Day 2

Particle Accelerators and Beam Optics

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Outline

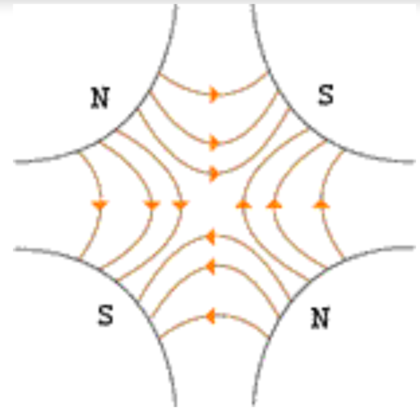
- Lecture 1
  - Overview of types and uses of accelerators
  - Single-pass vs. repetitive systems
  - Transverse vs. longitudinal motion
  - Beams and particle distributions
  - Transverse beam optics

Questions?

- Lecture 2
  - Dispersion
  - Longitudinal beam dynamics
    - » bunchers, re-bunchers; buckets and bunches
  - Optics modules
  - Accelerators for nuclear and high energy physics
  - Future directions

Quick Review

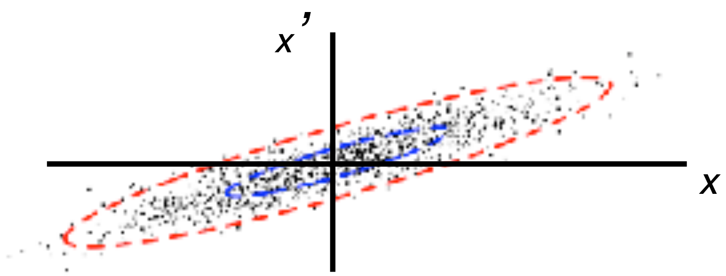
$$B\rho = \frac{A}{Q}\left(\frac{1}{300}\,\frac{\mathrm{T\cdot m}}{\mathrm{MeV/c/u}}\right)\;p_u$$



$$\vec{B} = B' y\,\hat{x} + B' x\,\hat{y}$$

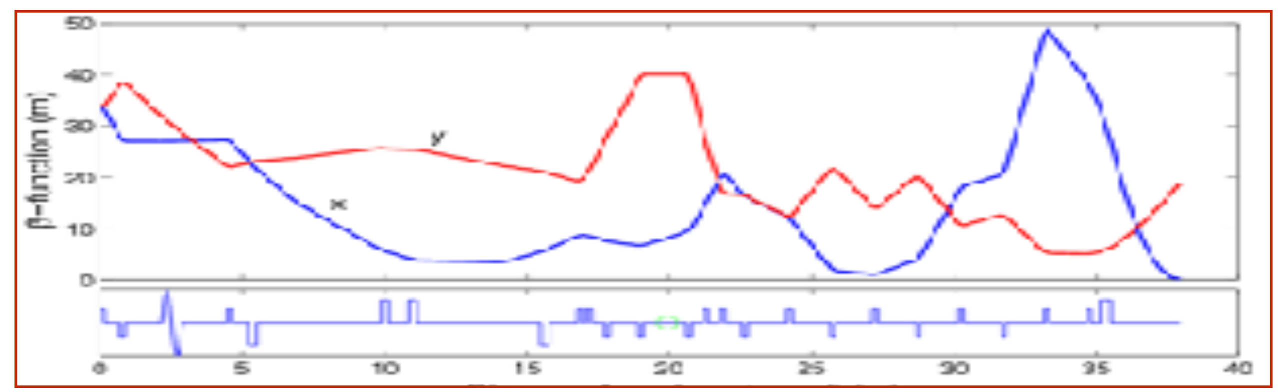
$$B' = \frac{\partial B_y}{\partial x}\Big|_{x=0}$$

$$\gamma x^2 + 2\alpha x x' + \beta x'^2 = \epsilon/\pi$$



$$KL = \frac{B'L}{B\rho} = \frac{1}{F}$$

$$\begin{pmatrix} x \\ x' \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ -\frac{1}{F} & 1 \end{pmatrix}\begin{pmatrix} x_0 \\ x_0' \end{pmatrix}$$



$$\beta \equiv \frac{\langle x^2\rangle}{\epsilon/\pi} \qquad \epsilon = \pi\sqrt{\langle x^2\rangle\langle x'^2\rangle - \langle x x'\rangle^2}$$

$$\begin{pmatrix} x_N \\ x_N' \end{pmatrix} = M_N M_{N-1}\cdots M_2 M_1 \begin{pmatrix} x_0 \\ x_0' \end{pmatrix}$$

$$\gamma \equiv \frac{\langle x'^2\rangle}{\epsilon/\pi} \qquad K \equiv \begin{pmatrix} \beta & -\alpha \\ -\alpha & \gamma \end{pmatrix}$$

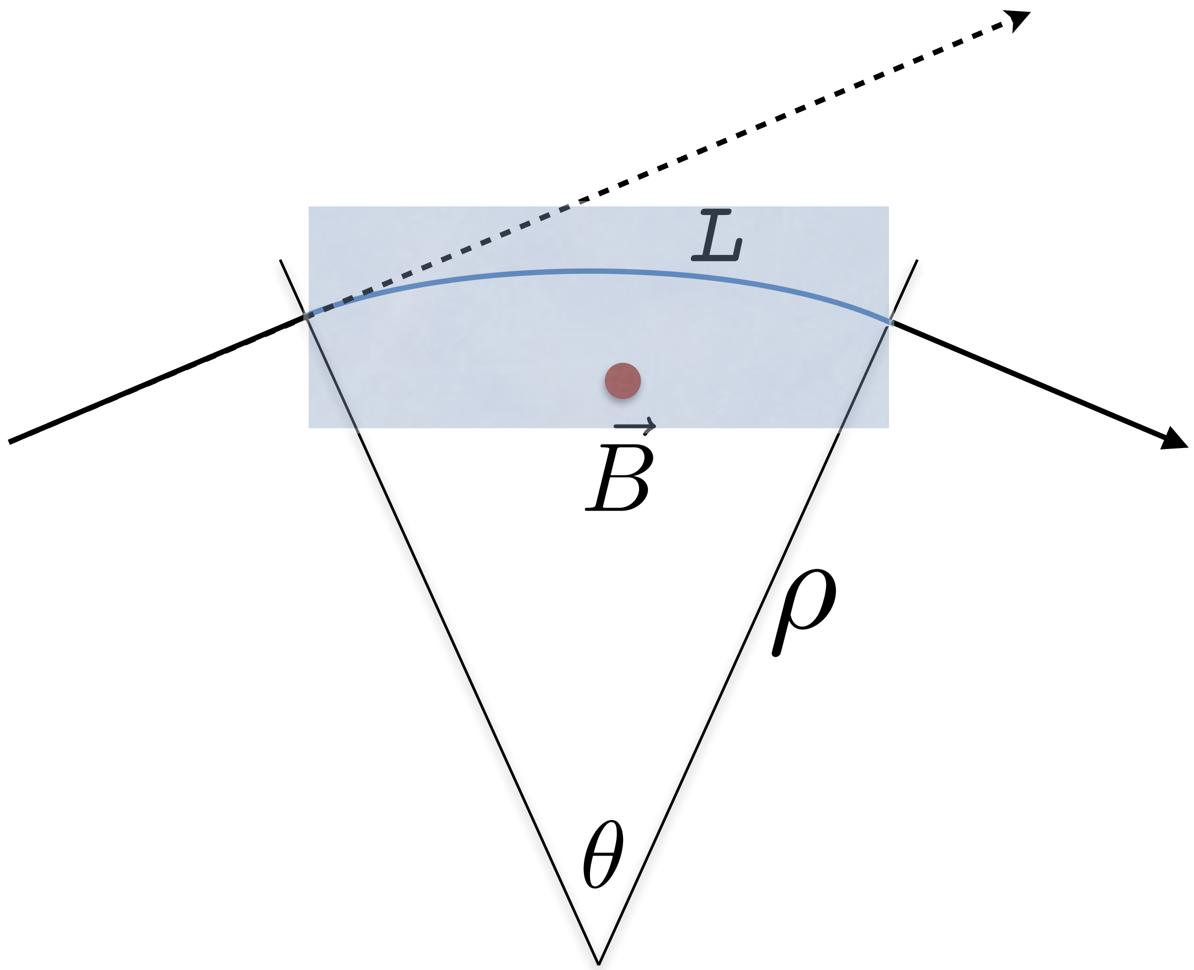
$$\boxed{K_i = M_i\,K_{i-1}\,M_i^T}$$

$$\alpha \equiv -\frac{\langle x x'\rangle}{\epsilon/\pi}$$

$$x_{rms}(s) = \sqrt{\epsilon\beta(s)/\pi}$$

$$\boxed{\epsilon \propto \frac{1}{p} \qquad x_{rms} \propto \frac{1}{\sqrt{p}}}$$

Bending through Dipole Field



$$\theta = \frac{L}{\rho} = \frac{B \cdot L}{(B\rho)}$$

$$= \frac{q \cdot B \cdot L}{p}$$

Dispersion

$$B\rho = \frac{p}{q} \qquad \theta = \frac{qB \cdot L}{p}$$

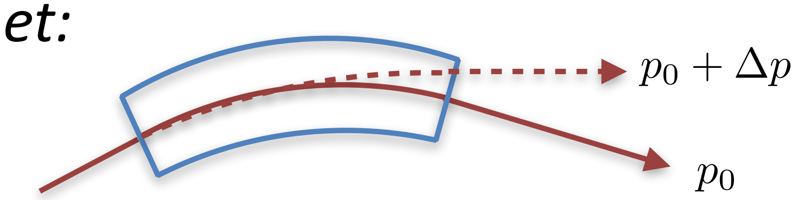
The bend angle (and/or focusing strength) depends upon momentum

Similar to index of refraction depending upon frequency

dipole steering “error” due to a different momentum —> “dispersion”

focusing “error” due to a different momentum —> “chromatic aberration"

dipole magnet:

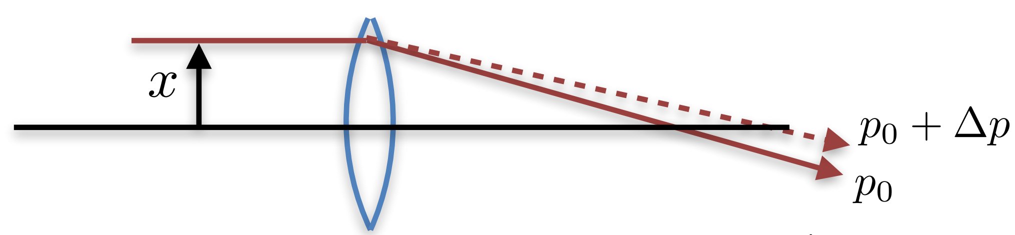


$$\frac{\Delta\theta}{\theta_0} = -\frac{\Delta p}{p}$$

[i.e., in “opposite” direction of bend]

at exit, to lowest order,
$$\Delta x' = \theta_0 \frac{\Delta p}{p}$$
$$\Delta x \approx \frac{1}{2}\ell\theta_0 \frac{\Delta p}{p}$$

likewise, for quadrupole:



$$f = f_0(1 + \frac{\Delta p}{p})$$

Trajectory differences due to momentum differences referred to as “dispersion”

and,
$$\boxed{D(s, \Delta p/p) \approx D(s) \equiv \frac{\Delta x(s)}{\Delta p/p}}$$
“dispersion function”

Dispersion [2]

(see E&S text for details…)

Equation of Motion:

$$x'' + (\frac{B'}{B\rho} + \frac{1}{\rho^2})x = 0 \quad \text{becomes} \quad x'' + \left\{ \left(\frac{1}{1+\Delta p/p} \right) \frac{B'}{B\rho} + \frac{p_0 - \Delta p}{p_0 + \Delta p} \cdot \frac{1}{\rho_0(s)^2} \right\} x = \frac{1}{\rho_0} \frac{\Delta p}{p_0 + \Delta p}$$

let $x = D\ \Delta p/p,$ particular solution

(must add the homogeneous solution, which we have found previously)

betatron oscillation

then,

$$D'' \frac{\Delta p}{p_0} + \left\{ \left(\frac{1}{1+\Delta p/p} \right) \frac{B'}{B\rho} + \frac{p_0 - \Delta p}{p_0 + \Delta p} \cdot \frac{1}{\rho_0(s)^2} \right\} D \frac{\Delta p}{p_0} = \frac{1}{\rho_0} \frac{\Delta p}{p_0 + \Delta p}$$

keep only terms linear in the relative momentum deviation,

$$D'' \frac{\Delta p}{p_0} + \left(\frac{B'}{B\rho} + \frac{1}{\rho_0^2} \right) D \frac{\Delta p}{p_0} = \frac{1}{\rho_0} \frac{\Delta p}{p_0}$$

a driven betatron oscillation, with a *constant* driving term. The "driver" is the dipole field within a bending magnet

$$\boxed{D'' + K\ D = \frac{1}{\rho_0}}$$

so, solutions are $\sin\sqrt{K}\ell$ & $\cos\sqrt{K}\ell$ plus *const.*

Dispersion [3]

$$D'' + K\ D = \frac{1}{\rho_0}$$

In terms of matrices…

$\boxed{K = 0 :}$

$$D'' = \frac{1}{\rho}, \quad D' = \frac{s}{\rho} + D'_0$$

$$D = D_0 + D'_0 s + \frac{1}{2}\frac{s^2}{\rho}$$

$$\begin{pmatrix} D \\ D' \\ 1 \end{pmatrix} = \begin{pmatrix} 1 & s & \frac{1}{2}s^2/\rho \\ 0 & 1 & s/\rho \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} D_0 \\ D'_0 \\ 1 \end{pmatrix}$$

in the limit of short, or "thin" elements, a bending magnet primarily changes the slope of the dispersion function by an amount equal to the bend angle of the magnet

$\boxed{1/\rho = 0 :}$

same 2x2 as before

$$\begin{pmatrix} D \\ D' \\ 1 \end{pmatrix} = \begin{pmatrix} M & \begin{pmatrix} 0 \\ 0 \end{pmatrix} \\ 0 & 1 \end{pmatrix} \begin{pmatrix} D_0 \\ D'_0 \\ 1 \end{pmatrix}$$

otherwise, the *D* transports roughly like a betatron oscillation

So, can use matrix methods (3x3 now; and 2x2 in "vertical" plane) to solve for:

$$\beta_x, \quad \alpha_x, \quad \psi_x$$

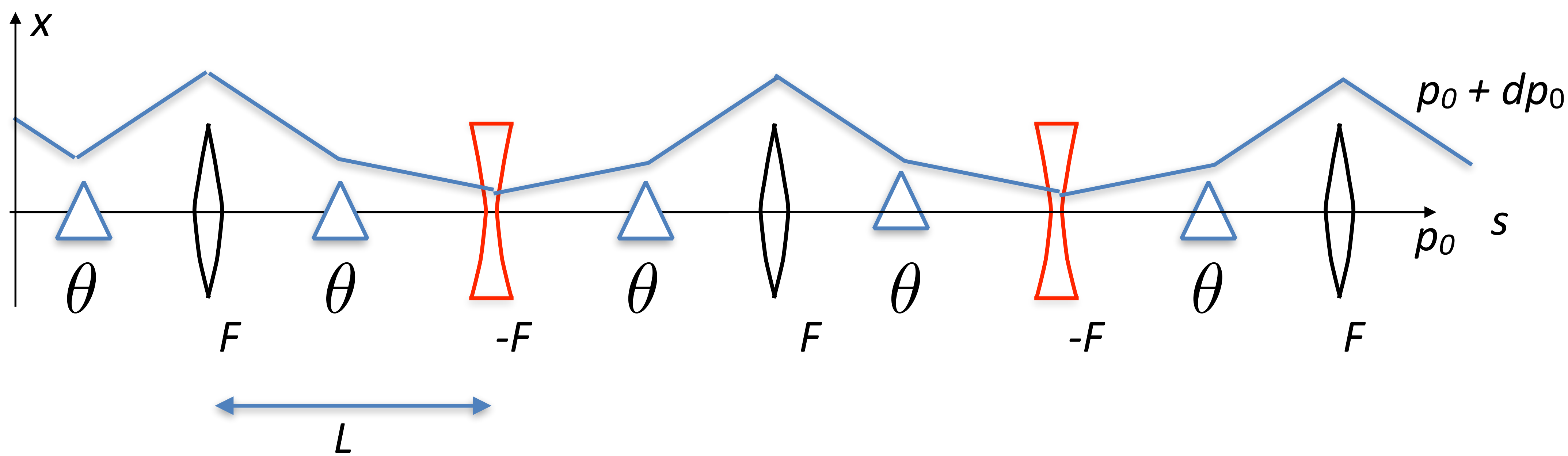
$$\beta_y, \quad \alpha_y, \quad \psi_y$$

$$D_x, \quad D'_x$$

(& D_y, D'_y , if also have vertical bending)

FODO Channel

- System of quadrupoles with alternating-sign gradients (F, D, F, …) separated by distance *L*, and with bending magnets in-between…



- Can show …

$$D_{max,min} = \frac{2F^2\theta}{L}\left(1 \pm \frac{L}{4F}\right)$$

typically, on order of a few meters

Ex: D = 4 m, dp/p = 0.1%, then Δx = 4 mm

Beam Size Including Dispersion

- Total excursion due to “off momentum” plus betatron oscillation:

$$x = x_\beta + D\ \delta \qquad\qquad \delta \equiv \Delta p/p$$

$$x^2 = x_\beta^2 + 2x_\beta D\delta + D^2\delta^2$$

- Assuming no correlation between x_β and particle’s momentum:

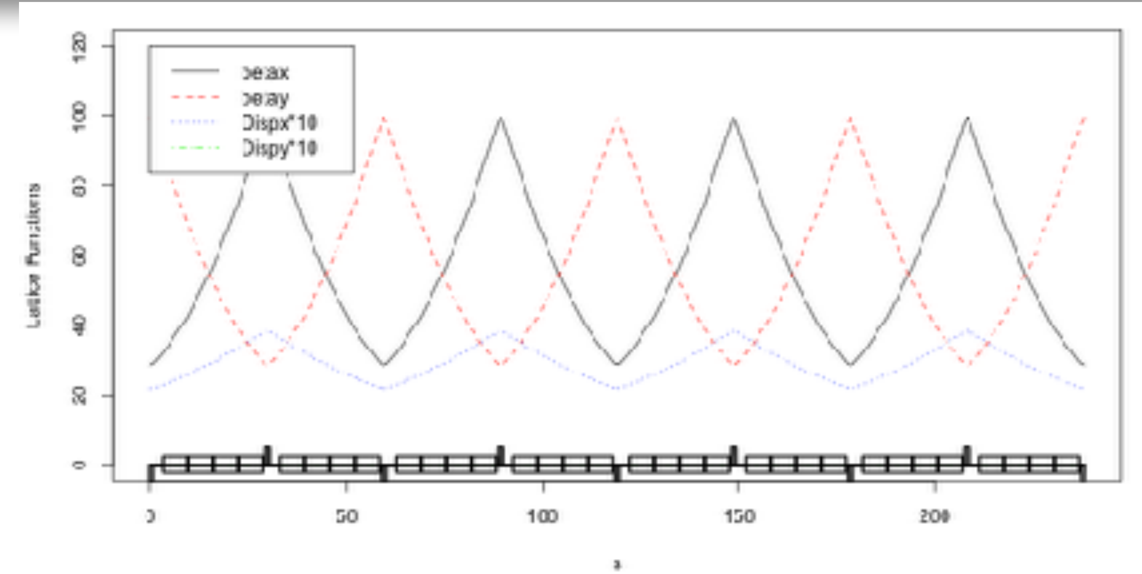
$$\langle x^2\rangle = \langle x_\beta^2\rangle + D^2\langle\delta^2\rangle$$

$$\langle x^2\rangle = \epsilon\beta/\pi + D^2\langle\delta^2\rangle$$

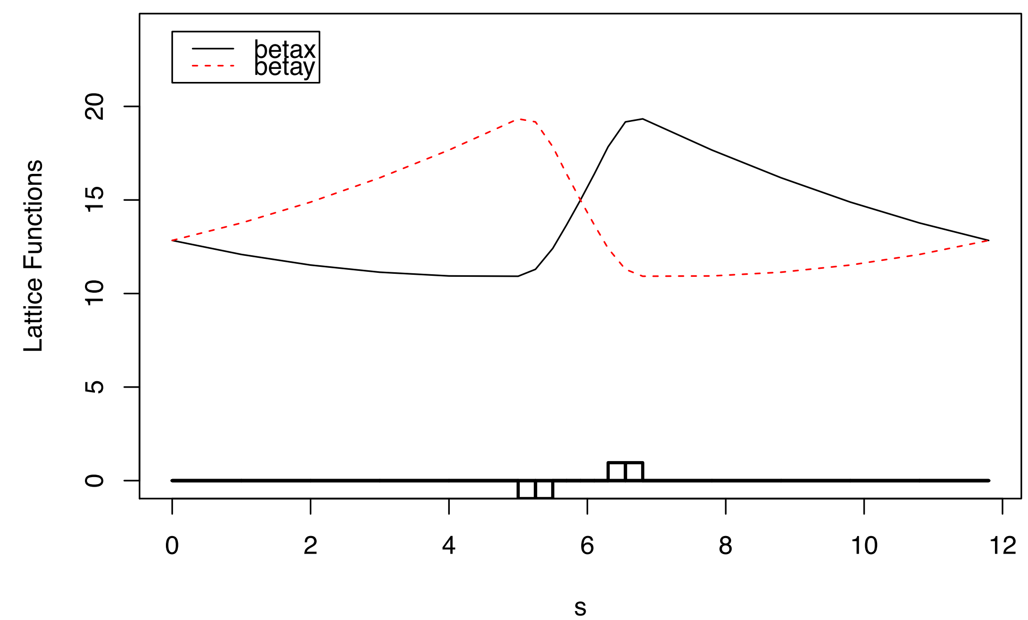
Optical Modules

- Very often useful to think of optical systems in terms of modules
- Each module has a purpose and/or special conditions to be met
  - general beam transport; achromatic; large dispersion for momentum selection, charge selection; small dispersion for isochronous transit; final focus onto target; long drift space for equipment; compact bending; etc. ...
- Large/long systems are best generated with (stable!) periodic lens systems -- may or may not have bending
- Often need longer spaces for instrumentation, RF, switching magnets, experiments, etc.
- May need to match one focusing structure into a different focusing structure (e.g., change of cryomodule lengths, etc.)
- Simultaneously trying to control $(\alpha, \beta, D, D')_{x,y}$ [and sometimes $\psi_{x,y}$] as well as X,Y,Z and X',Y',Z' of the ideal trajectory along the beam line!
  - *various computer programs are good at this*

