The Drell-Yan process: Theoretical aspects

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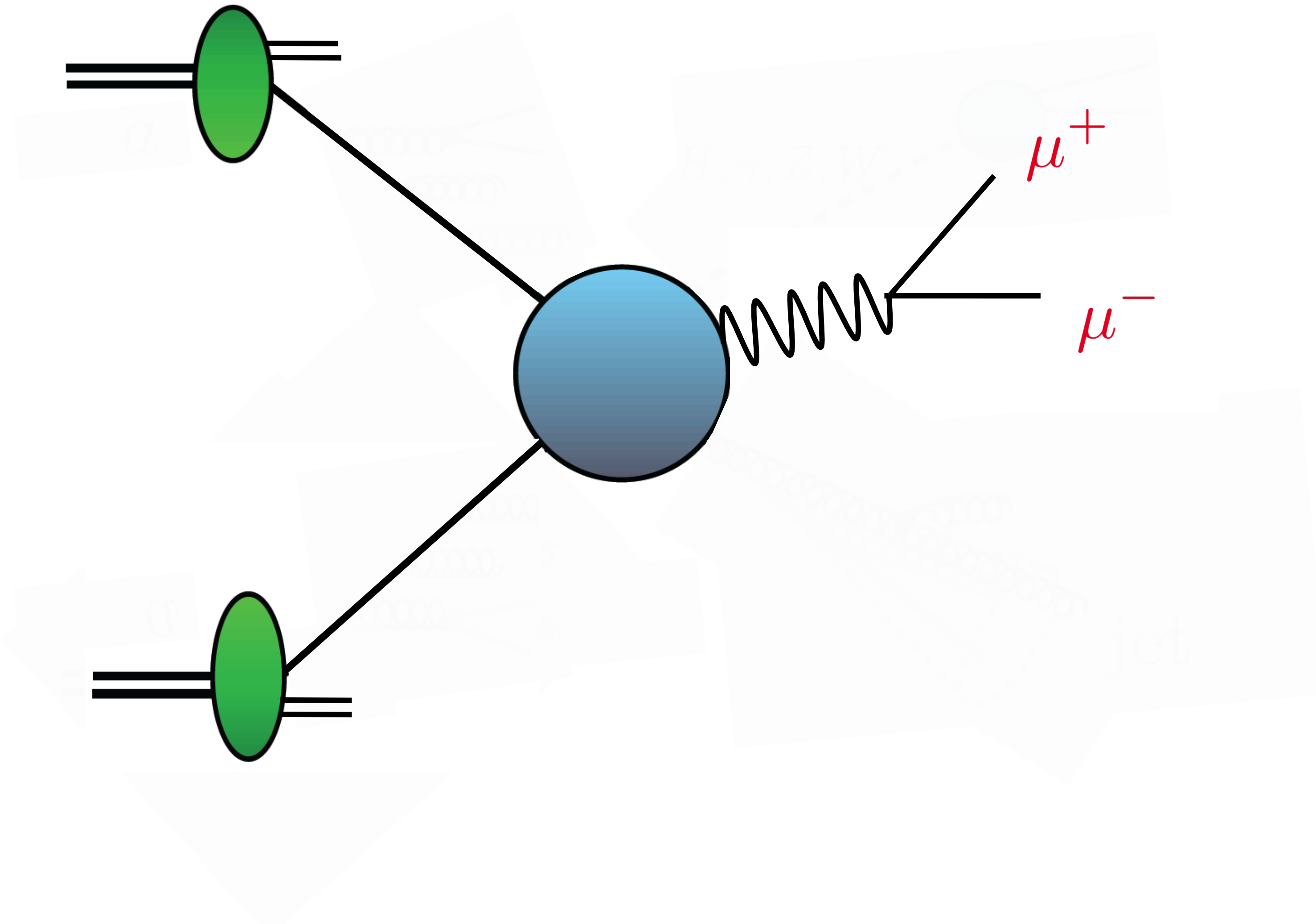
Trento, 21/05/12


μ+
μ−

Drell-Yan arguably is theoretically best explored process in hadronic scattering:

- probe of partonic structure of hadrons:
  anti-quarks, valence distribution of pion, …

- interface of QCD and QED/el.weak interactions:
  precision probes (e.g. W-mass at Tevatron)

- important spin phenomena

- LO is color-singlet annihilation $q\bar{q} \to \gamma^*$

  - higher-orders under control

  - higher-order computations "easier"

- many techniques relevant for $gg \to H$

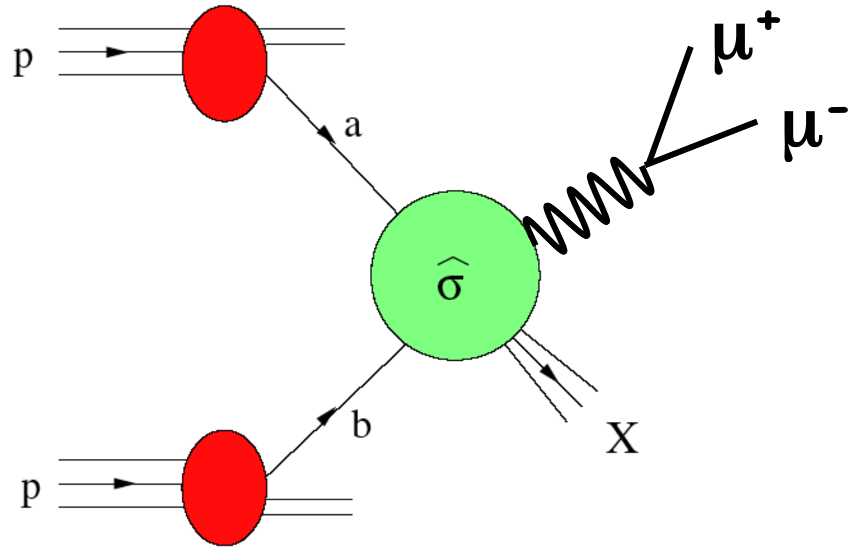
This talk:

Focus on some perturbative-QCD aspects

Outline:

- Introduction: Factorized hadronic scattering
- Threshold resummation
- Drell-Yan in πN scattering
- Drell-Yan with transverse momentum
- Conclusions

Introduction:
Factorized hadronic scattering



hard scale Q

$$Q^4 \frac{d\sigma}{dQ^2} = \sum_{ab} \int dx_a dx_b \, f_a(x_a,\mu) \, f_b(x_b,\mu) \, \omega_{ab}\left(z = \frac{Q^2}{\hat{s}}, \alpha_s(\mu), \frac{Q}{\mu}\right) + \ldots$$

universal pdfs

partonic hard scatt.
perturbative QCD

$\mu \sim Q$ fact./ren. scale

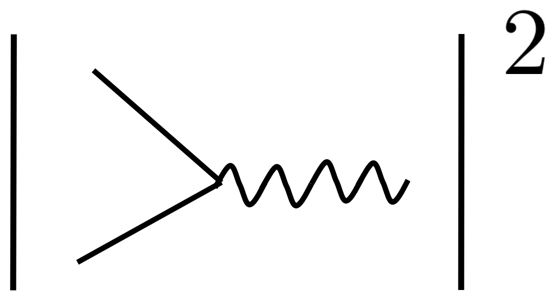
$$\omega_{ab} = \omega_{ab}^{(\mathrm{LO})} + \frac{\alpha_s}{2\pi}\,\omega_{ab}^{(\mathrm{NLO})} + \ldots$$

(up to power corrections $1/Q^2$)

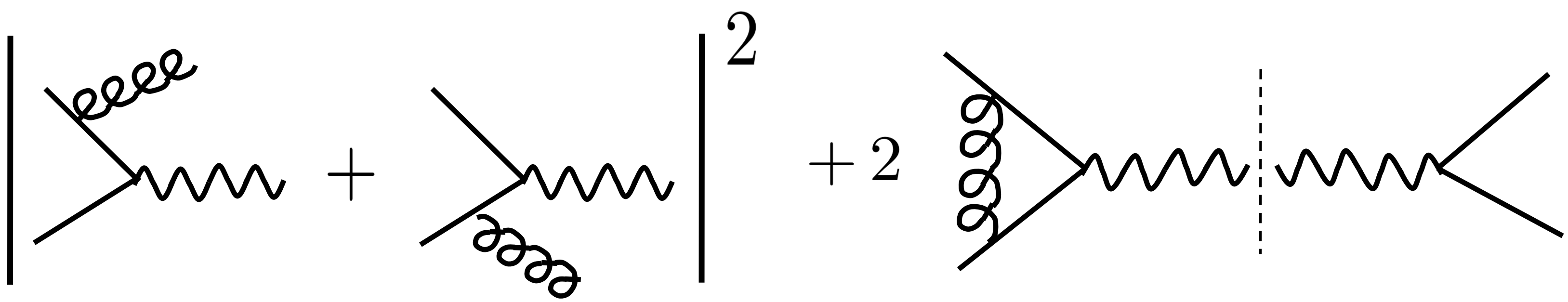
$$\omega_{ab} = \omega_{ab}^{(\mathrm{LO})} + \frac{\alpha_s}{2\pi}\,\omega_{ab}^{(\mathrm{NLO})} + \left(\frac{\alpha_s}{2\pi}\right)^2 \omega_{ab}^{(\mathrm{NNLO})} + \cdots$$

	Unpol.	Long. pol.	Trans. pol.
NLO	Kubar et al. Altarelli, Ellis,Martinelli Harada et al.	Ratcliffe Weber Gehrmann Kamal de Florian, WV	Weber, WV WV Contogouris et al. Barone et al.
NNLO	Hamberg, van Neerven, Matsuura Harlander, Kilgore Anastasiou, Dixon, Melnikov, Petriello Catani, Cieri, Ferrera, de Florian, Grazzini	Smith, v.Neerven, Ravindran	

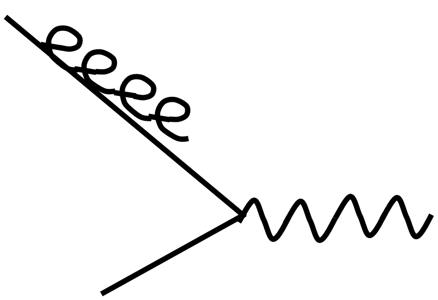
LO:



NLO:



Real + Virtual: IR singularities cancel



Collinear singularity

→ factorization into PDFs

→ scheme dependence

LO:

$$\omega_{q\bar{q}}^{(\mathrm{LO})} \sim \delta(1-z) \qquad z = \frac{Q^2}{\hat{s}}$$

NLO:

$$\omega_{q\bar{q}}^{(\mathrm{NLO})} \sim \frac{\alpha_s}{2\pi} C_F \left[4(1+z^2) \left(\frac{\ln(1-z)}{1-z} \right)_+ - 2\frac{1+z^2}{1-z} \ln z \right.$$

$$\left. + \left(\frac{2}{3}\pi^2 - 8 \right) \delta(1-z) + 2P_{qq}(z) \ln \frac{Q^2}{\mu^2} \right]$$

$$\int_0^1 dz\, f(z) \left(\frac{\ln(1-z)}{1-z} \right)_+ \equiv \int_0^1 dz\, (f(z) - f(1)) \frac{\ln(1-z)}{1-z}$$

- DGLAP evolution:

