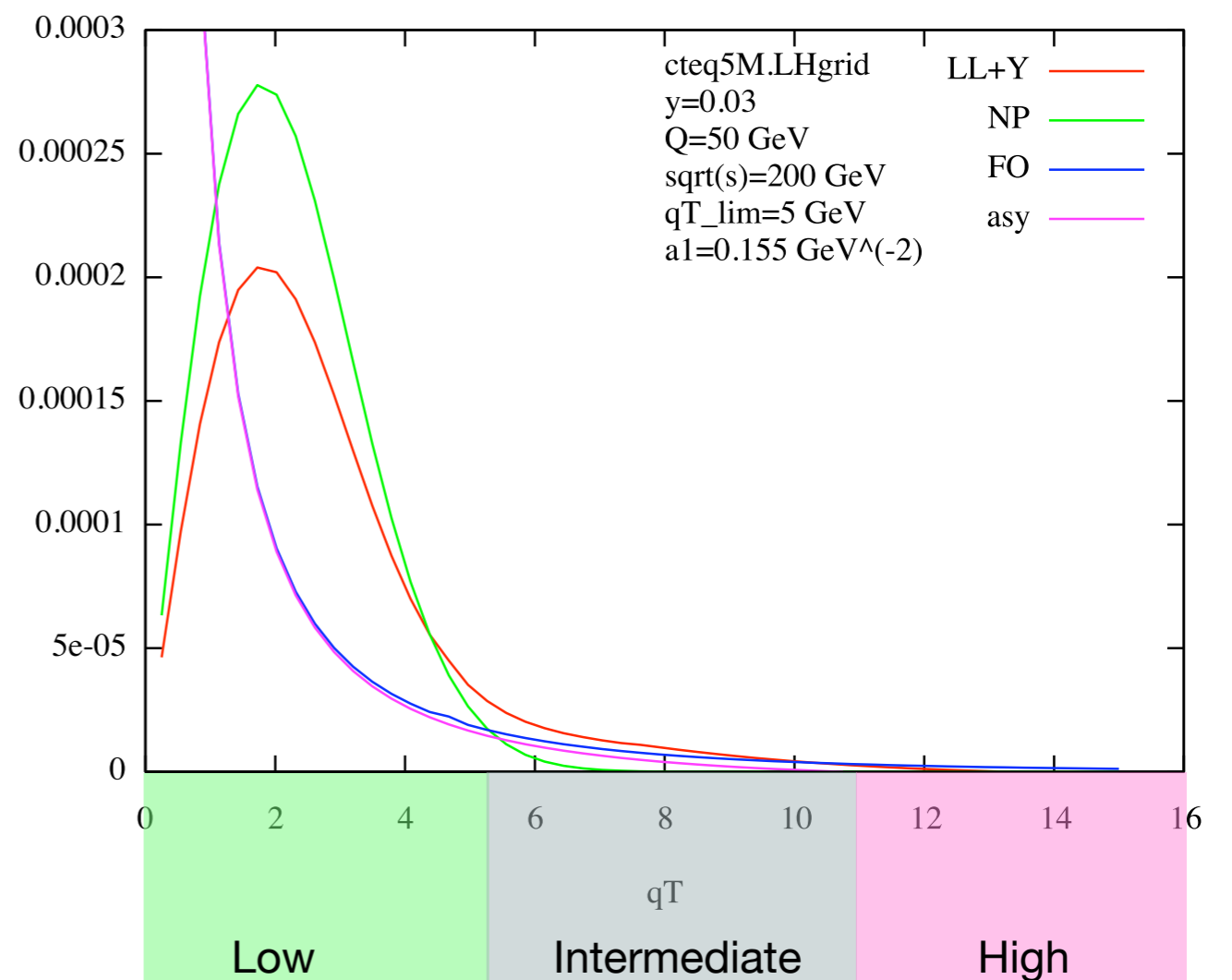
RHIC

see Werner's talk

Low and high transverse momentum



Tevatron

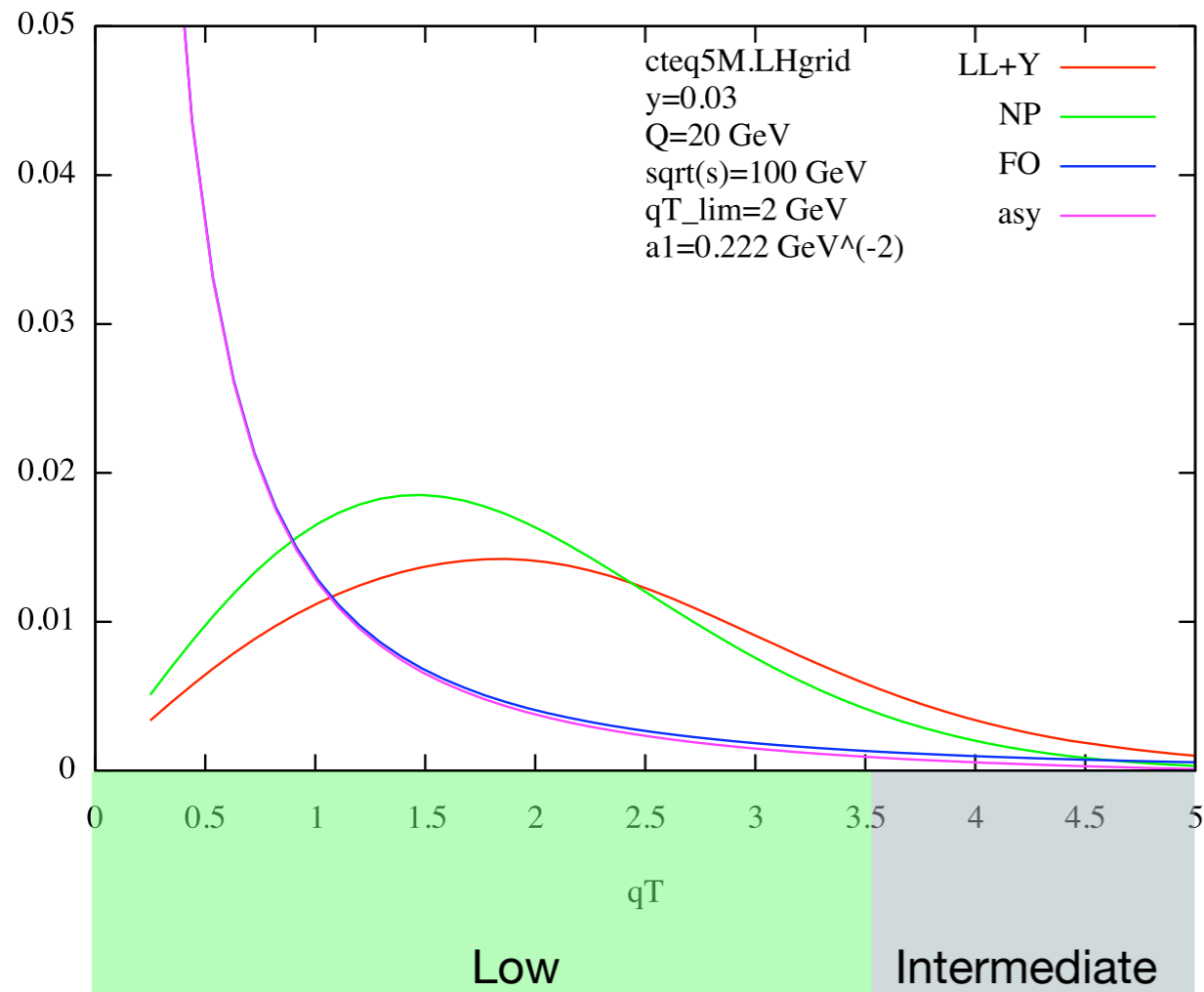


RHIC

see Werner's talk

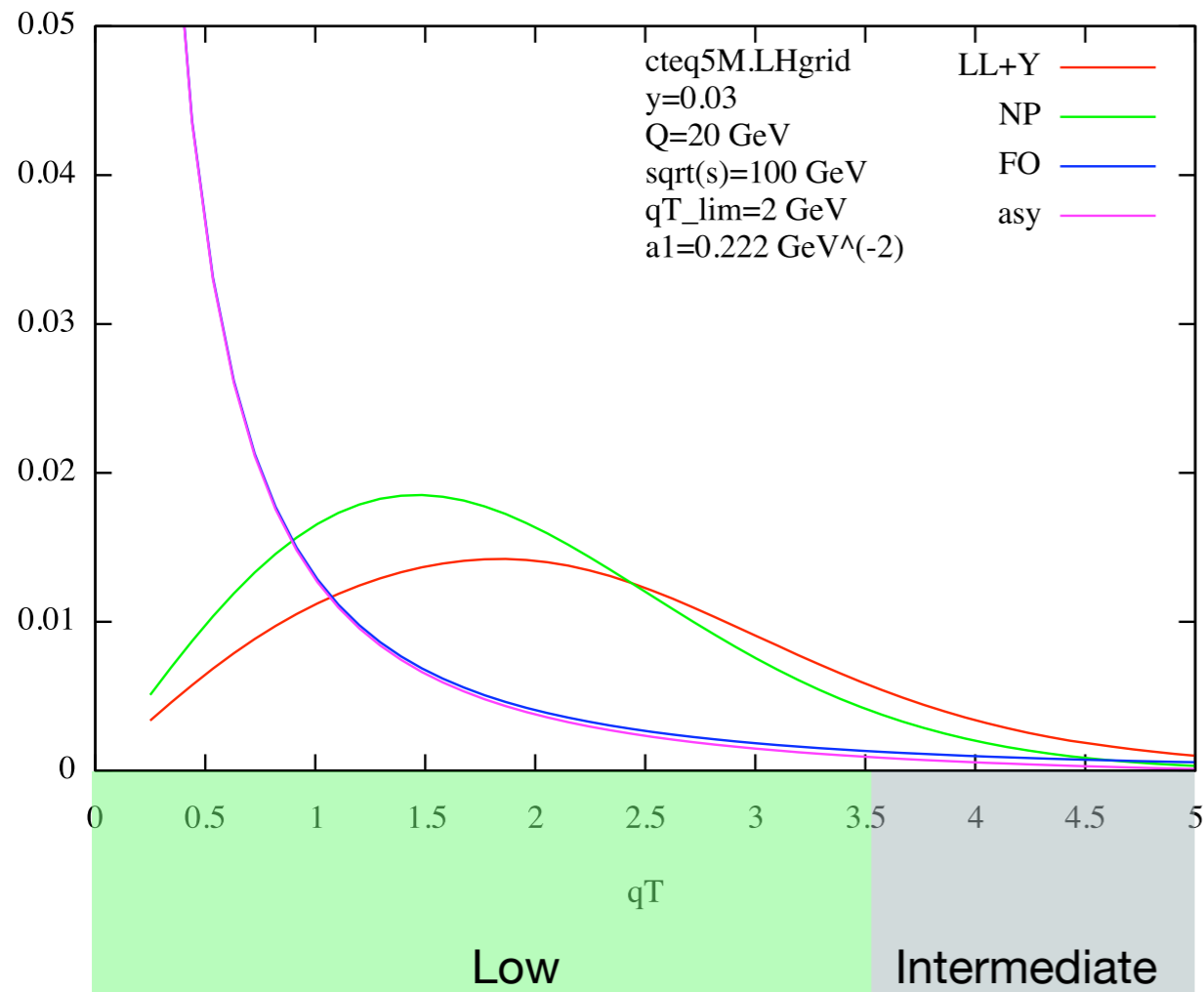
Low and high transverse momentum

Low and high transverse momentum

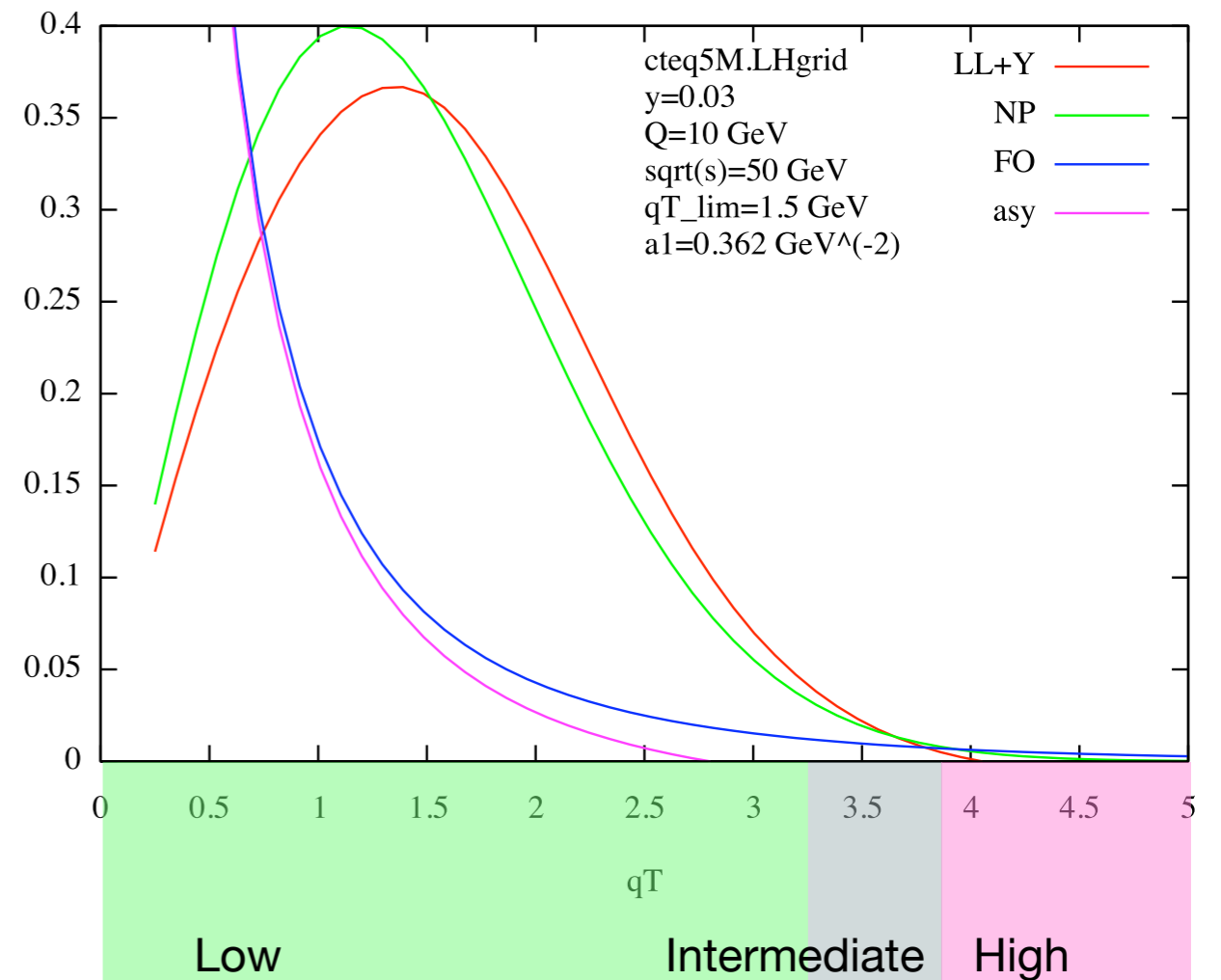


Medium energy

Low and high transverse momentum



Medium energy



Low energy

TMD part

$$F_{UU}^1 = \mathcal{C} [f_1 \bar{f}_1]$$

$$\sum_q e_q^2 \int d^2 \vec{k}_{aT} d^2 \vec{k}_{bT} \delta^{(2)}(\vec{q}_T - \vec{k}_{aT} - \vec{k}_{bT}) \left[f_1^q(x_a, \vec{k}_{aT}^2; Q) f_2^{\bar{q}}(x_b, \vec{k}_{bT}^2; Q) + (q \leftrightarrow \bar{q}) \right]$$

Unpol. TMD “state of the art”

$$f_1(x, k_T; Q) = \frac{1}{2\pi} \int d^2b_T e^{-ik_T \cdot b_T} [C \otimes f_1](x, b_T) e^{-S'(b_T, Q)} e^{-S'_{\text{NP}}(x, b_T, Q, \alpha_i)}$$

