Nuclear Reaction Theory: concepts and applications – Part II

Exotic Beams Summer School 2012, Argonne National Laboratory, 5th – 9th August 2012

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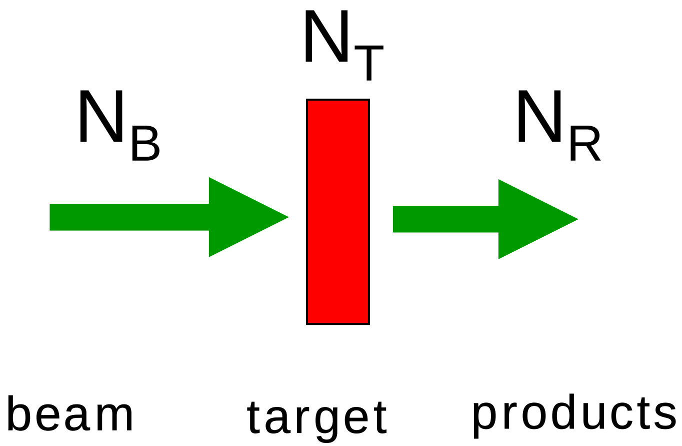
Reactions: rarest beams - ‘few’ nuclei per second



- Fast exotic beams allow for
  – thick secondary targets
  – event-by-event identification
  – clean product selection
  – nevertheless …..

- $N_R = s \times N_T \times N_B$
  - s cross section
  - N_T atoms in target
  - N_B beam rate
  - N_R reaction rate

- Example
  - s = 100 millibarn
  - $N_T = 10^{21}$
  - $N_B = 3$ Hz
  - $N_R = 26$/day
    $= 3\times10^{-4}$ Hz

From Thomas Glasmacher

Point particles: partial wave S-matrix

<u>Scattering states</u>

$$E_{cm} > 0 \quad k = \sqrt{\frac{2\mu E_{cm}}{\hbar^2}}$$

$$\left(\frac{d^2}{dr^2} - \frac{\ell(\ell+1)}{r^2} - \frac{2\mu}{\hbar^2} U_{\ell j}(r) + k^2\right) u_{k\ell j}(r) = 0$$

and beyond the range of the nuclear forces, then

$$\left(\frac{d^2}{dr^2} - \frac{\ell(\ell+1)}{r^2} - \frac{2\eta k}{r} + k^2\right) u_{k\ell j}(r) = 0, \quad \eta = \frac{\mu Z_c Z_v e^2}{\hbar k}$$

$F_\ell(\eta, kr), \; G_\ell(\eta, kr)$ regular and irregular Coulomb functions

$$\boxed{\begin{aligned} u_{k\ell j}(r) &\rightarrow e^{i\delta_{\ell j}}\left[\cos\delta_{\ell j} F_\ell(\eta, kr) + \sin\delta_{\ell j} G_\ell(\eta, kr)\right] \\ &\rightarrow (i/2)\left[H_\ell^{(-)}(\eta, kr) - S_{\ell j} H_\ell^{(+)}(\eta, kr)\right] \end{aligned}}$$

$$H_\ell^{(\pm)}(\eta, kr) = G_\ell(\eta, kr) \pm i F_\ell(\eta, kr)$$

Phase shift and partial wave S-matrix: Recall

$$u_{k\ell j}(r) \quad \rightarrow \quad e^{i\delta_{\ell j}} \underbrace{[\cos\delta_{\ell j} F_\ell(\eta, kr) + \sin\delta_{\ell j} G_\ell(\eta, kr)]}$$

If *U(r)* is real, the phase shifts $\delta_{\ell j}$ are real, and [...] also

$$u_{k\ell j}(r) \quad \rightarrow \quad (i/2)[H_\ell^{(-)}(\eta, kr) - S_{\ell j} H_\ell^{(+)}(\eta, kr)]$$

Ingoing waves

outgoing waves

$$S_{\ell j} = e^{2i\delta_{\ell j}}$$

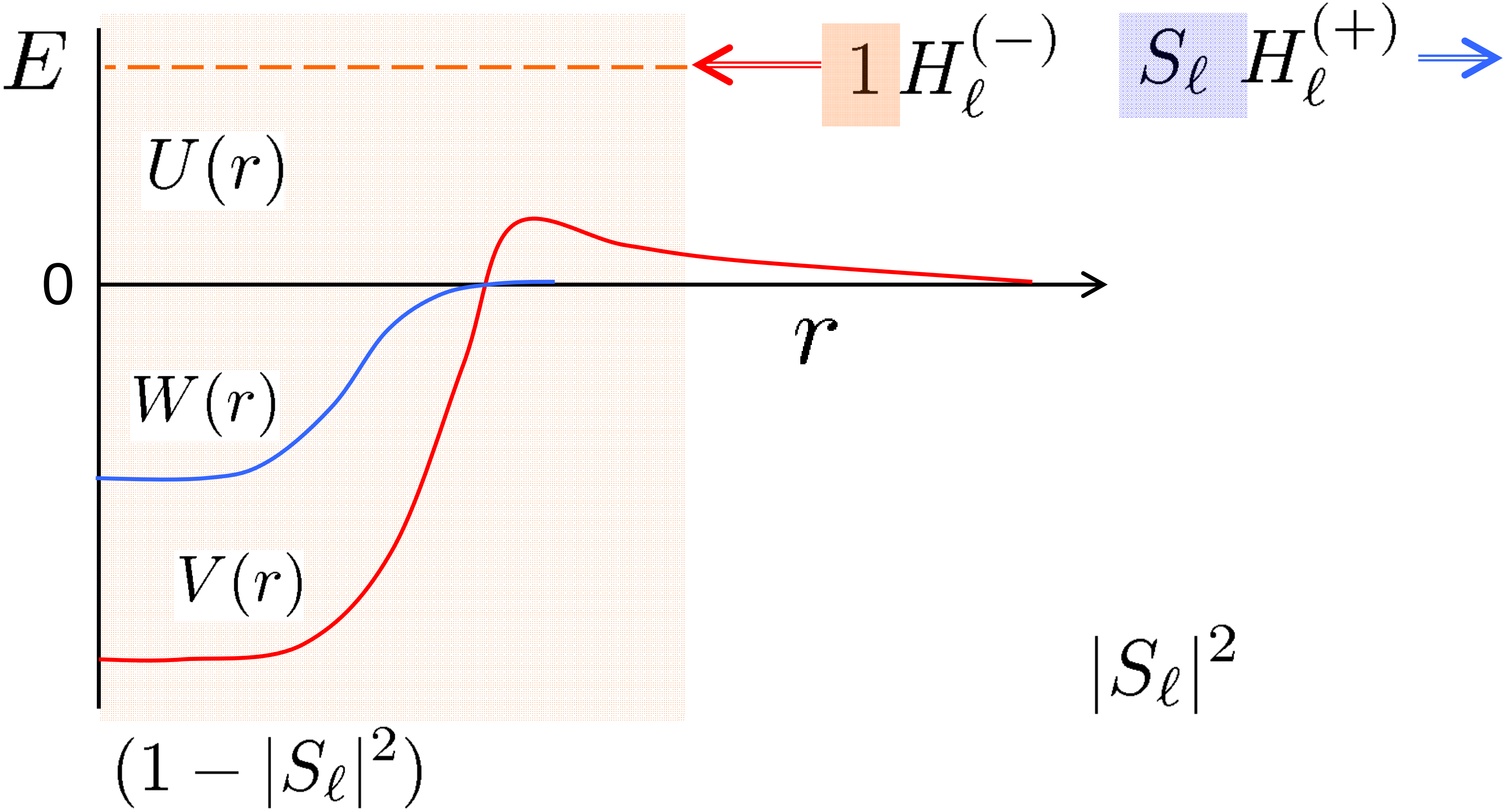
$|S_{\ell j}|^2$ survival probability in the scattering

$(1 - |S_{\ell j}|^2)$ absorption probability in the scattering

Having calculate the phase shifts and the partial wave S-matrix elements we can then compute all scattering observables for this energy and potential (but later).

Ingoing and outgoing waves amplitudes

$$u_{k\ell}(r) \rightarrow (i/2)[\boxed{1}\, H_\ell^{(-)} - \boxed{S_\ell} H_\ell^{(+)}]$$



Eikonal approximation: for point particles (1)

Approximate (semi-classical) scattering solution of

$$\left(-\frac{\hbar^2}{2\mu}\nabla_r^2 + U(r) - E_{cm}\right)\chi_{\vec{k}}^{+}(\vec{r}) = 0, \qquad \mu = \frac{m_c m_v}{m_c + m_v}$$

$$\left(\nabla_r^2 - \frac{2\mu}{\hbar^2}U(r) + k^2\right)\chi_{\vec{k}}^{+}(\vec{r}) = 0$$

valid when $\boxed{|U|/E \ll 1, \quad ka \gg 1}$ → small wavelength high energy

Key steps are: (1) the distorted wave function is written

$$\chi_{\vec{k}}^{+}(\vec{r}) = \exp(i\vec{k}\cdot\vec{r})\,\boxed{\omega(\vec{r})}$$

all effects due to *U(r)*, modulation function

(2) Substituting this product form in the Schrodinger Eq.

$$\left[2i\vec{k}\cdot\nabla\omega(\vec{r}) - \frac{2\mu}{\hbar^2}U(r)\omega(\vec{r}) + \nabla^2\omega(\vec{r})\right]\exp(i\vec{k}\cdot\vec{r}) = 0$$

Eikonal approximation: point particles (2)

$$\left[2i\vec{k}\cdot\nabla\omega(\vec{r}) - \frac{2\mu}{\hbar^2}U(r)\omega(\vec{r}) + \cancel{\nabla^2\omega(\vec{r})}\right]\exp(i\vec{k}\cdot\vec{r}) = 0$$

The conditions $|U|/E \ll 1, \quad ka \gg 1$ → imply that

$$\boxed{2\vec{k}\cdot\nabla\omega(\vec{r}) \gg \nabla^2\omega(\vec{r})}$$

Slow spatial variation cf. k

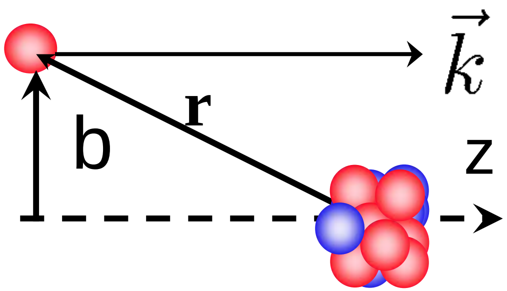
and choosing the z-axis in the beam direction $\vec{k}$

$$\frac{d\omega}{dz} \approx -\frac{i\mu}{\hbar^2 k}U(r)\omega(\vec{r})$$

phase that develops with z

with solution

$$\boxed{\omega(\vec{r}) = \exp\left[-\frac{i\mu}{\hbar^2 k}\int_{-\infty}^{z} U(r)dz'\right]}$$



1D integral over a straight line path through U at the impact parameter b

Eikonal approximation: point particles (3)

$$\chi^+_{\vec{k}}(\vec{r}) = \exp(i\vec{k}\cdot\vec{r})\ \omega(\vec{r}) \approx \exp(i\vec{k}\cdot\vec{r})\ \boxed{\exp\left[-\frac{i\mu}{\hbar^2 k}\int_{-\infty}^{z} U(r)dz'\right]}$$

So, after the interaction and as $z \to \infty$

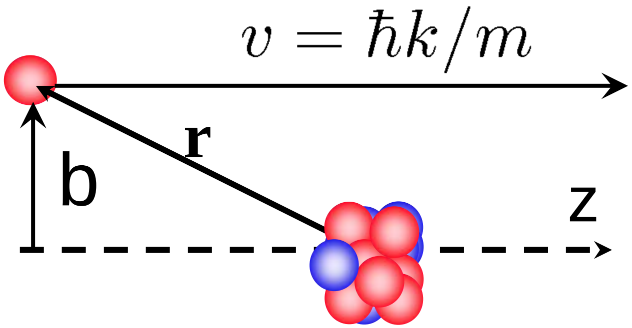
$$\chi^+_{\vec{k}}(\vec{r}) \to \exp(i\vec{k}\cdot\vec{r})\ \exp\left[-\frac{i\mu}{\hbar^2 k}\int_{-\infty}^{\infty} U(r)dz'\right] = S(b)\exp(i\vec{k}\cdot\vec{r})$$

$$\boxed{\chi^+_{\vec{k}}(\vec{r}) \to S(b)\exp(i\vec{k}\cdot\vec{r})}$$

S(b) is amplitude of the forward going outgoing waves from the scattering at impact parameter b

Eikonal approximation to the S-matrix S(b)

$$\boxed{S(b) = \exp\left[-\frac{i}{\hbar v}\int_{-\infty}^{\infty} U(r)dz'\right]}$$

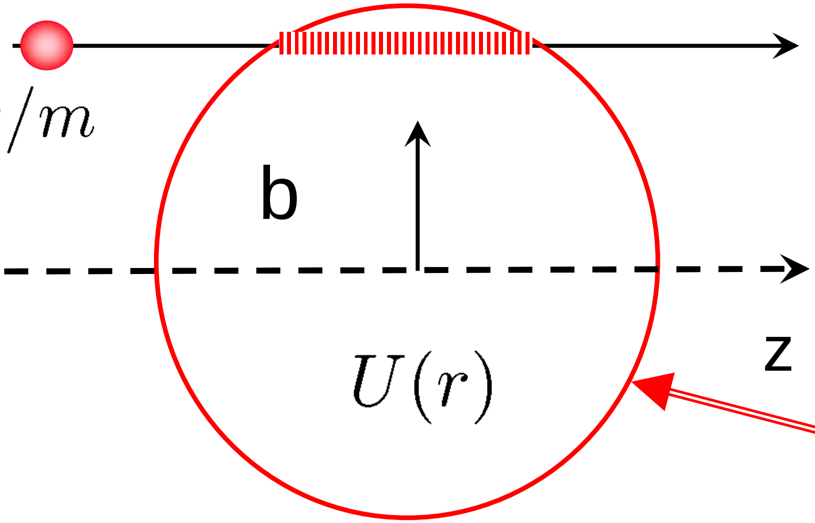


Moreover, the structure of the theory generalises simply to few-body projectiles

Eikonal approximation: point particles - summary

$$\chi^{+}_{\vec{k}}(\vec{r}) = \exp(i\vec{k}\cdot\vec{r})\ \exp\left[-\frac{i\mu}{\hbar^2 k}\int_{-\infty}^{z} U(r)dz'\right]$$

$$v = \hbar k/m$$



$$\chi(b) = -\frac{1}{\hbar v}\int_{-\infty}^{\infty} U(r)dz$$

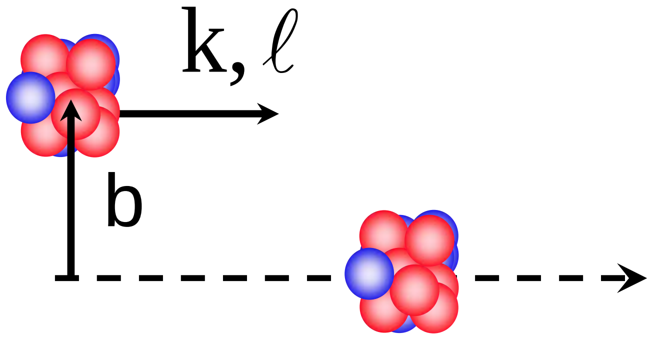
limit of range of finite ranged potential

$$\chi^{+}_{\vec{k}}(\vec{r}) \rightarrow S(b)\exp(i\vec{k}\cdot\vec{r})$$

$$\boxed{S(b) = \exp\left[i\chi(b)\right] = \exp\left[-\frac{i}{\hbar v}\int_{-\infty}^{\infty} U(r)dz'\right]}$$

