Reactions Theory II

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Non-elastic Cross Sections

Non-elastic Cross Sections

How?

A Code and a Book

Multi-channel Scattering

Use for inelastic, transfer, breakup channels (etc)
in addition to elastic.

Two channel (1=elastic, 2=reaction) make coupled channels:

$$\begin{aligned} [T_1 + U_1 - E_1]\psi_1(\mathbf{r}) + V_{12}\psi_2(\mathbf{r}) &= 0 \\ [T_2 + U_2 - E_2]\psi_2(\mathbf{r}) + V_{21}\psi_1(\mathbf{r}) &= 0. \end{aligned} \quad (1)$$

Forward coupling:

$V_{21}\psi_1(\mathbf{r})$ gives effect of channel 1 on channel 2,

Back coupling:

$V_{12}\psi_2(\mathbf{r})$ gives effect of channel 2 on channel 1.

These equations can be solved as coupled channels.

Simplified Multi-channel Scattering gives DWBA

If channel 2 is weak, we can neglect the $V_{12}\psi_2(\mathbf{r})$ term: the back effect on channel 1.

$$\begin{aligned} [T_1 + U_1 - E_1]\psi_1(\mathbf{r}) &= 0 \\ [T_2 + U_2 - E_2]\psi_2(\mathbf{r}) + V_{21}\psi_1(\mathbf{r}) &= 0. \end{aligned} \quad (2)$$

This equals the Born Approximation:

$$\begin{aligned} &[T_1 + U_1 - E_1]\psi_1(\mathbf{r}) = 0 \\ &\psi_2(\mathbf{r}) = -[T_2 + U_2 - E_2]^{-1} V_{21}\psi_1(\mathbf{r}) \end{aligned} \quad (3)$$

So the DWBA scattering amplitude in channel 2 is

$$f_{21}(\theta) = -\frac{m_2}{2\pi\hbar^2} \langle \mathbf{k}_2 \mid V_{21} \mid \psi_1 \rangle \quad (4)$$

DWBA is often useful for non-elastic channels.

Coupled Channels Calculations

Fresco

Coupled Reaction Channels Calculations

www.fresco.org.uk

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About Fresco

Fresco is a program developed by Ian Thompson over the period 1983 - 2006, to perform coupled-reaction channels calculations in nuclear physics. It uses Fortran 90 or Fortran 95 on Unix, Linux, Vax and Windows machines.

Sfresco is an additional version of Fresco, to provide Chi-squared searches of potential and coupling parameters, and to fit additional R-matrix terms in hybrid models.

Free!

Theory Book!

Nuclear Reactions for Astrophysics

Principles, Calculation and Applications of Low-Energy Reactions

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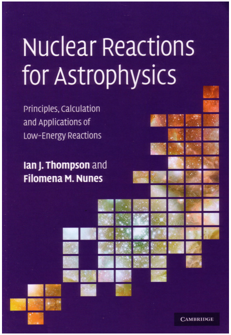


Cambridge University Press: http://www.cambridge.org/9780521856355

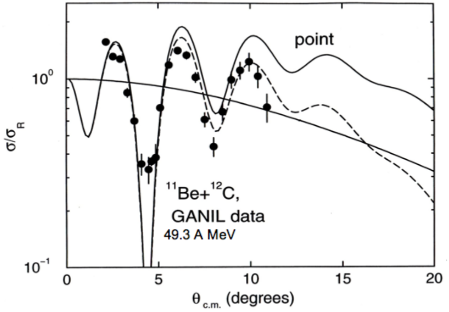
Physics of Nuclear Reactions

- Halo Scattering: Elastic
- Halo Total Reaction Cross Section
- Transfer Reactions
- Breakup Reactions
- Halo Fusion Reactions

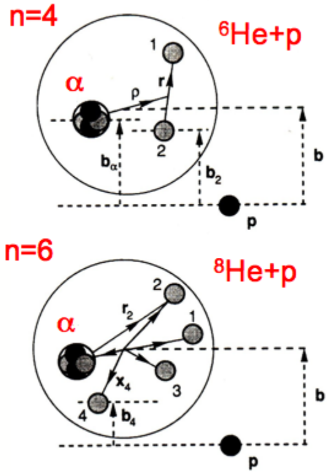
Halo Scattering: Elastic

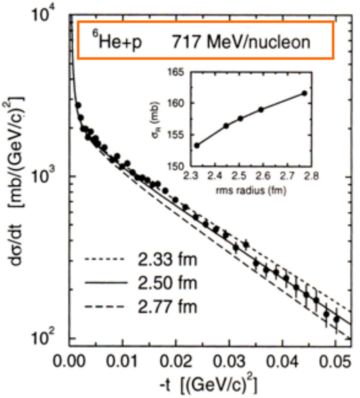
Depends on

- Folded potential from densities
- Halo breakup effects, i.e.
- Polarisation potential from breakup channel