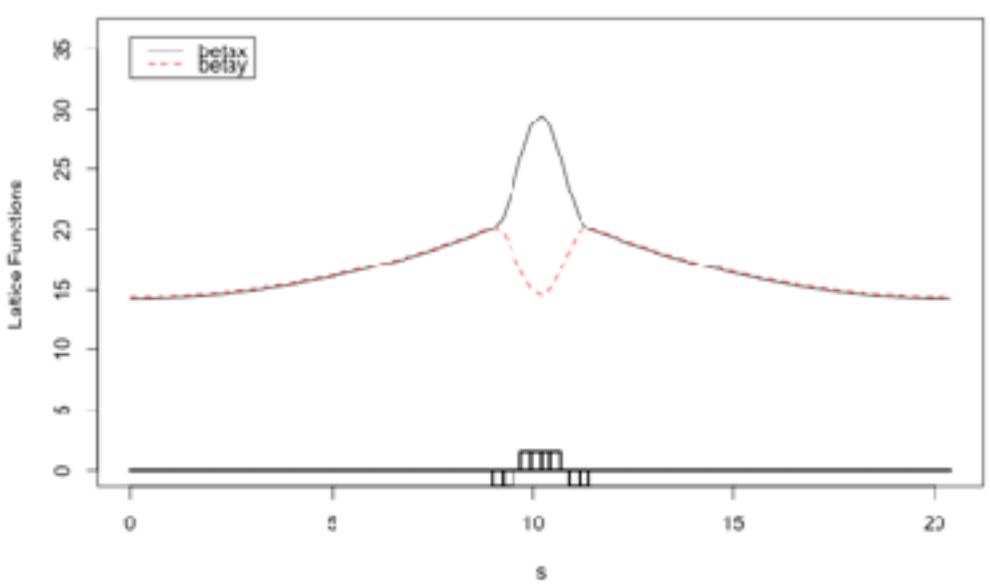
Doublets and Triplets

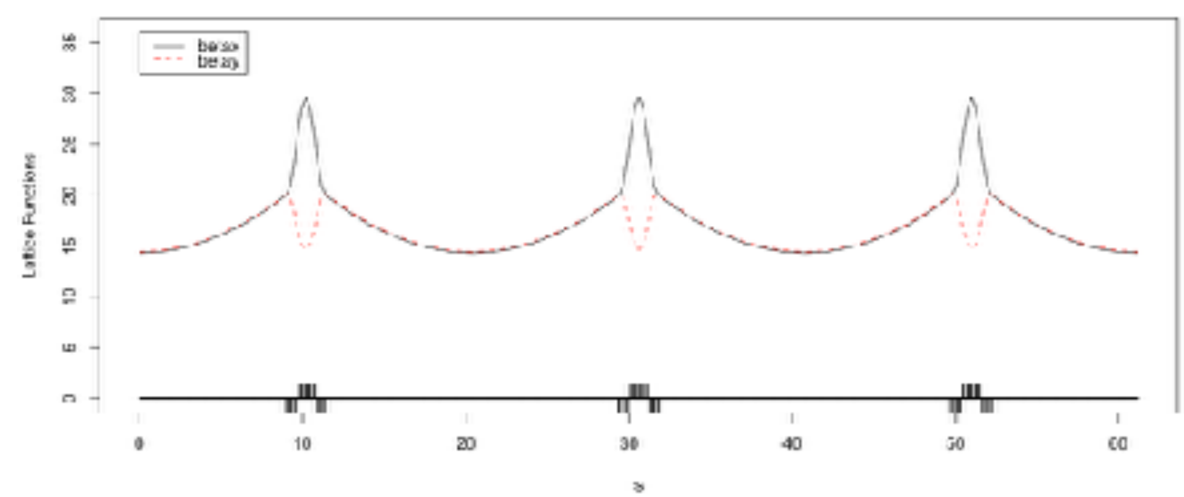


- FODO Cells
  - basic transport; equal spacing; easy analysis



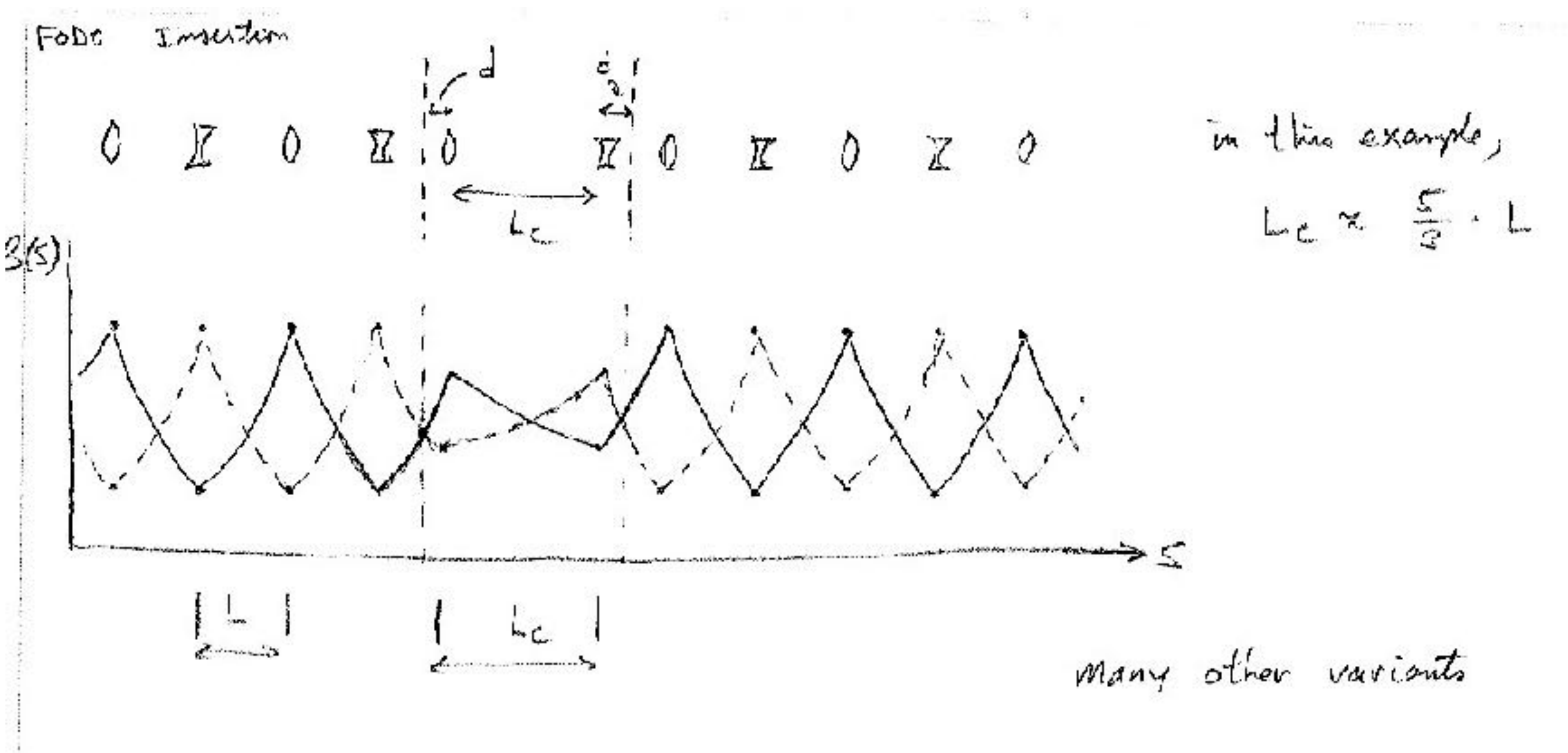
- Doublets
  - can be used to generate more space
- Triplets
  - can be used to keep beam “round”



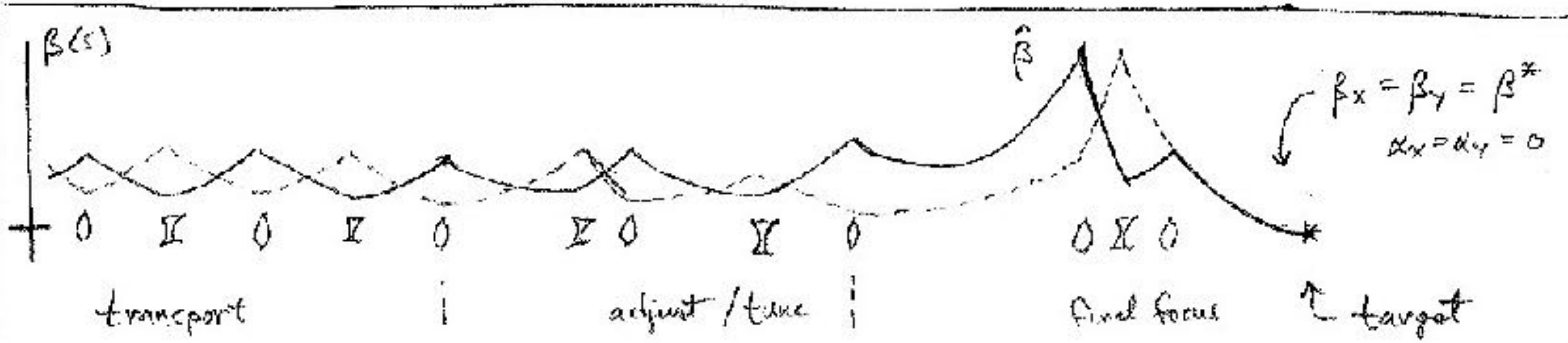


Collins Straight Section Insertion

- Wish to increase space between quadrupoles, perhaps to insert special element into the beam line. In order to match H and V optics simultaneously, we want $\alpha_x = -\alpha_y$ and $\beta_x = \beta_y$ at the match point(s).
- Where can we use this?



Final Focus



telescope-like : "eyepiece" "objective"

$|\leftarrow L^* \rightarrow|$

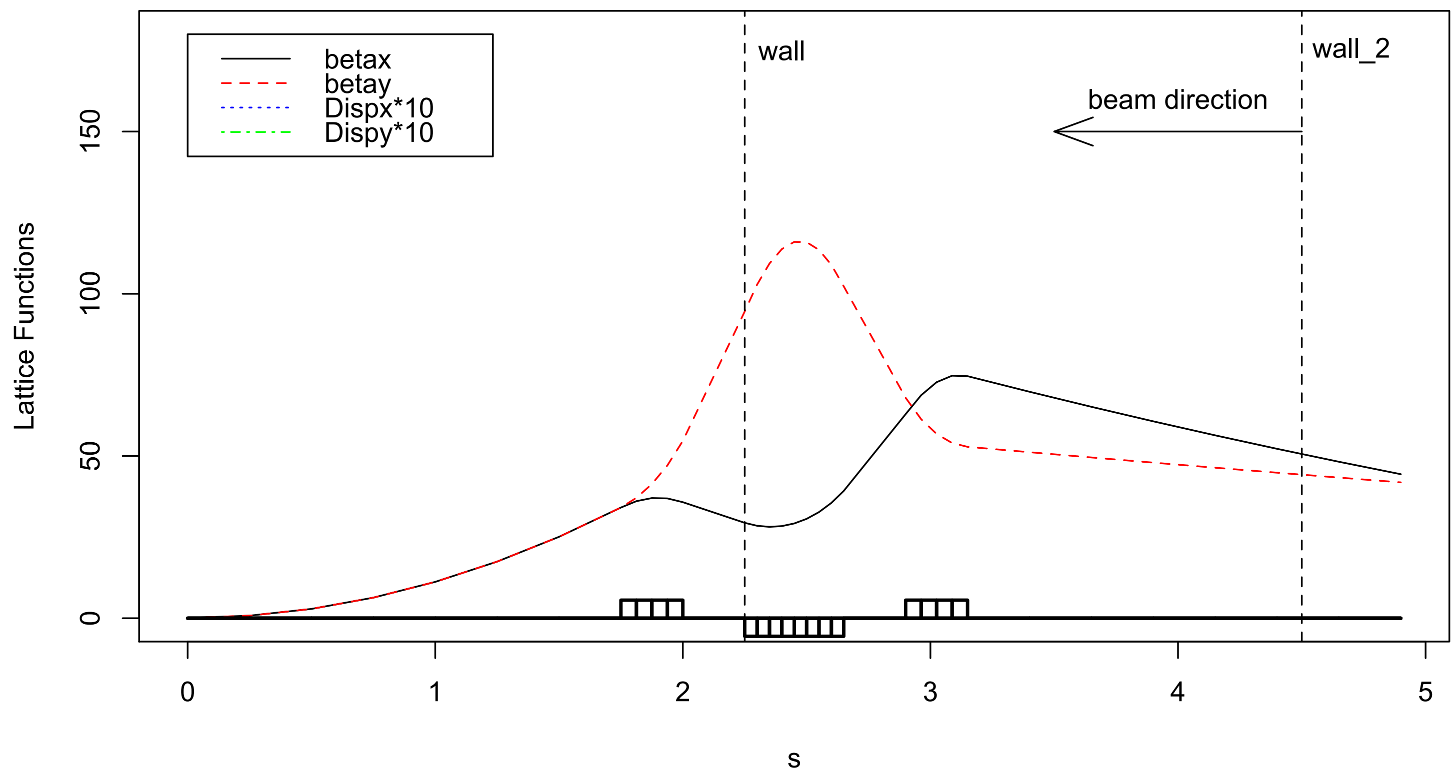
To tune, consider starting w/ $\beta^*, \alpha^* = 0$, and working backward to find values for tuning quads that match β_x, β_y back into the transport region

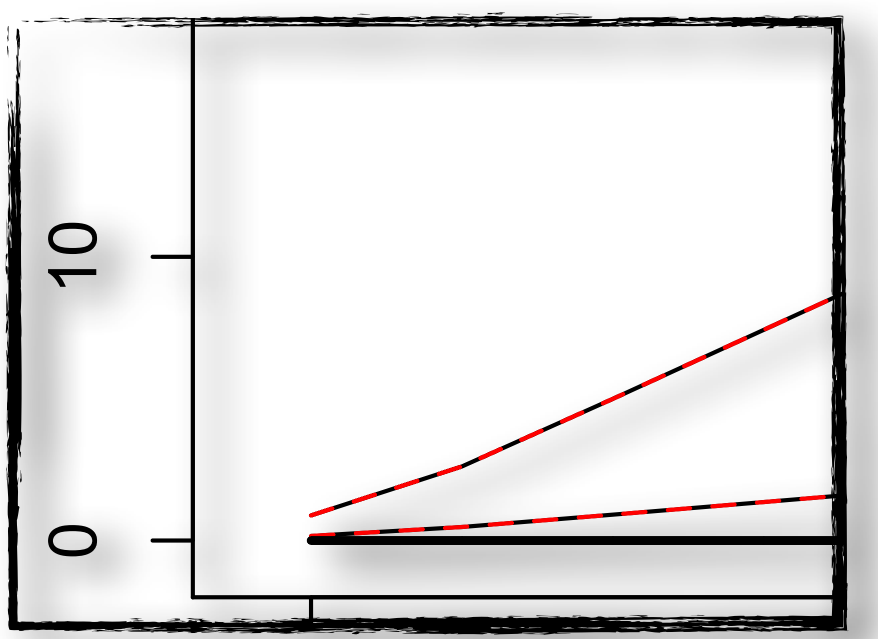
$$\hat{\beta} \approx L^{*2}/\beta^*$$

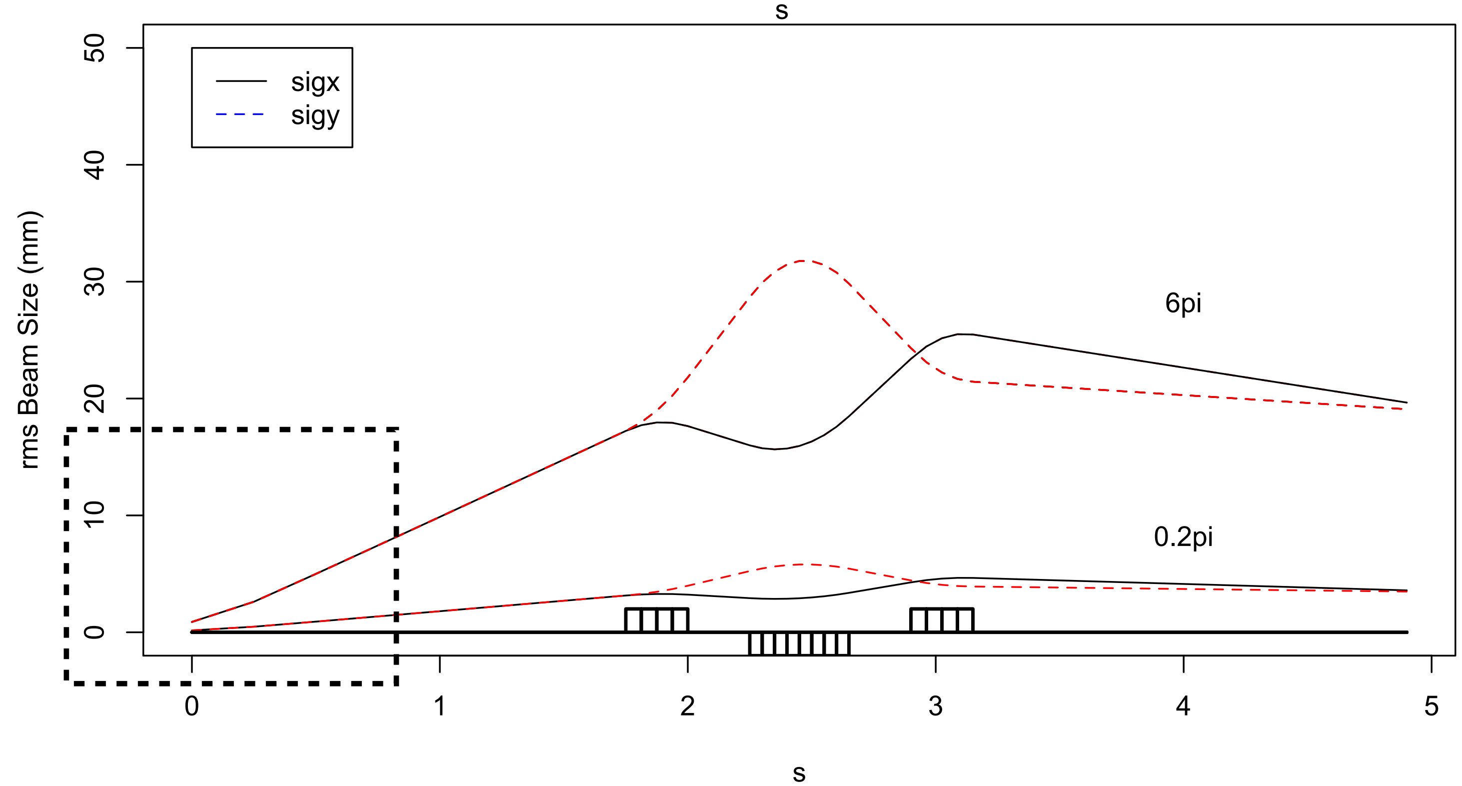
tune 4 quads to match 4 parameters $(\beta, \alpha)_{x,y}$

Final Focus [2]

- FRIB Final Focus:
- Beam size max/min
  - max/min ~ 35
  - max(linac)/min(target) ~ 10







Double-Bend Achromat

Achromat: entr / exit w/ $D = D' = 0$ (D = dispersion)

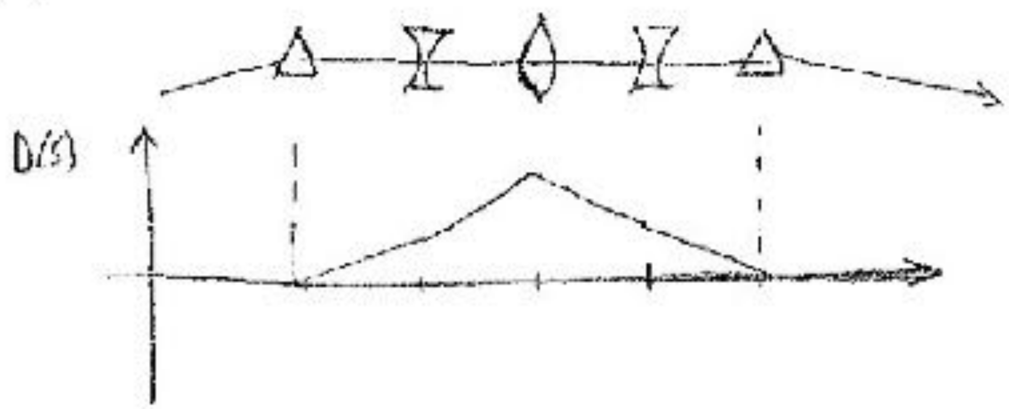
simple concept: use 2 bend magnets w/ quad(s) in between

if $\underline{d = 2f}$, then $\Delta D'$ at lens: $\Delta D' = -\frac{d\theta}{f}$

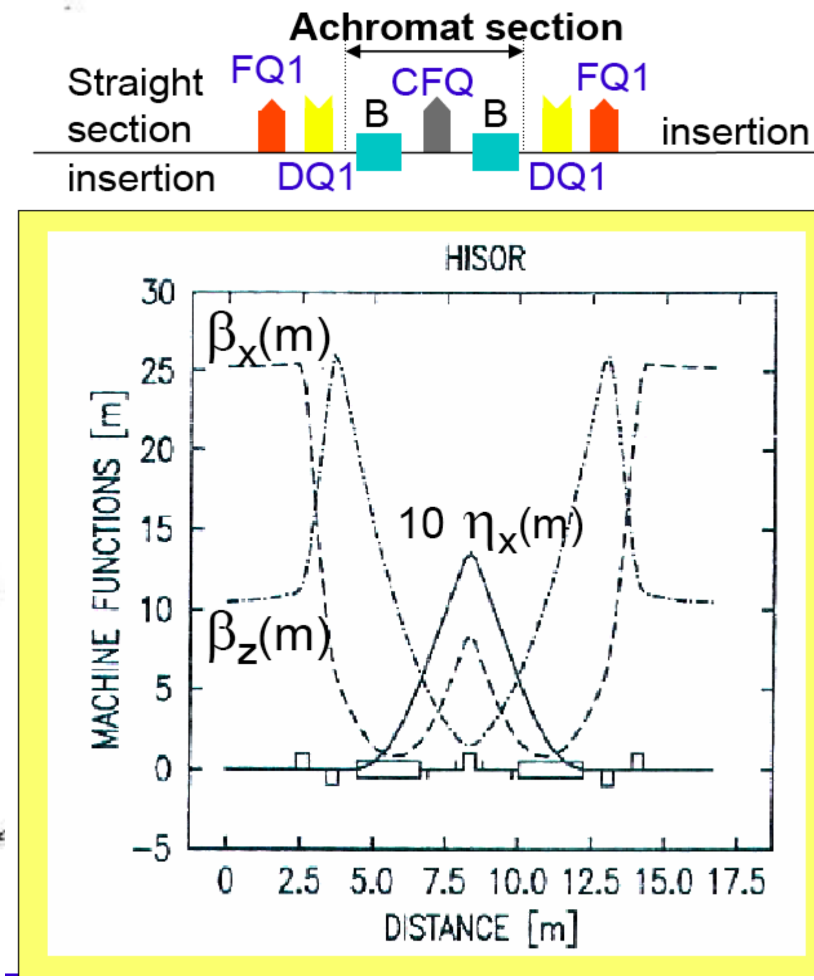
$= -2\theta$

Note: since lens focuses in x, defocuses in y,

can use a triplet:

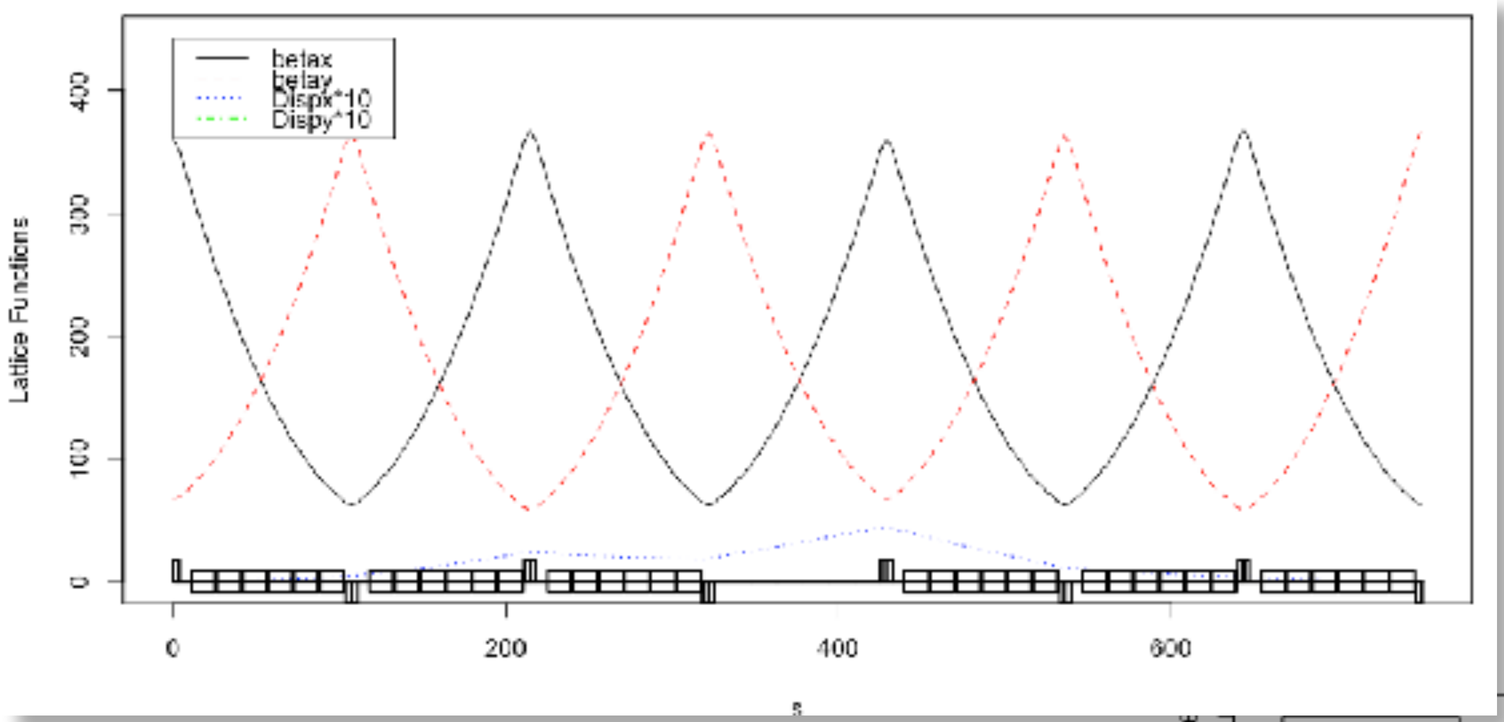


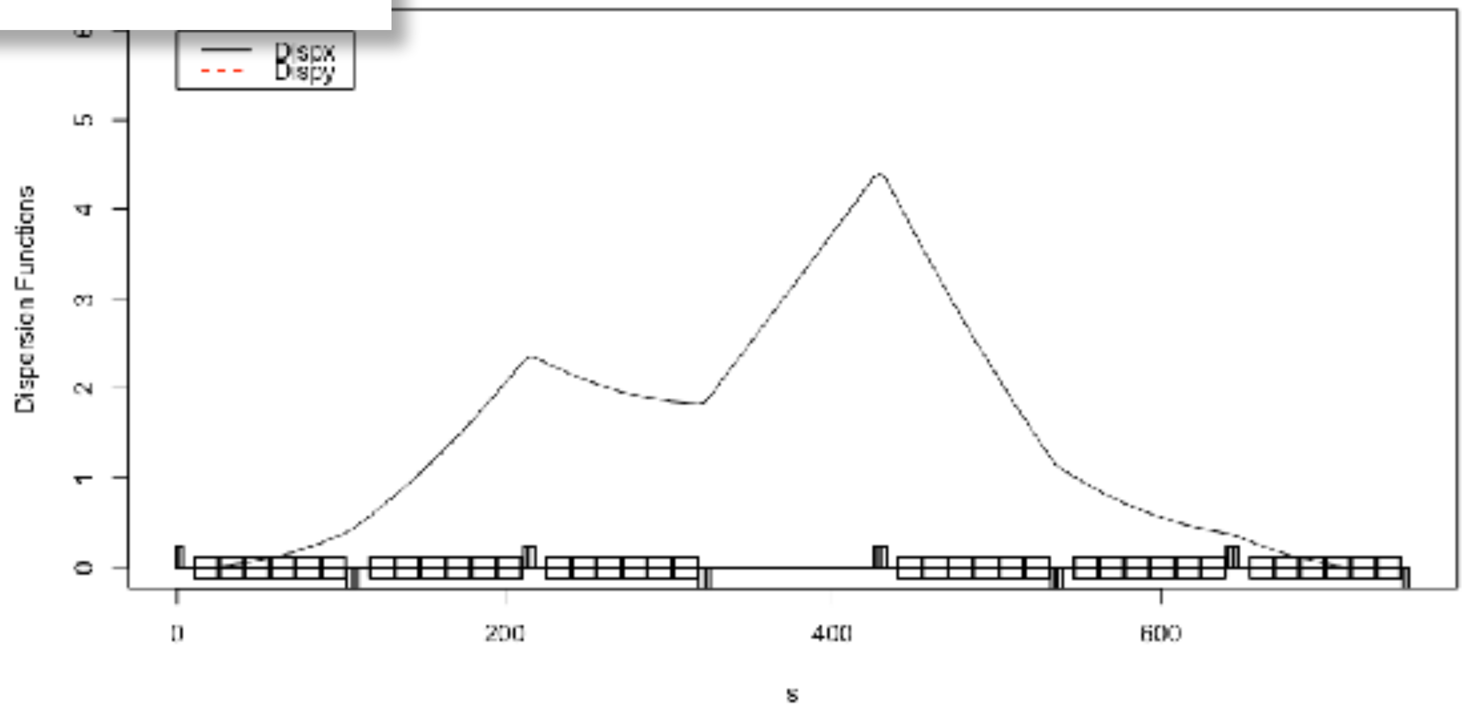
Chasman-Gree



Achromatic Sections

- FODO Achromatic Bend Section:





Chromatic Corrections and Chromaticity

- Focusing effects from the magnets will also depend upon momentum:

$$x'' + K(s,p)x = 0 \qquad K = e(\partial B_y(s)/\partial x)/p$$

- To give all particles the similar optics, regardless of momentum, need a "gradient" which depends upon momentum. Orbits spread out horizontally (or vertically) due to dispersion, can use a sextupole field:

$$\vec{B} = \frac{1}{2}B''[2xy\ \hat{x} + (x^2 - y^2)\ \hat{y}]$$

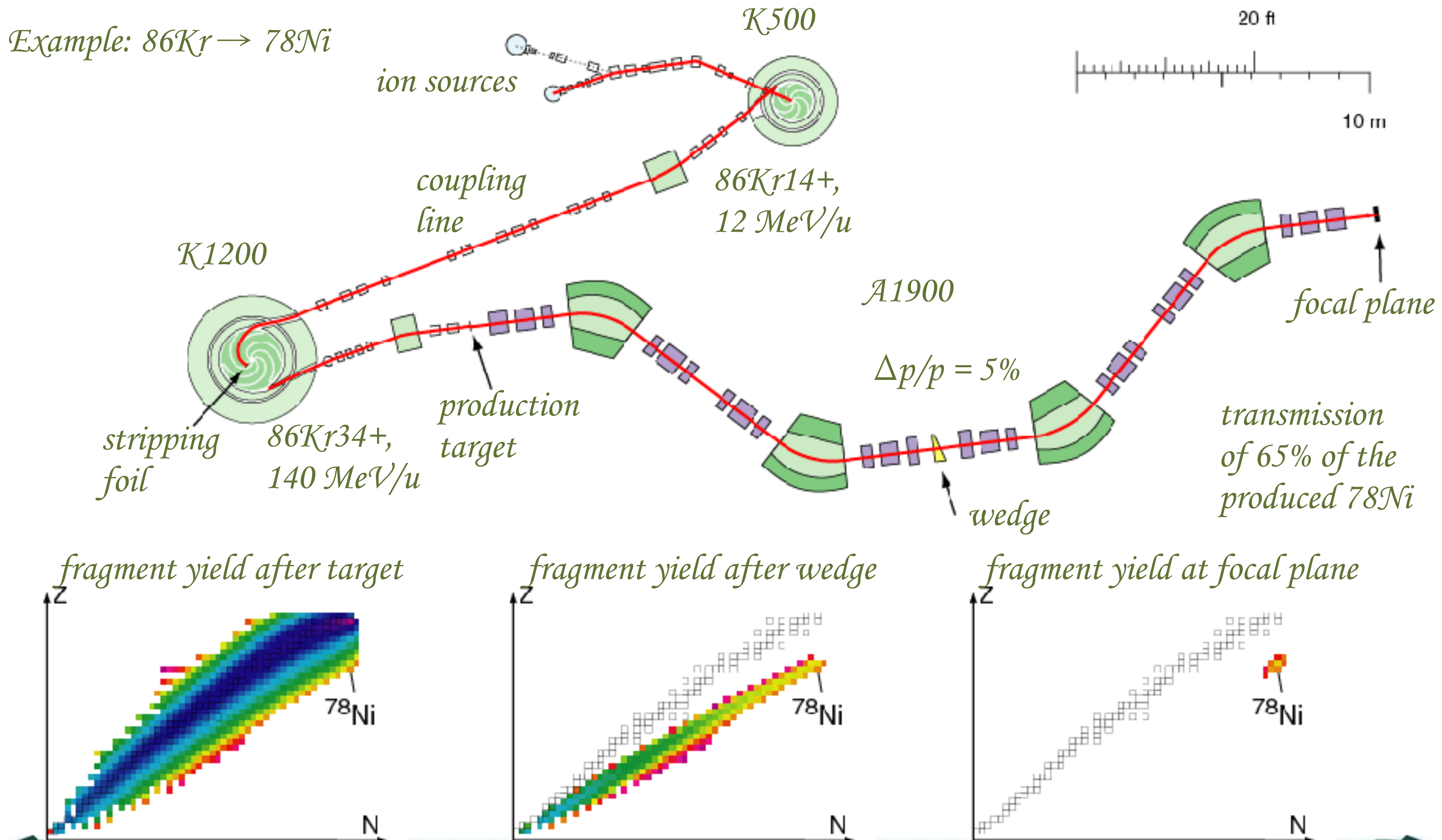
- which gives $\partial B_y/\partial x = B''x = B''D(\Delta p/p)$ (for $y = 0$)
  -
  - i.e., a field gradient which depends upon momentum

- Chromaticity* is the variation of optics with momentum; use sextupole magnets to control/adjust; but, now introduces a nonlinear transverse field ...
  - can have a transverse dynamic aperture!

*In a synchrotron, "the" chromaticity is the variation of the transverse oscillation frequency with momentum

In-Flight Production Example: NSCL's CCF

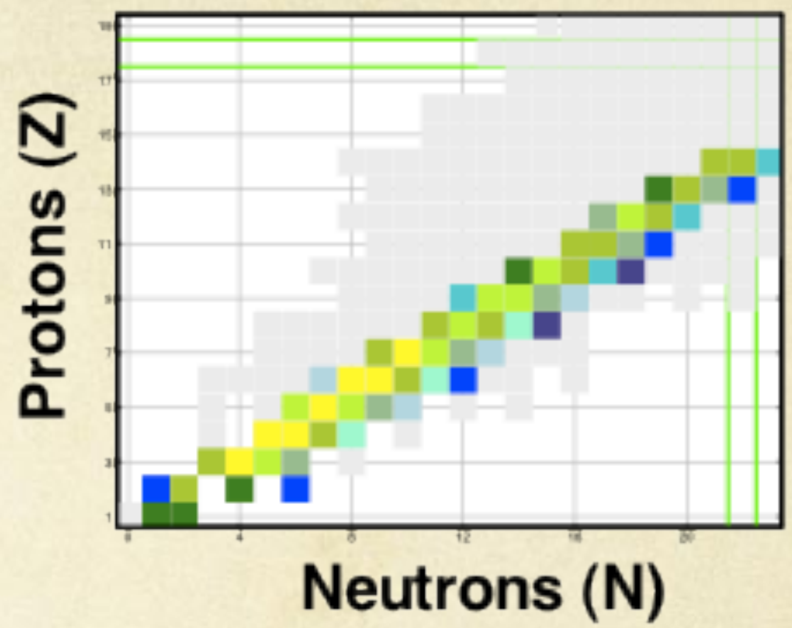
D.J. Morrissey, B.M. Sherrill, Philos. Trans. R. Soc. Lond. Ser. A. Math. Phys. Eng. Sci. 356 (1998) 1985.



Principle of Fragment Separator [2]

M. Hausmann, FRIB

- Magnetic separation alone insufficient
- Numerous nuclides with similar A/Z
- But with different proton number (Z)
- Energy loss in m Z dependent
  - Bethe formula (above)
- Interaction of beam with degrader (a piece of metal) leads to different velocity changes for different fragments
  - Previously similar magnetic rigidities get "dispersed"
- This allows to separate these by magnetic rigidity ⟶ mass selection



Longitudinal Focusing

- sometimes referred to as “phase focusing” or “time focusing”
- particles of different energy (momentum) move at different speeds, so tend to “spread out” relative to the “ideal” particle which is assumed to exist traveling with perfect synchronism with respect to the oscillating fields
- wish to study the (longitudinal) motion of particles relative to this “synchronous particle”

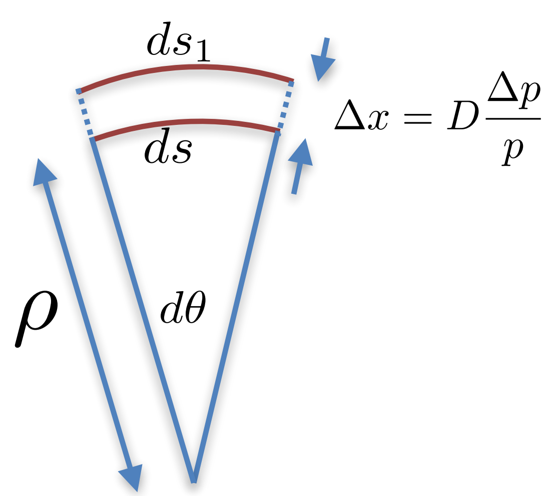
Longitudinal Focusing

- time of flight — the “slip factor”
- Evolution due to *dp/p* or *dW/W*
- Longitudinal focusing, time of arrival:
  - bunchers, rebunchers, debunchers

Momentum Compaction Factor

- How does path length along the beam line depend upon momentum?
  - in straight sections, no difference; in bending regions, *can* be different

Look closely at an infinitesimal section along the ideal trajectory…



$$d\theta = \frac{ds}{\rho} = \frac{ds_1}{\rho + \Delta x}$$

$$ds_1 - ds = \left(\frac{\rho + \Delta x}{\rho} - 1\right) ds$$

$$= \frac{\Delta x}{\rho} ds = \frac{D}{\rho}\,\frac{\Delta p}{p}\,ds$$

if L = path length along ideal trajectory between 2 points, then

$$\frac{\Delta L}{L} = \boxed{\frac{\int \frac{D(s)}{\rho(s)} ds}{\int ds}} \cdot \frac{\Delta p}{p}$$

The relative change in path length, per relative change in momentum, is called the *momentum compaction factor,*

$\alpha_p = \langle D/\rho \rangle$ along the ideal path

The Slip Factor

$$t = \frac{L}{v}$$

$$\frac{dt}{t} = \frac{dL}{L} - \frac{dv}{v}$$

$$\frac{dv}{v} = \frac{1}{\gamma^2}\frac{dp}{p}$$

$$\frac{dt}{t} = \left(\alpha_p - \frac{1}{\gamma^2}\right)\frac{dp}{p}$$

$$\frac{dt}{t} = \eta\,\frac{dp}{p}$$

Momentum Compaction Factor:

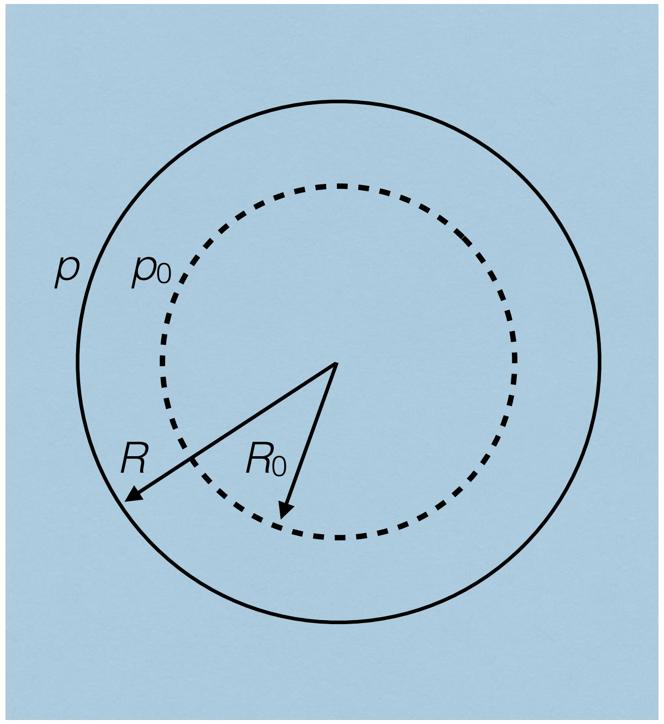
$$\alpha_p \equiv \left(\frac{dL/L}{dp/p}\right)$$

$$\alpha_p = \frac{\int [D(s)/\rho(s)]ds}{\int ds} = \langle D/\rho \rangle$$

D = dispersion

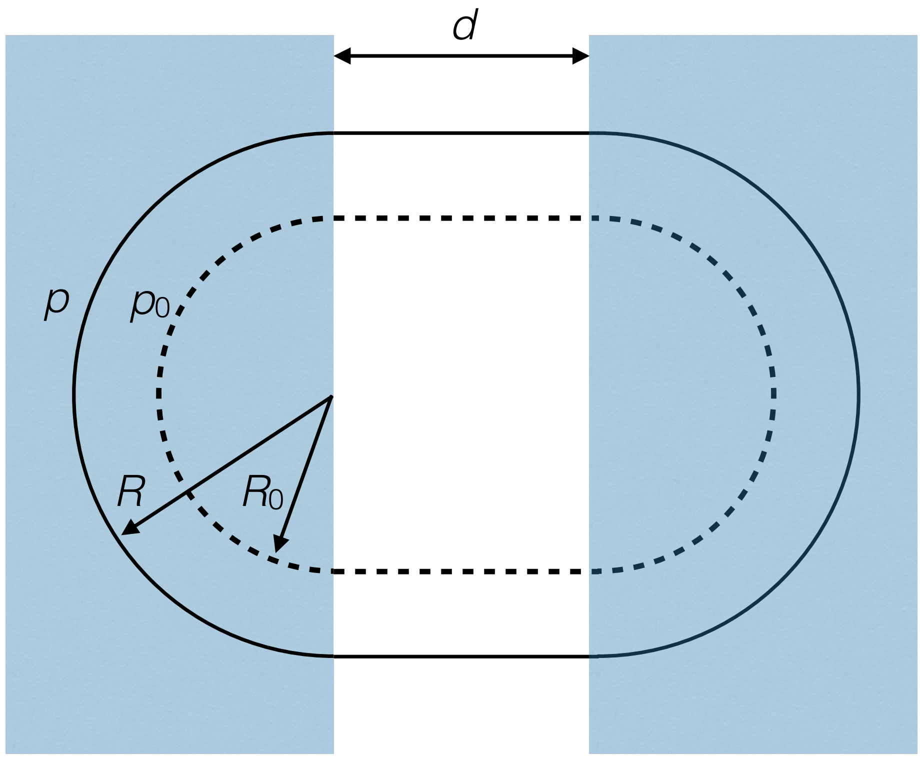
The Slip Factor: $\boxed{\eta \equiv \alpha_p - \frac{1}{\gamma^2}}$

A Simple Example…



$$R/R_0 = p/p_0$$

$$\frac{\Delta v}{v} = \frac{1}{\gamma^2}\frac{\Delta p}{p}$$



$$\tau = 2\pi R/v$$

$$\frac{\Delta L}{L_0} = \frac{\Delta R}{R_0} = \frac{\Delta p}{p_0}$$

$$\frac{\Delta \tau}{\tau_0} = (1 - \frac{1}{\gamma_0^2})\frac{\Delta p}{p_0}$$

$$\tau = (2\pi R + 2d)/v$$

$$\frac{\Delta L}{L_0} = \frac{2\pi(R-R_0)}{2\pi R_0 + 2d} = \frac{1}{1+d/\pi R_0}\frac{\Delta p}{p_0}$$

$$\frac{\Delta \tau}{\tau_0} = \left(\frac{1}{1+d/\pi R_0} - \frac{1}{\gamma_0^2}\right)\frac{\Delta p}{p_0}$$

Implications of the Slip Factor

- Suppose no bending in the line (e.g., linac), or, perhaps have bending yet $\gamma^2 < 1/\alpha_p$
  - then, the slip factor is negative, and particles of higher momentum take less time to traverse the same distance as the ideal particle

$$\eta = \alpha_p - \frac{1}{\gamma^2} = \left\langle \frac{D}{\rho} \right\rangle - \frac{1}{\gamma^2}$$

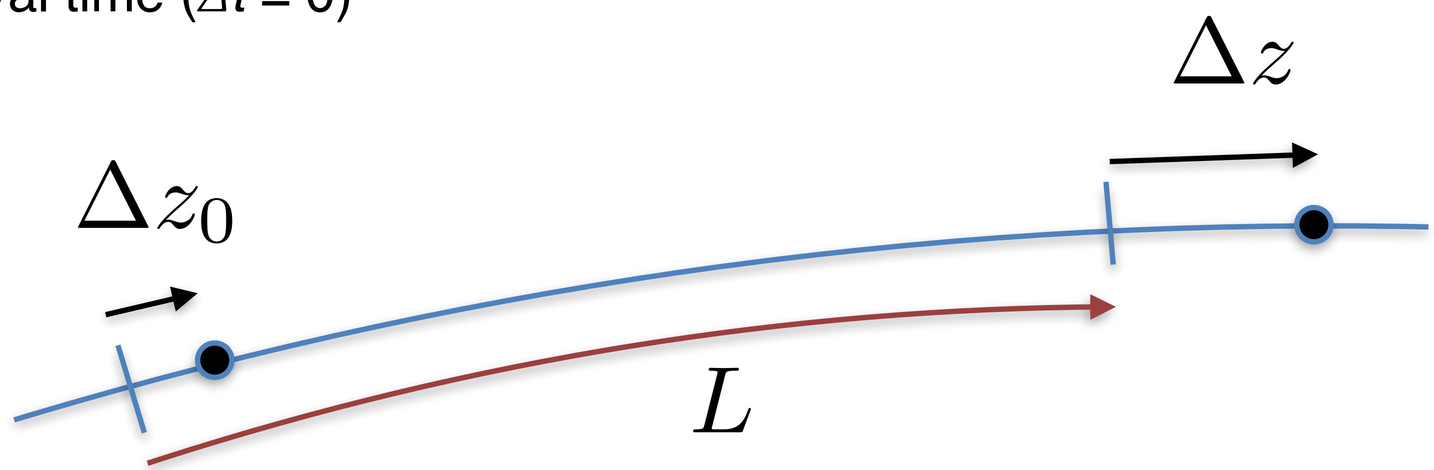
- If the energy of the particles is high enough in the presence of bending, then can have $\gamma^2 > 1/\alpha_p$
  - in this case, the slip factor is positive — the changes in path length outweigh the changes in speed when determining the time of flight difference
  - here, a higher-momentum particle will actually take ***longer*** to traverse the same distance as the ideal particle, even though it's moving faster

Linear Motion Very Near the Ideal Particle

- Particles moving along the ideal trajectory move toward or away from the ideal particle according to their speed (momentum/energy) and path length differences

Δt = arrival time relative to the ideal arrival time ($\Delta t = 0$)

$$\Delta z = -\beta c\, \Delta t$$



$$\tau = L/v = L/(\beta c)$$

$$\boxed{\begin{aligned} \Delta z &= \Delta z_0 - \eta L \frac{\Delta p}{p} \\ \Delta t &= \Delta t_0 + \eta \frac{L}{\beta c} \frac{\Delta p}{p} \end{aligned}}$$

$$\text{Note :} \quad \frac{\Delta p}{p} = \frac{1}{\beta^2} \frac{\Delta E}{E} = \frac{1}{\beta^2} \frac{\gamma - 1}{\gamma} \frac{\Delta W}{W}$$

Linear Motion Very Near the Ideal Particle [2]