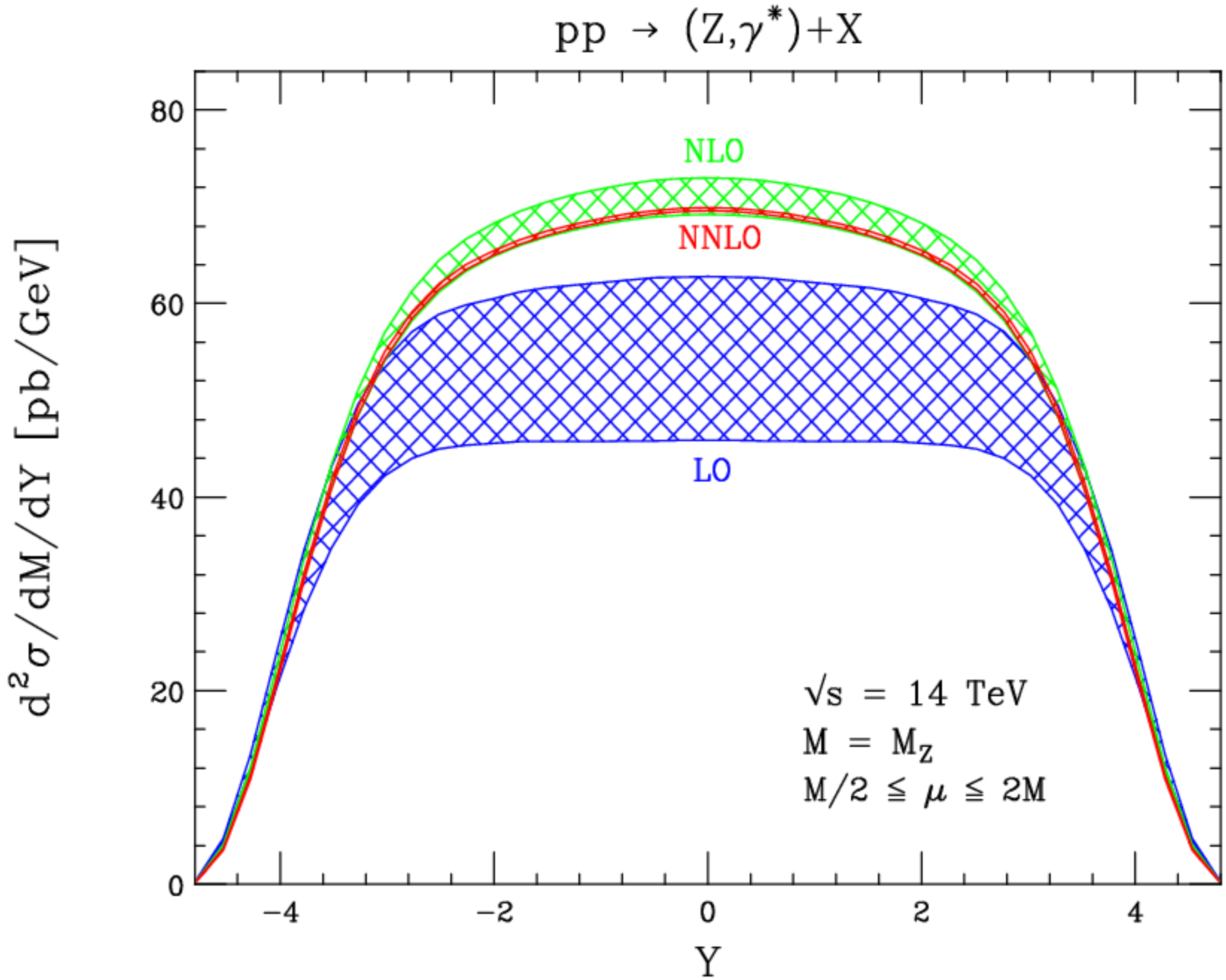
$$\mu^2 \frac{\mathrm{d}}{\mathrm{d}\mu^2} \begin{pmatrix} q(x,\mu^2) \\ g(x,\mu^2) \end{pmatrix} = \int_x^1 \frac{dz}{z} \begin{pmatrix} \mathcal{P}_{qq} & \mathcal{P}_{qg} \\ \mathcal{P}_{gq} & \mathcal{P}_{gg} \end{pmatrix} \begin{pmatrix} q \\ g \end{pmatrix} \left(\frac{x}{z},\mu^2\right)$$

$$\mathcal{P}_{ij} = \frac{\alpha_s}{2\pi} \mathcal{P}_{ij}^{\mathrm{LO}} + \left(\frac{\alpha_s}{2\pi}\right)^2 \mathcal{P}_{ij}^{\mathrm{NLO}} + \left(\frac{\alpha_s}{2\pi}\right)^3 \mathcal{P}_{ij}^{\mathrm{NNLO}} + \ldots$$

Ahmed,Ross
Altarelli,Parisi,...

Curci, Furmanski,
Petronzio
Antoniadis,Kounnas,
Lacaze
Mertig, van Neerven
WV
Kumano et al.
Koike et al.
WV

Moch,Vermaseren,
Vogt, Rogal



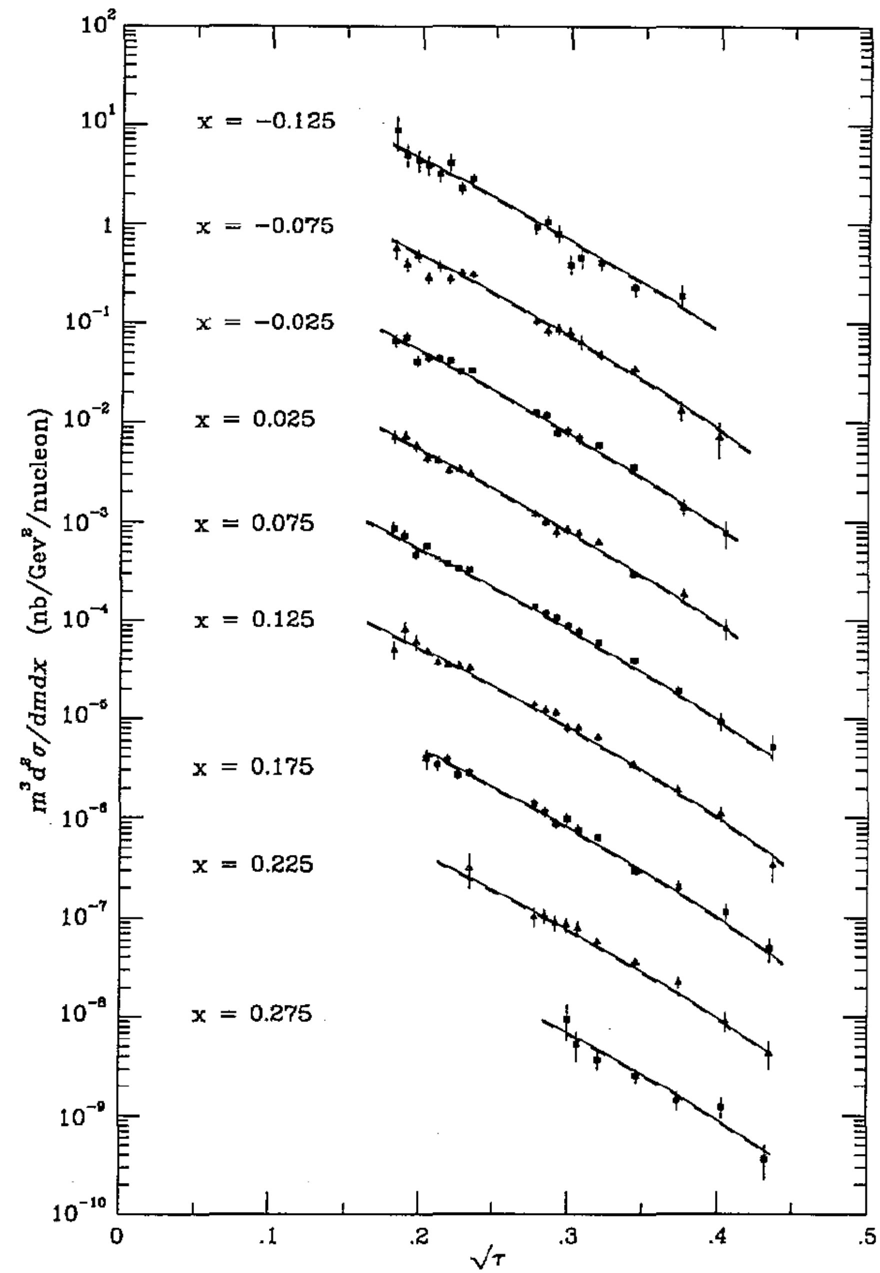
Anastasiou et al.

NLO quite successful overall for inclusive DY:

E605 (800 GeV pC)