*T. Rogers, M. Aybat, arXiv:1101.5057
see also M. Garcia's talk*

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collinear PDF



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collinear PDF

pQCD

*T. Rogers, M. Aybat, arXiv:1101.5057
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collinear PDF

pQCD

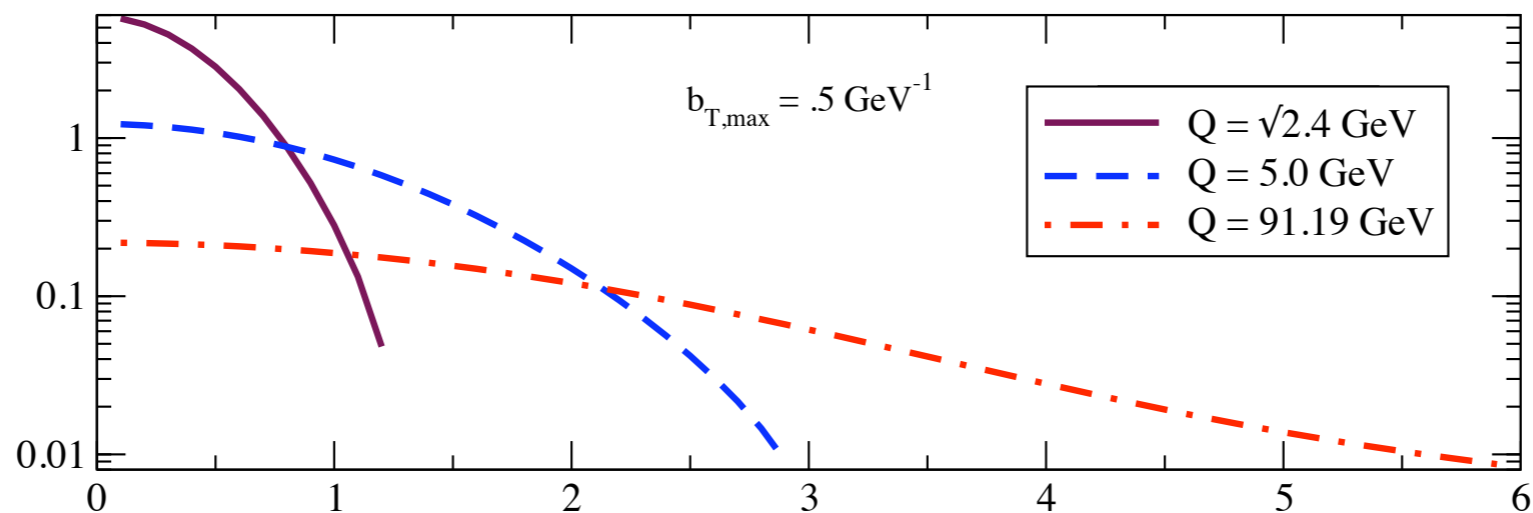
nonperturbative part of TMD

*T. Rogers, M. Aybat, arXiv:1101.5057
see also M. Garcia's talk*

Unpol. TMD “state of the art”

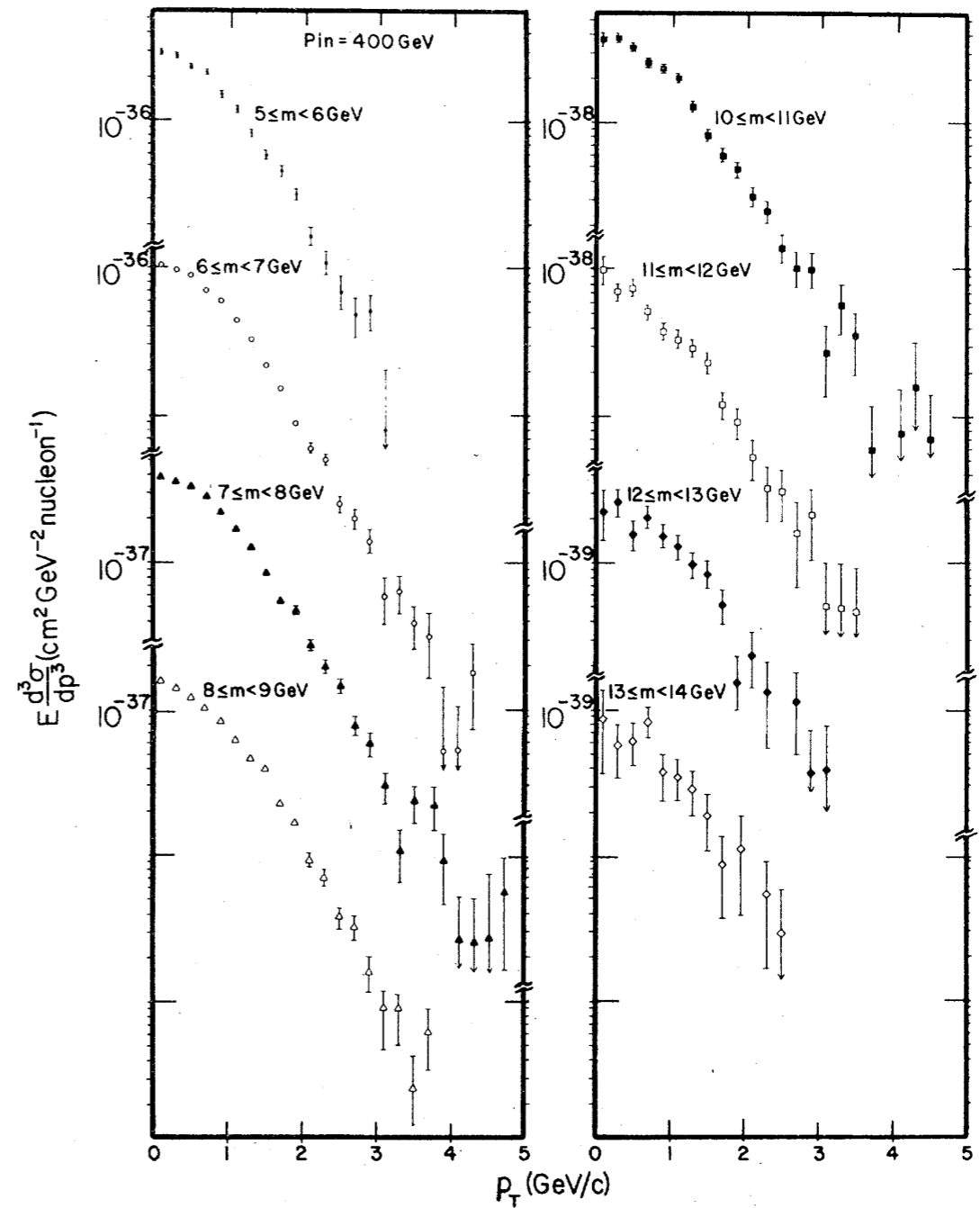
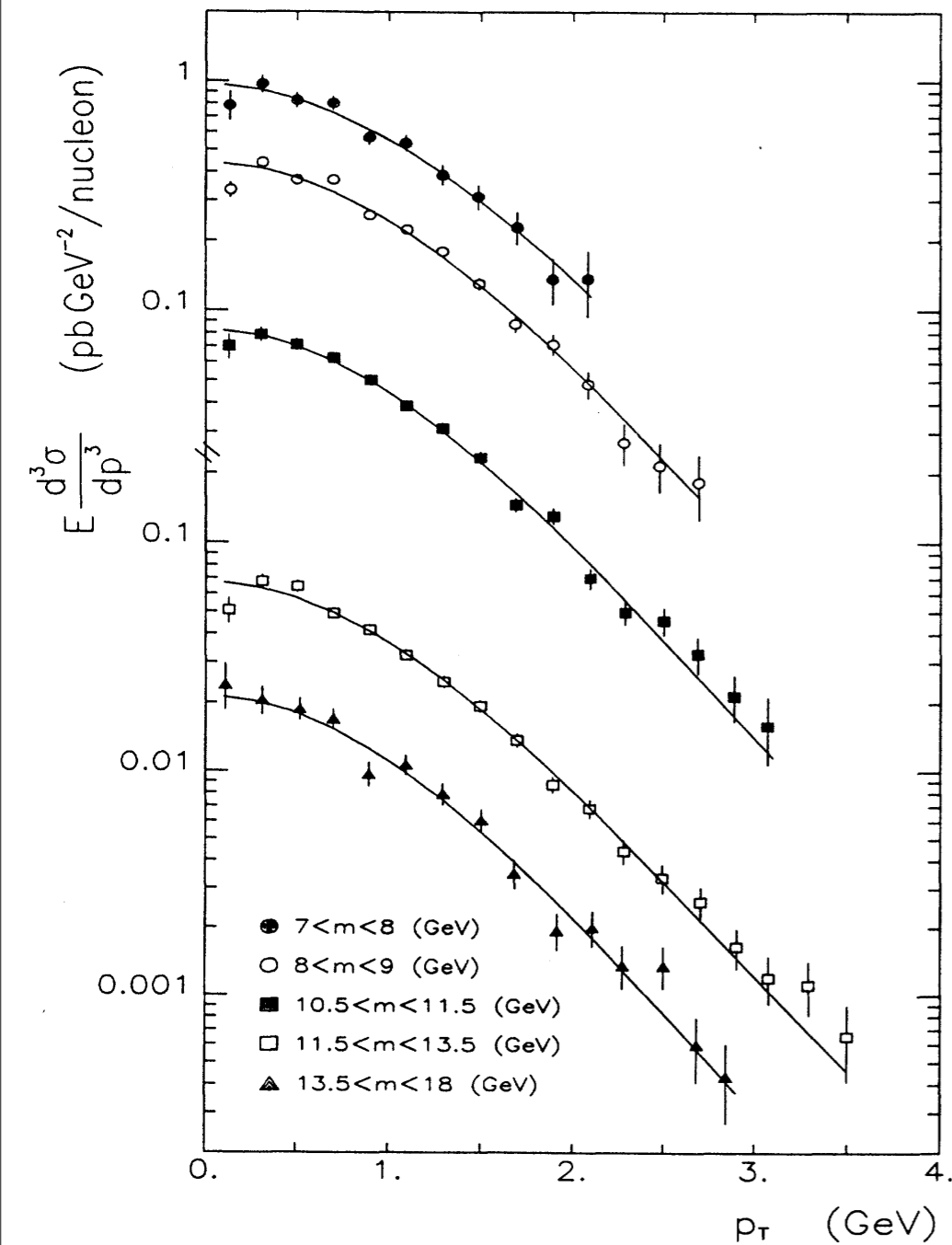
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Up Quark TMD PDF, $x = .09$

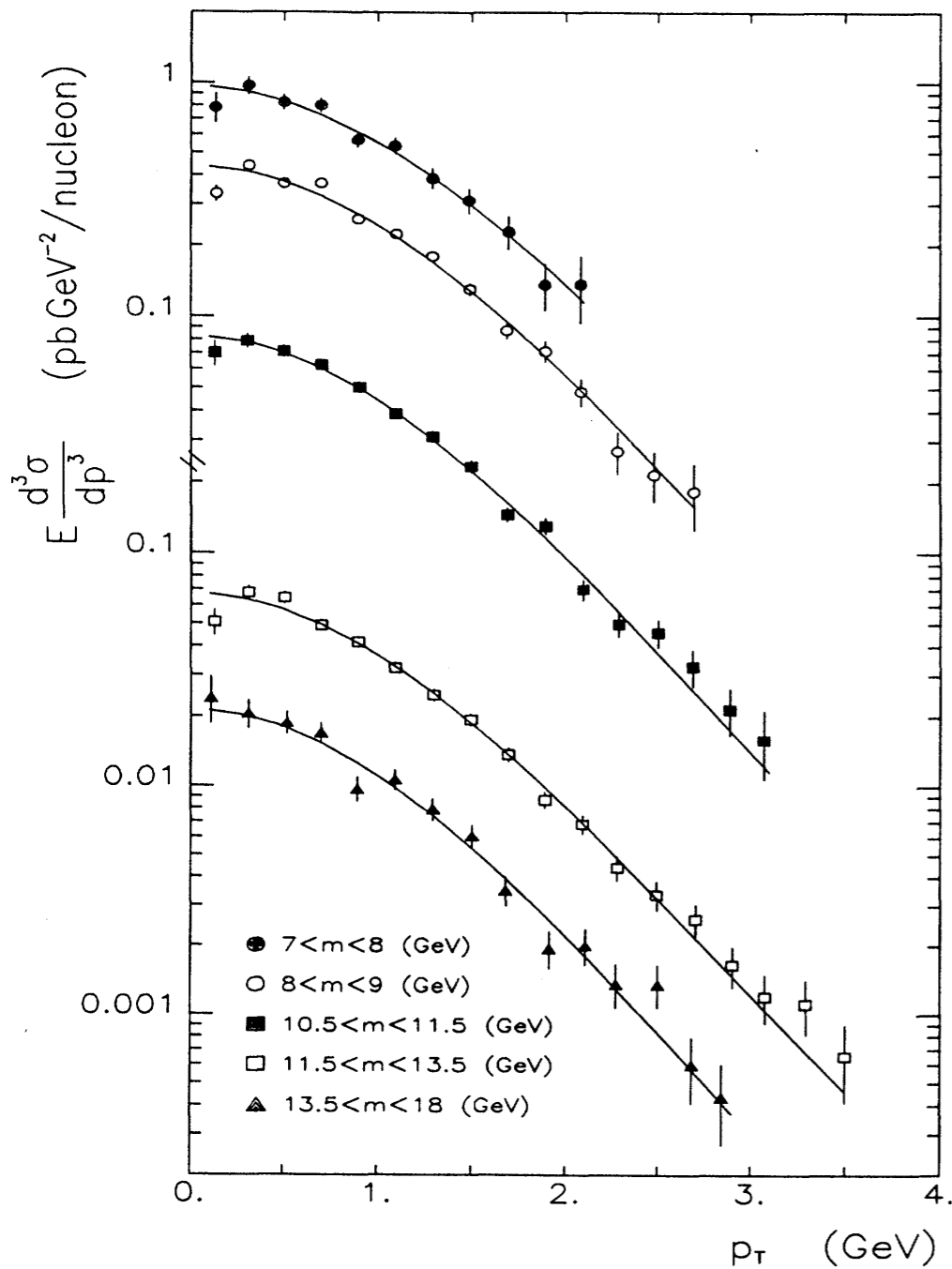


T. Rogers, M. Aybat, arXiv:1101.5057
Landry, Brock, Nadolsky, Yuan, PRD67 (03)
P. Schweitzer, T. Teckentrup, A. Metz, PRD81(10)