- Imagine a particle on the ideal trajectory and that has the ideal energy, W_s. A second particle on the ideal trajectory, but with a different energy, W, may be ahead of or lagging behind the ideal particle.
- We will use **radio frequency (RF) cavities** to provide an accelerating voltage to the particles as they pass by.
- The ideal particle will arrive at the cavity at the "ideal" time or, equivalently, at an ideal phase, ϕ_s, to receive an appropriate increase in its energy (which might be an increase of "0").
- We will keep track of the "difference" in energy between our test particle and the ideal particle:

$$W_s = \text{“ideal” energy}$$

$$\Delta W \equiv W - W_s$$

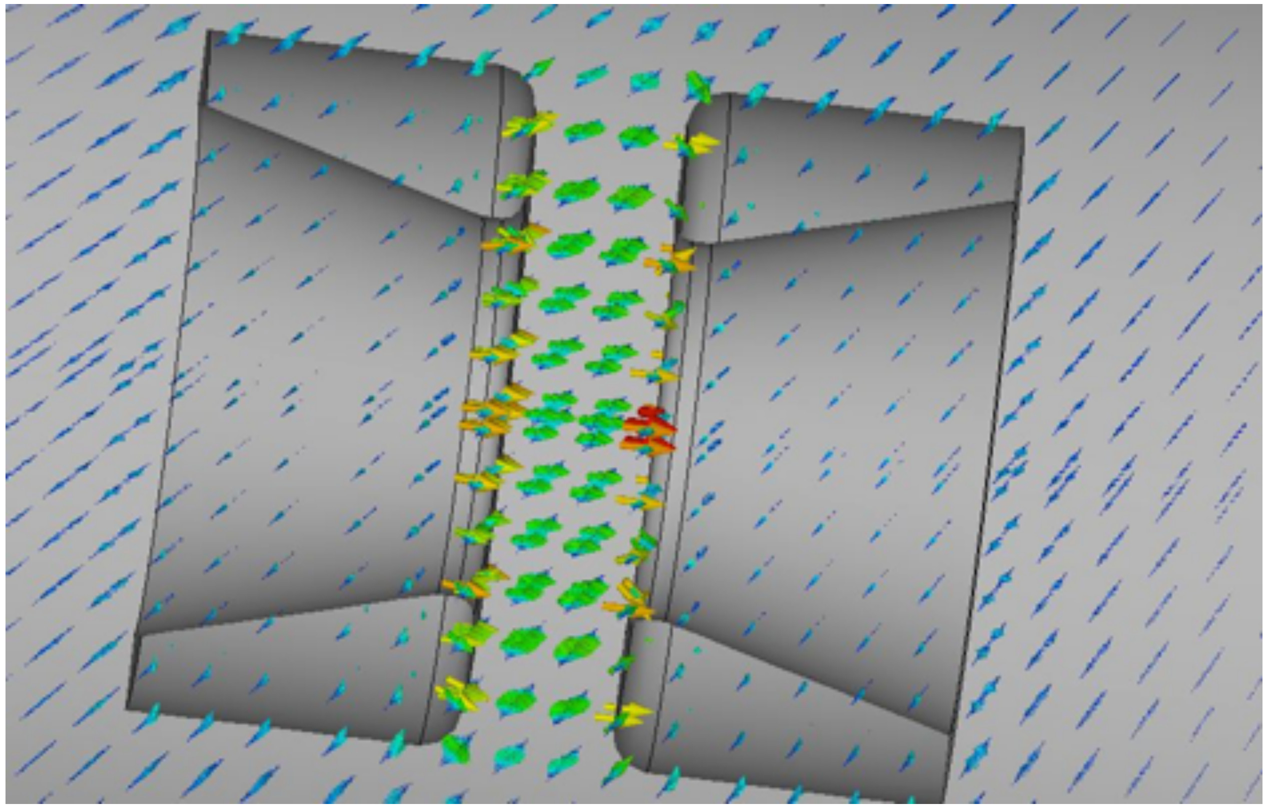
Acceleration using AC Fields

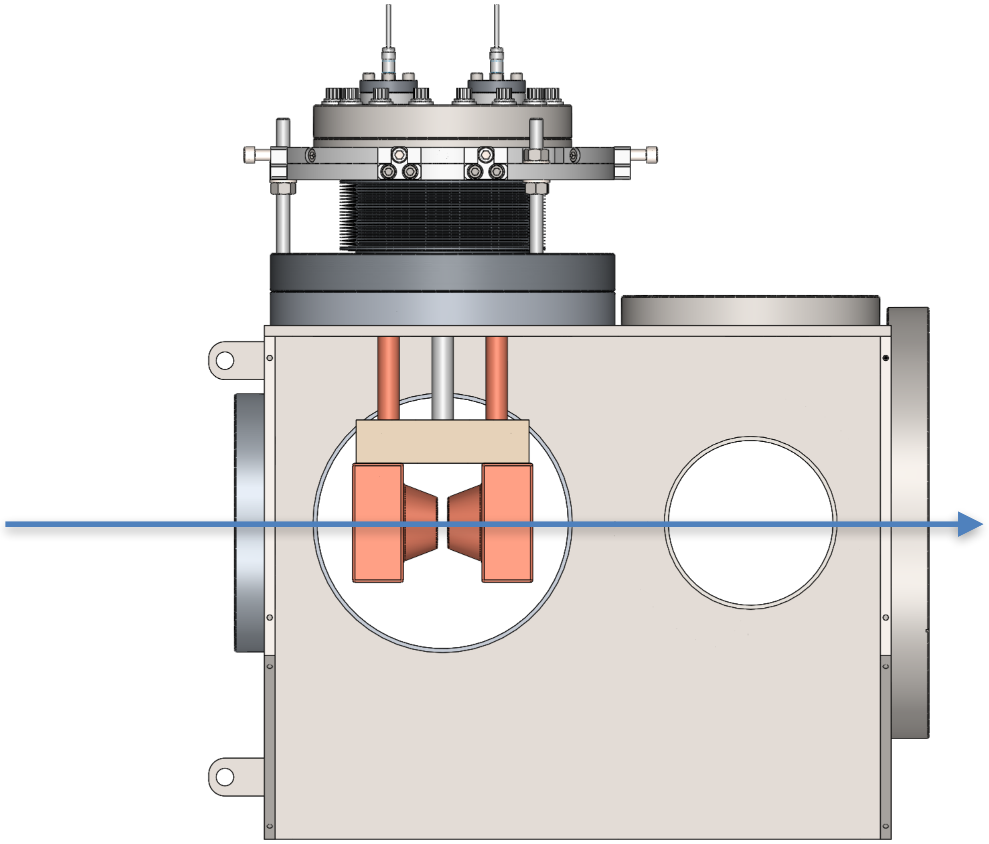
- Pass through a gap with an oscillating field…
- $$\Delta W = q\ E\ d = qV \qquad\qquad \Delta W/A\ [\mathrm{eV/u}] = (Q/A)\ eV$$
- But here, *V* is an "average" or "effective" potential; depends upon the frequency of the field in the gap, the incoming speed of the particle (due to the field varying with time), and the phase of the oscillation relative to the particle arrival time:

$$\Delta W\ [\mathrm{per\ nucleon}] = (Q/A)\ T(\beta)\ eV_0 \cos(\phi)$$

Accelerating Gap

- Create electric field / potential across two “plates”





- Vary the field with frequency, *f*

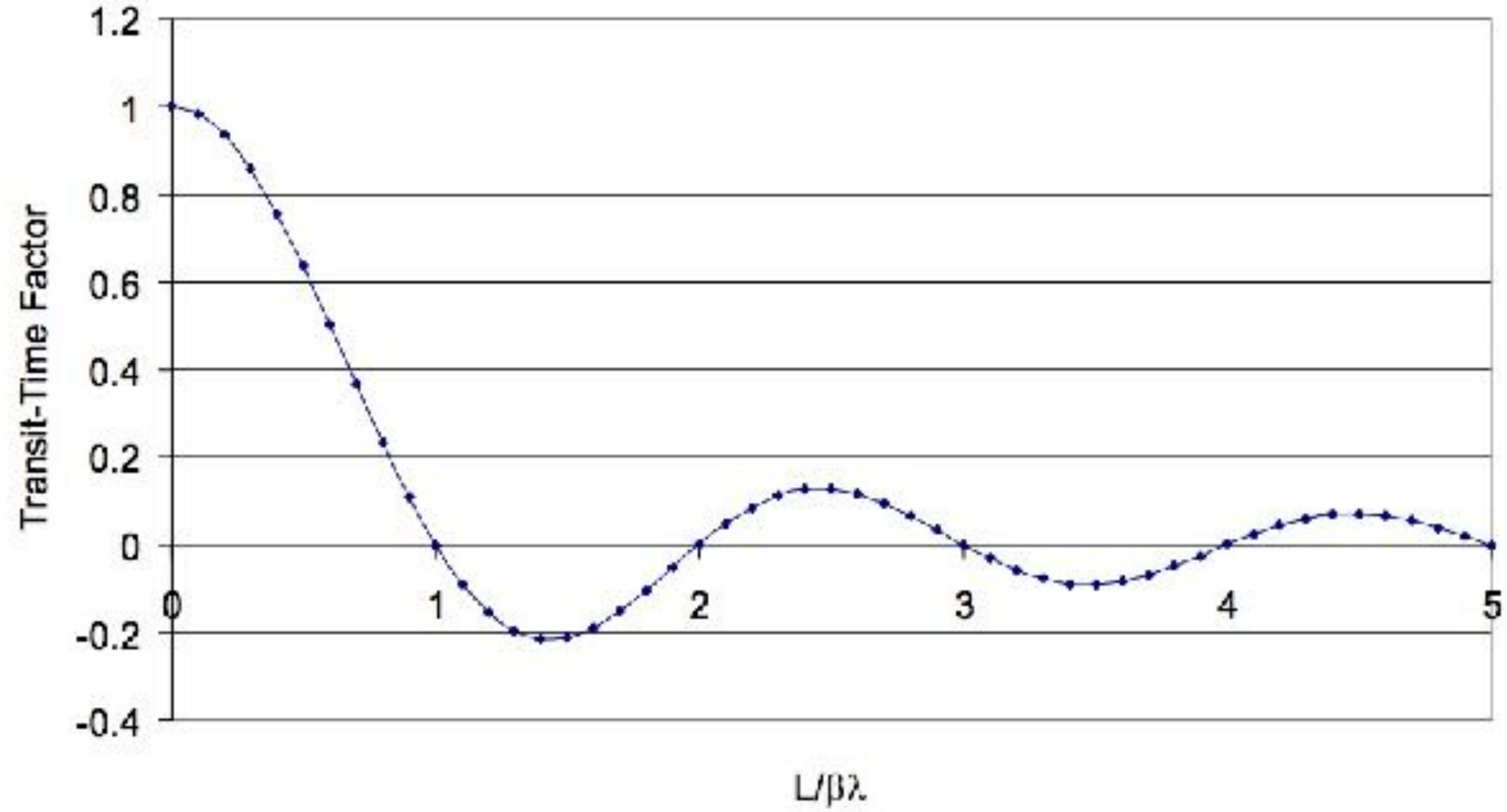
D. Alt, et al., MSU

Transit Time Factor

-

$$T(\beta) = \frac{1}{V_0}\int_{-g/2}^{g/2} E(0,s)\cos(ks)ds$$

$$V_0 = \int_{-g/2}^{g/2} E(0,s)ds \qquad k \equiv \frac{2\pi}{\beta\lambda}$$



- For $v = c$, and for a gap = $\lambda/2$, the TTF will be

$$\frac{1}{\lambda/2}\int_{-\lambda/4}^{\lambda/4}\cos(2\pi z/\lambda)dz = \frac{2}{\pi}$$

- Once get up to higher velocities ($v \sim c$), then can consider multiple-cell cavities since the velocity is no longer changing.

Resonant Structures

low-β cavities: **Just cavities that accelerate efficiently particles with β <1…**

low-β cavities are often further subdivided in low-, medium-, high- β

β=1 SC resonators:
"elliptical" shapes

Courtesy A. Facco

β<1 resonators, from very low (β~0.03) to intermediate (β~0.5):
many different shapes and sizes

Normal vs. Superconducting Cavities

DTL tank - Fermilab

Normal conducting

Cu cavity @ 300K

$R_s \sim 10^{-3}\ \Omega$

$Q \sim 10^4$

Superconducting

Nb Cavity @ 4.2K

$R_s \sim 10^{-8}\ \Omega$

$Q \sim 10^9$

LNL PIAVE 80 MHz, β =0.047 QWR

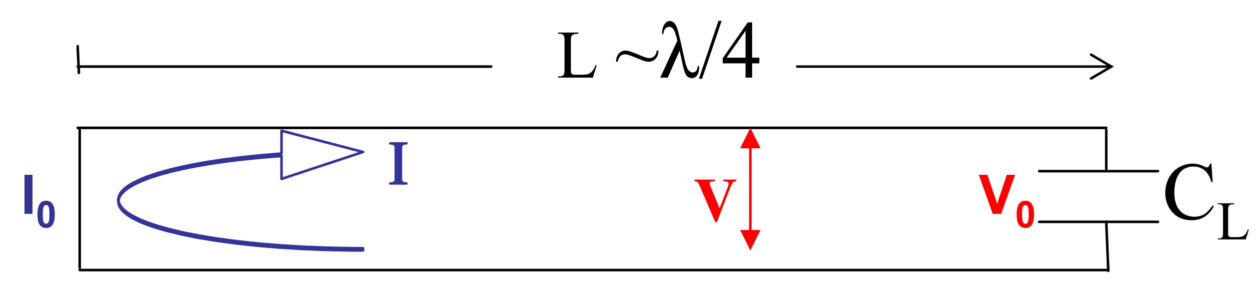
Superconductivity allows

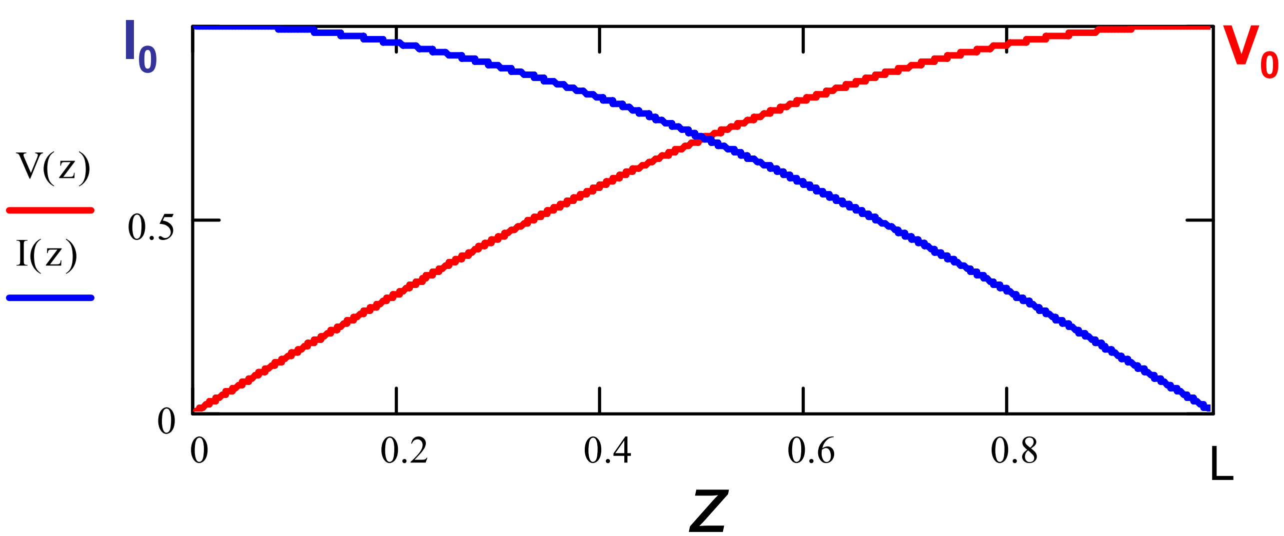
- *great reduction of rf power consumption even considering cryogenics (1W at 4.2K ~ 300W at 300K)*
- *the use of short cavities with wide velocity acceptance*

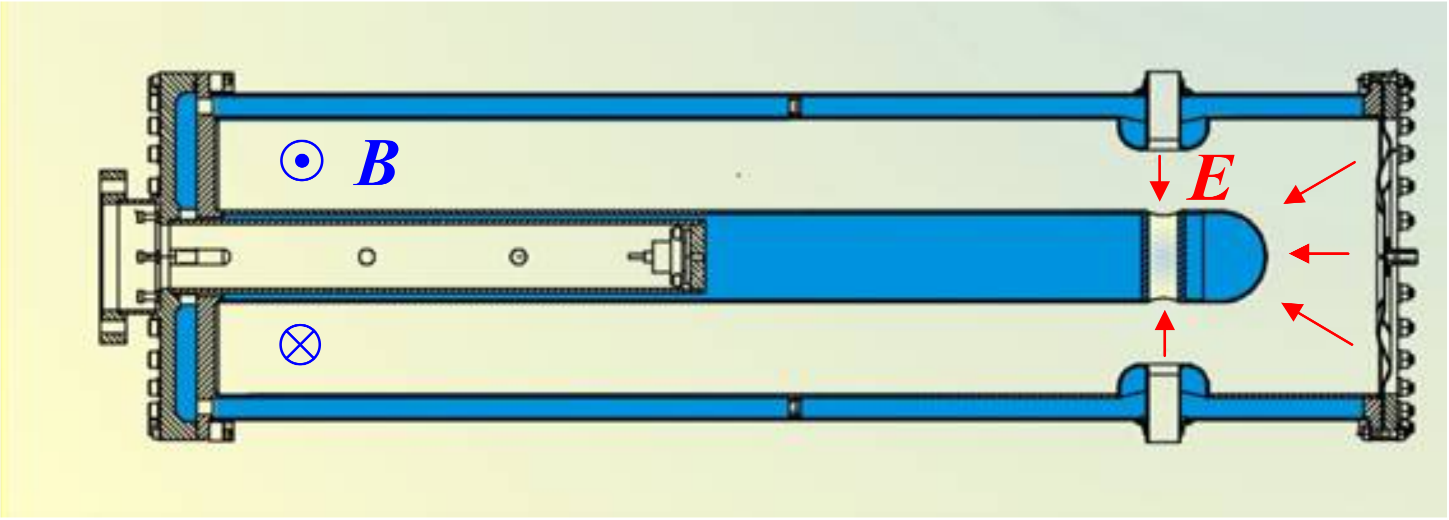
A. Facco –FRIB and INFN *SRF Low-beta Accelerating Cavities for FRIB* *MSU 4/10/2011*

Quarter-wave Structures

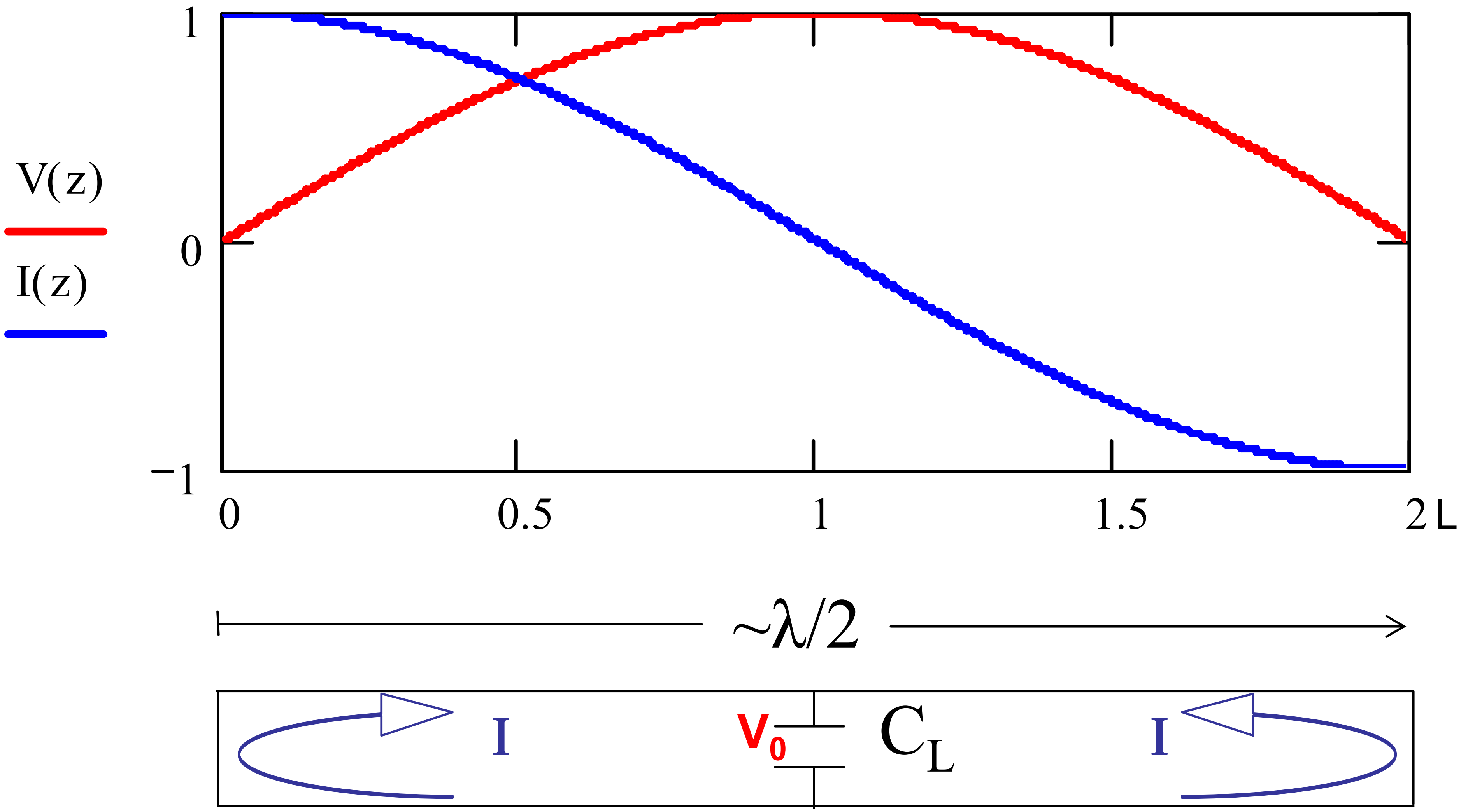
- Imagine a coaxial waveguide set to transmit an EM wave of frequency *f*, and wavelength $\lambda = c/f$
- Make the waveguide of length $L = \lambda/4$, and "close" one end
  - create a "standing wave" within the structure
- At the end where the E field is strong, allow the beam to pass through two gaps
  - separate gaps by distance $\beta\lambda/2$

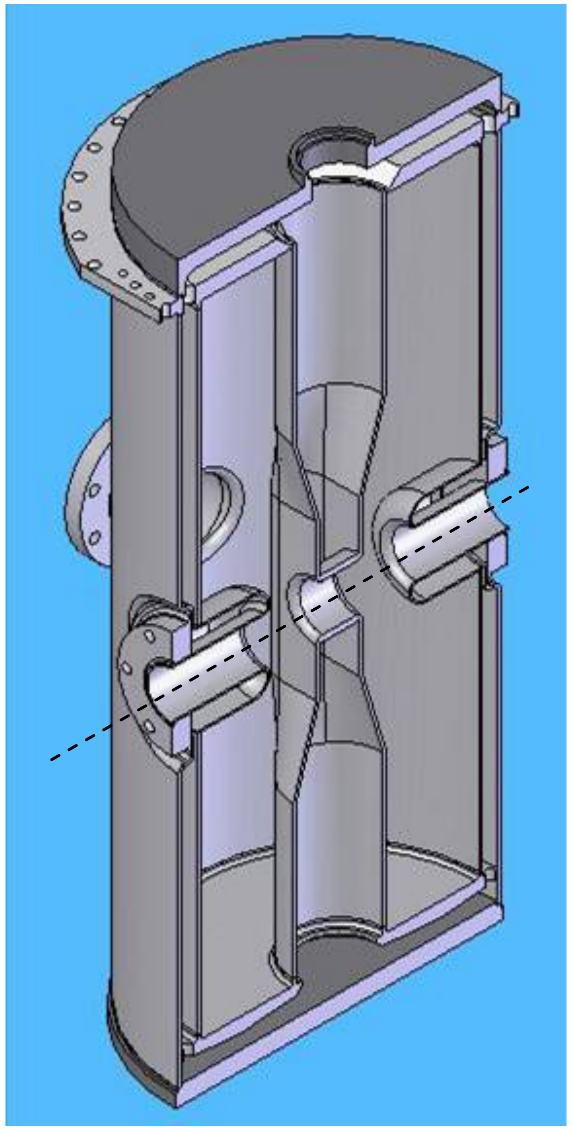






Half-wave Structures -- More Symmetry



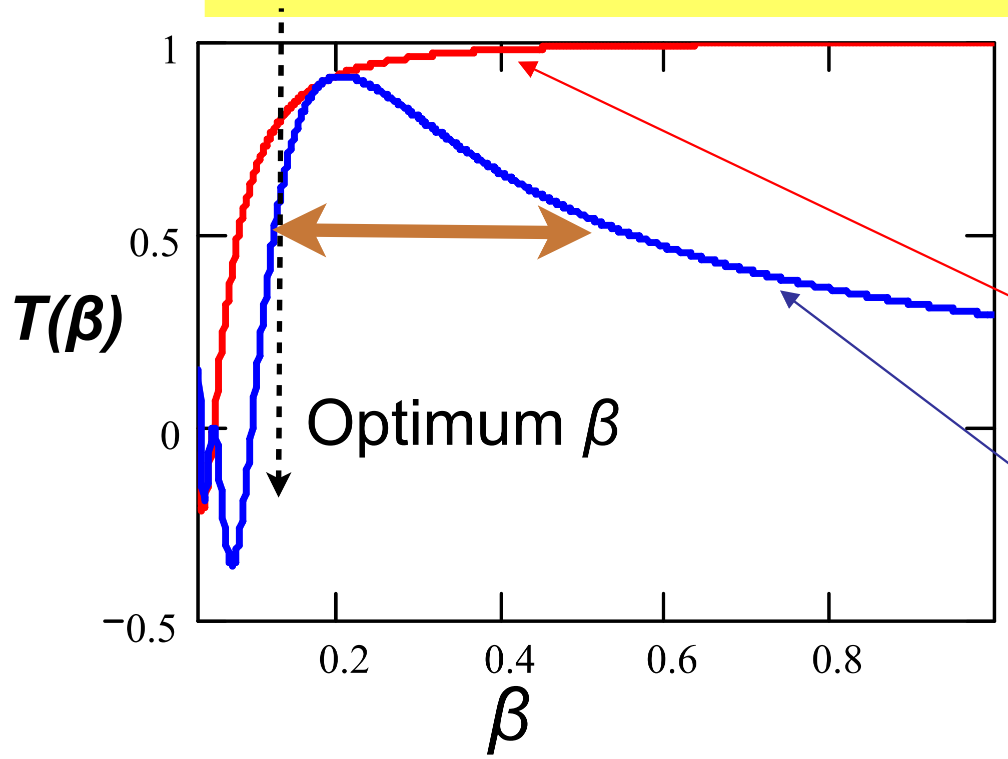


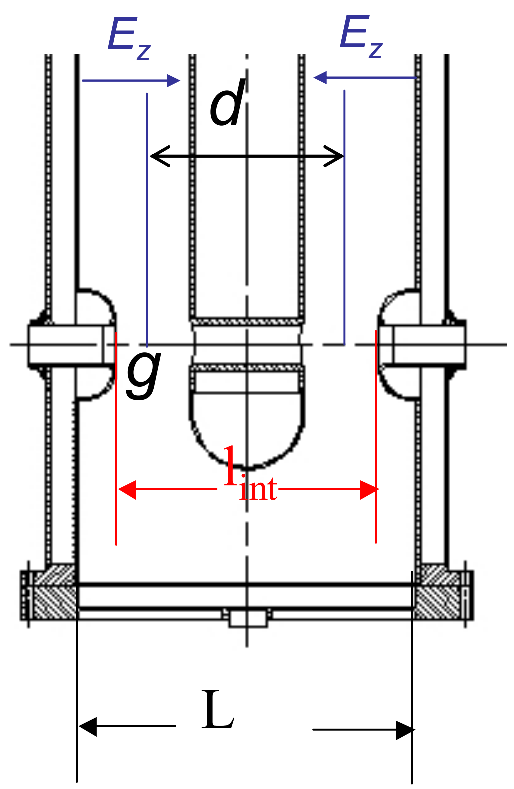
- A half-wave resonator is equivalent to 2 QWRs facing each other and connected