NLO scaled by 1.071

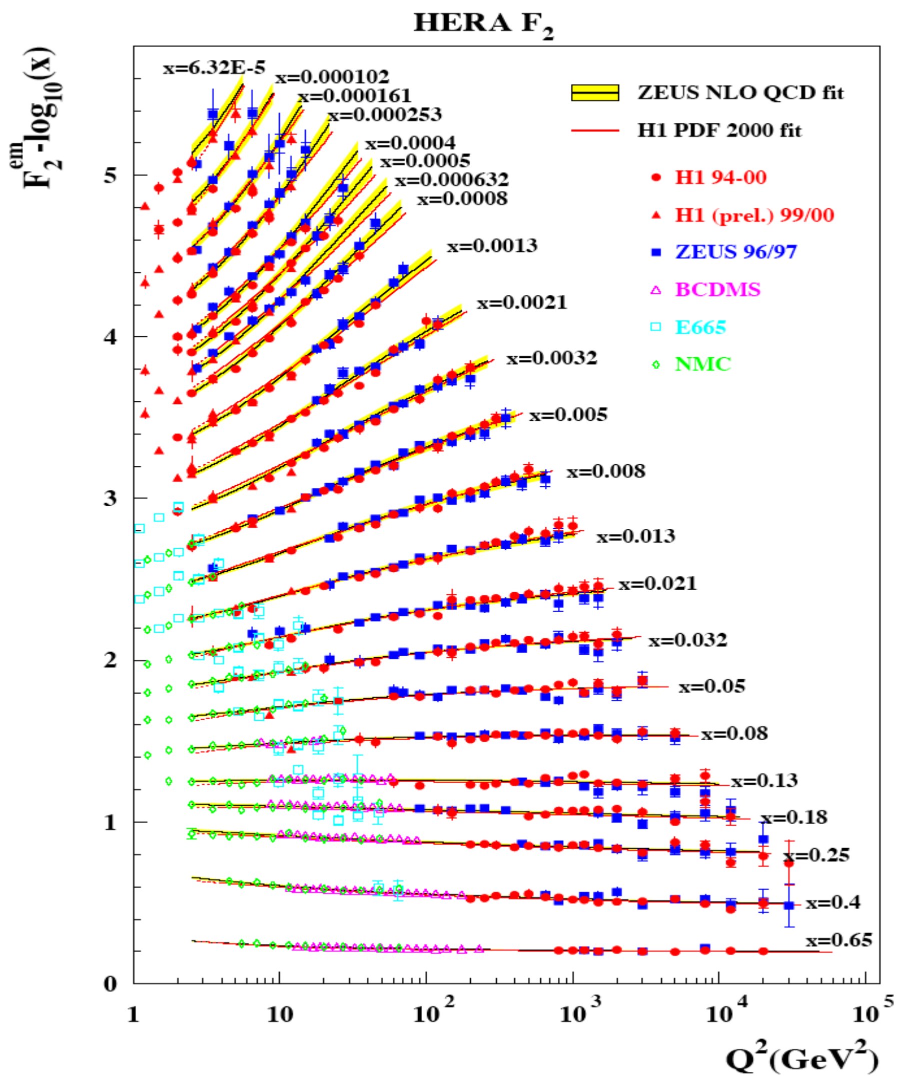
$$x = \frac{2p_L}{\sqrt{S}}$$

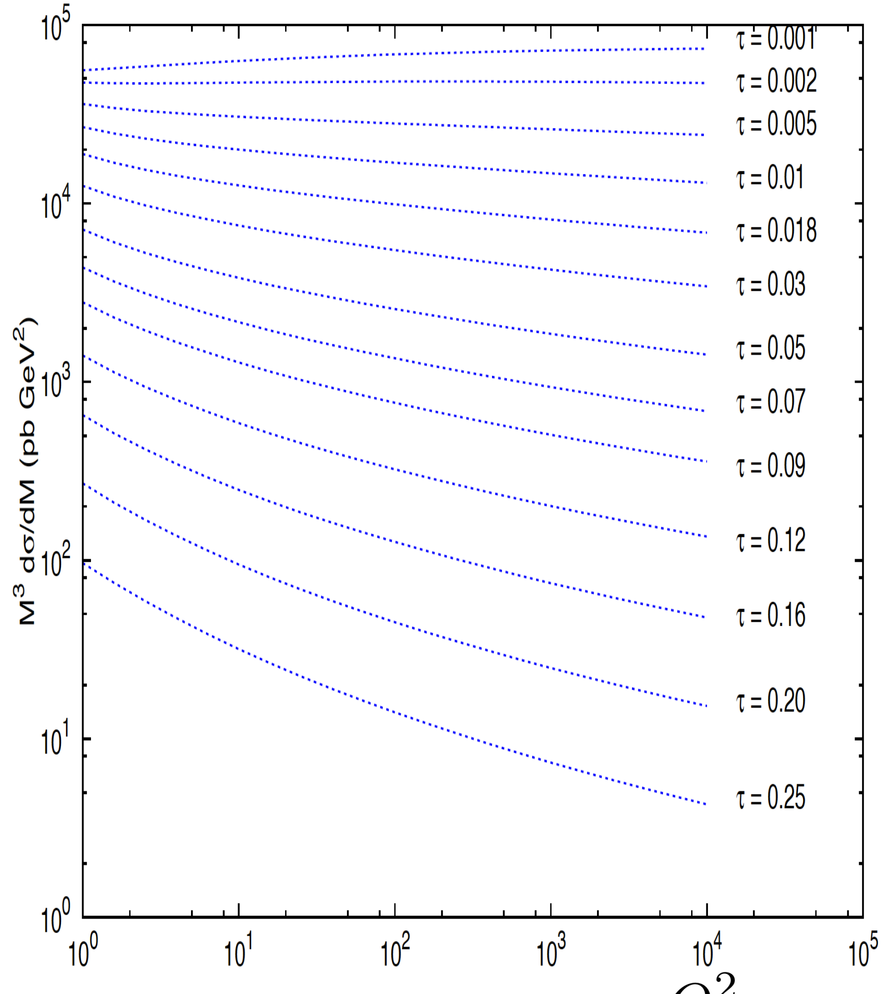


Stirling/Whalley

$$Q^3 \frac{d\sigma}{dQ}$$

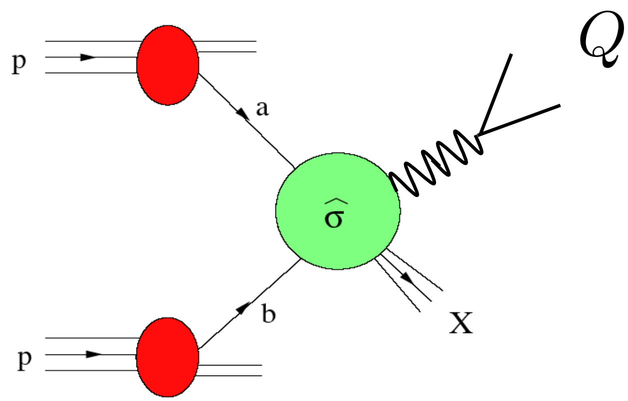
scaling violations
in DY at NLO



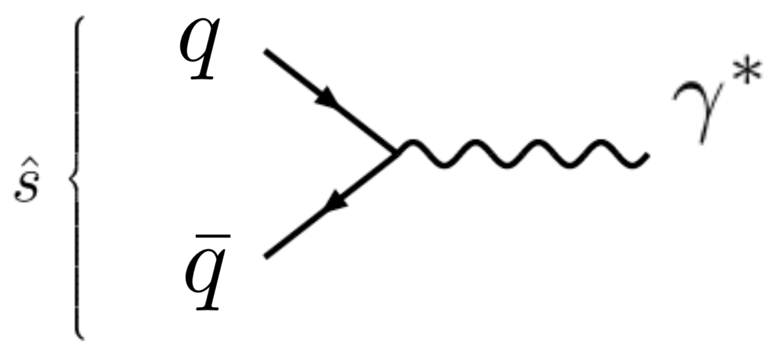


(M.Aicher)

Threshold resummation



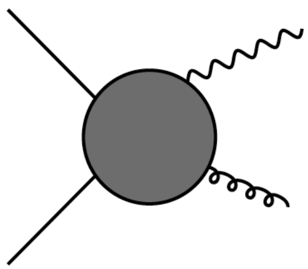
LO :



$$z = \frac{Q^2}{\hat{s}}$$

$$\omega_{q\bar{q}}^{(\mathrm{LO})} \propto \delta(1-z)$$

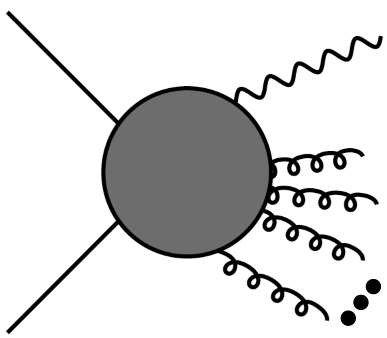
- NLO correction:



$$z \;\longrightarrow\; 1:$$

$$\omega_{q\bar{q}}^{(\mathrm{NLO})} \;\propto\; \alpha_s \left(\frac{\log(1-z)}{1-z}\right)_+ + \ldots$$

- higher orders:



$$\omega_{q\bar{q}}^{(\mathrm{N^kLO})} \;\propto\; \alpha_s^k \left(\frac{\log^{2k-1}(1-z)}{1-z}\right)_+ + \ldots$$

"threshold logarithms"

- for $z \to 1$ real radiation inhibited

- logs emphasized by parton distributions :

$$d\sigma \sim \int_{\tau}^{1} \frac{dz}{z}\, \mathcal{L}_{q\bar{q}}\left(\frac{\tau}{z}\right) \omega_{q\bar{q}}(z) \qquad \tau = \frac{Q^2}{S}$$

z = 1 relevant,
in particular as $\tau \to 1$

- logs more relevant at lower hadronic energies

- large logs may spoil perturbative series, unless taken into account to all orders

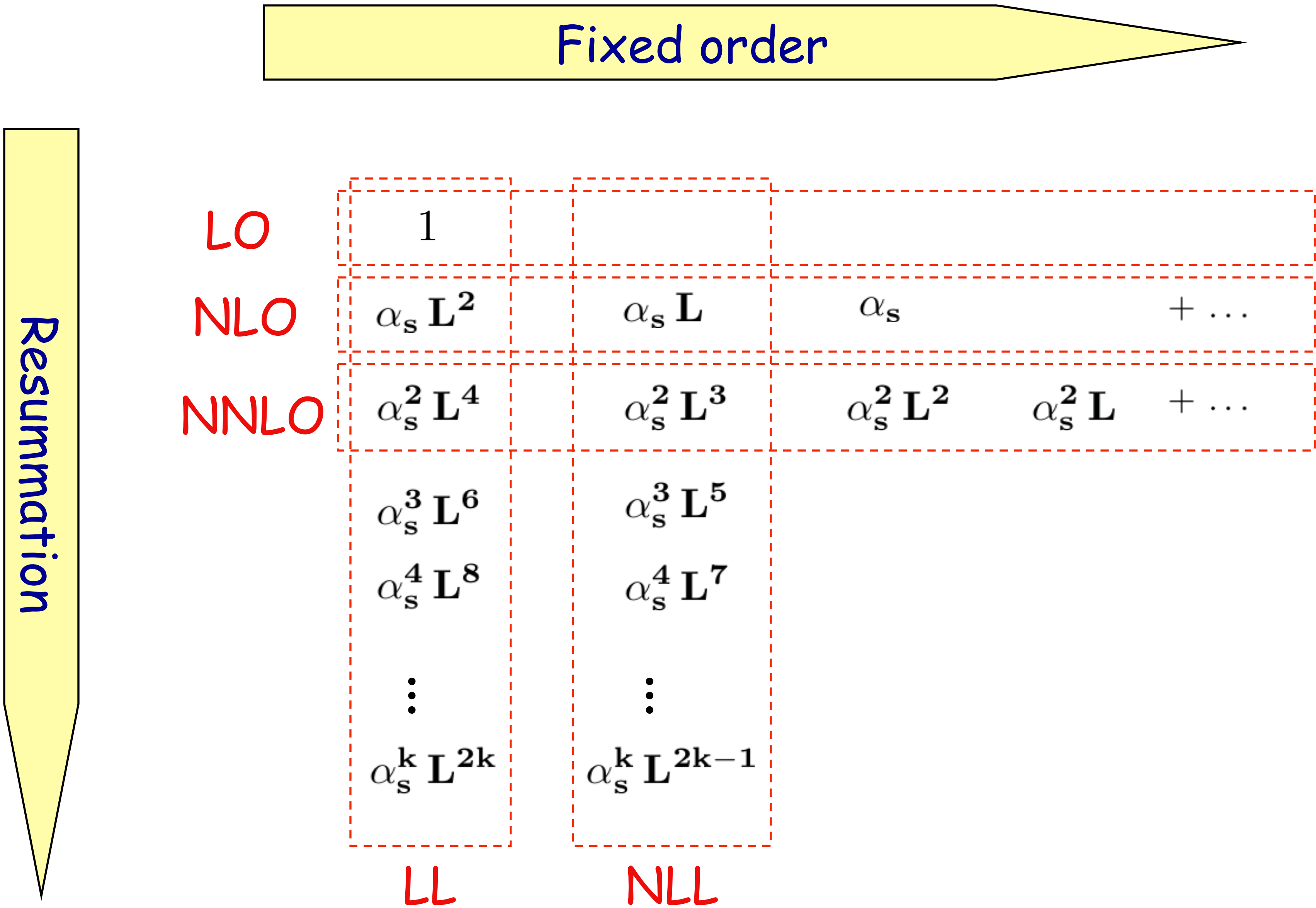
  = (Threshold) Resummation !

- particularly relevant for (lower-energy) fixed-target

- work began in the '80s with Drell-Yan process

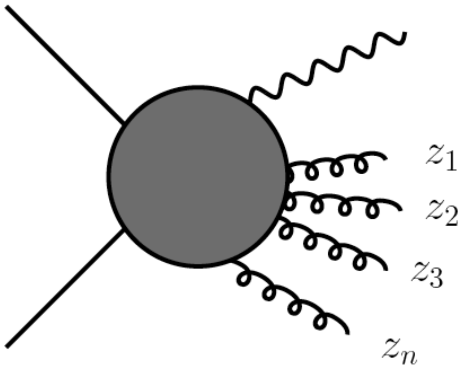
  Sterman; Catani, Trentadue

  various new techniques: Laenen, Sterman, WV
  Forte, Ridolfi; Becher, Neubert
  van Neerven, Smith, Ravindran
  Laenen, Magnea


Fixed order
Resummation
LO
NLO
NNLO
1
$\alpha_s L^2$
$\alpha_s L$
α_s
$+ \cdots$
$\alpha_s^2 L^4$
$\alpha_s^2 L^3$
$\alpha_s^2 L^2$
$\alpha_s^2 L$
$+ \cdots$
$\alpha_s^3 L^6$
$\alpha_s^3 L^5$
$\alpha_s^4 L^8$
$\alpha_s^4 L^7$
$\vdots$
$\vdots$
$\alpha_s^k L^{2k}$
$\alpha_s^k L^{2k-1}$
LL
NLL

Resummation relies on:

- factorization of QCD matrix elements in soft limit
- and of phase space when integral transform is taken:



$$\delta\left(1 - z - \sum_{i=1}^{n} z_i\right) = \frac{1}{2\pi i}\int_C dN\, \mathrm{e}^{N\left(1-z-\sum_{i=1}^{n} z_i\right)}$$

$$z_i = \frac{2E_i}{\sqrt{\hat{s}}}$$

$\overline{\text{MS}}$ scheme

$$\tilde{\omega}_{q\bar{q}}^{(\mathrm{res})}(N) \propto \exp\left[2\int_0^1 dy\, \frac{y^N - 1}{1-y}\int_{\mu^2}^{Q^2(1-y)^2} \frac{dk_\perp^2}{k_\perp^2}\, A_q\left(\alpha_s(k_\perp^2)\right) + \ldots\right]$$

$$A_q(\alpha_s) = C_F\left\{\frac{\alpha_s}{\pi} + \left(\frac{\alpha_s}{\pi}\right)^2\left[\frac{C_A}{2}\left(\frac{67}{18} - \zeta(2)\right) - \frac{5}{9}T_R n_f\right] + \ldots\right\}$$

- logs enhance cross section !