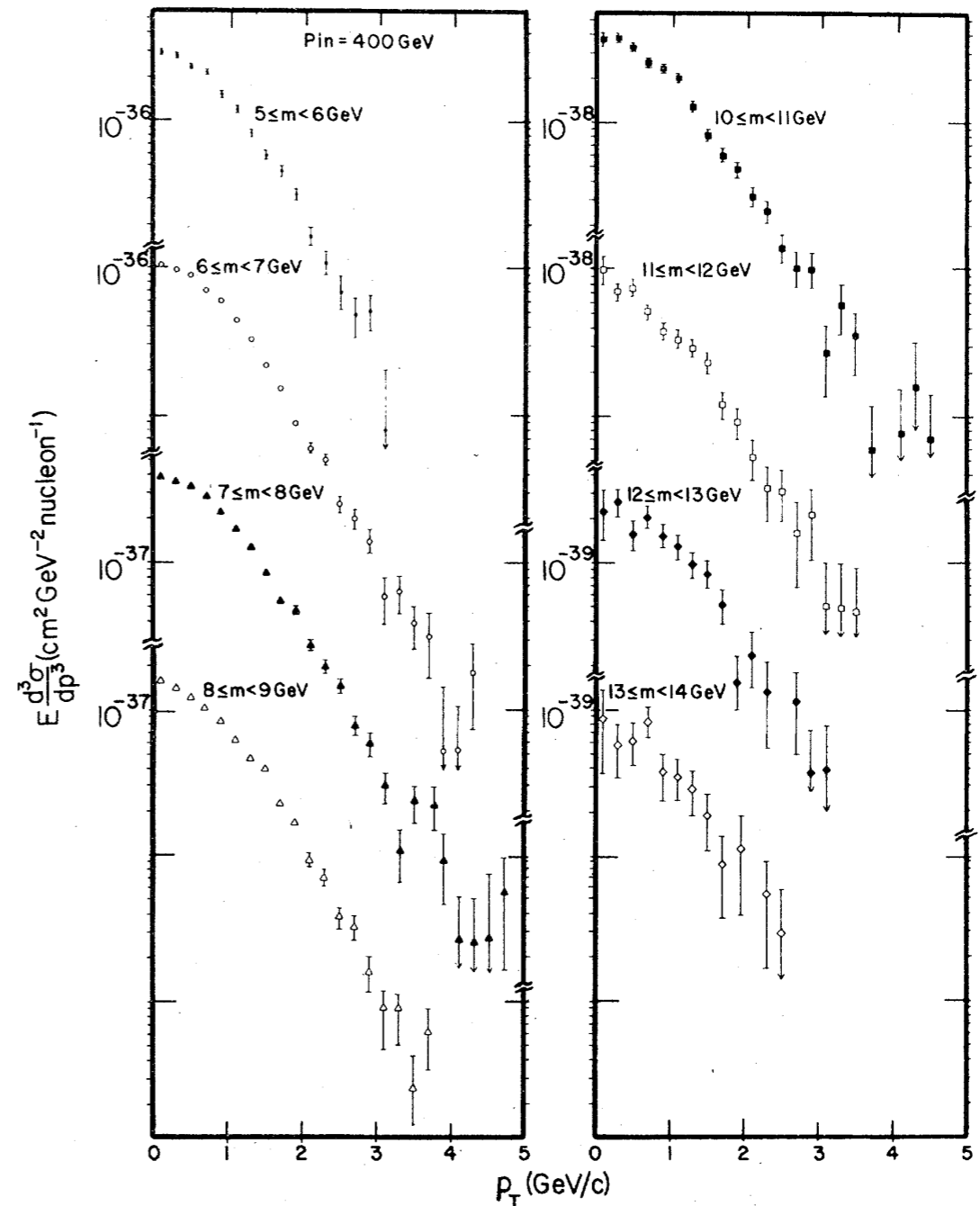
Data and TMD extractions



Data and TMD extractions

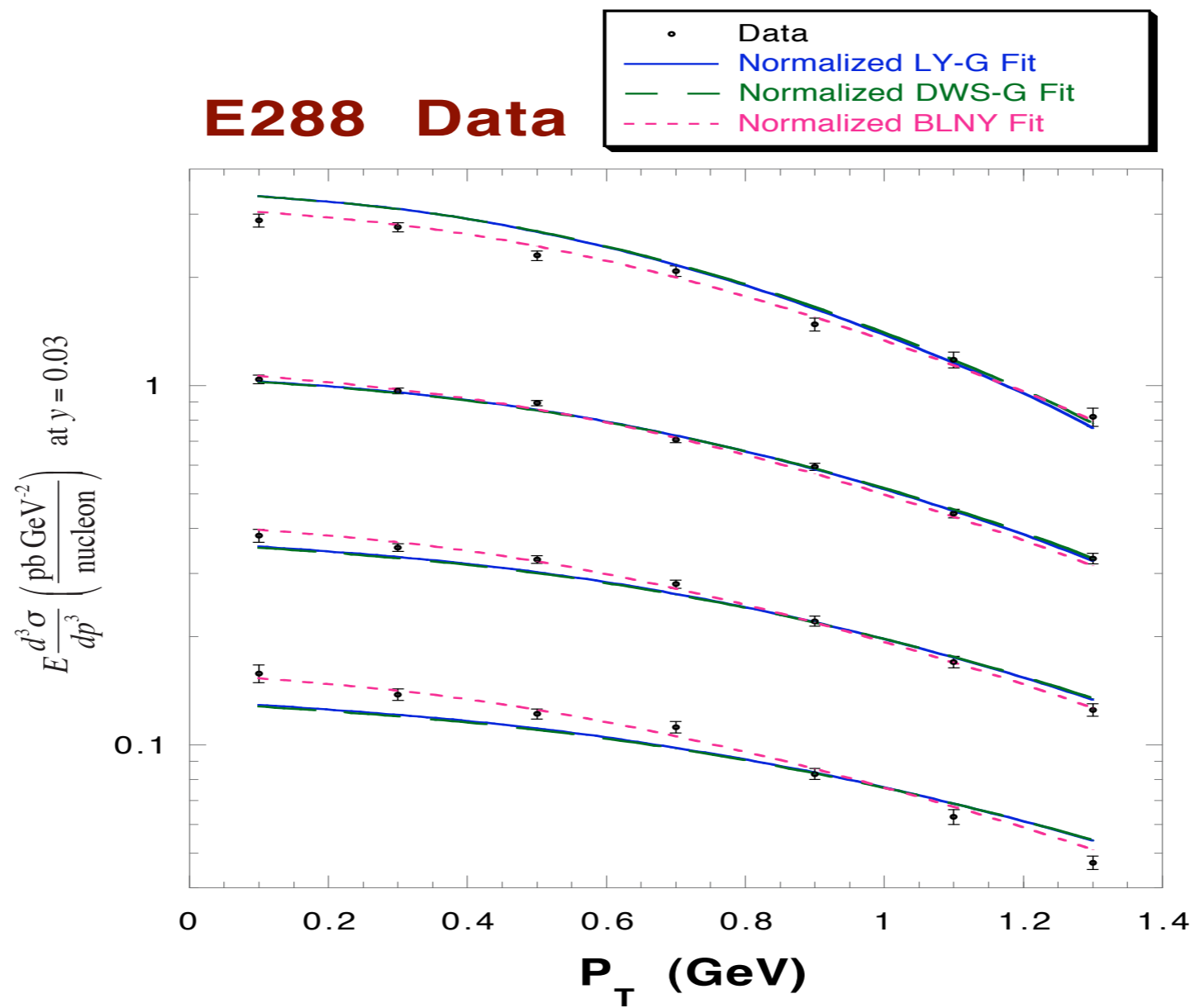


E605, Moreno et al. PRD43 (91)



E288, Ito et al., PRD23 (81)

BLNY fit



BLNY fit

Landry, Brock, Nadolsky, Yuan, PRD67 (03)

Experiment	Reference	Reaction	\sqrt{S} (GeV)	δN_{exp}
R209	[14]	$p + p \rightarrow \mu^+ \mu^- + X$	62	10%
E605	[15]	$p + Cu \rightarrow \mu^+ \mu^- + X$	38.8	15%
E288	[16]	$p + Cu \rightarrow \mu^+ \mu^- + X$	27.4	25%
CDF-Z (Run-0)	[17]	$p + \bar{p} \rightarrow Z + X$	1800	–
DØ -Z (Run-1)	[18]	$p + \bar{p} \rightarrow Z + X$	1800	4.3%
CDF-Z (Run-1)	[19]	$p + \bar{p} \rightarrow Z + X$	1800	3.9%

BLNY fit

Landry, Brock, Nadolsky, Yuan, PRD67 (03)

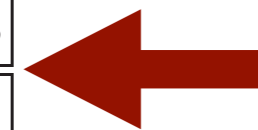
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D-Y (including Z production) is the most important source of information for unpolarized TMDs

BLNY fit

Landry, Brock, Nadolsky, Yuan, PRD67 (03)

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COMPASS,
E906, NICA

D-Y (including Z production) is the most important source of information for unpolarized TMDs

BLNY fit

Landry, Brock, Nadolsky, Yuan, PRD67 (03)

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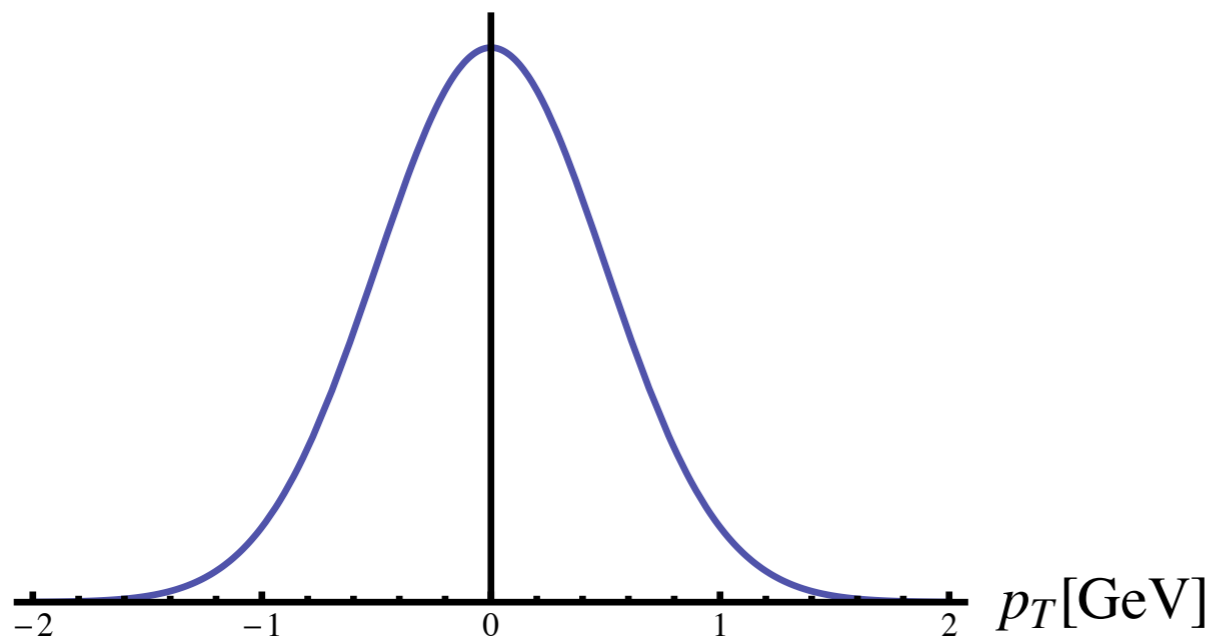
← COMPASS,
E906, NICA

← RHIC

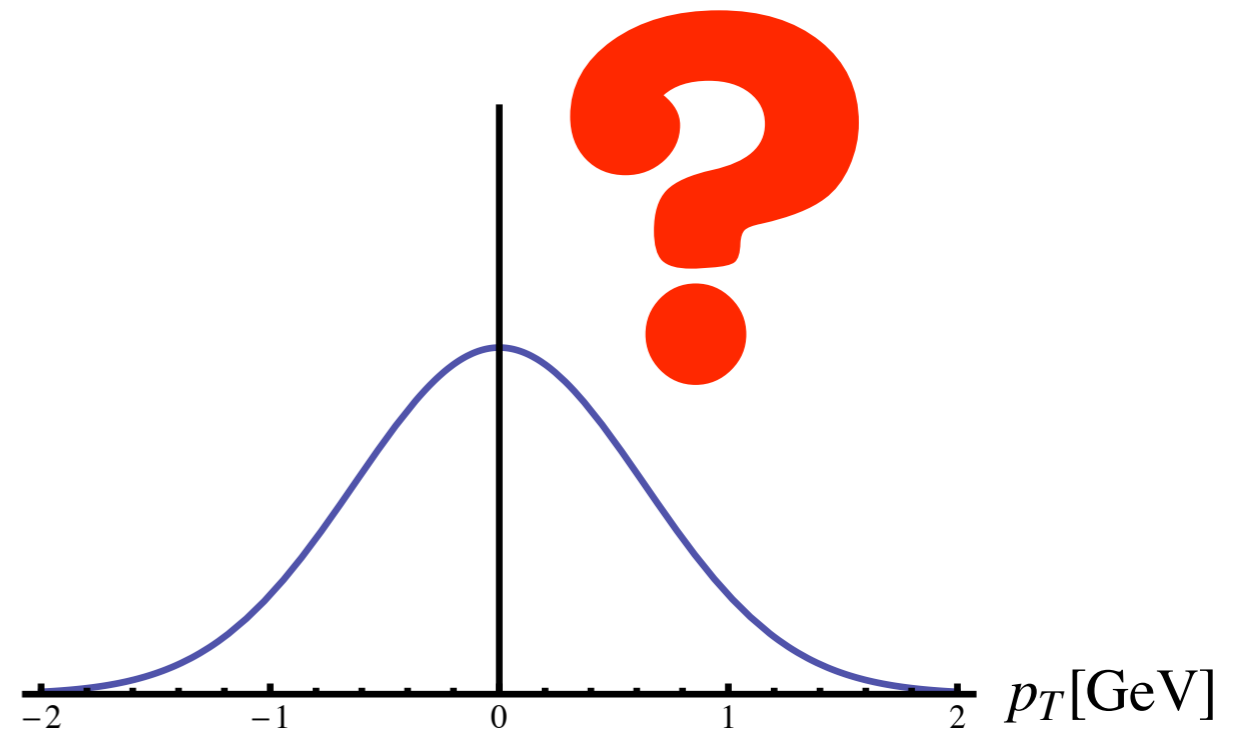
D-Y (including Z production) is the most important source of information for unpolarized TMDs

x dependence of TMDs

$x=0.1$

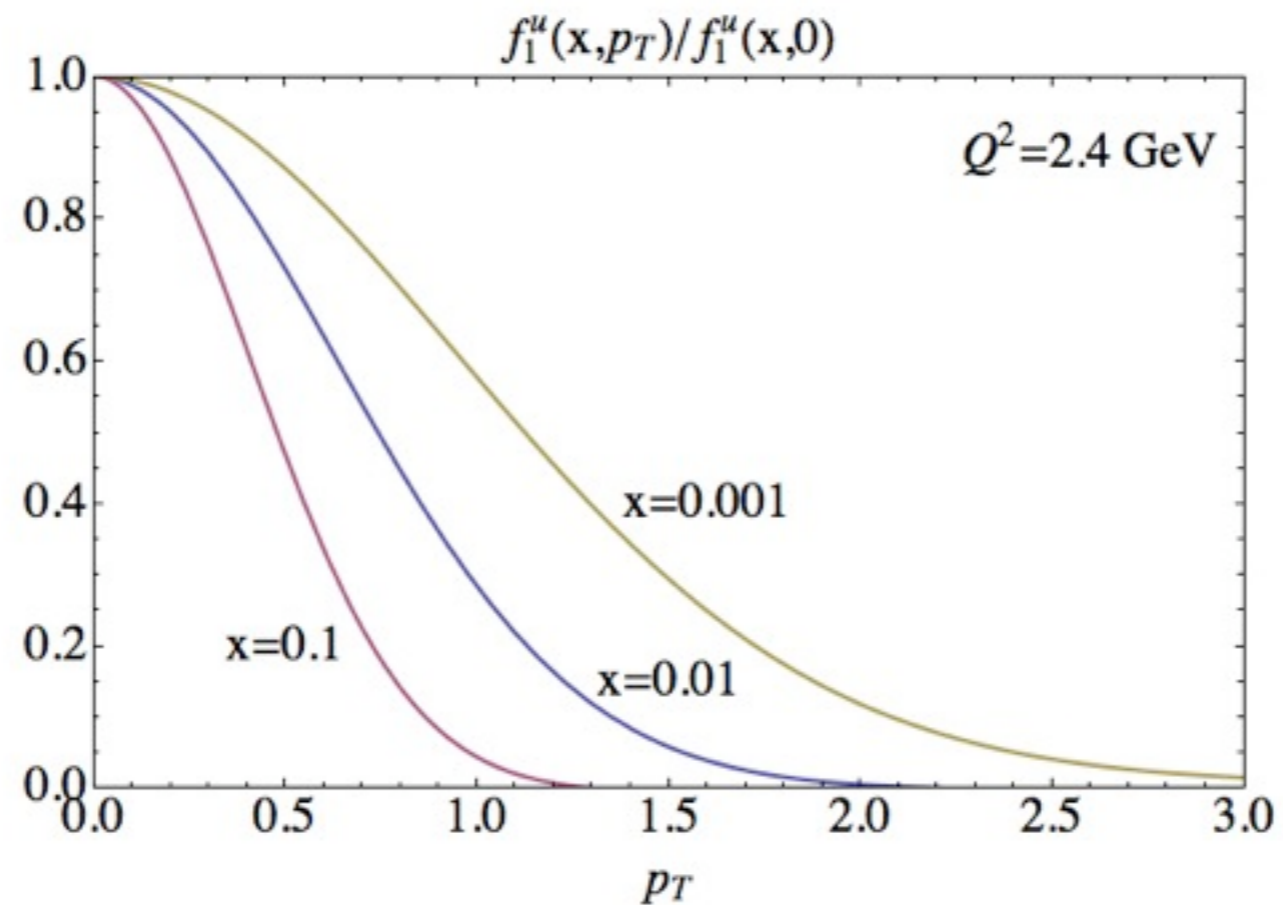


$x=0.001$



x dependence of TMDs

T. Rogers, M. Aybat, arXiv:1101.5057
Landry, Brock, Nadolsky, Yuan, PRD67 (03)
P. Schweitzer, T. Teckentrup, A. Metz, PRD81(10)

