

Schiffer Fest  
Argonne  
September 20-22, 2006

# Structure of Exotic Nuclei and the Nuclear Force

Takaharu Otsuka  
RIKEN

University of Tokyo /

I met John for the first time in 1976 or 77 in Tokyo.  
We walked together in the Ueno park in an evening.  
I was a Ph.D. student in Tokyo, but courageous enough to asked  
to produce more 2-body matrix elements.

## The effective interaction between nucleons deduced from nuclear spectra\*

John P. Schiffer

*Argonne National Laboratory, Argonne, Illinois 60439  
and University of Chicago, Chicago, Illinois 60637*

William W. True

*University of California, Davis, California 95616*

Two-body matrix elements of the residual nucleon-nucleon interaction are extracted from experimental data throughout the periodic table and are used to determine the ranges and well depths of various components of a local interaction. The  $T = 1$  even and odd components of the central interaction both definitely require two wells with different ranges; a shorter-range attractive well with a longer-range repulsive one. The need for a tensor interaction and a two-body spin-orbit interaction is also explored and their inclusion improves the fit slightly.

But, we did not communicate for a quarter century.

In the meantime, .....

New things have emerged  
in nuclear physics.....

Let me discuss one of them,  
shell structure of exotic nuclei.

## Single particle motion in the nucleus

- is determined basically by a potential like a Woods-Saxon with a spin-orbit splitting term,
- has magic numbers 2, 8, 20, 28, 50, 82 ,..., as Mayer and Jensen have proposed,
- can be basically reproduced by theories (Skyrme or Gogny) consisting of central (finite-range and density-dependent) and 2-body  $LS$  forces.



The relative locations among single-particle orbits do not change much.

The magic numbers are common for all nuclei, stable or unstable (exotic), while can be broken by deformation occasionally.

This has been widely conceived for half a century.

In 1990's, extensive studies on exotic nuclei have started.

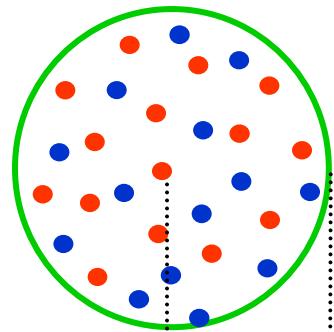
A generally conceived idea or hope :

Excess neutrons in halo or skin have more diffuse surface.

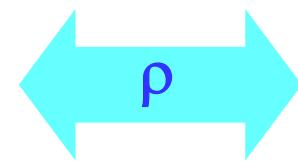
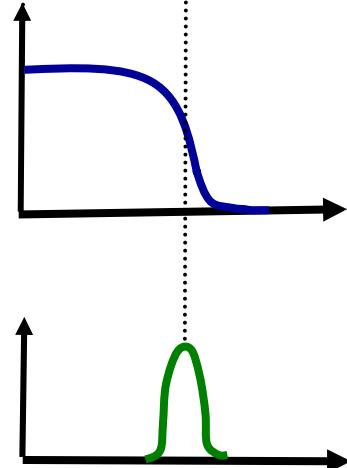
See next 2 pages (*LS* cartoon)

# Conventional image of $1s$ splitting change

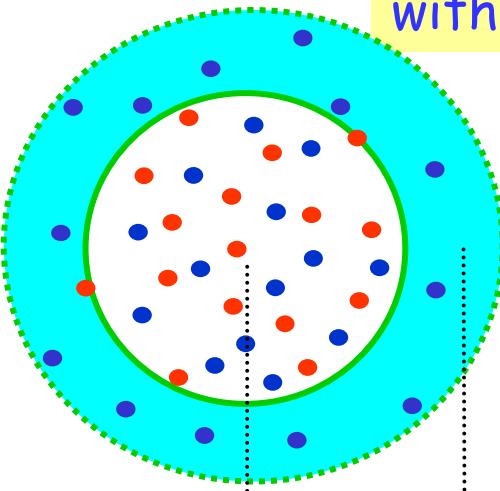
stable nucleus



- proton
- neutron



exotic nucleus  
with neutron skin

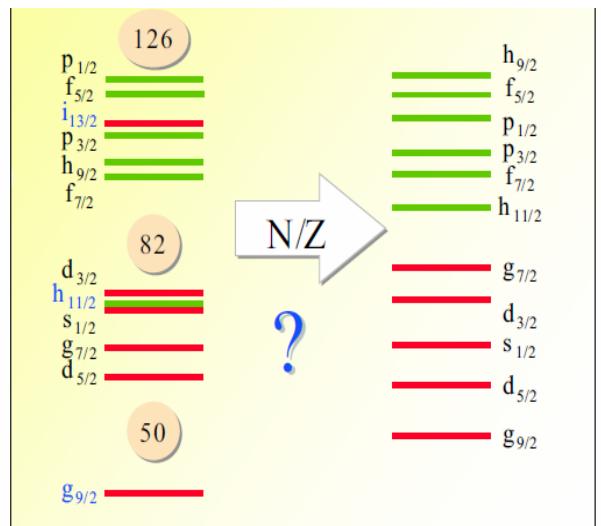


$1s$  splitting smaller

(Scale-type  $1s$  quenching)



## From RIA Physics White Paper



shell structure. The bunching of the energy levels that is endemic to shell structure depends on the form and the shape of the average mean field potential in which the hadrons are moving. With a diffuse surface region, the spin-orbit force may be weakened. Some

In 1990's, extensive studies on exotic nuclei have started.

A generally conceived idea or hope :

Excess neutrons in halo or skin have more diffuse surface.

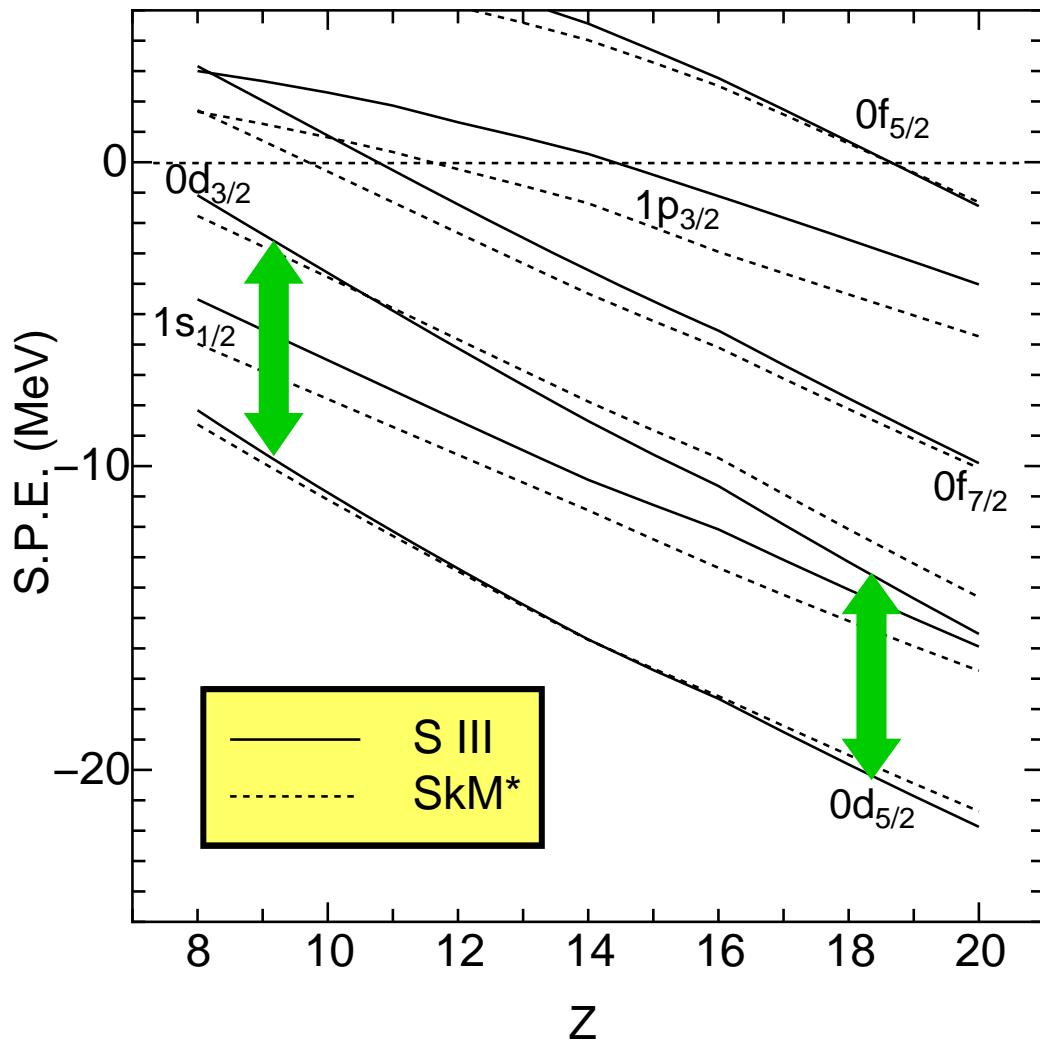
See next 2 pages (*LS* cartoon)

This mechanism was expected to be a major driving force.

But, in exotic nuclei studied actually, → HF results

# Hartree-Fock energies by Skyrme Hartree-Fock

Neutron Single-Particle Energies at  $N=20$



The shell structure remains rather unchanged

- orbitals shifting together
- change of potential depth
- ~ Woods-Saxon.

In 1990's, extensive studies on exotic nuclei have started.

A generally conceived idea or hope :

Excess neutrons in halo or skin have more diffuse surface.

See next 2 pages (*LS* cartoon)

This mechanism was expected to be a major driving force.

But, in exotic nuclei studied actually, → HF results

Nothing particular would happen between stability line and dripline ?

Too sad

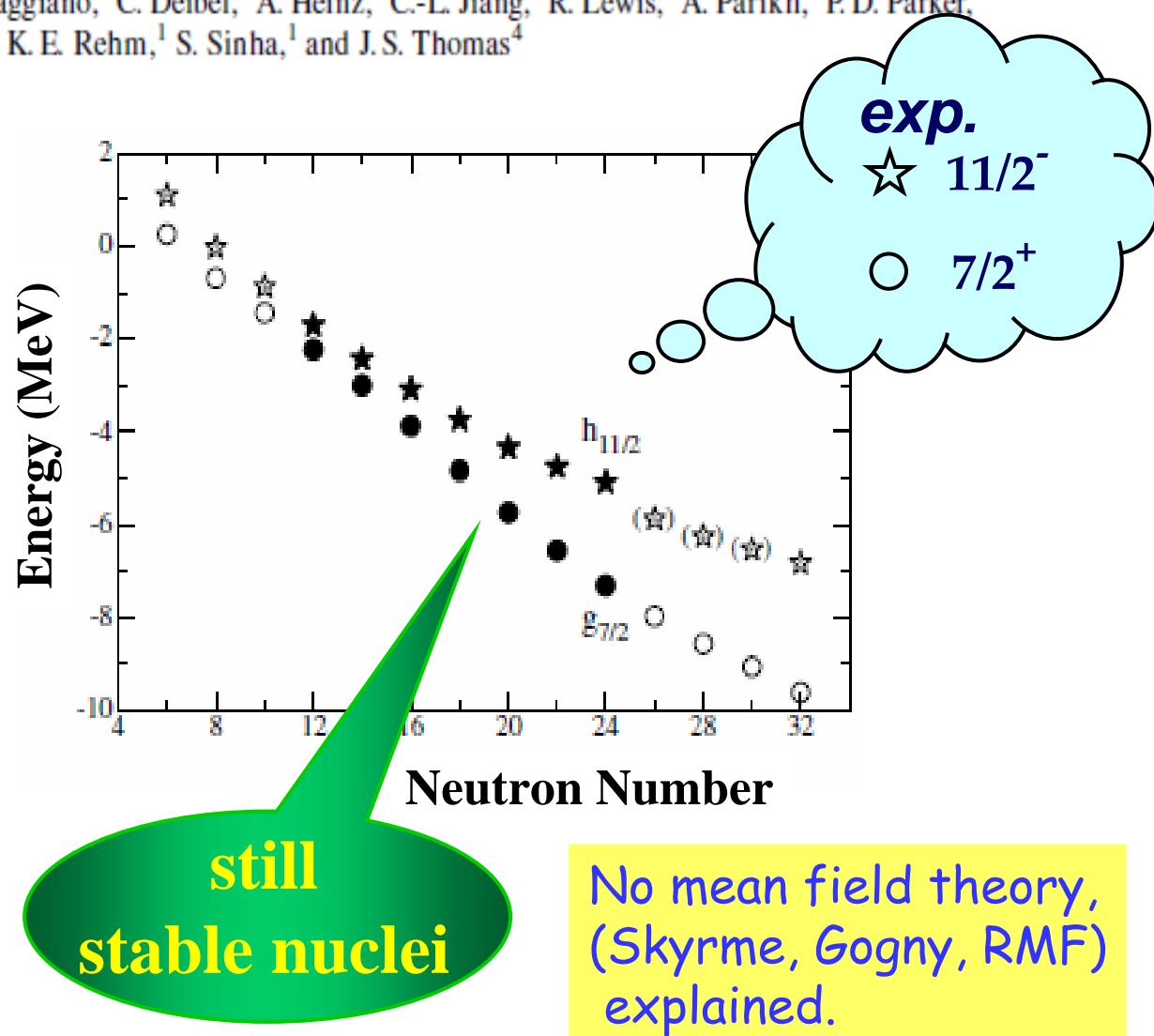
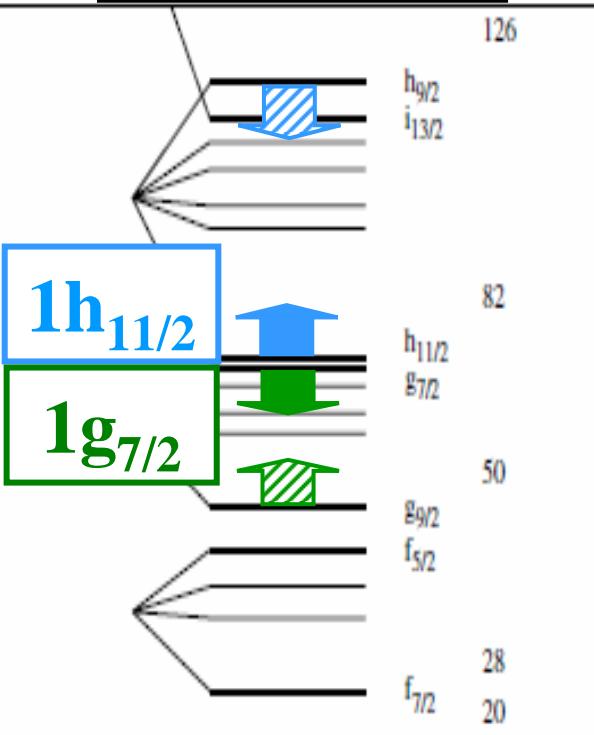
Ozawa, Tanihata et al. "discovered" that neutron separation energies shows strange behavior, and tried to explain it in connection to neutron halo (PRL 84, 5493 (2000)). But, it happens without halo.

A few years later,  
the quenching of spin-orbit  
splitting was studied  
by John and his company  
with John's favorite toy,  
transfer reaction.

## Is the Nuclear Spin-Orbit Interaction Changing with Neutron Excess?

J. P. Schiffer,<sup>1</sup> S. J. Freeman,<sup>1,2</sup> J. A. Caggiano,<sup>3</sup> C. Deibel,<sup>3</sup> A. Heinz,<sup>3</sup> C.-L. Jiang,<sup>1</sup> R. Lewis,<sup>3</sup> A. Parikh,<sup>3</sup> P. D. Parker,<sup>3</sup>  
K. E. Rehm,<sup>1</sup> S. Sinha,<sup>1</sup> and J. S. Thomas<sup>4</sup>

## Proton orbits



Independently,  
I was trying to see whether or not  
the shell structure can be changed  
by the  $NN$  interaction  
even if the dripline is still far away,  
and if so, how it occurs.

*Finally, the key is found to be the tensor force.*

Let me jump to this point,  
skipping some history.