Transit Time Factor for 2-gap π-mode Cavity

(constant E_z approximation)

$$T(\beta) \cong \frac{\sin\left(\frac{\pi g}{\beta\lambda}\right)}{\left(\frac{\pi g}{\beta\lambda}\right)} \sin\left(\frac{\pi d}{\beta\lambda}\right)$$





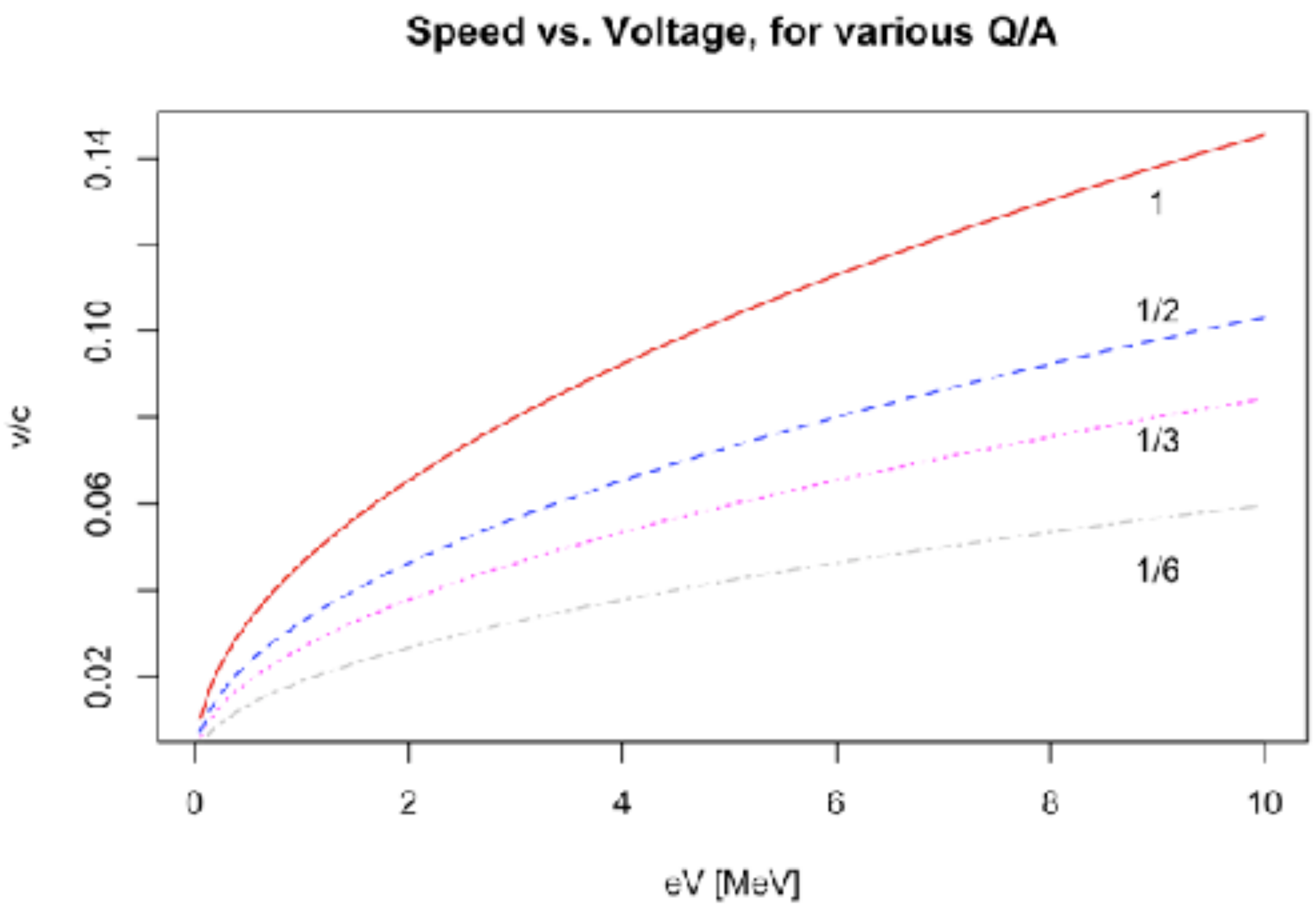
1° term: 1-gap effect → *$g<\beta\lambda/2$*

2° term: 2 gap effect → *$d\sim\beta\lambda/2$*

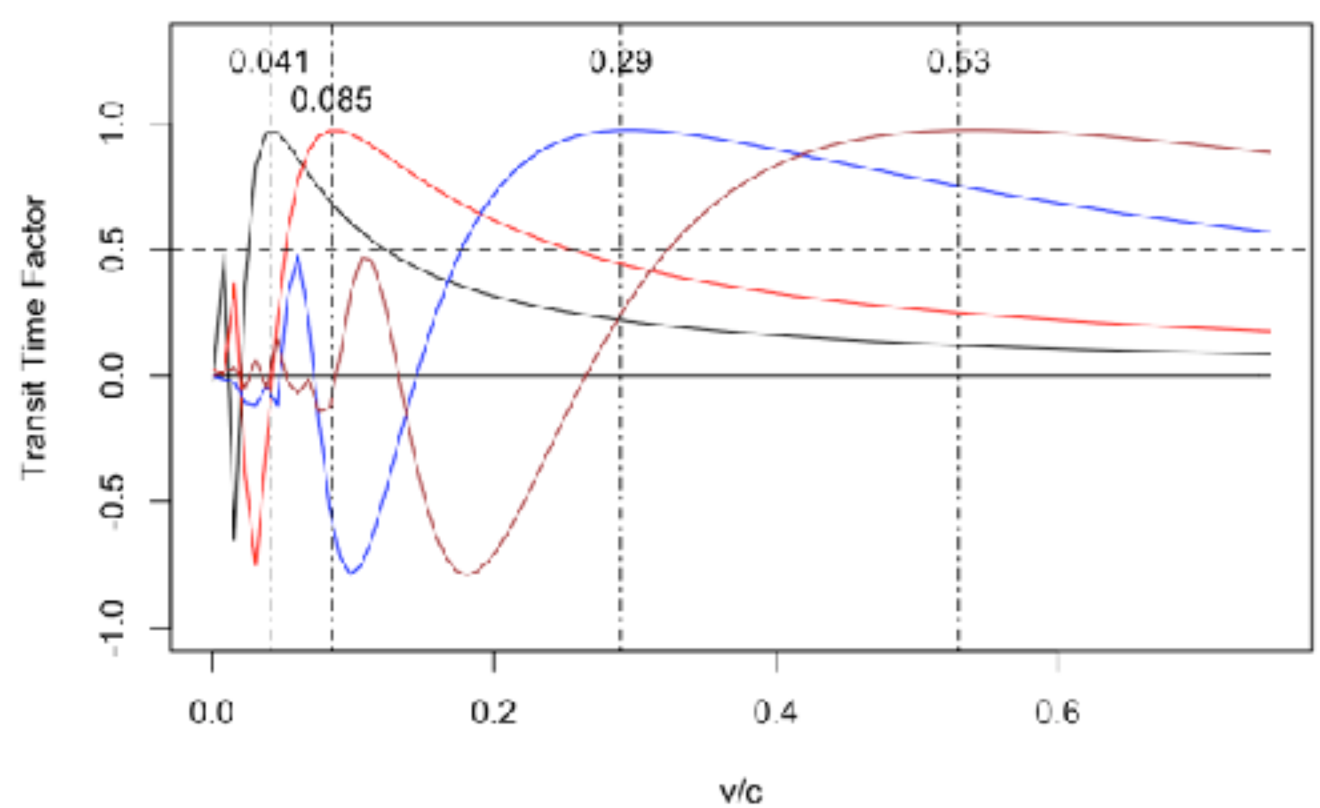
1°+ 2° term TTF curve

(For more than 2 equal gaps in π mode, the formulas change only in the 2° term)

Use in Heavy ion Accelerators



Would like ability to accelerate various isotopes, i.e., a variety of energies and Q//A ratios



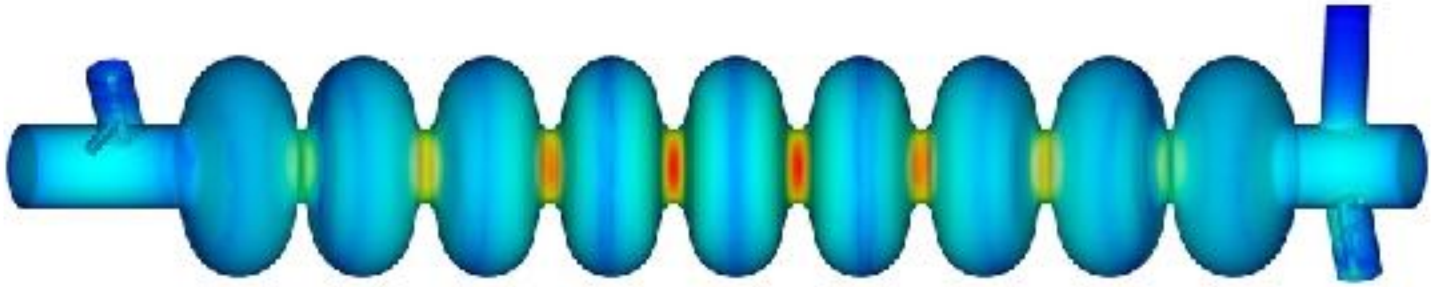
FRIB will use a variety of cavity frequencies, with each cavity voltage and phase being independently variable

Multi-cell Cavities

- Here: "ILC" (International Linear Collider) 9-cell style cavity

as ***v —> c***, can use multiple cells in succession

cells space by RF half-wavelength



for v = c, the TTF will be… $\frac{1}{\lambda/2}\int_{-\lambda/4}^{\lambda/4}\cos(2\pi z/\lambda)dz = \frac{2}{\pi}$

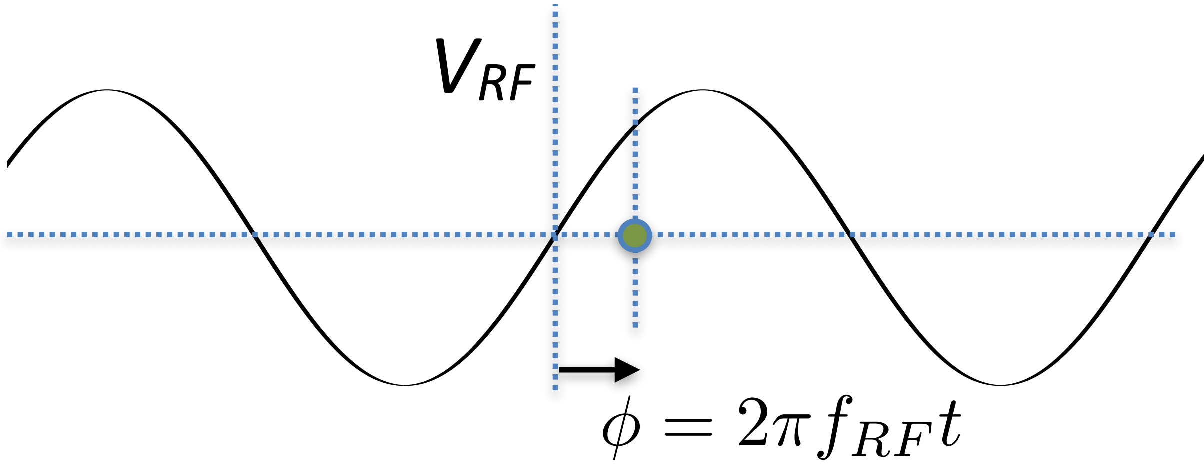
have achieved > 35 MV/m average accelerating gradient with superconducting cavities

(Note: even larger gradients achieved with non-SC, but very power intensive)

Linear Motion Very Near the Ideal Particle

- If a group of particles passes through a cavity such that the ideal (synchronous) particle receives no net energy gain, can give particles that are ahead/behind a decrease/increase in energy

$$W_s = \text{“ideal” energy}$$
$$\Delta W \equiv W - W_s$$



$$\begin{aligned}\Delta W &= \Delta W_0 + qV(\sin\phi - \sin\phi_s)\\ &= \Delta W_0 + qV[\sin(\phi_s + \Delta\phi) - \sin\phi_s]\end{aligned}$$
$$\Delta W \approx \Delta W_0 + qV\cos\phi_s\Delta\phi = \Delta W_0 + qV\cos\phi_s(2\pi f_{RF})\Delta t_0$$

- Can use matrix techniques to propagate the longitudinal motion

Linear Motion through Cavities and Drifts

- Keep track of time differences and energy differences…

drift:

$$\begin{pmatrix} \Delta t \\ \Delta W \end{pmatrix} = \begin{pmatrix} 1 & \eta \frac{L}{c} \frac{1}{\beta^3 \gamma} \frac{1}{mc^2} \\ 0 & 1 \end{pmatrix} \begin{pmatrix} \Delta t \\ \Delta W \end{pmatrix}_0$$

through cavity:
longitudinal focusing

$$\begin{pmatrix} \Delta t \\ \Delta W \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ (2\pi f_{RF}) qV \cos\phi_s & 1 \end{pmatrix} \begin{pmatrix} \Delta t \\ \Delta W \end{pmatrix}_0$$

remember —

$$\eta \equiv \alpha_p - \frac{1}{\gamma^2} \qquad \alpha_p = \frac{\int [D(s)/\rho(s)] ds}{\int ds}$$

Bunchers, Re-bunchers, Debunchers

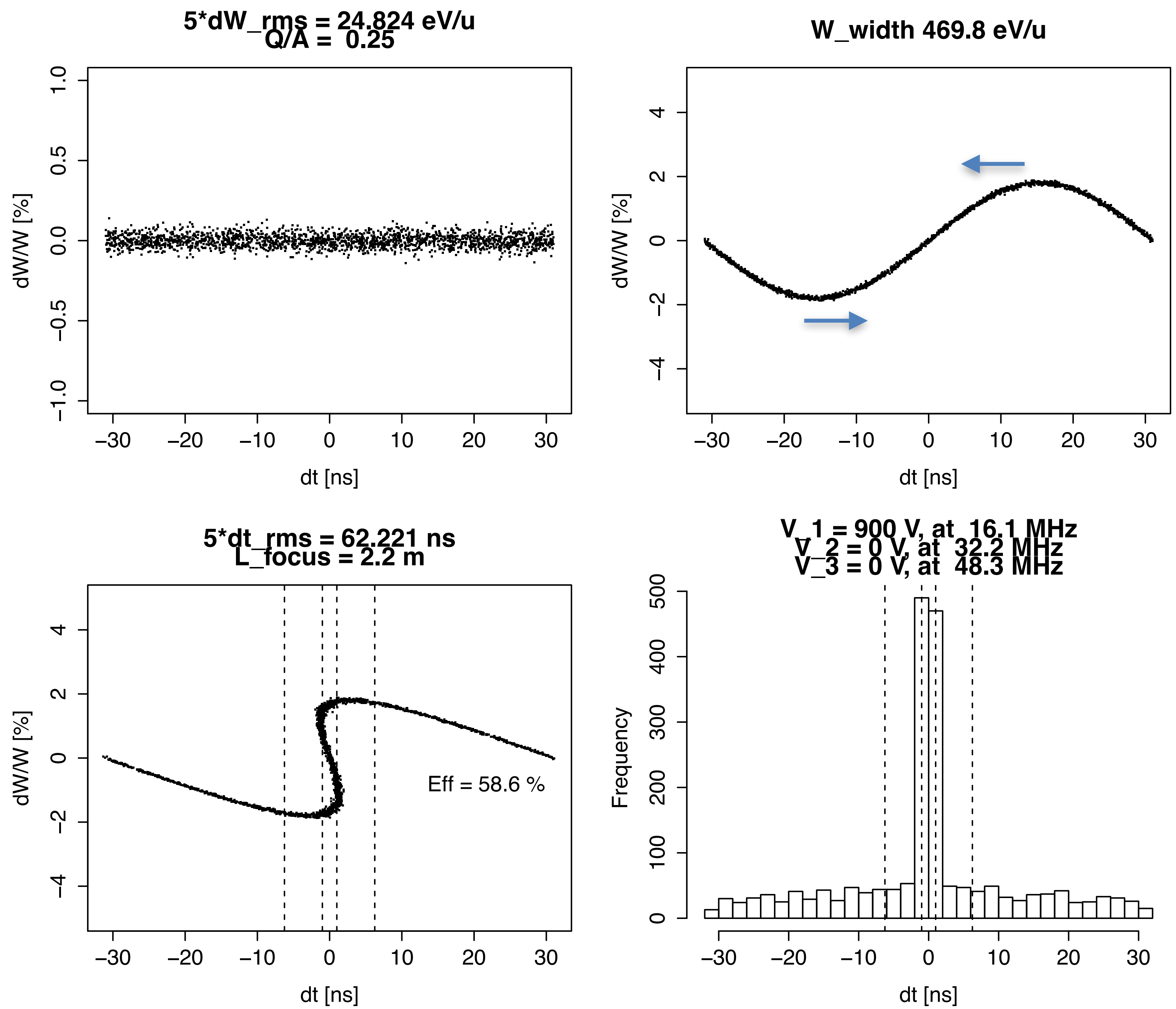
- If start with continuous stream of particles (DC current, with no strong “AC” component), can create bunches (AC beam) using a single cavity (buncher)

- If have bunched beam that is allowed to travel a certain distance, the particles within the bunch will begin to spread out due to the inherent spread in momentum
  - re-buncher: mitigate this effect
  - debuncher: enhance this effect

Beam Buncher

incoming DC beam

after the cavity

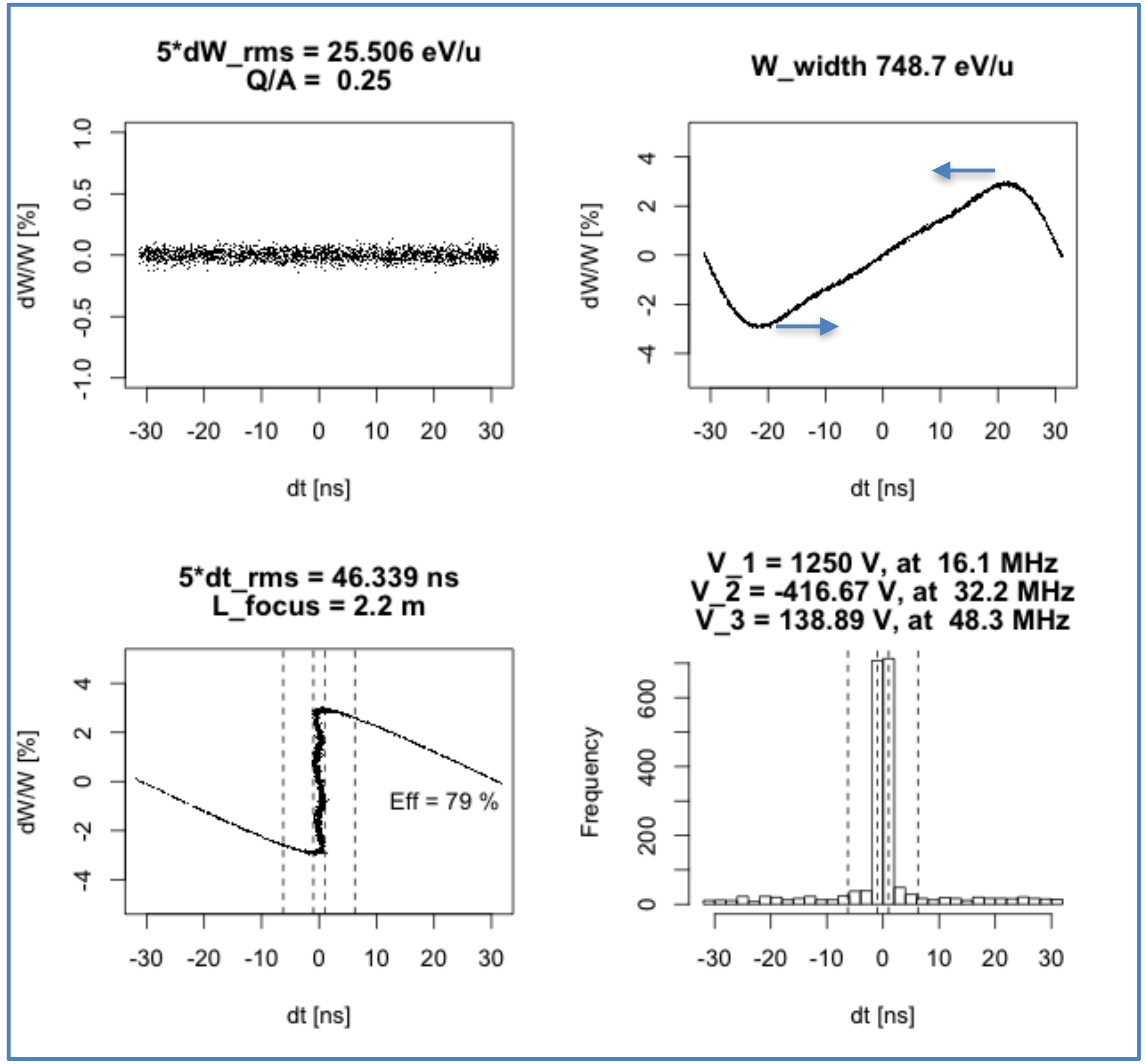


downstream of cavity

resulting time profile

Multi-harmonic Buncher

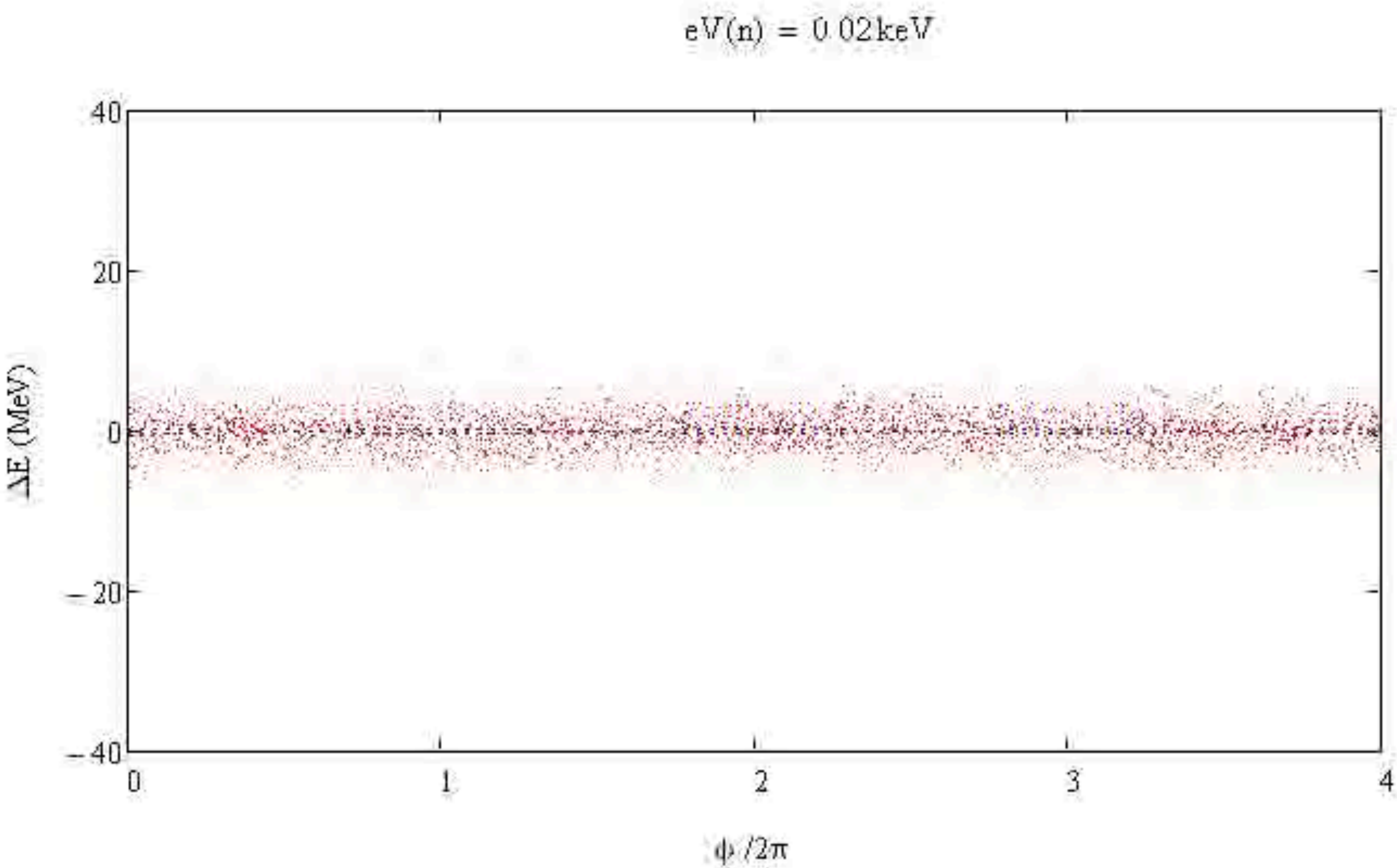
- Use 2, or 3 (or 4?) harmonics of the fundamental frequency to smooth out the sine wave into a more linear waveform



$$V(t) = V_1 \sin(2\pi f t) + V_2 \sin(4\pi f t) + V_3 \sin(6\pi f t) + V_4 \sin(8\pi f t) + \ldots$$