LL :

$$\tilde{\omega}_{q\bar{q}}^{(\mathrm{res})}(N) \;\propto\; \exp\left[+\frac{2C_F}{\pi}\,\alpha_s \ln^2 N + \ldots\right]$$

NLL :

Catani, Mangano, Nason, Trentadue

$$\tilde{\omega}_{q\bar{q}}^{(\mathrm{res})}(N) \;\propto\; \exp\left\{\underbrace{2\ln\bar{N}\; h^{(1)}(\lambda)}_{\text{LL}} + \underbrace{2h^{(2)}\left(\lambda, \frac{Q^2}{\mu^2}\right)}_{\text{NLL}}\right\}$$

$$\lambda = \alpha_s(\mu^2)\, b_0\, \log(N e^{\gamma_E})$$

$$h^{(1)}(\lambda) \;=\; \frac{A_q^{(1)}}{2\pi b_0 \lambda}\left[2\lambda + (1-2\lambda)\ln(1-2\lambda)\right] \qquad h^{(2)} \;=\; \ldots$$

Note,

$$\tau = \frac{Q^2}{S}$$

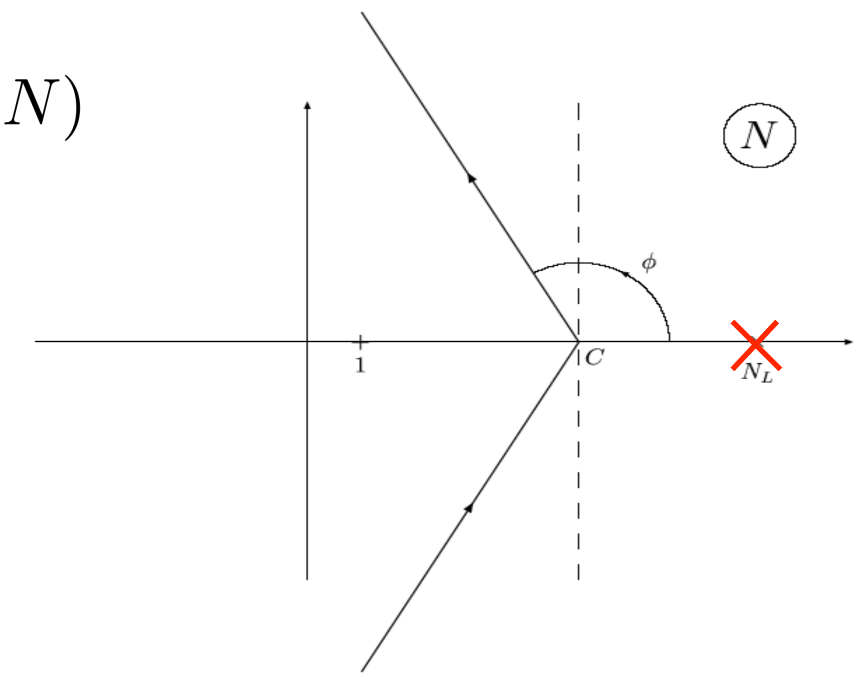
$$\int_0^1 d\tau\, \tau^{N-1} \frac{d\sigma}{dQ^2} \propto \sum_{ab} \left(\int_0^1 dx_a\, x_a^N\, f_a \right) \left(\int_0^1 dx_b\, x_b^N\, f_b \right) \tilde{\omega}_{ab}(N)$$

Inverse transform:

$$\sigma^{\rm res} = \frac{1}{2\pi i} \int_{C-i\infty}^{C+i\infty} dN\, \tau^{-N}\; \tilde{\sigma}^{\rm res}(N)$$

"Minimal prescription"

Catani, Mangano, Nason, Trentadue

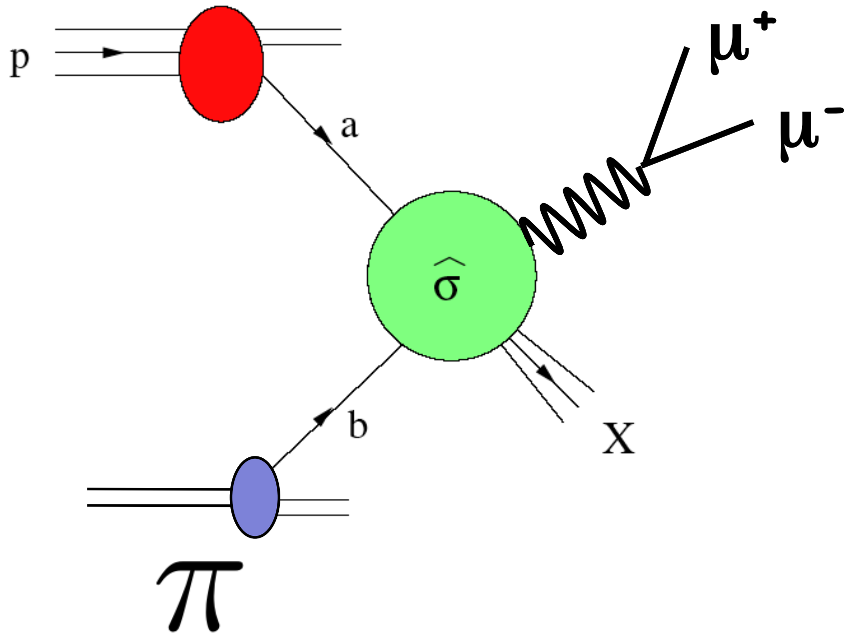


Drell-Yan process in πN scattering

M. Aicher, A.Schäfer, WV

- Drell-Yan process has been main source of information on pion structure:

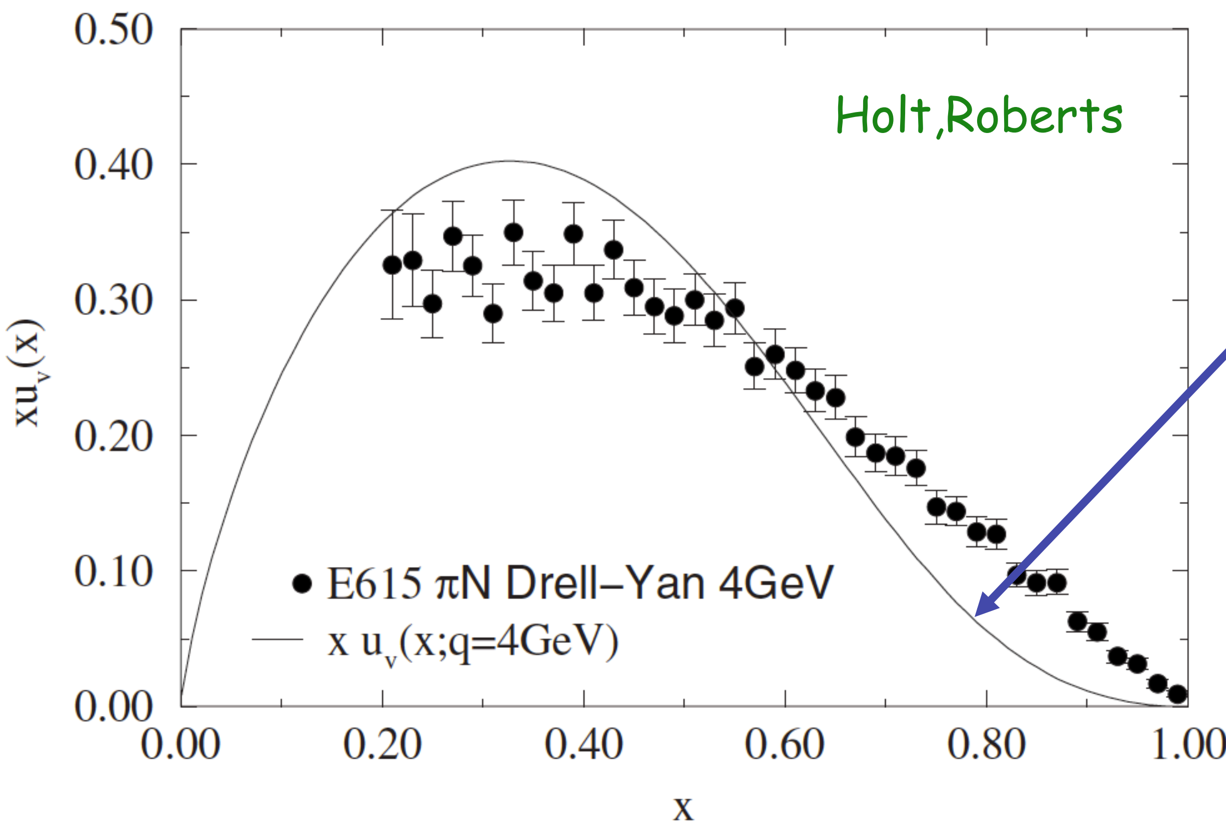
E615, NA10



$$d\sigma = \sum_{ab} \int dx_a \int dx_b \, f_a^{\pi}(x_a, \mu) f_b(x_b, \mu) \, d\hat{\sigma}_{ab}(x_a P_a, x_b P_b, Q, \alpha_s(\mu), \mu)$$

- Kinematics such that data mostly probe valence region: ~200 GeV pion beam on fixed target

• LO extraction of u_v from E615 data: $\sqrt{S} = 21.75\,\mathrm{GeV}$



$\sim\ (1-x)^2$

QCD counting rules

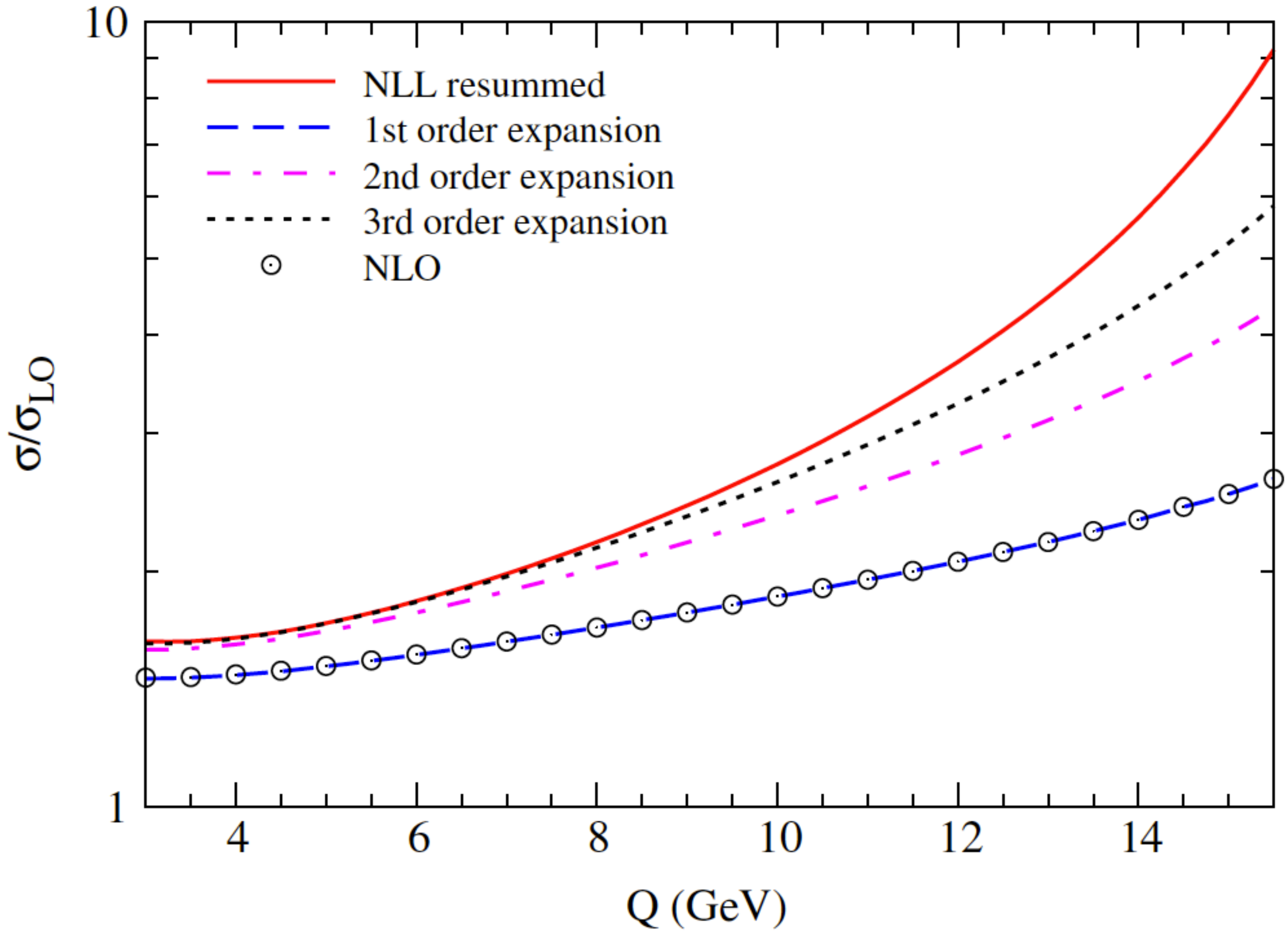
Farrar, Jackson; Berger, Brodsky; Yuan Blankenbecler, Gunion, Nason

Dyson-Schwinger

Hecht et al.