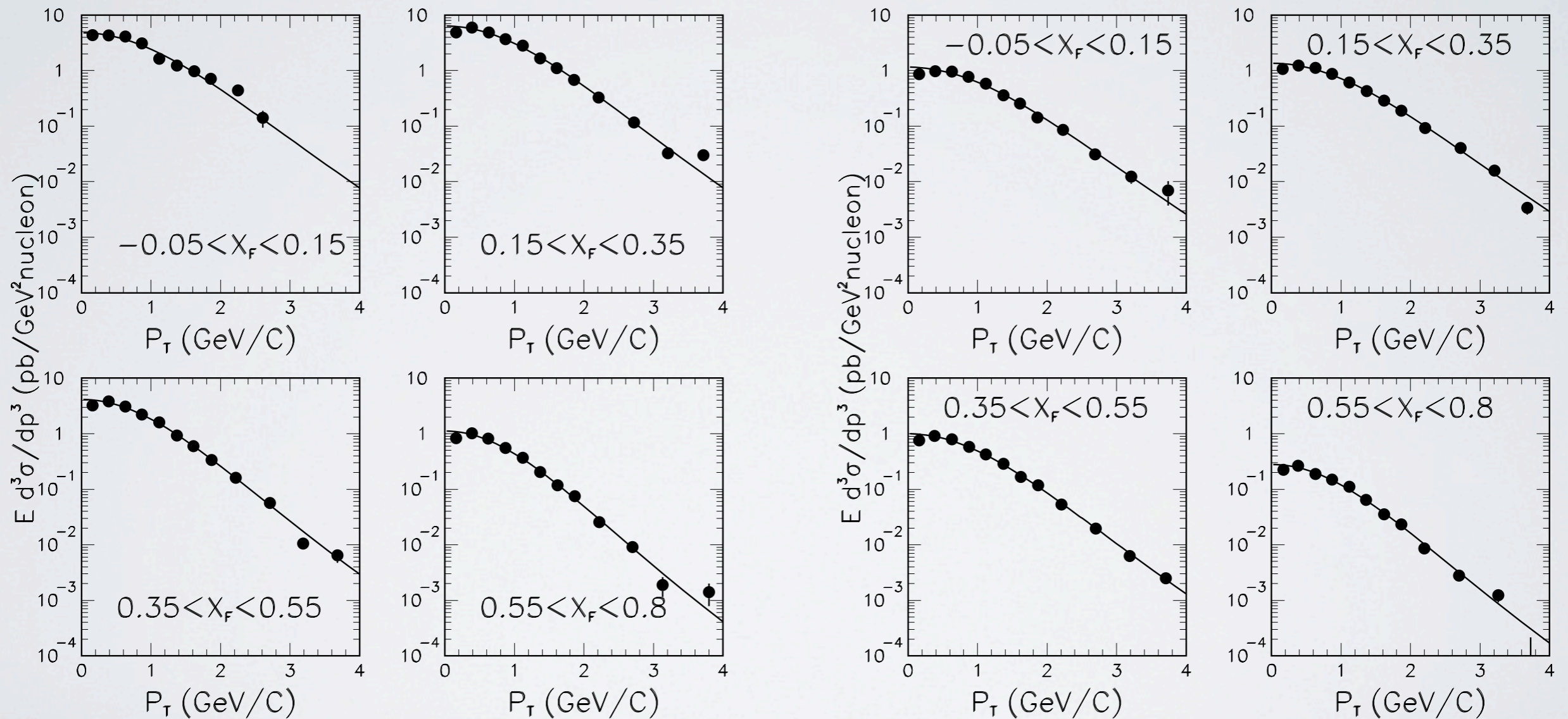


Widening driven by Tevatron data

Multidim. studies are needed

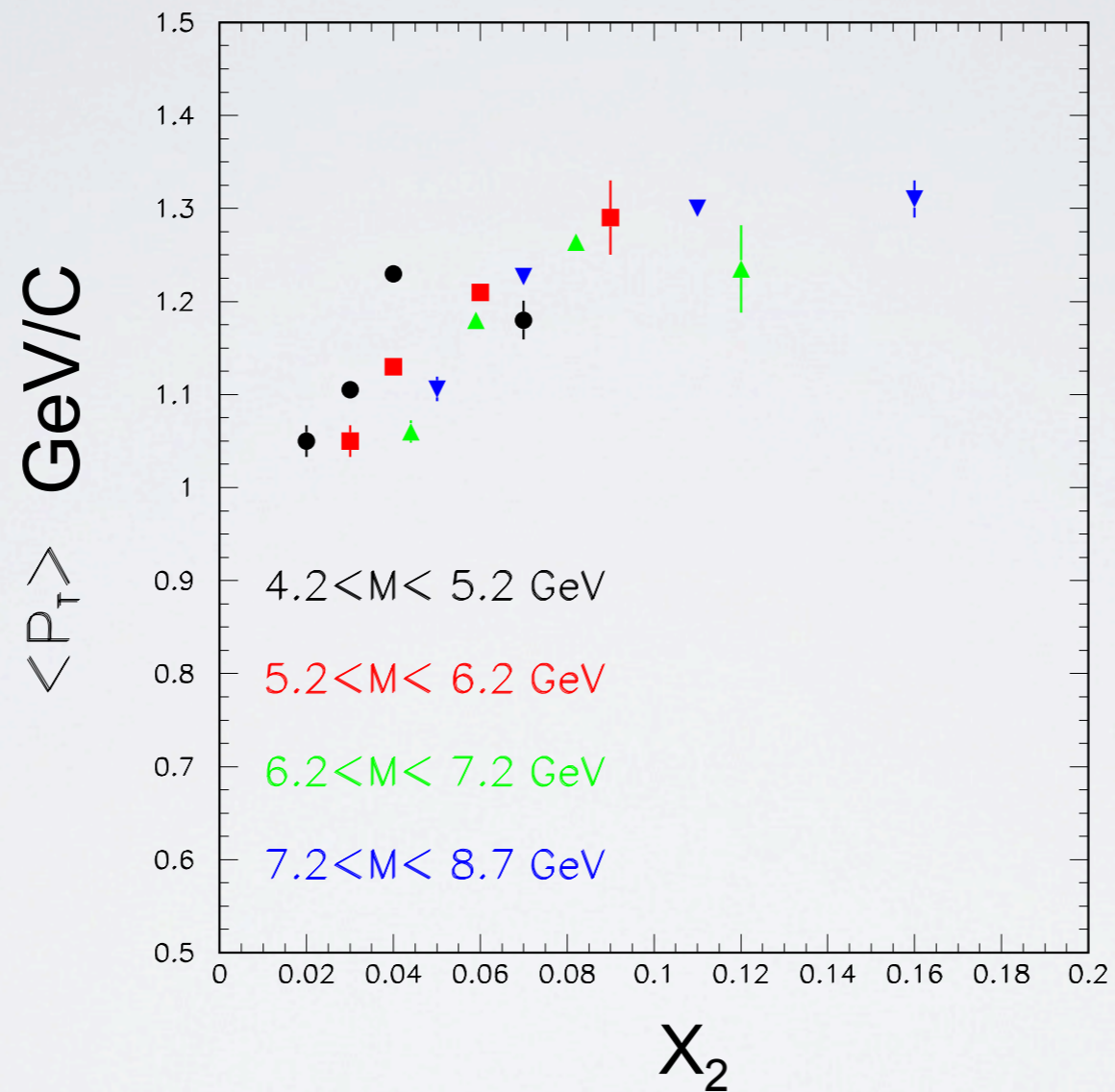
5.2 < M < 6.2 GeV

7.2 < M < 8.7 GeV



*E866/NuSea preliminary,
talk by J.-C. Peng at DY@BNL workshop*

New E866 data

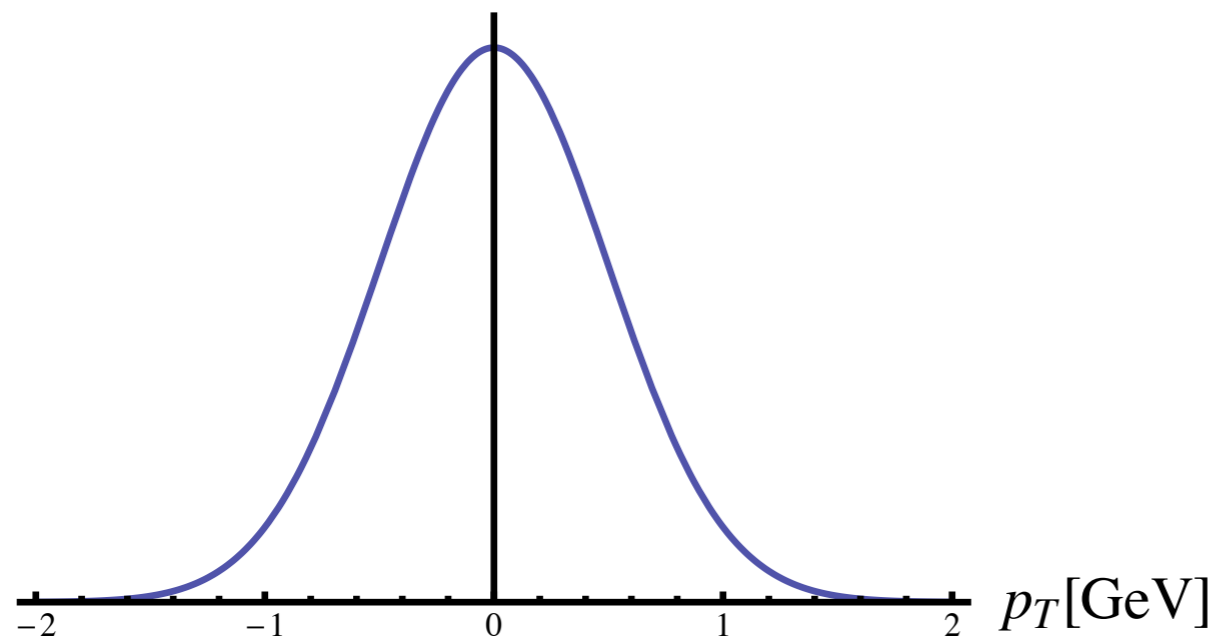


Behavior opposite to BLNY fit

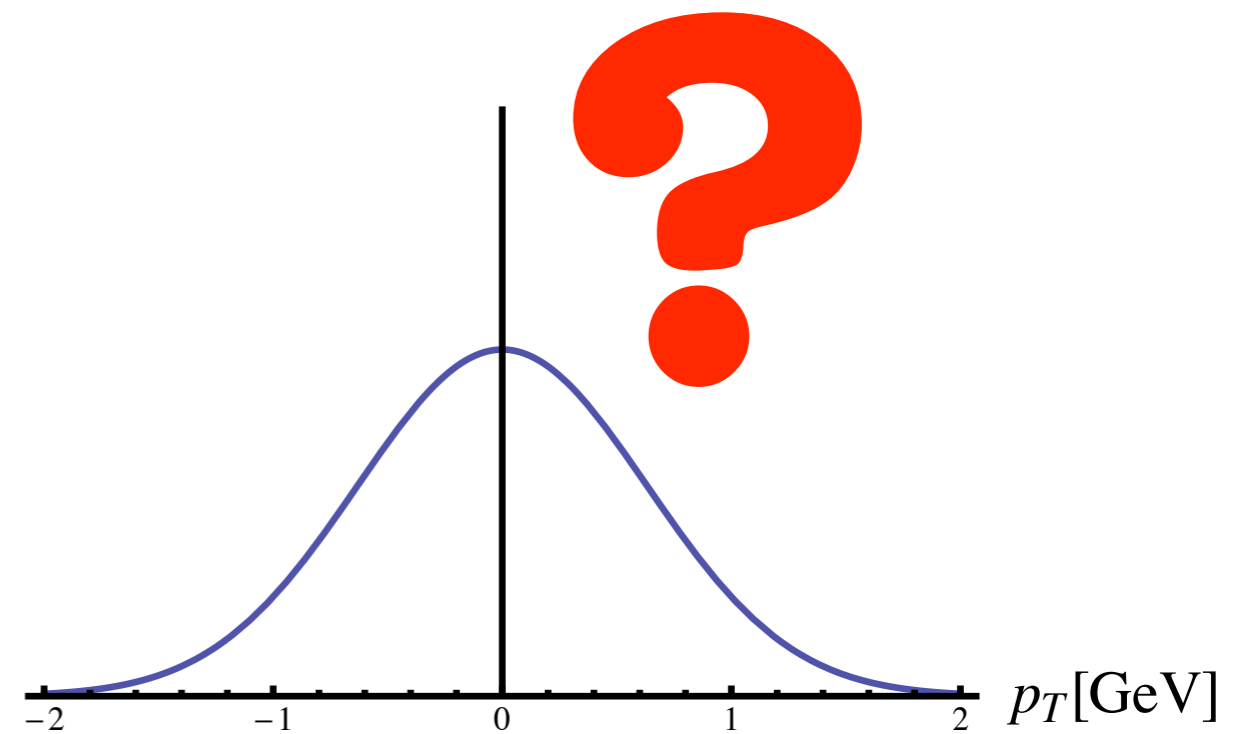
*E866/NuSea preliminary,
talk by J.-C. Peng at DY@BNL workshop*

Flavor-dependent TMDs

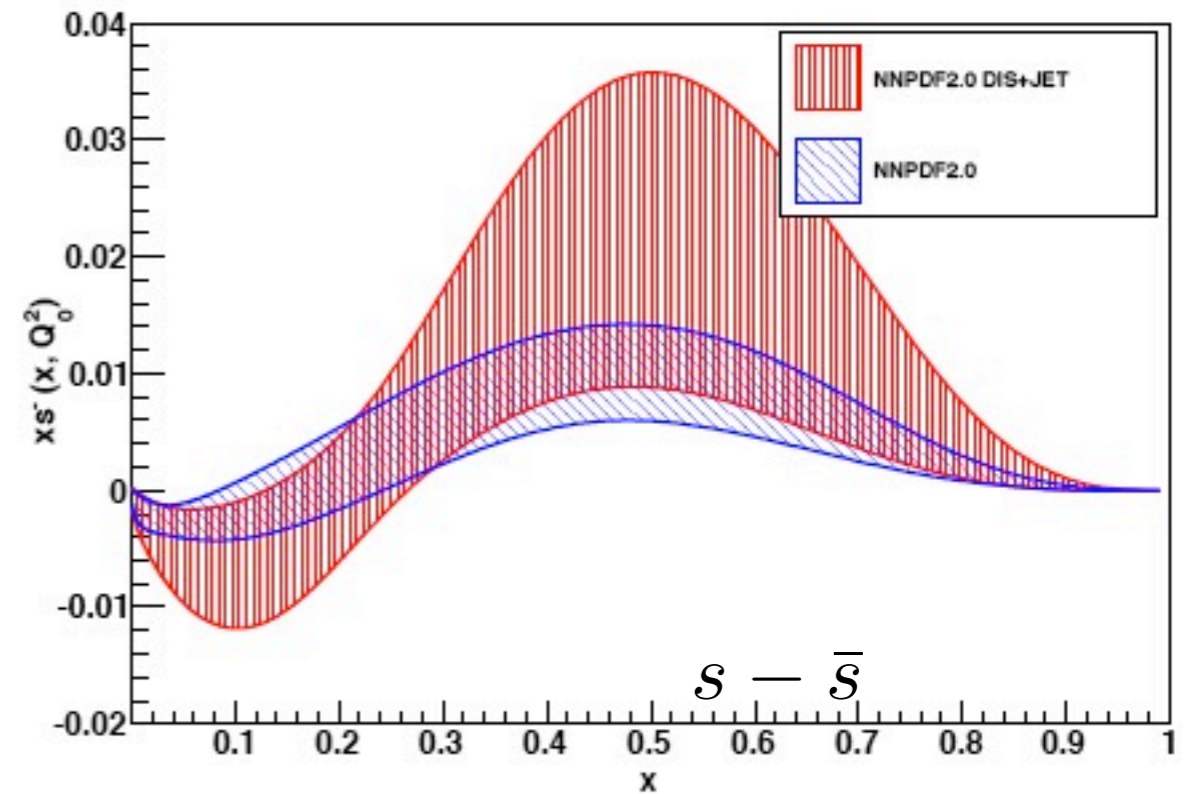
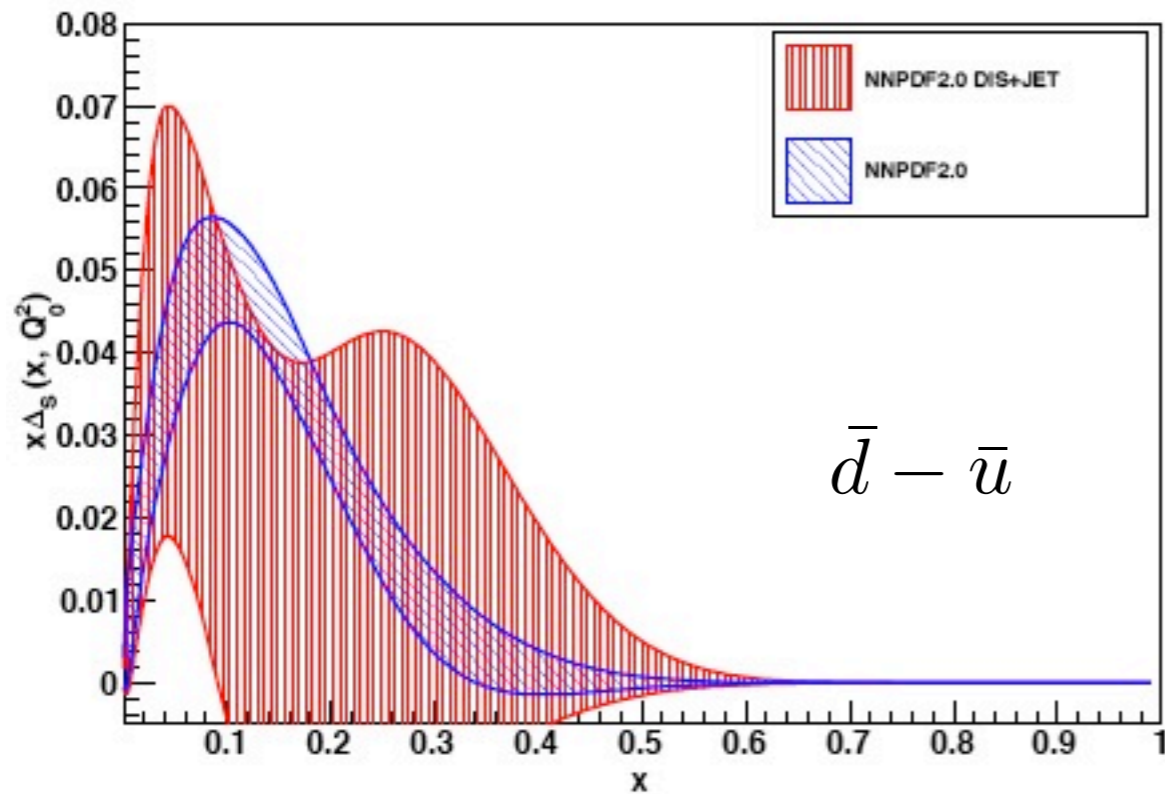
valence