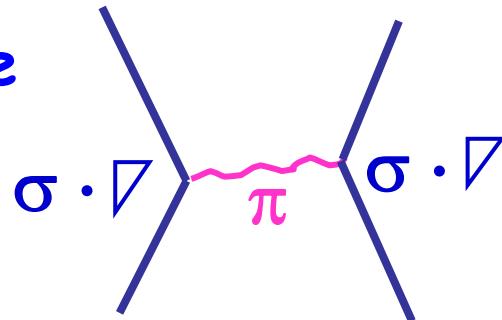
# Tensor Interaction

$$V_T = (\tau_1 \tau_2) ([\sigma_1 \sigma_2]^{(2)} Y^{(2)}(\Omega)) Z(r)$$

contributes  
only to  $S=1$  states

*relative motion*

$\pi$  meson : primary source



$\rho$  meson ( $\sim \pi + \pi$ ) : minor (~1/4) cancellation

Ref: Osterfeld, Rev. Mod. Phys. 64, 491 (92)

## Equivalent expressions of tensor force

The standard expression of tensor force may be

$$V_T = (\vec{\tau}_1 \cdot \vec{\tau}_2) S_{12} V(r)$$

$$\begin{aligned} S_{12} &= 3 (\vec{s}_1 \cdot \vec{r}/r) (\vec{s}_2 \cdot \vec{r}/r) - (\vec{s}_1 \cdot \vec{s}_2) \\ &= 3 ([\vec{s}_1 \times \vec{s}_2]^{(2)} \cdot [\vec{r} \times \vec{r}]^{(2)}/r^2) \end{aligned}$$

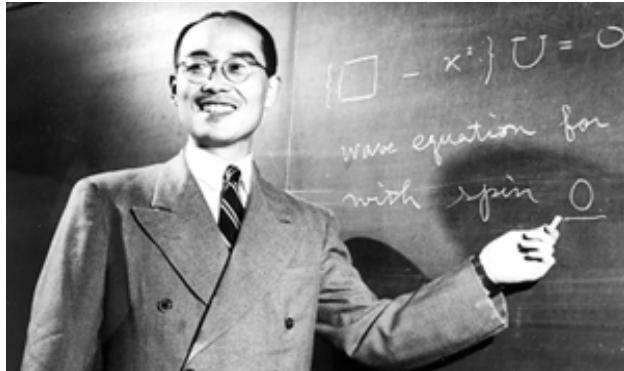
By using  $[\vec{r} \times \vec{r}]^{(2)}/r^2 = \sqrt{8\pi/15} Y^{(2)}$ , we get

$$S_{12} = \sqrt{24\pi/5} ([\vec{s}_1 \times \vec{s}_2]^{(2)} \cdot Y^{(2)})$$

We thus obtain an equivalent expression

$$V_T = \sqrt{24\pi/5} (\vec{\tau}_1 \cdot \vec{\tau}_2) ([\vec{s}_1 \times \vec{s}_2]^{(2)} \cdot Y^{(2)}) V(r)$$

The atomic nucleus is bound due to meson exchange.  
(Yukawa 1935)



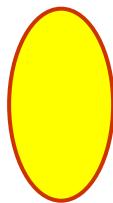
Multiple pion exchanges  
→ strong effective central forces in  $NN$  interaction  
(as represented by  $\sigma$  meson, etc.)  
→ nuclear binding

Where can we see one pion exchange ?

One pion exchange ~ Tensor force

First-order tensor force effect in spectroscopy  
→ manifestation of pions in nuclei

# *Intuitive Picture*

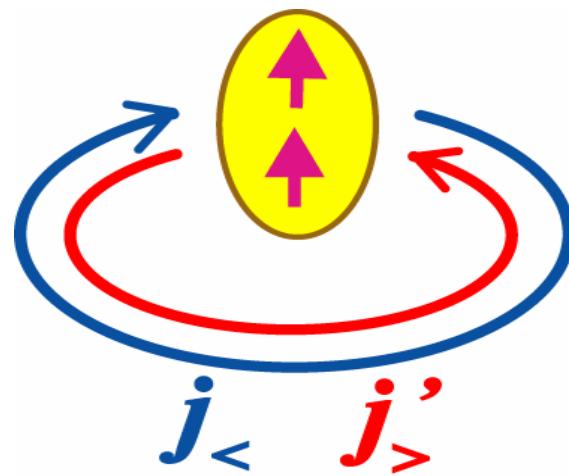


## wave function of relative motion

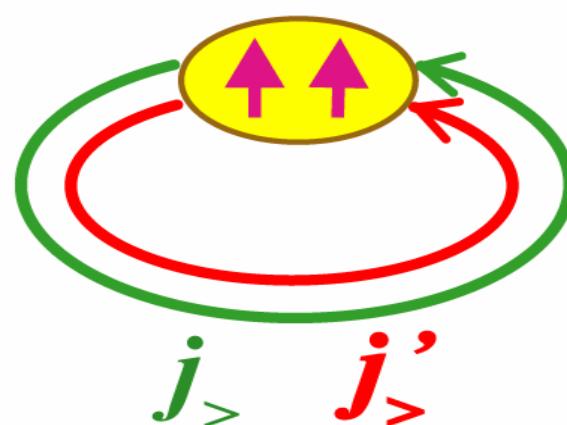


spin of nucleon

large relative momentum



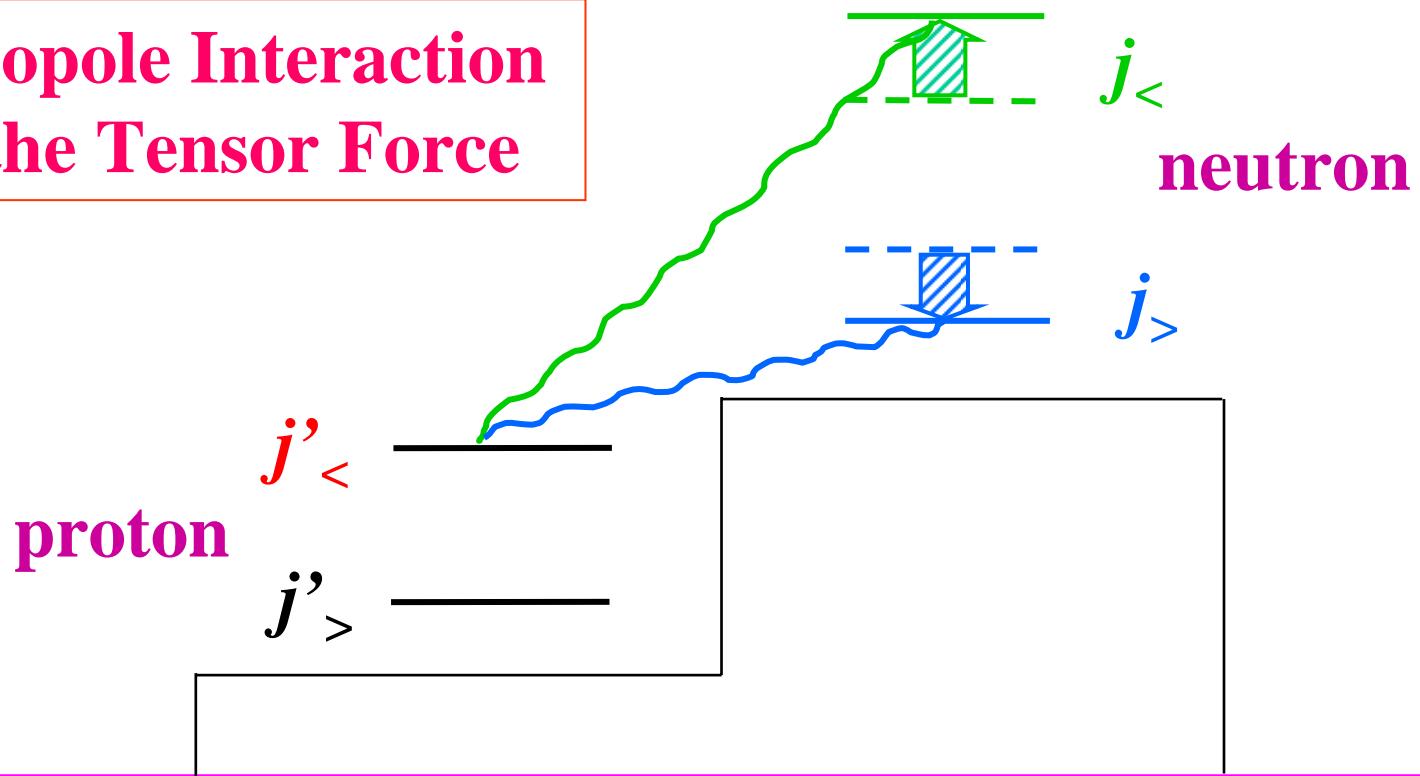
small relative momentum



deuteron  $\Rightarrow$  attractive

repulsive

## Monopole Interaction of the Tensor Force

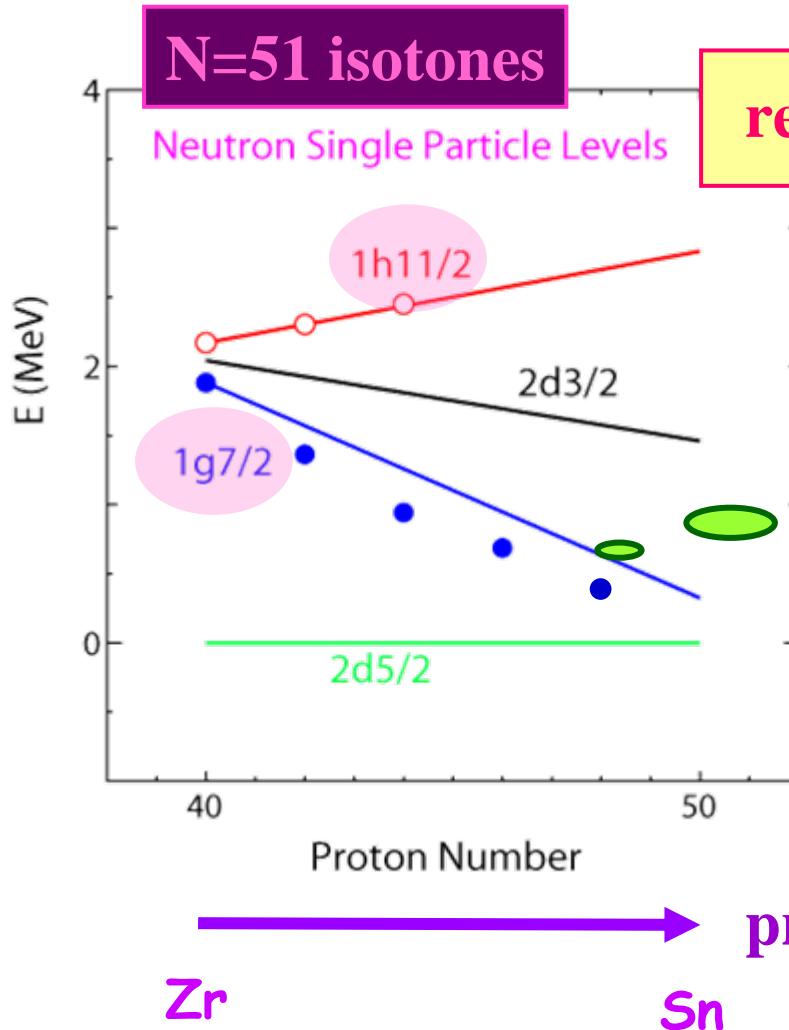


**Identity for tensor monopole interaction**

$$(2j_>+1) \ v_{m,T}^{(j' j_>)} + (2j_<+1) \ v_{m,T}^{(j' j_<)} = 0$$

$v_{m,T}$  : monopole strength for isospin  $T$

# Changes of N=51 neutron effective single-particle energies from Zr to Sn



repulsion between  $g_{7/2}$  and  $h_{11/2}$

Federman-Pittel mechanism

Lines :  $\pi + \rho$  meson tensor

Points : exp. level

protons in  $g_{9/2}$

shown relative to  $d_{5/2}$

## Reduction of the Spin-Orbit Splittings at the $N = 28$ Shell Closure

L. Gaudefroy,<sup>1,2</sup> O. Sorlin,<sup>2,1</sup> D. Beaumel,<sup>1</sup> Y. Blumenfeld,<sup>1</sup> Z. Dombrádi,<sup>3</sup> S. Fortier,<sup>1</sup> S. Franchoo,<sup>1</sup> M. Gélin,<sup>2</sup>  
 J. Gibelin,<sup>1</sup> S. Grévy,<sup>2</sup> F. Hammache,<sup>1</sup> F. Ibrahim,<sup>1</sup> K. W. Kemper,<sup>4</sup> K.-L. Kratz,<sup>5,6</sup> S. M. Lukyanov,<sup>7</sup> C. Monrozeau,<sup>1</sup>  
 L. Nalpas,<sup>8</sup> F. Nowacki,<sup>9</sup> A. N. Ostrowski,<sup>5,6</sup> T. Otsuka,<sup>10</sup> Yu.-E. Penionzhkevich,<sup>7</sup> J. Piekarewicz,<sup>4</sup> E. C. Pollacco,<sup>8</sup>  
 P. Roussel-Chomaz,<sup>2</sup> E. Rich,<sup>1</sup> J. A. Scarpaci,<sup>1</sup> M. G. St. Laurent,<sup>2</sup> D. Sohler,<sup>11</sup> M. Stanou,<sup>12</sup>  
 T. Suzuki,<sup>13</sup> E. Tryggestad,<sup>1</sup> and D. Verney<sup>1</sup>

### Neutron single-particle energies

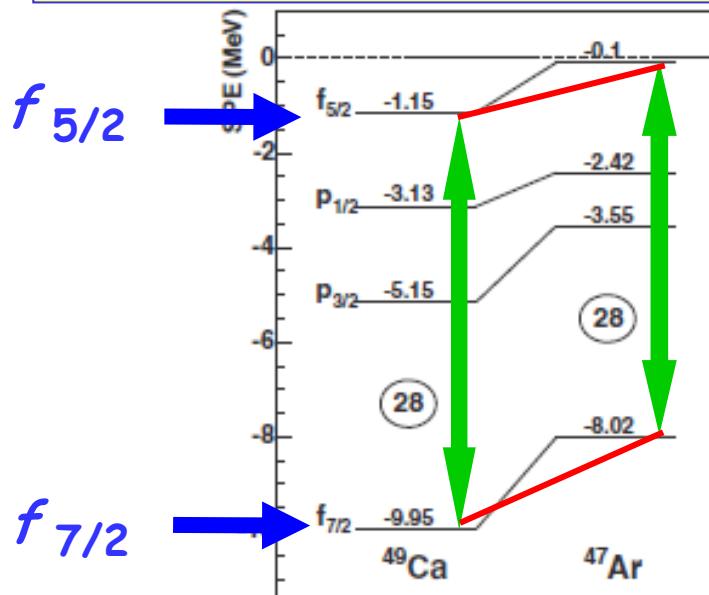
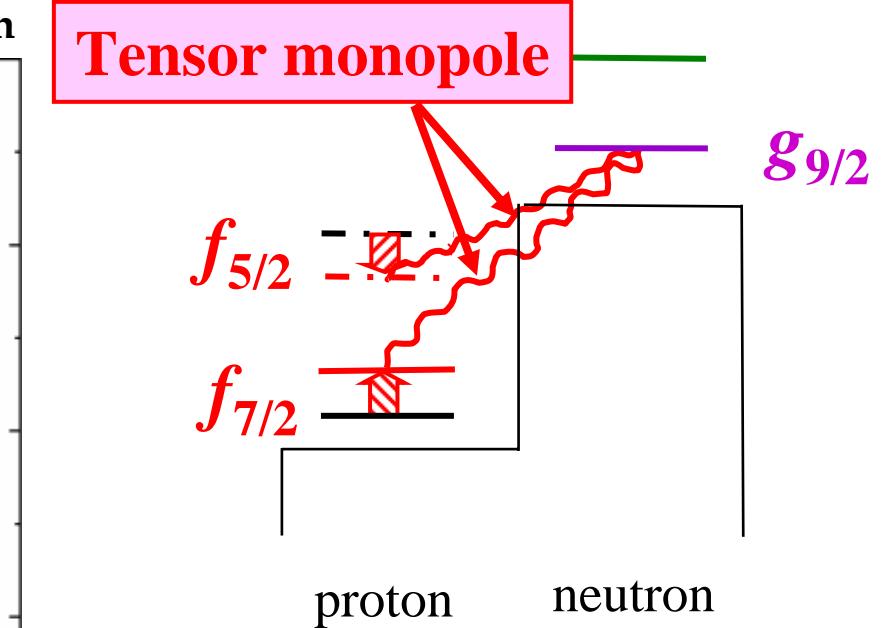
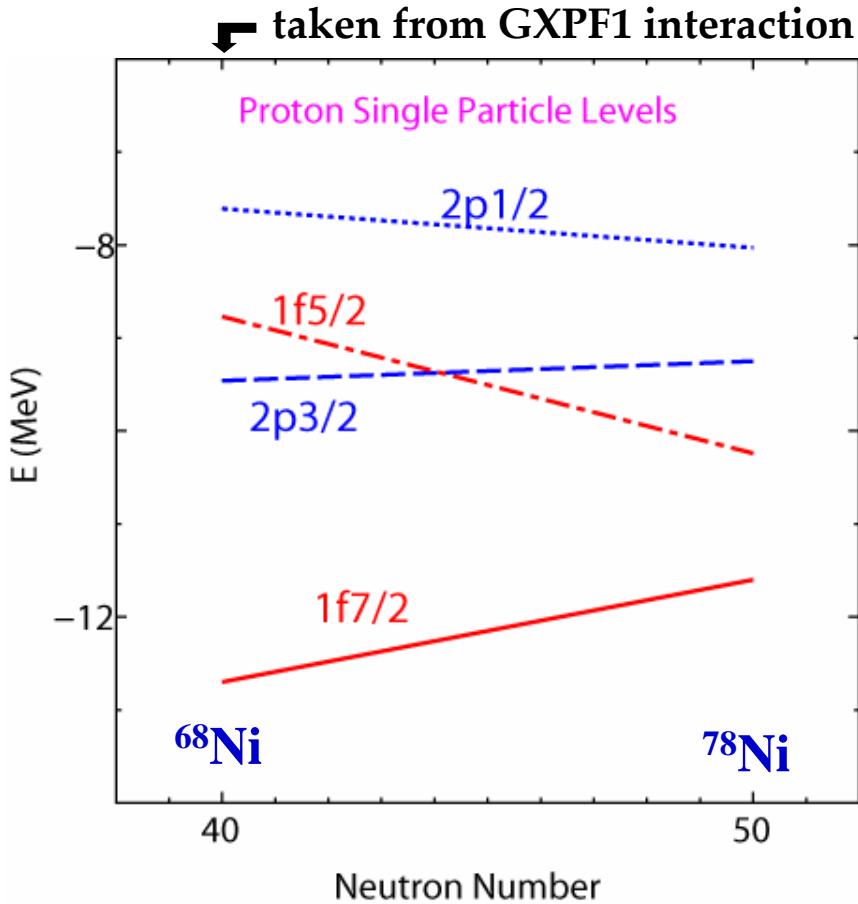


FIG. 3. Neutron single-particle energies (SPE) of the  $fp$  orbitals for the  $^{47}\text{Ar}_{29}$  and  $^{49}\text{Ca}_{29}$  nuclei (see text for details).