(Compass kinematics)

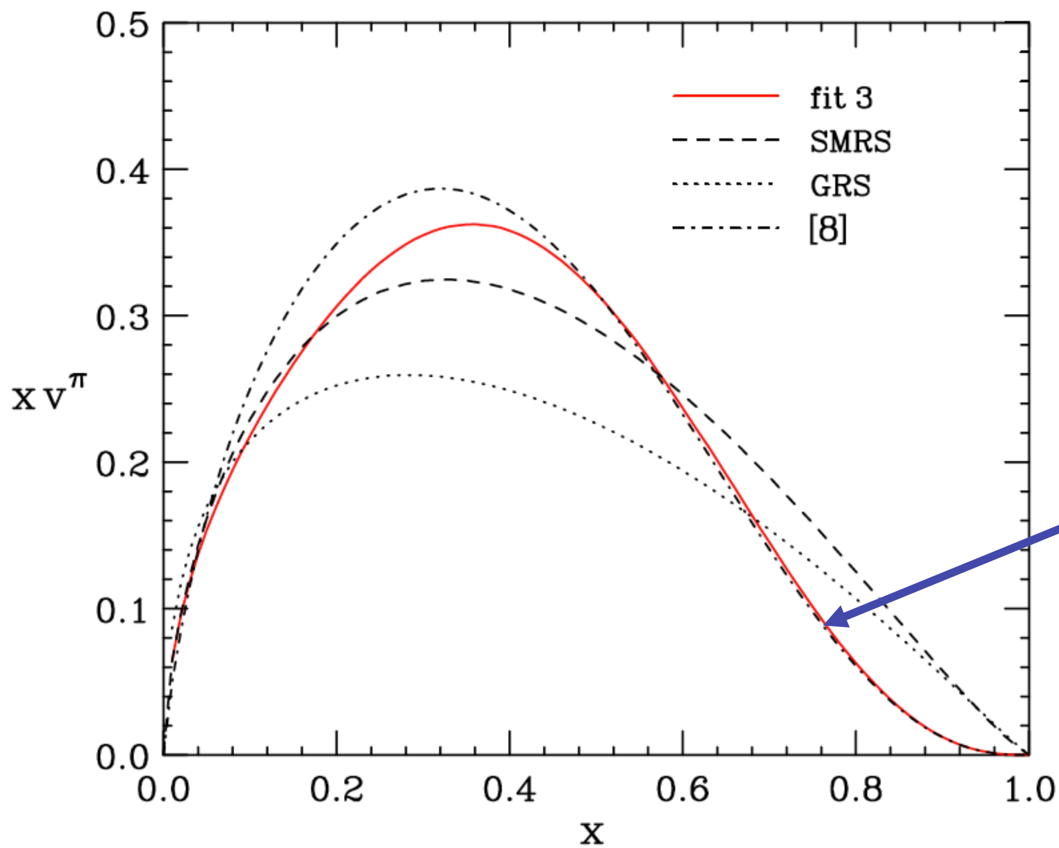
$\sqrt{S} = 19\,\mathrm{GeV}$



Aicher, Schäfer, WV
(earlier studies: Shimizu, Sterman, WV, Yokoya)

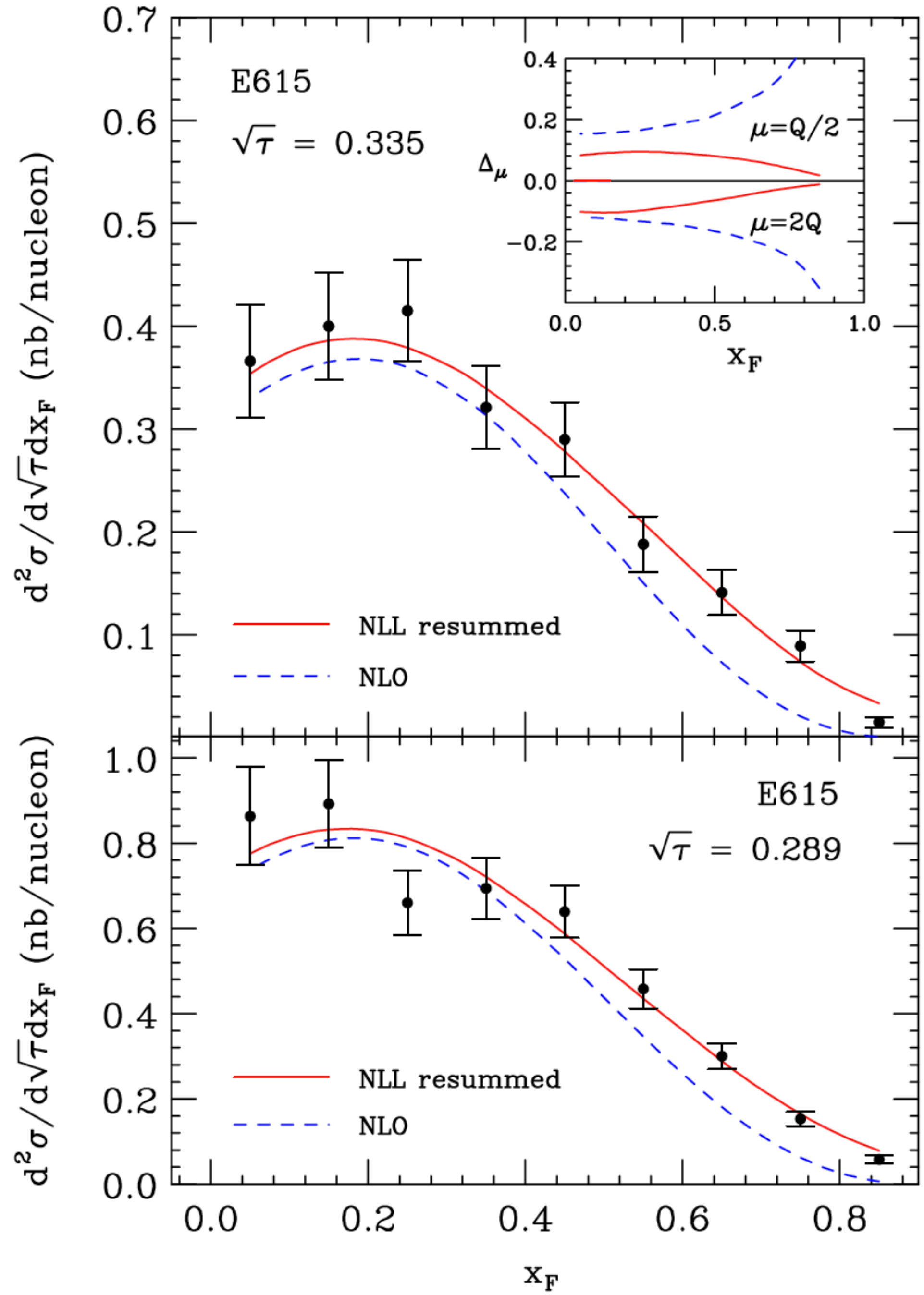
$$xv^\pi(x, Q_0^2) = N_v x^\alpha (1-x)^\beta (1+\gamma x^\delta)$$

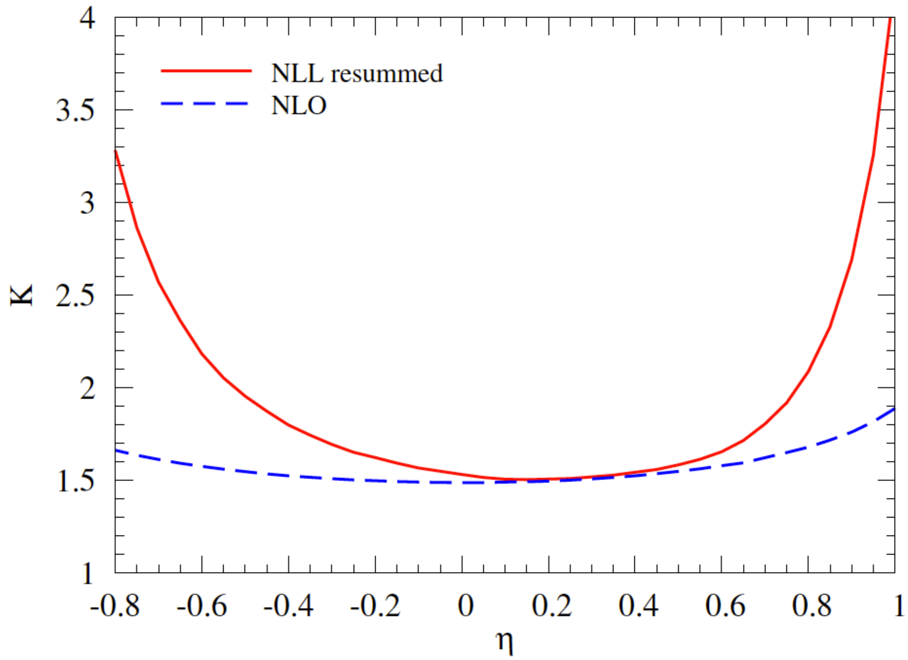
Fit	$2\langle xv^\pi \rangle$	α	β	γ	K	χ^2 (no. of points)
1	0.55	0.15 ± 0.04	1.75 ± 0.04	89.4	0.999 ± 0.011	82.8 (70)
2	0.60	0.44 ± 0.07	1.93 ± 0.03	25.5	0.968 ± 0.011	80.9 (70)
3	0.65	0.70 ± 0.07	2.03 ± 0.06	13.8	0.919 ± 0.009	80.1 (70)
4	0.7	1.06 ± 0.05	2.12 ± 0.06	6.7	0.868 ± 0.009	81.0 (70)



$Q = 4$ GeV

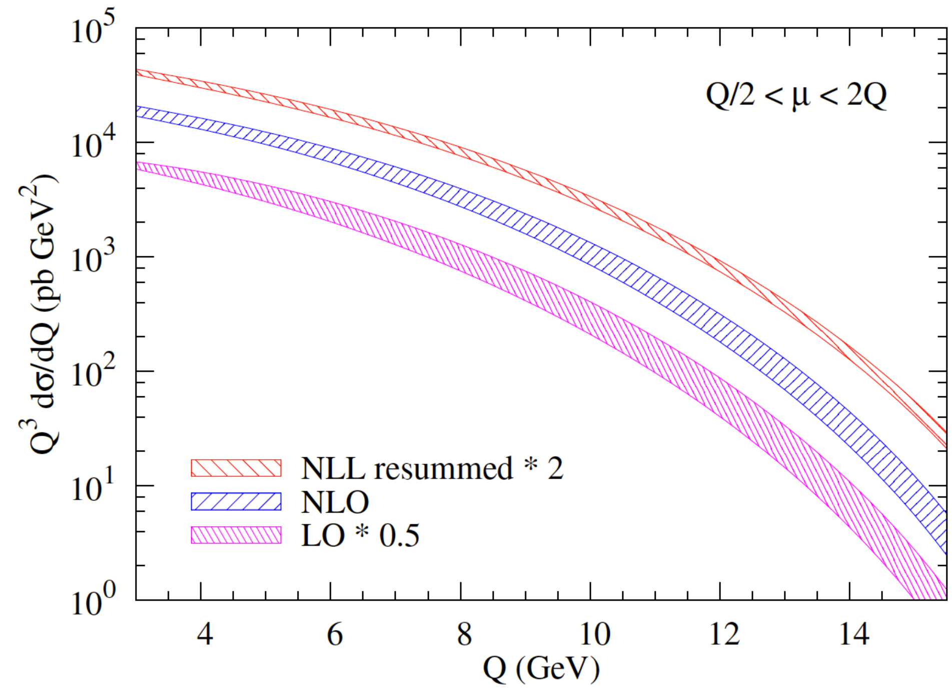
$\sim (1-x)^{2.34}$


E615
$\sqrt{\tau}$ = 0.335
Δ_μ
μ=Q/2
μ=2Q
x_F
NLL resummed
NLO
E615
$\sqrt{\tau}$ = 0.289
NLL resummed
NLO
$d^2\sigma/d\sqrt{\tau}dx_F$ (nb/nucleon)
x_F



NLL resummed
NLO
K
η

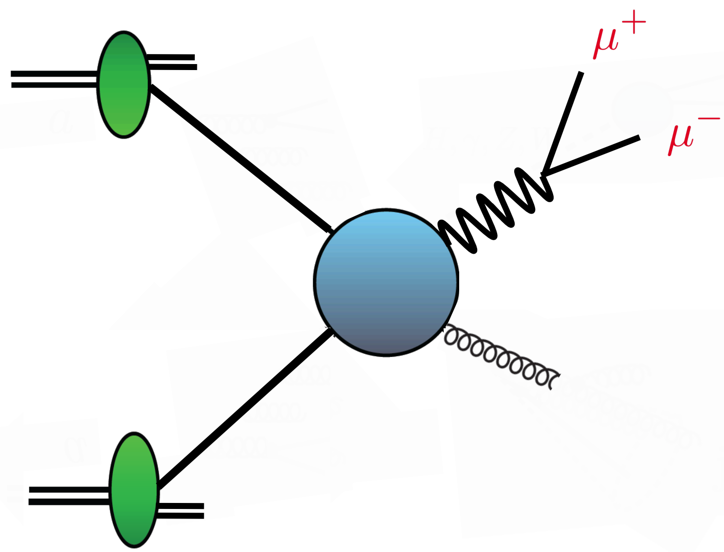
$$\sqrt{\tau} = 0.3$$



Q/2 < μ < 2Q
NLL resummed * 2
NLO
LO * 0.5
Q³ dσ/dQ (pb GeV²)
Q (GeV)

Drell-Yan with transverse momentum

(see talk by V. Barone)



$q_T \neq 0$

(measured !)

