

Nuclear Astrophysics: Lecture 1

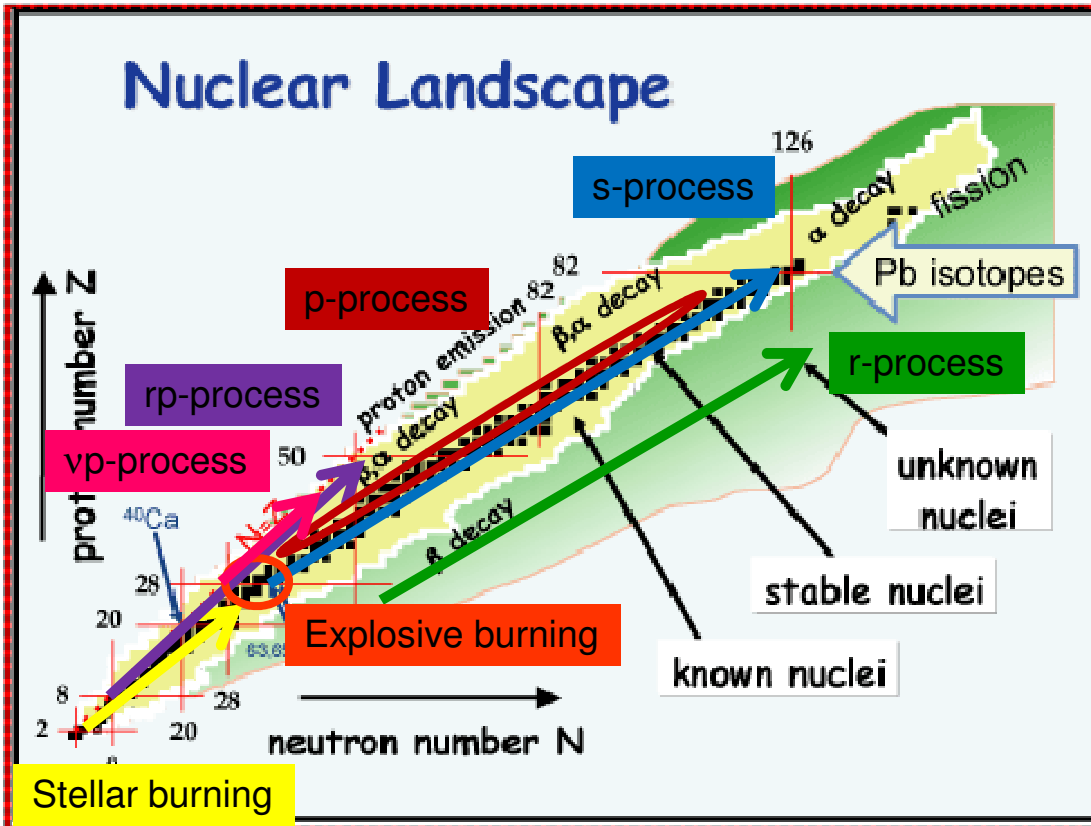
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Lecture plan

- Lecture 1
 - Solar system abundances
 - A tiny little bit of BBN
 - Hydrostatic nuclear burning
 - Thermonuclear reaction rates
- Lecture 2
 - Explosive nuclear burning
 - Heavy element synthesis
 - Spectroscopy and metal-poor stars

Origin of elements



Astrophysical sites:

Stellar evolution of low-mass and massive stars

AGB stars (main s-process)
 core He-burning of massive stars (weak s-process)

Supernovae

Core-collapse supernovae

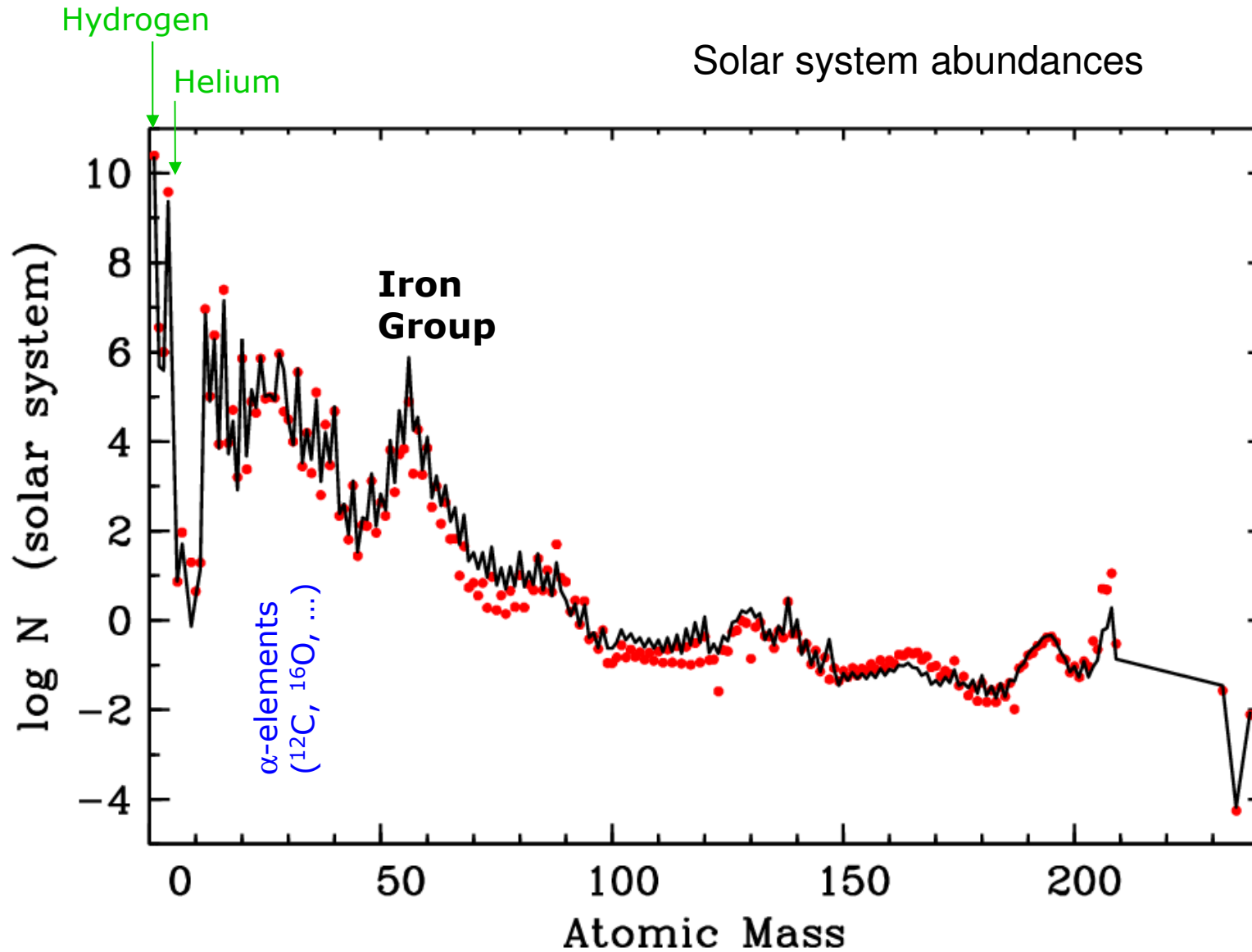
Core-collapse supernovae

Neutrino-driven winds in SNe?

NS mergers

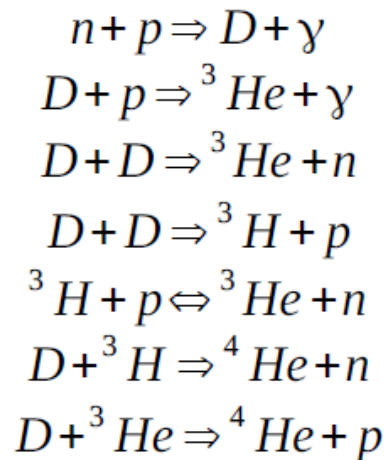
X-ray bursts

Origin of elements

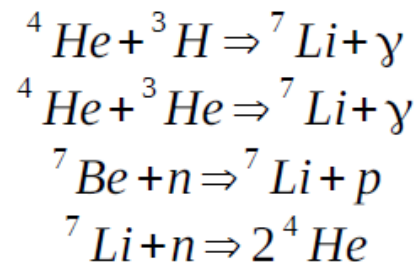


Big Bang Nucleosynthesis (BBN)

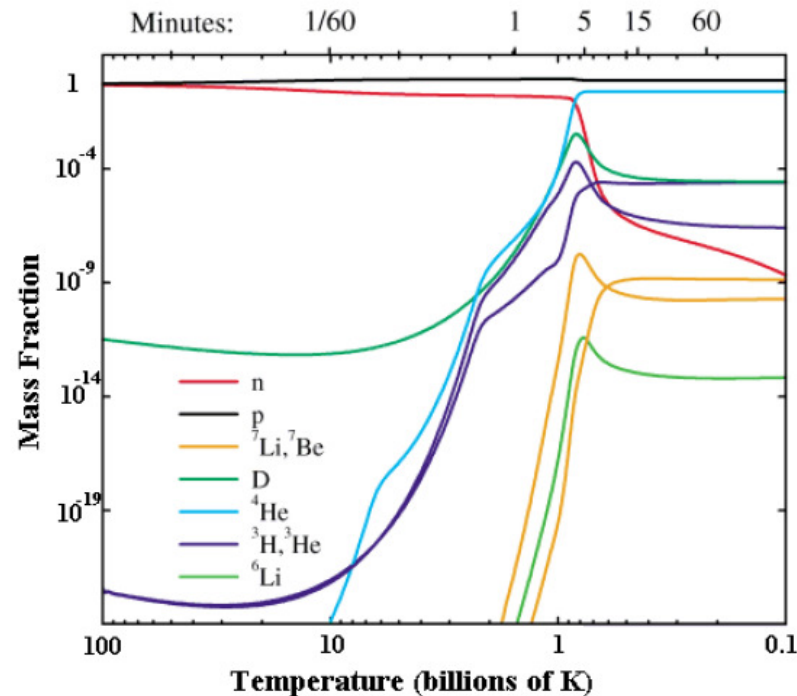
- After $t \sim 180$ s nuclear reactions begin to occur in earnest.
- There are essentially just 11 reactions.
- The most important 7 are:



- The other 4 reactions that occur are:

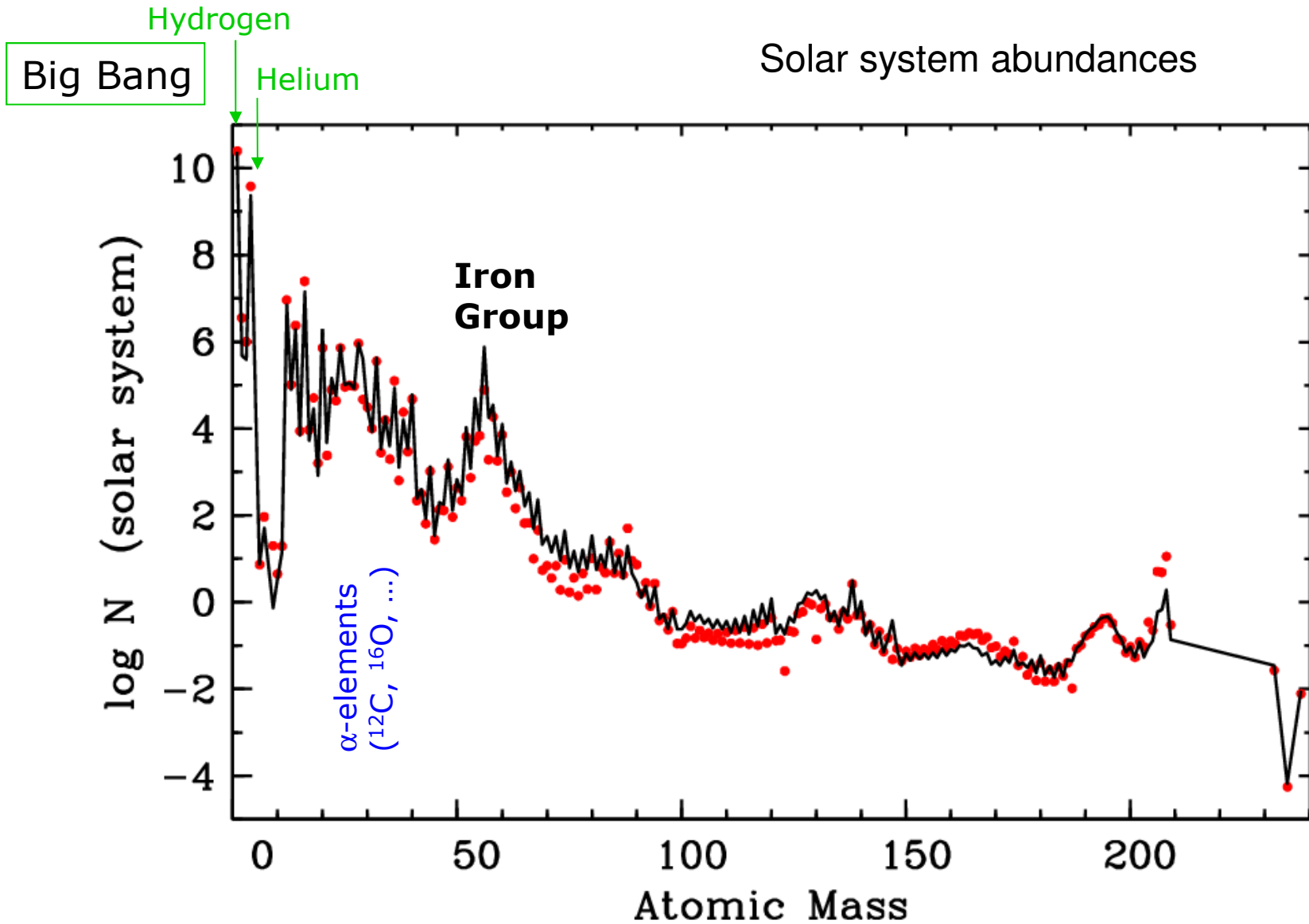


Big Bang Nucleosynthesis (BBN)

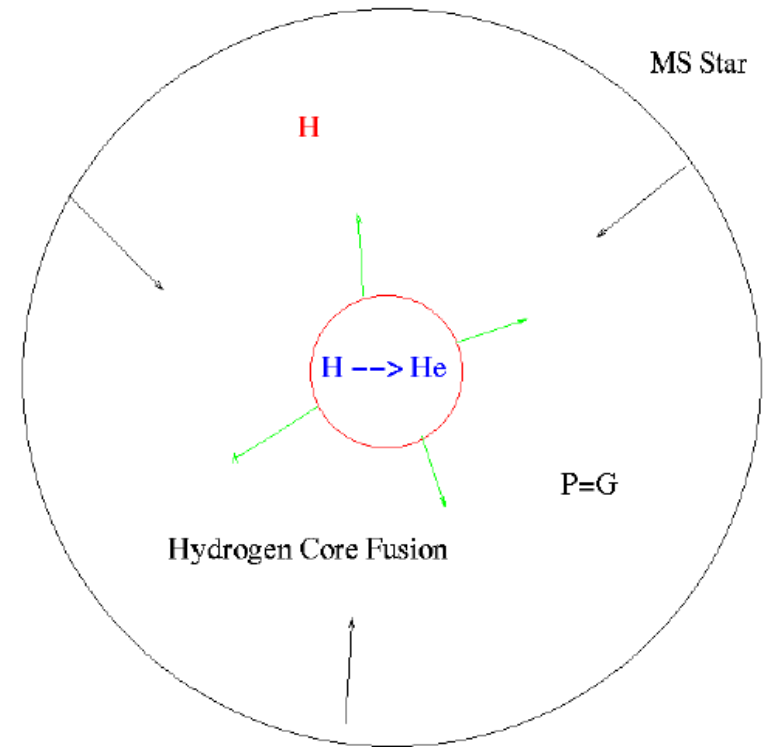
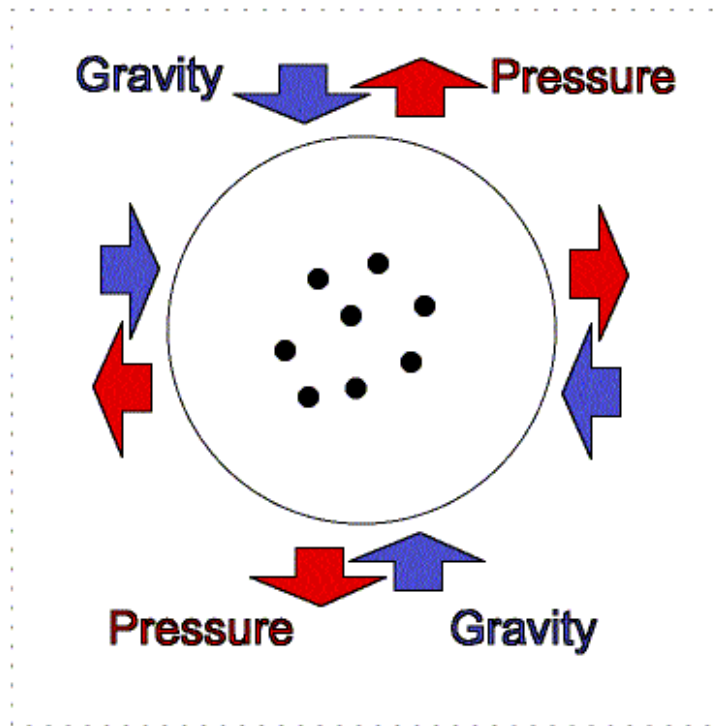


- After $t \sim 15$ minutes, BBN is over
- What is produced are lots of leftover free protons, ${}^4\text{He}$ and trace amounts of D, ${}^3\text{H} + {}^3\text{He}$, and ${}^7\text{Li} + {}^7\text{Be}$.
- essentially every neutron ended up in ${}^4\text{He}$.
- The first stars were born with this composition!

Origin of elements



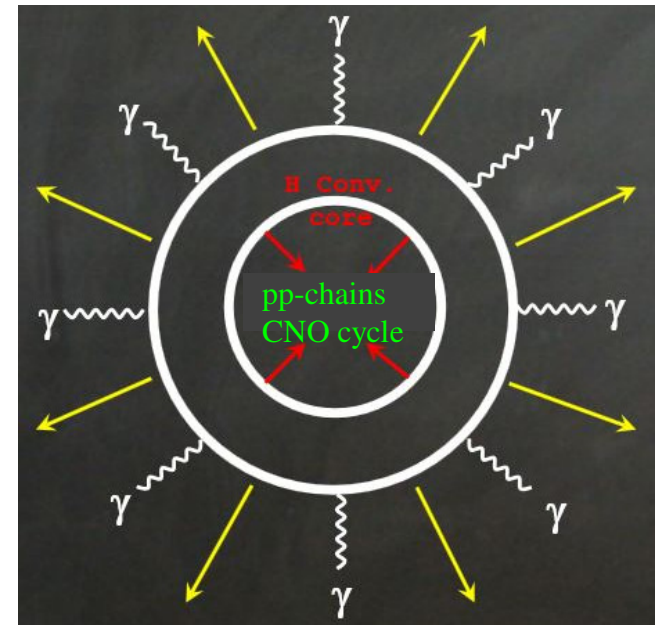
Stars (structure and evolution)



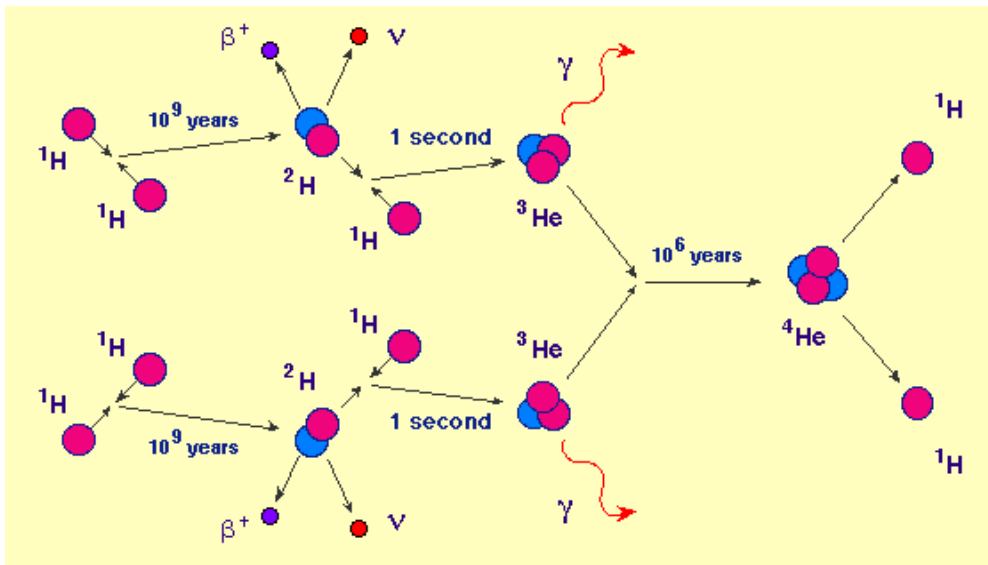
Not to scale!

H-burning

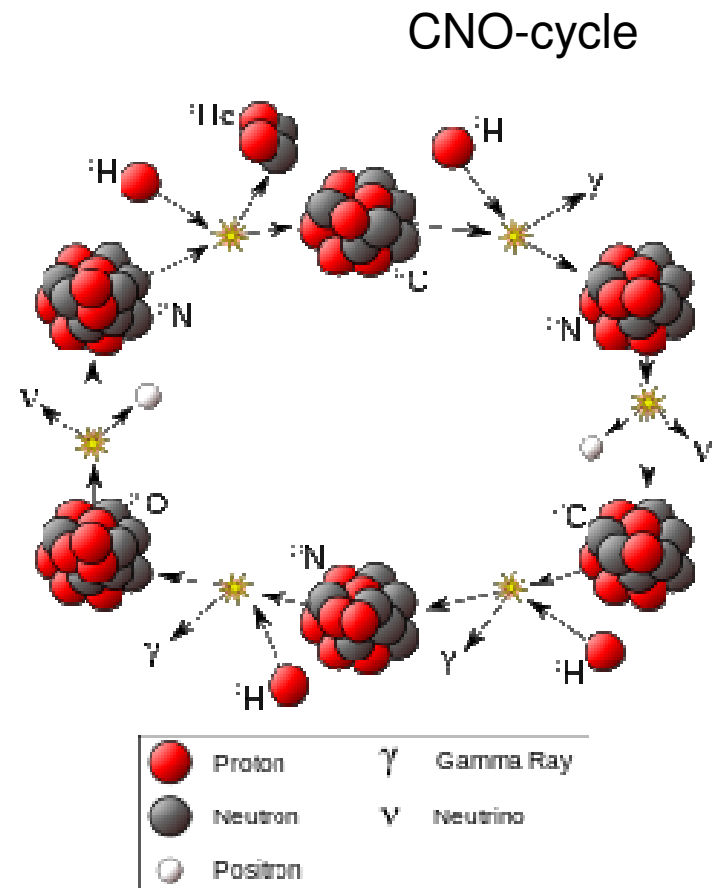
- Typical temperature: 10^7 K
- Net reaction: $4 p \rightarrow {}^4\text{He}$
 - Fuel: hydrogen
 - Main product: helium
 - Bottle neck: $p + p \rightarrow d + e^+ + \nu_e$ (Q-value: 0.42 MeV)
 - Lower mass stars: pp-chains
 - Higher mass stars: CNO cycle
- Duration:
12 billion (our Sun) to 10 million ($25M_{\text{sun}}$ star)