Mean-field models  
 (Skyrme or Gogny)  
 do not reproduce this  
 reduction.

Tensor force effect  
 due to vacancies of  
 proton  $d_{3/2}$  in  $^{47}\text{Ar}_{29}$  :  
 650 (keV) by  $\pi + p$  meson  
 exchange.

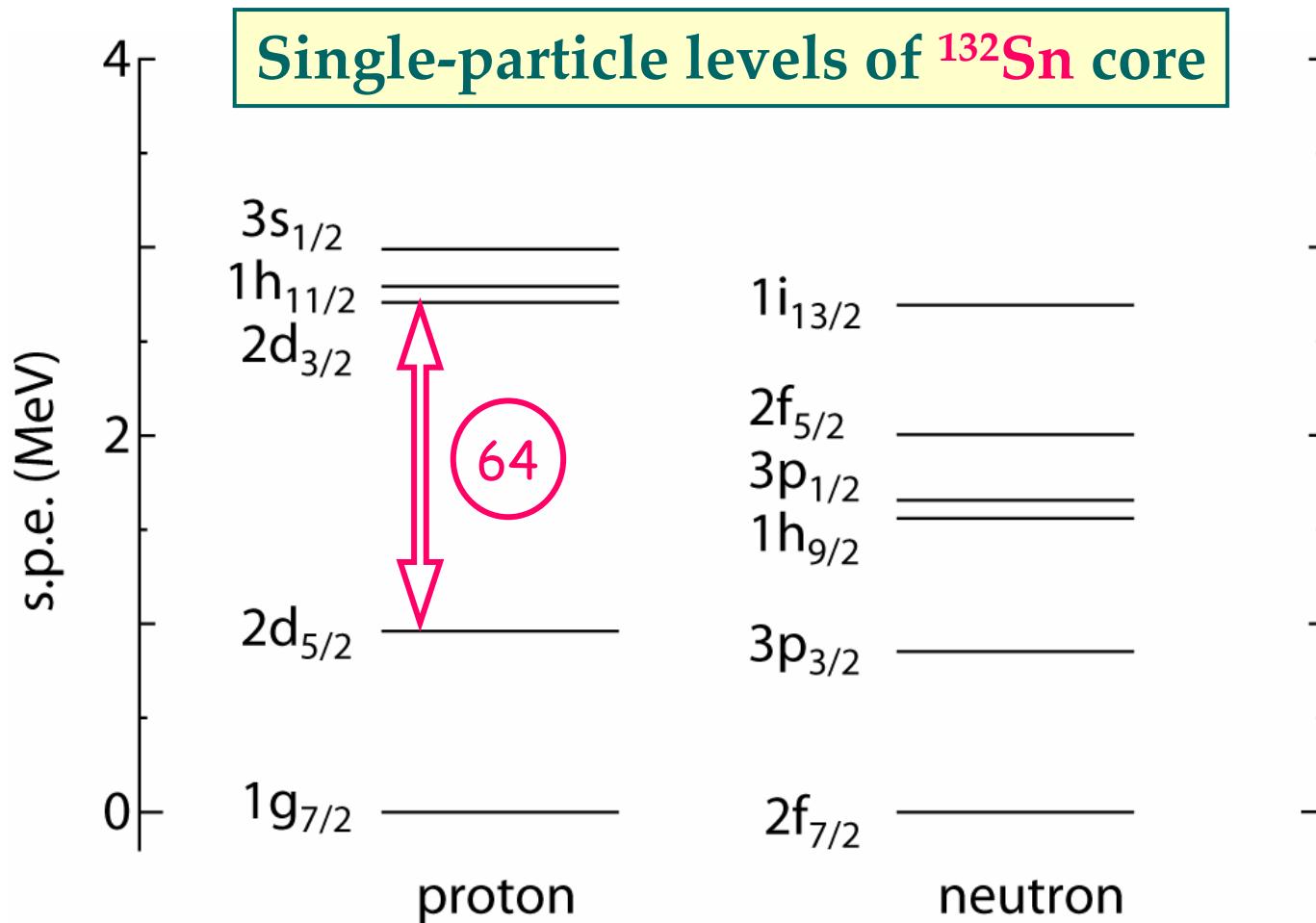
# Change of proton single-particle energies due to the tensor force ( $\pi + \rho$ meson exchange) *calculation only*



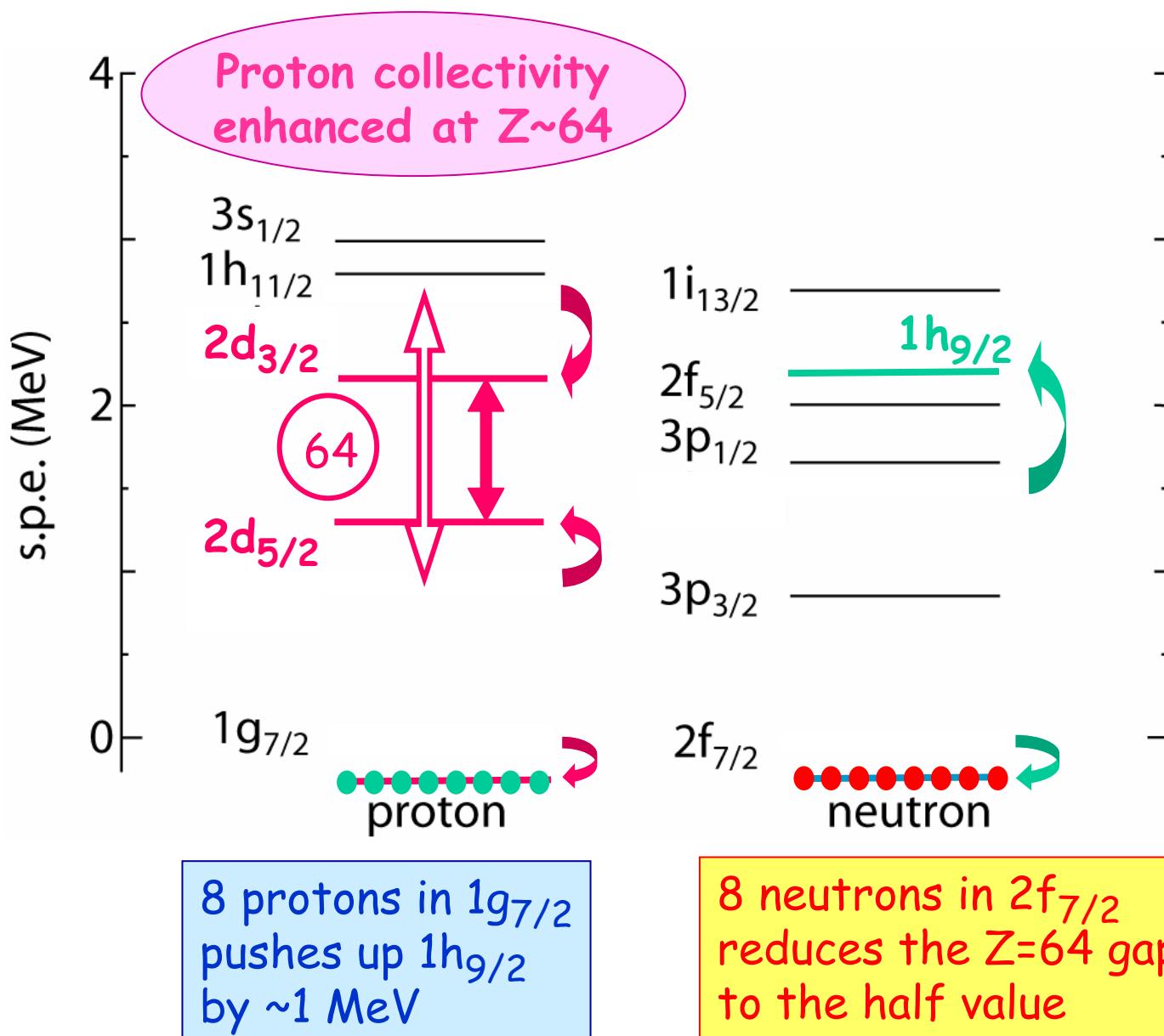
→ Low-lying  $2^+$  levels in Ni,  
M. Sawicka et al.,  
Phys. Rev. C68, 044304 (03)

neutrons in  $g_{9/2}$

# Weakening of Z=64 submagic structure for N~90



# Weakening of Z=64 submagic structure for N~90



# Implementation of tensor interaction into mean field calculations

## Gogny interaction (J. Decharge and D. Gogny, 1980)

$(1+\sigma\sigma+\tau\tau+\sigma\sigma\tau\tau)$  (Gauss1 + Gauss2) + Density Dep.  
**finite range**                    **zero range**

Successful descriptions of various properties with  
D1S interaction (J.F. Berger et al., Nucl. Phys. A428, 23c (84))

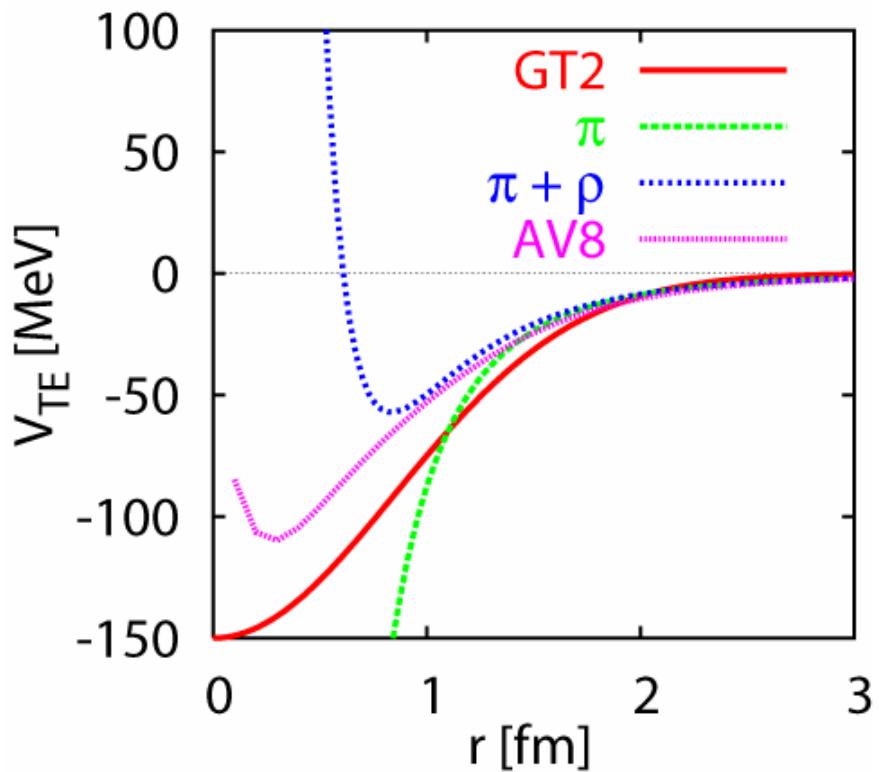
# Tensor interaction is added

**All parameters are readjusted**

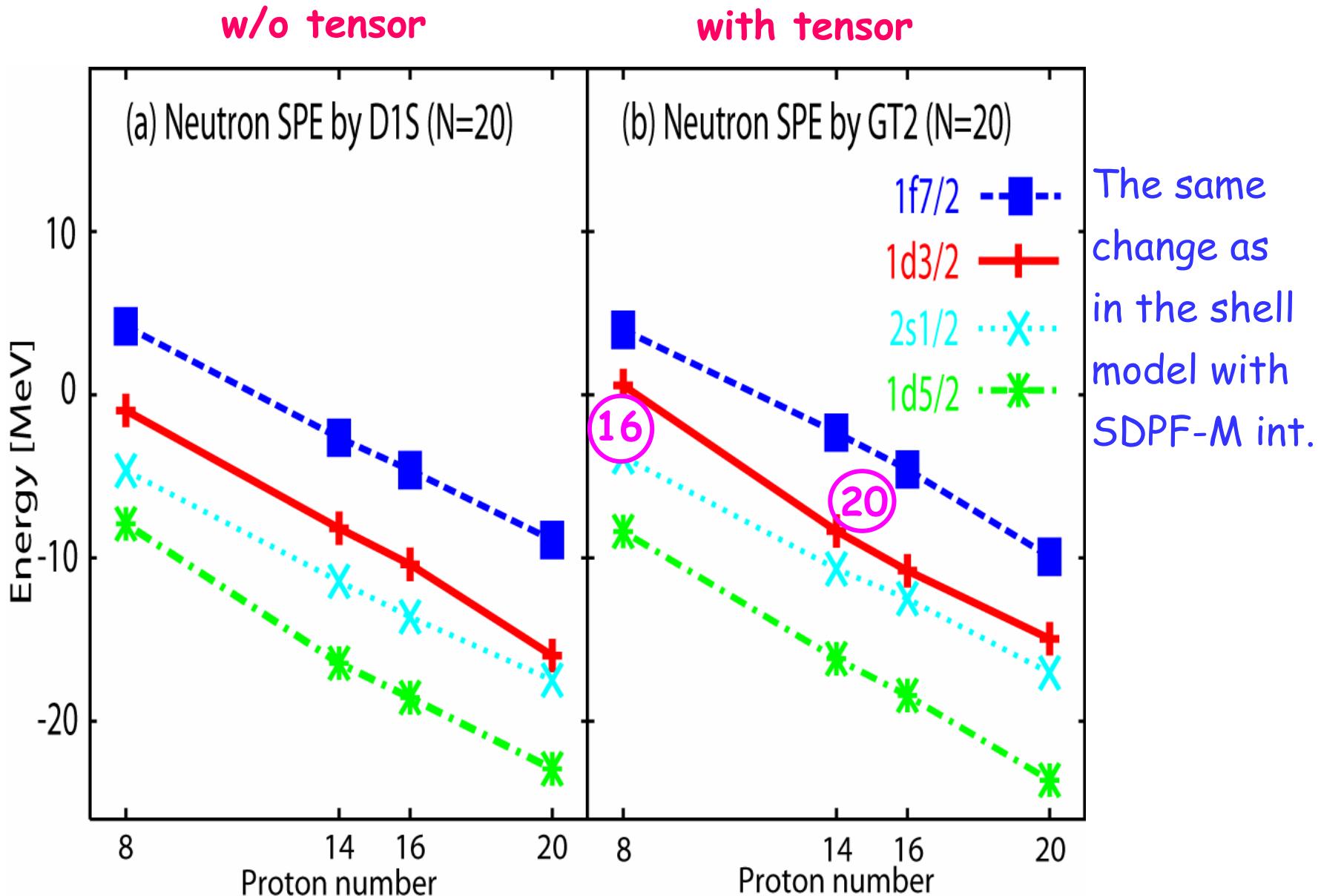
# Nuclear matter properties reproduced with improvement of incompressibility