sea



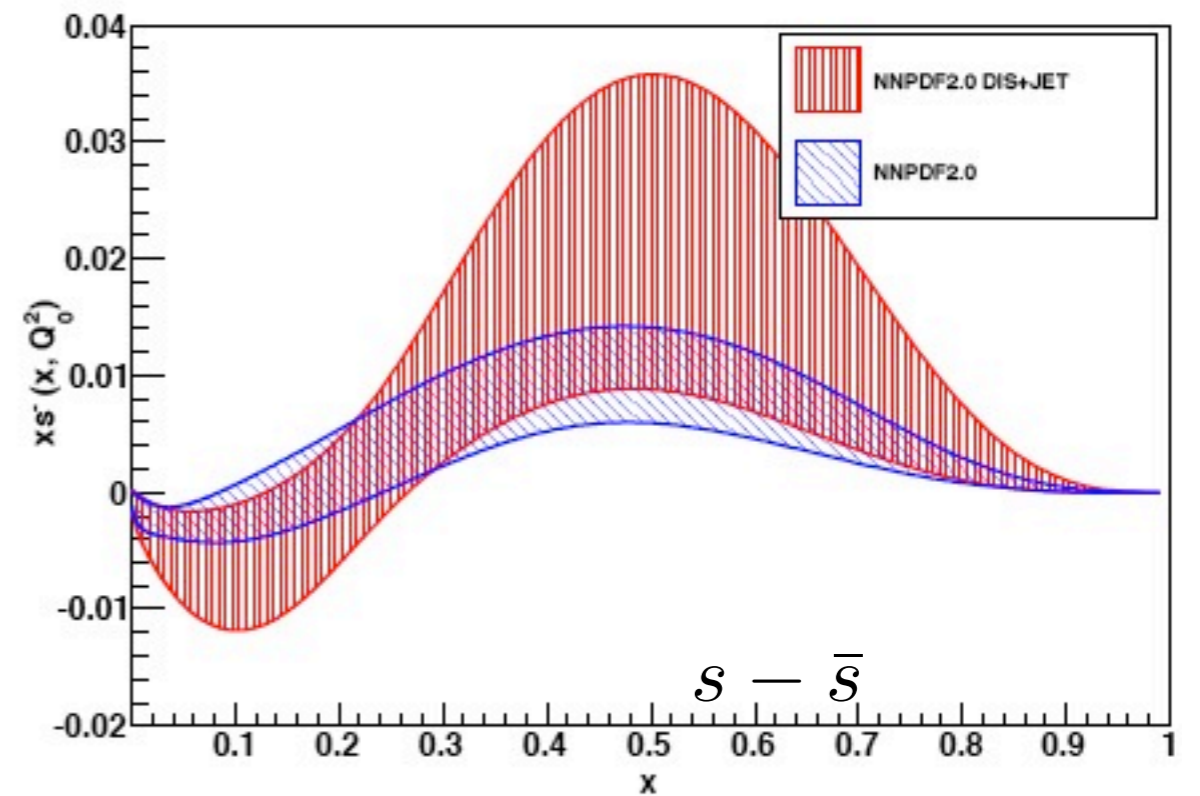
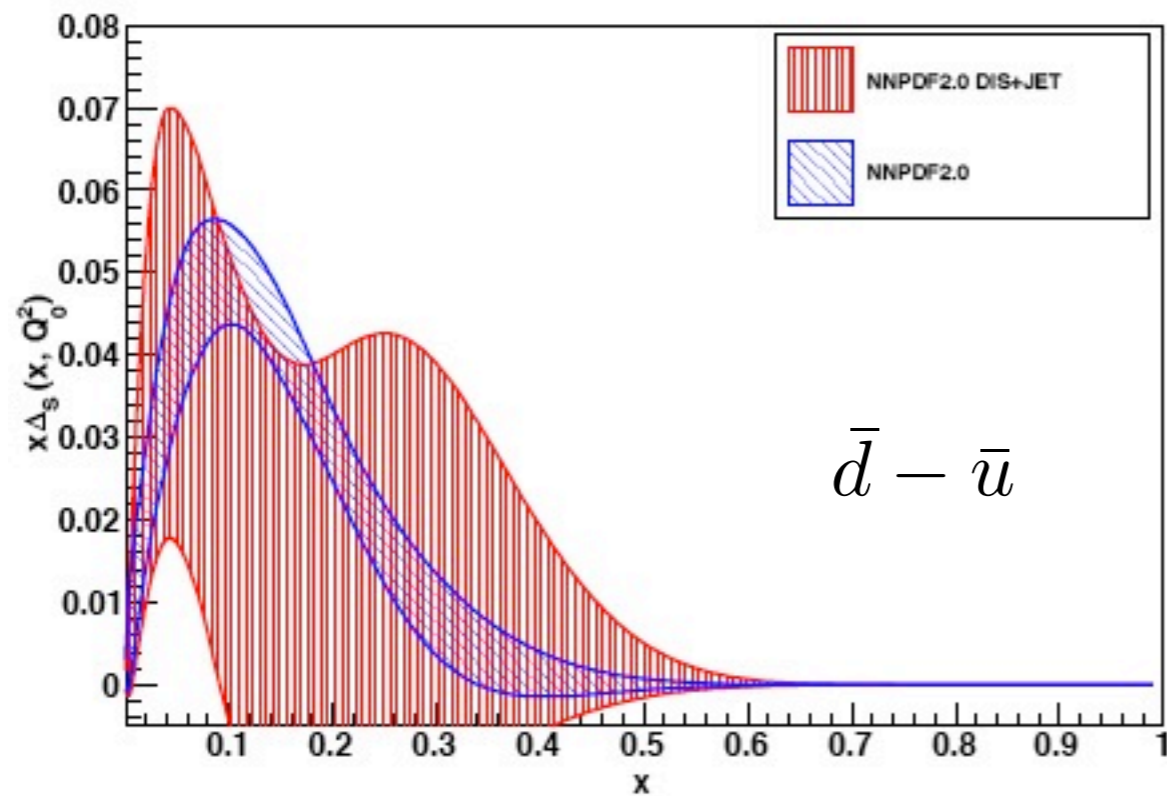
D-Y: impact on PDFs



Blue: adding E605 and E866 data

We can expect similar impact on TMDs

D-Y: impact on PDFs



Blue: adding E605 and E866 data

We can expect similar impact on TMDs

NNPDF Coll., NPB838 (10), see E. Nocera's talk

Future perspectives

- COMPASS pion-nucleon may be useful to constrain pion unpolarized TMDs (will it be useful for the proton ones?)
- To study flavor dependence, it would be nice to have different beam/targets (e.g., antiprotons, deuterons)
- W production should be extremely useful for flavor studies

2

$$F \cos \phi$$

UU

Matching?

Low q_T

High q_T

Tw 3

$$F \sim \frac{1}{Q} \frac{q_T}{M^2}$$

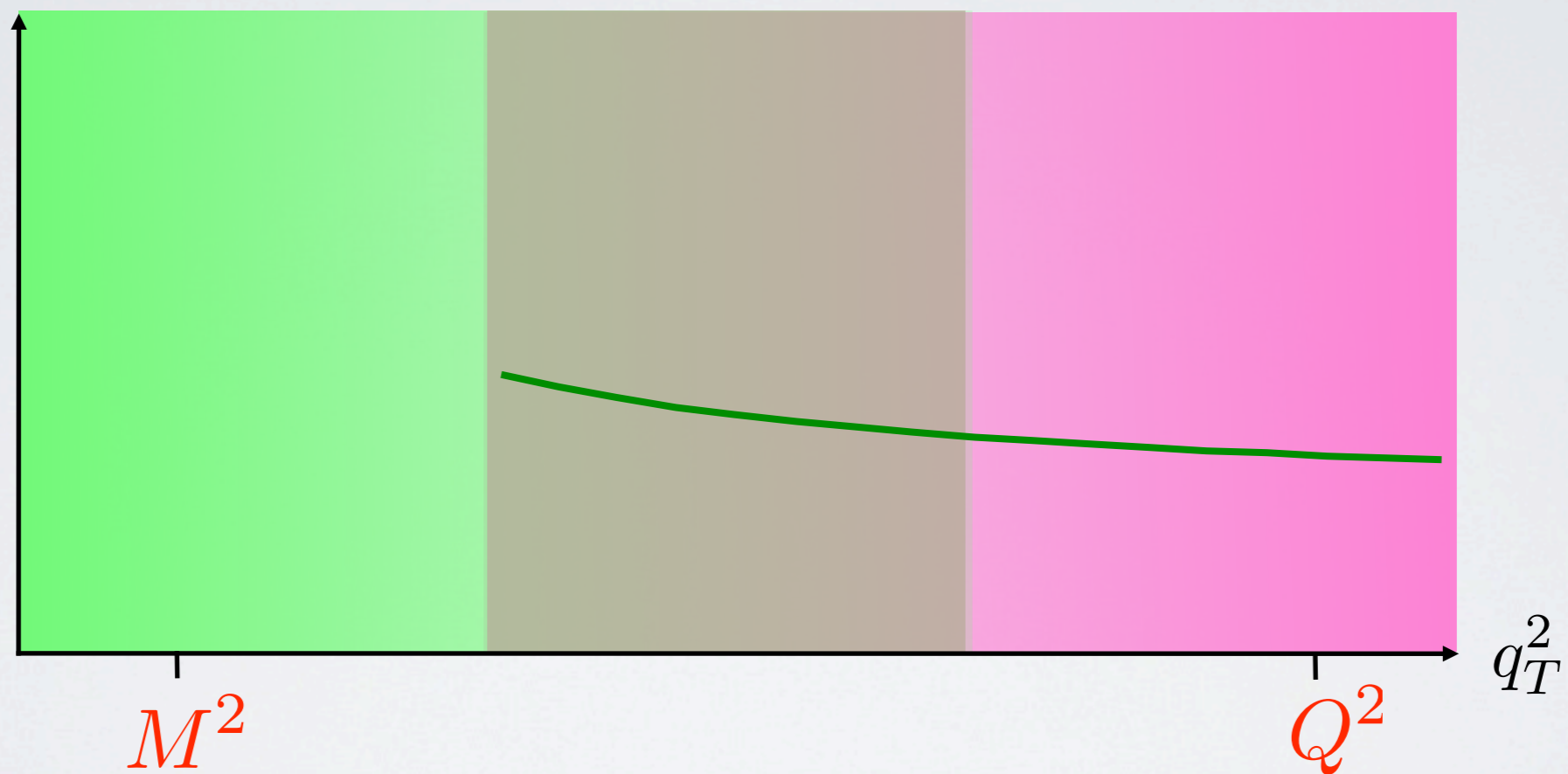
$$F \sim \frac{q_T}{Q} \frac{1}{M^2 + q_T^2}$$

$$F \sim \frac{1}{Q q_T}$$

Tw 2

Yes, but not quite...

Unexpected mismatch



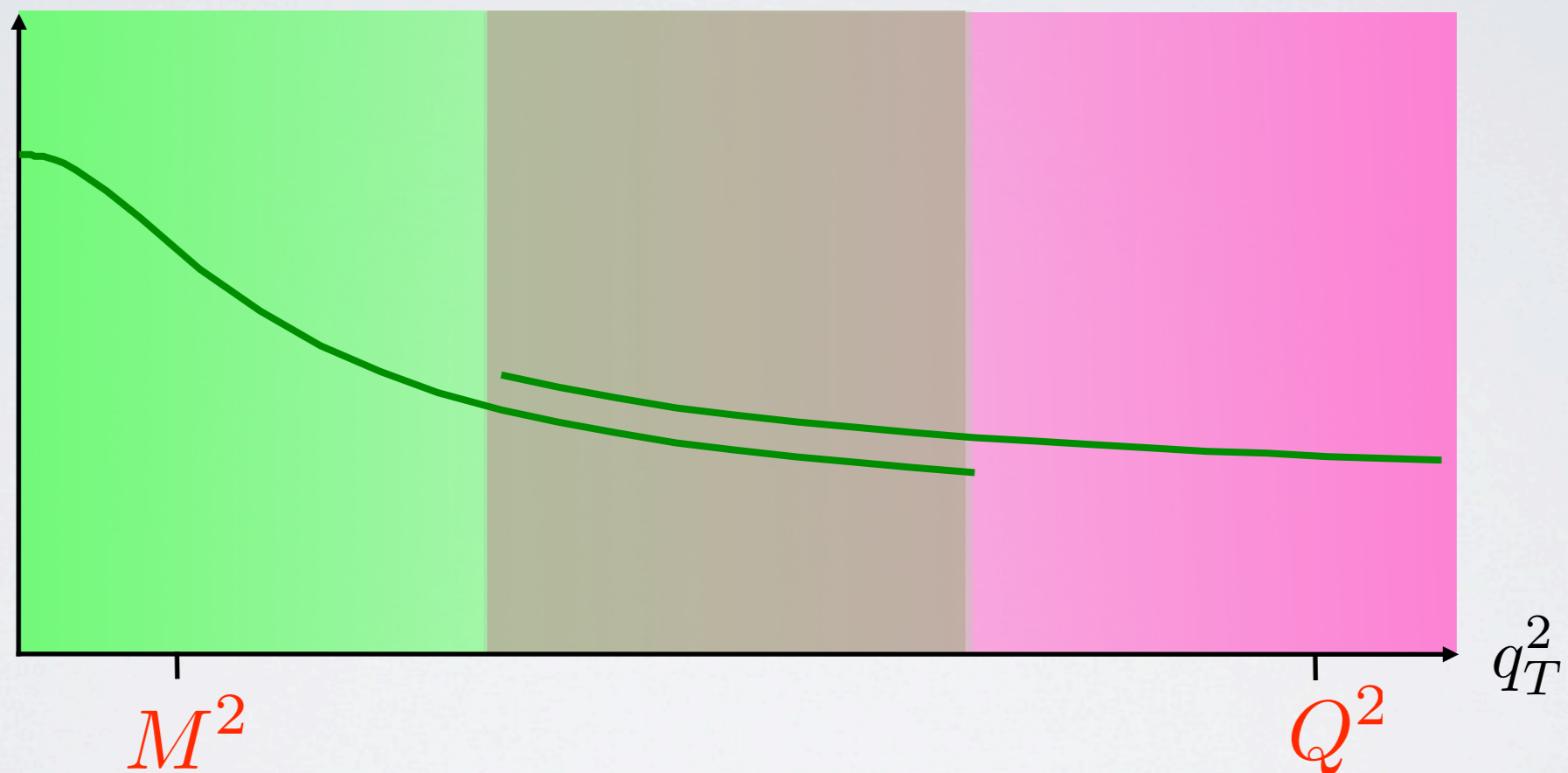
Bacchetta, Boer, Diehl, Mulders, JHEP08 (08)

Unexpected mismatch



Bacchetta, Boer, Diehl, Mulders, JHEP08 (08)

Unexpected mismatch



The TMD formalism at twist 3 is incomplete
(Is it possible to fix it?)

Bacchetta, Boer, Diehl, Mulders, JHEP08 (08)

Low transverse momentum

$$F_{UU,CS}^{\cos\phi} = \frac{2M_1}{Q} \mathcal{C} \left[\frac{\hat{\mathbf{h}} \cdot \mathbf{k}_{2T}}{M_2} \left(\frac{h + \tilde{h}}{2} \bar{h}_1^\perp - \frac{M_2}{M_1} f_1 \frac{\bar{f}^\perp + \tilde{f}^\perp}{2} \right) + \frac{\hat{\mathbf{h}} \cdot \mathbf{k}_{1T}}{M_1} \left(\frac{f^\perp + \tilde{f}^\perp}{2} \bar{f}_1 - \frac{M_2}{M_1} h_1^\perp \frac{\bar{h} + \tilde{h}}{2} \right) \right]$$

Z. Lu and I. Schmidt, PRD84 (11)

Low transverse momentum

Pure twist 3

$$F_{UU,CS}^{\cos\phi} = \frac{2M_1}{Q} \mathcal{C} \left[\frac{\hat{\mathbf{h}} \cdot \mathbf{k}_{2T}}{M_2} \left(\frac{h + \tilde{h}}{2} \bar{h}_1^\perp - \frac{M_2}{M_1} f_1 \frac{\bar{f}^\perp + \tilde{f}^\perp}{2} \right) + \frac{\hat{\mathbf{h}} \cdot \mathbf{k}_{1T}}{M_1} \left(\frac{f^\perp + \tilde{f}^\perp}{2} \bar{f}_1 - \frac{M_2}{M_1} h_1^\perp \frac{\bar{h} + \tilde{h}}{2} \right) \right]$$

Z. Lu and I. Schmidt, PRD84 (11)

Low transverse momentum

Pure twist 3

$$F_{UU,CS}^{\cos \phi} = \frac{2M_1}{Q} \mathcal{C} \left[\frac{\hat{\mathbf{h}} \cdot \mathbf{k}_{2T}}{M_2} \left(\frac{h + \tilde{h}}{2} \bar{h}_1^\perp - \frac{M_2}{M_1} f_1 \frac{\bar{f}^\perp + \tilde{f}^\perp}{2} \right) + \frac{\hat{\mathbf{h}} \cdot \mathbf{k}_{1T}}{M_1} \left(\frac{f^\perp + \tilde{f}^\perp}{2} \bar{f}_1 - \frac{M_2}{M_1} h_1^\perp \frac{\bar{h} + \tilde{h}}{2} \right) \right]$$

$$F_{UU,GJ}^{\cos \phi_h} = \frac{2M_1}{Q} \mathcal{C} \left[\frac{\hat{\mathbf{h}} \cdot \mathbf{k}_{2T}}{M_2} \left(h \bar{h}_1^\perp - \frac{M_2}{M_1} f_1 \tilde{f}^\perp \right) + \frac{\hat{\mathbf{h}} \cdot \mathbf{k}_{1T}}{M_1} \left(f^\perp \bar{f}_1 - \frac{M_2}{M_1} h_1^\perp \tilde{h} \right) \right]$$

Z. Lu and I. Schmidt, PRD84 (11)

Low transverse momentum

Pure twist 3

$$F_{UU,CS}^{\cos \phi} = \frac{2M_1}{Q} \mathcal{C} \left[\frac{\hat{\mathbf{h}} \cdot \mathbf{k}_{2T}}{M_2} \left(\frac{h + \tilde{h}}{2} \bar{h}_1^\perp - \frac{M_2}{M_1} f_1 \frac{\bar{f}^\perp + \tilde{f}^\perp}{2} \right) + \frac{\hat{\mathbf{h}} \cdot \mathbf{k}_{1T}}{M_1} \left(\frac{f^\perp + \tilde{f}^\perp}{2} \bar{f}_1 - \frac{M_2}{M_1} h_1^\perp \frac{\bar{h} + \tilde{h}}{2} \right) \right]$$

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$$F_{UU}^{\cos \phi_h} = \frac{2M}{Q} \mathcal{C} \left[-\frac{\hat{\mathbf{h}} \cdot \mathbf{k}_T}{M_h} \left(x_B h H_1^\perp + \frac{M_h}{M} f_1 \frac{\tilde{D}^\perp}{z} \right) - \frac{\hat{\mathbf{h}} \cdot \mathbf{p}_T}{M} \left(x_B f^\perp D_1 + \frac{M_h}{M} h_1^\perp \frac{\tilde{H}}{z} \right) \right] \quad \text{SIDIS}$$