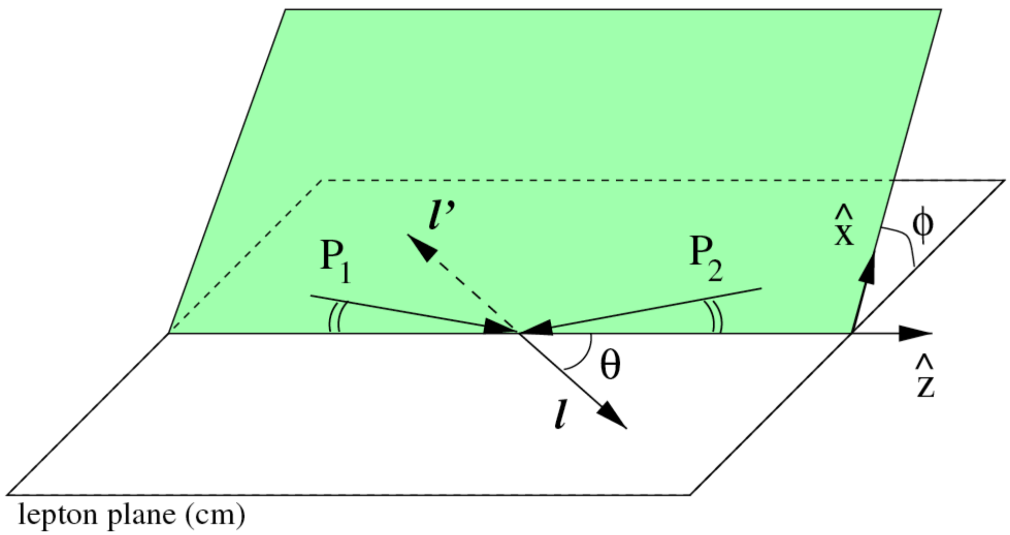
- more refined aspects of QCD hard scattering: TMD factorization

- (transverse) spin phenomena / nucleon TMD structure Sivers, Boer-Mulders, non-universality

$$\frac{d\sigma}{d\Omega d^4q} = \frac{\alpha^2}{2(2\pi)^4 Q^2 s^2}\{W_T(1+\cos^2\theta) + W_L(1-\cos^2\theta)$$

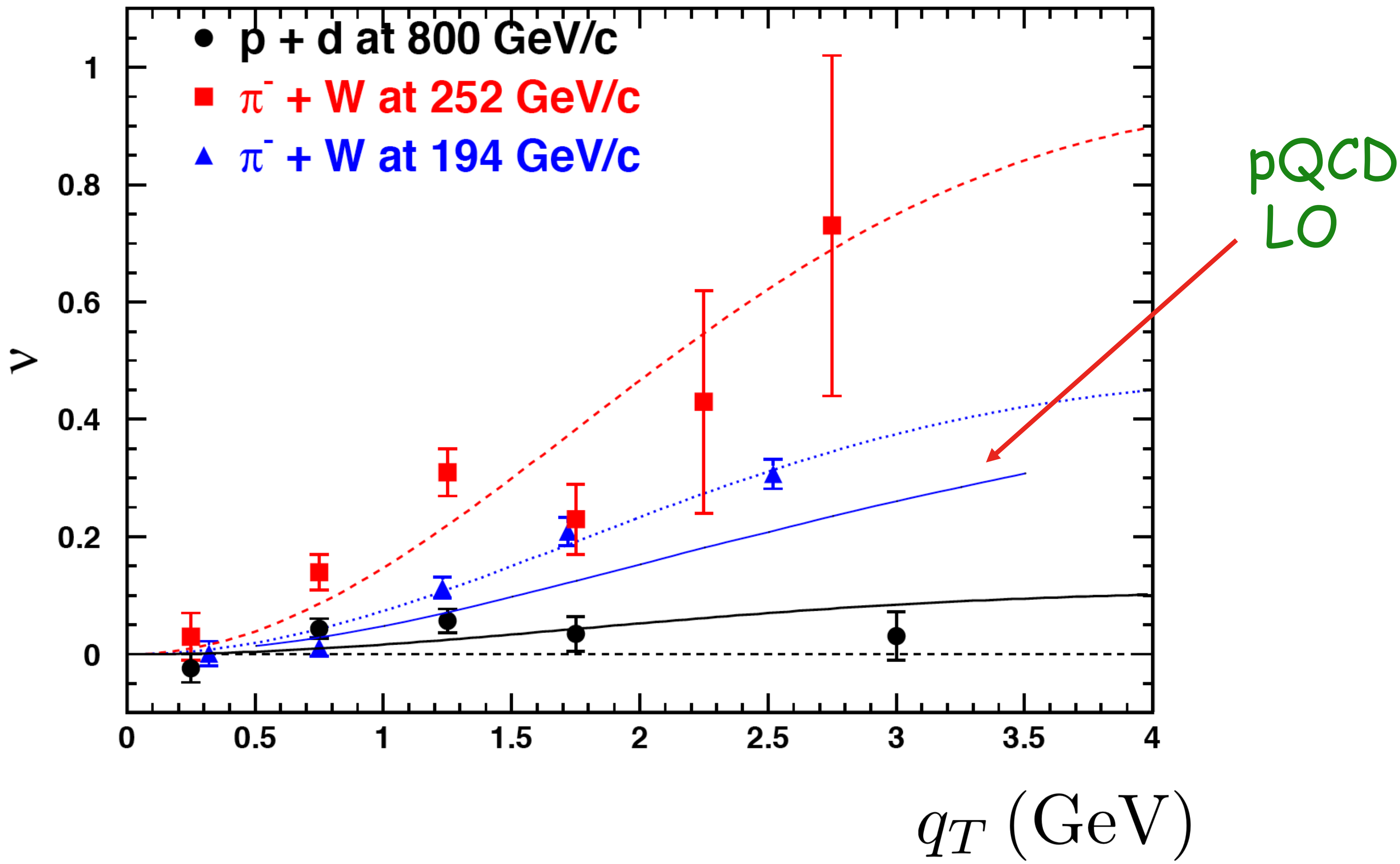
$$+ W_\Delta \sin 2\theta \cos\phi + W_{\Delta\Delta}\sin^2\theta \cos 2\phi\}$$

Lam, Tung; Collins

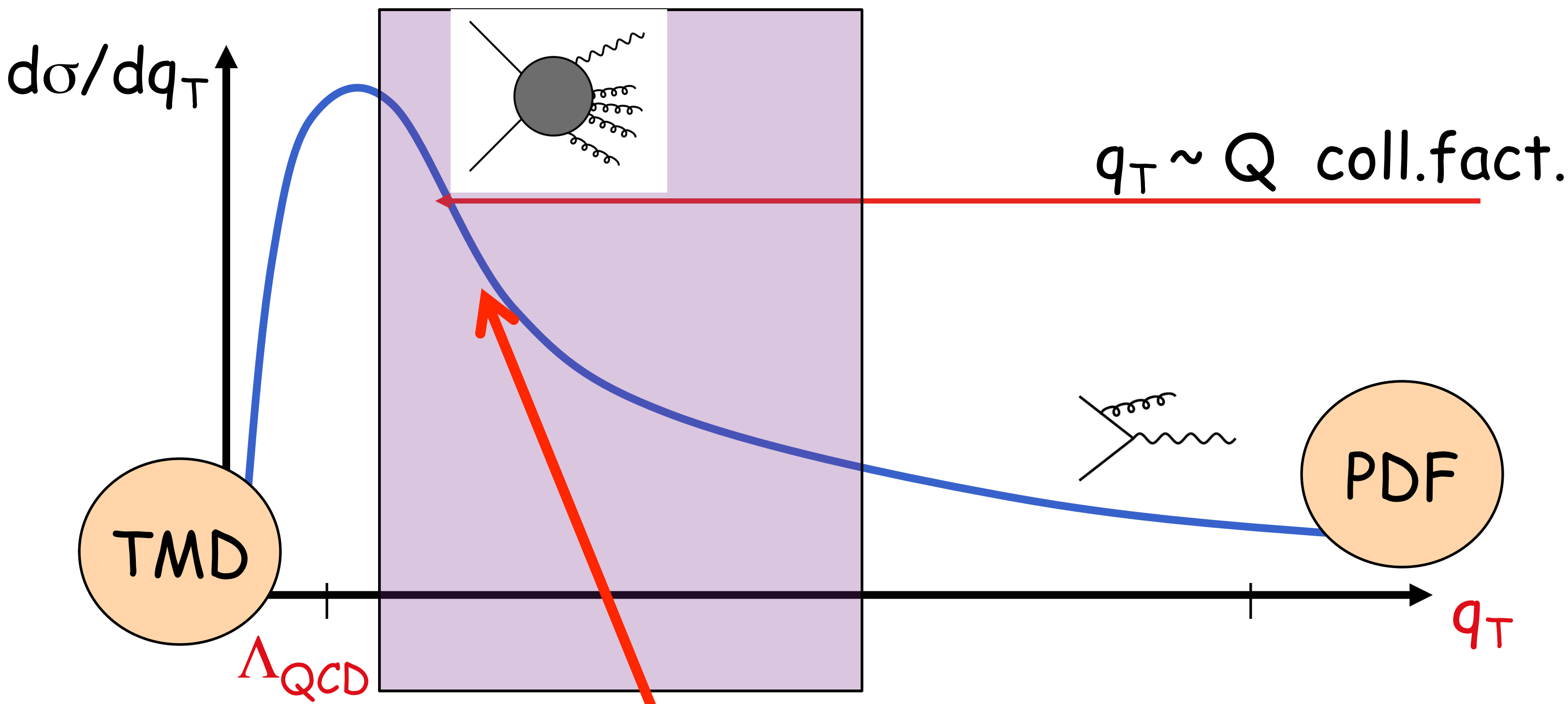


$$1 - \lambda - 2\nu = 0$$

Lam-Tung relation



Boer, WV;
Berger, Qiu, Rodriguez-Pedraza



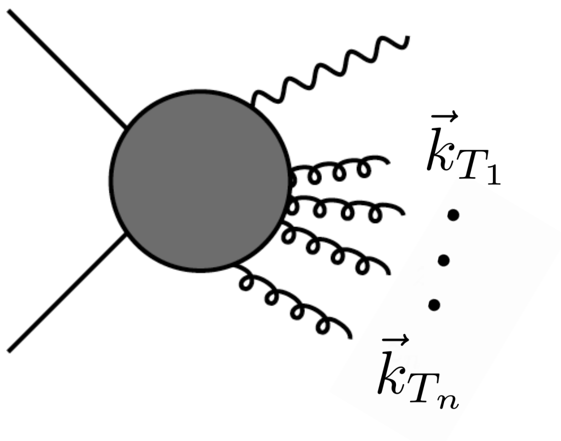
well-known feature: emergence of Sudakov logarithms

$$\alpha_s^k \frac{\log^{2k-1}\left(\frac{Q^2}{q_T^2}\right)}{q_T^2} + \dots$$

- these logs are related (although not identical) to the threshold logs

- all-order resummation for spin-av. cross section understood for long time

Collins, Soper, Sterman; ...



$$\delta^2\left(\vec{q}_T+\sum_i \vec{k}_{T_i}\right) = \frac{1}{(2\pi)^2}\int d^2b\, \mathrm{e}^{-i\vec{b}\cdot\left(\vec{q}_T+\sum_i \vec{k}_{T_i}\right)}$$