H-burning

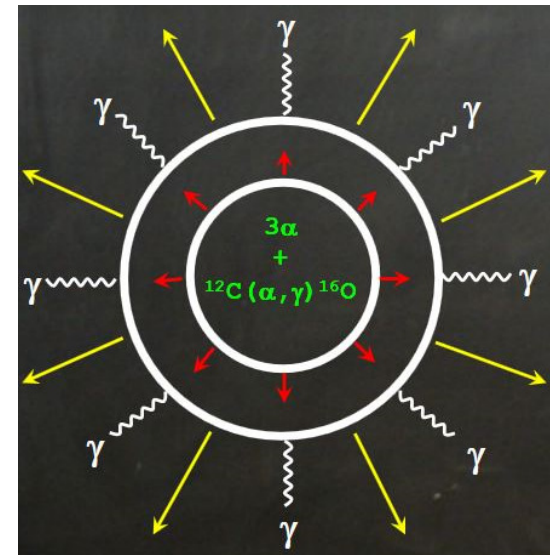


pp-chain

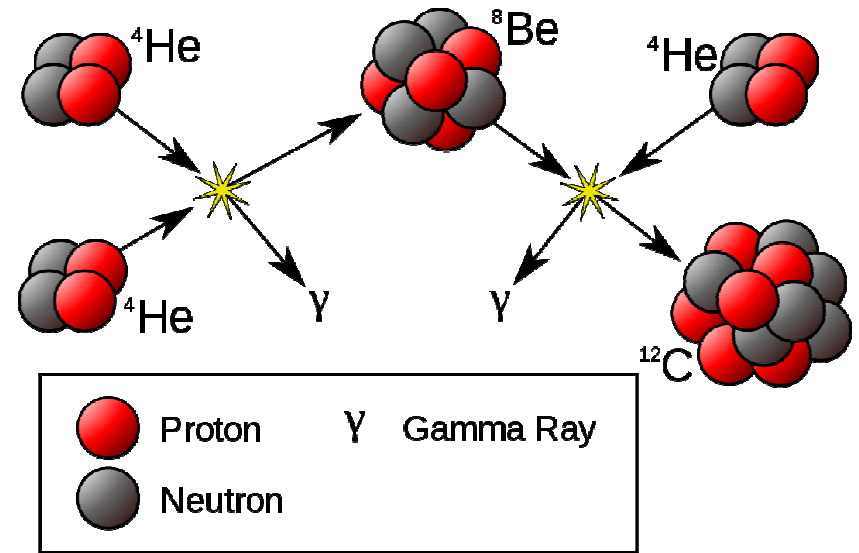
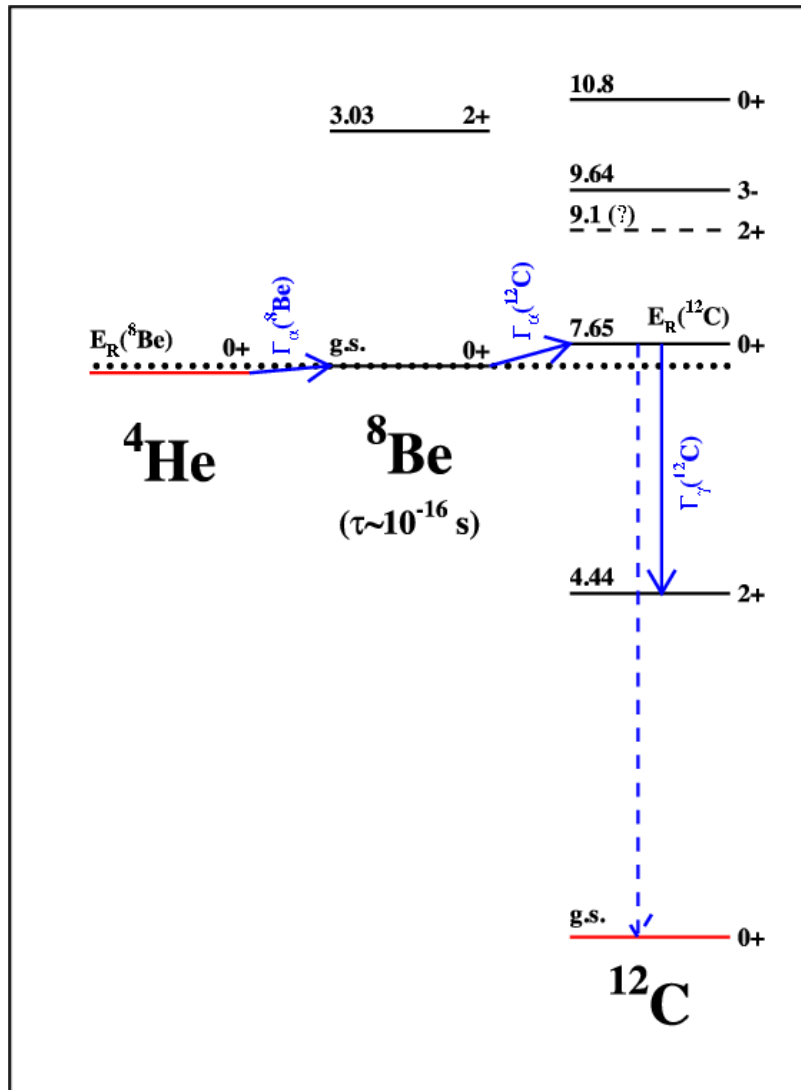


He-burning

- Typical conditions:
 - Temperature: $(1-2) \cdot 10^8$ K
 - Density: a few $10^2 - 10^4$ g/cm³
- Net reaction: $4\text{He} (2\alpha, \gamma) {}^{12}\text{C}$
 - Fuel: helium
 - Main products: carbon, oxygen
 - $4\text{He} + 4\text{He} \leftrightarrow {}^8\text{Be} + \gamma$
 - ${}^8\text{Be} + 4\text{He} \leftrightarrow {}^{12}\text{C} + \gamma$
 - And ${}^{12}\text{C} + 4\text{He} \rightarrow {}^{16}\text{O} + \gamma$
 - Difficulty: lifetime of ${}^8\text{Be} \sim 10^{-16}$ s
→ Hoyle state (resonance in ${}^{12}\text{C}$ at $E=7.68$ MeV)
 - Other products: ${}^{21,22}\text{Ne}$, ${}^{25,26}\text{Mg}$, ${}^{36}\text{S}$, ${}^{37}\text{Cl}$, ${}^{40}\text{K}$, ${}^{40}\text{Ar}$
 - ${}^{14}\text{N} (\alpha, \gamma) {}^{18}\text{F} (e^+, \nu) {}^{18}\text{O} (\alpha, \gamma) {}^{22}\text{Ne} (\alpha, n) {}^{25}\text{Mg}$