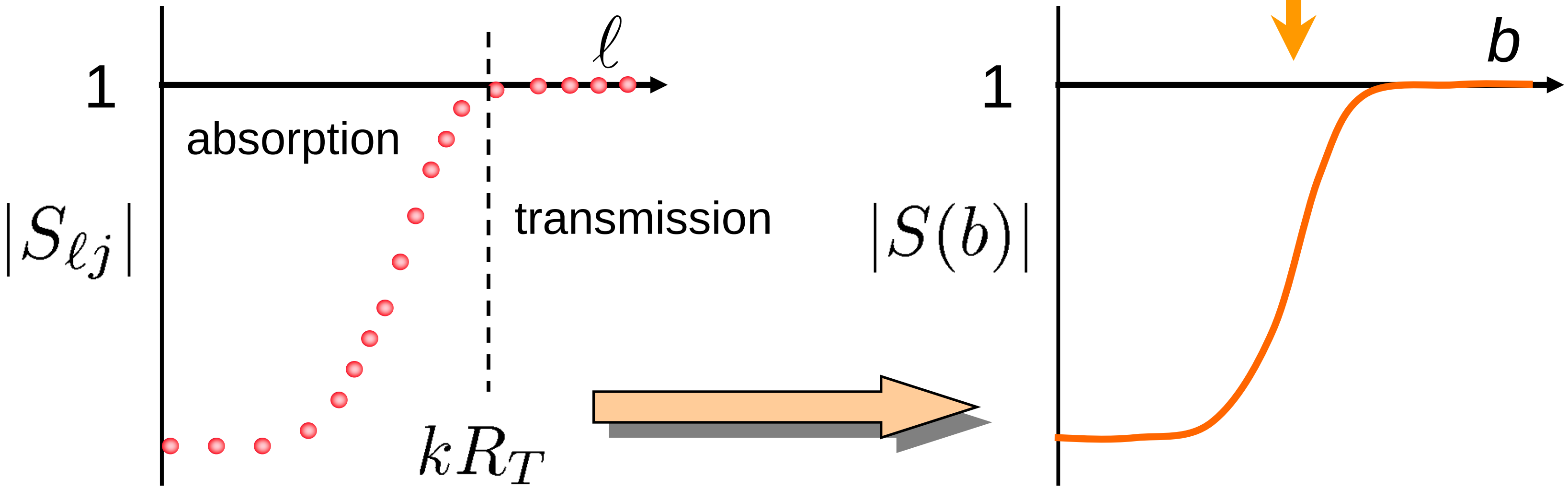
Semi-classical model for the S-matrix - S(b)

b=impact parameter



for high energy/or large mass, semi-classical ideas are good

$kb \simeq \ell$, actually $\Rightarrow \ell+1/2$



$$u_{k\ell j}(r) \to (i/2)[H_\ell^{(-)} - S_{\ell j} H_\ell^{(+)}] \qquad S(b) = \exp\left[-\frac{i}{\hbar v}\int_{-\infty}^{\infty} U(r)dz'\right]$$

Point particle scattering – cross sections

All cross sections, etc. can be computed from the S-matrix, in either the partial wave or the eikonal (impact parameter) representation, for example (spinless case):

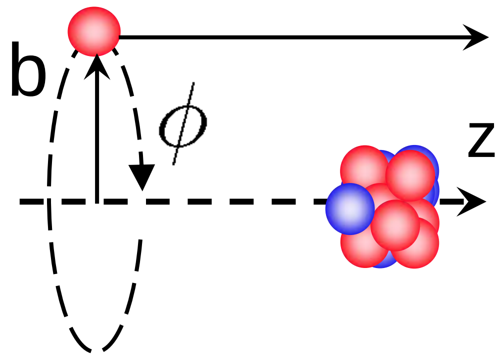
$$\sigma_{el} = \frac{\pi}{k^2}\sum_{\ell=0}^{\infty}(2\ell+1)|1-S_\ell|^2 \approx \int d^2\vec{b}\;|1-S(b)|^2$$

$$\sigma_R = \frac{\pi}{k^2}\sum_{\ell=0}^{\infty}(2\ell+1)(1-|S_\ell|^2) \approx \int d^2\vec{b}\;(1-|S(b)|^2)$$

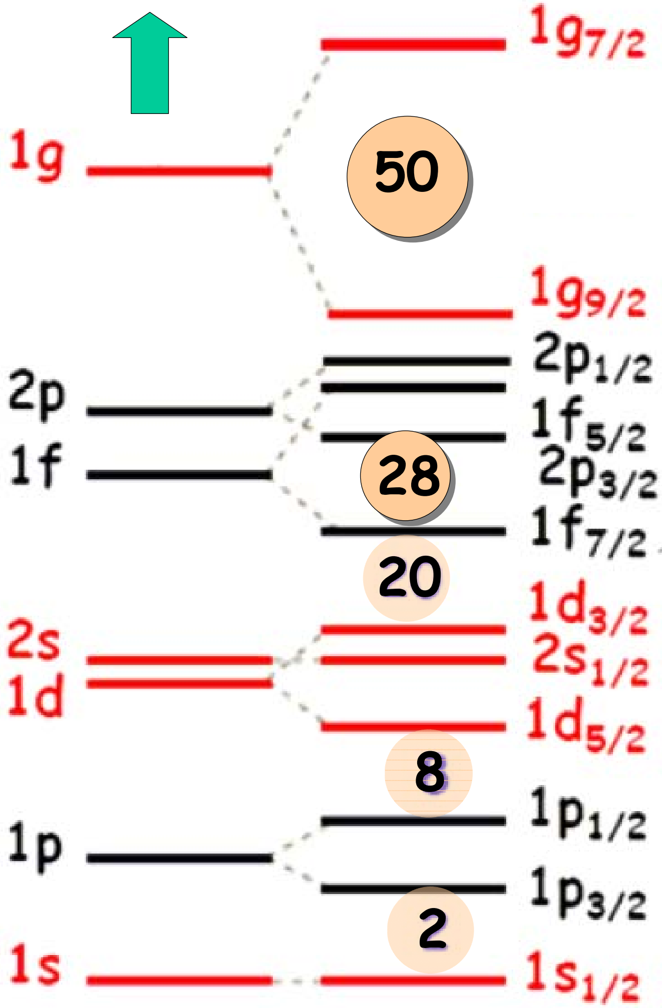
$$\sigma_{tot} = \sigma_{el} + \sigma_R = 2\int d^2\vec{b}\;[1-\mathrm{Re}.S(b)]$$ etc.

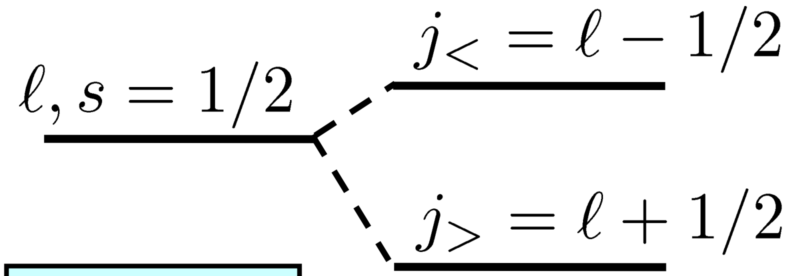
and where (cylindrical coordinates)

$$\int d^2\vec{b} \equiv \int_0^\infty bdb \int_0^{2\pi} d\phi = 2\pi \int_0^\infty bdb$$



What is involved in realistic reaction calculations?