ReA pre-buncher; Alt, *et al.* (MSU)

Adiabatic Capture in a Storage Ring



Repetitive Systems of Acceleration

- We will assume that particles are propagating through a system of accelerating cavities. Each cavity has oscillating fields with frequency f_{RF}, and maximum "applied" voltage V (i.e., this takes into account TTF's, etc.). The ideal particle would arrive at the cavity at phase ϕ_s.

- We will choose ϕ_s to be relative to the "positive zero-crossing" of the RF wave, such that the ideal particle acquires an energy gain of

$$\Delta E = \Delta W = qV \sin\phi_s = QeV \sin\phi_s$$

» this definition used for synchrotrons; linacs more often define ϕ_s relative to the "crest" of the RF wave

- apologies for this possible *further* confusion…

Acceleration of Ideal Particle

- Wish to accelerate the ideal particle. As the particle exits the (n+1)-th RF cavity/station we would have

$$E_s^{(n+1)} = E_s^{(n)} + QeV \sin \phi_s$$

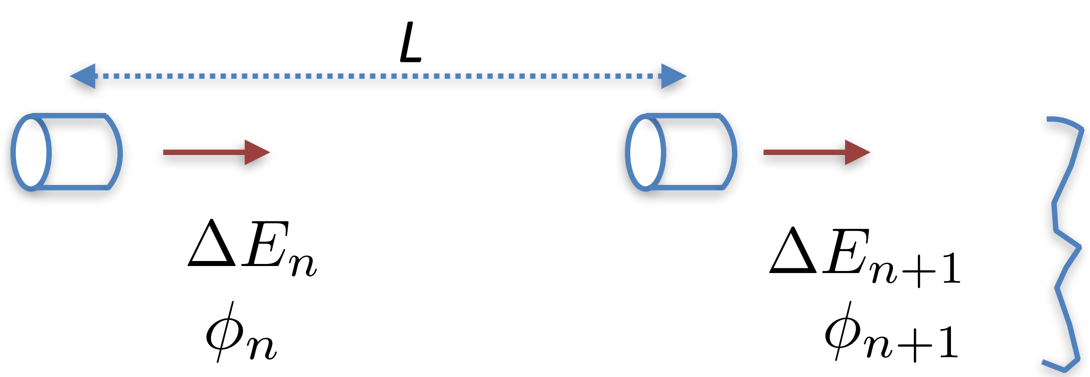
- If we are considering a synchrotron, we can consider the above as the total energy gain on the (n+1)-th revolution. The ideal energy gain per second would be:

$$dE_s/dt = f_0 QeV \sin \phi_s$$

- Next, look at (longitudinal) motion of particles near the ideal particle:
-
- ϕ = phase w.r.t. RF system
- $\Delta E \equiv E - E_s$ = energy difference from the ideal

Acceleration

- Assume that our accelerating system of cavities is set up so that the ideal particle always arrives at the next cavity when the accelerating voltage *V* is at the same phase (called the "synchronous phase")



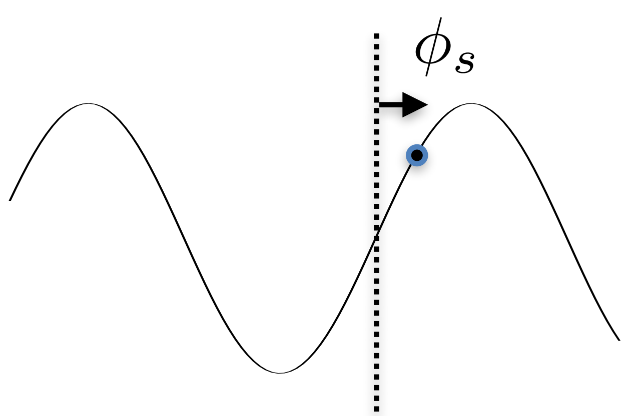
$$\boxed{\begin{aligned}\phi_{n+1} &= \phi_n + \frac{2\pi h \eta}{\beta^2 E}\Delta E_n \\ \Delta E_{n+1} &= \Delta E_n + QeV(\sin\phi_{n+1} - \sin\phi_s)\end{aligned}}$$

(*difference equations*)

Notes:

$$h = L/\beta\lambda, \qquad \lambda = c/f_{\rm rf}$$

If *L* is circumference of a synchrotron then: $h = f_{\rm rf}/f_0$
where f_0 is the revolution frequency,
In this case, *h* is called the "harmonic number"



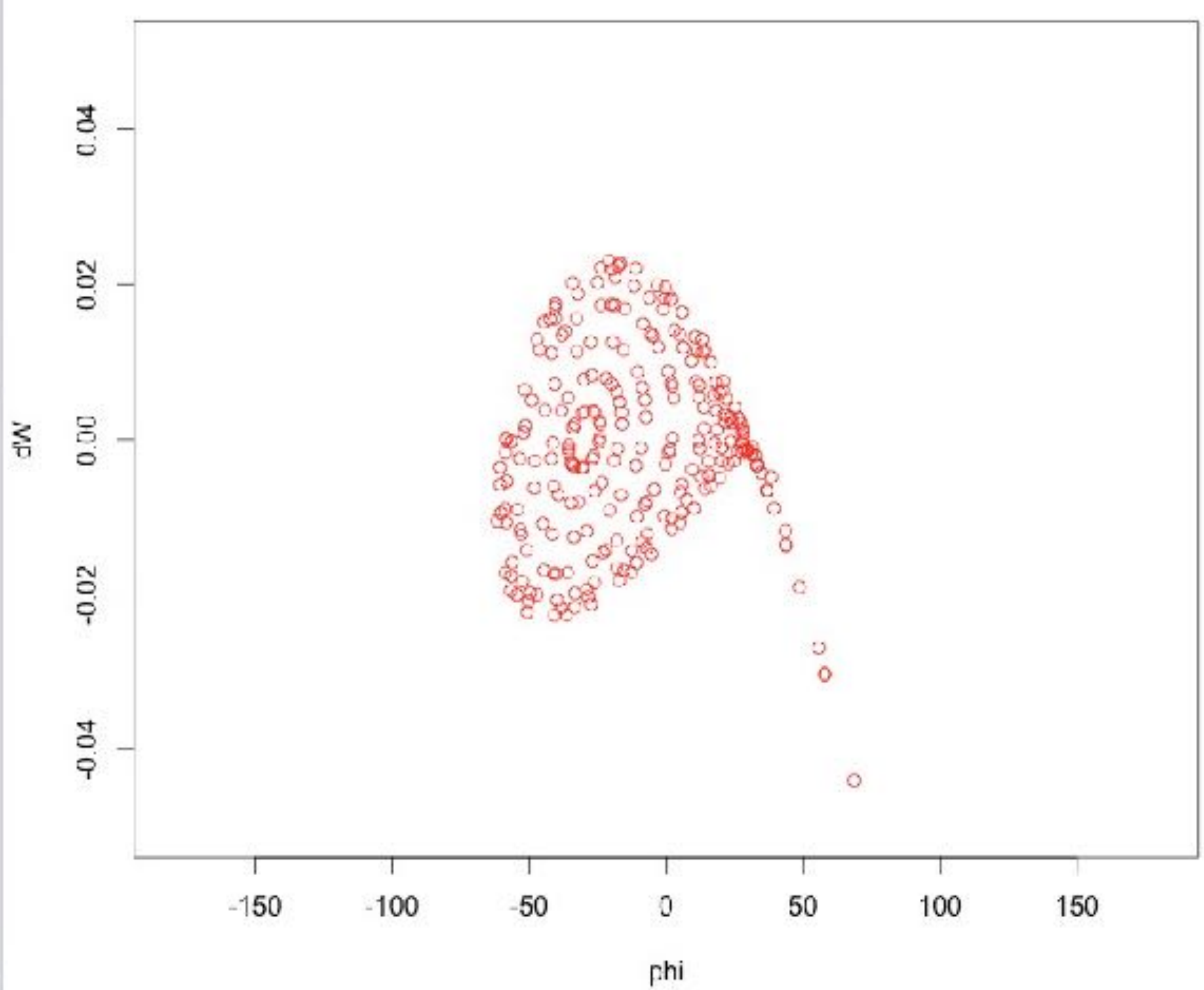
$$E = mc^2 + W; \qquad \Delta E \Leftrightarrow \Delta W$$

Applying the Difference Equations

```
while (i < Nturns+1) {
   phi = phi + k*dW
   dW  = dW + QonA*eV*(sin(phi)-sin(phis))
   points(phi*360/2/pi, dW, pch=21,col="red")
   i = i + 1
 }
```

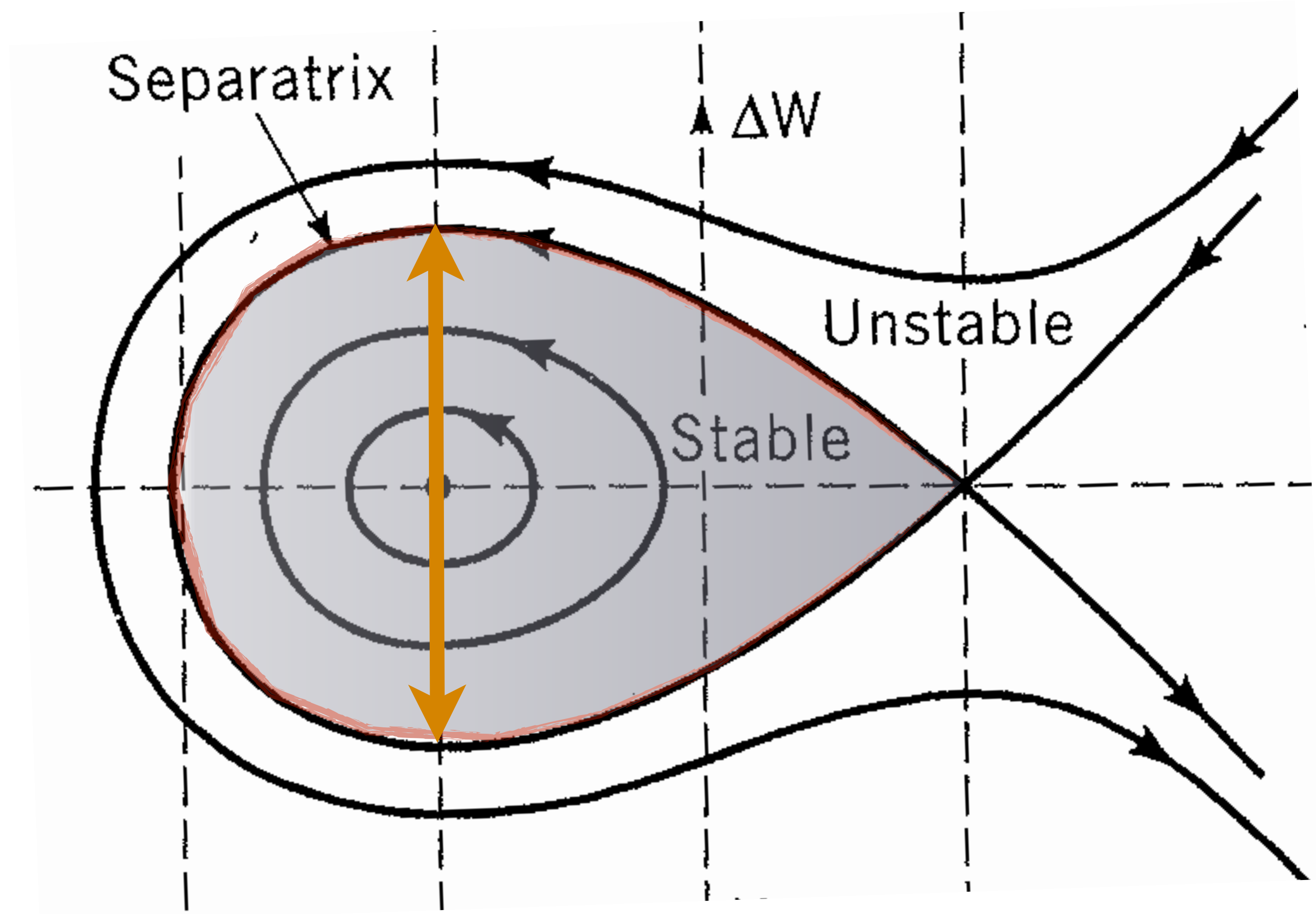
Let's run a code…

```r
# Program to plot longitudinal phase space motion
#  through a system of cavities (just an example...)

Nturns = 20

# Some Parameters
Ws = 1.0 # MeV/u
phis = -30*pi/180  # synchronous phase angle
eV = 0.2 # MeV/u
QonA = 0.25
gamma = (931+Ws)/931
beta = sqrt(1-1/gamma^2)
eta = -1/gamma^2
h = 1/(beta*3e8/80.5e6)
k = 2*pi*h*eta/beta^2*(gamma-1)/gamma/Ws

# initialize the phase space plot
phi  = -90
dW   = 0
plot(phi, dW, xlim=c(-180,180), ylim=c(-0.05,0.05),
typ="n")

trk = 1
while (trk < 16) {
# initialize particle positions in phase space
  u0 <- locator(1)
  phi   <- u0$x/180*pi
  dW    <- u0$y
# track the particle...
  i = 1
  while (i < Nturns+1) {
    phi = phi + k*dW
    dW  = dW - QonA*eV*(cos(phi)-cos(phis))
    points(phi*360/2/pi, dW, pch=21,col="red")
    i = i + 1
  }
  trk = trk + 1
}
```



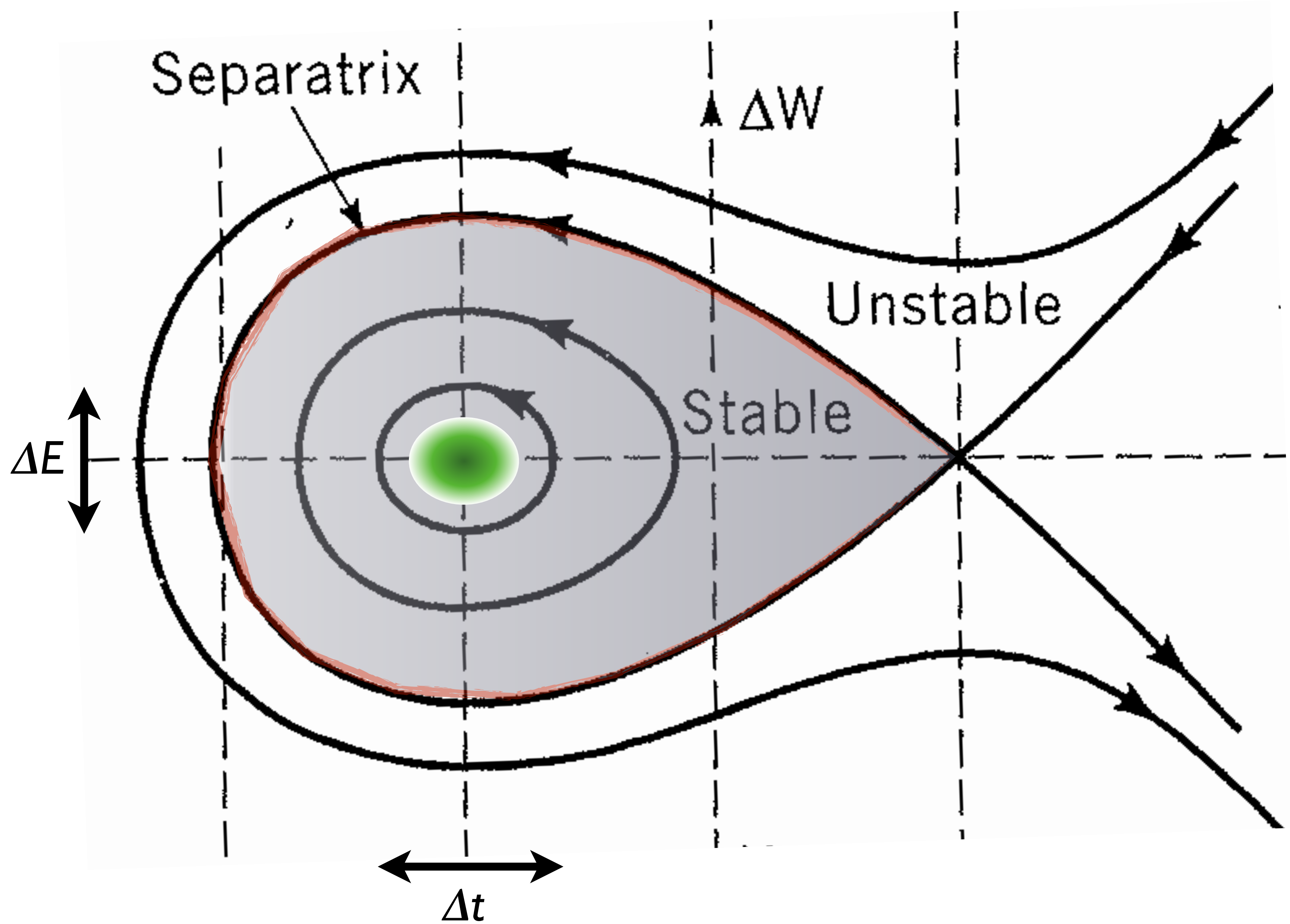
Acceptance and Emittance

- Stable region often called an RF “bucket”
  - “contains” the particles
- Maximum vertical extent is the maximum spread in energy that can be accelerated through the system



Acceptance and Emittance

- Stable region often called an RF "bucket"
  - "contains" the particles
- Maximum vertical extent is the maximum spread in energy that can be accelerated through the system
- Desire the beam particles to occupy much smaller area in the phase space



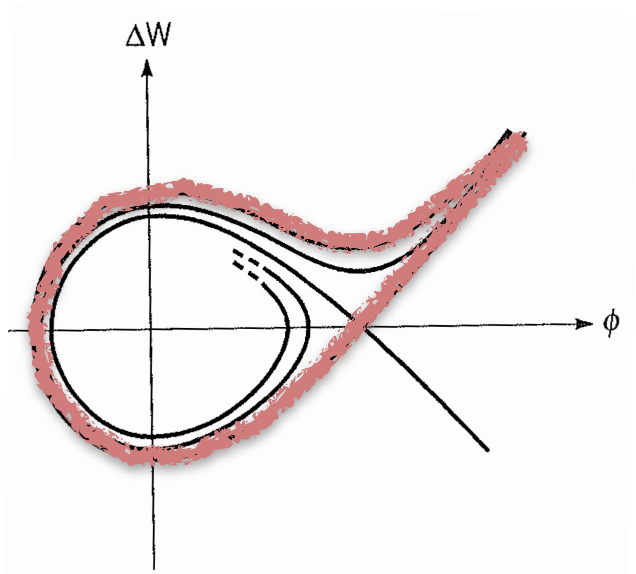
area: "eV-sec"
Note: *E, t canonical*

Golf Clubs *vs.* Fish

- Our analysis "assumes" slowly changing variables (including the energy gain!). Quite reasonable in many Alvarez-style linacs and in synchrotrons
- In linacs, fractional energy change can be large, and so this will distort the phase space
- Plots from Wangler's book:

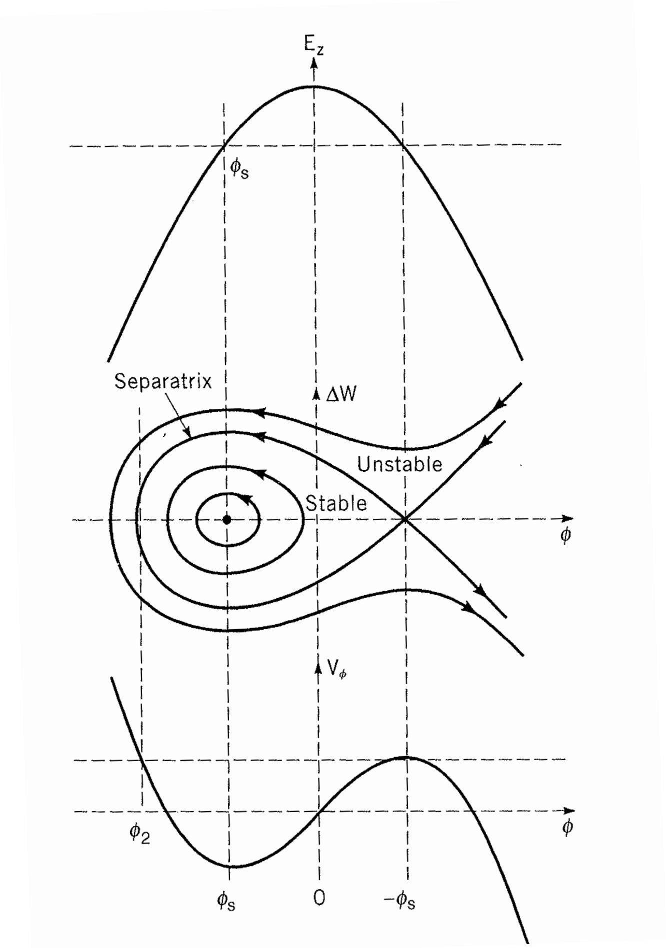
Here, a more rapid acceleration is included