Four- and Six-body Scattering



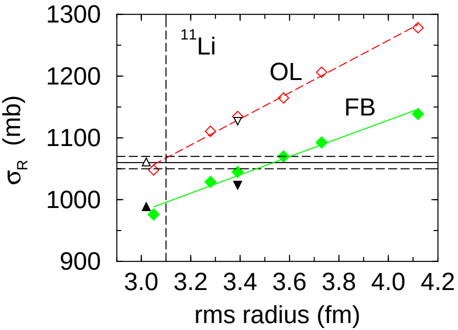


Al-Khalili and Tostevin, PRC **57** (1998) 1846
Tostevin et al., PRC **56** (1997) R2929

Halo Total Reaction Cross Section

Depends on

- Densities and NN scattering, as usual
- But: effects of Halo Breakup (virtual and real) are big!
- Use few-body Glauber, not Optical Limit Glauber
- Do we scatter from average positions? Or average scattering from positions?



New radii are larger.

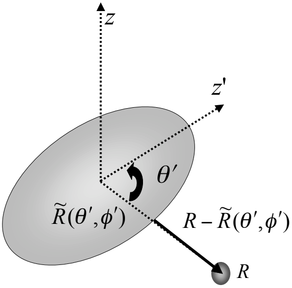
Inelastic Scattering

Need a structure model for the couplings: rotational or vibrational model.
Consider here the rotational model with excitation energies

$$\epsilon_I = \frac{\hbar^2}{2\mathcal{M}}\left[I(I+1) - K(K+1)\right] \qquad (5)$$

The coupling interaction of multipole λ depends on the derivative of the optical potential $U(r)$ as

$$V_{fi}^{\lambda}(r) = -\frac{\beta_\lambda R_0}{\sqrt{4\pi}}\; U'(r)\; \hat{I}_i \langle I_i K, \lambda 0 | I_f K \rangle, \qquad (6)$$

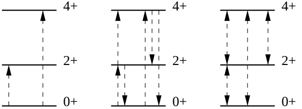


Example of Inelastic Scattering

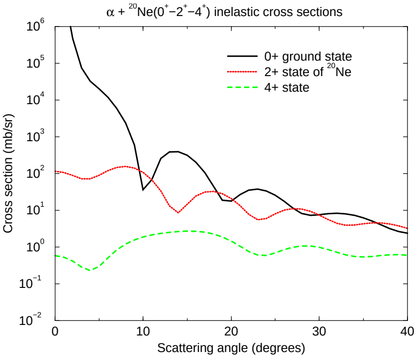
α-particle scattering on ^{20}Ne.

Choose here a rotational model: $\beta_2 = 0.205$.

Theory options:



First order Second order All orders

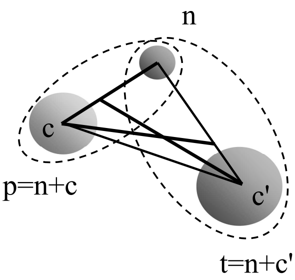


This is an all-order calculation.

Transfer Interaction

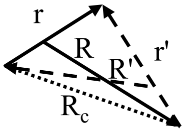
From $\phi_p(\mathbf{r})$ projectile bound state (p=n+c),
to $\phi_t(\mathbf{r}')$ target bound state (t=n+c'):

$$[H_p - \varepsilon_p]\phi_p(\mathbf{r}) = 0 \quad H_p = T_\mathbf{r} + V_p(\mathbf{r})$$
$$[H_t - \varepsilon_t]\phi_t(\mathbf{r}') = 0 \quad H_t = T_{\mathbf{r}'} + V_t(\mathbf{r}') \quad (7)$$



The transfer interaction has two forms:

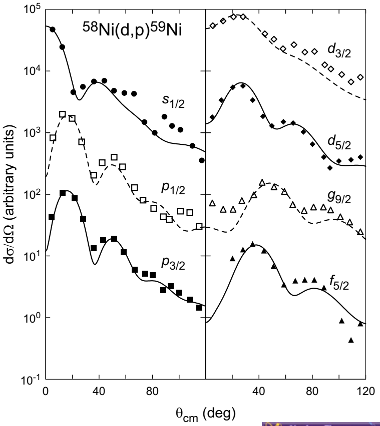
$$\mathcal{V}_{\rm prior}(\mathbf{R},\mathbf{r}) = V_t(\mathbf{r}') + U_{c'c}(\mathbf{R}_c) - U_i(R)$$
$$\mathcal{V}_{\rm post}(\mathbf{R}',\mathbf{r}') = V_p(\mathbf{r}) + U_{c'c}(\mathbf{R}_c) - U_f(R') \quad (8)$$



These should give the same cross sections.

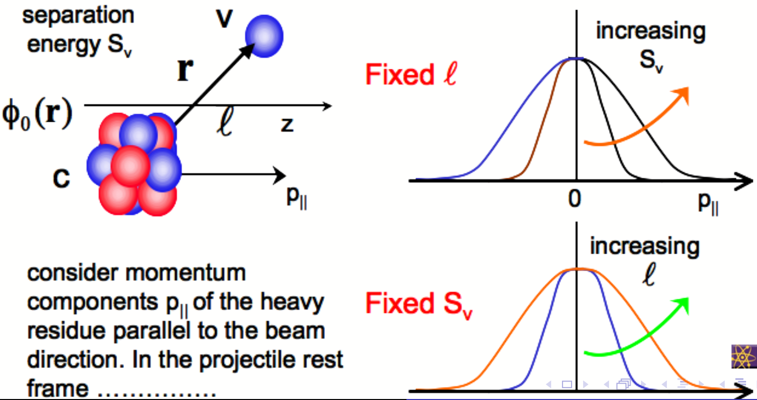
Transfer Reactions to Probe Single-Particle Structure

- Weak, so use DWBA
- One-nucleon transfers, (p,d) shape shows L-value of orbital magnitude gives spectroscopic factor
- Two-neutron transfers, (p,t) Magnitude depends on s-wave pairing in halo Only relative magnitudes reliably modeled.
- But: full analysis requires multi-step calculations



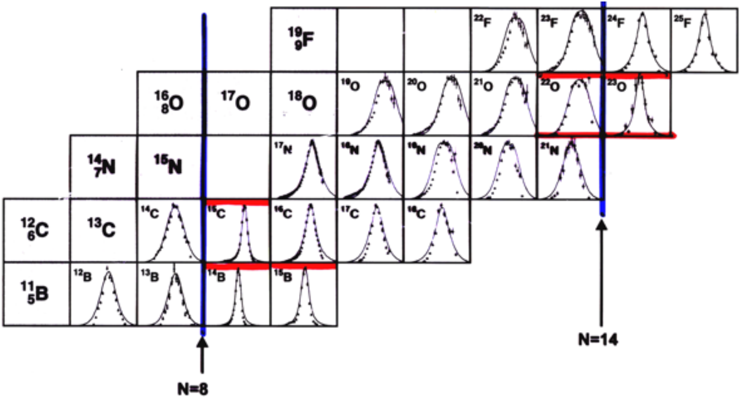
Stripping (Breakup) Reactions: Measuring Momentum

Probing the momentum content of bound states by breakup reactions



Stripping Reactions: Nuclear Structure

Glauber (eikonal) theory of breakup:

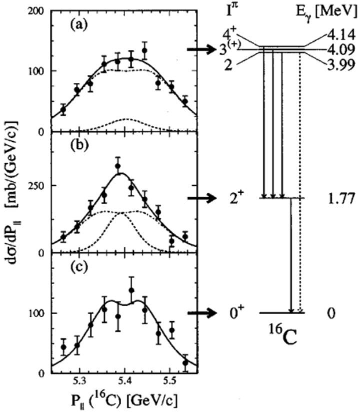


F. Sauvan et al., Phys. Lett. B 491 (2000) 1

Stripping Reactions: Removing a Neutron

Reaction ^{9}Be(^{17}C, ^{16}Cγ)X

Measured γ from core decays helps to fix the final state



ℓ=0,2 admixture

ℓ=0,2 admixture

pure ℓ=2 but large!!