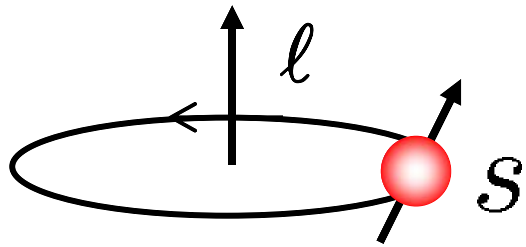




^{23}O

Z=8

N=15

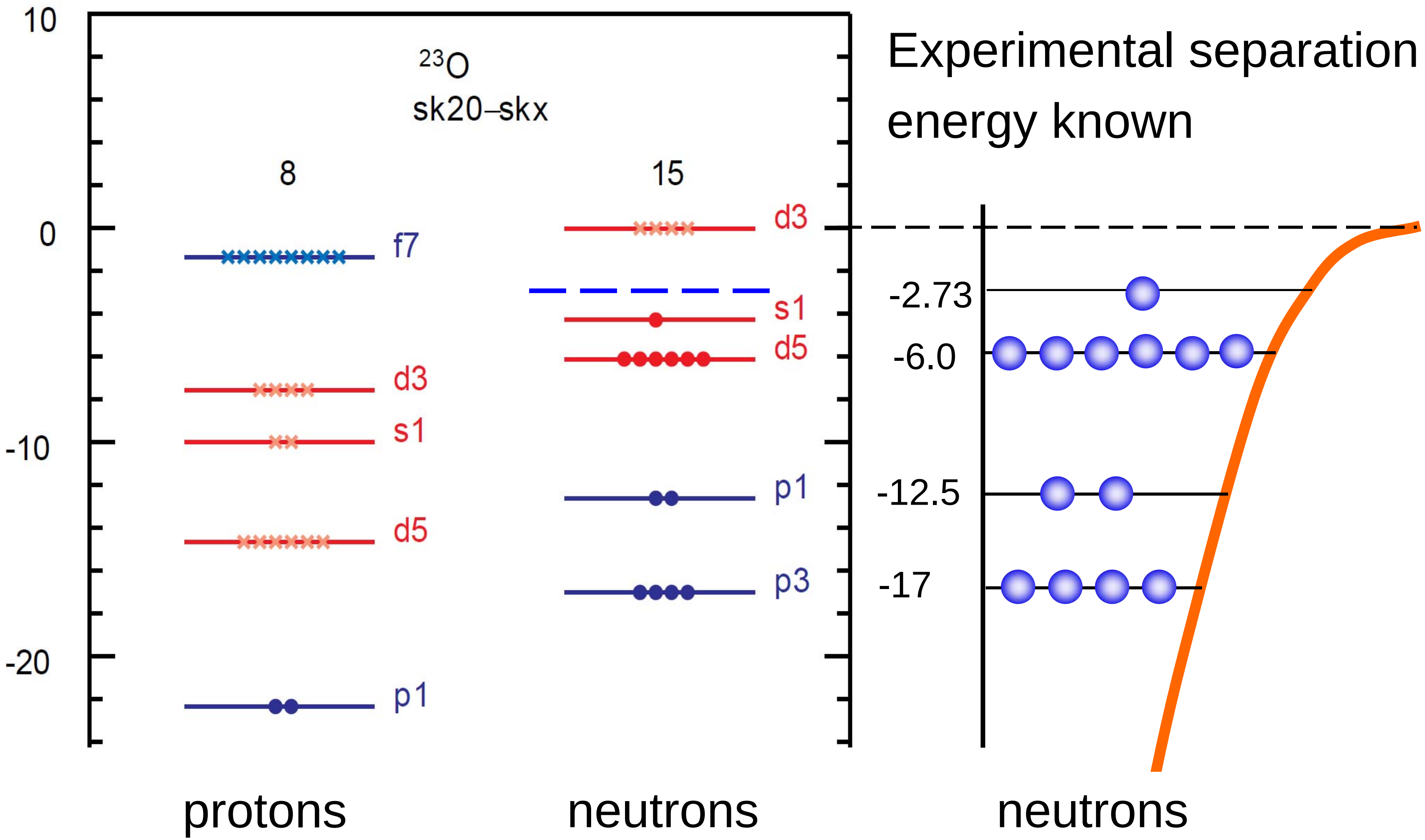


$$V_{\ell s}(r)\vec{\ell}\cdot\vec{s}$$

$$V(r) + V_{so}(r)\vec{\ell}\cdot\vec{s}$$

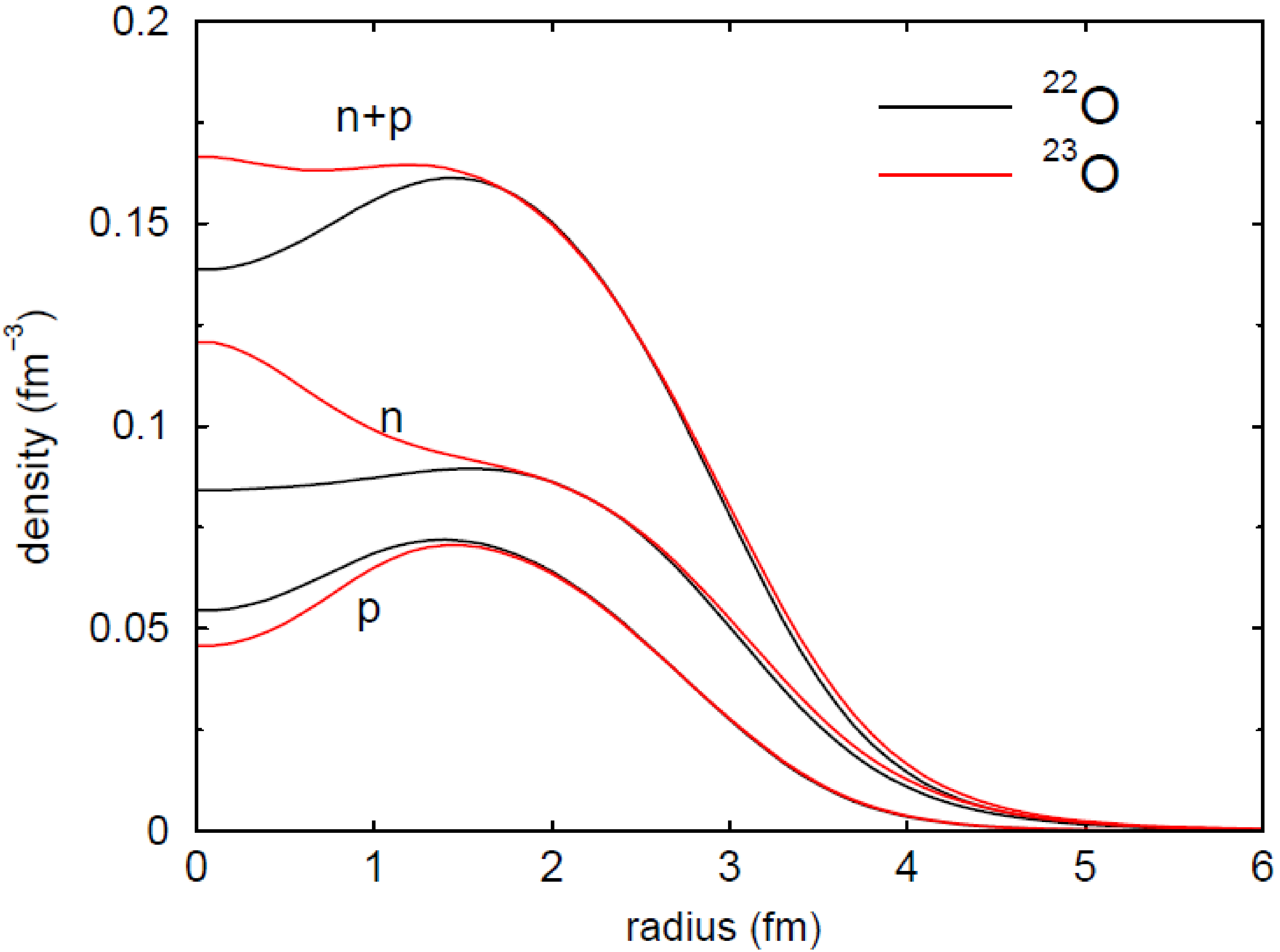
$$V_{so}(r) < 0$$

Examples: What is involved – take neutron from ^{23}O



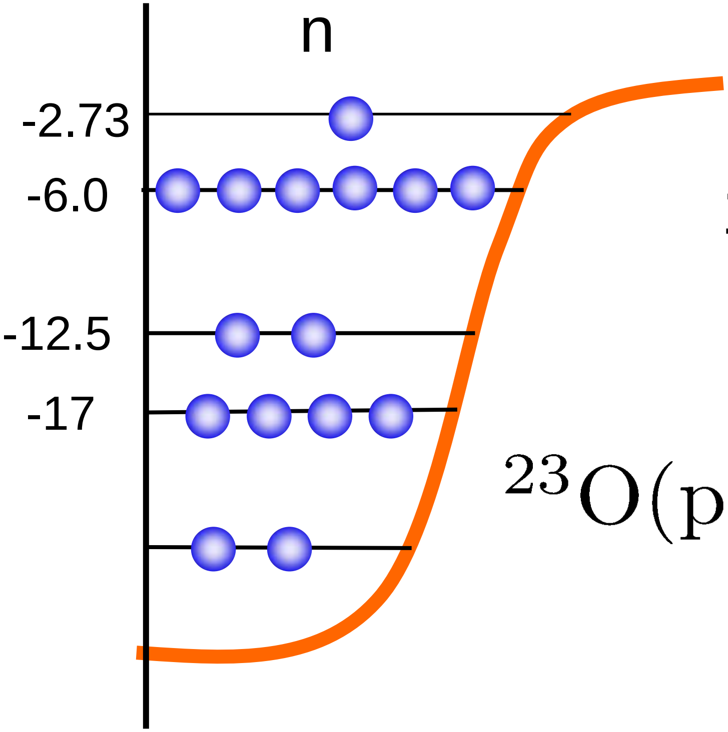
Hartree-Fock mean field calculation

Neutron: proton: nucleon radial densities (HF)



Orientation I – neutron transfer – (p,d) reaction

Single neutron removal from $^{23}O \equiv [1d_{5/2}]^6\,[2s_{1/2}]$



$2s_{1/2}$ S_n=2.73 MeV

$1d_{5/2}$ S_n=6.0 MeV

$$^{23}\mathrm{O(p,d)}^{22}\mathrm{O}(J_f^{\pi})$$

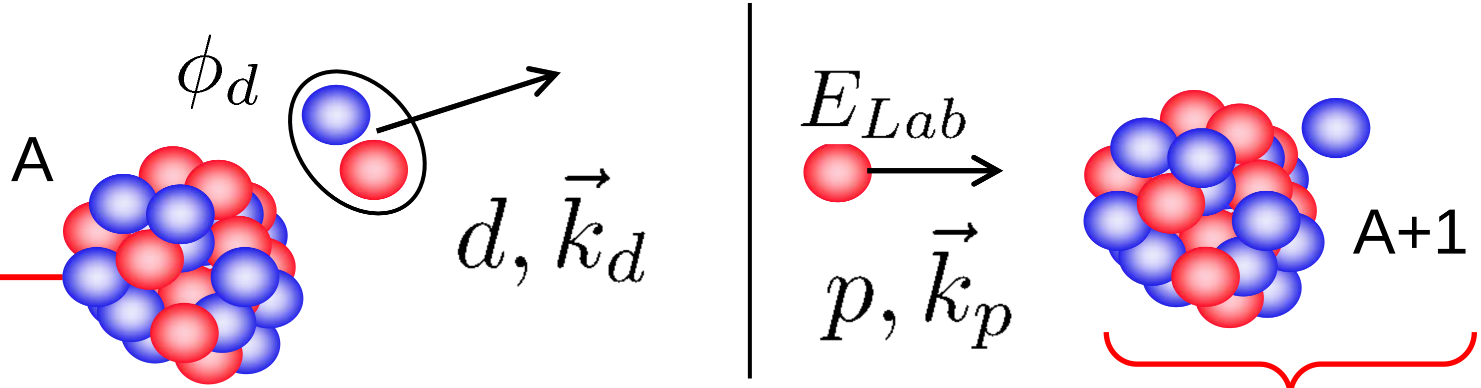
$J_f^{\pi} = 0^+$, g.s.
S_n=2.73 MeV

$J_f^{\pi} = 2^+$, 3.2 MeV
S_n=6.0 MeV

transfer reaction code(s) available at:
http://www.nucleartheory.net/NPG/code.htm

Transfer reaction transition amplitudes - DWBA

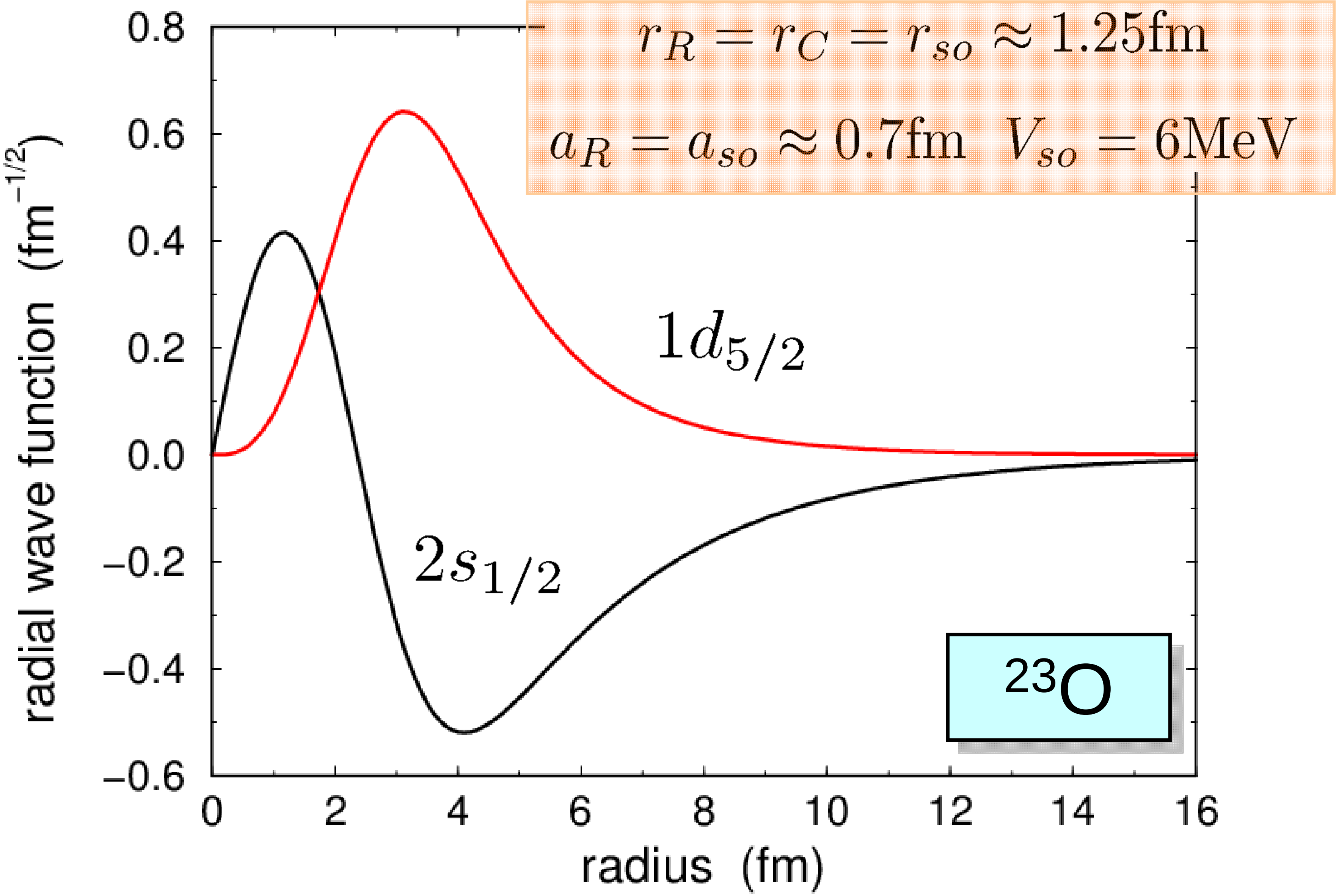
$$T(p,d) = \langle \underbrace{\chi^{(-)}_{d,\vec{k}_d}\, \Phi(A,J_f)\phi_d}_{\text{exit channel}} | V_{np} | \underbrace{\chi^{(+)}_{p,\vec{k}_p}\, \Phi(A+1,J_i)}_{\text{entrance channel}} \rangle$$



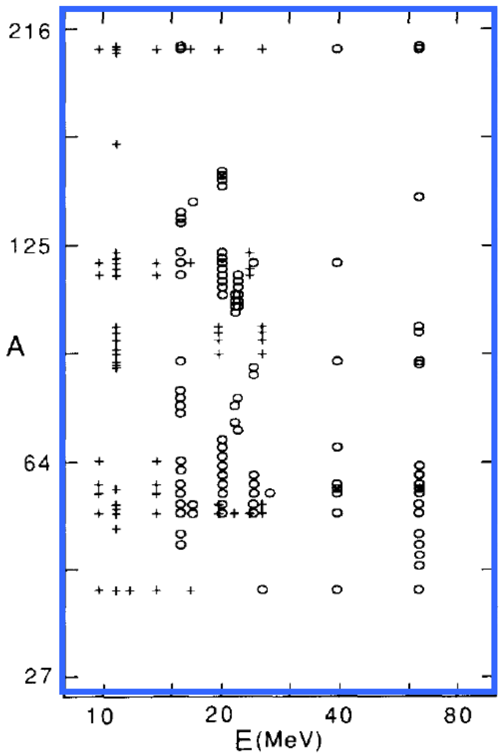
$$V_{np}\phi_d(\vec{r}) \approx D_0 \delta(\vec{r})$$ - short range

$$\phi_{n\ell j}(\vec{r}_n) = \langle \Phi(A,J_f) | \Phi(A+1,J_i) \rangle$$

Neutron bound state wave functions



Global optical potentials – e.g. CH91 for nucleons



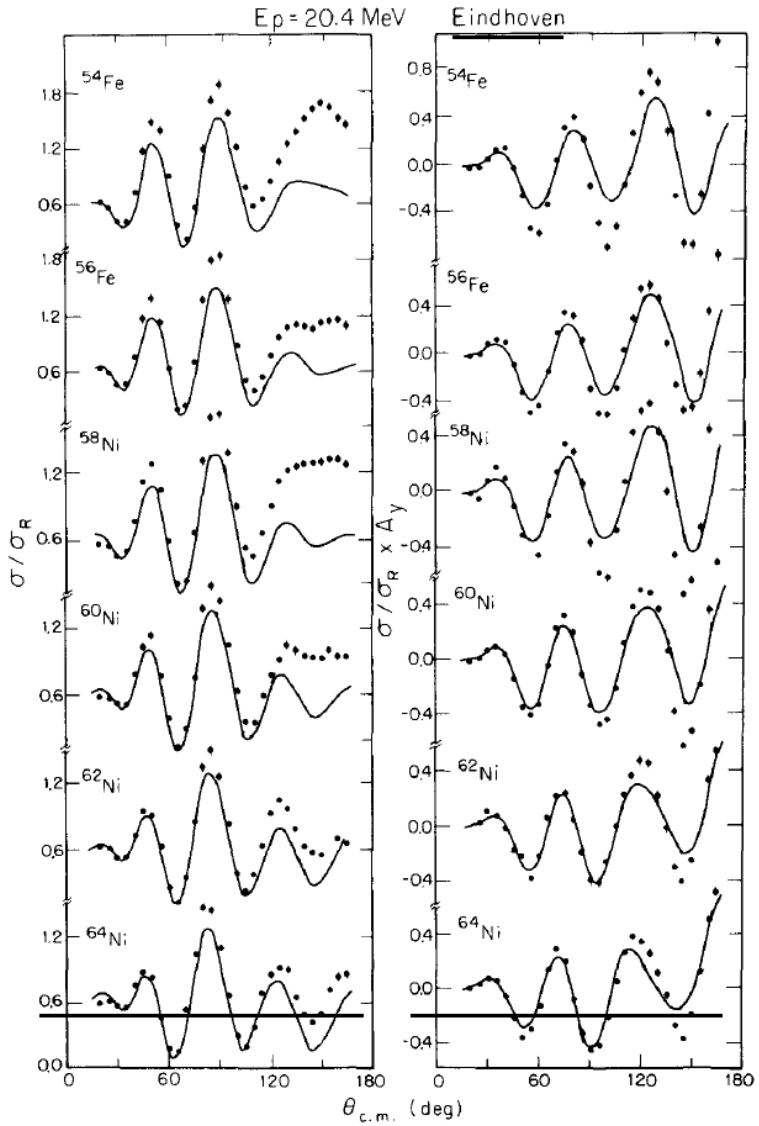
A GLOBAL NUCLEON OPTICAL MODEL POTENTIAL*