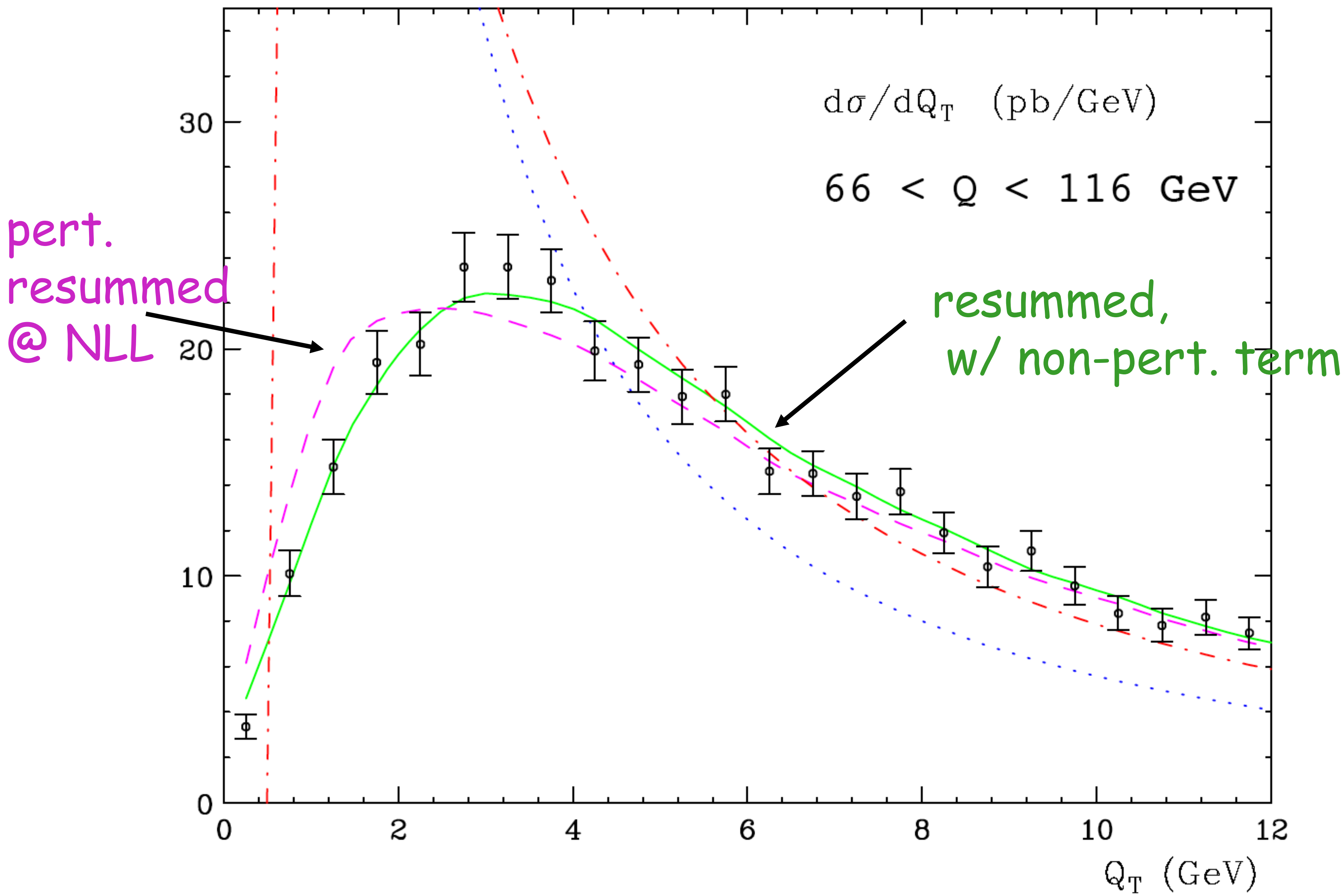
$$\frac{\log^{2k-1}\left(\frac{Q^2}{q_T^2}\right)}{q_T^2} \quad \leftrightarrow \quad \log^{2k}(bQ)$$

$$\hat{\sigma}^{(\mathrm{resum})}(b) \;\propto\; \exp\left[2\int_0^{Q^2}\frac{dk_\perp^2}{k_\perp^2}\left(J_0(bk_\perp)-1\right)\left\{A_q(\alpha_s(k_\perp^2))\,\log\frac{Q^2}{k_\perp^2}+\ldots\right\}\right]$$

(Sudakov exponent)

- logs suppress cross section !

Kulesza, Sterman, WV



- great strides forward recently on connection to scale evolution of TMDs and resummation formalism for single-spin observables

Aybat,Collins,Qiu,Rogers
Kang,Xiao,Yuan
Aybat,Prokudin,Rogers
Anselmino,Boglione,Melis

- can be formulated to give evolution of TMDs in terms of

$$\frac{d\sigma}{dq_\perp^2} \sim \sigma_0 \int d^2k_{\perp,1} \int d^2k_{\perp,2}\, F(x_1, k_{\perp,1}, Q)\, \bar{F}(x_2, k_{\perp,2}, Q)\, \delta^{(2)}(\vec{k}_{\perp,1} + \vec{k}_{\perp,2} - \vec{q}_\perp)$$

Mert Aybat,Rogers,
Collins,Qiu
Kang,Xiao,Yuan

- evolved TMD in b-space:

$$
\begin{aligned}
F(x,b,Q) \;=\; & F(x,b,Q_0) \times \\
& \times \exp\left\{ \ln\frac{Q}{Q_0}\tilde{K}(b_*;\mu_b) + \int_{Q_0}^{Q}\frac{d\mu'}{\mu'}\left[\gamma_F(g(\mu');1) \;-\ln\frac{Q}{\mu'}\gamma_K(g(\mu'))\right]\right. \\
& \left. + \int_{Q_0}^{\mu_b}\frac{d\mu'}{\mu'}\ln\frac{Q}{Q_0}\gamma_K(g(\mu')) - g_K(b_T)\ln\frac{Q}{Q_0}\right\}
\end{aligned}
$$

non-pert.
piece

$$
\mu_b \sim 1/b_* \qquad b_* = \frac{b}{\sqrt{1+b^2/b_{\max}^2}}
$$

Reason for non-perturbative piece:

$$\hat{\sigma}^{(\mathrm{resum})}(b) \;\propto\; \exp\left[\frac{2C_F}{\pi} \int_0^{Q^2} \frac{dk_\perp^2}{k_\perp^2} \left(J_0(bk_\perp) - 1\right) \left\{ \alpha_s(k_\perp^2) \log \frac{Q^2}{k_\perp^2} + \ldots \right\} \right]$$

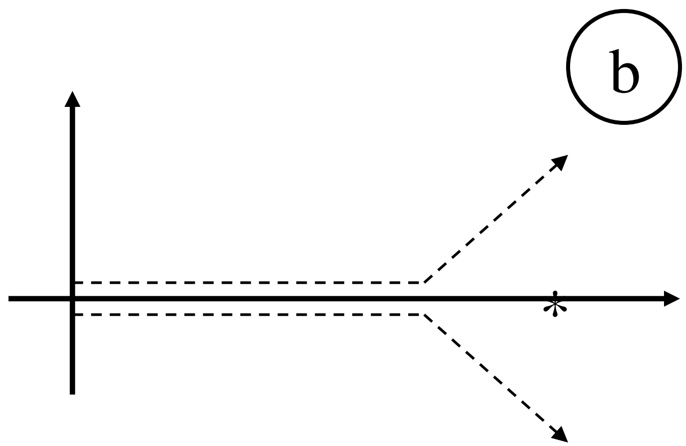
Logarithms are contained in

$$\exp\left[-\frac{2C_F}{\pi} \int_{1/b^2}^{Q^2} \frac{dk_\perp^2}{k_\perp^2} \left\{ \alpha_s(k_\perp^2) \log \frac{Q^2}{k_\perp^2} + \ldots \right\} \right]$$

→ need prescription for dealing with large-b

$$\int d^2b \, \mathrm{e}^{-i\vec{b}\cdot\vec{q}_T} \; [\ldots]$$

e.g. $\quad b^* \equiv \dfrac{b}{\sqrt{1 + b^2/b_{\max}^2}}$ or



Contribution from very low $k_\perp$

$$\exp\left[-b^2\,\frac{C_F}{\pi}\underbrace{\int dk_\perp^2\,\alpha_s(k_\perp^2)\,\log\left(\frac{Q}{k_\perp}\right)}_{g_1\;+\;g_2\,\log\left(\frac{Q}{Q_0}\right)}\right]$$

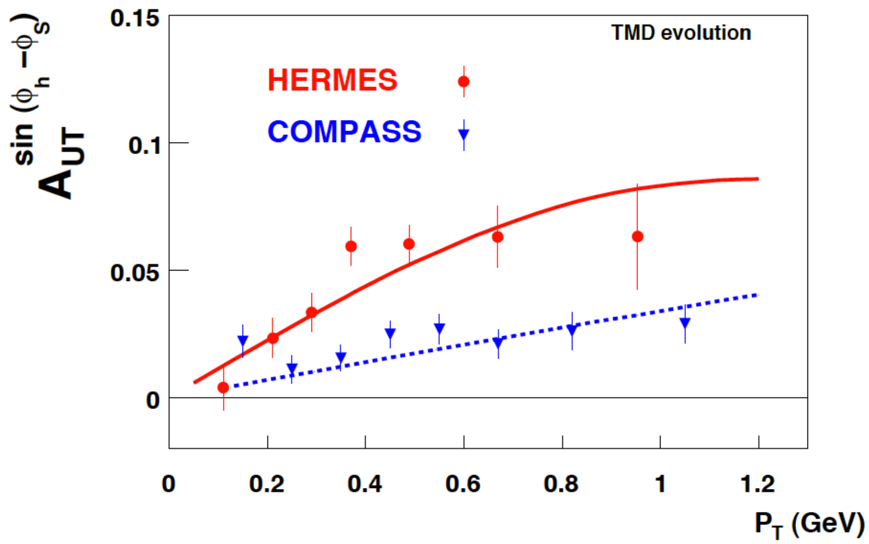
- suggests Gaussian non-pert. contribution with logarithmic Q dependence
- expected to be universal (unpol. <-> Sivers)
- "global" fits

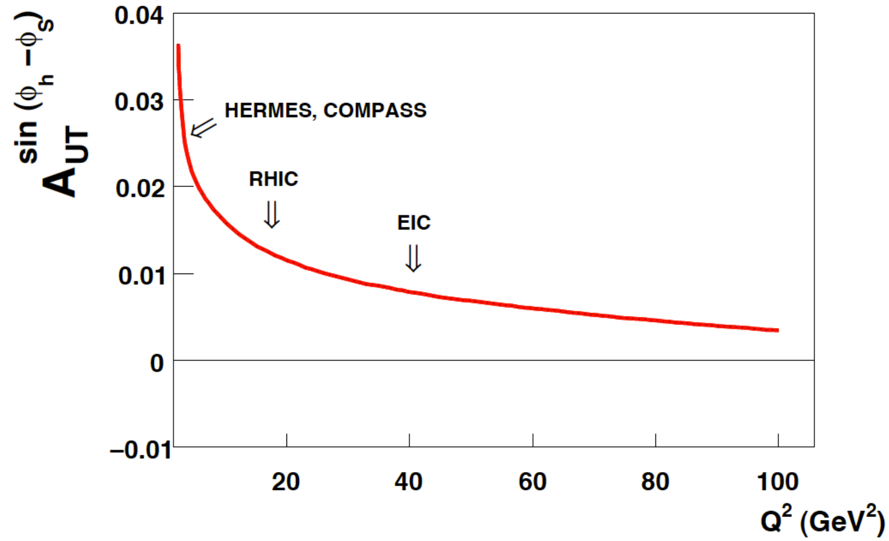
  Davies, Webber, Stirling; Landry et al., Ladinsky, Yuan; Qiu, Zhang; Konychev, Nadolsky

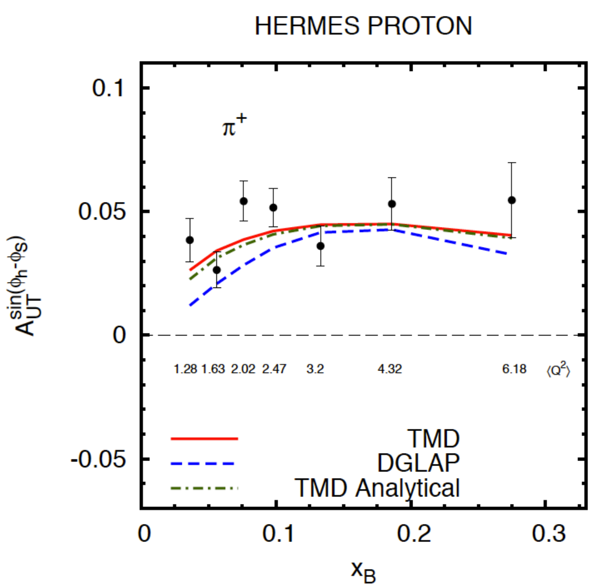
- values of g_1,g_2 depend on treatment of large-b region !