Z. Lu and I. Schmidt, PRD84 (11)

From low to high q_T

$$F_{UU,GJ}^{\cos \phi_h} = \frac{2M_1}{Q} c \left[\frac{\hat{\mathbf{h}} \cdot \mathbf{k}_{2T}}{M_2} \left(h \bar{h}_1^\perp - \frac{M_2}{M_1} f_1 \tilde{f}^\perp \right) + \frac{\hat{\mathbf{h}} \cdot \mathbf{k}_{1T}}{M_1} \left(f^\perp \bar{f}_1 - \frac{M_2}{M_1} h_1^\perp \tilde{h} \right) \right]$$

From low to high q_T

$$F_{UU,GJ}^{\cos \phi_h} = \frac{2M_1}{Q} \mathcal{C} \left[\frac{\hat{\mathbf{h}} \cdot \mathbf{k}_{2T}}{M_2} \left(h \bar{h}_1^\perp - \frac{M_2}{M_1} f_1 \tilde{f}^\perp \right) + \frac{\hat{\mathbf{h}} \cdot \mathbf{k}_{1T}}{M_1} \left(f^\perp \bar{f}_1 - \frac{M_2}{M_1} h_1^\perp \tilde{h} \right) \right]$$

$$x f_1 \sim \frac{1}{\mathbf{k}_T^2} \alpha_s \mathcal{F}[f_1]$$

$$x f^\perp \sim \frac{1}{\mathbf{k}_T^2} \alpha_s \mathcal{F}[f_1]$$

From low to high q_T

$$F_{UU,GJ}^{\cos \phi_h} = \frac{2M_1}{Q} \mathcal{C} \left[\frac{\hat{\mathbf{h}} \cdot \mathbf{k}_{2T}}{M_2} \left(h \bar{h}_1^\perp - \frac{M_2}{M_1} f_1 \tilde{f}^\perp \right) + \frac{\hat{\mathbf{h}} \cdot \mathbf{k}_{1T}}{M_1} \left(f^\perp \bar{f}_1 - \frac{M_2}{M_1} h_1^\perp \tilde{h} \right) \right]$$

$$x f_1 \sim \frac{1}{\mathbf{k}_T^2} \alpha_s \mathcal{F}[f_1]$$

$$h_1^\perp \sim \frac{M^2}{\mathbf{k}_T^4} \alpha_s \mathcal{F}[h_1^{\perp(1)}, \dots]$$

$$x f^\perp \sim \frac{1}{\mathbf{k}_T^2} \alpha_s \mathcal{F}[f_1]$$

$$h \sim \frac{1}{\mathbf{k}_T^2} \alpha_s \mathcal{F}[h_1^{\perp(1)}, \dots]$$

From low to high q_T

$$F_{UU,GJ}^{\cos \phi_h} = \frac{2M_1}{Q} \mathcal{C} \left[\frac{\hat{\mathbf{h}} \cdot \mathbf{k}_{2T}}{M_2} \left(\cancel{h_1^\perp \bar{f}_1^\perp} - \frac{M_2}{M_1} f_1 \tilde{f}^\perp \right) + \frac{\hat{\mathbf{h}} \cdot \mathbf{k}_{1T}}{M_1} \left(f^\perp \bar{f}_1 - \cancel{\frac{M_2}{M_1} k_{1T}^\perp \bar{h}} \right) \right]$$

$$x f_1 \sim \frac{1}{\mathbf{k}_T^2} \alpha_s \mathcal{F}[f_1]$$

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$$h \sim \frac{1}{\mathbf{k}_T^2} \alpha_s \mathcal{F}[h_1^{\perp(1)}, \dots]$$

SIDIS example

LOW q_T

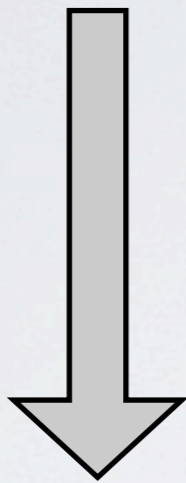
$$F_{UU}^{\cos \phi_h} = \frac{2M}{Q} \mathcal{C} \left[-\frac{\hat{\mathbf{h}} \cdot \mathbf{k}_T}{M_h} \left(x_B h H_1^\perp + \frac{M_h}{M} f_1 \frac{\tilde{D}^\perp}{z} \right) - \frac{\hat{\mathbf{h}} \cdot \mathbf{p}_T}{M} \left(x_B f^\perp D_1 + \frac{M_h}{M} h_1^\perp \frac{\tilde{H}}{z} \right) \right]$$

Intermediate q_T

SIDIS example

LOW q_T

$$F_{UU}^{\cos \phi_h} = \frac{2M}{Q} \mathcal{C} \left[-\frac{\hat{\mathbf{h}} \cdot \mathbf{k}_T}{M_h} \left(x_B h H_1^\perp + \frac{M_h}{M} f_1 \frac{\tilde{D}^\perp}{z} \right) - \frac{\hat{\mathbf{h}} \cdot \mathbf{p}_T}{M} \left(x_B f^\perp D_1 + \frac{M_h}{M} h_1^\perp \frac{\tilde{H}}{z} \right) \right]$$

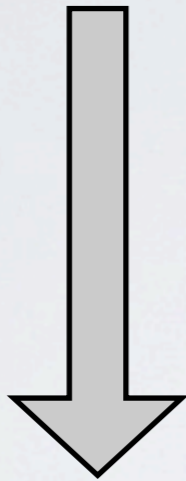


Intermediate q_T

SIDIS example

LOW q_T

$$F_{UU}^{\cos \phi_h} = \frac{2M}{Q} \mathcal{C} \left[-\frac{\hat{\mathbf{h}} \cdot \mathbf{k}_T}{M_h} \left(x_B h H_1^\perp + \frac{M_h}{M} f_1 \frac{\tilde{D}^\perp}{z} \right) - \frac{\hat{\mathbf{h}} \cdot \mathbf{p}_T}{M} \left(x_B f^\perp D_1 + \frac{M_h}{M} h_1^\perp \frac{\tilde{H}}{z} \right) \right]$$



Intermediate q_T

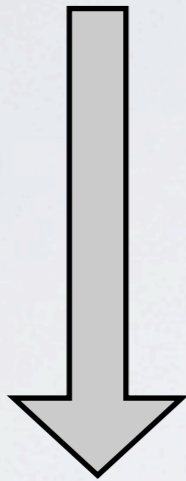
$$F_{UU}^{\cos \phi_h} = -\frac{1}{Qq_T} \frac{\alpha_s}{2\pi^2 z^2} \sum_a x e_a^2 \left[f_1^a(x) D_1^a(z) L\left(\frac{Q^2}{q_T^2}\right) + f_1^a(x) (D_1^a \otimes P'_{qq} + D_1^a \otimes P'_{gq})(z) \right. \\ \left. + (P'_{qq} \otimes f_1^a + P'_{qg} \otimes f_1^g)(x) D_1^a(z) - 2C_F f_1^a(x) D_1^a(z) \right]$$

Bacchetta, Boer, Diehl, Mulders, JHEP08 (08)

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Does not match exactly!

Bacchetta, Boer, Diehl, Mulders, JHEP08 (08)

“Wandzura-Wilczek” approx

$$f^\perp = \frac{f_1}{x} + \tilde{f}^\perp$$

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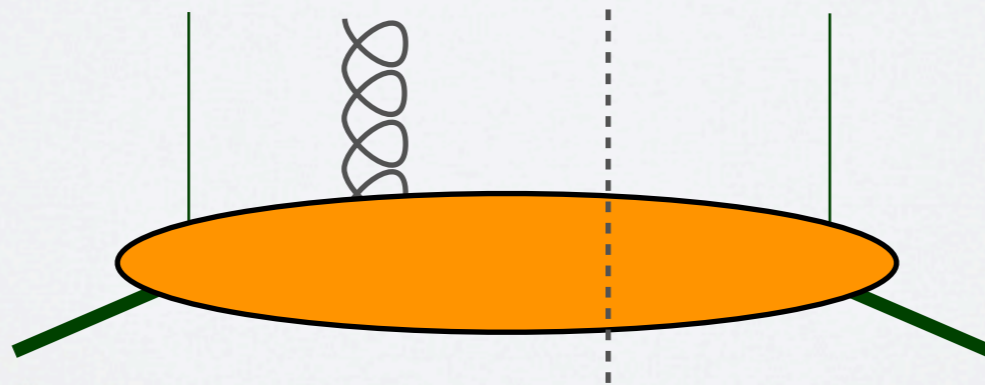
$$f^\perp = \frac{f_1}{x} + \cancel{f^\perp}$$

Formally “fine,” but probably is not realistic and misses nice physics of quark-gluon correlations

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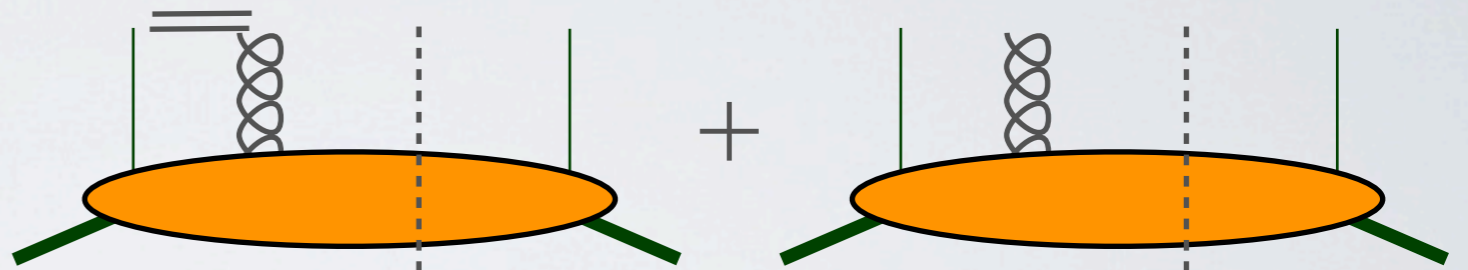
Formally wrong.

E.g., violates time-reversal condition:

$$\int d^2 k_T h(x, k_T^2) = 0$$

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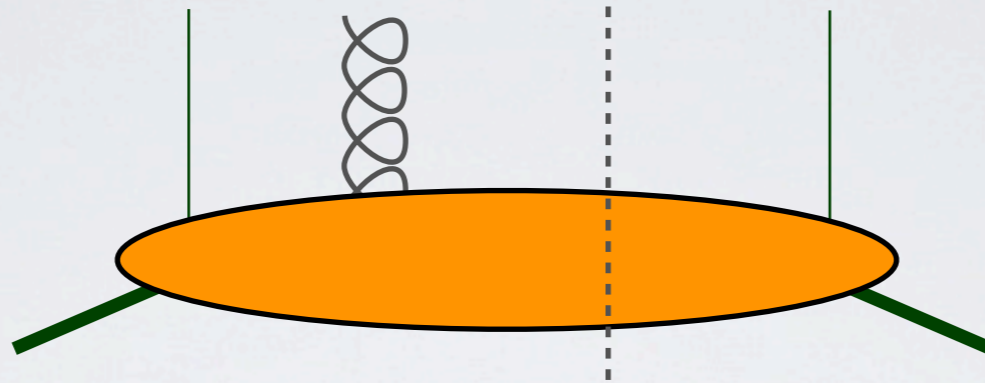
$$h = \frac{1}{M^2} \frac{h^\perp}{x} + \tilde{h}$$

“Wandzura-Wilczek” approx

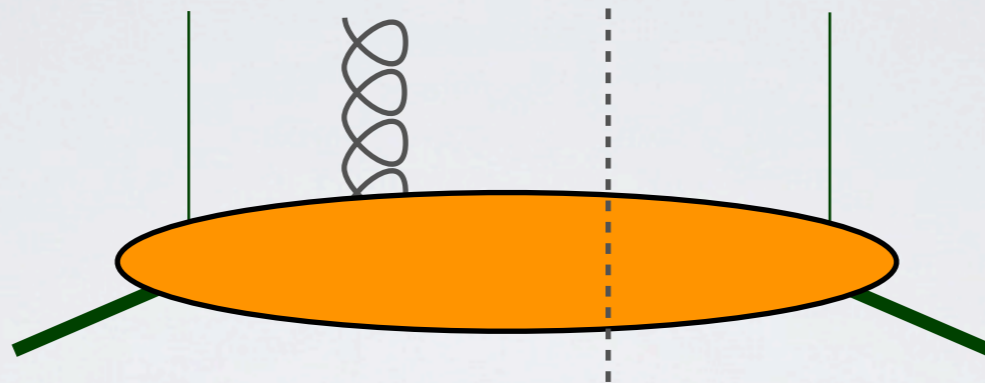
$$~~h = \frac{m_1^2}{M^2} \frac{h^\perp}{x} + \tilde{h}~~$$

Formally “fine,” but probably is not realistic and misses nice physics of T-odd functions.

Quark-gluon correlations



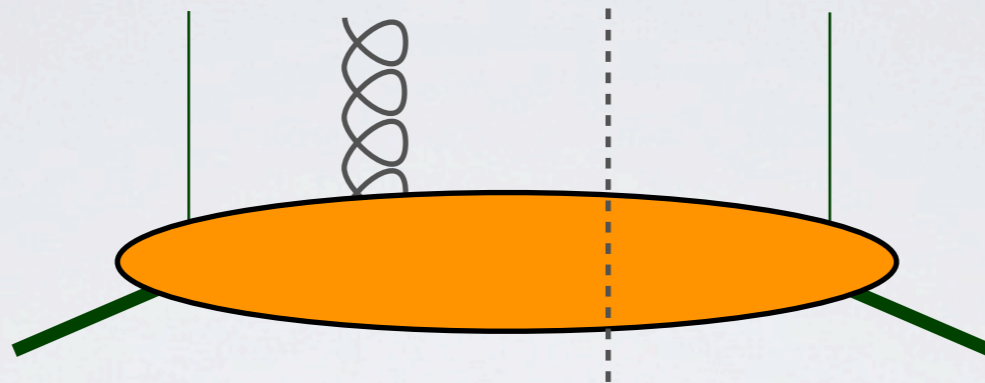
Quark-gluon correlations



- Describe physics effects of the presence of the gluons together with the quarks (quarks feeling the QCD interaction...)

see, e.g., Burkardt [arXiv:0810.3589](https://arxiv.org/abs/0810.3589)

Quark-gluon correlations



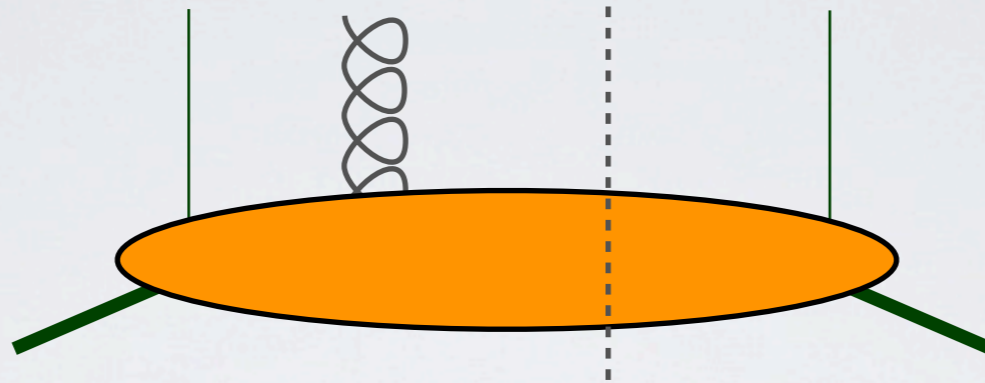
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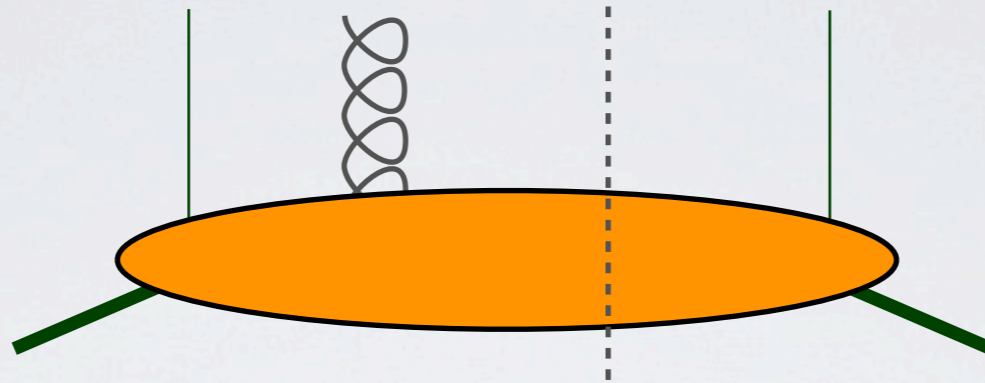
- Some models already exist

see, e.g., Lu, Schmidt, arXiv:1202.0700

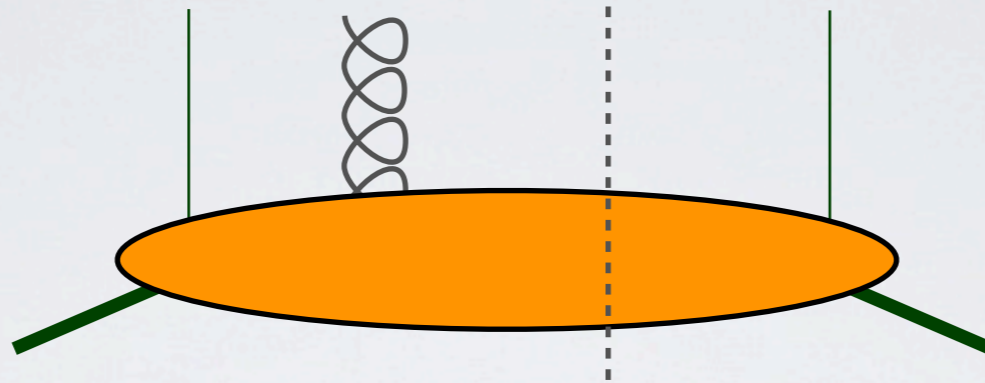
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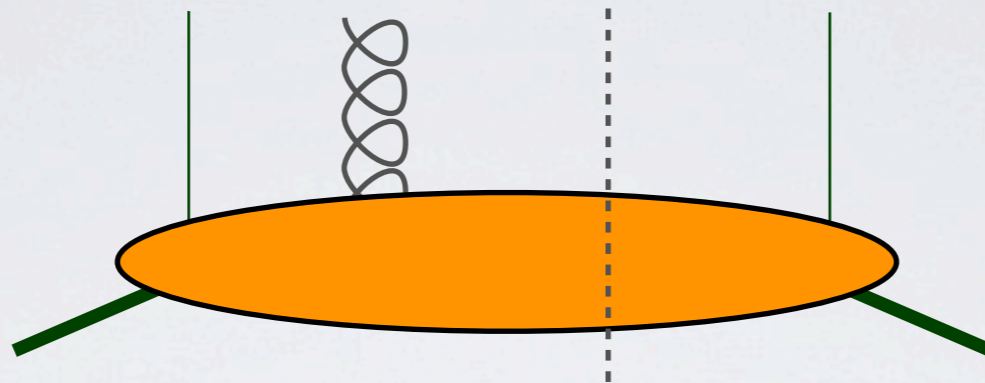


Quark-gluon correlations



- Important for the high-transverse-momentum tails of TMDs
see, e.g., Ji, Qiu, Vogelsang, Yuan, PLB638 (06)

Quark-gluon correlations



- Important for the high-transverse-momentum tails of TMDs
see, e.g., Ji, Qiu, Vogelsang, Yuan, PLB638 (06)
- Intriguing properties from the theoretical point of view (e.g., evolution)
see, e.g., Braun, Manashov, Pirnay, PRD80 (09)

3

$$F \cos 2\phi$$

UU

Matching?