R.L. VARNER

Oak Ridge National Laboratory, Oak Ridge, TN 37831-6368, USA
and Triangle Universities Nuclear Laboratory, Duke University, Durham, NC 27706, USA

and

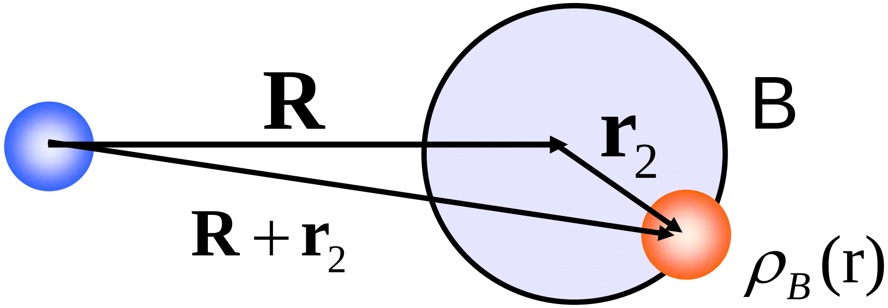
W.J. THOMPSON, T.L. McABEE**, E.J. LUDWIG and T.B. CLEGG

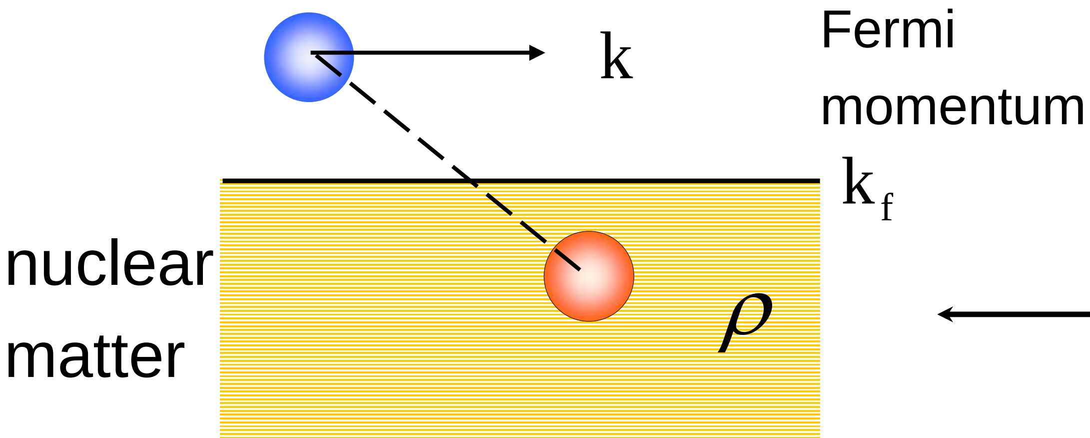


PHYSICS REPORTS (Review Section of Physics Letters) 201, No. 2 (1991) 57–119. North-Holland

Theoretical nucleon potential – based on density

$$V_{NB}(R) = \int d\mathbf{r}_2\ \rho_B(r_2)\ v_{NN}(\mathbf{R}+\mathbf{r}_2)$$





include the effect of NN interaction in the “nuclear medium” – Pauli blocking of pair scattering into occupied states

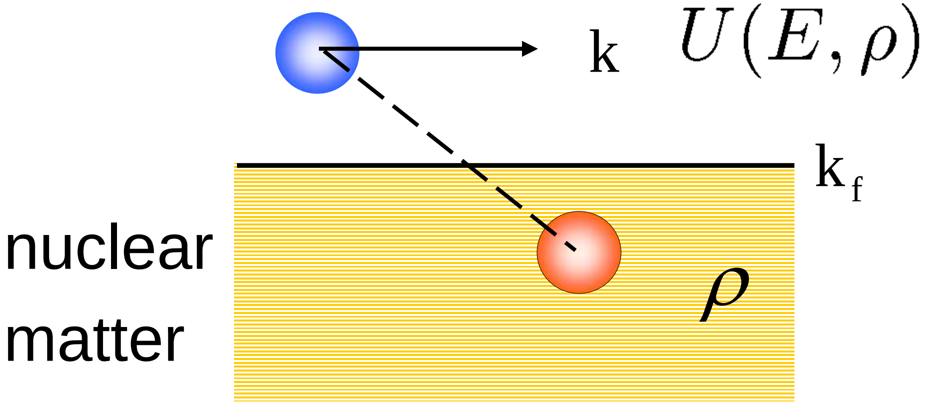
$$\rightarrow v_{NN}(\rho,\mathbf{r})$$

(e.g. M3Y, JLM)

But as E → high

$$v_{NN} \rightarrow v_{NN}^{free}$$

JLM interaction – local density approximation

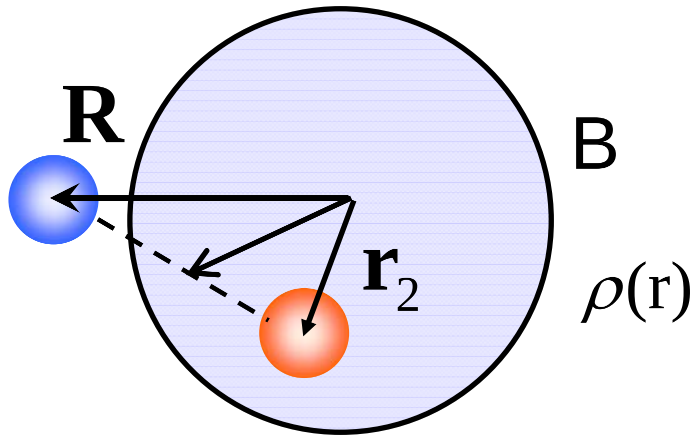


complex and density dependent interaction

$$v_{NN}(r) = \frac{U(E,\rho)}{\rho} f(r)$$

$$f(r) = (\sqrt{\pi}t)^{-3} \exp(-r^2/t^2)$$

For finite nuclei, what value of density should be used in calculation of nucleon-nucleus potential? Usually the <u>local density</u> at the mid-point of the two nucleon positions $\mathbf{r}_x$



$$\boxed{U_B(R) = V_B(R) + iW_B(R) = \int d\vec{r}_2\, \rho_B(r_2) \frac{U(E,\rho(r_x))}{\rho(r_x)} f(r)}$$

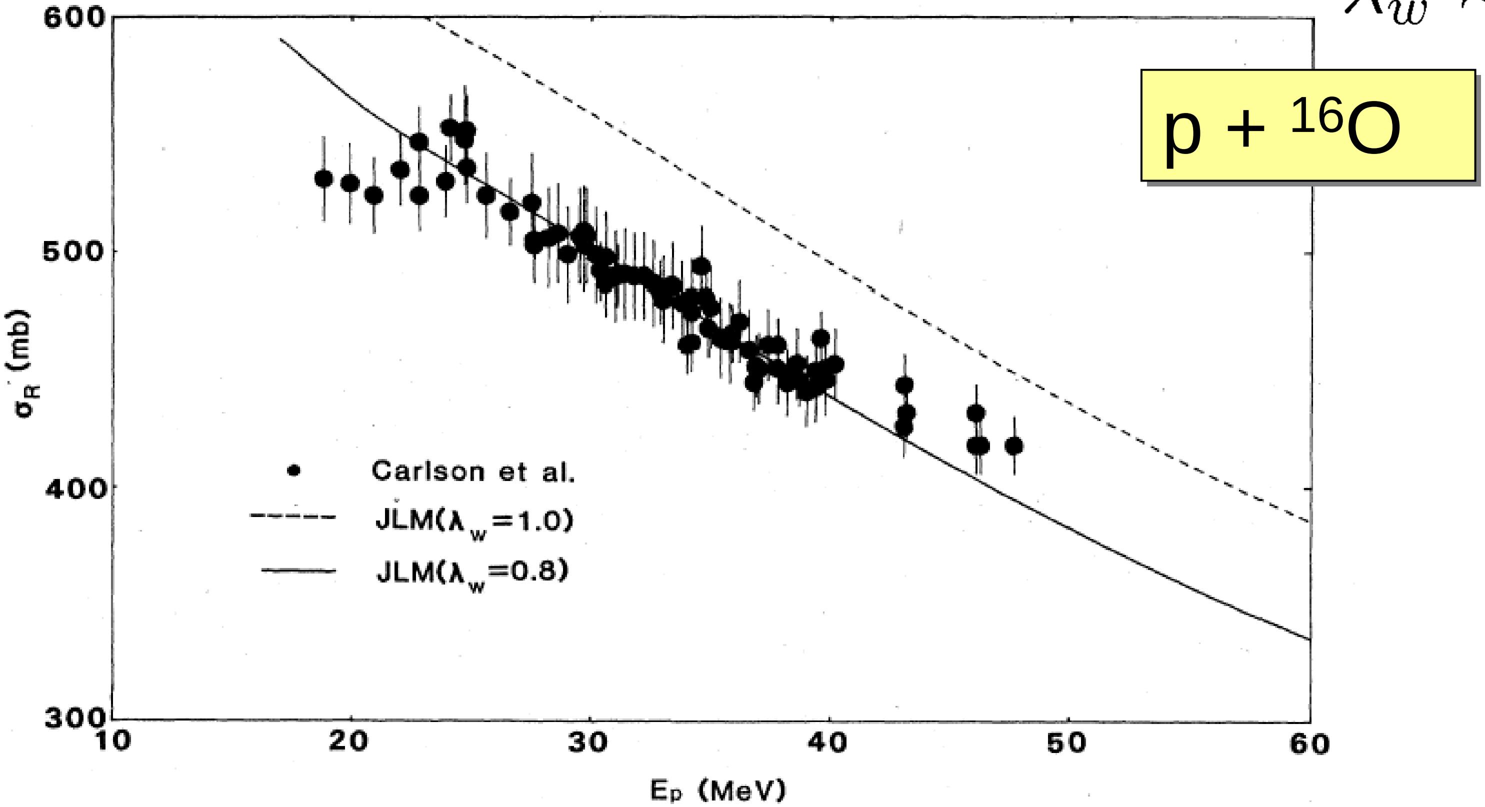
JLM interaction – fine tuning

Strengths of the real and imaginary parts of the potential can be adjusted based on experience of fitting data.

$$U_B(R) = \lambda_v V_B(R) + i\lambda_w W_B(R)$$

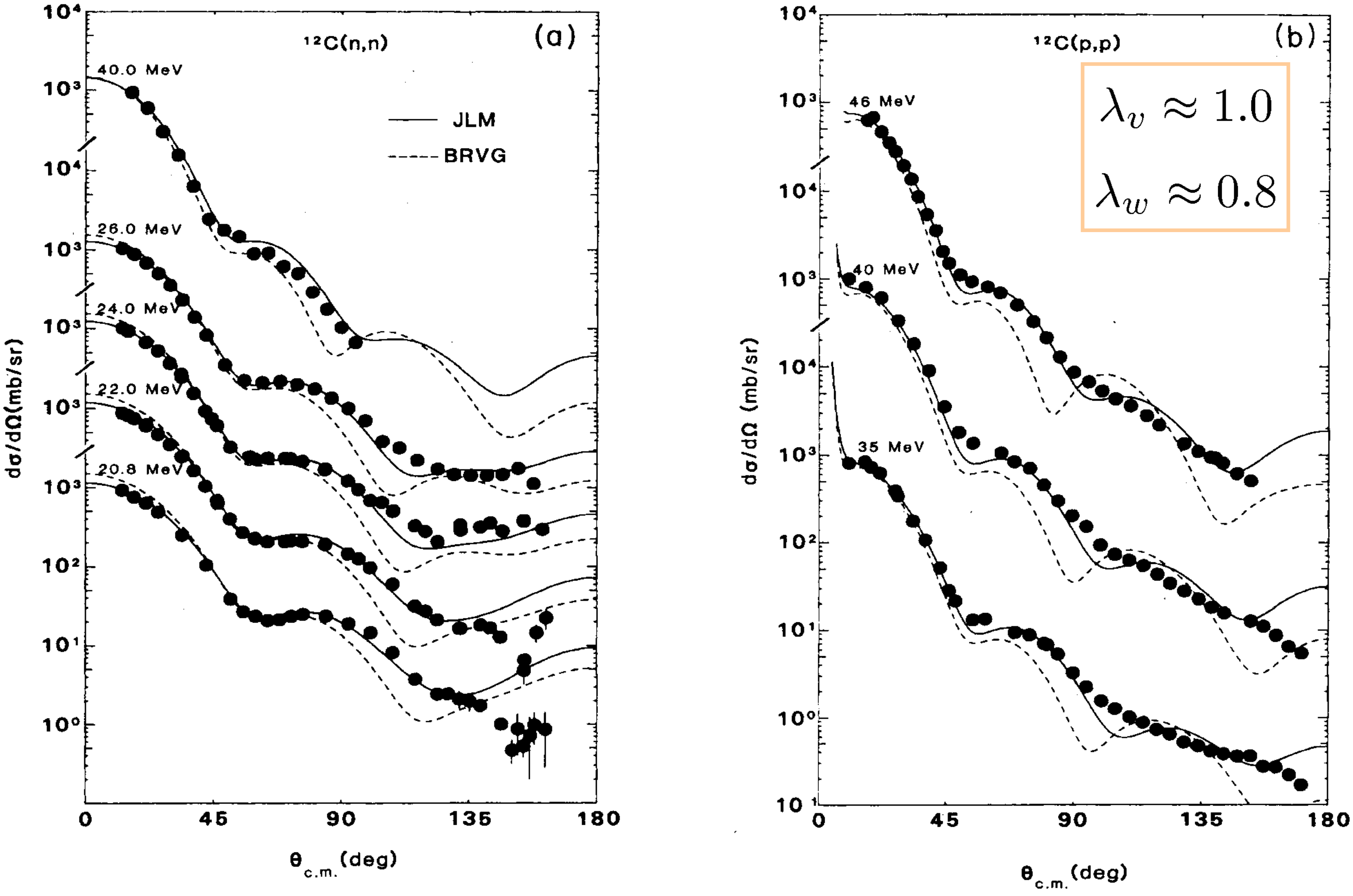
$$\lambda_v \approx 1.0$$

$$\lambda_w \approx 0.8$$



J.S. Petler et al. Phys. Rev. C **32** (1985), 673

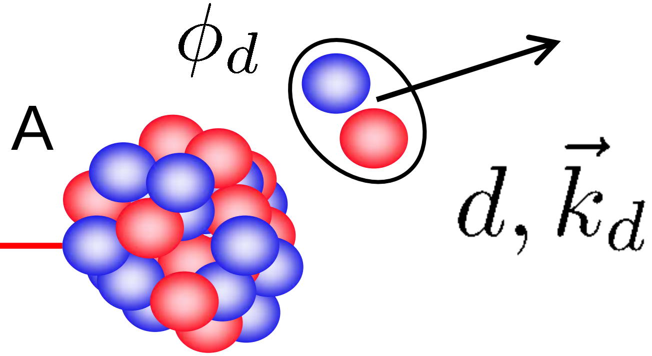
JLM folded nucleon-nucleus optical potentials