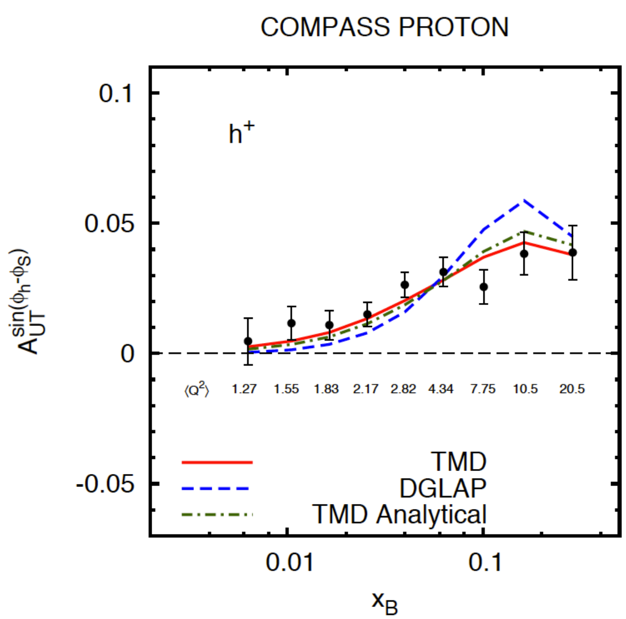
- initial phenomenology very encouraging

Aybat,Prokudin,Rogers



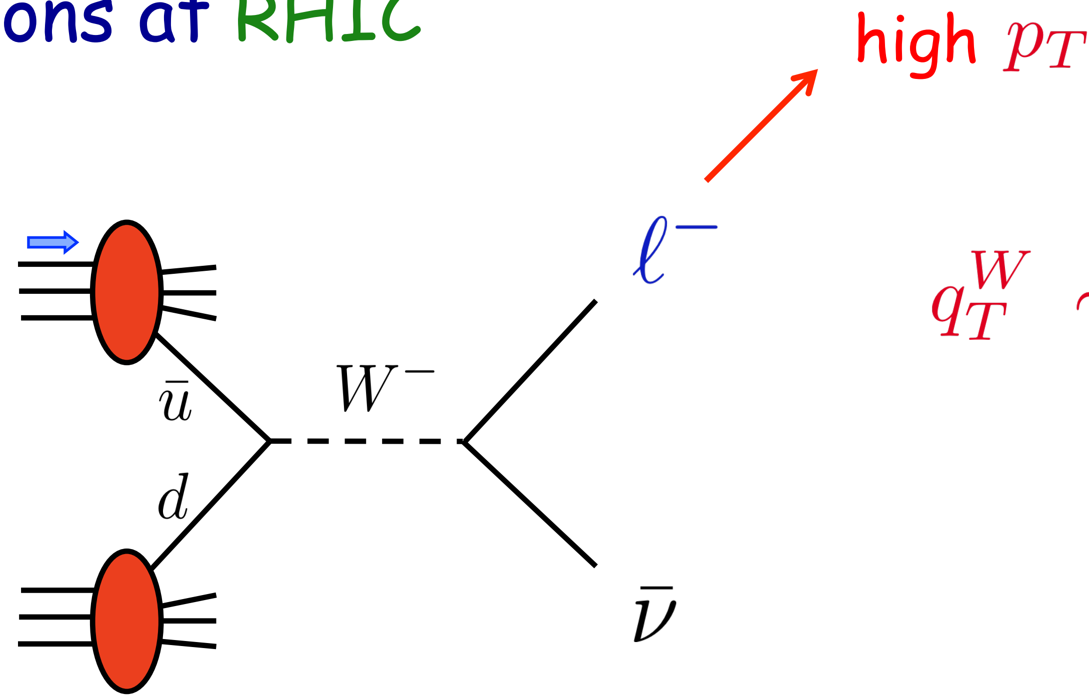






Anselmino, Boglione, Melis

W bosons at RHIC



$$q_T^W \sim 0$$

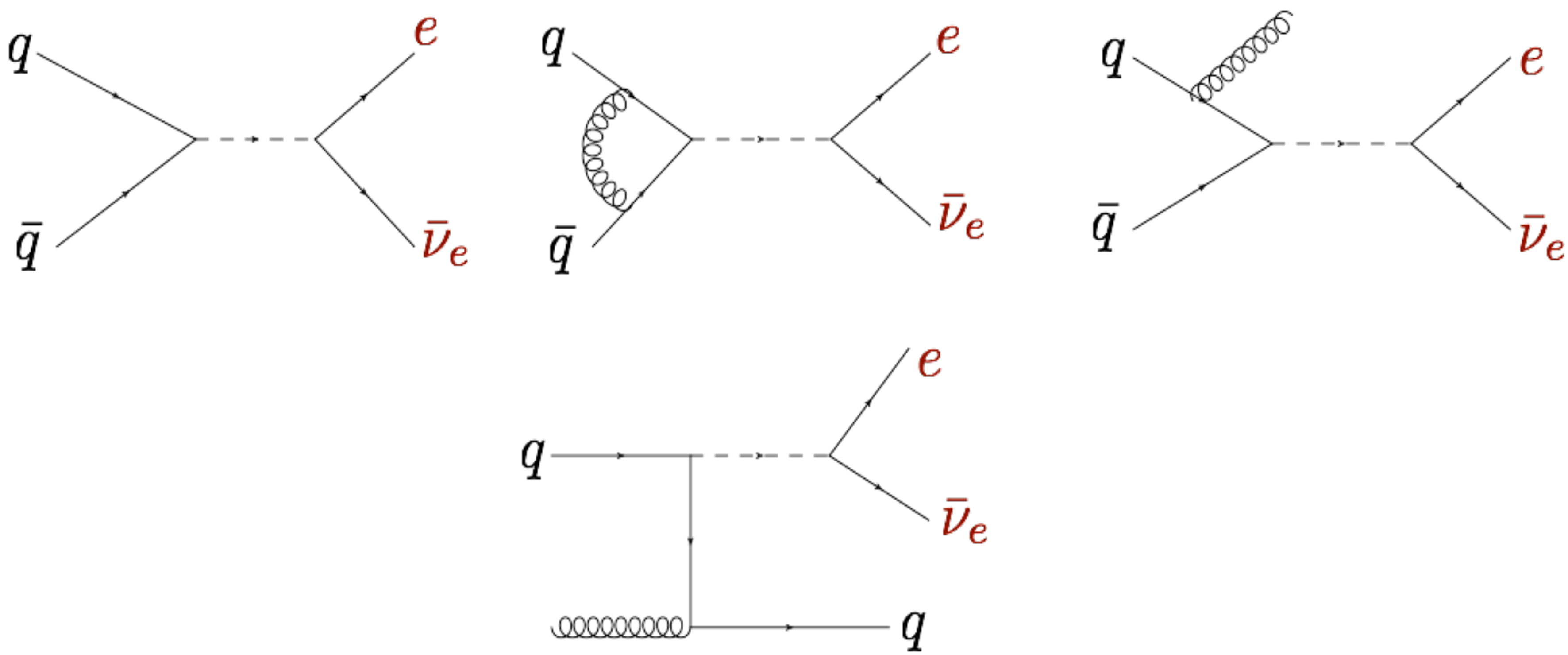
$$A_L = \frac{\sigma^+ - \sigma^-}{\sigma^+ + \sigma^-} \neq 0$$

$$A_L^{e^-} \sim \frac{\Delta\bar{u}(x_1)d(x_2)(1-\cos\theta)^2 - \Delta d(x_1)\bar{u}(x_2)(1+\cos\theta)^2}{\bar{u}(x_1)d(x_2)(1-\cos\theta)^2 + d(x_1)\bar{u}(x_2)(1+\cos\theta)^2}$$

- NLO for polarized case: Nadolsky, Yuan
  de Florian, WV
  Gehrmann, von Arx

- new analytical NLO calculation of Ringer, WV

  $$q\bar{q}' \to \ell^{\pm} X \;,\; qg \to \ell^{\pm} X$$

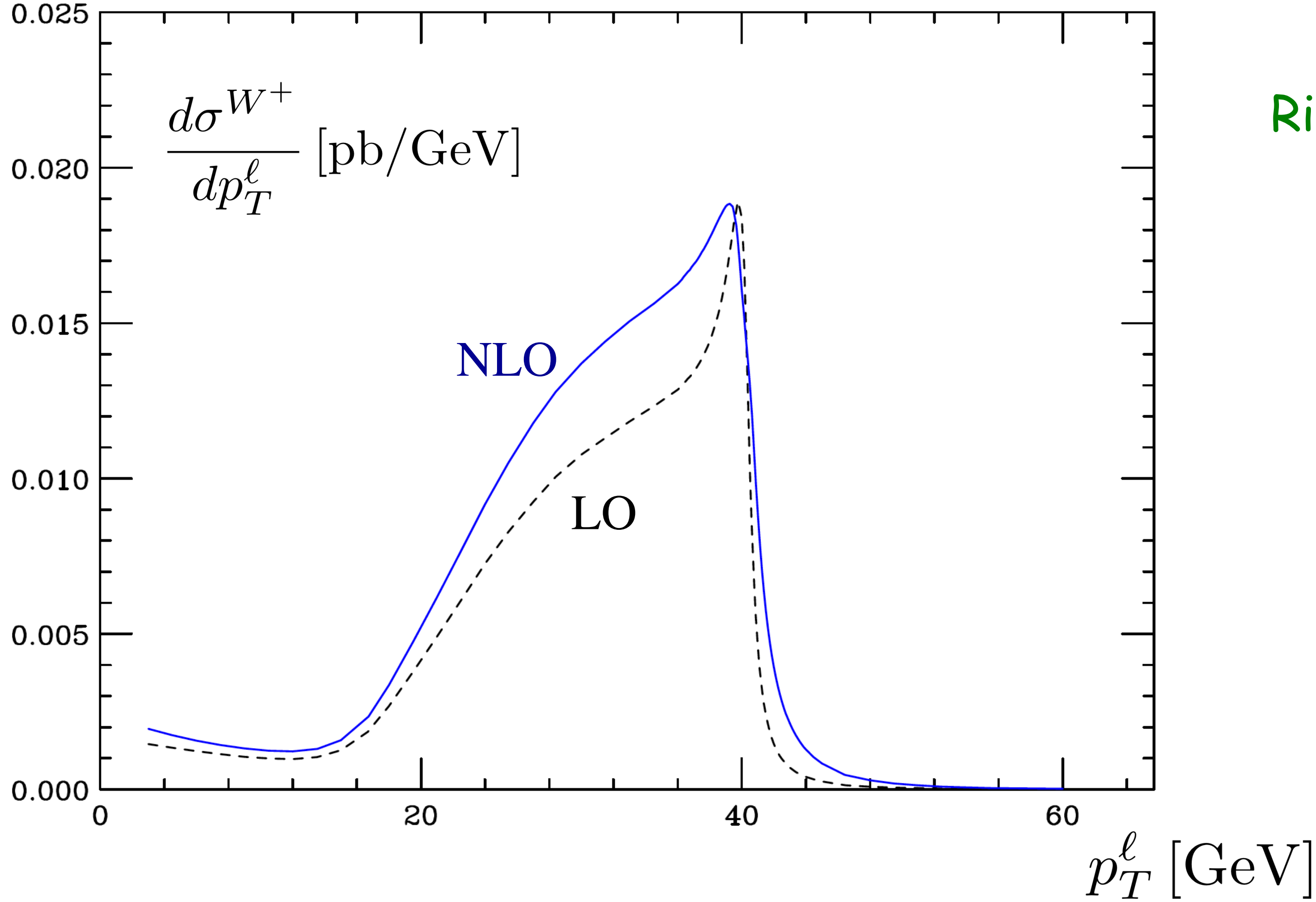


- **structure:** $1 - v = \frac{p_T^\ell}{\sqrt{s}}\,\mathrm{e}^{-\hat\eta}$ $\qquad vw = \frac{p_T^\ell}{\sqrt{s}}\,\mathrm{e}^{+\hat\eta}$

$$\frac{d\hat\sigma^{\mathrm{NLO}}}{dv\,dw} = \frac{\alpha_s}{2\pi}\left\{\frac{f_{\mathrm{LO}}(v)}{(s-M_W^2)^2+\Gamma^2M_W^2}\left[A\left(\frac{\ln(1-w)}{1-w}\right)_+ + B(v)\frac{1}{(1-w)_+}\right.\right.$$

$$\left. + C(v)\delta(1-w)\right]$$

$$\left. + \ldots + \frac{\ln\left(\frac{(ws-M_W^2)^2+\Gamma^2M_W^2}{M_W^4+\Gamma^2M_W^2}\right)}{(ws-M_W^2)^2+\Gamma^2M_W^2} + \ldots\right\}$$



Ringer, WV

- should help to understand behavior around $p_T^\ell \sim M_W/2$
  <-> interplay with resummation (Tevatron!)

- "joint" resummation:
  threshold and q_T logs can be resummed simultaneously

Laenen, Sterman, WV

$$\frac{d\sigma^{\text{res}}}{dQ^2\,dq_T^2} \propto \sum_q \int_C dN \left(\frac{Q^2}{S}\right)^{-N} \int d^2\mathbf{b}\ \mathrm{e}^{-i\,\tilde{\mathrm{q}}_T\cdot\tilde{\mathbf{b}}}\ f_q^N(Q)\ f_{\bar{q}}^N(Q)$$

$$\times\ \exp\left\{2\int_0^{Q^2} \frac{dk_\perp^2}{k_\perp^2} A(\alpha_s(k_\perp^2)) \left[J_0(b\,k_\perp)\,K_0\left(\frac{2N k_\perp}{Q}\right) + \ln\left(\frac{\bar{N} k_\perp}{Q}\right)\right]\right\}$$

- recover
  - -- threshold logs for $N \to \infty,\ b$ small
  - -- q_T logs for $b \to \infty,\ N$ small

- q_T-integrated cross section recovers threshold-resummed cross section

Conclusions

- understanding of higher-order QCD corrections in Drell-Yan cross section very advanced: NLO, NNLO, resummations to NLL, NNLL

- QCD corrections important, in particular in fixed-target regime: threshold logs

- recent progress toward TMD evolution

- many opportunities for fixed-target DY experiments:

  TMDs
  pion structure
  nuclear pdfs ?
  test of higher orders in pert. Theory