# Gogny-Tokyo interaction - 2 (GT2)

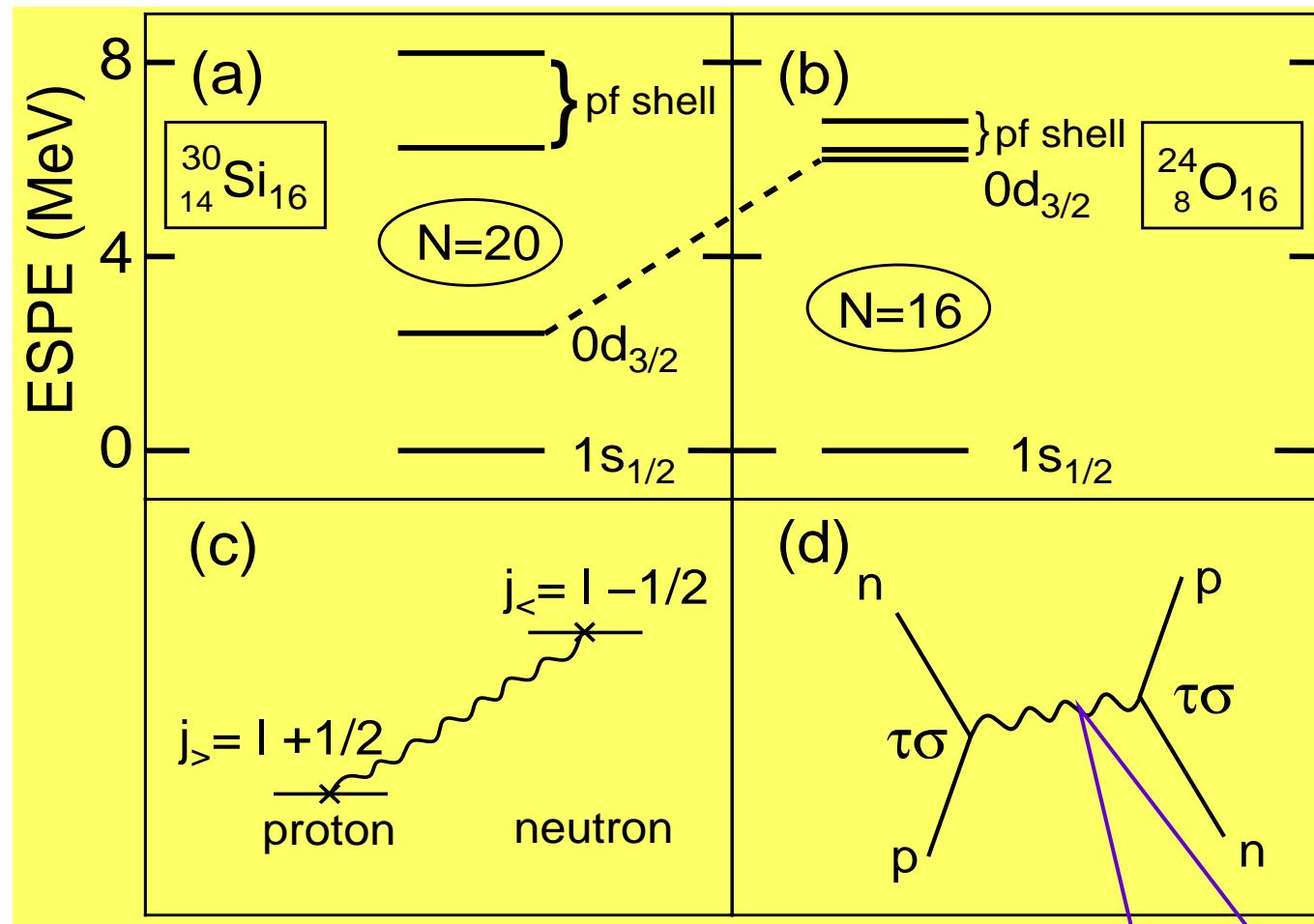
# Triplet-Even potential due to the Tensor force



## Two Gogny(-type) interactions : D1S and GT2



# Tensor interaction is the primary origin of the p-n $j_>$ - $j_<$ coupling.

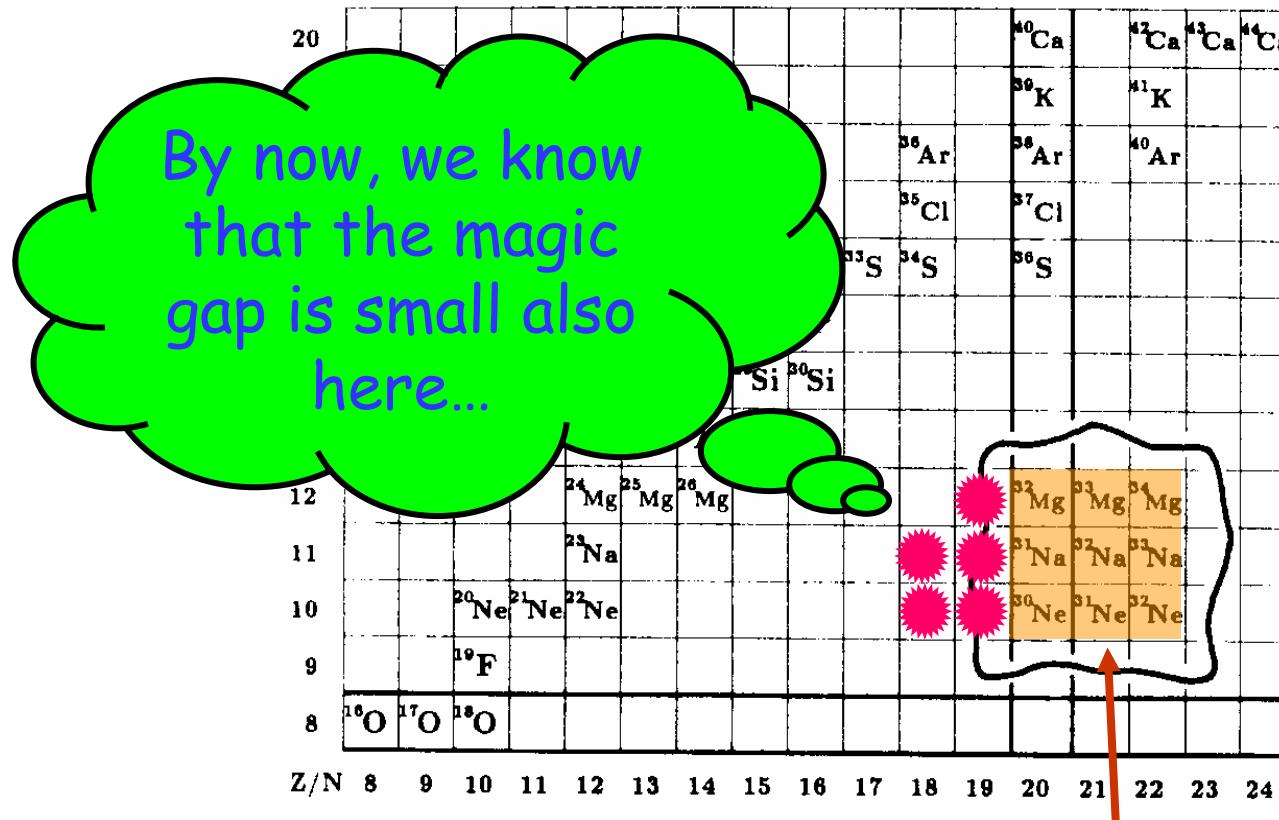


$\sigma\sigma\tau\tau$  central



Tensor

# Island of Inversion is being changed ...



By now, we know  
that the magic  
gap is small also  
here...

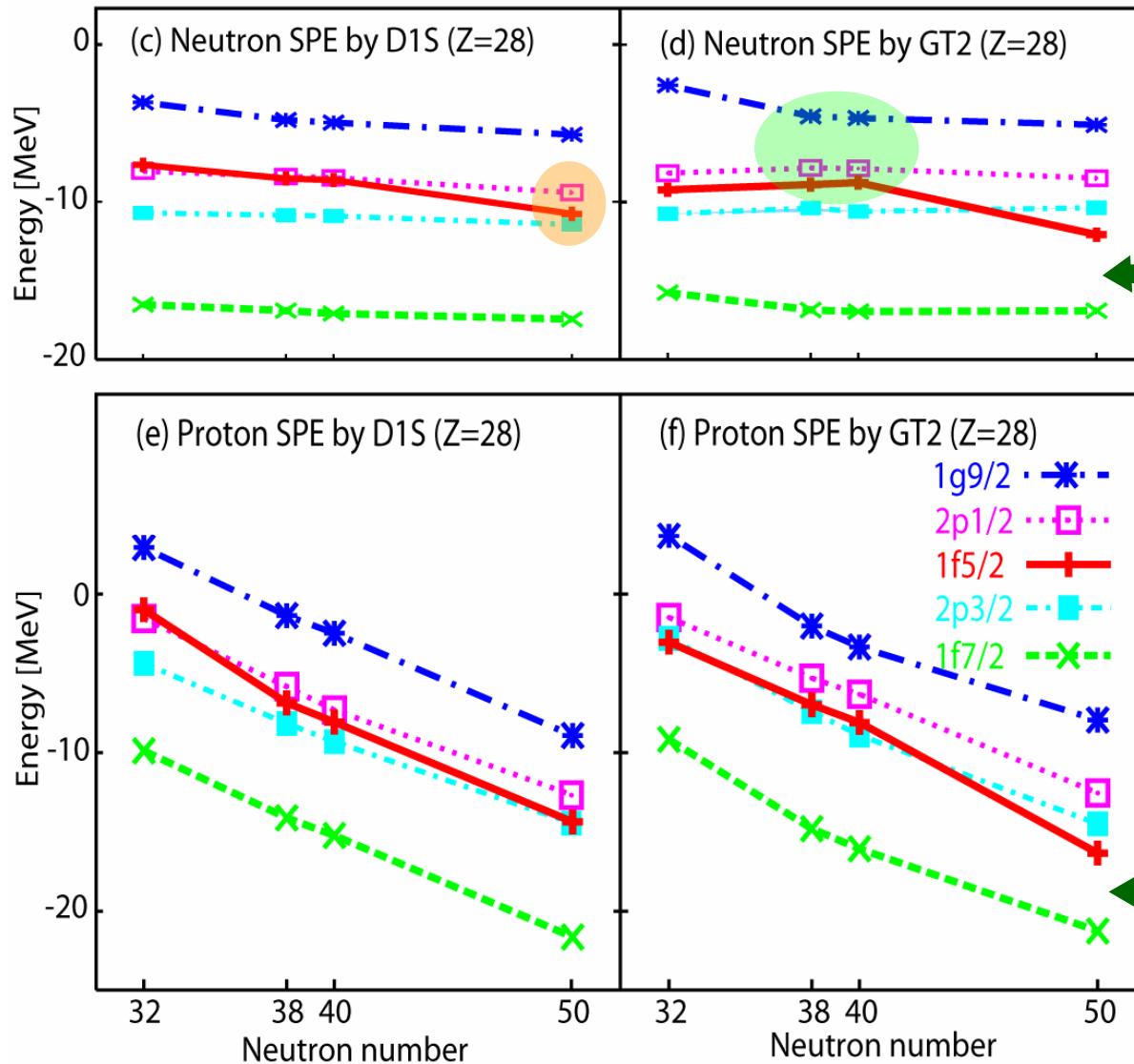
Island of Inversion :  
region of intruder ground states

Only 9 nuclei in the original model (1990)

# Single-particle energies of exotic Ni isotopes

w/o tensor

with tensor



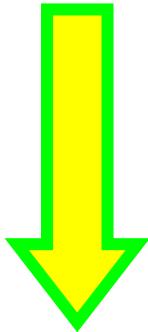
relevance to  
R-process  
s.p.e.'s  
binding energies

N=28

Z=28

## 2-body *LS* force

$$V_{LS} = V_{LS}(r) \mathbf{L}_{12} \cdot (\mathbf{s}_1 + \mathbf{s}_2)$$
$$\simeq iW_0(\boldsymbol{\sigma}_1 + \boldsymbol{\sigma}_2) \cdot \overleftarrow{\mathbf{k}} \times \delta(r) \overrightarrow{\mathbf{k}}$$



Vautherin - Brink

One-body mean potential

Proton

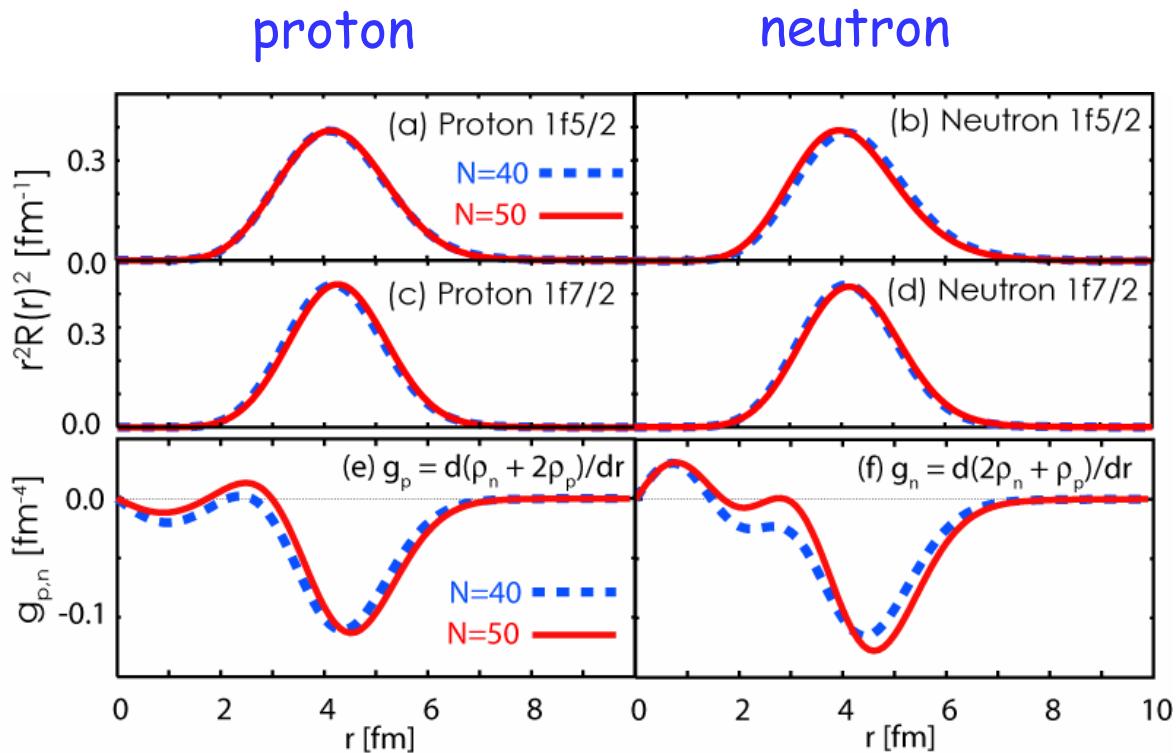
$$U_p \propto \frac{d}{dr}(\rho_n + 2\rho_p)$$

Neutron

$$U_n \propto \frac{d}{dr}(2\rho_n + \rho_p)$$

# Wave functions of f7/2 and f5/2 and derivatives of densities

--- N=40  
— N=50



f5/2

f7/2

derivatives of  
densities

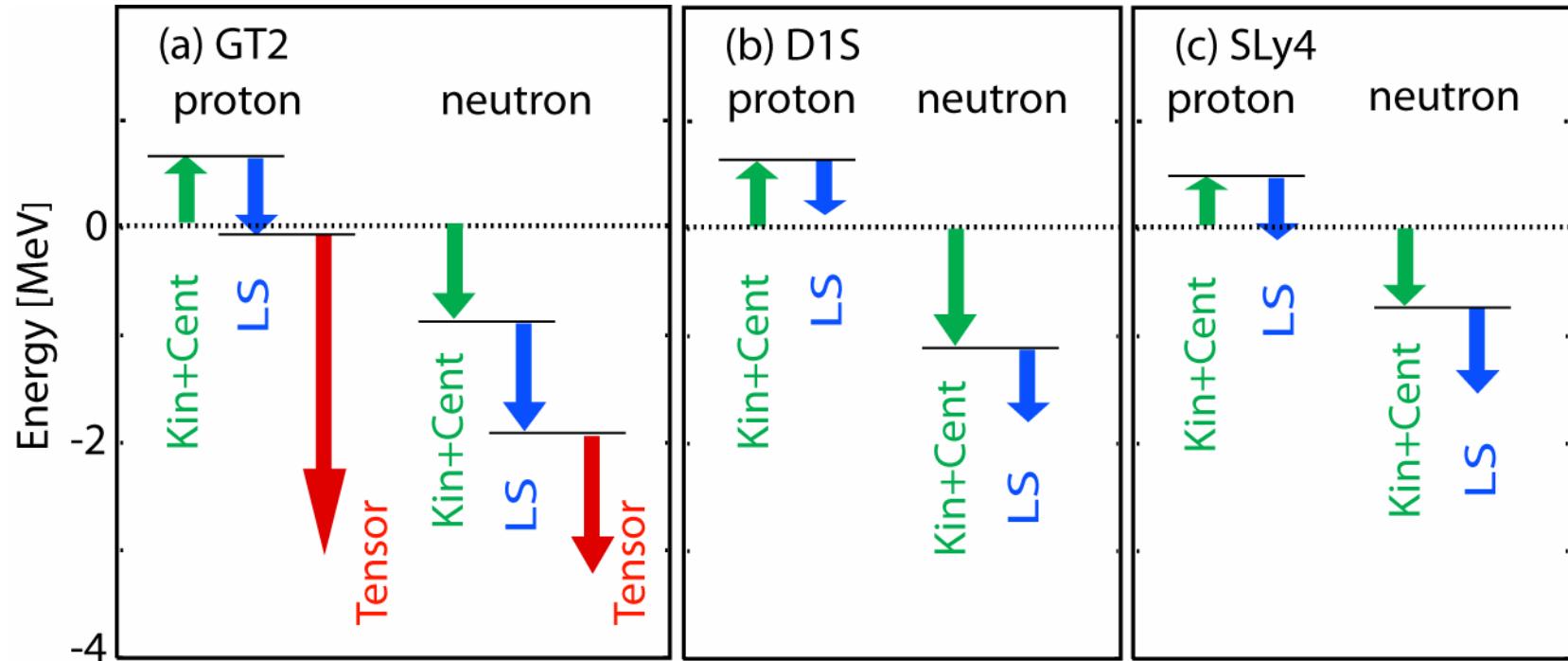
Peaks of derivatives do not  
get lower, but move outwards.