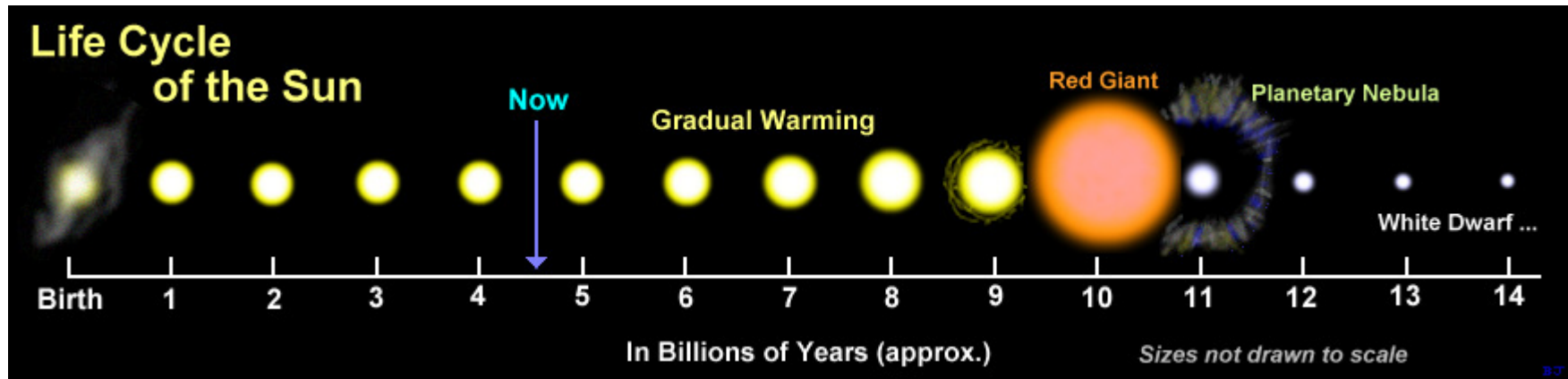


He-burning



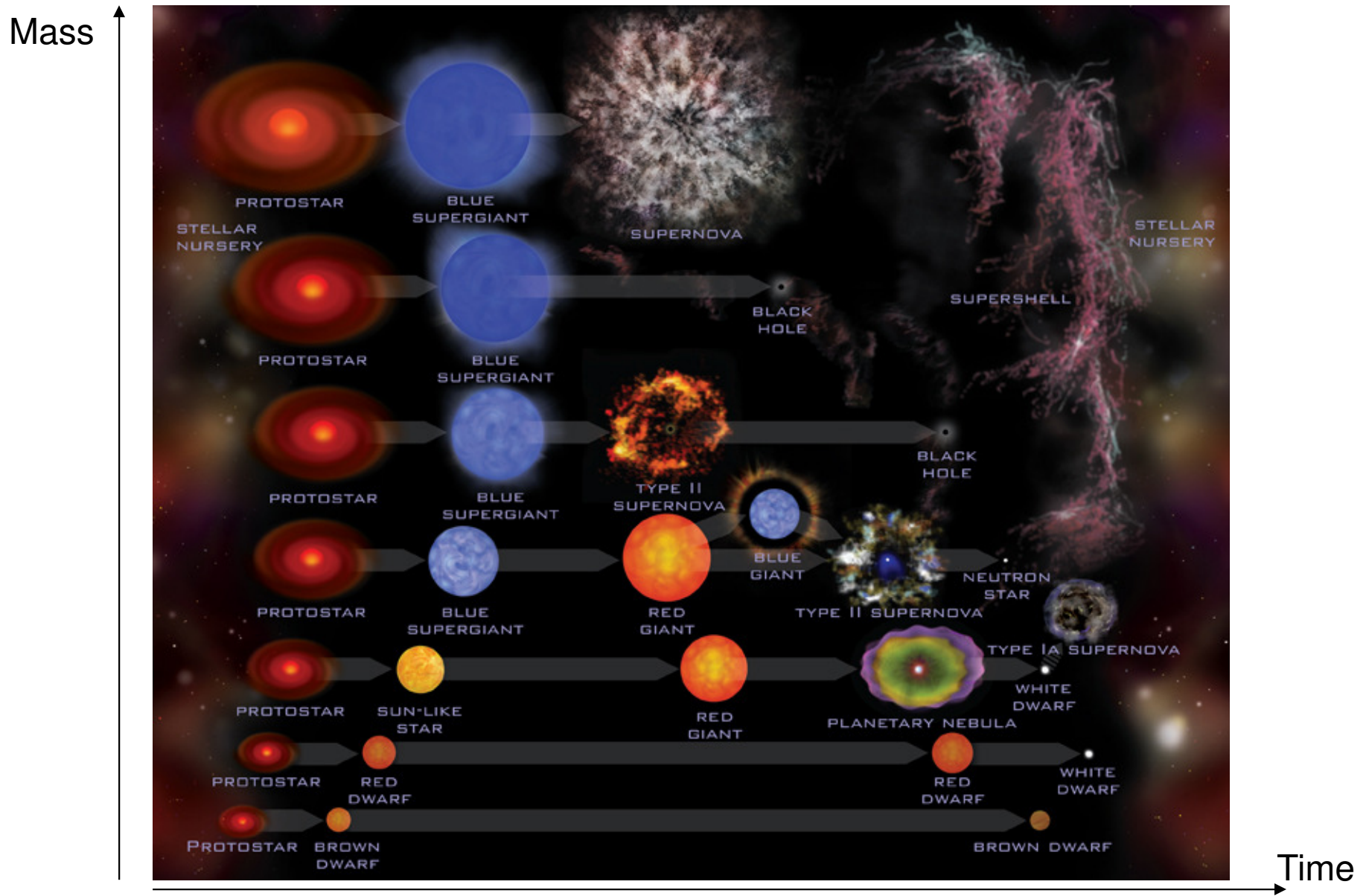
Low-mass stars

- End their life after He-burning
- Eg the Sun



- If WD is in a binary system → type Ia supernova

Stellar lifetimes

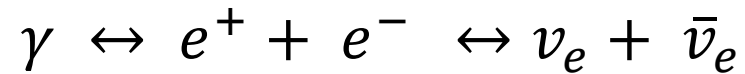


C-burning

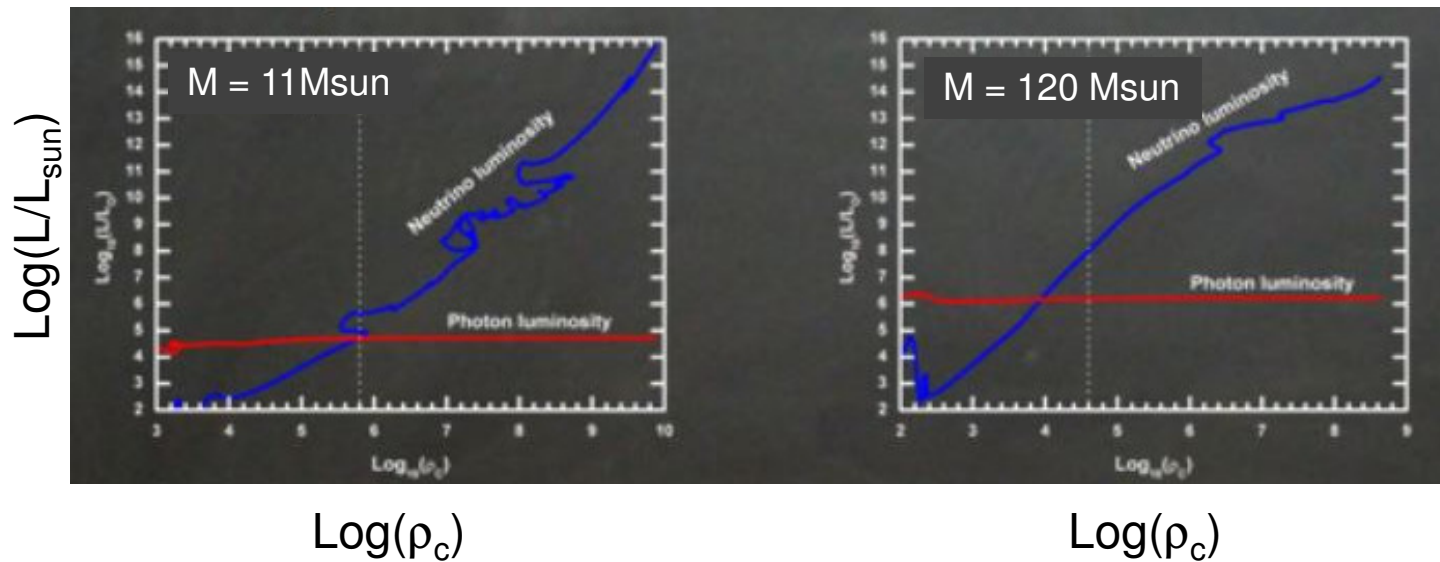
- Typical conditions:
 - Temperature: $(6-8) 10^8$ K
 - Density: 10^5 g/cm³
- Net reaction: $^{12}\text{C} + ^{12}\text{C}$
 - Fuel: carbon
 - Main products: neon, magnesium, oxygen
 - $^{12}\text{C} + ^{12}\text{C} \rightarrow \alpha + ^{20}\text{Ne}$ (Q=4.62 MeV)
 - $^{12}\text{C} + ^{12}\text{C} \rightarrow \text{p} + ^{23}\text{Na}$ (Q=2.24 MeV)
 - Other reactions: $^{23}\text{Na} + \text{p} \rightarrow \alpha + ^{20}\text{Ne}$
 $^{20}\text{Ne} + \alpha \rightarrow ^{24}\text{Mg}$
 - $^{12}\text{C} + \alpha \rightarrow ^{16}\text{O} + \gamma$ (Q=4.73 MeV)

Neutrino-losses

- At temperatures above $\sim 10^9$ K: pair-production



- Luminosity of **photons** and **neutrinos**



Ne-burning

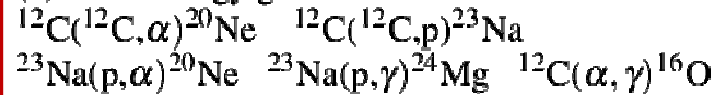
- Typical conditions:
 - Temperature: $(1-2) 10^9$ K
 - Density: 10^6 g/cm³
- Reactions:
 - Fuel: neon
 - Main products: oxygen, silicon
 - $^{20}\text{Ne} (\gamma, \alpha) ^{16}\text{O}$
 - Other reactions:
 - $^{20}\text{Ne} (\alpha, \gamma) ^{24}\text{Mg} (\alpha, \gamma) ^{28}\text{Si} (\alpha, \gamma) ^{32}\text{S}$
 - $^{21}\text{Ne} (\alpha, n) ^{24}\text{Mg} (n, \gamma) ^{25}\text{Mg} (\alpha, n) ^{28}\text{Si}$
 - $^{23}\text{Na} (\alpha, p) ^{25}\text{Mg} (\alpha, n) ^{28}\text{Si}$
 - $^{25}\text{Mg}(p, \gamma) ^{25}\text{Al}$
 - $^{23}\text{Na} (p, \alpha) ^{20}\text{Ne}$

O-burning

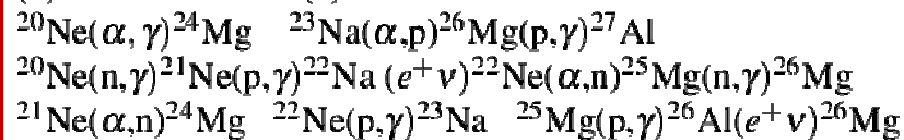
- Typical conditions:
 - Temperature: $(1.5-2.2) 10^9$ K
 - Density: 10^7 g/cm³
- Reaction:
 - Fuel: oxygen
 - Main products: silicon
 - $^{16}\text{O} + ^{16}\text{O} \rightarrow \text{p} + ^{31}\text{P} \quad (Q=7.676 \text{ MeV})$
 - $^{16}\text{O} + ^{16}\text{O} \rightarrow \alpha + ^{28}\text{Si} \quad (Q=9.593 \text{ MeV})$
 - $^{16}\text{O} + ^{16}\text{O} \rightarrow \text{n} + ^{31}\text{S} \quad (Q=1.459 \text{ MeV})$
 - Other reactions:
 - $^{31}\text{P} (p, \alpha) ^{28}\text{Si}$
 - $^{33}\text{S} (e^-, \nu_e) ^{33}\text{P}$
 - $^{35}\text{Cl} (e^-, \nu_e) ^{35}\text{P}$

C-, Ne-, O-burning reactions (details)

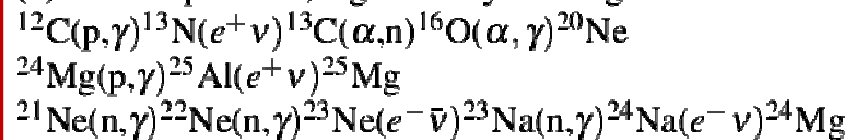
(a) basic energy generation



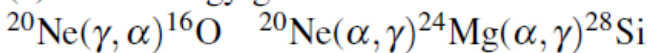
(b) fluxes $> 10^{-2} \times$ (a)



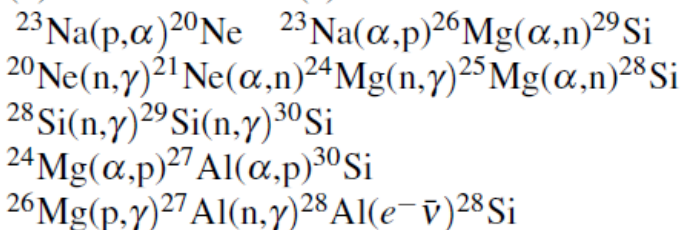
(c) low temperature, high density burning



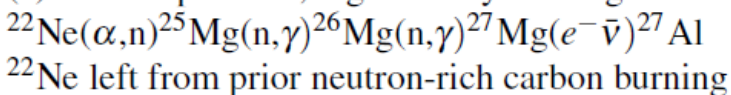
(a) basic energy generation



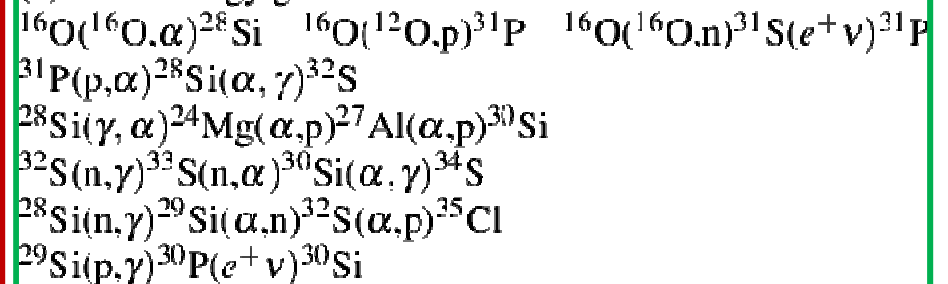
(b) fluxes $> 10^{-2} \times$ (a)



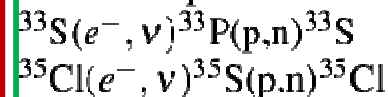
(c) low temperature, high density burning



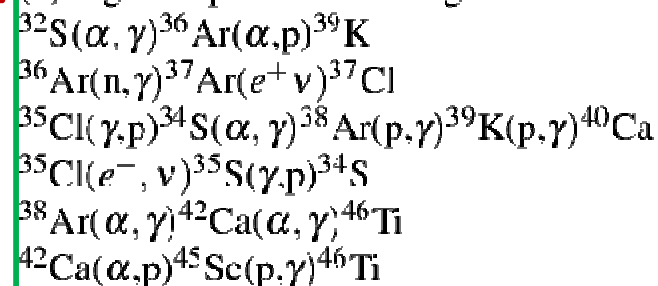
(a) basic energy generation



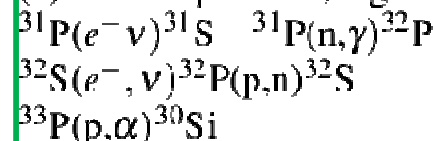
electron captures



(b) high temperature burning



(c) low temperature, high density burning



Si-burning

- Typical temperature: $(3-4) 10^9$ K
- Net reaction: $^{28}\text{Si} + ^{28}\text{Si}$
 - Fuel: silicon
 - Main products: Fe-group elements ($A = 50-60$ nuclei)
 - Other reactions:
 $^{28}\text{Si} + \gamma \rightarrow \text{p} + ^{27}\text{Al}$
 $^{28}\text{Si} + \gamma \rightarrow \alpha + ^{24}\text{Mg}$
 $^{28}\text{Si} + \gamma \rightarrow \text{n} + ^{27}\text{Si}$
- Balance between forward and reverse reactions for increasing number of processes: $a + b \leftrightarrow c + d$
→ Nuclear statistical equilibrium (NSE)

Nuclear Statistical Equilibrium (NSE)