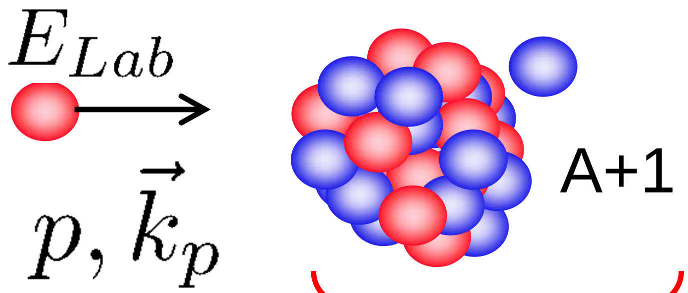


J.S. Petler et al. Phys. Rev. C **32** (1985), 673

Transfer reaction transition amplitudes - DWBA

$$T(p,d) = \langle \underbrace{\chi^{(-)}_{d,\vec{k}_d}\,\Phi(A,J_f)\phi_d}_{\text{exit channel}}|V_{np}|\underbrace{\chi^{(+)}_{p,\vec{k}_p}\,\Phi(A+1,J_i)}_{\text{entrance channel}}\rangle$$





$$V_{np}\phi_d(\vec{r}) \approx D_0\delta(\vec{r})$$ - short range

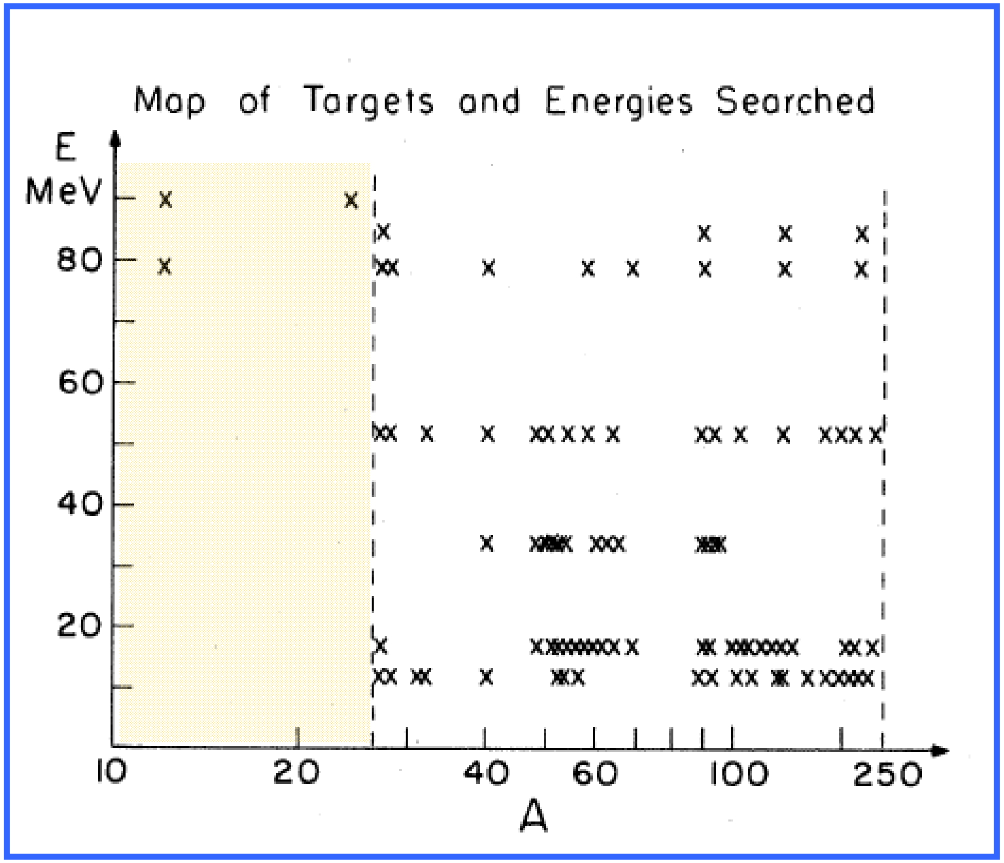
$$\phi_{n\ell j}(\vec{r}_n) = \langle\Phi(A,J_f)|\Phi(A+1,J_i)\rangle$$

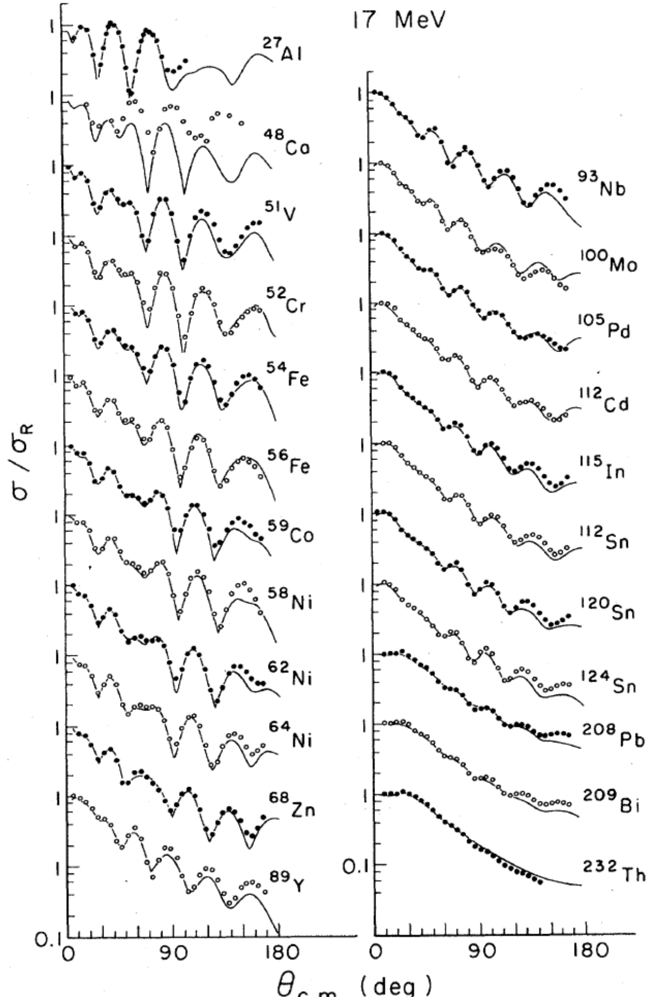
Global optical potentials – e.g. for deuterons

PHYSICAL REVIEW C VOLUME 21, NUMBER 6 JUNE 1980

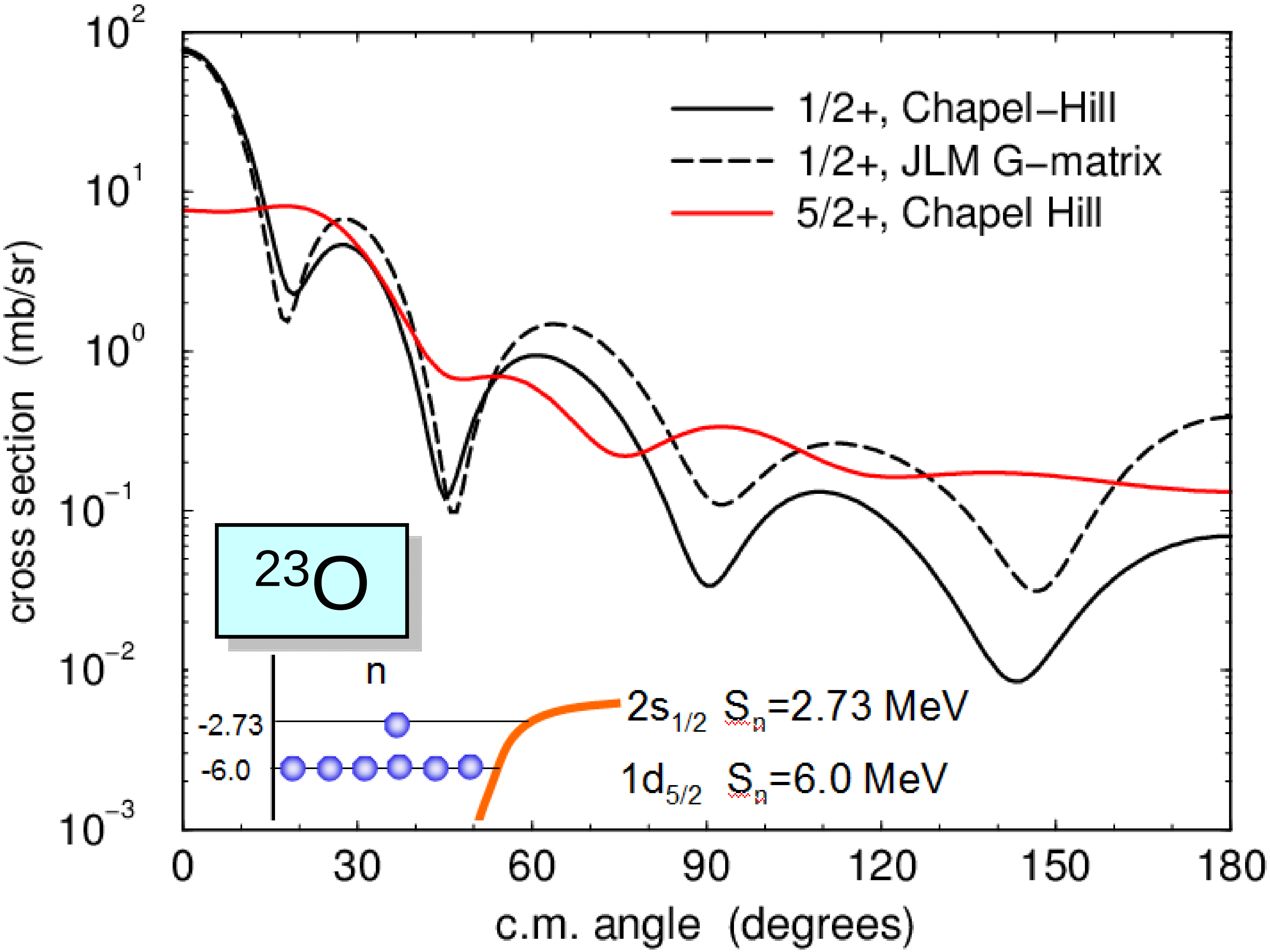
Global optical model potential for elastic deuteron scattering from 12 to 90 MeV