Position-type *ls* quenching



Scale-type *ls* quenching

Contributions of **Kinetic+Central**, **2-body LS**, and **Tensor**  
 components to the change of  $f7/2 - f5/2$  gap  
 in going from N=40 to N=50 ( $g9/2$  occupancy)

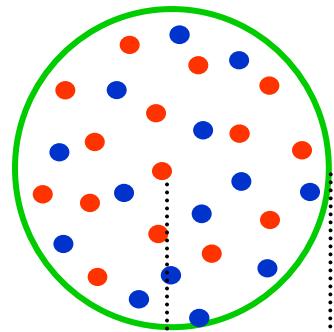


**Kin+Cent** and **LS** : almost the same among three calculations

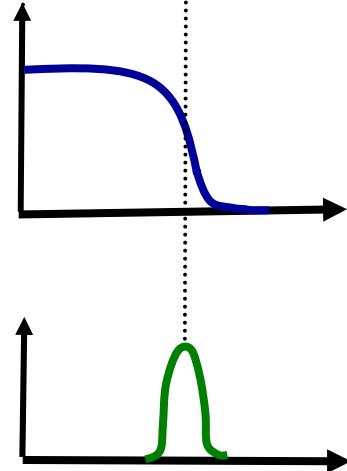
**Tensor** : largest effect

# Conventional image of $1s$ splitting change

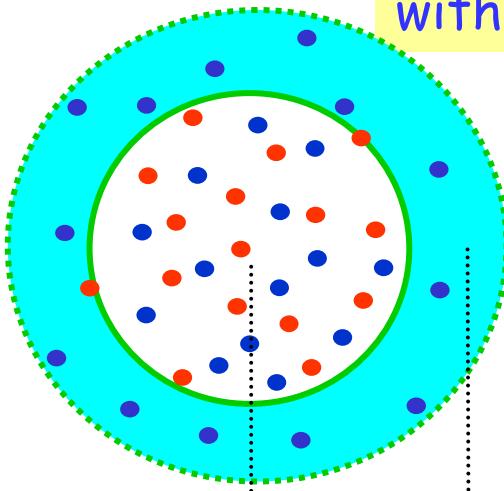
stable nucleus



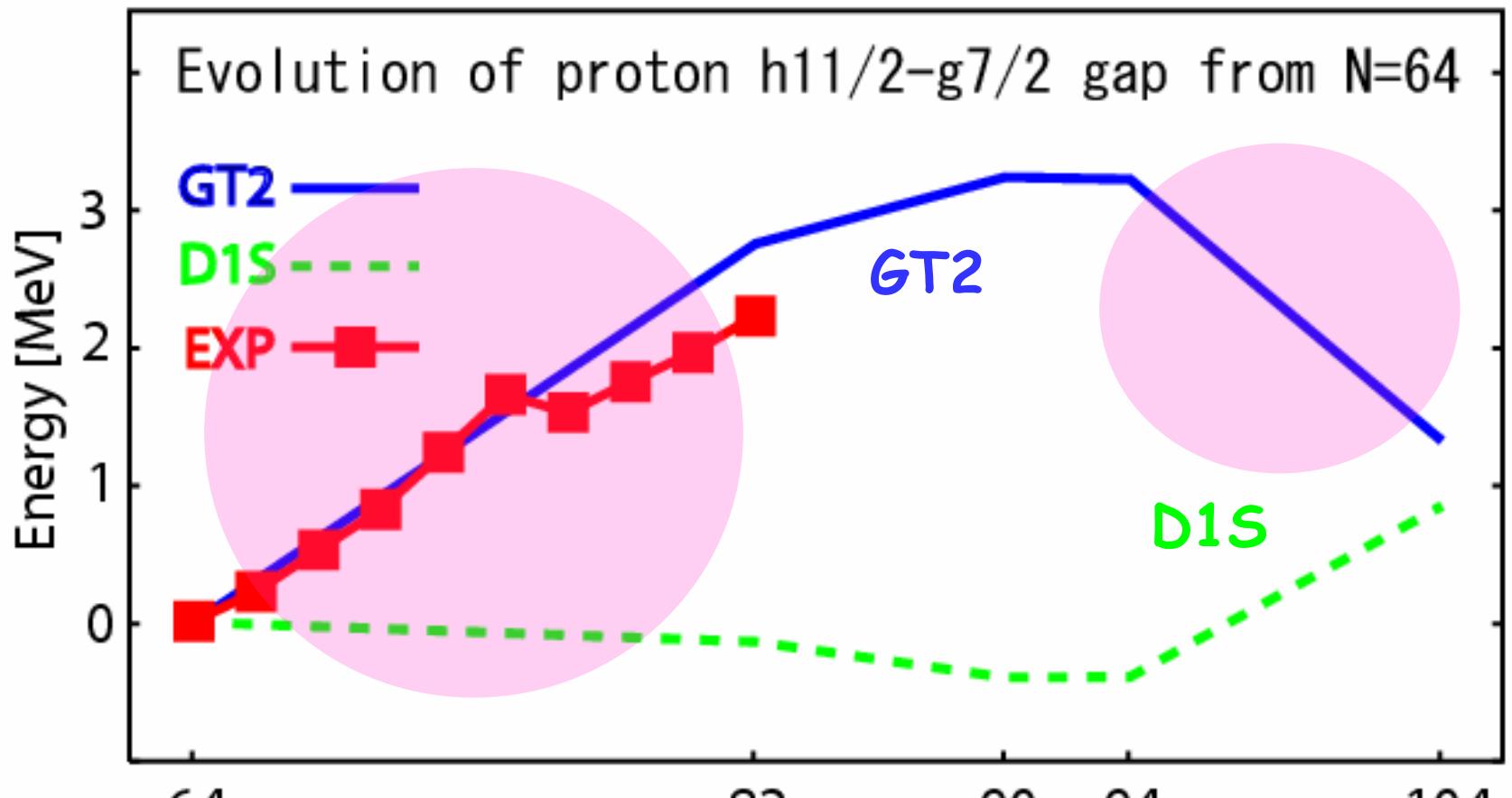
- proton
- neutron



exotic nucleus  
with neutron skin

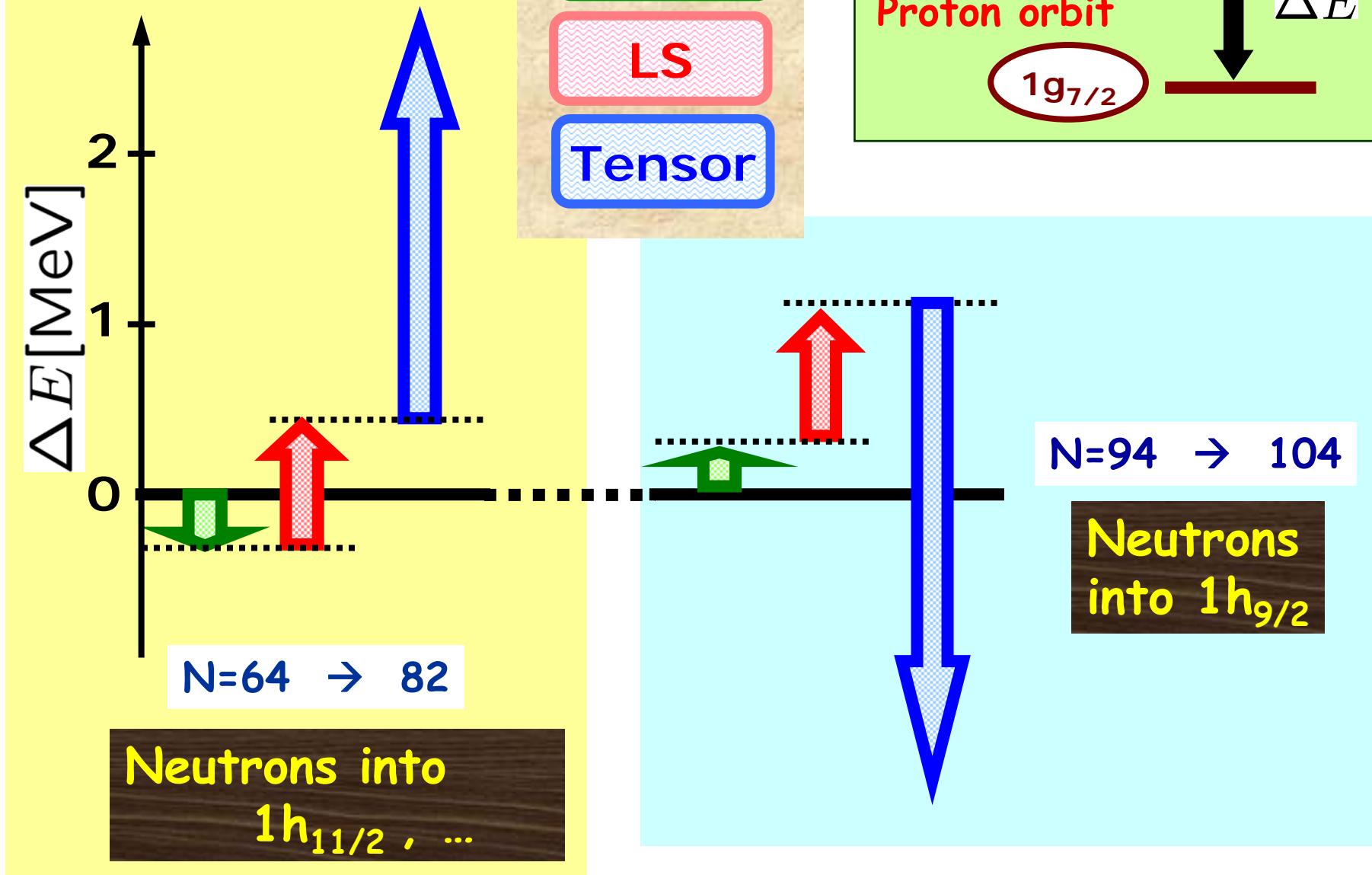


$1s$  splitting smaller  
(Scale-type  $1s$  quenching)



RIBF region

post-RIBF region



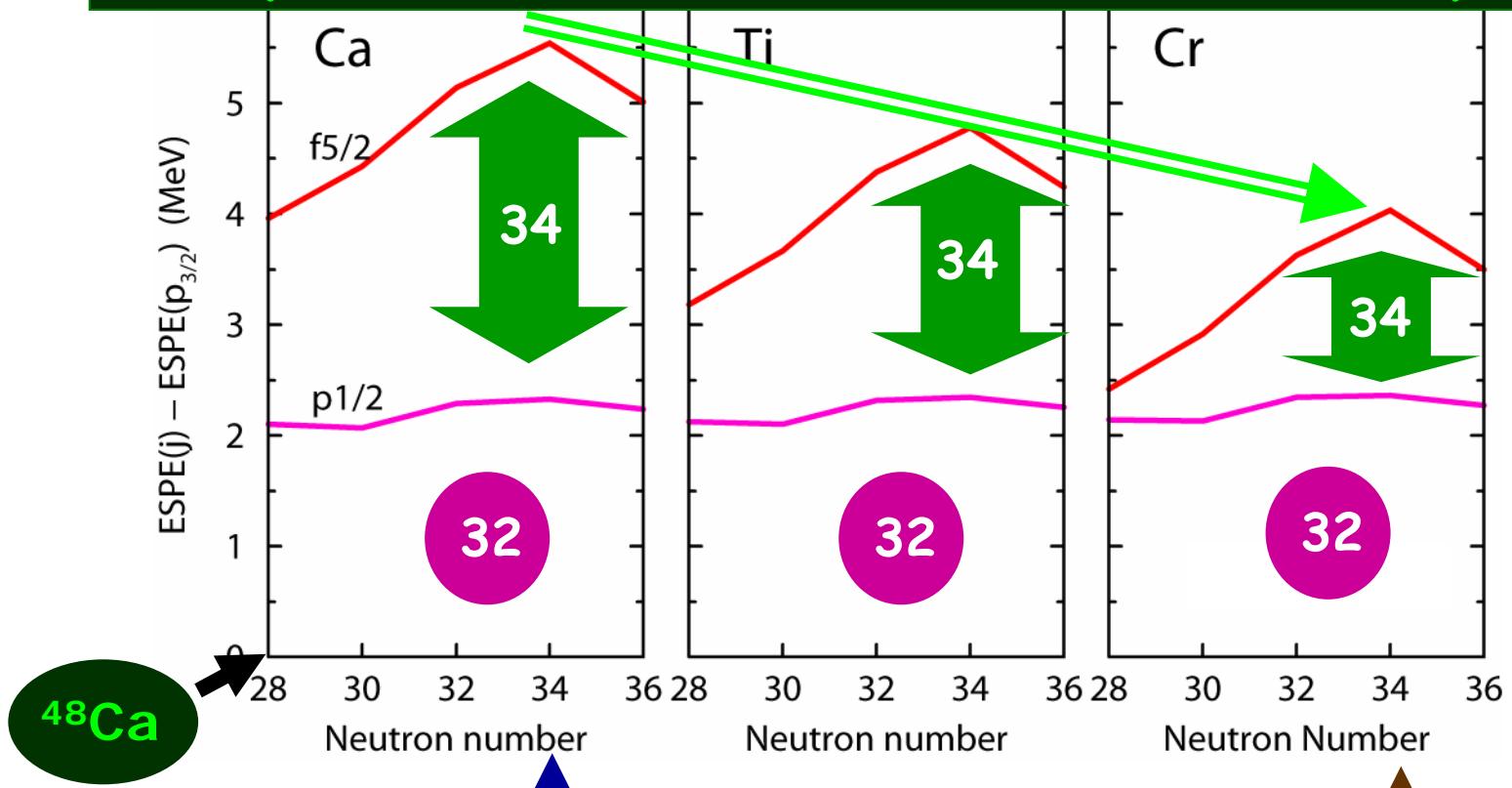
New magic numbers N=32 and 34 :

Their appearance and disappearance

*Frontier of RI-beam experiments  
(currently impossible)*

# Effective single-particle energies of Ca, Ti and Cr isotopes calculated by GXPF1B interaction

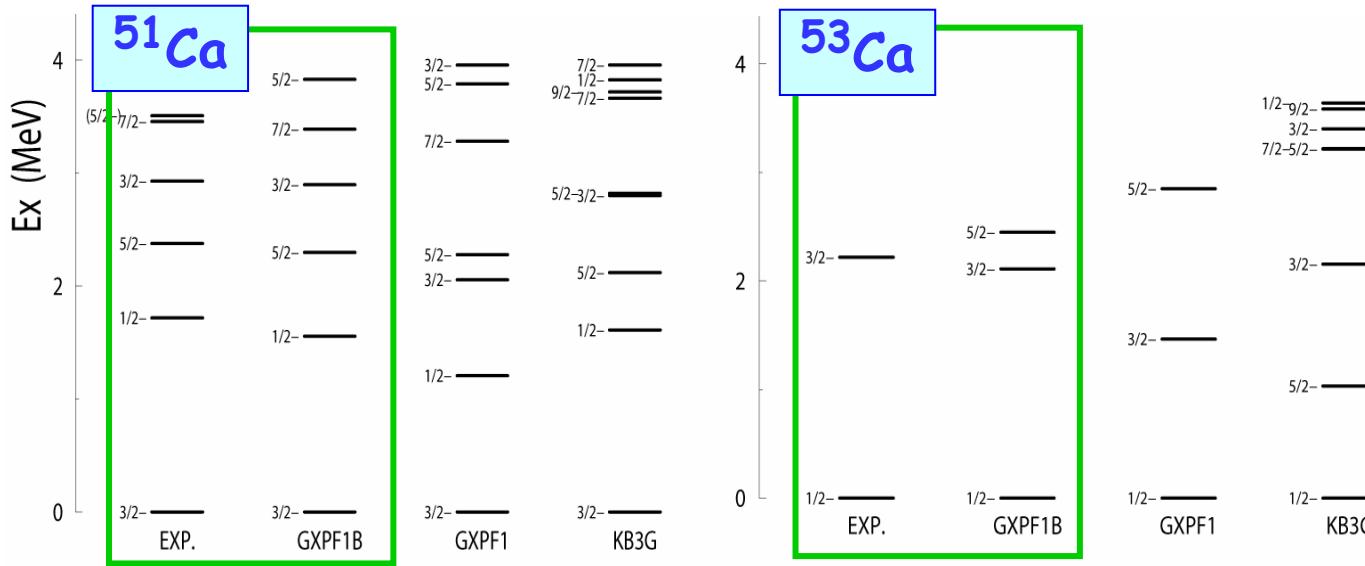
## Monopole effect of tensor force (f7/2 occupancy)