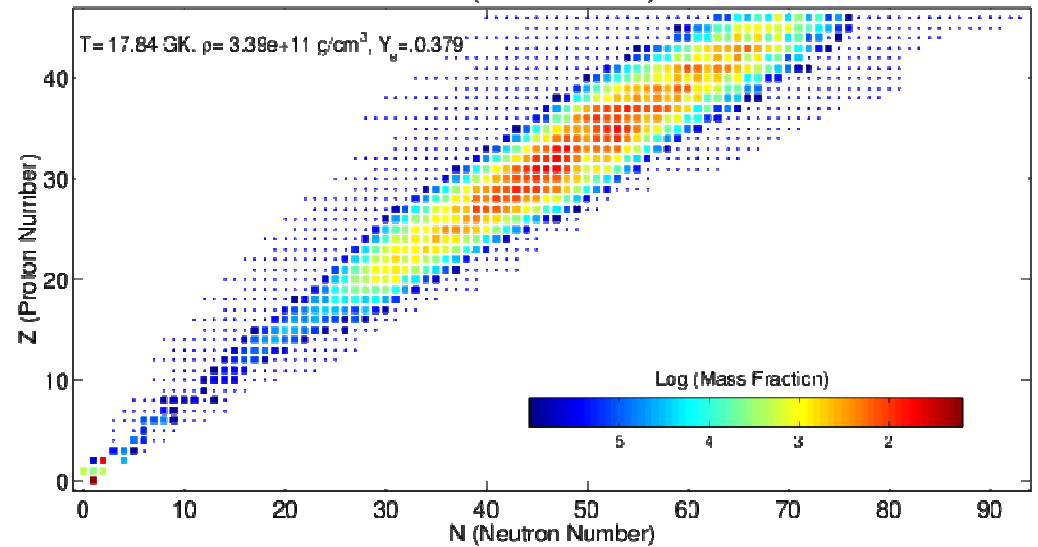
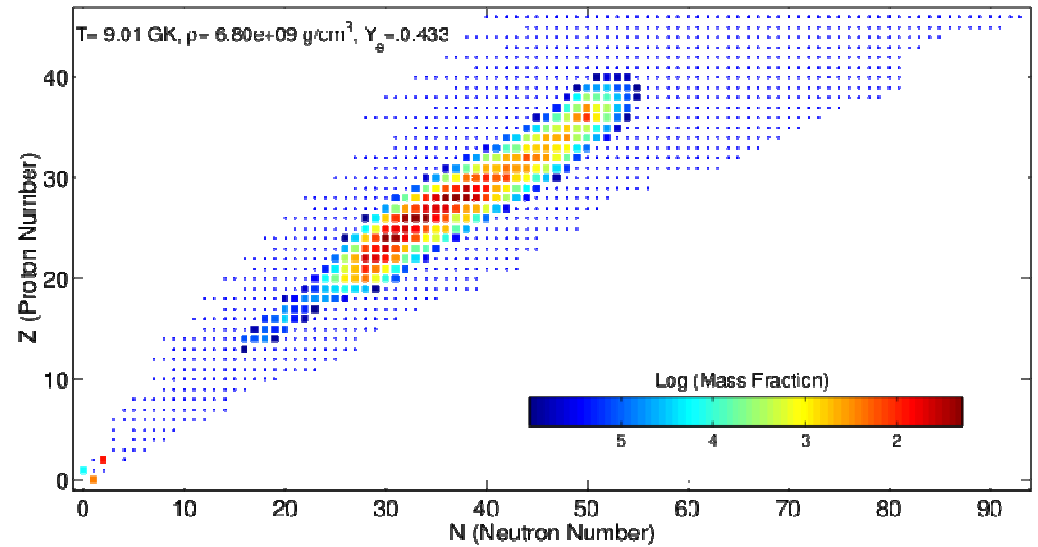
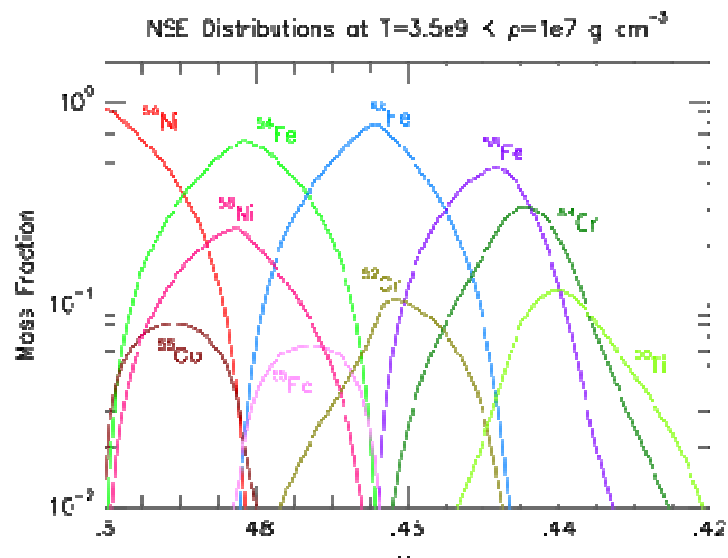
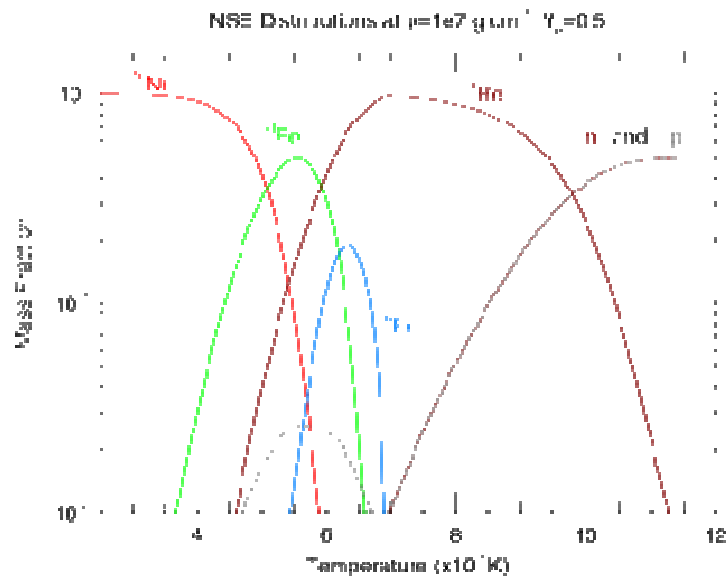
- Chemical equilibrium (for species i):

$$Z_i \mu_p + N_i \mu_n = \mu_i$$

- Abundances given by:

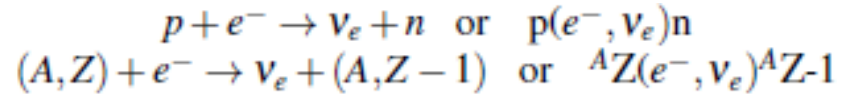
$$Y(Z, N) = G_{Z,N} (\rho N_A)^{A-1} (A^{2/3}/2^A) \left(\frac{2\pi h}{m_n kT} \right)^{3/2(A-1)} Y_n^N Y_p^Z e^{B_{Z,N}/kT}$$

Nuclear Statistical Equilibrium (NSE)



Weak Interactions

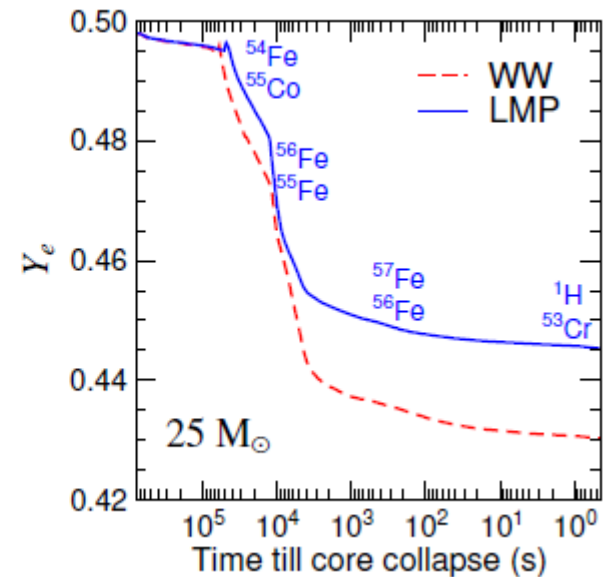
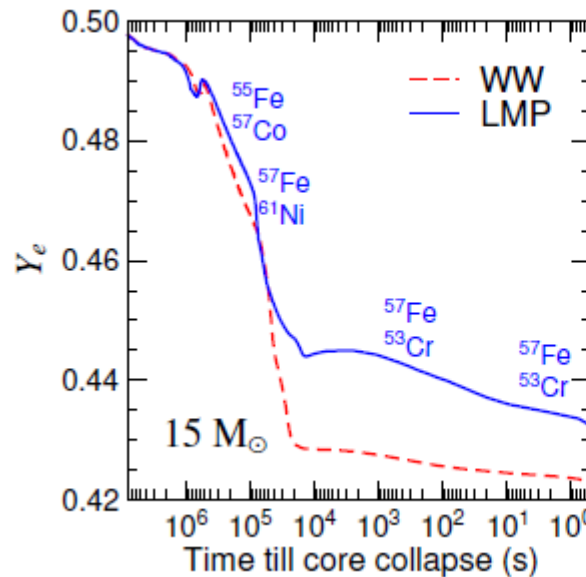
- Become relevant in later burning stages
- Lead to neutronization
- Electron captures:



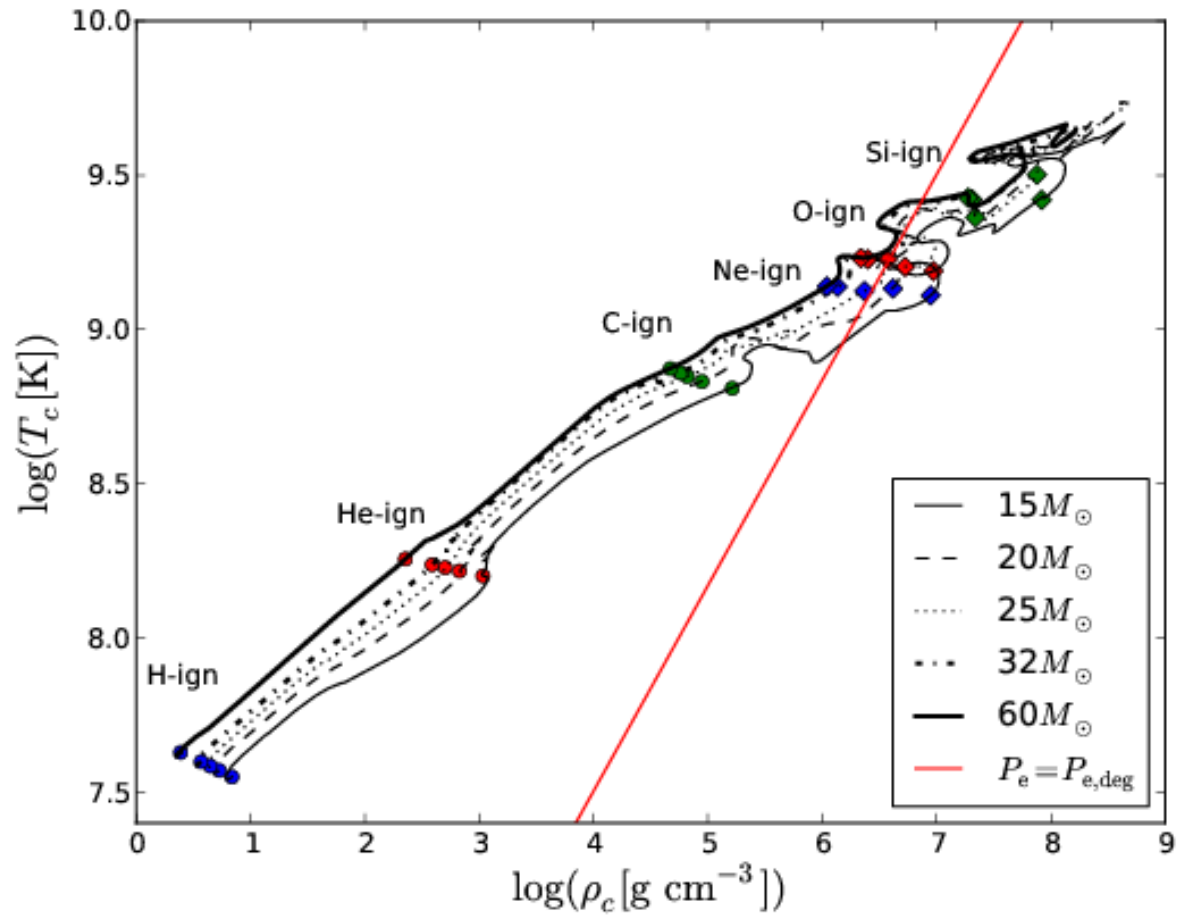
$$E_F(\rho Y_e = 10^7 \text{ gcm}^{-3}) = 0.75 \text{ MeV}$$

$$E_F(\rho Y_e = 10^9 \text{ gcm}^{-3}) = 4.70 \text{ MeV}$$

- Electron fraction:

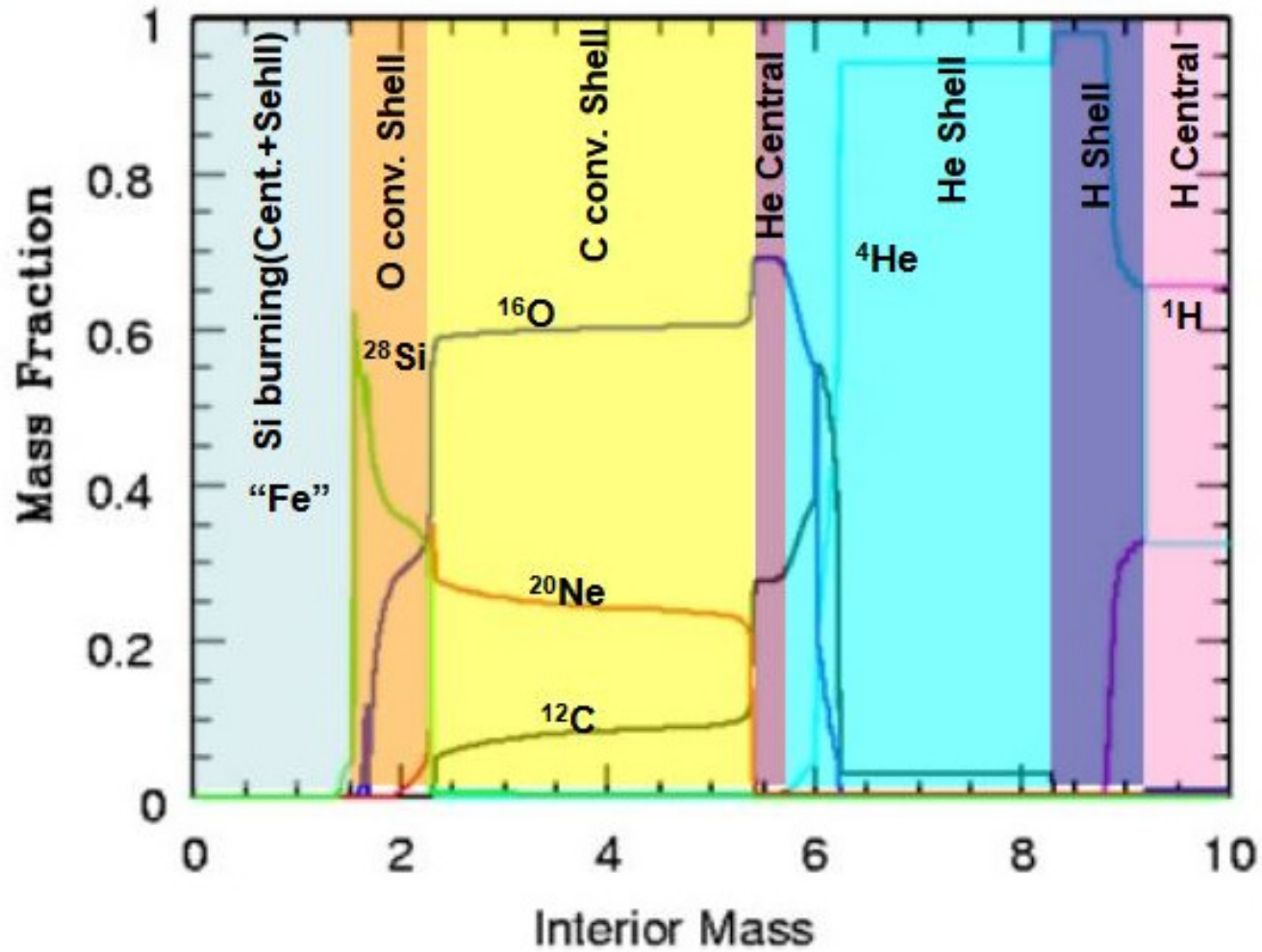


Central evolution

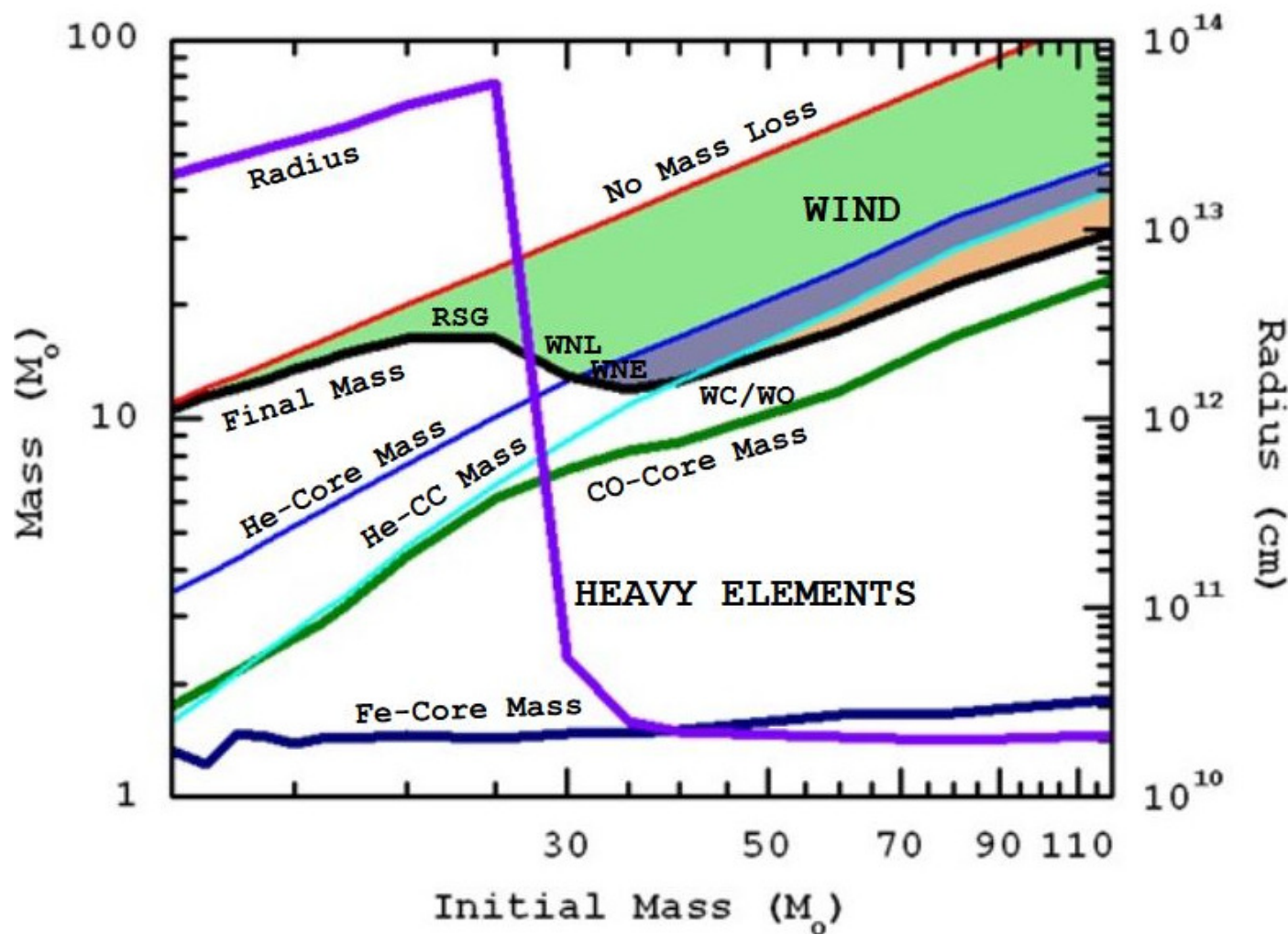


Pre-supernova stage

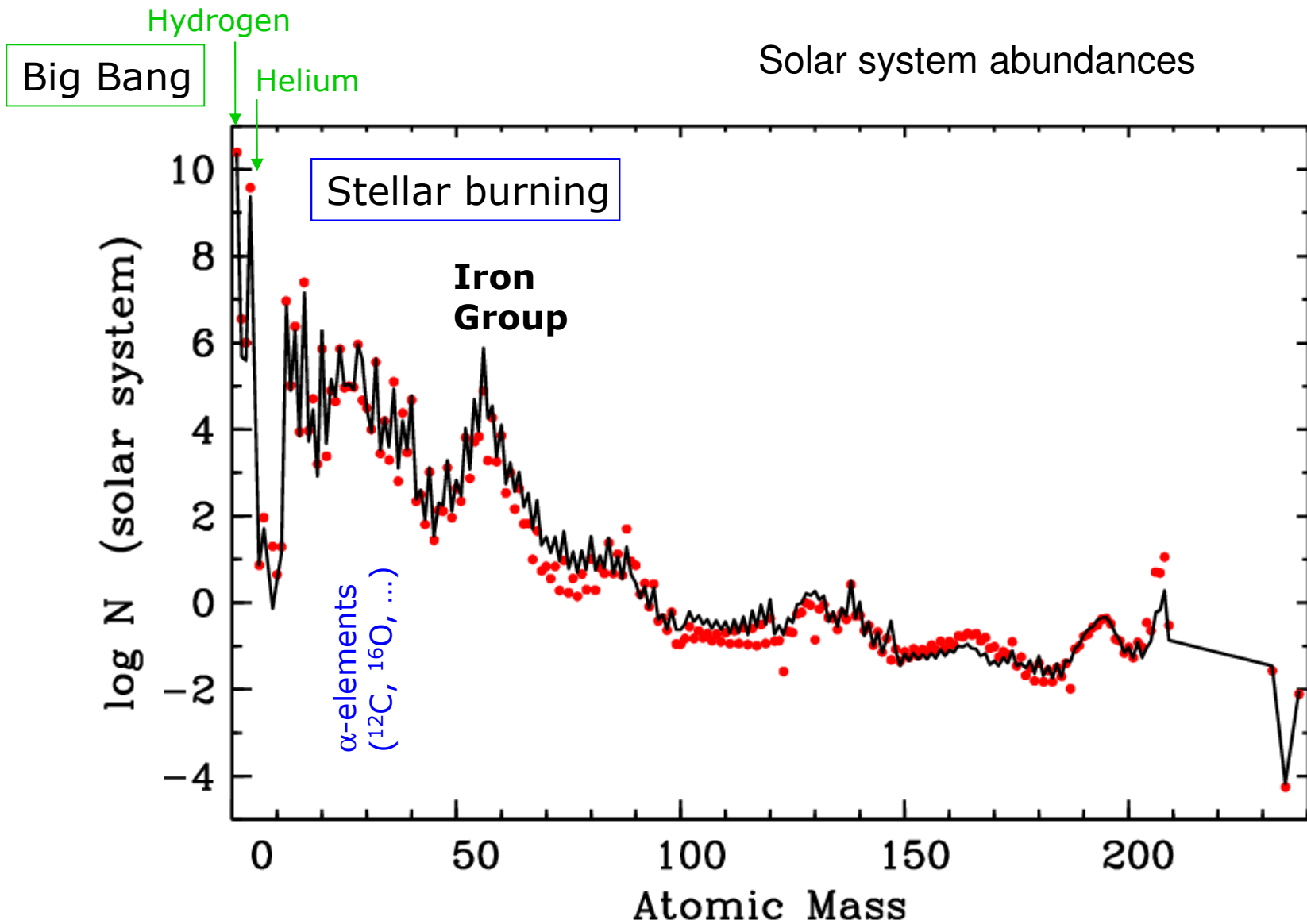
Composition



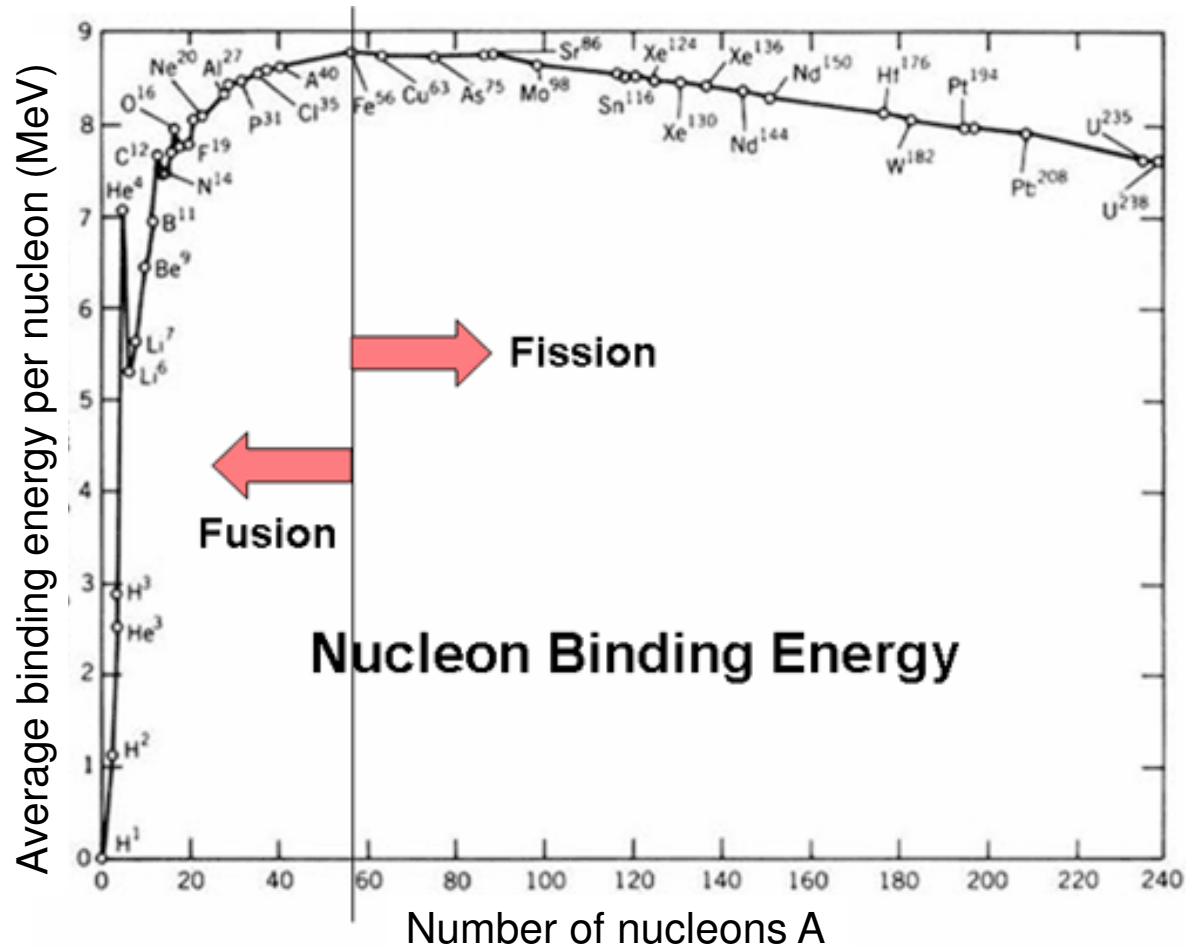
Pre-supernova stage



Origin of elements



Nuclear binding energy



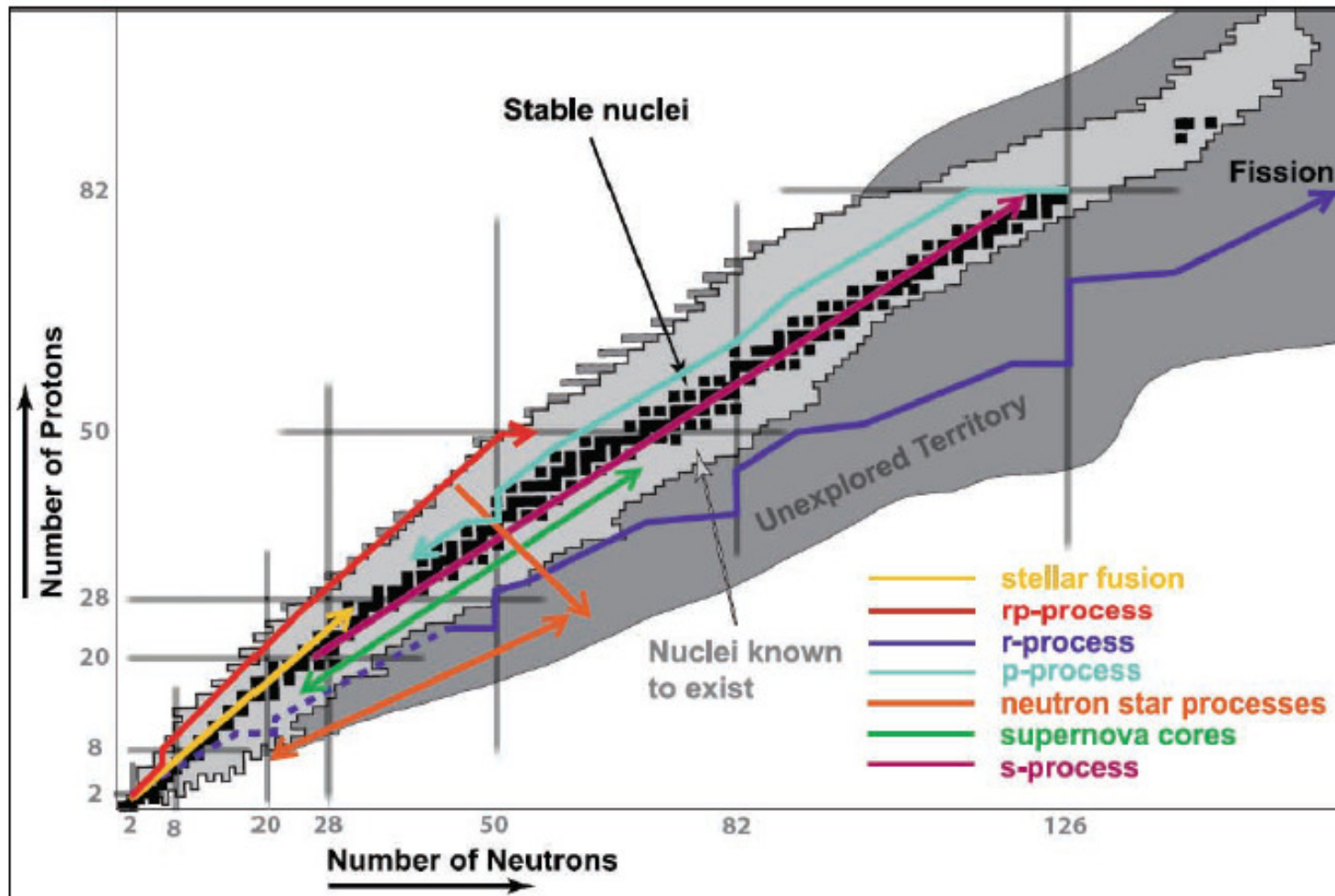
H → He → C → O → → Fe

Nuclear fusion in stellar cores

Need mechanisms other than charged-particle fusion:

E.g. neutrons, photons, neutrinos

Origin of elements



How are nuclei made? Where? Through what processes?

Nuclear physics

- Need to know the relevant nuclear physics:
 - Properties of nuclei (mass, half-life, spin, levels, etc)
 - Properties of reactions between nuclei (and leptons, photons)

Reaction rates

Consider:

- n_i : number density of particles of type i cm^{-3}
- n_j : number density of particles of type j cm^{-3}