(linac)

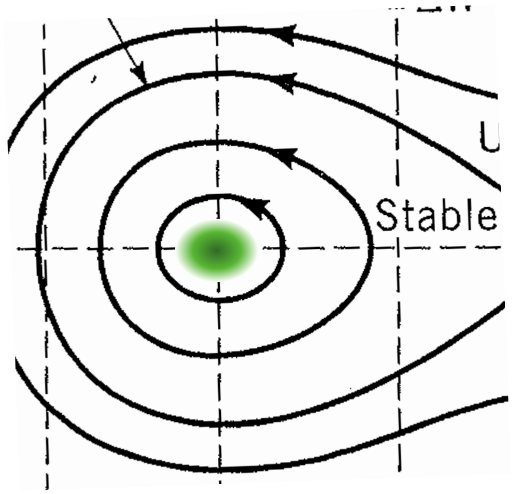


Here, assume that energy is "constant" or varying very slowly

(synchrotron)



Motion Near the Ideal Particle



Linearize the motion, and write in matrix form…

$$
\begin{aligned}
\phi_{n+1} &= \phi_n + \frac{2\pi h\eta}{\beta^2 E}\Delta E_n \\
\Delta E_{n+1} &= \Delta E_n + QeV(\sin\phi_{n+1} - \sin\phi_s) \\
&= \Delta E_n + QeV(\sin\phi_s \cos\Delta\phi_{n+1} + \sin\Delta\phi_{n+1}\cos\phi_s) - \sin\phi_s) \\
&= \Delta E_n + QeV\cos\phi_s\ \Delta\phi_{n+1} \\
&= \Delta E_n + QeV\cos\phi_s\left[\Delta\phi_n + \frac{2\pi h\eta}{\beta^2 E}\Delta E_n\right]
\end{aligned}
$$

Thus,

$$
\boxed{
\begin{aligned}
\Delta\phi_{n+1} &= \Delta\phi_n + \frac{2\pi h\eta}{\beta^2 E}\Delta E_n \\
\Delta E_{n+1} &= QeV\cos\phi_s\Delta\phi_n + \left(1 + \frac{2\pi h\eta}{\beta^2 E}QeV\cos\phi_s\right)\Delta E_n
\end{aligned}
}
$$

or,

$$\begin{pmatrix} \Delta\phi \\ \Delta E \end{pmatrix}_{n+1} = \begin{pmatrix} 1 & \frac{2\pi h\eta}{\beta^2 E} \\ QeV\cos\phi_s & \left(1 + \frac{2\pi h\eta}{\beta^2 E} QeV\cos\phi_s\right) \end{pmatrix} \begin{pmatrix} \Delta\phi \\ \Delta E \end{pmatrix}_n$$

$$= \begin{pmatrix} 1 & 0 \\ QeV\cos\phi_s & 1 \end{pmatrix} \begin{pmatrix} 1 & \frac{2\pi h\eta}{\beta^2 E} \\ 0 & 1 \end{pmatrix} \begin{pmatrix} \Delta\phi \\ \Delta E \end{pmatrix}_n$$

$$M = M_c \cdot M_d$$

"thin" cavity — *drift*

(*acts as longitudinal focusing element*)

Note: for $\eta < 0$, M_d is a "backwards" drift; i.e., $\Delta\phi$ decreases for $\Delta E>0$

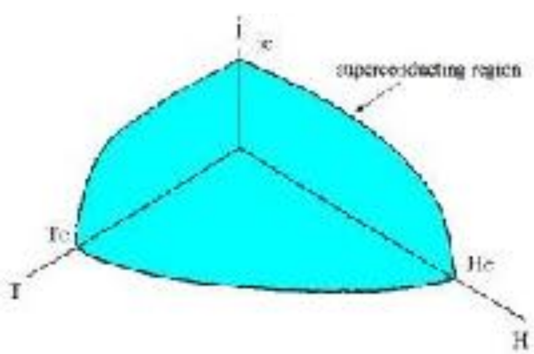
(when no bending)

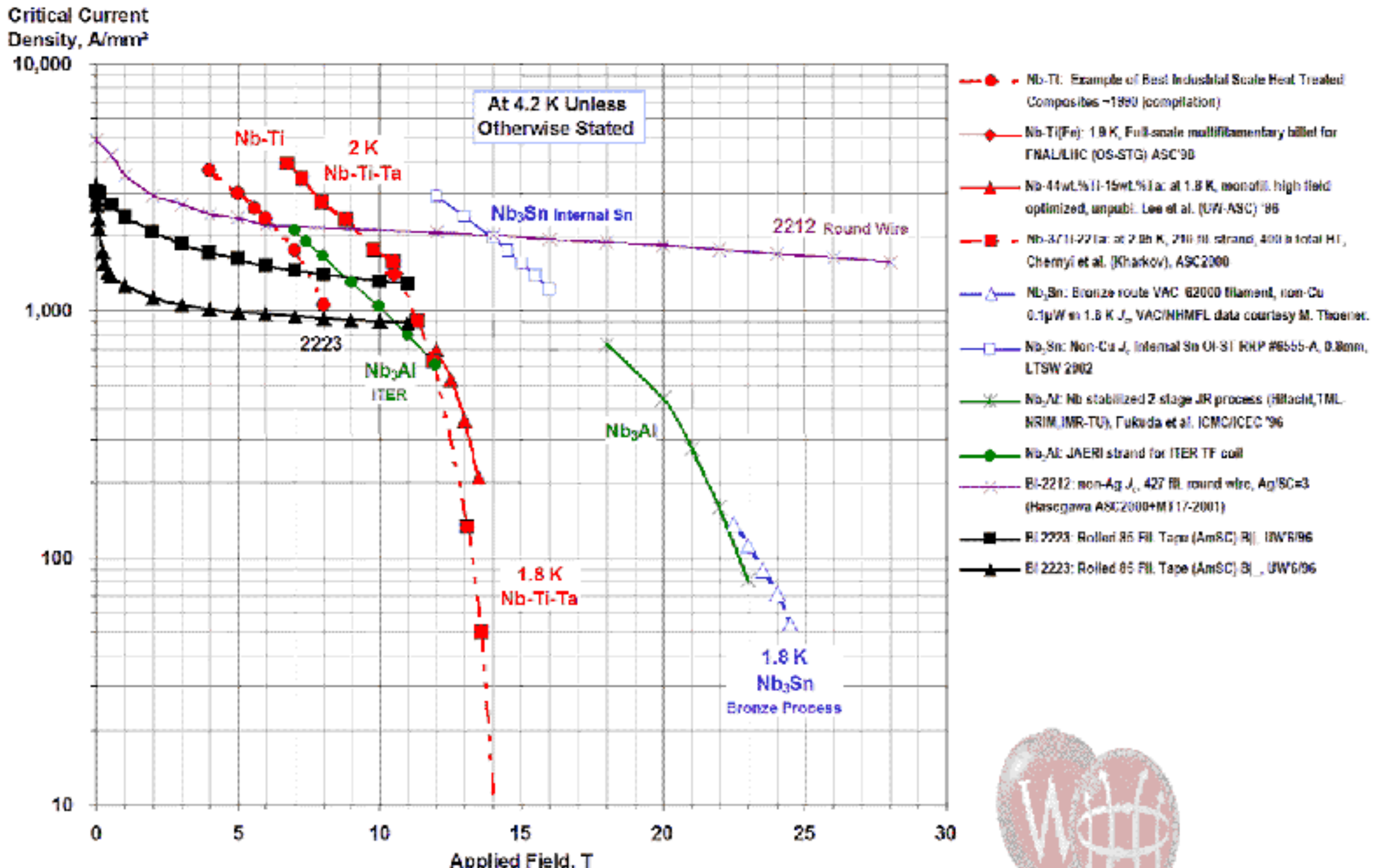
$\eta = -1/\gamma^2$ in straight region (linac)

Outlook for the Field

Advancing Critical Currents in Superconductors

University of Wisconsin-Madison
Applied Superconductivity Center





note: "engineering" current densities will be less than these "critical" current densities

Accelerator Magnets

quench training:

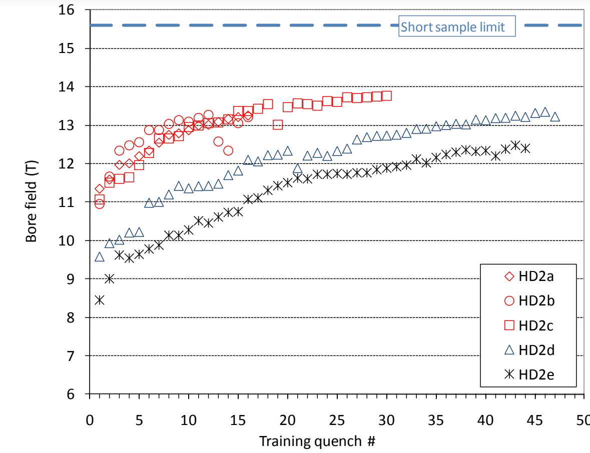
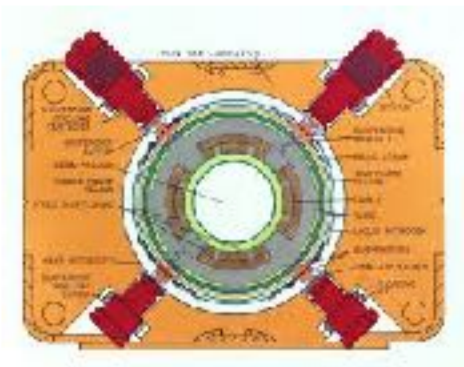


Figure 12: Bore field (T) as a function of training

magnetic pressure:

$$P = \frac{B^2}{2\mu_0}$$

$$\frac{(4\ \mathrm{T})^2}{8\pi 10^{-7}} = 6.4\mathrm{MN/m}^2 = 64\ \mathrm{atm}$$



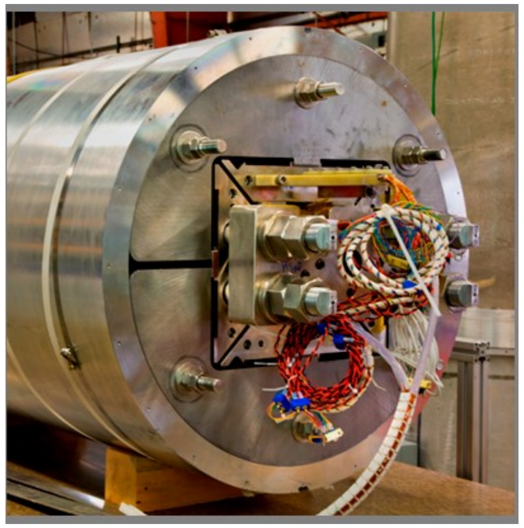


Figure 10: Magnet HD2

$$\frac{(13\ \mathrm{T})^2}{8\pi 10^{-7}} = 670\ \mathrm{atm}$$

$$\frac{(8\ \mathrm{T})^2}{8\pi 10^{-7}} = 250\ \mathrm{atm}$$

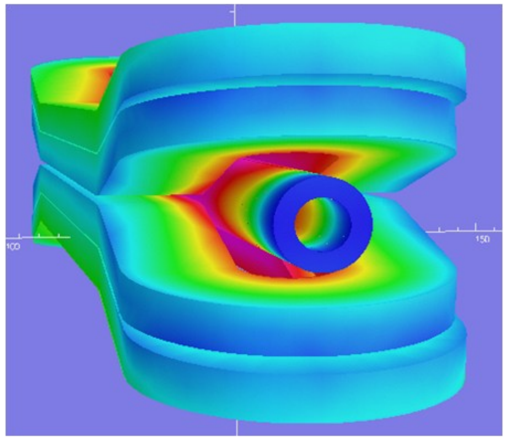


Figure 11: Computed field magnitude of HD2

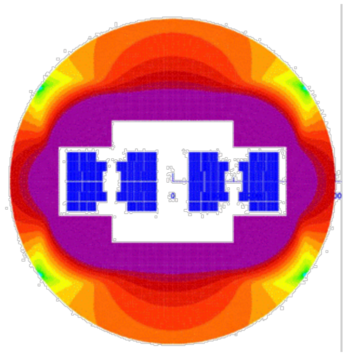
$$\frac{(16\ \mathrm{T})^2}{8\pi 10^{-7}} = 1000\ \mathrm{atm}$$

$$\frac{(20\ \mathrm{T})^2}{8\pi 10^{-7}} = 1600\ \mathrm{atm}$$

Advanced Magnet Design

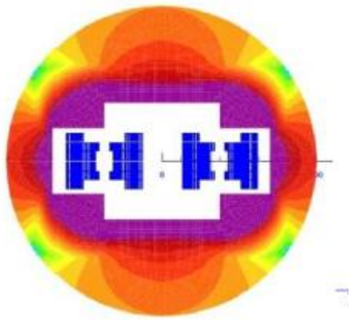
Texas A&M, LBNL, BNL, CERN …

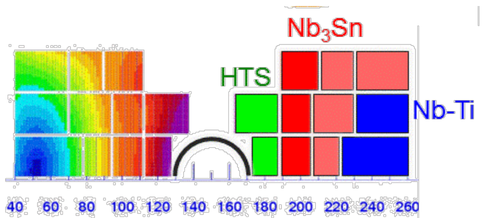
- "stress management"
  - "block" coils
  - end designs are critical



J_{eng} ~ 400 A/mm²

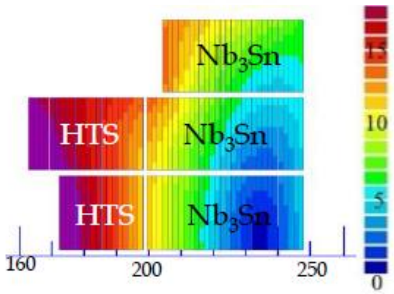
Nb-Ti up to	8 T
Nb_3Sn up to	13 T
HTS up to	20 T



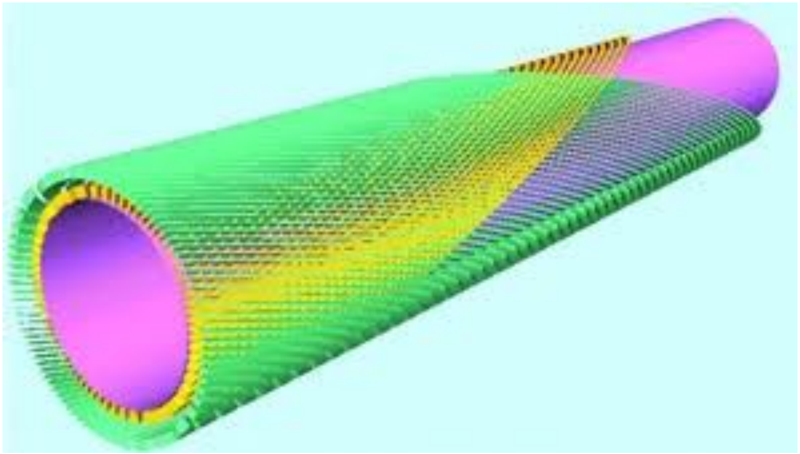


Cost optimized, graded winding

HTS: more than **1500 tons** procurement by 2030
↓
Concept and models now



- Canted Cosine Theta Design



nested arrangement of canted coils can possibly reach fields up to 16-20 T

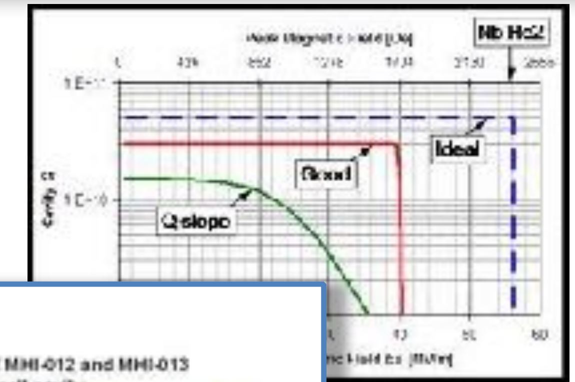
LBNL

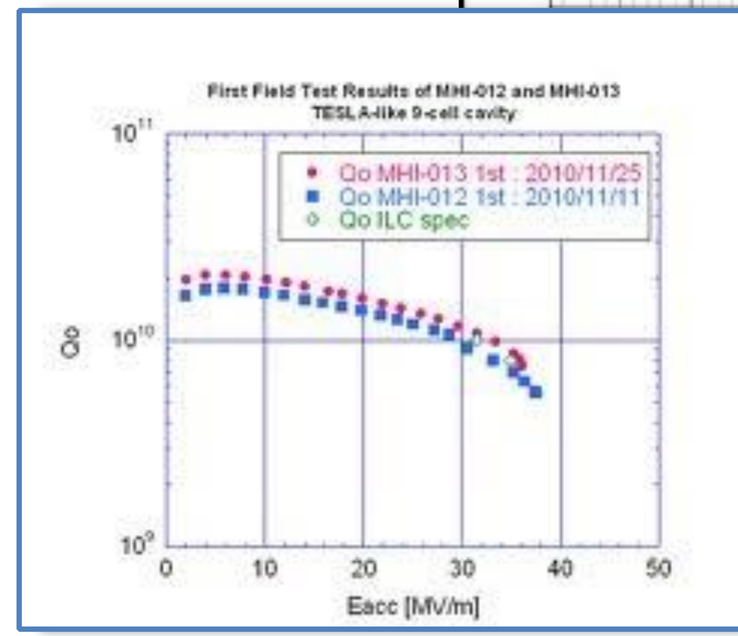
Accelerator Cavities

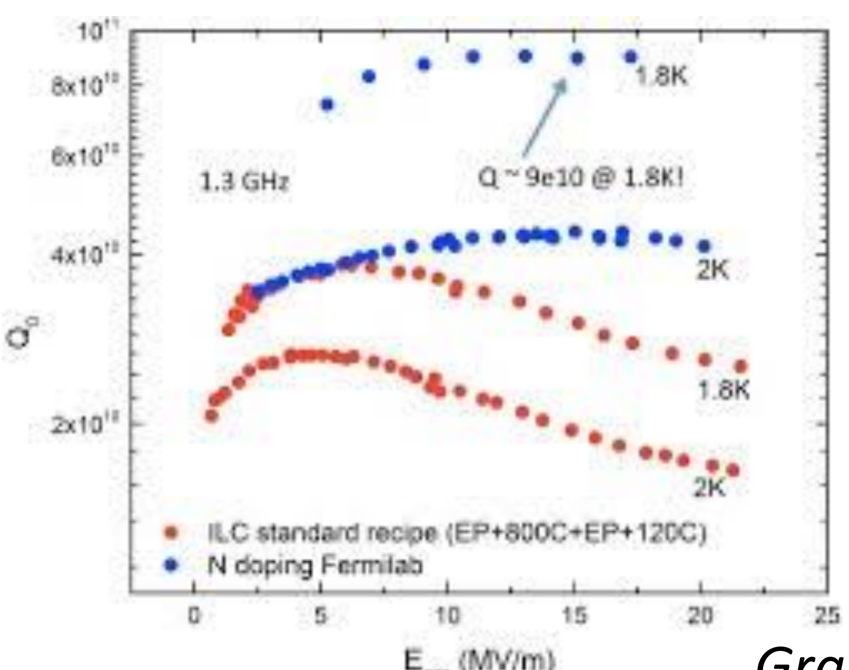
- SRF (efficient, 30-50 MV/m)
- vs. Cu (100-150 MV/m)

- development of production techniques
  - surface treatments, doping of Nb to reduce Q-slope, etc.

- Nb-coated materials — lower cost SRF?







Grassellino, et al.

Some Projects — Current and Envisioned

- FRIB (MSU) 200 MeV/u, 400 kW Facility for Rare Isotope Beams
- LCLS-II (SLAC) 4 GeV Linac Coherent Light Source upgrade
- FAIR (GSI) 34 GeV/u, 100 kW
- ESS (Lund) 5 MW European Spallation Source (2 GeV p linac)
- PIP-II (FNAL) 800 MeV p, MW-scale linac
- ILC (?? Japan ??) 500 GeV electron-positron collider
- CLIC (?? CERN??) 3 TeV electron-positron collider
- e^- — ion collider (?? BNL ??) 5-20 GeV e- / 50-250 GeV ions
- FCC (?? CERN ??) 100 km Future Circular Collider 100 TeV pp
- China: 50-80 km ring(s), 240 GeV ee collider; 70-100 TeV pp
- DAEdALUS cyclotrons, 800 MeV
- NuSTORM / NuFactory / Muon Collider — neutrino, muon sources
- …

Future Applications of Accelerators

- Energy — ADS
- Homeland Security
- Defense
- New Medical Techniques
- Isotope Production
- New industrial applications?

Extrapolating to the Future: Livingston Plot

- In 1954, M. Stanley Livingston produced a curve in his book *High Energy Accelerators*, indicating exponential growth in particle beam energies over "past" ~25 years;
  - the 33 "Bev" (GeV) AGS at Brookhaven and 28 GeV PS at CERN were underway, and kept up the trend
- The advent of Strong Focusing (A-G focusing) was key to keeping this trend going...

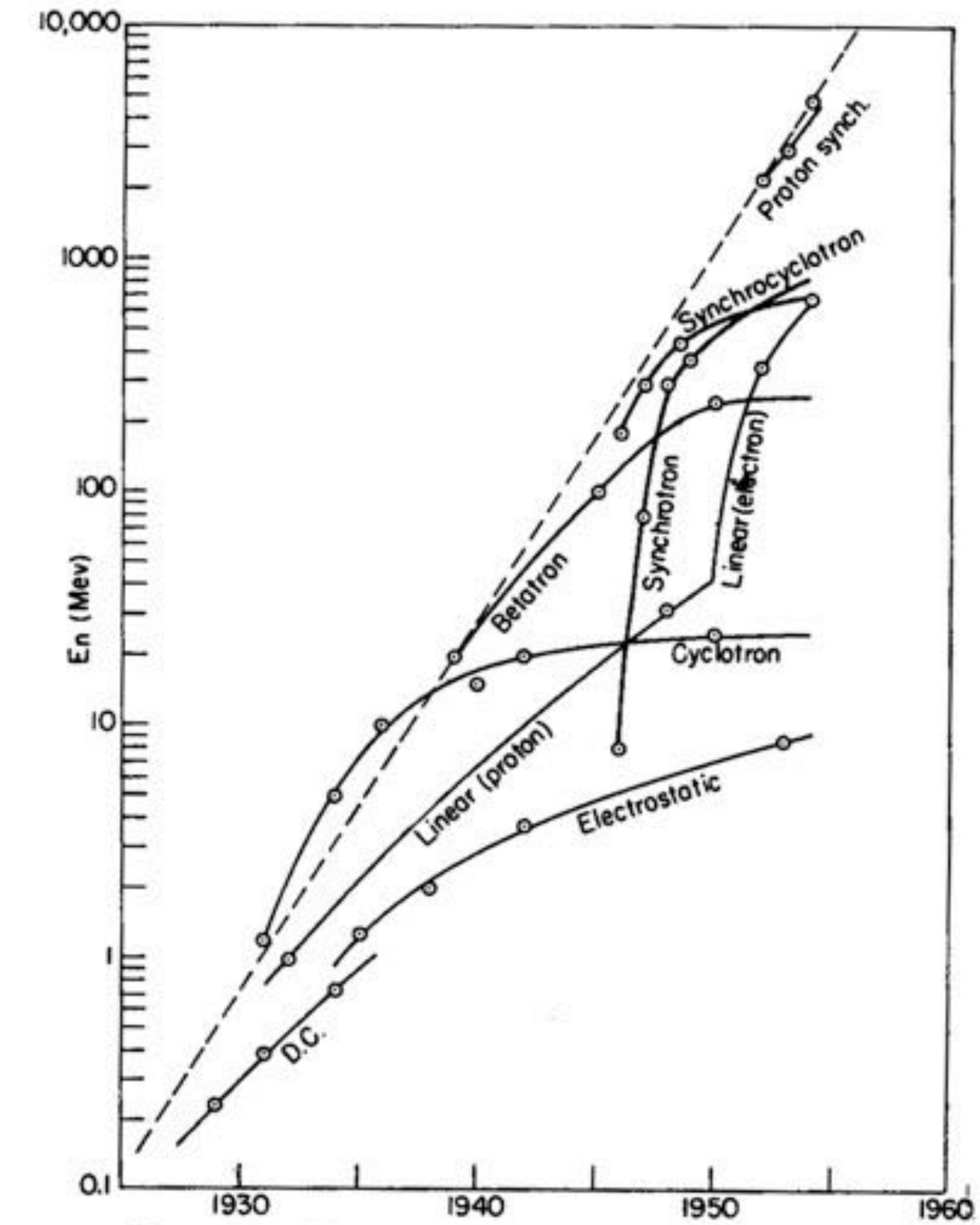


FIG. 7-8. Exponential rise in energy attained with accelerators during the past 25 years.