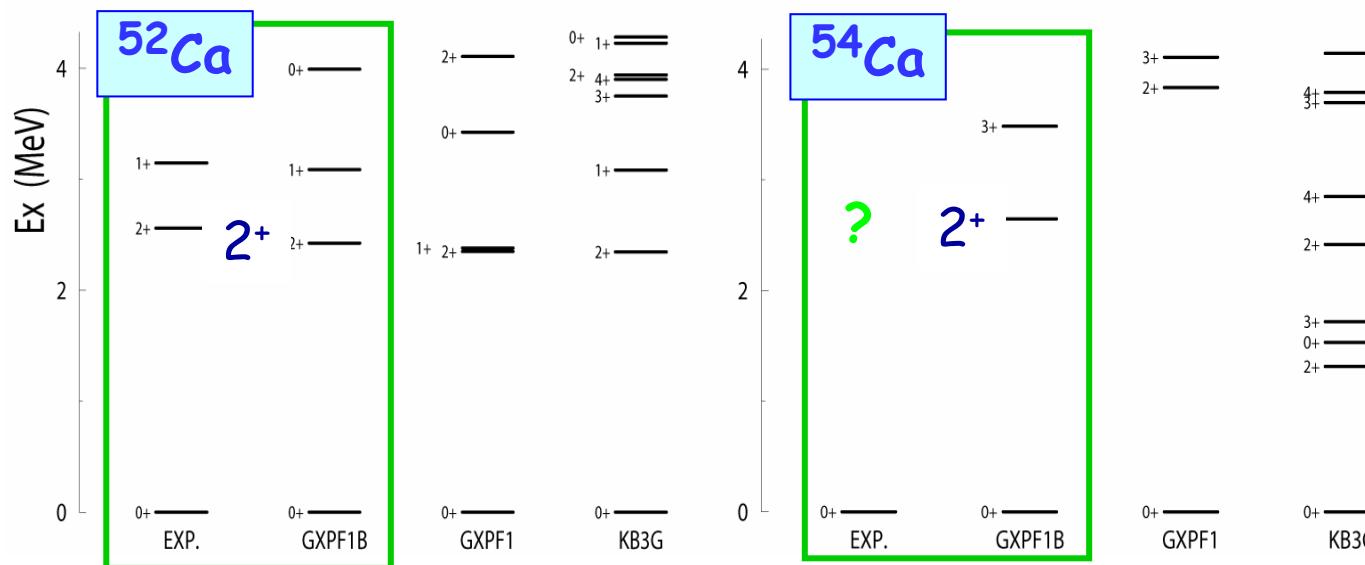
Gap = 2 MeV at N=32 and 3 MeV at N=34

Gap = 2 MeV at N=32 and 1.5 MeV at N=34

# Exotic Ca Isotopes : N = 32 and 34 magic numbers ?



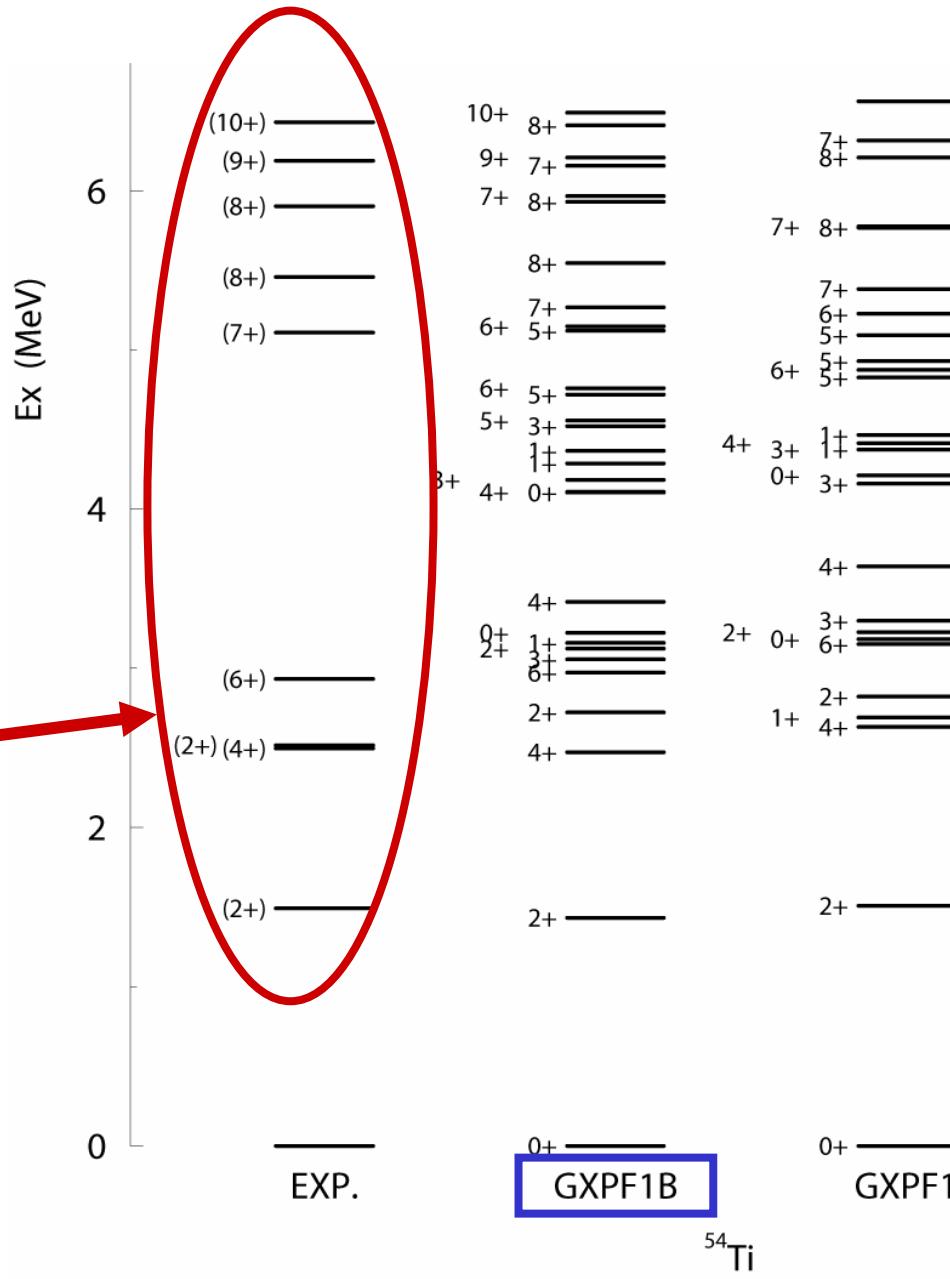
GXPF1B int.:  
p3/2-p1/2  
part refined  
from  
GXPF1 int.  
(G-matrix  
problem)



Some exp.  
levels :  
priv. com.

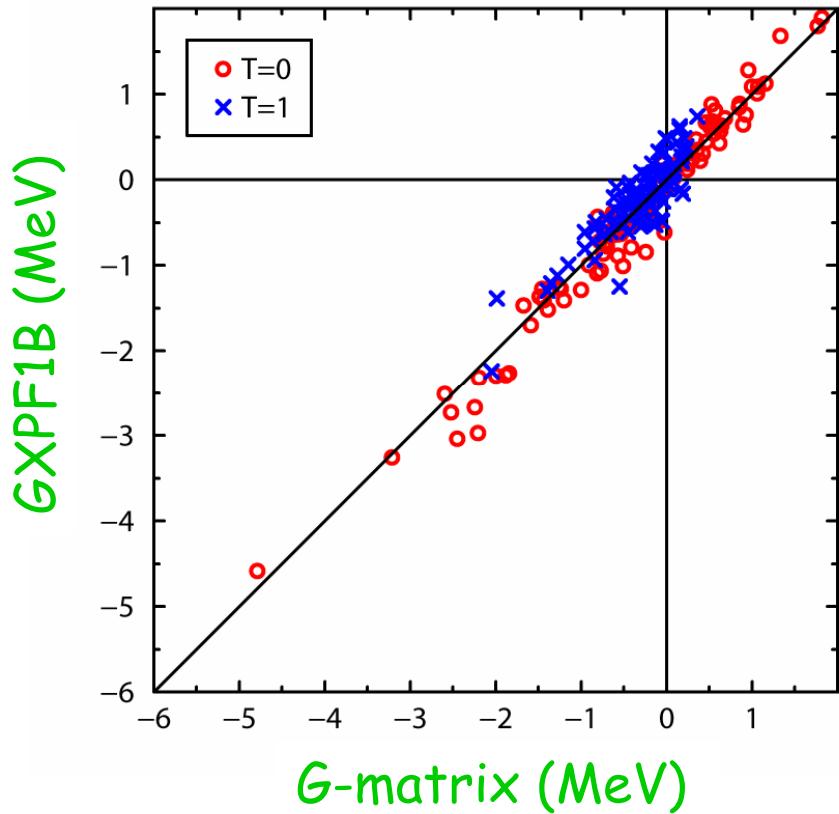
$^{54}\text{Ti}$

R.V.F. Janssens  
et al. PLB546  
(2002) 55

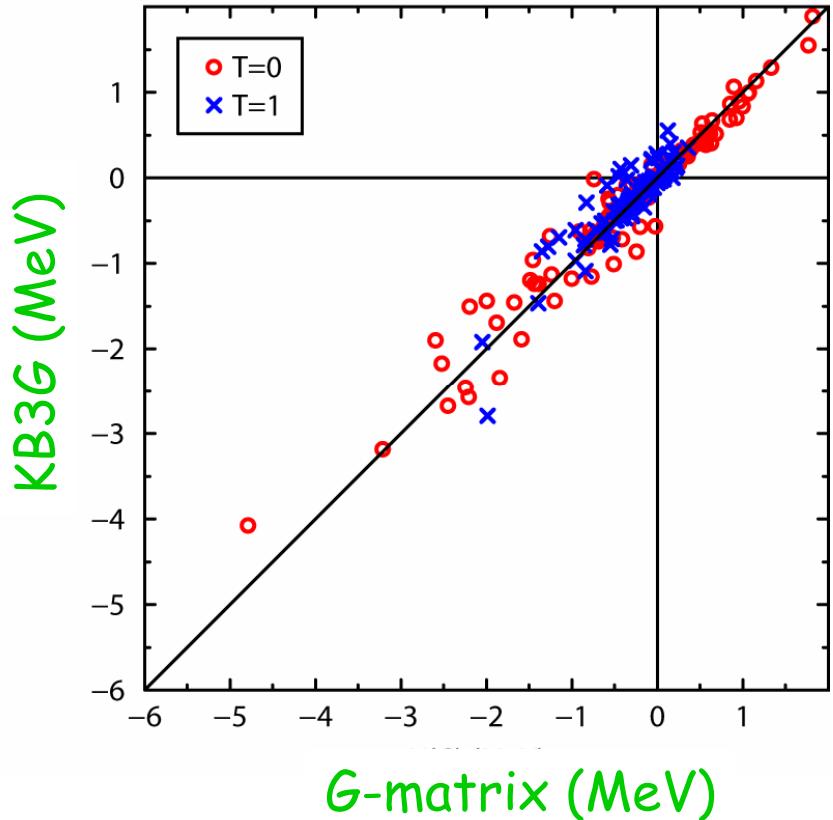


## Comparison with $G$ -matrix + polarization correction

$G_{XPF1B}$  vs.  $G$ -matrix (Bonn- $C$ )



$KB3G$  vs.  $G$ -matrix (Bonn- $C$ )



each point = a 2-body matrix element

## Summary

Nuclear shell evolves in unique ways as compared to other physical systems, particularly in exotic nuclei (note that  $\Delta N$  must be  $> 10$  to see this → RNB ! ).

### Shell evolution due to tensor interactions

- drives  $j_>$  or  $j_<$  levels in a specific and robust way  
*intuitive picture* → many cases expected from  $p$ -shell to superheavies
- is the dominant origin of shell evolution

John's Sb (Sn) experiment → first test  
excluding other possibilities  
connection to exp. in RIBF, ...., hopefully "RIA"

### Shell evolution due to 2-body LS interactions

Naïve picture (scale-type) may not be relevant

Real phenomena may be of position-type →  $l_s$  splitting decreases

## Remark on forces

Central forces : basic binding

Complex origins :    Multiple meson exchange,  
                            hard core (quarks, QCD may be needed)  
                            3-body forces

Tensor force : amusing changes in single particle properties

Simpler origin

dominated by one pion exchange (of course p, higher order ...)

↔ consistent with Chiral Perturbation (Weinberg)

Physics opened by upcoming RNB machines  
can be connected to QCD

New principle for further shell-model and mean-field studies

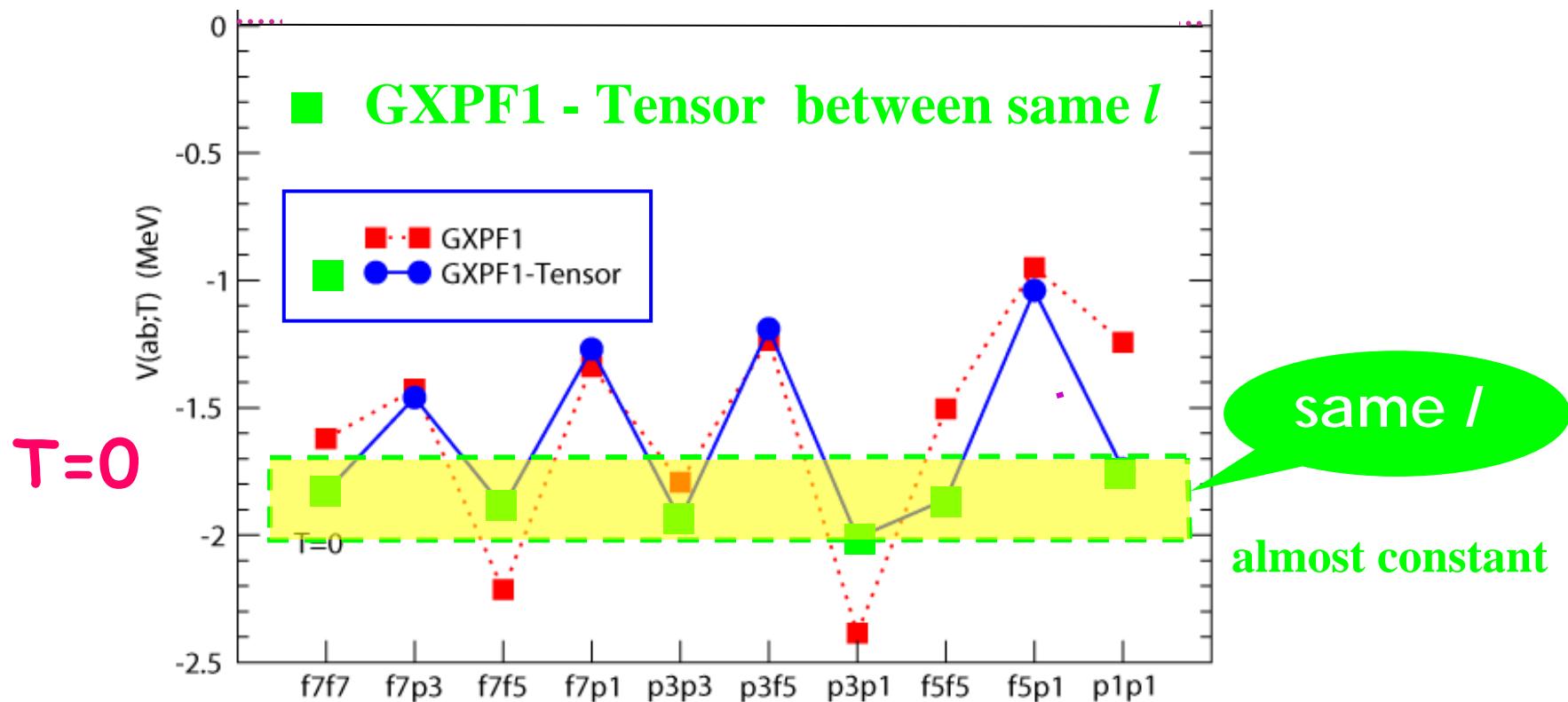
# Collaborators

D. Abe	Tokyo
T. Matsuo	Hitachi Ltd.
T. Suzuki	Nihon U.
R. Fujimoto	U. Tokyo
M. Honma	U. Aizu
H. Grawe	GSI
Y. Akaishi	KEK

END

# Monopole interaction after subtraction of tensor part

GXPF1 = shell model interaction for pf-shell  
(G-matrix + fit)