W. W. Daehnick, J. D. Childs,* and Z. Vrcelj

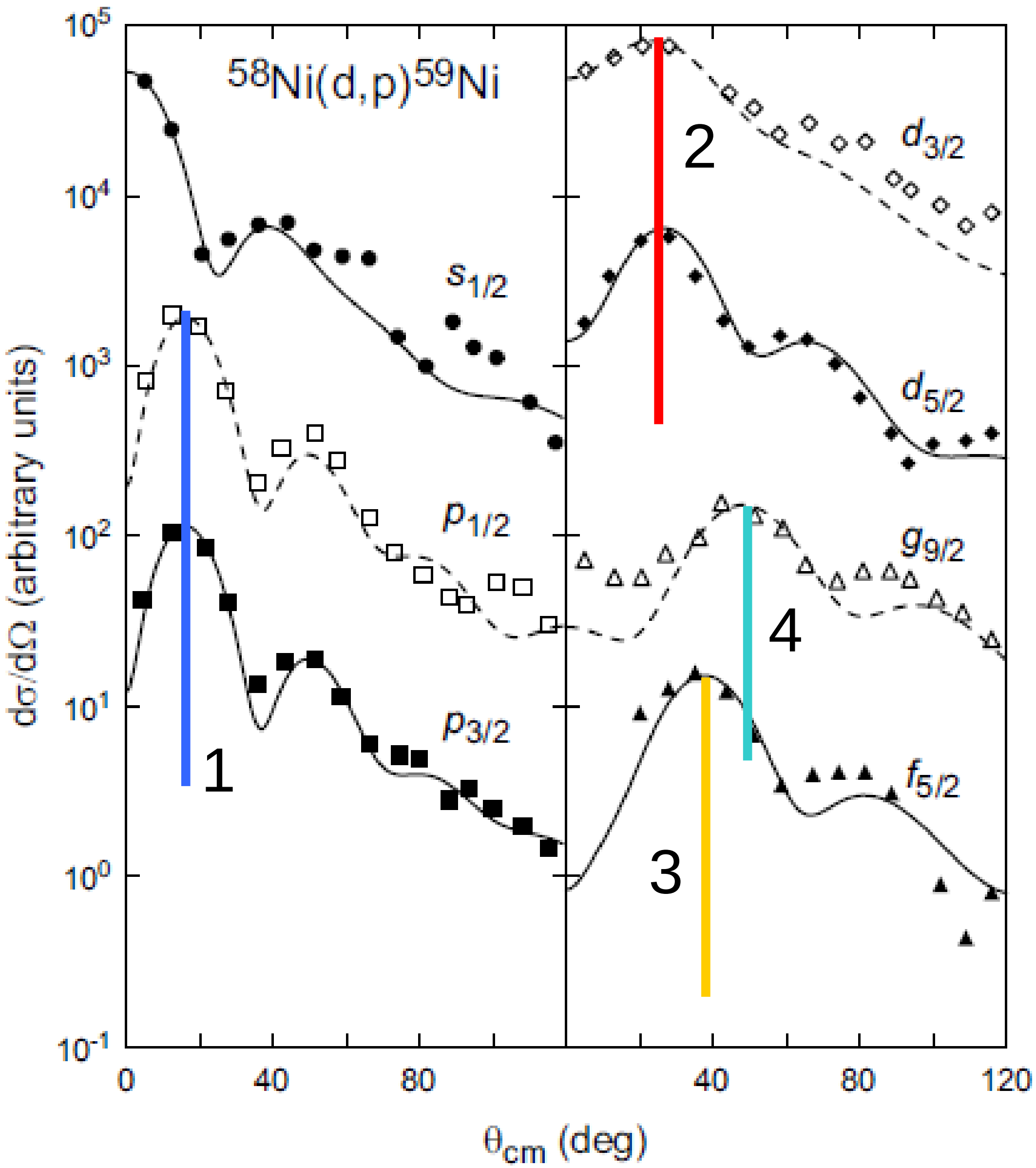




Calculated (p,d) transfer (pick-up) cross sections

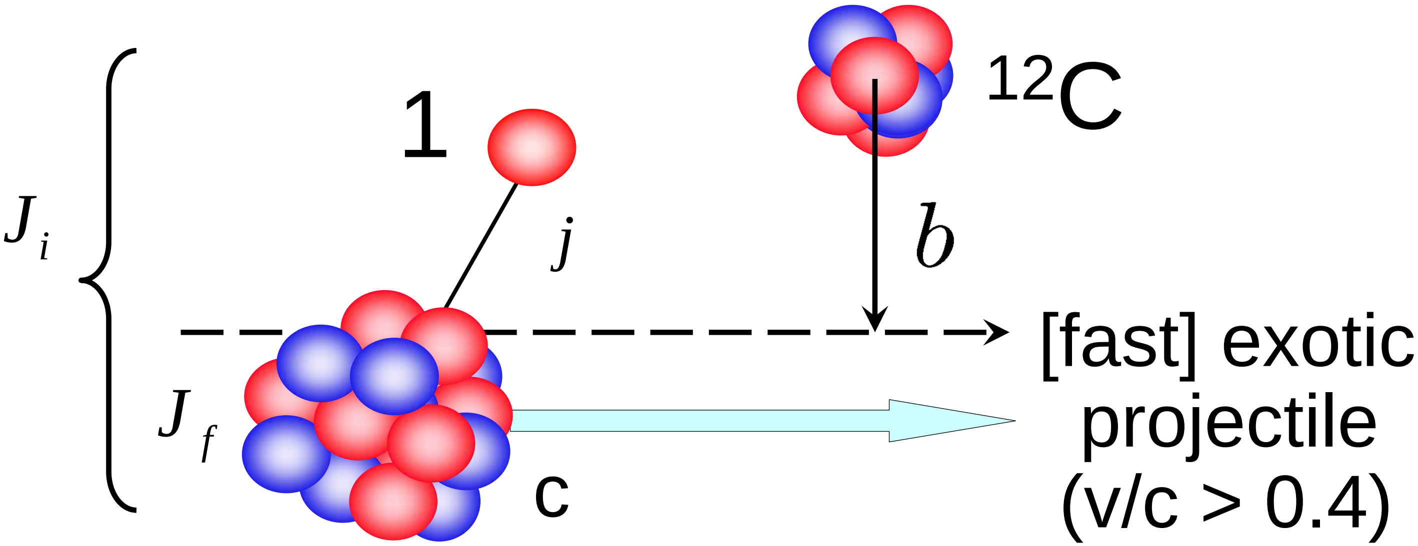


Example – from Ian Thompson EBSS 2011 slides



Orientation II – neutron removal – or knockout

Another experimental option is one-nucleon removal – at ~100 MeV/nucleon and greater – fragmentation beams

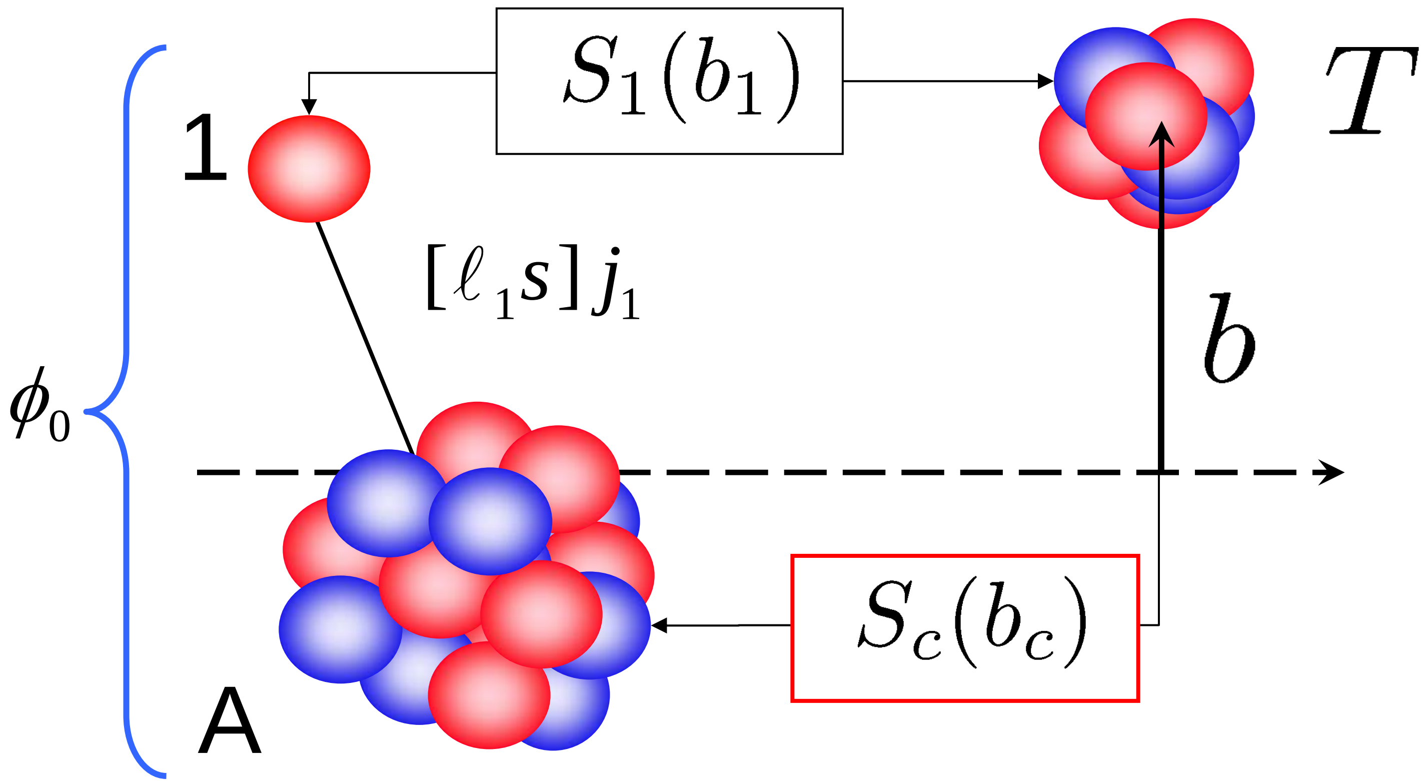


Experiments do not measure target final states. Final state of core c measured – using decay gamma rays.

How to describe and what can we learn from these?

P.G. Hansen and J.A. Tostevin, Ann Rev Nucl Part Sci **53** (2003) 219

Use for reactions – stripping/knockout of a nucleon



$$\sigma_{\text{strip}} = \int d\mathbf{b} \ \langle \phi_0 \,|\, |S_C(b_c)|^2 \,(1-|S_1(b_1)|^2)\,|\, \phi_0 \rangle$$

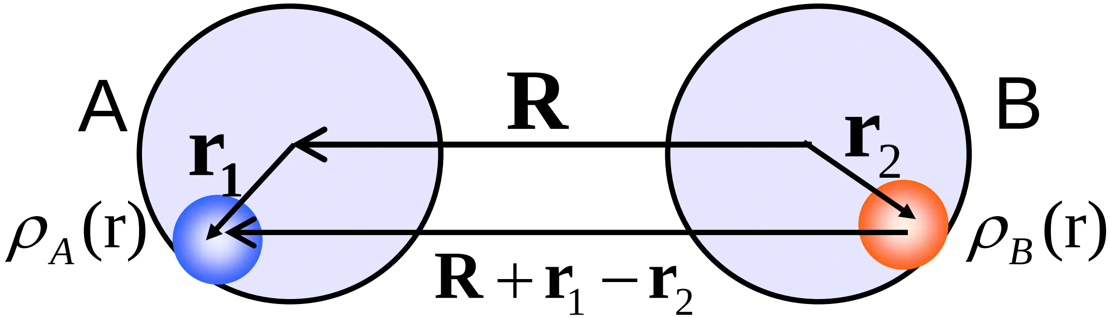
Have also assumed the sudden/adiabatic approximation

Effective interactions – Folding models

Double folding

$$U_{AB}(R) = \int d\mathbf{r}_1 \int d\mathbf{r}_2 \; \rho_A(r_1)\, \rho_B(r_2)\, v_{NN}(\mathbf{R} + \mathbf{r}_1 - \mathbf{r}_2)$$

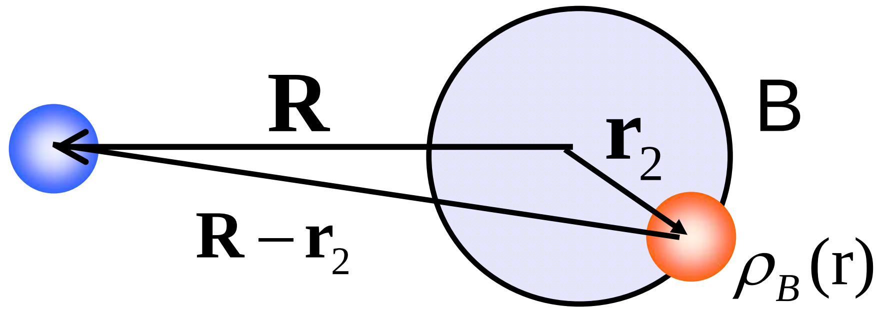
U_{AB}



Single folding

$$U_B(R) = \int d\mathbf{r}_2 \; \rho_B(r_2)\, v_{NN}(\mathbf{R} - \mathbf{r}_2)$$

U_B



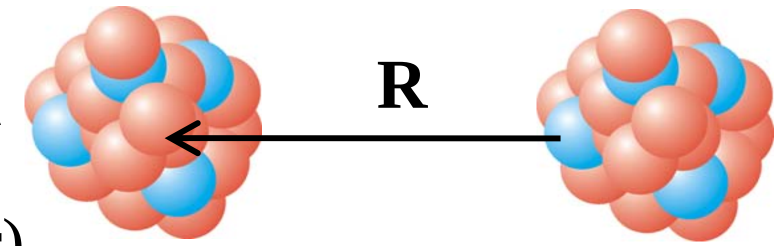
Core-target effective interactions – for $S_c(b_c)$

Double folding

$$U_{AB}(\mathrm{R}) = \int d\mathbf{r}_1 \int d\mathbf{r}_2\ \rho_A(\mathrm{r}_1)\,\rho_B(\mathrm{r}_2)\,\mathrm{t}_{\mathrm{NN}}(\mathbf{R} + \mathbf{r}_2 - \mathbf{r}_1)$$

$$U_{AB}$$

A



B

$\rho_A(\mathrm{r})$ $\rho_B(\mathrm{r})$

At higher energies – for nucleus-nucleus or nucleon-nucleus systems – first order term of multiple scattering expansion

$$t_{NN}(r) = \left[-\frac{\hbar v}{2}\sigma_{NN}(i + \alpha_{NN})\right] f(r), \quad \int d\vec{r} f(r) = 1$$