The Past 40 Years

Some limiting factors ...

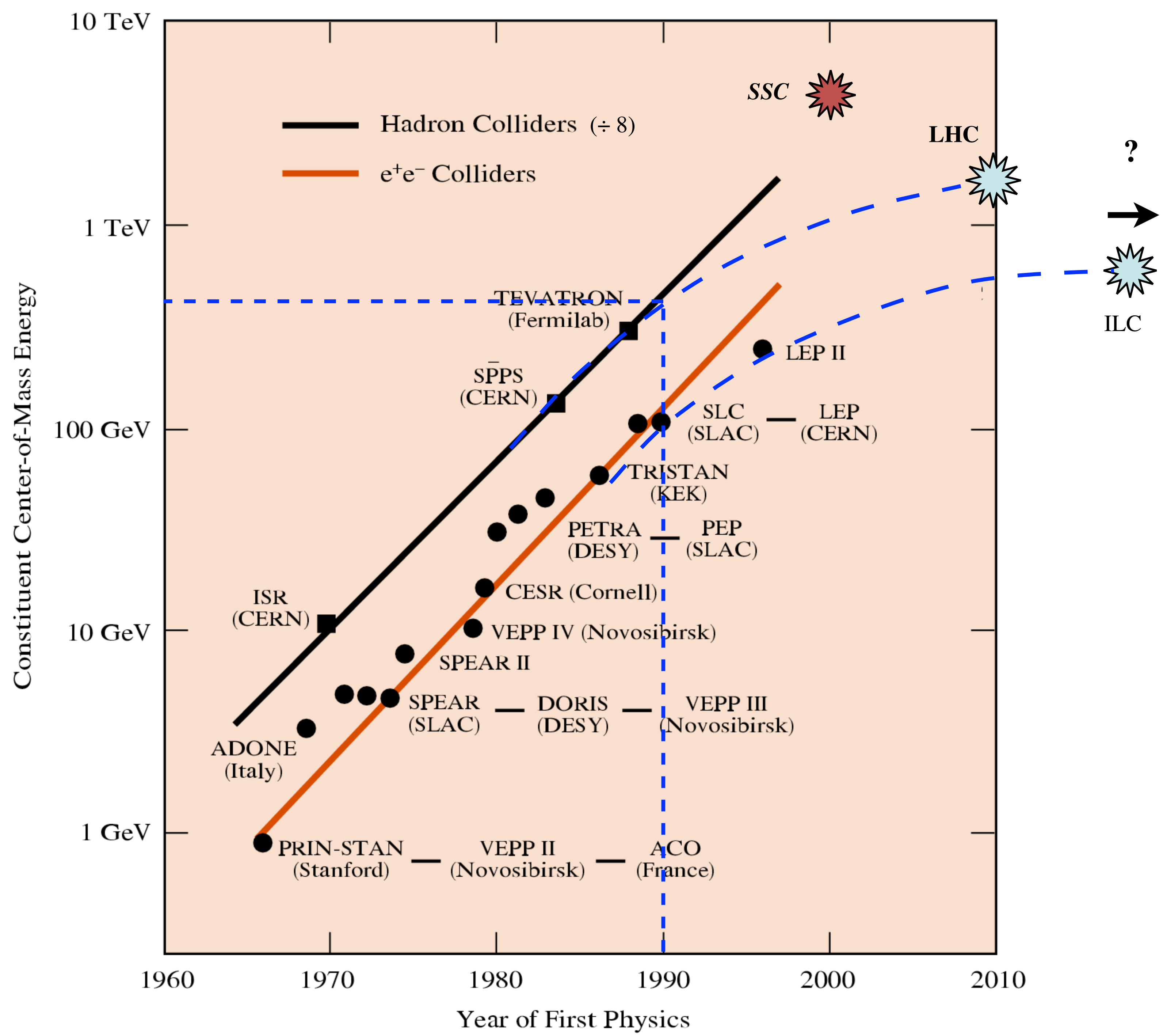
- superconducting technology -- accel. cavities this time, not magnets
- high accelerating gradient (>35 MeV/m)

- Synchrotron Radiation
  - effects obvious in e+e-; hence, the **L** in **ILC**
  - real estate vs. electric field strength
- stored energy an issue in LHC;   *beam power* issue in linac
- energy deposition in targets, Interaction Points; backgrounds
- small apertures --> alignment tolerances (micron scale)
- requires very small beam sizes — approaching nm scale
  - damping rings -- S.R. put to good use
  - emittance exchange -- eliminate need for damping rings?

The Livingston Curve Again

- In attempt to compare e- & p, switch to C-of-M view of constituents
- seeing a new roll-off happening
- driven by budgets, if constrained to present technology
- thus, need new technologies to make affordable...

adopted from W. Panofsky. *Beam Line* (SLAC) 1997



Lawrence Berkeley Lab Laser Wakefield Acceleration

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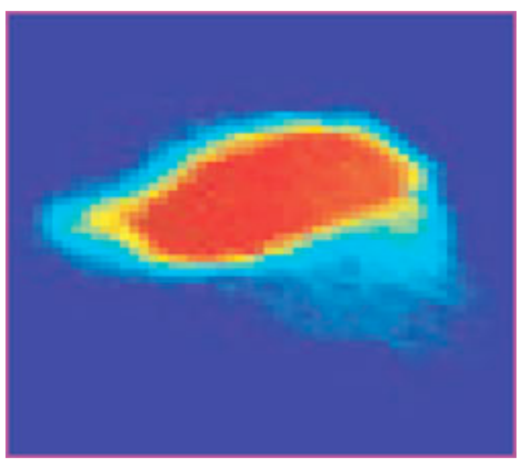
From Zero to a Billion Electron Volts in 3.3 Centimeters
Highest Energies Yet From Laser Wakefield Acceleration

Contact: Paul Preuss, (510) 486-6249, paul_preuss@lbl.gov

BERKELEY, CA — In a precedent-shattering demonstration of the potential of laser-wakefield acceleration, scientists at the Department of Energy's Lawrence Berkeley National Laboratory, working with colleagues at the University of Oxford, have accelerated electron beams to energies exceeding a billion electron volts (1 GeV) in a distance of just 3.3 centimeters. The researchers report their results in the October issue of *Nature Physics*.



Billion-electron-volt, high-quality electron beams have been produced with laser wakefield acceleration in recent experiments by Berkeley Lab's LOASIS group, in collaboration with scientists from Oxford University.

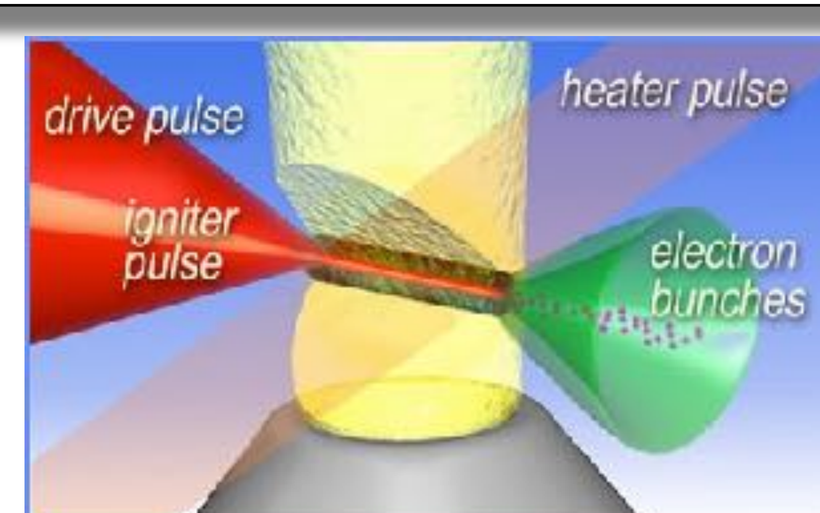
By comparison, SLAC, the Stanford Linear Accelerator Center, boosts electrons to 50 GeV over a distance of two miles (3.2 kilometers) with radiofrequency cavities whose accelerating electric fields are limited to about 20 million volts per meter.

The electric field of a plasma wave driven by a laser pulse can reach 100 *billion* volts per meter, however, which has made it ... Oxford ... energy ...

This is ... Lab's A... "Billion... accelerators open the way to very compact high-energy experiments and superbright free-electron lasers."

Also, similar (but different) efforts at U Texas, U Michigan, Stanford/SLAC, elsewhere...

- 30 *G*eV/m, compared to 30 *M*eV/m in present SRF cavity designs
- ... and, *small* momentum spread (2-5%) as well



Lawrence Berkeley Lab Laser Wakefield Acceleration

- Staging Demo

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NATURE | LETTER

Multistage coupling of independent laser-plasma accelerators

S. Steinke, J. van Tilborg, C. Benedetti, C. G. R. Geddes, C. B. Schroeder, J. Daniels, K. K. Swanson, A. J. Gonsalves, K. Nakamura, N. H. Matlis, B. H. Shaw, E. Esarey & W. P. Leemans

Affiliations | Contributions | Corresponding author

Nature (2016) | doi:10.1038/nature16525

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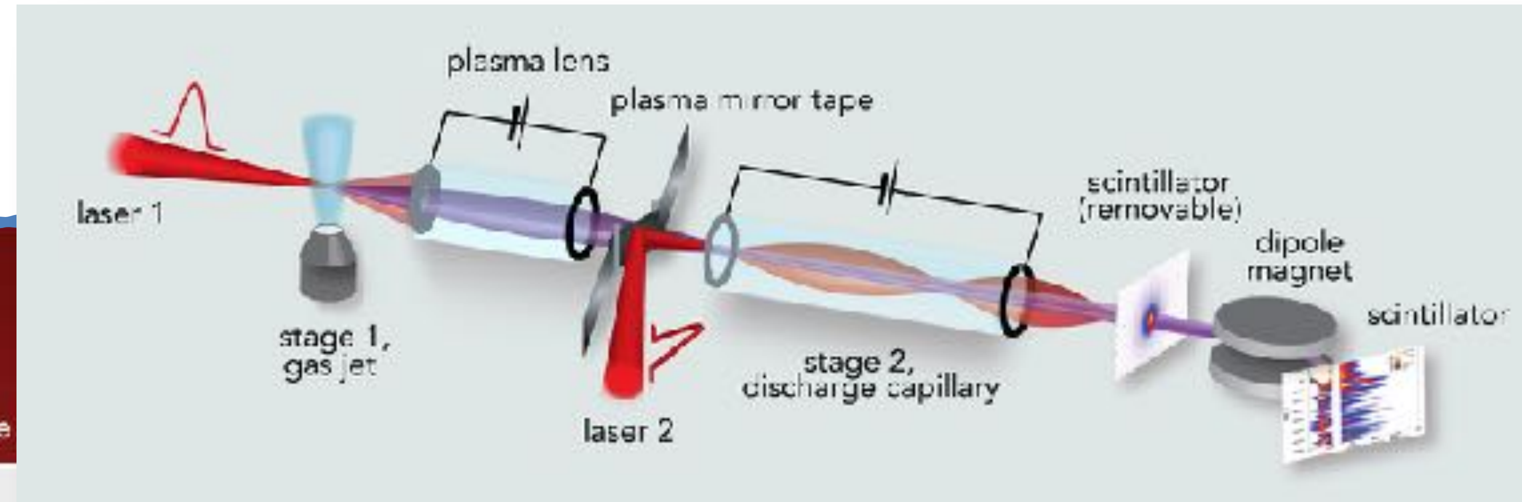
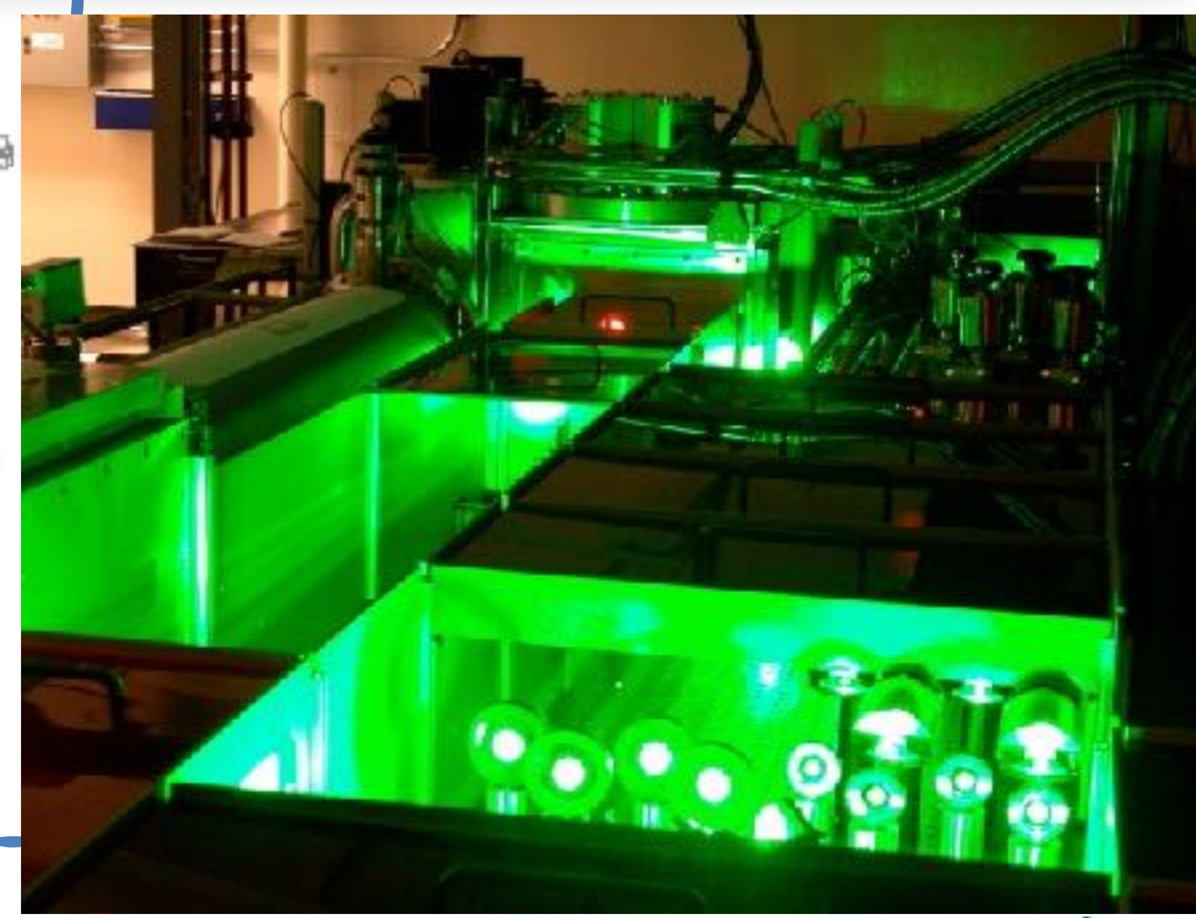




Summer 2017 MJS EBSS 2017

In the Meantime, … FCC?

- Coming off of the successful first running of LHC, and the Nobel-Prize-winning Higgs discovery, Europe is engaging the international community in discussions/studies of a **Future Circular Collider**
- China is also looking at large rings for its future

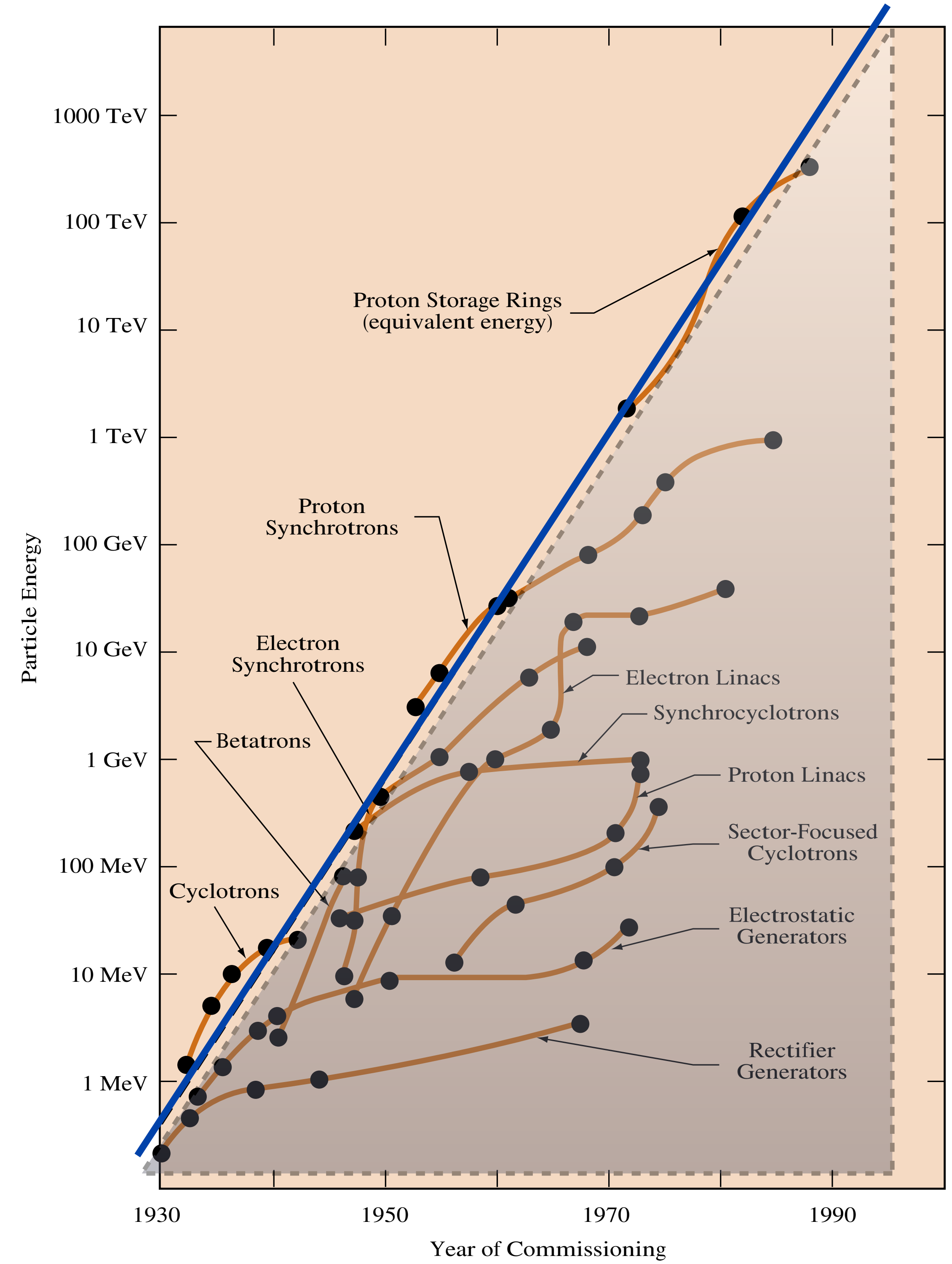
Parameters are a 50 TeV x 50 TeV p-p collider, with circumference ~100 km (about same size as the Texas SSC, which was a 20 TeV x 20 TeV collider)

View from France into Switzerland, showing existing LHC complex (orange) and a possible 100 TeV collider ring (yellow)

photo courtesy J. Wenninger (CERN)

Looking Below the Curve

- Accelerator Facilities, and the need for scientists to develop, build, commission, operate, improve them have seen an enormous growth over the decades
- While peak accelerator energies continue to drive particle physics, much work to do and applications to develop at lower energies
- Many, many facilities and industrial uses are not shown here, but flood the area “below the curve”



A “Final” word...

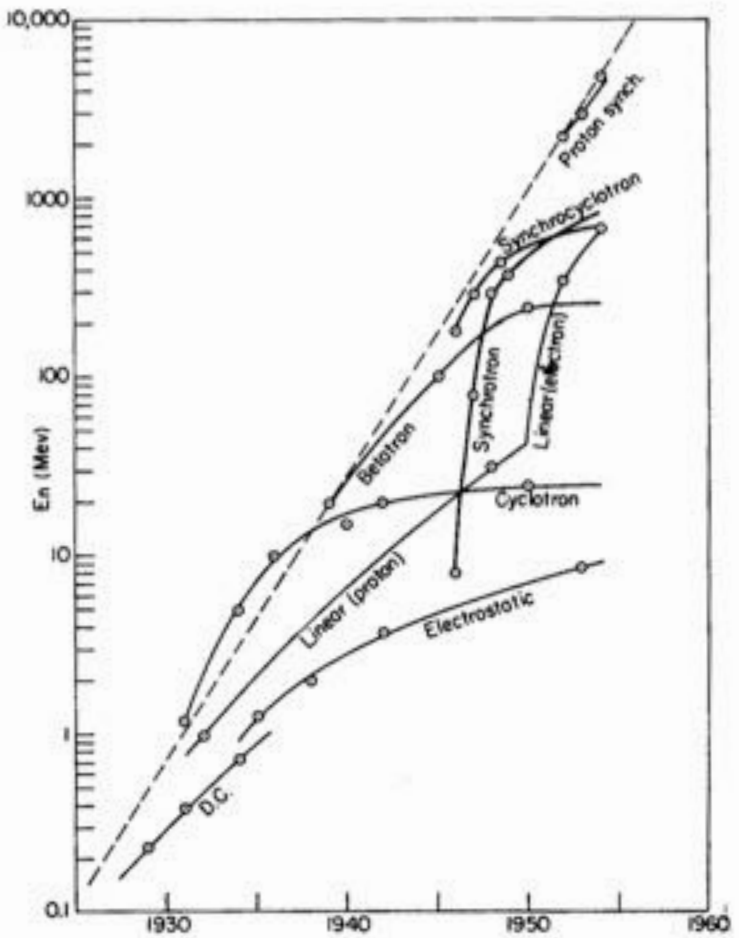


FIG. 7-8. Exponential rise in energy attained with accelerators during the past 25 years.

of the plot is the approximately linear slope of this envelope, which means that energy has in fact increased exponentially with time. The rate of rise is such that the energy has increased by a factor of 10 every six years, from a start at 100 kv in 1929 to 3 billion volts in 1952.

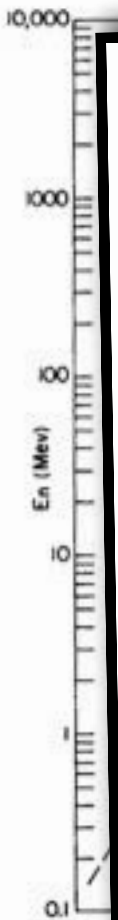
It is interesting to extrapolate this curve into the future, to predict the energy of accelerators after another six years. We have reason to hope that either the Brookhaven or the CERN A-G proton synchrotrons will have reached 25 Bev by that

time. Further extrapolation of this exponentially rising curve would predict truly gigantic accelerators which would exceed any possible budgets, even those of government laboratories. So we will postpone such speculation until the present machines can demonstrate their value to science.

Those of us in the accelerator field are frequently asked, “When will this development of higher-and-higher-energy accelerators stop?” Yet it must be recognized that it is not the urge to higher voltage which inspires this growth, but the pressure of the continuously expanding horizons of science. As long as there are unsolved problems in Nature which might be answered by higher-energy particles, and as long as the scientific urge to know the answers continues, there will be a steady and persistent demand to develop the tools and instruments required.

A “Final” word...

ALTERNATE GRADIENT FOCUSING 151



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152 HIGH-ENERGY ACCELERATORS

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M. Stanley Livingston, 1954

THANKS!

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syphers@fnal.gov

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  - D. A. Edwards and M. J. Syphers, *An Introduction to the Physics of High Energy Accelerators*, John Wiley & Sons (1993)
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  - H. Padamsee, J. Knobloch, T. Hays, *RF Superconductivity for Accelerators*, John Wiley & Sons (1998)
  - S. Y. Lee, *Accelerator Physics*, World Scientific (1999)
  - and many others…
- Many Conference Proceedings — visit http://www.jacow.org