Low q_T

High q_T

Tw 4

$$F \sim \frac{1}{Q^2} \frac{q_T^2}{M^2}$$

$$F \sim \frac{q_T^2}{Q^2} \frac{1}{M^2 + q_T^2}$$

$$F \sim \frac{1}{Q^2}$$

Tw 2

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Tw 2 $F \sim \frac{q_T^2}{M^4}$

$$F \sim \frac{M^2 q_T^4}{M^6 + q_T^6}$$

$$F \sim \frac{1}{q_T^4} \quad \text{Tw 4}$$

+

Matching?

Low q_T

High q_T

No

Tw 4 $F \sim \frac{1}{Q^2} \frac{q_T^2}{M^2}$

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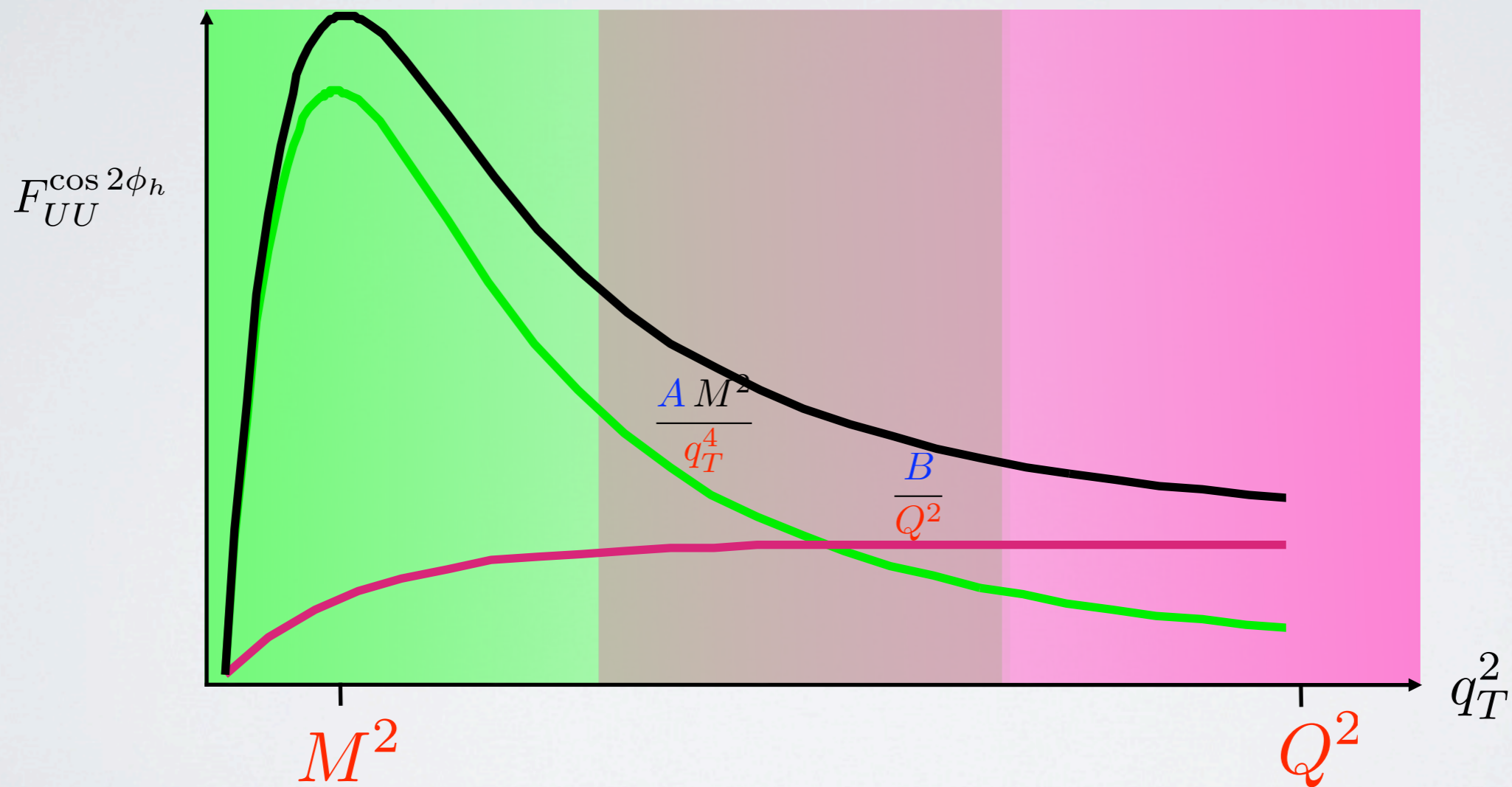
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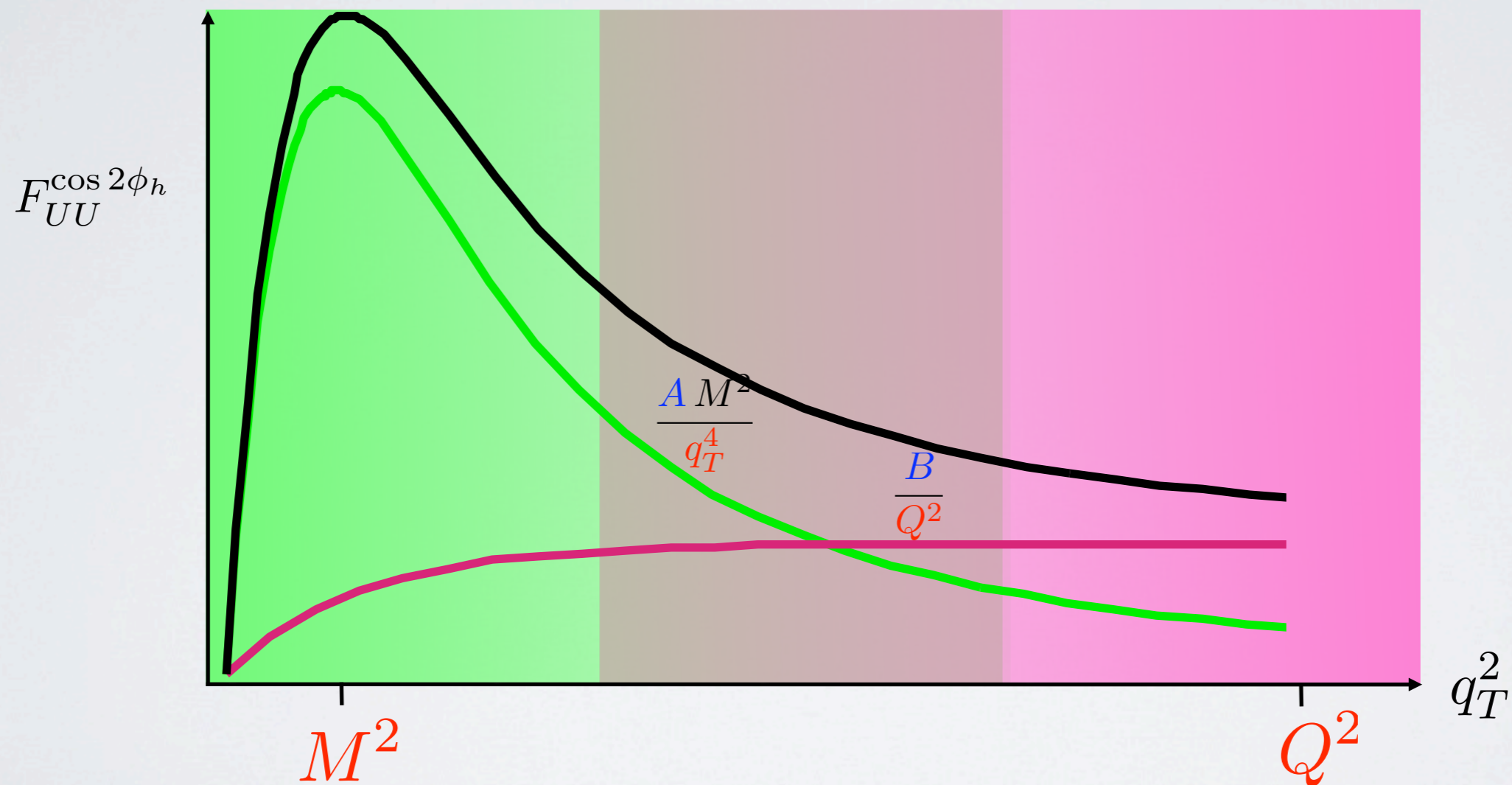
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Expected mismatch



Expected mismatch

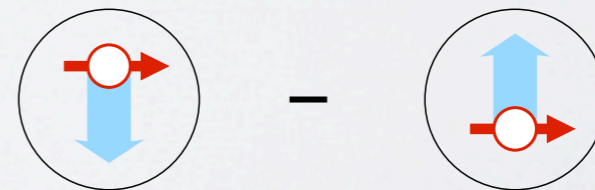


Two distinct mechanisms are involved

TMD description

$$F_{UU}^{\cos 2\phi} = c \left[\frac{2(\vec{h} \cdot \vec{k}_{aT})(\vec{h} \cdot \vec{k}_{bT}) - \vec{k}_{aT} \cdot \vec{k}_{bT}}{M_a M_b} h_1^\perp \bar{h}_1^\perp \right]$$

Boer-Mulders TMD
“spin without spin”



TMD description

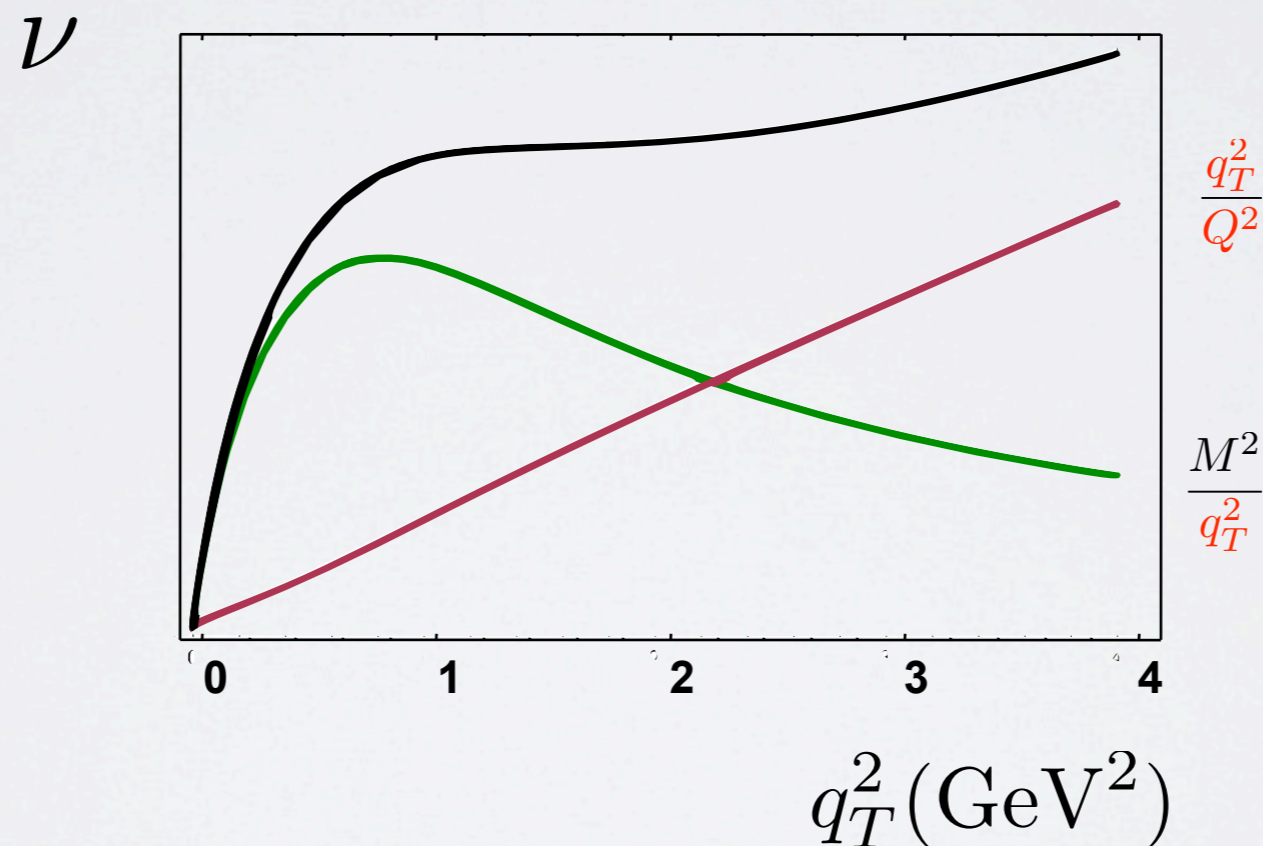
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Boer-Mulders TMD
“spin without spin”

$$h_1^\perp = \begin{array}{c} \text{red circle with right arrow} \\ \text{blue circle with down arrow} \end{array} - \begin{array}{c} \text{blue circle with up arrow} \\ \text{red circle with right arrow} \end{array}$$