nucleon-nucleon cross section

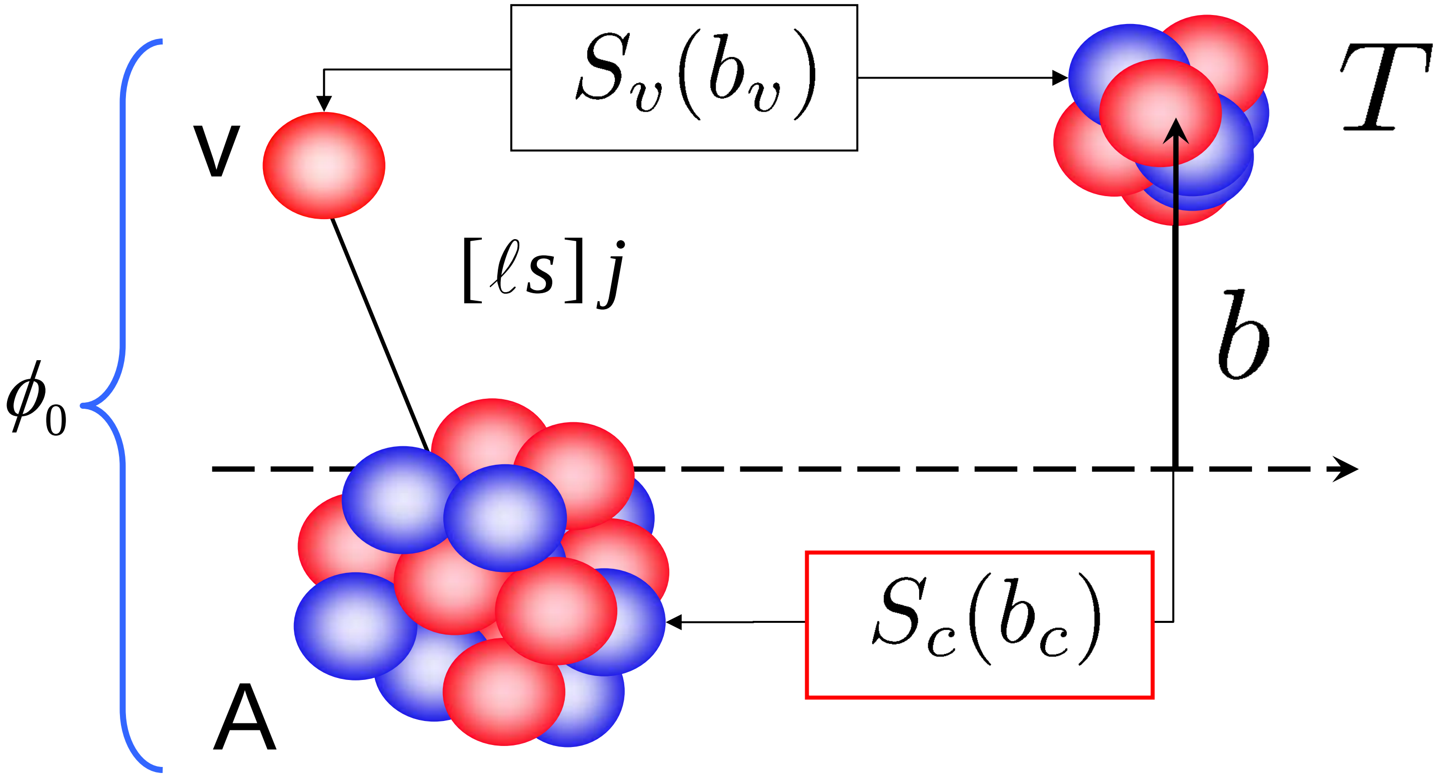
$$\text{e.g. } f(r) = \delta(r)$$

$$f(r) = (\sqrt{\pi}t)^{-3}\exp(-r^2/t^2)$$

resulting in a COMPLEX nucleus-nucleus potential

M.E. Brandan and G.R. Satchler, Phys. Rep. **285** (1997) 143-243.

Diffractive (breakup) removal of a nucleon

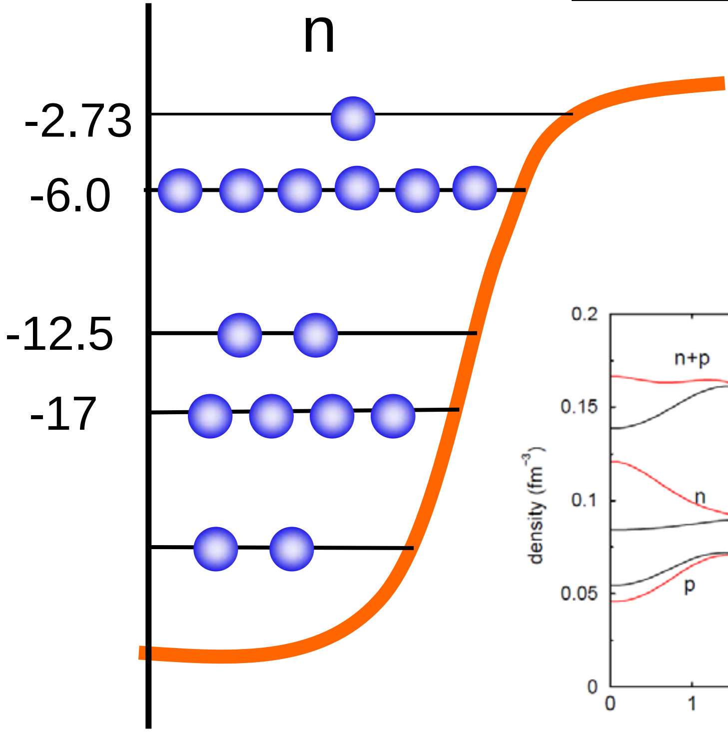


$$\sigma_{\mathrm{diff}} = \int d\mathbf{b} \left\{ \langle \phi_0 \,||\, \mathrm{S_c}\, \mathrm{S_v}\, |^2\, | \phi_0 \rangle - | \langle \phi_0 \,|\, \mathrm{S_c}\, \mathrm{S_v} \,|\, \phi_0 \rangle |^2 \right\}$$

Orientation II – neutron removal – cross sections

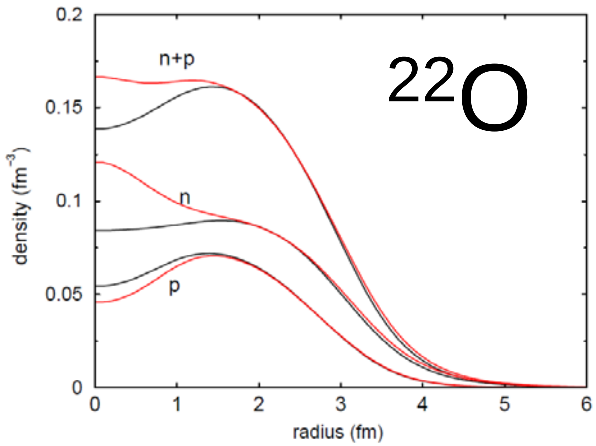
Single neutron removal from $^{23}O \equiv [1d_{5/2}]^6\,[2s_{1/2}]$

at <u>72 MeV/nucleon</u> on a ^{12}C target

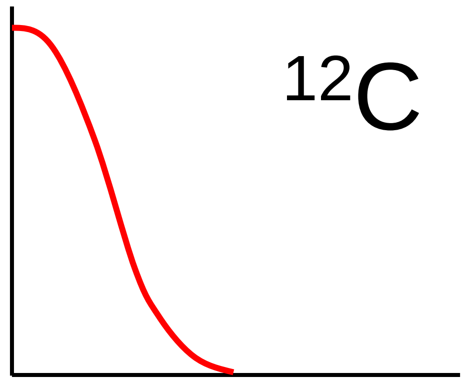


$2s_{1/2}$ 2.73 MeV

$1d_{5/2}$ 6.0 MeV

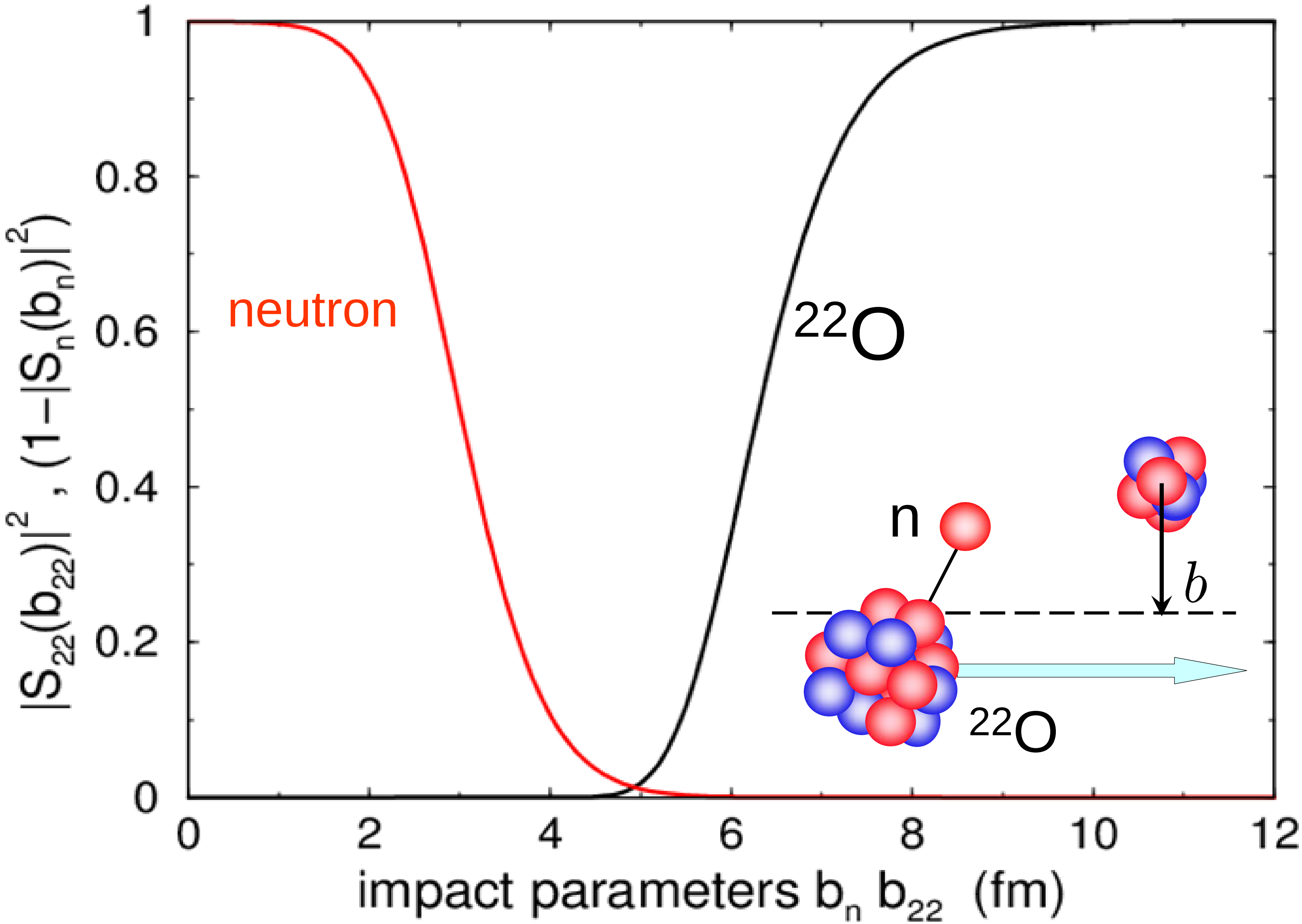


Gaussian density
rms radius 2.32 fm

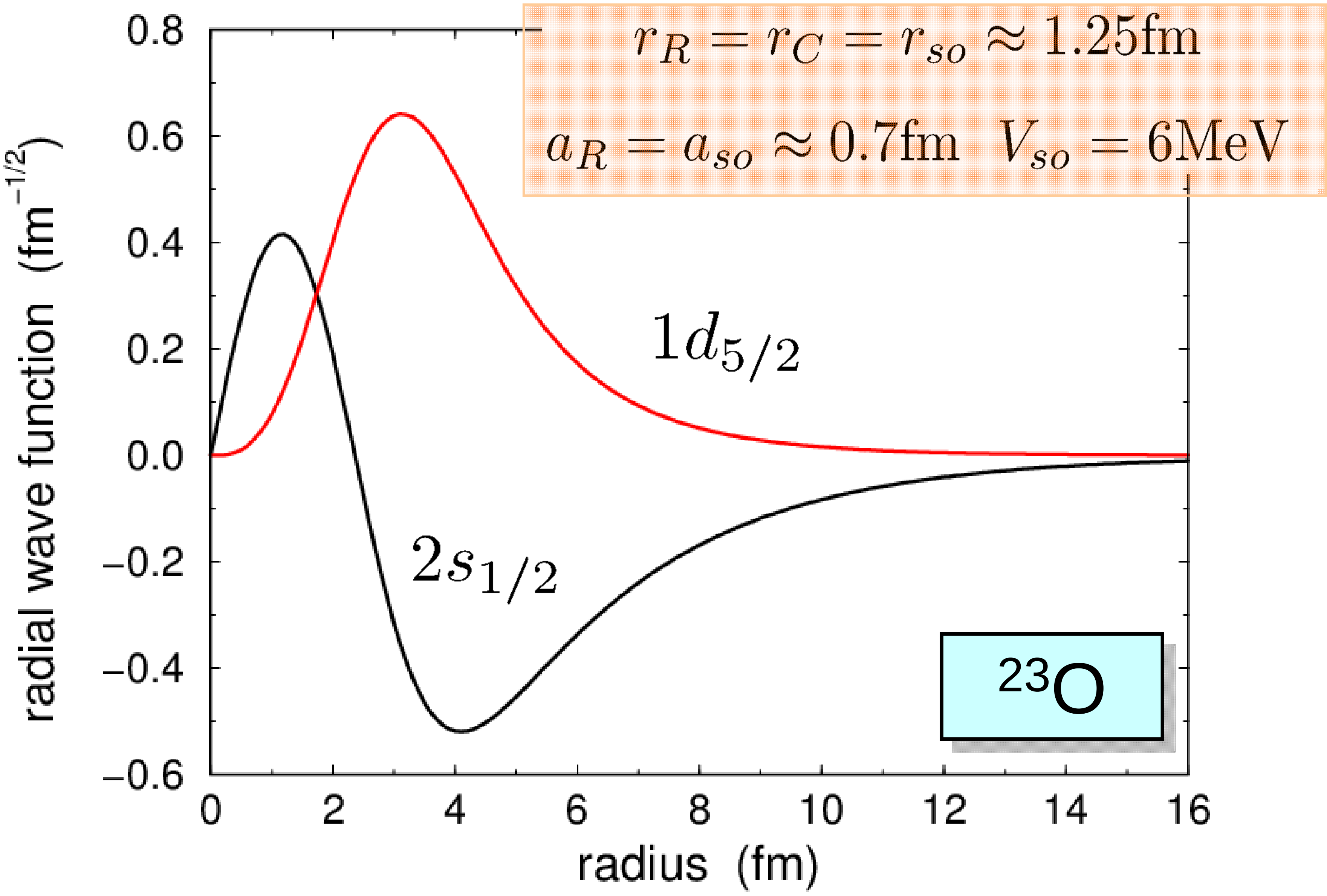


$$t_{NN}(r) = \left[-\frac{\hbar v}{2}\sigma_{NN}(i+\alpha_{NN})\right] f(r), \quad \int d\vec{r}\, f(r) = 1$$