- σ : cross section (effective area for reaction) cm^2



- Reactions per time per volume
 = relative flux of particles i $\text{cm}^{-3} \text{cm s}^{-1}$
 × number of particles j cm^{-3}
 × cross section cm^2
 $r = n_i v n_j \sigma(v)$ $\text{cm}^{-3} \text{s}^{-1}$

Reaction rates

- Previously: particles i move at constant v
- For constant relative velocity between particles i and j

→ reacts / vol / time: $r_{i,j} = \int \sigma \cdot |\vec{v}_i - \vec{v}_j| dn_i dn_j$

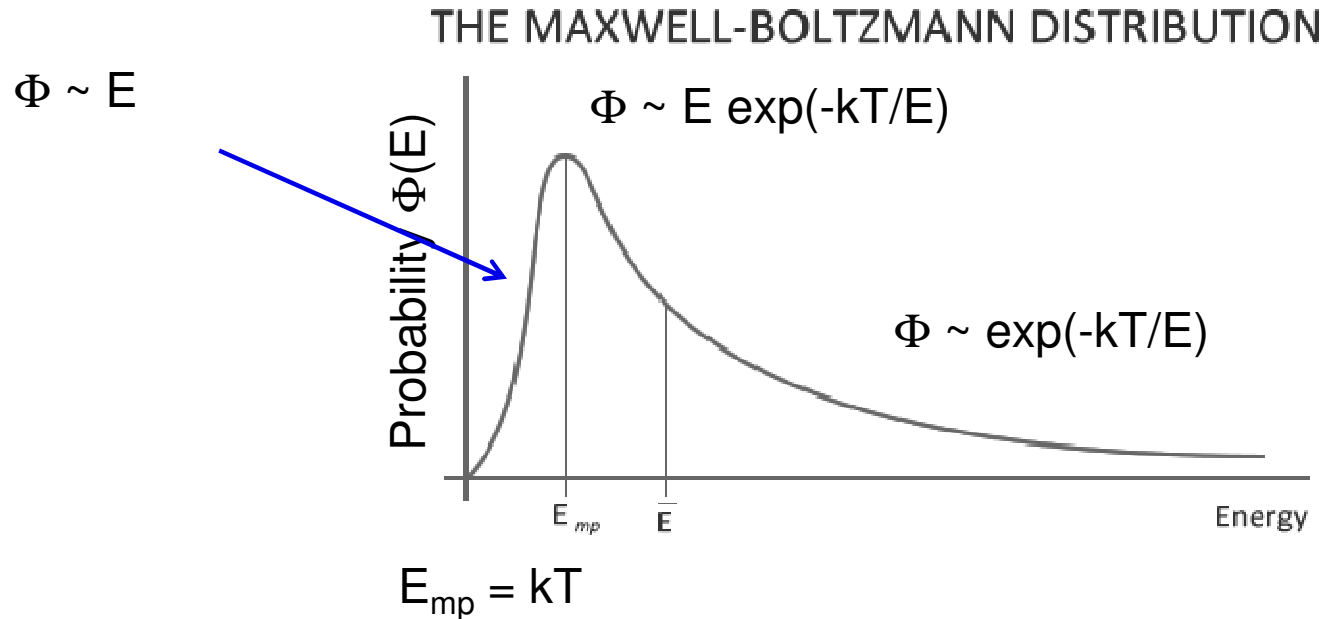
- General: projectiles and targets follow velocity distribution

$$r_{i,j} = n_i n_j \int \sigma(|\vec{v}_i - \vec{v}_j|) |\vec{v}_i - \vec{v}_j| \phi(\vec{v}_i) \phi(\vec{v}_j) d^3 v_i d^3 v_j$$

Integral depends on type of particles and distribution

Maxwell-Boltzmann distribution

- Nuclei in astrophysical plasma are not mono-energetic
- They obey MB distribution