[Maddalena et al., PRC63(01)024613]

Halo Fusion: an Unsolved Problem

In low-energy Halo Fusion (near the Coulomb barrier):
Halo neutrons should affect fusion:

- Increase fusion, from neutron attractions & neutron flow
- Decrease complete fusion, from breakup
- Increase fusion, from molecular states & resonances

So: need experiments + good theories!
Some experiments already performed with ^{6}He and ^{9}Be, but theoretical interpretations are still unclear.

Compound Nucleus Decays (after fusion)

Flux does not 'disappear' the nuclei fuse together, but reappears as mixture of narrow resonances of the compound system.

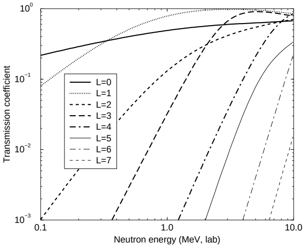
- Narrow resonances ⇒ long-lived ⇒ many oscillations to decay
- Bohr hypothesis: decay independent of production method
- So decay by all possible means α:
  emission of γ, n, p, α, maybe fission.
- Average the cross sections over (say) 0.1 MeV, $\langle\sigma_{\alpha'\alpha}\rangle$ to cover many resonances
- Hauser-Feshbach theory gives the statistical branching ratios between the channels α.

So we can calculate residual nuclear ground states after all emissions are finished.

Transmission coefficients for CN production

'Transmission coefficient' $\mathcal{T}_\alpha(E) = 1 - |S_\alpha(E)|^2$ is the probability of CN production for scattering at energy E.

Transmission coefficients for neutrons incident on ^{90}Zr in various partial waves L, using a global optical potential:



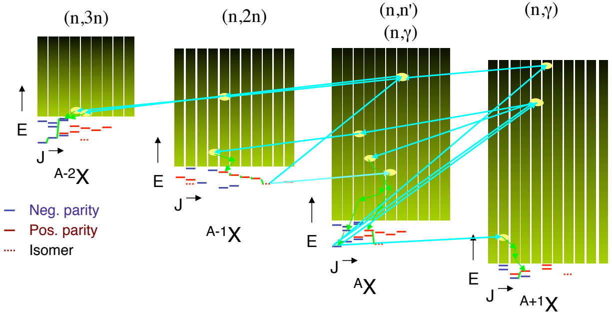
Decay paths and Branching Probabilities

- So consider all possible exit channels α'' and normalize to total
- Hauser-Feshbach cross section $\alpha \to \alpha'$ (simple form):

$$\langle \sigma_{\alpha'\alpha}(L;E) \rangle = \frac{\pi}{k^2}(2L+1)\frac{\mathcal{T}_\alpha \mathcal{T}_{\alpha'}}{\sum_{\alpha''} \mathcal{T}_{\alpha''}}$$

- The same $\mathcal{T}_\alpha$ are used for producing as for decaying.
- If we do not know all the α, average over a level density $\rho(E)$

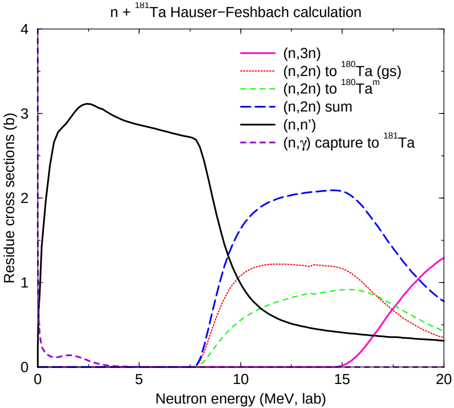
Decay paths starting from neutron + ^{A}X:



This is the framework for Hauser-Feshbach calculations. They ignore interference effects between successive steps, so are more semi-classical than quantum.

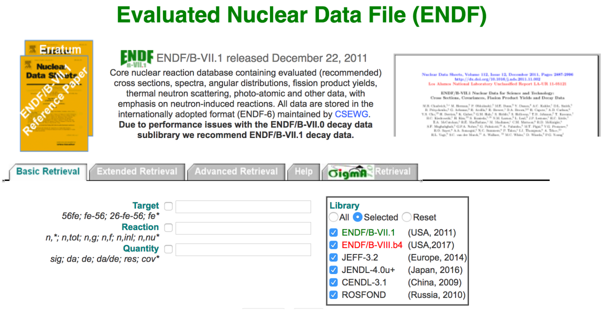
Result of a Hauser-Feshbach Calculation

Using the code TALYS:



Evaluated Data for Nuclear Reactions

National libraries available, such as
ENDF: Evaluated Nuclear Data File, at NNDC.



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