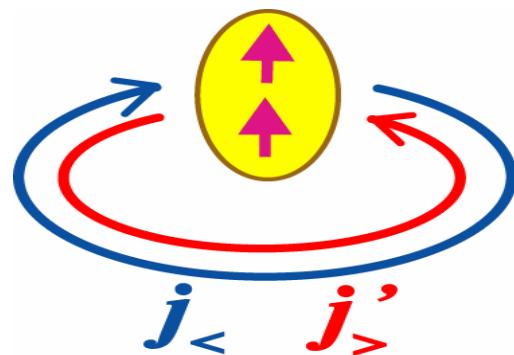


## Zero-range spin-momentum tensor coupling term

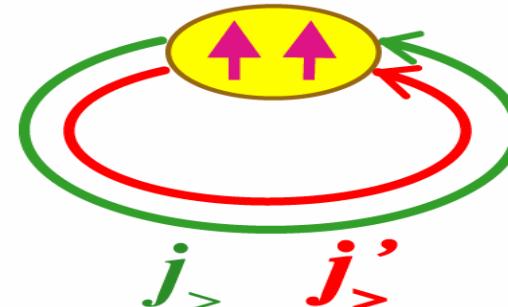
$$\begin{aligned}
 v_T = & \frac{1}{2} T \{ [(\sigma_1 \cdot k')(\sigma_2 \cdot k') - \frac{1}{3}(\sigma_1 \cdot \sigma_2)k'^2] \delta(r_1 - r_2) \\
 & + \delta(r_1 - r_2) [(\sigma_1 \cdot k)(\sigma_2 \cdot k) - \frac{1}{3}(\sigma_1 \cdot \sigma_2)k^2] \} \\
 & + U \{ (\sigma_1 \cdot k') \delta(r_1 - r_2) (\sigma_1 \cdot k) \\
 & - \frac{1}{3}(\sigma_1 \cdot \sigma_2) [k' \cdot \delta(r_1 - r_2) k] \}, \quad (1)
 \end{aligned}$$

This is not be a good approximation to the tensor force itself, but may simulate the monopole effect of the tensor shown below, picking up differences in relative momenta.

large relative momentum      small relative momentum

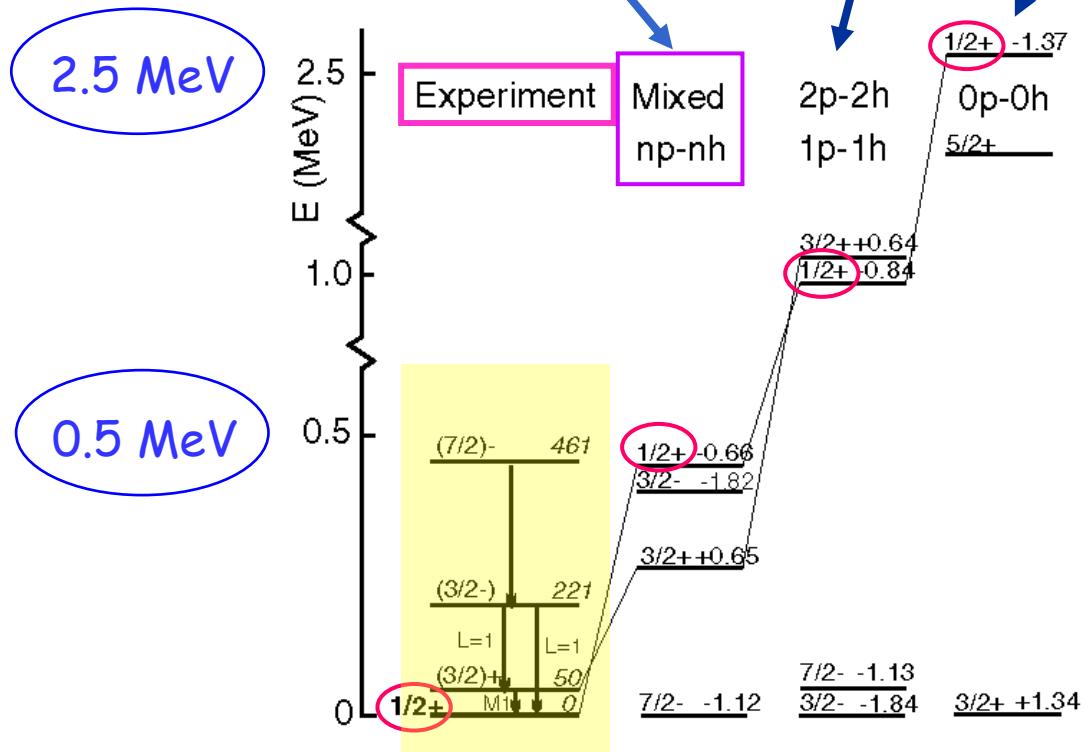


**deuteron**  $\Rightarrow$  **attractive**



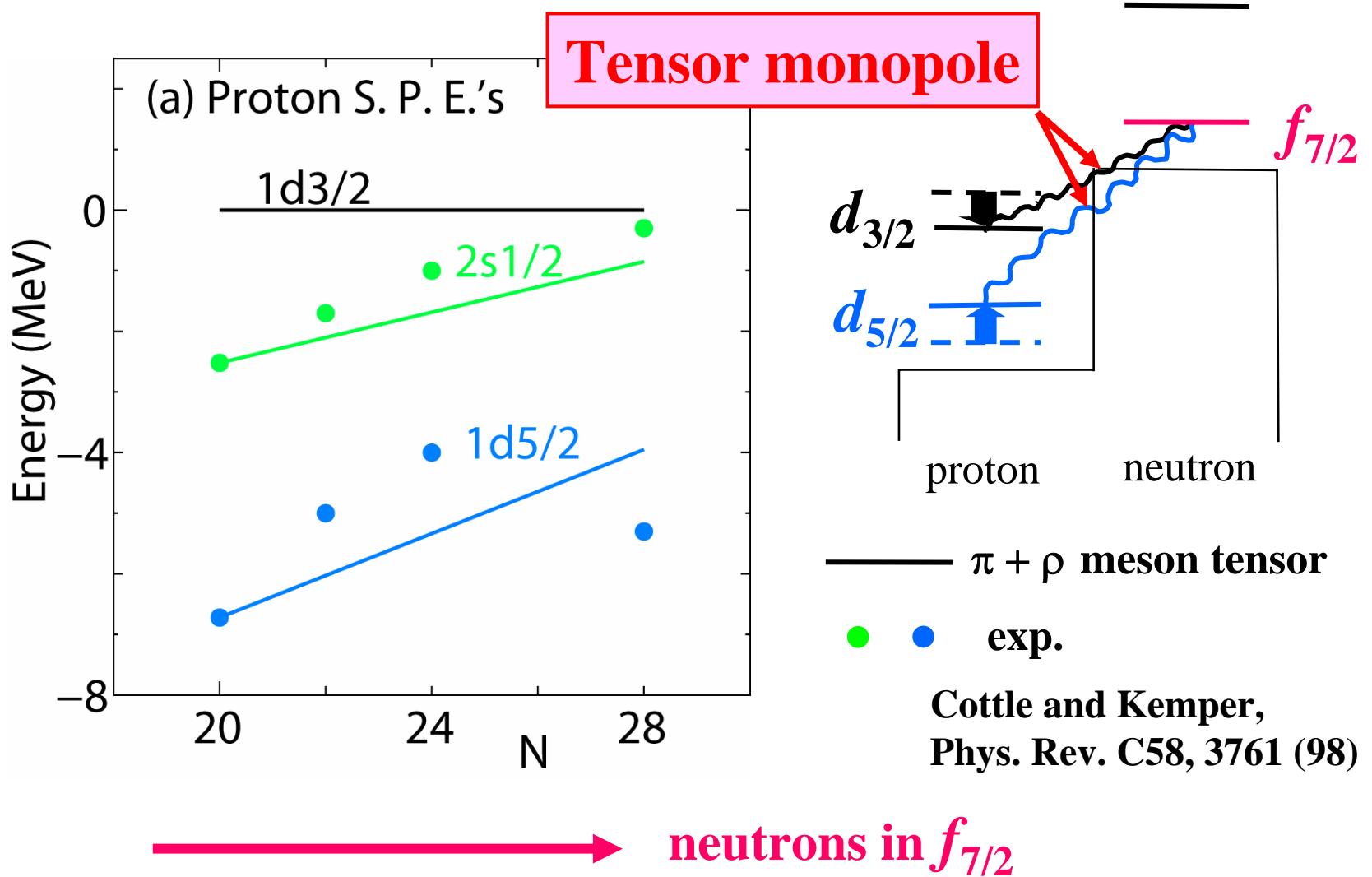
**repulsive**

Outside the *original*  
Island of Inversion



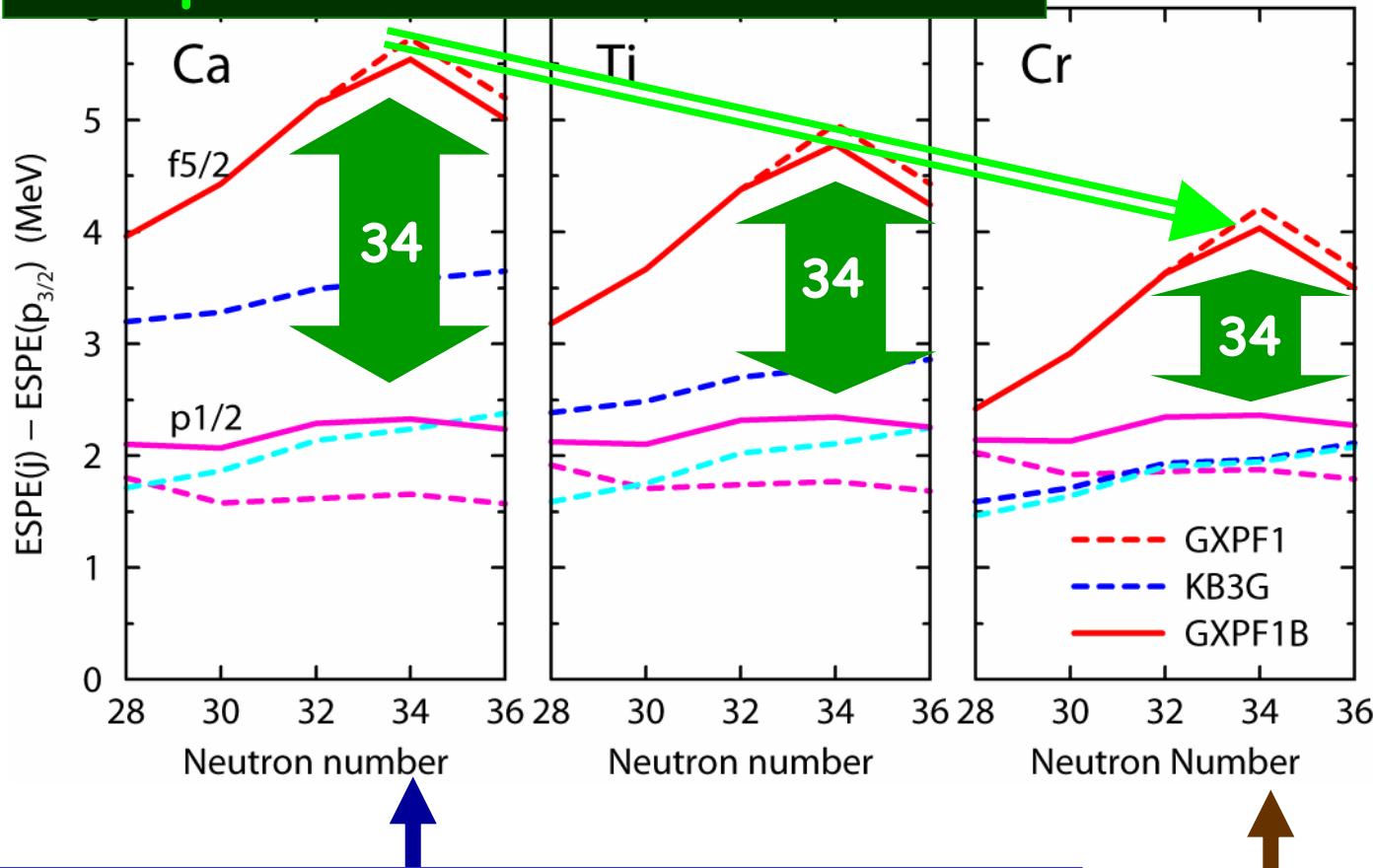
1/2<sup>+</sup> state comes down by 2 MeV from USD result by  
strong mixing between *sd* and *pf* shell configurations  
due to **narrower  $N=20$  gap**  
→ fine details to be refined (odd-*A* nuclei are difficult)

# Proton effective single-particle levels (relative to $d_{3/2}$ )



# Effective single-particle energies of Ca, Ti and Cr isotopes

## Monopole effect of tensor force



Gap = 2 MeV at  $N=32$  and 3 MeV at  $N=34$

Gap = 2 MeV at  $N=32$  and 1.5 MeV at  $N=34$

The shell structure is the basis of quantum many-body systems and also reflects the underlying mechanism to make the system bound.

If the shell is created by the potential source at the Center (like Coulomb potential of Hydrogen-like atoms), the shell structure does not change much.

However, if the shell structure is made by the particles themselves, the shell may evolve lively as the number of particles changes. Is this the case with exotic nuclei ?

But, the most relevant point today is that John Schiffer likes single-particle motion.

Among many possible issues, e.g. loose binding effect, on the shell evolution, I would like to focus on the following two points.

- As  $N$  increases from the  $\beta$  stability line, the spin-orbit splitting becomes smaller due to the change of the density distribution.

Is this true ? If so, how does it happen ?

- More recently, variations of the shell structure off the  $\beta$  stability line but still far away from the drip line have emerged due to the **tensor force**.

How does the tensor force work in old and new problems ?

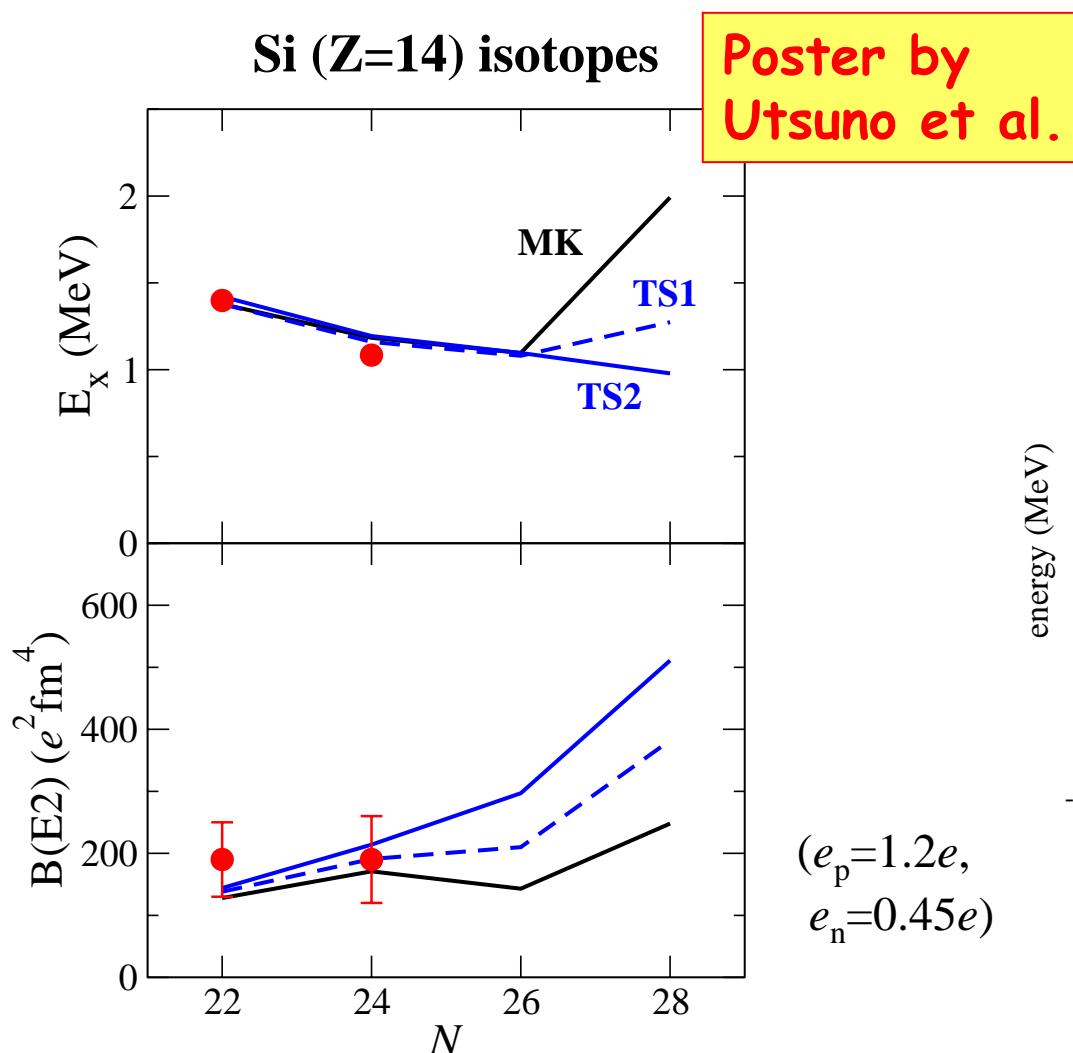
How do these two effects compare in their sizes ?