Reaction rates

- Use center-of-mass coordinates, carry out integration, and remember that $\int \phi(\vec{V}) d^3V = 1$

reaction rate becomes $r_{i;j} = n_i n_j \langle \sigma v \rangle_{i;j}$

with the thermonuclear cross section $\langle \sigma v \rangle$

$$\langle \sigma v \rangle (T) = \left(\frac{8}{\mu\pi} \right)^{1/2} \frac{1}{(kT)^{3/2}} \int_0^{\infty} E \sigma(E) \exp(-E/kT) dE$$

- Only depends on temperature
- If we know $\sigma(E)$, we can get $\langle \sigma v \rangle$

Astrophysical S-factor

- Use known energy dependence of $\sigma(E)$
- For charged particles: $\sigma(E)$ is proportional to:
 - Coulomb barrier penetration $\sim \exp(-E^{-1/2})$
 - Nuclear size $\sim 1/E$
- All other energy dependencies are lumped together into astrophysical S-factor $S(E)$
- Why?
 - For non-resonant reactions: $S(E)$ is slowly varying
→ better to work with $S(E)$ if extrapolations are needed

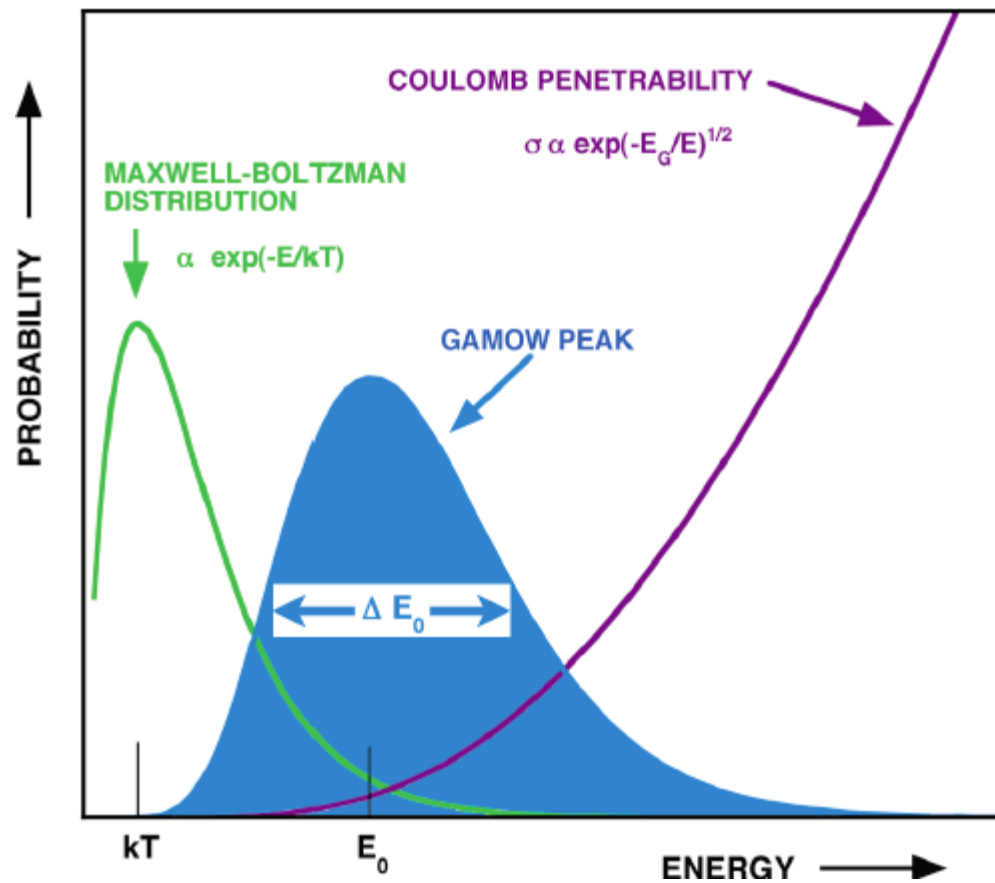
Astrophysical S-factor

- Cross section $\sigma = E^{-1} \times \exp(-E^{1/2}) \times S(E)$
- Reaction rate becomes

$$\begin{aligned}\langle \sigma v \rangle &= \left(\frac{8}{\mu\pi} \right)^{1/2} \frac{1}{(kT)^{3/2}} \int_0^\infty E \sigma(E) \exp(-E/kT) dE \\ &= \left(\frac{8}{\mu\pi} \right)^{1/2} \frac{1}{(kT)^{3/2}} \int_0^\infty S(E) \exp(-bE^{-1/2}) \exp(-E/kT) dE.\end{aligned}$$

- $S(E)$ is slowly varying with E , so integral is dominated by the two exponentials

Gamow peak



Most effective stellar energy

Nuclear reaction networks

- Turn number of reactions per volume and time into differential equation, for a reaction $i(j, o)m$

$$r_{i;j} = \frac{1}{1 + \delta_{ij}} n_i n_j \langle \sigma v \rangle \quad \longrightarrow \quad \begin{aligned} \left(\frac{\partial n_i}{\partial t}\right)_\rho &= \left(\frac{\partial n_j}{\partial t}\right)_\rho = -r_{i;j} \\ \left(\frac{\partial n_o}{\partial t}\right)_\rho &= \left(\frac{\partial n_m}{\partial t}\right)_\rho = +r_{i;j} \end{aligned}$$

- Total rate of change of number density:

$$\dot{n}_i = \left(\frac{\partial n_i}{\partial t}\right)_\rho + n_i \frac{\dot{\rho}}{\rho}$$

- Includes changes due to density change (we are not interested in those)

Abundances, mass fractions

- Matter density ρ (g cm^{-3})
- Number density n depends on matter density
- Can we separate dependence on matter density?
→ Define abundance $Y = n / \rho N_A$
- Units of abundance: mole g^{-1}
- Mass fraction $X_i = A_i Y_i$ with normalized sum

Nuclear reaction networks

- Use abundance $Y_i = \frac{n_i}{\rho N_A}$ $\dot{Y}_i = \frac{\dot{n}_i}{\rho N_A} - \frac{n_i}{\rho N_A} \frac{\dot{\rho}}{\rho}$

- Derivative becomes:

$$\dot{Y}_i = \frac{1}{\rho N_A} \left(\frac{\partial n_i}{\partial t} \right)_{\rho} = -\frac{r_{i;j}}{\rho N_A} = -\frac{1}{1 + \delta_{ij}} \rho N_A \langle \sigma v \rangle_{i;j} Y_i Y_j$$

- For decays (and reactions with photons and leptons):
 - “decay rate” λ
 - Derivate becomes $\dot{Y}_i = -\lambda_i Y_i$

Inverse reactions

- Many reactions are the inverse of an other reaction
- Forward and inverse reactions are linked by time reversal invariance
- For reaction $i(j,o)m$ the thermonuclear cross section depends on
 - Q-value (energy difference between products and reactants)
 - Partition functions (Energy weighted density of states)

$$\langle \sigma v \rangle_{i;j,o} = \frac{1 + \delta_{ij}}{1 + \delta_{om}} \frac{G_m g_o}{G_i g_j} \left(\frac{\mu_{om}}{\mu_{ij}} \right)^{3/2} \exp(-Q_{o,j}/kT) \langle \sigma v \rangle_{m;o,j}$$

Nuclear reaction networks

- Set of coupled differential equations

$$\dot{Y}_i = \sum_j N_j^i \lambda_j Y_j + \sum_{j,k} N_{jk}^i \rho N_A \langle \sigma v \rangle_{jk} Y_j Y_k + \sum_{j,k,l} N_{jkl}^i \rho^2 N_A^2 \langle \sigma v \rangle_{jkl} Y_j Y_k Y_l$$

Specify number of particles created or destroyed; take into account reactions between the same (indistinguishable) species

Thermonuclear cross section

Y .. Abundance
 λ ...decay rate

Nuclear reaction networks

- Set of coupled differential equations

$$\dot{Y}_i = \underbrace{\sum_j N_j^i \lambda_j Y_j}_{\text{Decays, photodisintegrations, reactions with leptons}} + \underbrace{\sum_{j,k} N_{jk}^i \rho N_A \langle \sigma v \rangle_{jk} Y_j Y_k}_{\text{Two-particle reactions}} + \underbrace{\sum_{j,k,l} N_{jkl}^i \rho^2 N_A^2 \langle \sigma v \rangle_{jkl} Y_j Y_k Y_l}_{\text{Three-particle reactions}}$$

- Decays, photodisintegrations, reactions with leptons (e^- , e^+ , ν)
- Two-particle reactions
- Three-particle reactions (e.g. triple- α reaction)
- Right-hand side is sum of all reactions either creating or destroying species i
- Group into 1-body, 2-body, and 3-body reactions programming reasons

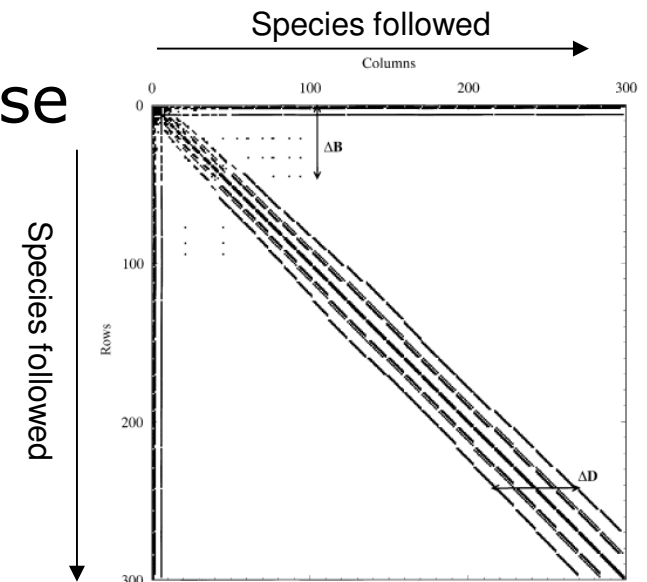
Nuclear reaction networks

- Set of coupled differential equations

$$\dot{Y}_i = \sum_j N_j^i \lambda_j Y_j + \sum_{j,k} N_{jk}^i \rho N_A \langle \sigma v \rangle_{jk} Y_j Y_k + \sum_{j,k,l} N_{jkl}^i \rho^2 N_A^2 \langle \sigma v \rangle_{jkl} Y_j Y_k Y_l$$

- One equation for each species followed
 - ~2000 for supernova nucleosynthesis
 - ~6000 for r-process nucleosynthesis

- Matrix for system of DE is sparse
 - Use special solvers and matrix storage format



How to Model Nucleosynthesis

In principle: need 3D hydro in order to follow convection, mixing, explosion

Problems:

- Coupling of hydro to reaction networks (nucleosynthesis, energy generation)
- Explosions

Compromise:

- (1D) hydro with reduced energy generation network
- Mixing length theory, convection criteria
- Parameterized explosions (mass cut and/or explosion energy as free parameters)

Nevertheless: mostly reliable nucleosynthesis expected
(except for nuclides dependent on explosion mechanism)

Implementation of Networks

- Fully coupled
 - Energy feedback + abundances
- Operator splitting
 - Reduced network for energy generation
 - Abundances in full network (mixing, convection)
- Post-processing
 - Reduced network for energy generation
 - Other abundances from post-processing

Explosive burning

- Similar to hydrostatic burning, but
 - Shorter timescales
 - Higher temperatures
- H-burning:
 - Hot CNO-cycle (pp-chains are too slow), where $^{13}\text{N}(\beta)$ becomes $^{13}\text{N}(p,\gamma)$
- He-burning:
 - N-rich isotopes ^{15}O , ^{18}O , ^{19}F , ^{21}Ne
- C- and Ne-burning:
 - Simultaneously occurring

Explosive burning

- O-burning:
 - Quasi-equilibrium (regions of equilibrium, connected by individual reactions)
- Si-burning:
 - Complete destruction of silicon
 - Details depend on peak temperature and density:
 - Complete Si-burning
 - Incomplete Si-burning (p-rich)
 - Incomplete Si-burning (α -rich)

Explosive burning