Eikonal S-matrix spatial selectivity

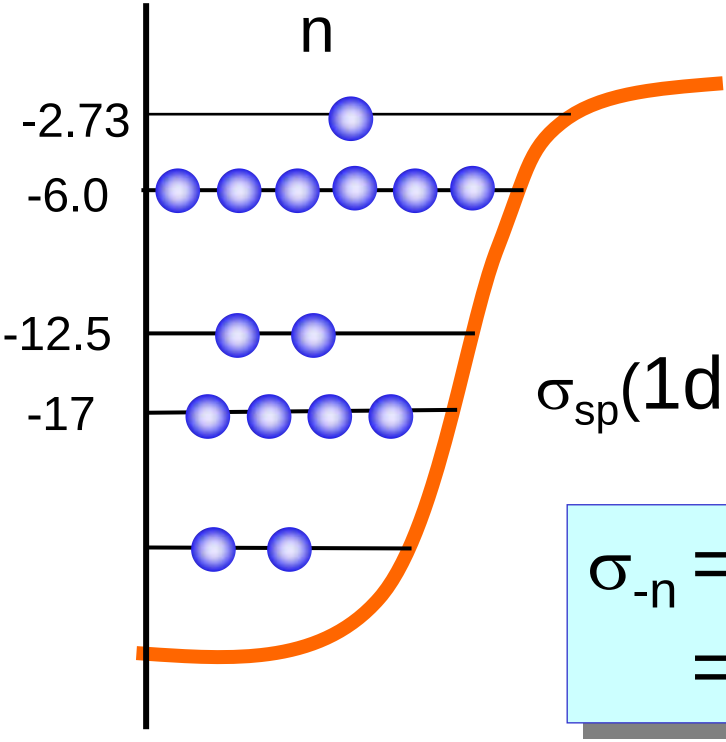


Neutron bound state wave functions



Orientation II – neutron knockout – cross sections

Single neutron removal from $^{23}O \equiv [1d_{5/2}]^6\,[2s_{1/2}]$



$2s_{1/2}$ S_n=2.73 MeV

$1d_{5/2}$ S_n=6.0 MeV

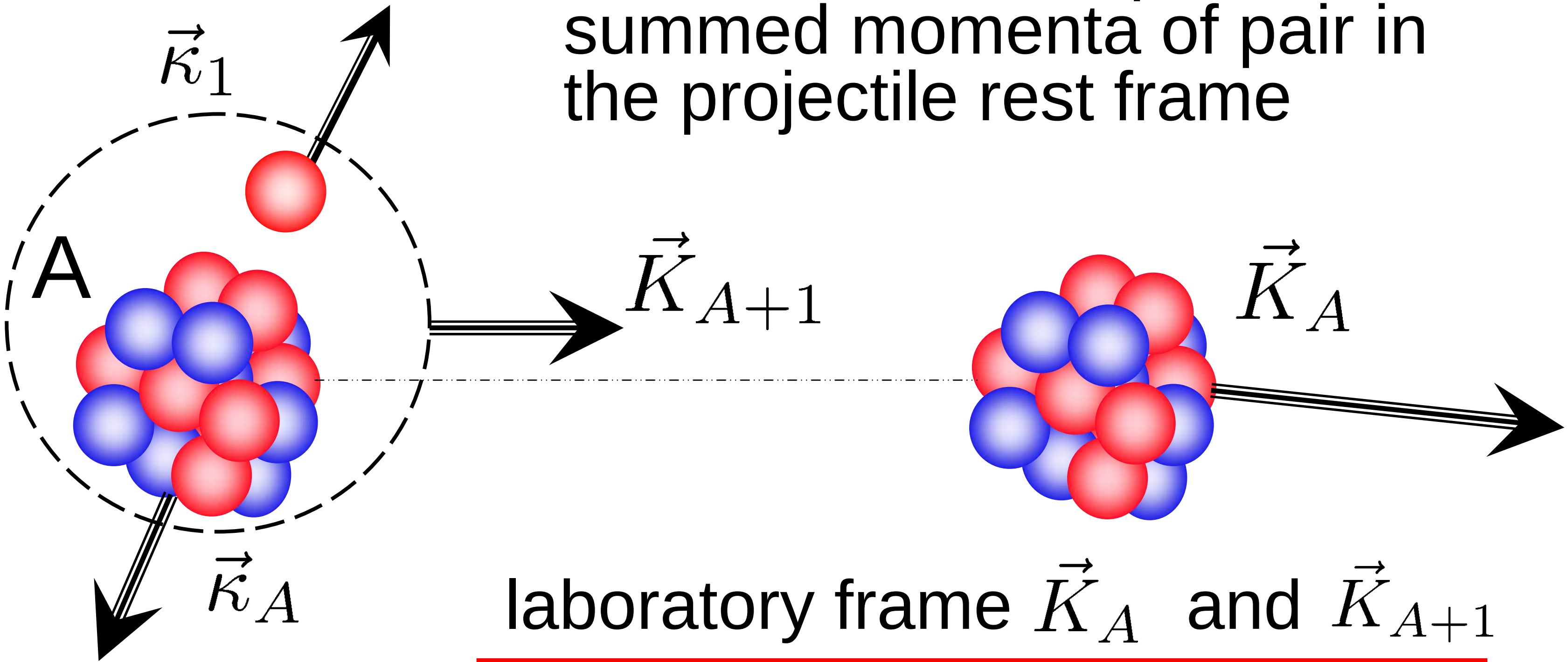
$\sigma_{sp}(1d_{5/2})$=28.3 mb $\sigma_{sp}(2s_{1/2})$=72.7 mb

$$\sigma_{-n} = 6\,\sigma_{sp}(1d_{5/2}) + \sigma_{sp}(2s_{1/2})$$
$$= 242.5\text{ mb}$$

<u>Measurement at RIKEN</u> [Kanungo *et al.* PRL **88** ('02) 142502]
at 72 MeV/nucleon on a ^{12}C target; σ_{-n} = 233(37)mb

Sudden 1N removal from the mass A residue

Sudden removal: residue momenta probe the summed momenta of pair in the projectile rest frame



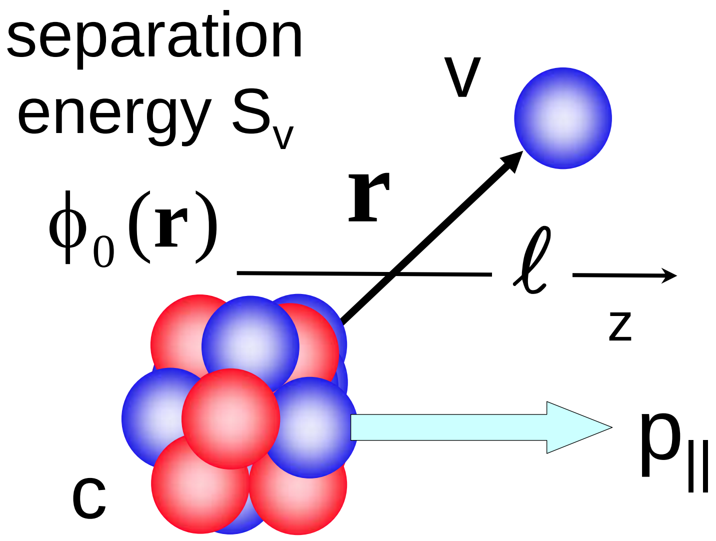
laboratory frame $\vec{K}_A$ and $\vec{K}_{A+1}$

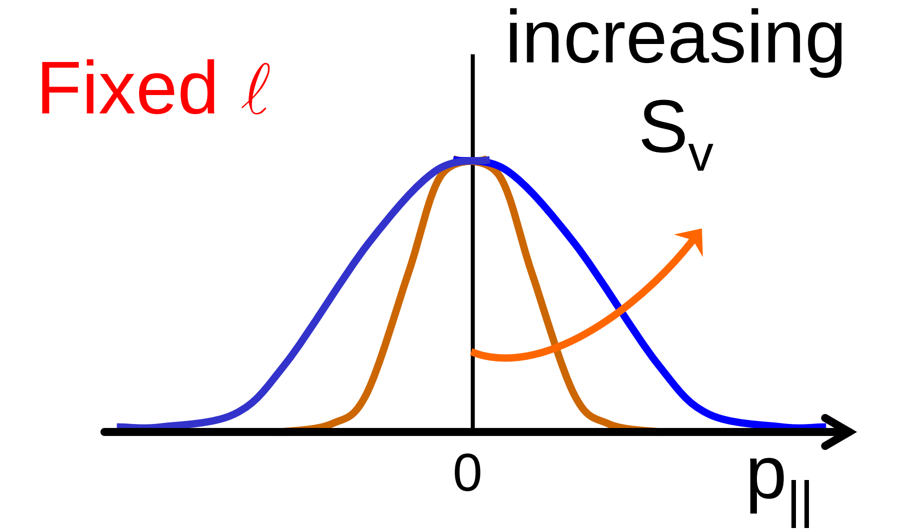
Projectile rest frame

$$\boxed{\vec{K}_A = \frac{A}{A+1}\vec{K}_{A+1} - \vec{\kappa}_1}$$

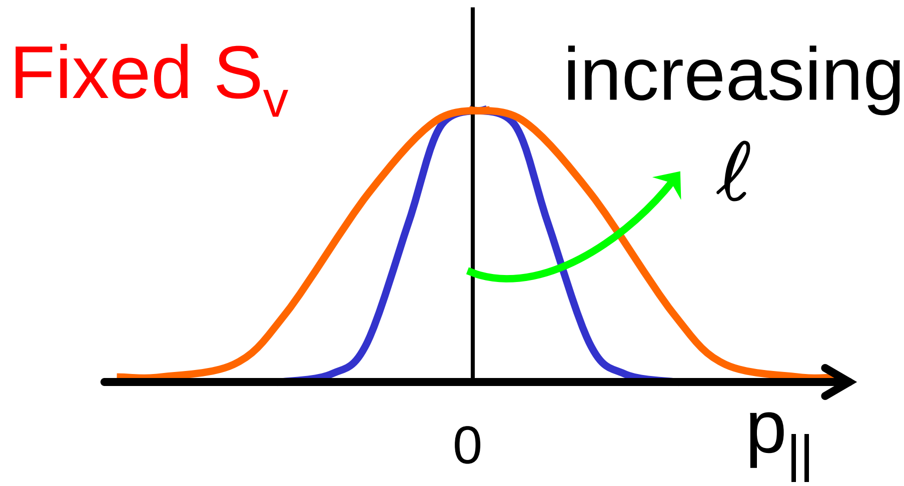
and component equations

Measurement of the residue's momentum



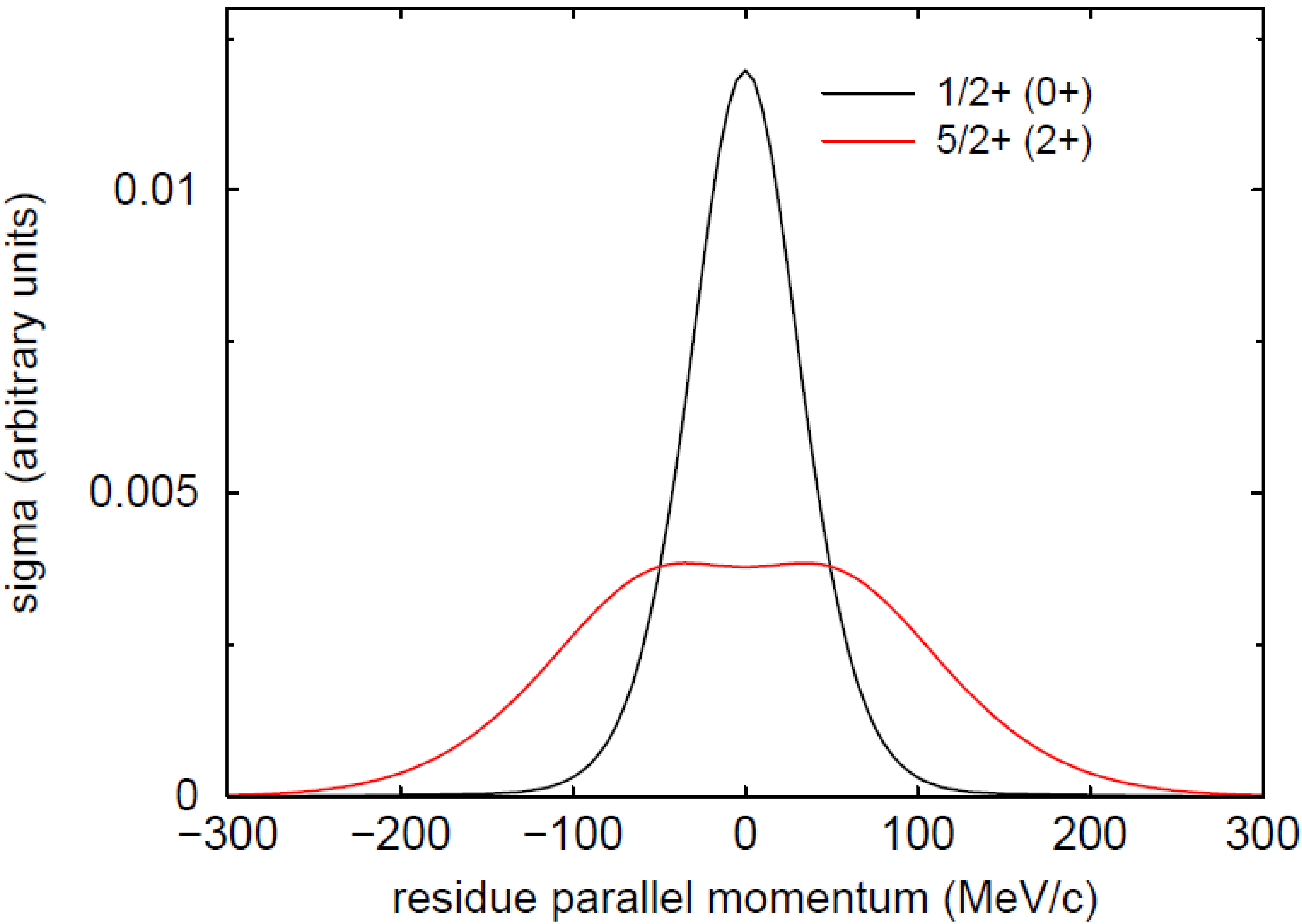


consider momentum components $p_{||}$ of the core parallel to the beam direction, in the projectile rest frame



$$\Delta p \Delta z > \hbar/2$$

Forward momentum distributions of ^{22}O residues



End of Part II: thanks for your attention

Exotic Beams Summer School 2012, Argonne National Laboratory, 5th - 9th August 2012