# • Structure of neutron-rich Si isotopes

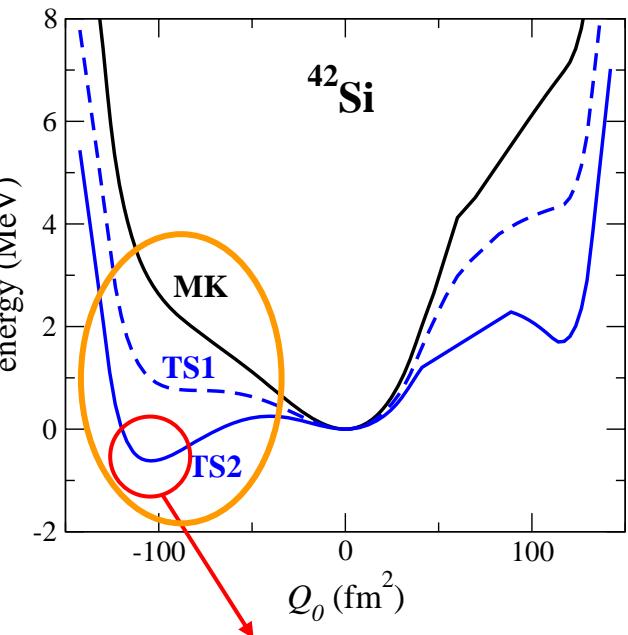
—

■ by the tensor force

24 MeV (TS2) vs. 1.12 MeV (MK).



Constrained HF calculation  
in the shell-model space



About 1.5 protons are excited  
across the  $Z=14$  shell gap.

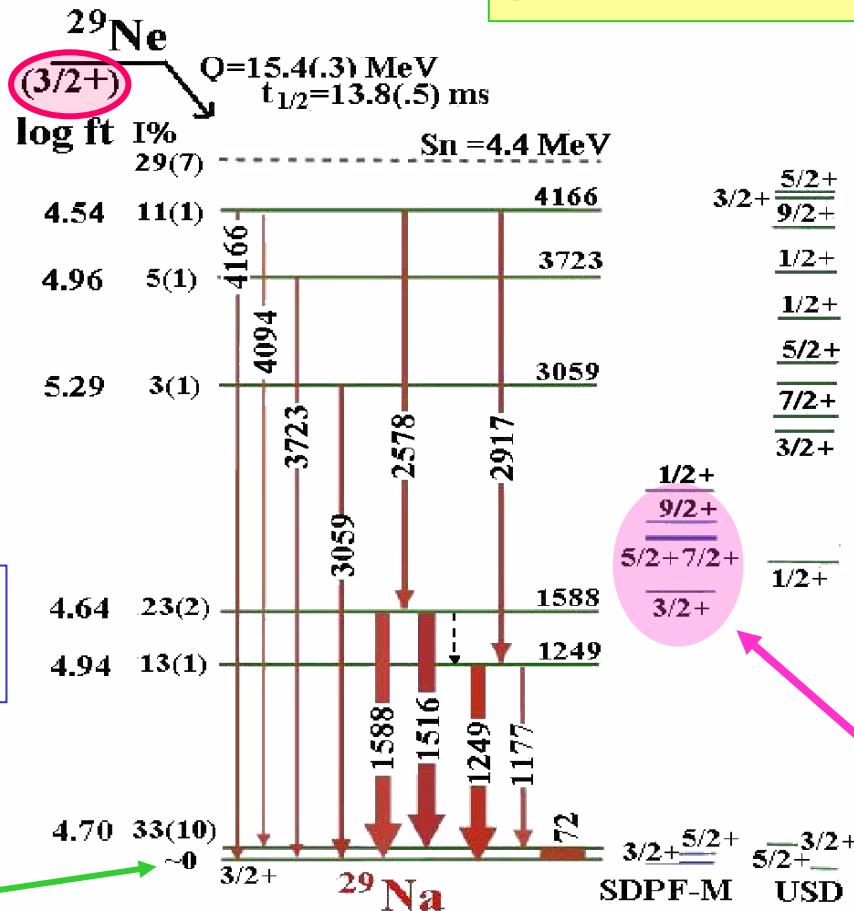
The following points were raised as new aspects of shell evolution in exotic nuclei :

- As  $N$  increases from the  $\beta$  stability line, mean potential has a more **diffuse surface**, and the **spin-orbit splitting becomes smaller**.
- In dripline nuclei, extremely loosely bound orbits, particularly s-orbit, remain bound by **tunnel effect** (neutron halo).
- Also near the drip line, coupling to continuum may produce new phenomena, e.g., those like BCS/BEC (pairing dynamics).

More recently, variations of the shell structure **off the  $\beta$  stability line but still far away from the drip line** have emerged. Such variations appears to be due to the spin-isospin component of the NN interaction, particularly the **tensor**.

Lowest states are intruders (> 50%)

$^{29}\text{Na}_{18}$



$3/2^+, 5/2^+$ ?  
(from log ft)

zero BR

only state below  
2.8 MeV by USD  
(large log ft)

4 additional states  
of intruder config.  
by SDPF-M int.  
(MCSM calc.)

FIG. 2 (color online). Proposed level scheme for  $^{29}\text{Na}$  populated following the  $\beta^-$  decay of  $^{29}\text{Ne}$ . The absolute  $\beta$ -decay branching to each level per 100 decays is indicated along with the calculated  $\log ft$  values. Shown on the right are shell model calculations with the USD and SDPF-M interactions.

## Level scheme of $^{28}\text{Ne}$

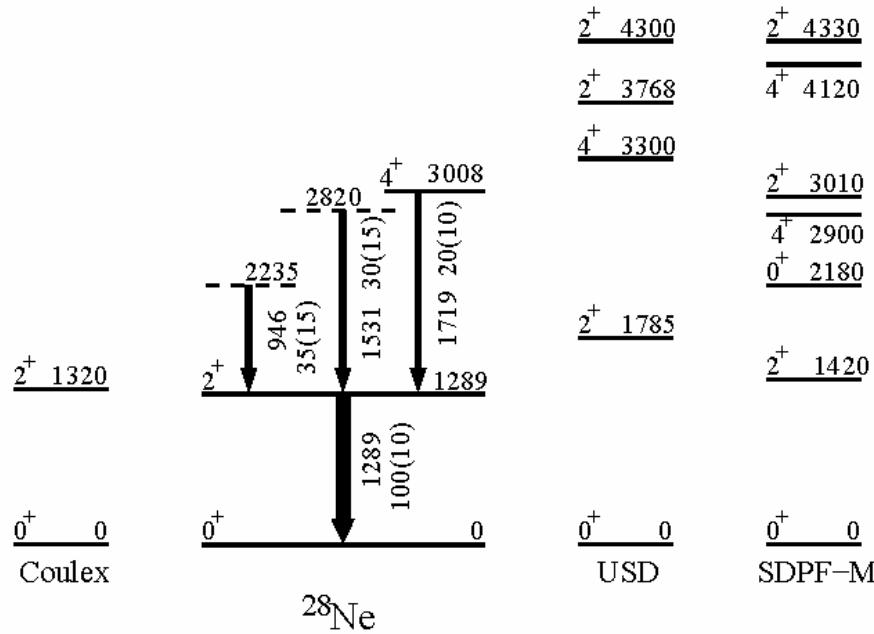


FIG. 9: Proposed level scheme of  $^{28}\text{Ne}$ . The results of different shell model calculations [7, 22] are included in the right part of the figure. The energy of the first excited state observed in Coulomb excitation experiment is taken from Ref. [20]. The spin 4 assignment to the 3008 keV state is taken from Ref. [21].

Federman and Pittel, Phys. Lett. B 69, 385 (1977)

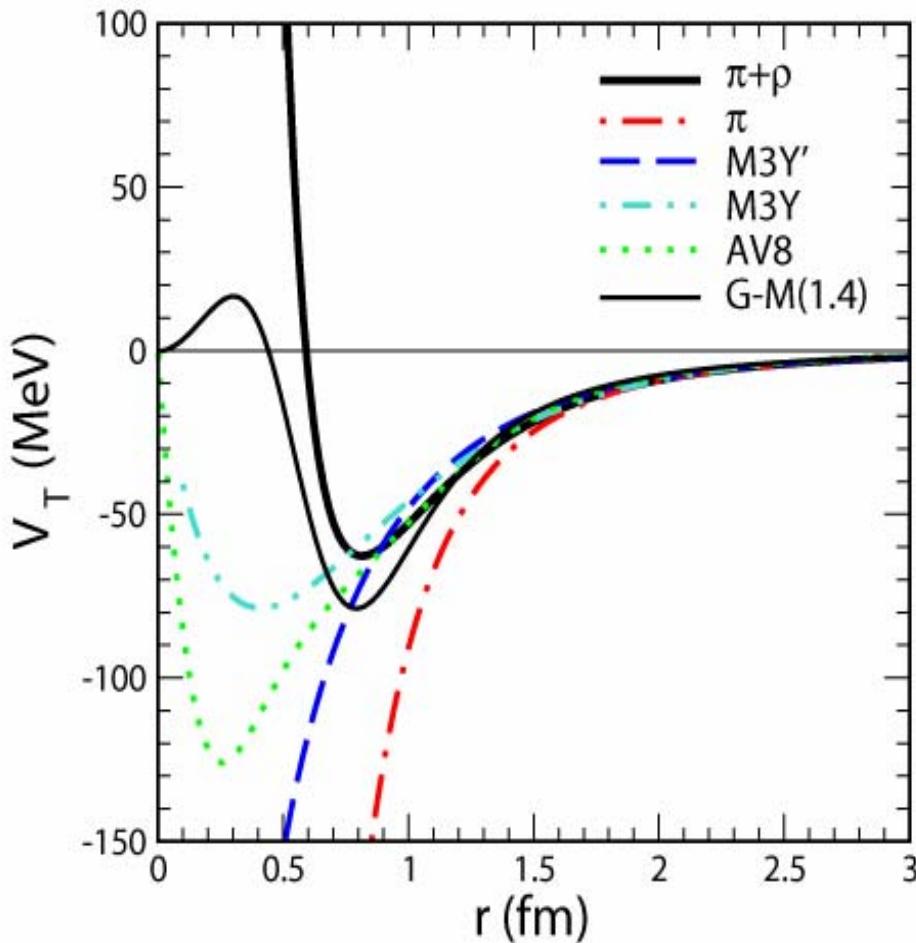
- Overlap of radial wave functions is emphasized -

they can simultaneously fill the  $1g_{9/2}$  proton and  $1g_{7/2}$  neutron orbitals.

The strong overlap of these spin-orbit-partner orbitals can lead to important n-p correlations in this region and thus to deformation.

At this point it is useful to generalize our earlier remarks as to when strong n-p correlations should occur. As noted earlier, the crucial criterion is that the neutrons and protons occupy orbitals with good overlap. It was pointed out long ago [8] that the overlap between two orbitals ( $n_N l_N j_N$ ) and ( $n_p l_p j_p$ ) is maximum if  $n_N = n_p$  and  $l_N \approx l_p$ . So far, we have focussed on cases in which  $n_N = n_p$  and  $l_N = l_p$ , although we have emphasized that  $j_N$  need not be the same as  $j_p$ .

# Tensor potential



tensor



**no s-wave to  
s-wave  
coupling**



**differences in  
short distance :  
irrelevant**

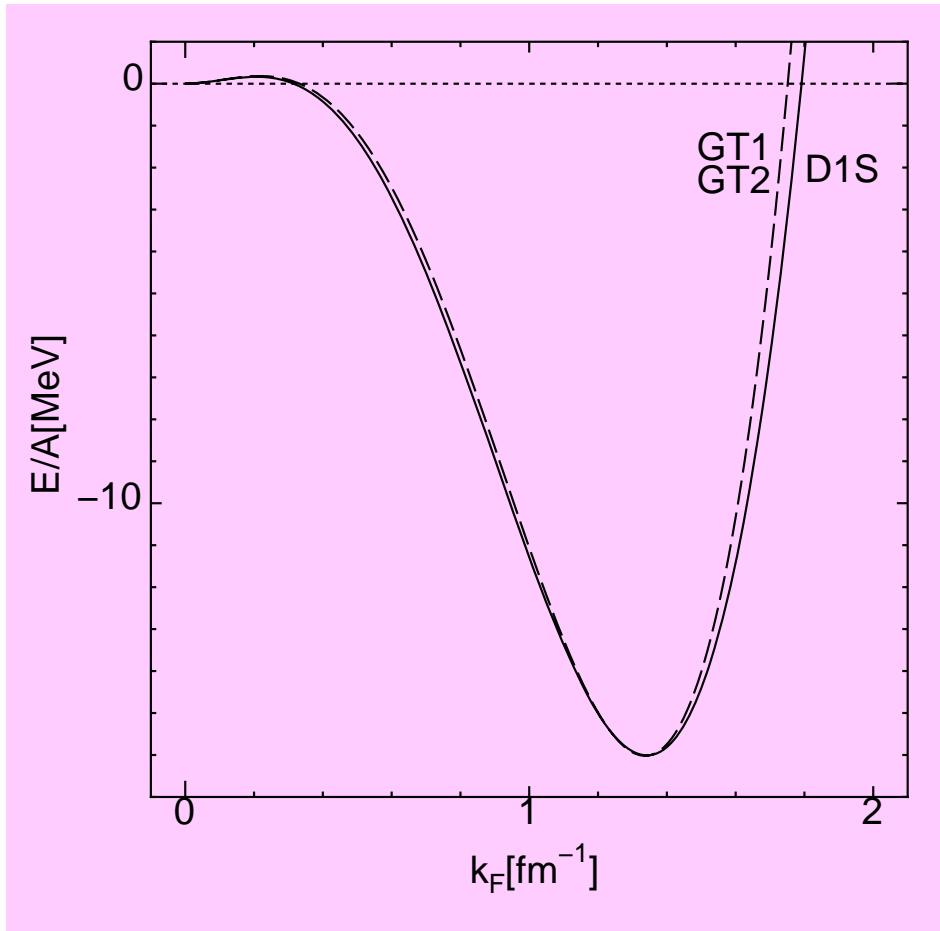
## Basic ideas to fix GT2 parameters

1. Include tensor
  - follow meson exchange results
  - but cut out singularity at short distance
2. Imcompressibility
3. T=1 mean field is made more repulsive, while the pairing is made stronger
  - See the result for oxygen isotopes
4. Parameters fitted by B.E.'s of  $^{16}\text{O}$ ,  $^{40,48}\text{Ca}$ , ( $^{56}\text{Ni}_s$ )  $^{132}\text{Sn}$ ,  $^{208}\text{Pb}$ , and by single-particle structure of neutron rich oxygen

Far from perfection, particularly in pairing correlation

# Nuclear matter property

E/A



## Parameters

D1S

i	1	2
$\mu$	0.7	1.2

$V^0$	-512.94	-3.52
-------	---------	-------

$V^\sigma$	300.60	-25.76
------------	--------	--------

$V^\tau$	557.37	-25.42
----------	--------	--------

$V^{\sigma\tau}$	-349.40	55.98
------------------	---------	-------

$W_0$	130
-------	-----

$t_3$	1390.6
-------	--------

$x_3$	1
-------	---

GT2

i	1	2
$\mu$	0.7	1.2

$V^0$	-210	-25
-------	------	-----

$V^\sigma$	-1040	129
------------	-------	-----

$V^\tau$	-781	120
----------	------	-----

$V^{\sigma\tau}$	700	-65
------------------	-----	-----

$W_0$	160
-------	-----

$t_3$	1400
-------	------

$x_3$	1
-------	---

## Potential in each spin-isospin channel