On the $\cos 2\phi$ modulation

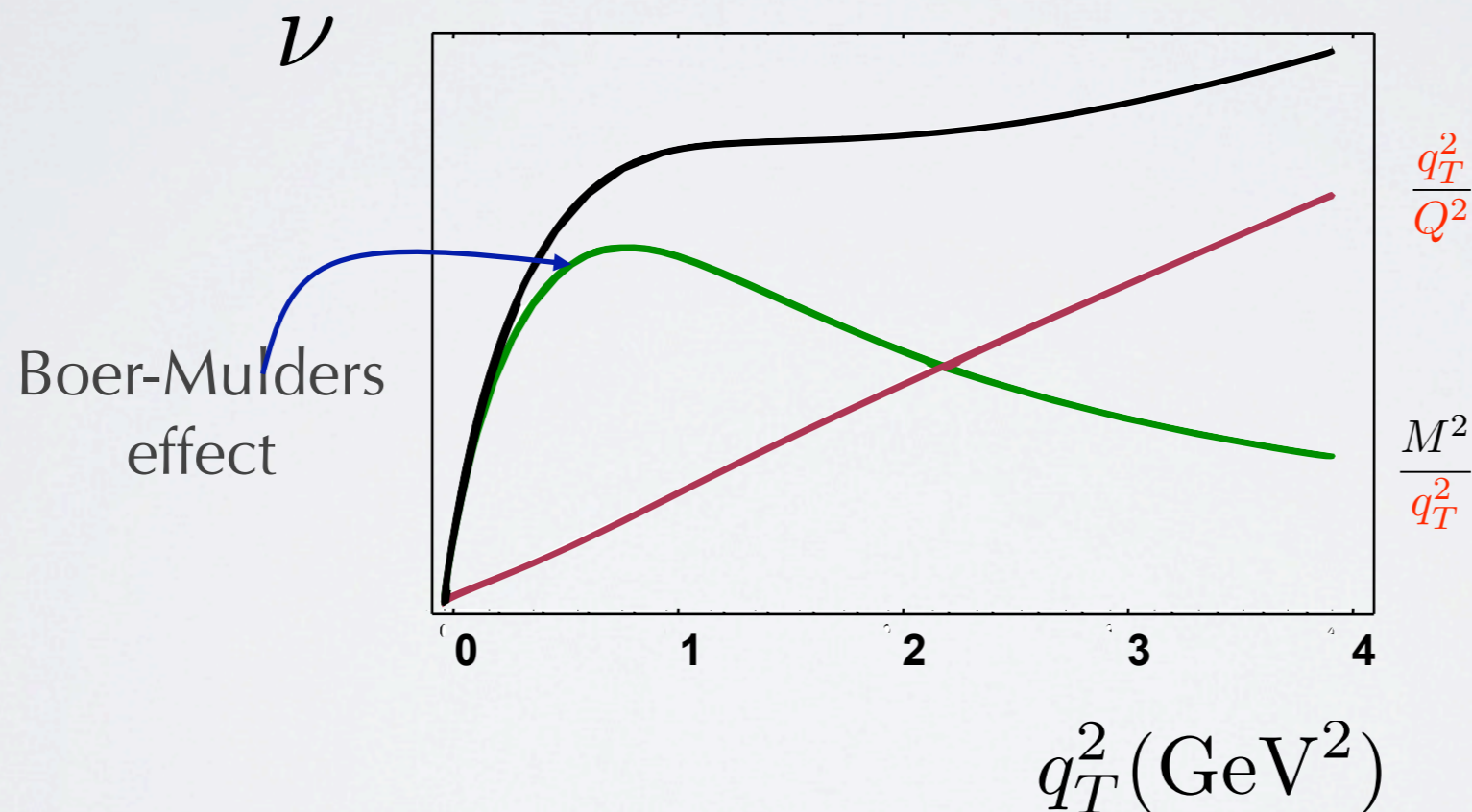
$$\frac{d\sigma}{d\Omega} \propto 1 + \lambda \cos^2 \theta + \mu \sin 2\theta \cos \phi + \frac{\nu}{2} \sin^2 \theta \cos 2\phi$$



Bacchetta, Boer, Diehl, Mulders, JHEP08 (08)

On the $\cos 2\phi$ modulation

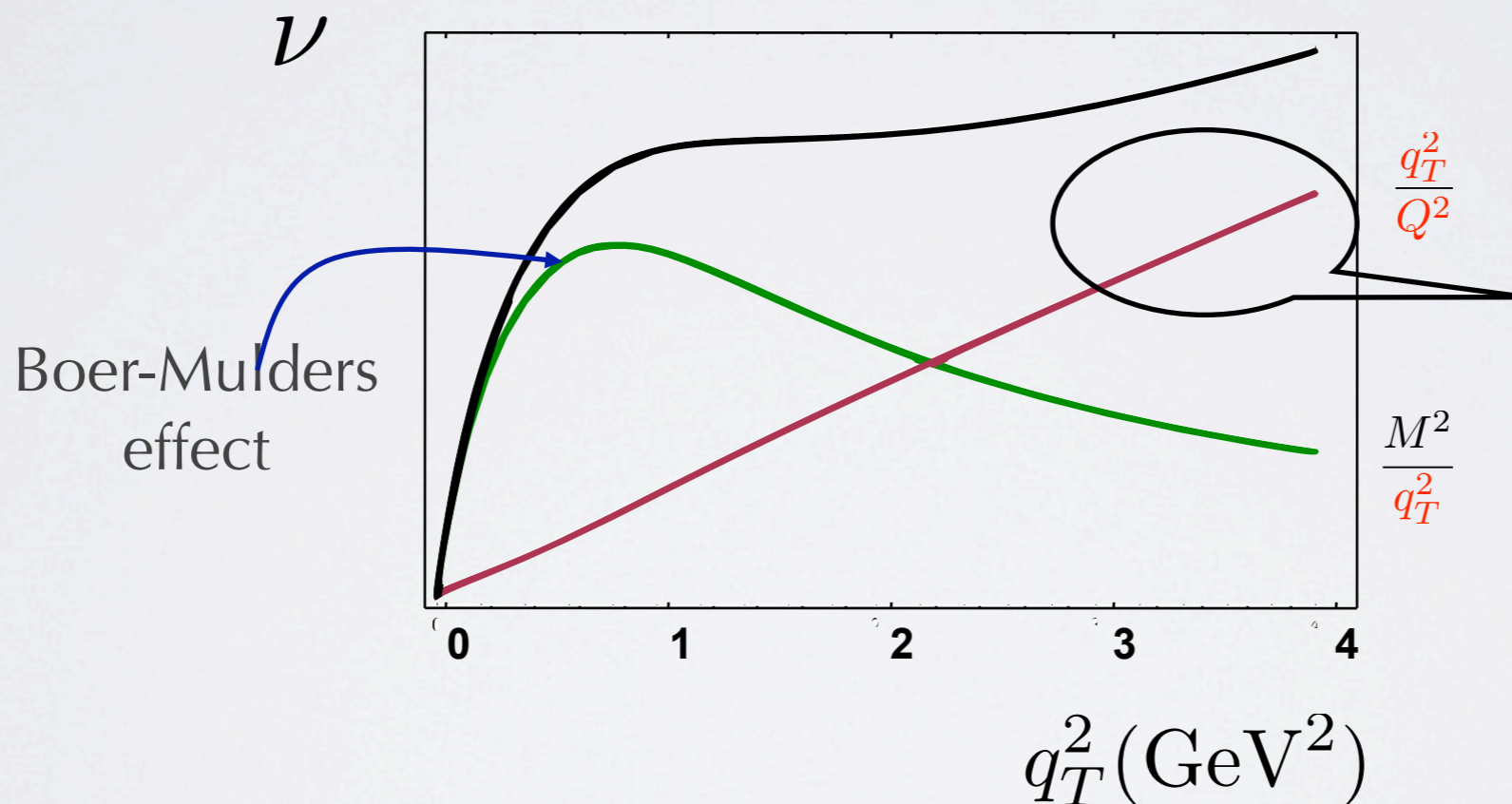
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Can be calculated with pQCD. Resummation important.

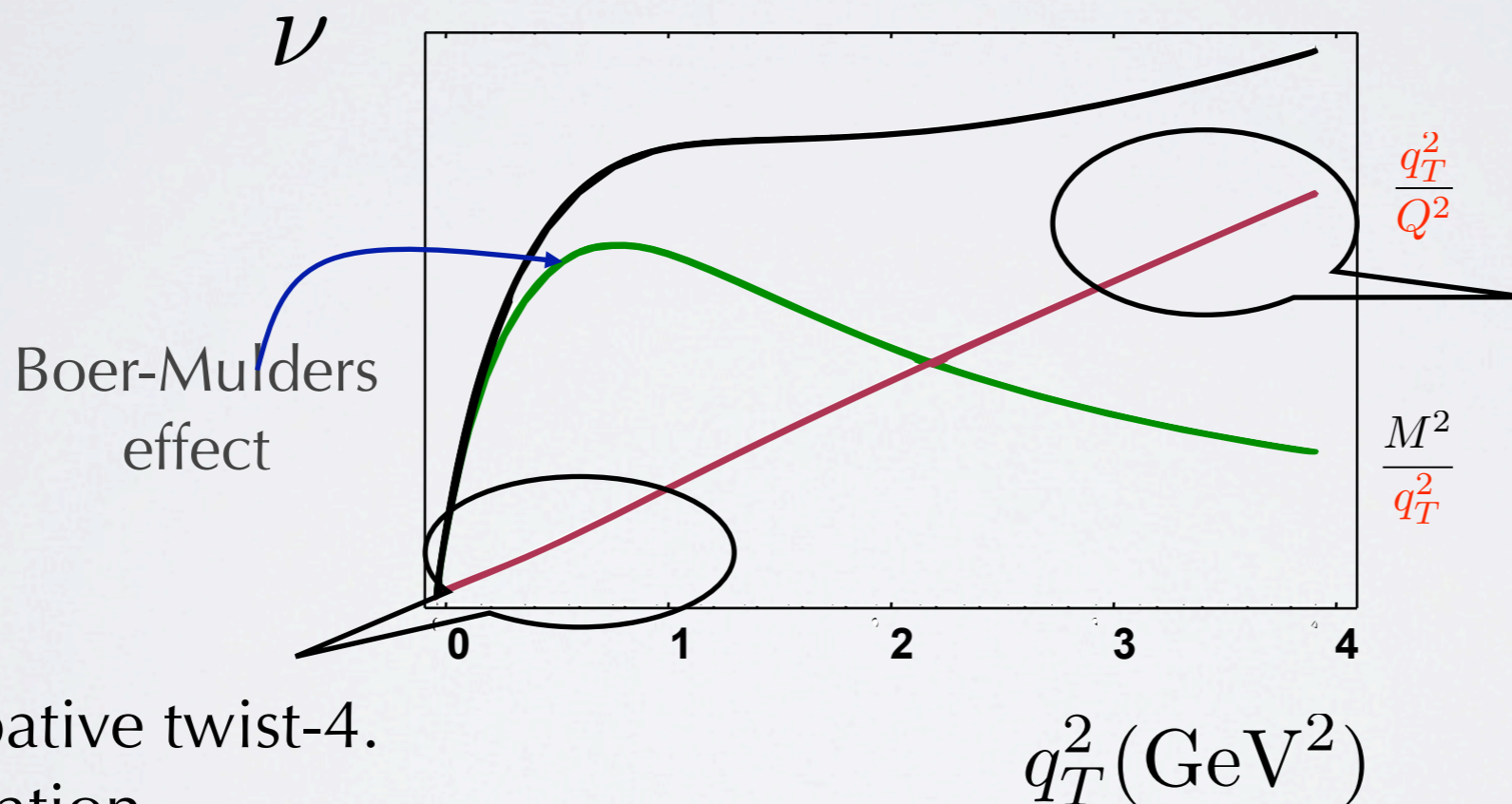
Boer, Vogelsang, PRD74 (06)

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Bacchetta, Boer, Diehl, Mulders, JHEP08 (08)

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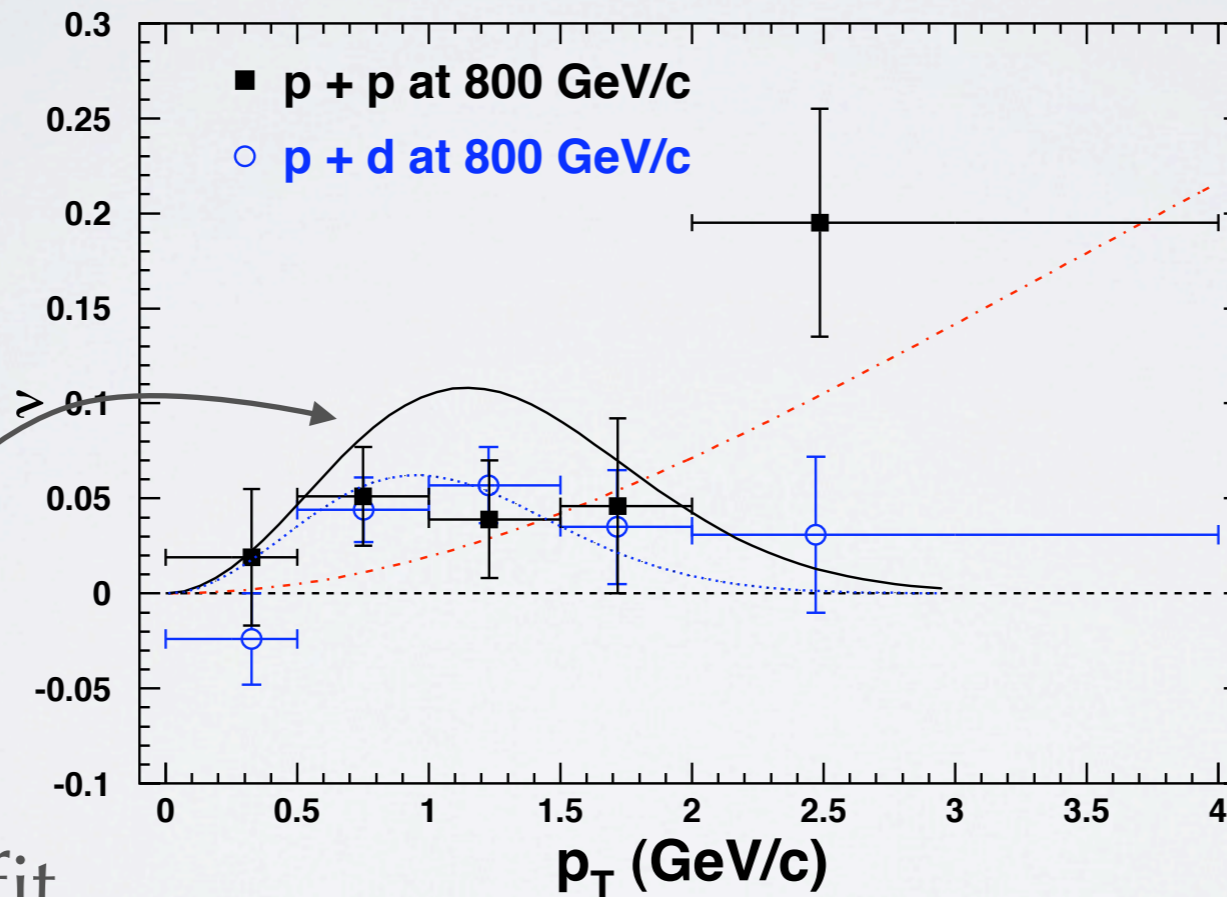
Boer, Vogelsang, PRD74 (06)

Berger, Qiu, Rodrigues-Pedraza, PRD76 (07)

Nonperturbative twist-4.
No factorization.
(Cahn twist-4 can be a model?)

Bacchetta, Boer, Diehl, Mulders, JHEP08 (08)

Phenomenology



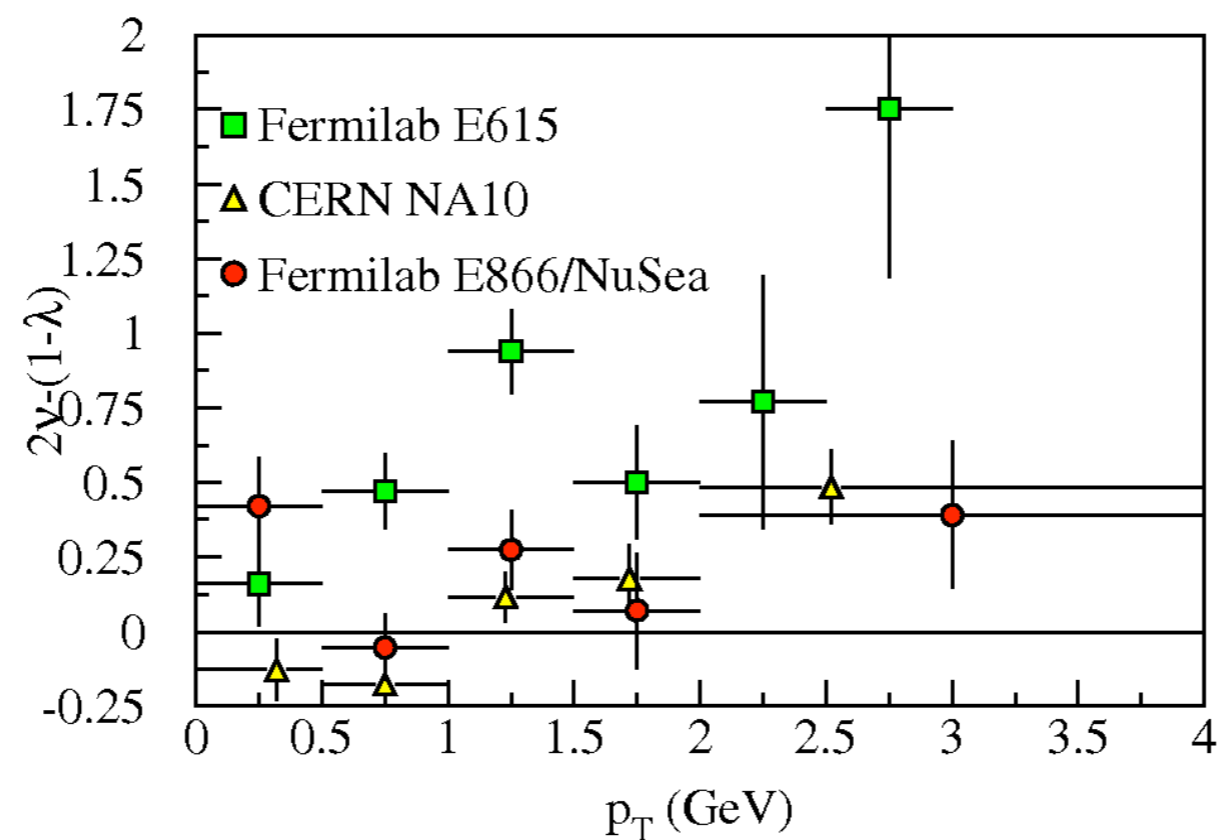
Boer-Mulders fit

Zhang, Lu, Ma, Schmidt, PRD78 (08)

Enzo Barone's talk

Violation of Lam-Tung relation

$$1 - \lambda = 2\nu$$



talk by P. Reimer at DY@BNL workshop

May be a better way to study Boer-Mulders function,
since pQCD contributions may cancel

Conclusions

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Already unpolarized Drell-Yan has many rich features related to TMDs...

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see next talk and tomorrow's talks